

Fate of Bosonic Topological Edge Modes in the Presence of Many-Body Interactions

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Many magnetic materials are predicted to exhibit bosonic topological edge modes in their excitation spectra, because of the nontrivial topology of their magnon, triplon, or other quasiparticle band structures. However, there is a discrepancy between theory prediction and experimental observation, which suggests some underlying mechanism that intrinsically suppresses the expected experimental signatures, like the thermal Hall current. Many-body interactions that are not accounted for in the noninteracting quasiparticle picture are most often identified as the reason for the absence of the topological edge modes. Here we report persistent bosonic edge modes at the boundaries of a ladder quantum paramagnet with gapped triplon excitations in the presence of the full many-body interaction. We use tensor network methods to resolve topological edge modes in the time-dependent spin-spin correlations and the dynamical structure factor, which is directly accessible experimentally. We further show that signatures of these edge modes survive even when the noninteracting quasiparticle theory breaks down; we discuss the topological phase diagram of the model, demonstrate the fractionalization of its low-lying excitations, and propose potential material candidates.

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The enormous success of the description of electronic topological phases of matter quickly inspired research that lead to the generalization of the theory to bosonic quasiparticles, like photons [1–6], phonons [7–10], magnons [11–19], or triplons [20–22]. Analogous to the quantum or spin Hall effect for electrons, the nontrivial topology of magnetic excitations contributes to unusual responses like the thermal Hall or spin Nernst effect in the form of bosonic edge currents.

These topologically protected edge spin waves are a key ingredient for many future spintronic applications, which are a promising alternative to conventional computing, but with a minimal ecological footprint [23]. There is a multitude of blueprints that make use of their properties to build next-generation devices, like spin-wave diodes, beam splitters, interferometers, and others [23–26].

Even though a significant amount of material candidates has been proposed to host nontrivial bosonic

excitations [11,14,20,25,27–34], their detection remains elusive. While some experiments indeed show signatures of the predicted edge modes or infer their existence through bulk band topological arguments [35–40], others have not been able to produce evidence of topological quasiparticles [41,42], and for most of the proposed candidates a clear indication of their presence is yet to be discovered.

Many reasons that could make the effects of the topological edge modes fail to materialize have been theorized. Among other things, they include thermal damping, magnon-phonon coupling, and domain formation. But what is most often conjectured to be the source of the suppression of thermal Hall effect are many-body effects, because they obstruct the application of the standard classification tools which rely on an effectively noninteracting theory. In contrast to their fermionic counterparts, even small many-body interactions [41,43–45] or exchange anisotropies that are typically present in realistic models [46] seem to substantially affect bosonic topological transport properties. How interactions affect topological bosonic edge modes in general is yet to be determined, and while many studies go beyond the noninteracting quasiparticle limit [44,47–49], only a few studies account for the full many-body interaction [50,51].

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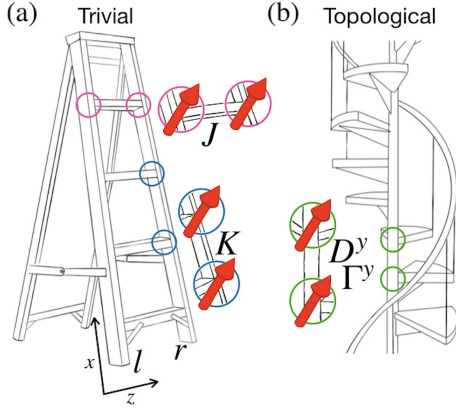


FIG. 1. (a) Schematic of the spin ladder. On each rung, two spin-1/2 sites are (strongly) coupled through antiferromagnetic Heisenberg interaction. Along the side rails, the spins interact (weakly) antiferromagnetically. In this limit the spins form singlet pairs along the rungs. (b) The same model but with antisymmetric (DM) and symmetric (pseudodipolar) exchange in the y direction given by D_y and Γ_y . These terms introduce a winding and open a topological gap in the excitation spectrum.

In this Letter, we report bosonic topological edge modes that are stable in the presence of the full many-body interaction using density matrix renormalization group (DMRG) and time-evolution methods. We exclude other potential sources of suppression and focus on the consequences of the many-body effects alone. The results presented assume zero temperature and no coupling to any other type of quasiparticles. Moreover, we choose a one-dimensional model, which allows us to study localized edge modes that are not subject to scattering off edge defects present in a real sample.

