

Global Becomes Local: Efficient Many-Body Dynamics for Global Master Equations

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This Letter makes progress on the issue of global vs local master equations. Global master equations like the Redfield master equation (following from standard Born and Markov approximation) require a full diagonalization of the system Hamiltonian. This is especially challenging for interacting quantum many-body systems. We discuss a short-bath-correlation-time expansion in reciprocal (energy) space, leading to a series expansion of the jump operator, which avoids a diagonalization of the Hamiltonian. For a bath that is coupled locally to one site, this typically leads to an expansion of the global Redfield jump operator in terms of local operators. We additionally map the local Redfield master equation to a novel local Lindblad form, giving an equation which has the same conceptual advantages of traditional local Lindblad approaches while being applicable in a much broader class of systems. Our ideas give rise to a nonheuristic foundation of local master equations, which can be combined with established many-body methods.

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Introduction—Foundational questions of the theory of open quantum systems have recently experienced a resurgence due to the high control and resolution of recent experiments on open quantum matter [1–13] and a new push from the theory side to study the dynamics of open quantum many-body systems [14–30]. While the limit of ultraweak system-bath coupling is well understood [31–33], leading to the quantum-optical master equation in Lindblad form, the underlying secular approximation is often violated by genuine quantum many-body systems where close degeneracies in the many-body spectrum are expected from exponentially large densities of states and vanishing finite-size gaps. Hence, one has to resort to finite system-bath coupling master equations where a multitude of master equations have been proposed [34–57], each with their advantages and drawbacks [58,59].

Nevertheless, the well-established Redfield master equation [31–33,60] still often outperforms other approaches [58,59,61] (despite known problems regarding positivity violation and inaccurate steady states [59,61–68], which, however, can be cured efficiently [53,69]). It belongs to the class of global master equations [70], meaning that it involves the global eigenstates and eigenenergies of the system. This is challenging for extended quantum many-body systems [46,57,71], especially far from equilibrium, where an effective low-energy theory in terms of free

quasiparticles is not possible. Also, it prevents one from directly applying established many-body methods [26]. Alternatively, one therefore often resorts to local Lindblad equations [70], where for site-locally coupled baths one introduces local Lindblad jump operators [16–18,70,72–76]. Nevertheless, such approaches often violate consistency with statistical mechanics [59]. Additionally, for out-of-equilibrium systems, e.g., coupled to multiple baths, and strongly coupled systems, the specific details of the environment and how the system is coupled to it influence the steady state [2,53,65,77], so that one needs microscopically derived equations. Here, we show an alternative scheme that maps the Redfield master equation into a local form, which on leading order can be further approximated to Lindblad form with local jump operators.

Redfield master equation—Consider a typical open quantum system setup with total Hamiltonian $H_{\text{tot}} = H_S + H_{\text{SB}} + H_B$, where H_S and H_B are Hamiltonians that act on the system and bath Hilbert space only and H_{SB} is the system-bath interaction Hamiltonian, which for simplicity we assume is a direct product $H_{\text{SB}} = v \otimes B$ of system and bath coupling operators v and B , respectively. The generalization to arbitrary interaction Hamiltonians is discussed in the Supplemental Material [78]. Performing Born and Markov approximations, the dynamics of the reduced density matrix of the system $\rho(t) = \text{Tr}_B \rho_{\text{tot}}(t)$ is given by the Redfield quantum master equation [31–33,53,60,79,80]

$$\partial_t \rho(t) = -i[H_S, \rho(t)] + ([u\rho(t), v] + \text{H.c.}). \quad (1)$$

As sketched in Fig. 1(a), the operator $u = \int_0^\infty d\tau \tilde{v}(-\tau)C(\tau)$ convolutes the coupling operator in the interaction picture $\tilde{v}(t) = e^{iH_S t} v e^{-iH_S t}$ with the bath-correlation

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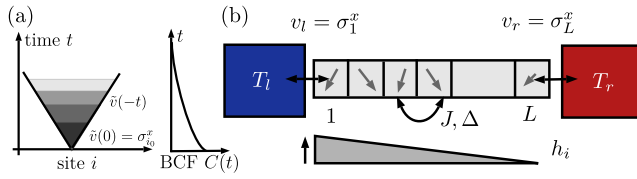


FIG. 1. (a) Sketch of Redfield jump operator u . In the interaction picture, an initially local coupling operator $\tilde{v}(0)$ is spreading in space. This operator is convoluted with the bath-correlation function (BCF) $C(t)$. (b) System under investigation: An XXZ chain is connected at both ends to heat baths of temperature T_l and T_r with coupling operators v_l and v_r . Additionally, an external field gradient h_i in the z direction is applied.

function $C(t) = \langle \tilde{B}(t)\tilde{B}(0) \rangle_B$, where $\tilde{B}(t) = e^{iH_B t} B e^{-iH_B t}$. Throughout this Letter, we set $\hbar = k_B = 1$.

While the Redfield equation (1) generally leads to more accurate results when compared to heuristic Lindblad descriptions (like local Lindblad equations), it has two major deficiencies that prohibit its straightforward application to genuine quantum many-body systems: (i) Since it does not obey Lindblad structure, its dynamics will generally violate positivity (this issue can be mitigated by making connections to statistical mechanics [53]). And (ii) to implement it, one generally introduces the system's eigenbasis $H_S = \sum_k E_k |k\rangle\langle k|$, giving $u = \sum_{kq} \hat{v}_{kq} W(E_k - E_q)$ [78] with eigenspace-projected coupling operator $\hat{v}_{kq} = |k\rangle\langle k| v |q\rangle\langle q|$ and Fourier-Laplace transform $W(E) = \int_0^\infty d\tau e^{-iE\tau} C(\tau)$ of the bath-correlation function. This is problematic, since it requires a full diagonalization of the many-body Hamiltonian, which is generally a hard task. And even then, if the coupling operator v can be represented as a sparse matrix (e.g., if v is an operator that is local in space), the matrix u is generally a dense matrix, making the numerical solution of the equation memory and time consuming. We, therefore, call the Redfield equation (1) a global master equation, since it involves the global eigenstates of the system. Put differently, this practically hinders us from even “writing down” the global Redfield master equation in a way that we can combine it with standard many-body methods. Note that, different from here, the term “global master equation” is sometimes used specifically for the quantum optical master equation [31–33], a Lindblad master equation that follows from the Redfield equation after rotating-wave approximation.

Correlation-time expansion—To write the Redfield equation (1) in local form, we use ideas similar to derivations of quantum Brownian motion [31], where one expands $\tilde{v}(-\tau)$ in first order for short times τ , which is valid for short bath-correlation times; cf. Fig. 1(a). Here, we expand to arbitrary order [46] and additionally insert $1 = e^{-i(E-\epsilon_0)\tau}|_{E=\epsilon_0}$ to rewrite

$$u = \int_0^\infty d\tau e^{-i([H_S, \cdot] - \epsilon_0)\tau} [v] e^{-iE\tau}|_{E=\epsilon_0} C(\tau), \quad (2)$$

where in the exponential it enters a superoperator acting as $([H_S, \cdot] - \epsilon_0)[v] = [H_S, v] - \epsilon_0 v$. By expanding the first exponential in a series in τ , which is valid as long as $C(\tau)$ decays on a bath-correlation timescale that is short against the timescale of operator spreading of $\tilde{v}(-\tau)$ [typically confined by Lieb-Robinson bounds [57,81,82]; cf. Fig. 1(a)], then using $-i\tau = (\partial/\partial E)$ and exchanging the integral and the derivative, we find a Taylor-like expansion:

$$u \approx \sum_{n=0}^N \frac{W^{(n)}(\epsilon_0)}{n!} ([H_S, \cdot] - \epsilon_0)^n [v] \equiv T_N^{\epsilon_0}[v], \quad (3)$$