Model—We consider a quantum $S = 1/2$ Heisenberg model on a ladder with spin-orbit interaction and an external magnetic field, as shown schematically in Fig. 1. This model, first considered in Ref. [21], is given by the following Hamiltonian:

$$\hat{H} = \hat{H}_{\text{rung}} + \hat{H}_{\text{rail}} + \hat{H}_{\text{SOC}} + \hat{H}_Z, \quad (1)$$

$$\hat{H}_{\text{rung}} = J \sum_i \hat{\mathbf{S}}_{li} \cdot \hat{\mathbf{S}}_{ri}, \quad (2)$$

$$\hat{H}_{\text{rail}} = K \sum_i \left(\hat{\mathbf{S}}_{li} \cdot \hat{\mathbf{S}}_{li+1} + \hat{\mathbf{S}}_{ri} \cdot \hat{\mathbf{S}}_{ri+1} \right), \quad (3)$$

$$\begin{aligned} \hat{H}_{\text{SOC}} = & D_y \sum_{\alpha=l,r} \sum_i \left(\hat{S}_{ai}^z \hat{S}_{ai+1}^x - \hat{S}_{ai}^x \hat{S}_{ai+1}^z \right), \\ & + \Gamma_y \sum_{\alpha=l,r} \sum_i \left(\hat{S}_{ai}^z \hat{S}_{ai+1}^x + \hat{S}_{ai}^x \hat{S}_{ai+1}^z \right), \end{aligned} \quad (4)$$

$$\hat{H}_Z = h_y \sum_i \left(\hat{S}_{li}^y + \hat{S}_{ri}^y \right), \quad (5)$$

where $\hat{\mathbf{S}}_{ai} = (\hat{S}_{ai}^x, \hat{S}_{ai}^y, \hat{S}_{ai}^z)^\top$ are the usual spin operators with $\alpha = l, r$ corresponding to the left and right rail of the ladder and i running over all rungs at positions r_i .

In the limit of strong antiferromagnetic rung coupling, this model is the prototypical example of a gapped quantum paramagnet [52,53]. In that limit the ground state of the model is close to a product state of spin singlets

$$|\psi_0\rangle \sim \bigotimes_{i=1}^L (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}, \quad (6)$$

with low-lying triplet excitations, the corresponding quasiparticles being triplons. In the presence of $SU(2)$ symmetry ($D_y = \Gamma_y = h_y = 0$), the triplons are degenerate, but can be split by either applying an external magnetic field or through spin-orbit interactions.

In the paramagnetic phase of the model, an effective low-energy spectrum can be calculated by first rewriting the model in terms of triplons through bond-operator theory [54], and then discarding all resulting terms that are not bilinear, which has been shown to be a controlled approximation [55,56] for low triplon densities. Within the harmonic approximation, it was shown in Ref. [21] that the so-obtained excitation spectrum can be characterized by a topological winding number that classifies the topology of the magnet's bulk and enforces—if nontrivial—triplon modes at the ends of the ladder with fractional particle number. The model follows the design philosophy of many topological toy models. That is, SOC is used to open a topological gap by entangling an internal degree of freedom [12,57–61].

The model is suitable to describe the magnetism of potentially many materials and belongs to the well-studied class of two-leg ladder compounds. Examples of spin ladder materials are BiCu_2PO_6 [62], NaV_2O_5 [63,64], or multiple cuprate ladders [65–68]. The recipe for finding topological edge modes in these systems requires mainly two ingredients: (i) strong antiferromagnetic rung coupling (stronger than along the rails), and (ii) spin-orbit interaction, the relative strengths of which can be tuned by doping, gate voltages, or structural manipulations like skewing and stretching [69–73]. The abundance of experimental handles suggests that a topological phase transition might easily be switchable, which again expands the potential applications of these systems in future devices.

Tensor networks and dynamical response—To relate our model directly to an experimentally measurable quantity, we compute the dynamic structure factor (DSF), which is most directly accessed through inelastic neutron scattering [74,75] or inverse spin Hall noise spectroscopy (ISHNS) [76]. Because we are investigating the effect of interactions, and since our model is (quasi) one-dimensional, we use the DRMG algorithm to obtain the exact many-body ground state of the finite spin ladder [77].

In contrast to electronic systems, the band topology of bosonic quasiparticles is not a ground state property, so in order to access the low-lying excitations, we apply time-evolution methods [78–83] to compute the odd part [84] of the time-dependent spin-spin correlation

$$C_{ij}^{\gamma\gamma'}(t) = \langle \psi_0 | \hat{S}_i^\gamma \hat{U}(t) \hat{S}_j^{\gamma'} | \psi_0 \rangle, \quad (7)$$

with $\hat{S}_i^\gamma = \hat{S}_{li}^\gamma - \hat{S}_{ri}^\gamma$ and the unitary time-evolution operator $\hat{U}(t)$. Time-resolved data is typically unavailable for larger (interacting) systems, and the evolution of topological edge states remains largely unexplored. In the Appendix on Hilbert space fragmentation, we show that the topological boundary modes have strongly enhanced time coherence and that the model's Hilbert space is fragmented by additional conserved charges in some limit.

The DSF, which encodes the dynamical susceptibility of the paramagnet is defined as the Fourier transform of the spin-spin correlations in space and time

$$S_k^{\gamma\gamma'}(\omega) = \frac{1}{2\pi L} \int_{-\infty}^{\infty} dt e^{i(\omega - \omega_0)t} \sum_{i,j} e^{i(r_j - r_i)k} C_{ij}^{\gamma\gamma'}(t). \quad (8)$$

It is a positive and real quantity and can be calculated using DMRG and time-evolution methods [85]. Usually, the state is evolved only for as long as the excitation does not reach the boundary of the system to avoid finite size effects. In addition, by imposing translational invariance, one of the spatial degrees of freedom of $C_{ij}^{\gamma\gamma'}(t)$ can be fixed [85–88]. Because we are explicitly studying the excitations in the presence of a boundary, we require the “full” real-space correlation function, as well as a “long-enough” time evolution. $S_k^{\gamma\gamma'}(\omega)$ obeys sum rules [88] that we track to diagnose the severity of finite-time and finite-size effects, which lead to numerical artifacts (see Supplemental Material [89]).