where we call the corresponding master equation the *ad hoc local Redfield equation*. It enters the n th derivative $W^{(n)}(\epsilon_0)$, and $([H_S, \cdot] - \epsilon_0)^n$ denotes n times the application of the superoperator. We observe that the terms in Eq. (3) represent an expansion in small parameter $E_k - E_q - \epsilon_0$, which is the deviation of the transition energy $E_k - E_q$ (due to the commutator structure) from the expansion energy ϵ_0 of a thought quantum jump from q to k that is induced by u . Equation (3) has the significant advantage that, if the expansion is truncated, a *diagonalization* of the system Hamiltonian is *not required*, which is a remarkable simplification in the case of interacting many-body systems. Also, if v and H_S have a sparse representation, the nested commutators remain sparse. In the case where v is a site-local operator and H_S involves nearest-neighbor terms only, Eq. (3) can be understood as an expansion in local operators, exactly reflecting our intuition from Fig. 1(a).

Let us consider a pure Ohmic bath, $H_B = \sum_\alpha \omega_\alpha b_\alpha^\dagger b_\alpha$, $B = \sum_\alpha c_\alpha (b_\alpha^\dagger + b_\alpha)$, with a continuum of modes α and spectral density $J(E) = \sum_\alpha c_\alpha^2 [\delta(E - \omega_\alpha) - \delta(E + \omega_\alpha)] = \gamma E$ with dissipation strength γ . In this case, $W(E)$ has no imaginary part and takes the form $W(E) = J(E)/[e^{E/T} - 1]$, where T is the temperature of the bath. Calculating the derivatives $W^{(n)}(\epsilon_0)$, we observe that each order involves higher terms in $1/T$. Hence, Eq. (3) can be regarded as a high-temperature expansion of the Redfield jump operator. At high temperatures, when compared to the typical transition energies in the system, as we show in Supplemental Material [78], this expansion energy is, in principle, arbitrary. This is also confirmed by our intuition that at high temperatures $C(t)$ decays rapidly and for $T \rightarrow \infty$ tends to a delta function, giving the leading $N = 0$ term, in Eq. (3). Also note that, by setting $N = 1$ and $\epsilon_0 = 0$, we recover the standard result for quantum Brownian motion [31]. At intermediate temperatures, however, and if the bath is coupled locally to the many-body system, we can use additional information about the coupling operator v .

Optimal expansion energy—What remains is to specify the optimal expansion energy ϵ_0 . As it turns out, for coupling operators v that are site local, an optimal choice can be found from introducing *local* transition energies at the coupling site. We, thus, assume a generic quantum many-body system, whose Hamiltonian can typically be understood as the sum of on-site terms and the residual interaction or tunneling terms, i.e., $H_S = \sum_{i=1}^L H_S^{\text{loc},i} + H_S^{\text{resid}}$. Here, $H_S^{\text{loc},i} = I_1 \otimes \cdots \mathcal{H}_i \otimes \cdots I_L$ acts nontrivially only on site i of a many-body system with L sites, and I_j is the identity on site j . Even though exact diagonalization is generally a hard task for interacting many-body systems, the site-local Hamiltonian can typically be diagonalized straightforwardly: $H_S^{\text{loc},i} |k^i\rangle = E_k^{\text{loc},i} |k^i\rangle$.

To derive the local Redfield master equation, we assume that the coupling operator $v = I_1 \otimes \cdots \mathcal{V}_{i_0} \otimes \cdots I_L$ is acting site locally at site i_0 only. We then perform a spectral decomposition of the operator v with respect to this local on-site basis $v = \sum_{kq} \hat{v}_{kq}^i$, with $\hat{v}_{kq}^i = |k^{i_0}\rangle \langle k^{i_0}| v |q^{i_0}\rangle \langle q^{i_0}|$. Then, using the arguments of Eq. (3) at each of the terms in the sum, we finally arrive at the *local Redfield equation*

$$u \approx u_{\text{loc}} = \sum_{kq} \sum_{n=0}^N \frac{W^{(n)}(\epsilon_{kq}^{i_0})}{n!} ([H_S, \cdot] - \epsilon_{kq}^{i_0} \cdot)^n [\hat{v}_{kq}^i], \quad (4)$$

where we have used the freedom to choose different Taylor expansion energies $\epsilon_{kq}^{i_0}$ for each term of the spectral decomposition. Note that, in the limit of vanishing $H_S^{\text{resid}} = 0$, we observe that we can achieve $([H_S, \cdot] - \epsilon_{kq}^{i_0} \cdot) [\hat{v}_{kq}^i] = 0$ simply by setting the expansion energies to the local transition energies $\epsilon_{kq}^{i_0} = E_k^{\text{loc},i_0} - E_q^{\text{loc},i_0}$. Thus, for $H_S^{\text{resid}} = 0$, we reproduce the *exact* operator u at all temperatures T already in the lowest order $N = 0$ and all higher orders vanish. Thus, we expect good convergence as long as $H_S^{\text{resid}} \ll H_S$.

To illustrate this, we study an XXZ model with external field h_i in the z direction:

$$H_S = -J \sum_{i=1}^{L-1} (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z) + \sum_{i=1}^L H_S^{\text{loc},i}, \quad (5)$$

with $H_S^{\text{loc},i} = h_i \sigma_i^z$, where the field $h_i = h + (i-1)\delta$ can be tuned to have a gradient for $\delta \neq 0$; cf. Fig. 1(b). We couple the spin system to two pure Ohmic baths, with temperature T_l (T_r) to the leftmost (rightmost) site operator $v_l = \sigma_l^x$ ($v_r = \sigma_r^x$). Here, the dissipator is given as the sum $\partial_t \rho = -i[H_S, \rho] + \sum_{j=l,r} ([u_j \rho, v_j] + \text{H.c.})$. In this case, the decomposition in terms of the local eigenstates $|\downarrow^1\rangle, |\uparrow^1\rangle$ of σ_l^z reads $v_l = \sigma_l^- + \sigma_l^+$, with $\sigma_l^\pm = (\sigma_l^x \pm i\sigma_l^y)/2$. We find the local Redfield $u_l \approx \mathcal{T}_N^{\epsilon_l}[\sigma_l^-] + \mathcal{T}_N^{\epsilon_l^+}[\sigma_l^+]$ with local transition energies $\epsilon_l^\pm = \pm 2h_l$.

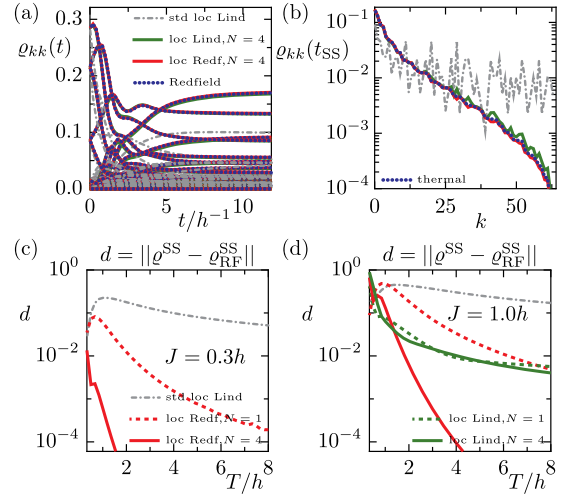


FIG. 2. (a) Population dynamics of the many-body eigenstates k of the boundary-driven XXZ chain with $T_l = T_r = T = 2.2h$ and $J = h$. Exact Redfield (dotted blue line), local Redfield (solid red line), local Lindblad (solid green line), and both $N = 4$ and standard local Lindblad (dash-dotted line) dynamics. Coherences in Supplemental Material [78]. (b) Steady-state populations for the parameters and methods of (a). (c), (d) Trace distance d of the steady state of the local Redfield (red lines) and local Lindblad (green lines) to the exact Redfield equation as a function of temperature T for orders $N = 1$ (dashed line), $N = 4$ (solid line), and (c) $J = 0.3h$ and (d) $J = h$, as well as the standard local Lindblad (dash-dotted line). Other parameters $\gamma = 0.25h$, $L = 6$, $\Delta = 0.7$, and $\delta = -0.07h$; the initial state is pure $|\psi(0)\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}^{\otimes L}$.

In Fig. 2(a), we plot the typical relaxation dynamics of the XXZ spin chain. The dotted blue lines show the dynamics of the many-body eigenstates under the exact Redfield master equation for an intermediate temperature $T_l = T_r = T = 2.2h$ and interaction strength $J = h$. The solid red lines show the dynamics under the local Redfield equation of the order of $N = 4$. We observe exceptionally close agreement throughout both the complete relaxation dynamics and the steady state [Fig. 2(b)], where even the correct thermal distribution (which here is the steady state of the exact Redfield equation) is reproduced. Therefore, all observables, e.g., entropy production, are captured with high accuracy. Figures 2(c) and 2(d) show the trace distance $\|A\| = \text{Tr} \sqrt{A^\dagger A}$ of the steady state of the local Redfield at the order of $N = 1$ (dashed red line) and $N = 4$ (solid red line) to the exact Redfield equation at interaction strength (c) $J = 0.3h$ and (d) $J = h$. Note that the error plots in Figs. 2(c) and 2(d) qualitatively do not change by increasing the system size L . In order to validate our intuition about the short bath-memory time expansion, in Fig. 3(a), we plot the trace distance d_{tr} averaged over the initial relaxation dynamics up to time $\tau_R = \gamma^{-1}$ as a function of bath memory time $\tau_B = (2\pi T)^{-1}$ [78] relative to the timescale $\tau_S = J^{-1}$ of operator spreading the system.

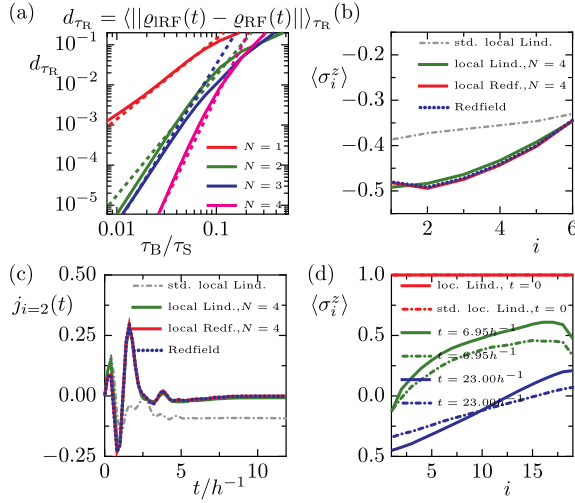


FIG. 3. (a) Error of the local Redfield averaged over the initial relaxation time τ_R vs bath memory time τ_B in units of operator-spreading time τ_S for different orders N . Dashed lines are power laws $\propto (\tau_B/\tau_S)^{N+1}$. (b) Steady-state average magnetization $\langle \sigma_i^z \rangle$ and (c) local magnetization current $j_{i=2}(t)$ at site $i = 2$ as a function of time t for methods and parameters in Fig. 2(a), i.e., $L = 6$. (d) Snapshots of the magnetization profile $\langle \sigma_i^z \rangle$ for our new, $N = 1$ (solid line) and the traditional local Lindblad (dash-dotted line) equation for chain length $L = 19$ at three different times (red, green, and blue lines). Results obtained using quantum trajectory unravelings with $N_{\text{traj}} = 5000$ trajectories. Other parameters as in Fig. 2(a), but $|\psi(0)\rangle = |\uparrow, \dots, \uparrow\rangle$.

This distance follows the dashed power laws $(\tau_B/\tau_S)^{N+1}$, which is the expected error scaling of our short-time expansion up to the order of N in Eq. (2). Details are discussed in Supplemental Material [78], where we also show similar good performance for other bath models, e.g., a Lorentz-Drude bath, as well as other quantum many-body systems, a Bose-Hubbard chain coupled to a thermal particle reservoir.

Local Lindblad form—The analytical form of u also allows us to approximate the dynamics with a Lindblad form. For our purposes, the standard rotating-wave approximation is unfeasible, since it relies on the knowledge of the full many-body spectrum. Instead, we follow Ref. [50] and first rewrite Eq. (1) in the form $\partial_t \varrho = -i[H_{\text{eff}}, \varrho] + \mathcal{D}[\varrho]$, with effective Hamiltonian $H_{\text{eff}} = H_S - (i/2)(vu - u^\dagger v)$ and dissipator $\mathcal{D}[\varrho] = D_{L_+}[\varrho] - D_{L_-}[\varrho]$ with $D_L[\varrho] = (L\varrho L^\dagger - \frac{1}{2}\{L^\dagger L, \varrho\})$. Here enter jump operators $L_\pm = (u \pm \lambda v)/\sqrt{2\lambda}$, where there occurs a free parameter $\lambda \in \mathbb{R}$ [50,78,83]. Note that the second term in the Redfield dissipator $\mathcal{D}$ has a negative “rate,” which is why it violates Lindblad form. However, for the *ad hoc* local Redfield equation (3), we observe that, by setting the free parameter to $\lambda = W(\epsilon_0)$, L_- vanishes in leading order $N = 0$ and, thus, can be neglected at high temperatures [78]. Hence, the expansion presented here also sheds light on the recently proposed truncation [50].

For the local Redfield equation (4), an optimal parameter choice can be found by averaging the above optimal value with weights according to local thermal equilibrium [78], $\lambda_{\text{loc}} = \sqrt{\sum_{kq} W(\epsilon_{kq}^{i_0})^2 r_{kq}^2} / \sqrt{\sum_{kq} r_{kq}^2}$ with $r_{kq} = \langle k^{i_0} | v | q^{i_0} \rangle e^{-E_q^{\text{loc}, i_0}/T}$. The local Redfield equation can, therefore, approximately be rewritten in Lindblad form $\partial_t \varrho \approx -i[H_{\text{eff}}, \varrho] + D_{L_+}[\varrho]$, with $L_+ = (u_{\text{loc}} + \lambda_{\text{loc}} v)/\sqrt{2\lambda_{\text{loc}}}$ and the local approximation u_{loc} [Eq. (4)]. We call this our *local Lindblad equation*. As we show in Figs. 2(a) and 2(b), it predicts the dynamics as well as the steady state quite accurately at intermediate and high temperatures. We plot the corresponding error in green in Fig. 2(d). Surprisingly, for $N = 1$, by neglecting the negative jump operator, the error decreases when compared to the local Redfield equation at the same order $N = 1$. Nevertheless, by increasing the order to $N = 4$, the accuracy does increase only slightly and there is no convergence toward the exact Redfield result, due to the residual error that we make by neglecting the negative jump operator. This local Lindblad form is highly desired, because it allows for stochastic unraveling with standard quantum trajectory algorithms [70,84–86]. Since the resulting jump operator L_+ is sparse and analytically obtainable, it is of great advantage over the full-rank exact many-body operator u or the Lindblad jump operators that follow after rotating-wave approximation. Both of these forms with local jump operators also allow for combining with other approaches like mean-field equations or tensor network methods [86]. Also note that there exist quantum trajectory unravelings directly of the Redfield equation [83,87–90].

Benchmark against traditional local Lindblad—Our local Lindblad equation provides an alternative to the widely used traditional local Lindblad equations while, at the same time, not suffering from typical problems of the standard local Lindblad equations [70]. Using a standard local Lindblad approach yields the master equation $\partial_t \varrho = -i[H_S, \varrho] + \sum_{i=1,L} (\gamma_i^- D_{\sigma_i^-}[\varrho] + \gamma_i^+ D_{\sigma_i^+}[\varrho])$ [70], with rates $\gamma_i^\pm = 2W(\pm 2h_i)$. For pure Ohmic baths, there is no Lamb shift contribution to the Hamiltonian since $W(E)$ is real. The XXZ chain with local Lindblad equation has attracted a lot of attention, because it can have exactly solvable steady states [16,17].

As we observe in Fig. 2(a), the populations of the many-body eigenstates are only poorly described by the standard local Lindblad equation. This poor performance is confirmed by Figs. 2(c), 2(d), and 3(a), where we plot the measure d for the distance of its steady state to the exact Redfield steady state. Indeed, it is expected since one way to derive the standard local Lindblad equation microscopically [70] is in the limit $J \rightarrow 0$. Nevertheless, such local jumps have attracted considerable attention [91,92], also in regimes where there is no microscopic justification [16,17,70], as well as in the context of collisional models [93,94]. Our local Lindblad equation is an alternative that

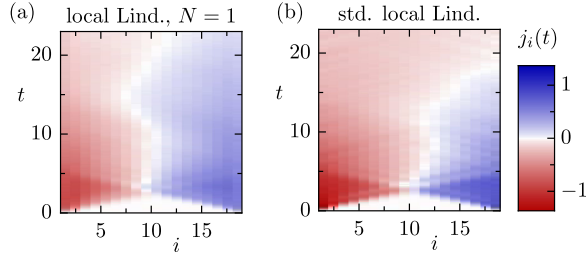


FIG. 4. Site-resolved magnetization current $j_i(t)$ (a) for the local Lindblad with the order of $N = 1$ and (b) for the traditional local Lindblad equation. Results obtained from quantum trajectory unraveling for the parameters in Fig. 3(d).

has similar properties but can overcome some of its flaws: As we show in Fig. 3(b), it predicts a steady-state magnetization profile that is much closer to the actual thermal result predicted by the Redfield equation [78]. In Fig. 3(c), we additionally show the site-resolved magnetization current [70] $j_i(t) = -i\langle[H_S^{i,i+1}, \sigma_i^z]\rangle$ with $H_S^{i,i+1}$ denoting the nearest-neighbor interaction term in Eq. (5) at site i . We observe that, even though the system is connected to two heat baths with identical temperature $T_l = T_r = T$, the standard local Lindblad master equation, due to the tilted external field h_i , predicts a finite steady-state current through the system which is unphysical. The Redfield, local Redfield, and our local Lindblad equation do not suffer from such unphysical behavior [95]. Hence, our local Lindblad equation fulfills all criteria for a consistent Markovian master equation that were formulated in Ref. [61]: It obeys (i) complete positivity, (ii) local conservation laws, and (iii) shows thermalization. In Supplemental Material [78], we also show that our local master equations describe nonequilibrium setups (where $T_l \neq T_r$) with similar high accuracy.

Quantum trajectories for local Lindblad form—We combine our new and the traditional local Lindblad equation with established quantum trajectory methods [70,84–86], where we implement the algorithm of Ref. [84]. Since both forms yield jump operators that have a sparse representation, they allow us to treat relatively large systems. In Figs. 3(d) and 4, we demonstrate this by increasing the length of the spin chain to $L = 19$, where the Hilbert space dimension is $2^L = 524288$. With global master equations, such system sizes are numerically out of reach and typically limit one to Hilbert space dimensions of $\sim 10^4$ on current hardware [53,83]. We observe again that the magnetization profile $\langle\sigma_i^z\rangle$ in Fig. 3(d) that is predicted by the traditional local Lindblad approach deviates strongly from the more accurate novel local Lindblad dynamics. Also, the traditional local Lindblad equation overestimates the magnetization currents that we plot in Fig. 4 in the initial relaxation dynamics. By combining our approach with matrix product state methods for quantum trajectories, we expect that even larger system sizes are attainable.

Summary—Even though the Redfield master equation is typically considered a global master equation, we emphasize that, for coupling to local baths with short bath-correlation time, the jump operator will remain localized in space. We provide a Taylor-like expansion of the Redfield jump operator in terms of local operators and show convergence for high and intermediate bath temperatures. This additionally has the advantage that a diagonalization of the full Hamiltonian is not required. However, there might exist other expansions [57] with better convergence especially in the case of bath spectral densities with a low cutoff. We show that our method can be further approximated to a Lindblad form with local jump operators, outperforming the traditional local Lindblad ansatz. We combine our results with quantum trajectories to simulate the Redfield dynamics for Hilbert space dimensions that were previously challenging. Our method can be augmented with mean-field or matrix-product methods to solve the Redfield equation for extended many-body systems.

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