We study the z component of the field-susceptible part of Eq. (8) which is given by $S_k(\omega) = S_k^{zz}(\omega)$ to which we simply refer to as DSF from now.

Results—Before moving on to the case of strong SOC, we review the key properties of the model's excitation spectrum in the weakly interacting limit. We set $D_y = \Gamma_y$, and express the Hamiltonian through operators t_i^γ that create a triplon by acting on the singlet state on the i th dimer [54]. The full Hamiltonian is given in the Appendix on the model's topological properties, but, in particular,

$$\hat{H}_{\text{SOC}} = D_y \sum_i t_i^{z\dagger} P_i P_{i+1} t_{i+1}^x - t_i^{z\dagger} P_i t_{i+1}^{x\dagger} P_{i+1} + \text{H.c.}, \quad (9)$$

with $P_i = 1 - \sum_\gamma t_i^\gamma t_i^{\gamma\dagger}$. In Fig. 2(a) we plot the DSF of the topological paramagnet along with the triplon bands obtained within the harmonic approximation in Ref. [21]. For small SOC the harmonic approximation is accurate,

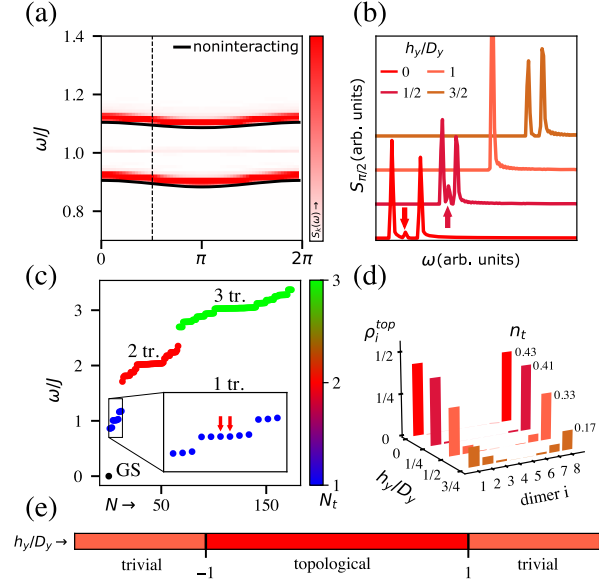


FIG. 2. (a) DSF of the topological quantum paramagnet with $L = 32$ rungs and $K/J = 0.01$, $D_y/J = \Gamma_y/J = 0.1$, and $h_y = 0$ [corresponding to the red plot in (b)]. The solid lines show the spectrum of the effective low-energy models from Ref. [21] for the same parameters. There is a localized boundary mode at $\omega/J = 1$. (b) DSF for different values of h_y/D_y at $k = \pi/2$ [dashed line in (a)]. The first two curves lie in the topological phase region [see (e)] with the in-gap modes marked by arrows. At $h_y/D_y = 1$ the gap is closed. The last curve lies in the trivial region and is gapped, with no mode in between. All peaks are centered at $\omega \approx J$ (and shifted for clarity). (c) Many-body spectrum of a 4-rung system with the same parameters as in (a). The spectrum is separated into sectors of different triplon particle numbers. The color indicates N_t . The inset shows the 1-triplon sector with the edge modes marked by arrows. (d) ρ_i^{top} for different values of h_y/D_y , but with $L = 8$. The in-gap modes are localized at the two boundaries of the system with fractional particle numbers n_t at each termination. For finite magnetic fields the in-gap mode spreads into the bulk and vanishes at the topological phase transition at $h_y/D_y = 1$. (e) The topological phase diagram of the model.

because the density of triplons is small leading to only minor renormalizations from triplon-triplon interactions. In the topologically nontrivial case (small values of magnetic field) an additional mode appears between the two bulk bands that we hereafter refer to as *in-gap mode*. It is absent in the trivial phase (large values of magnetic field). Figure 2(b) shows the topological phase transition at $k = \pi/2$ with the external magnetic field h_y as a tuning parameter. At $h_y = |D_y|$ the gap closes, and further increasing the magnetic field it reopens, but with no topological in-gap mode. We did finite size scaling with ladders of up to $L = 64$ rungs to make sure that the in-gap mode retains finite spectral weight.

As expected, the harmonic approximation and the tensor networks agree on the dispersion relation of the

single-triplon bulk excitations. That both predict a topological edge mode is less conspicuous, even for small SOC. While signatures of chiral edge modes (on a cylinder) have been observed using tensor networks [50], the harmonic approximation is not obviously adiabatic. For example, in one-dimensional electron systems finite interactions force a Mott transition through which the topological properties of the system cannot be tracked [96]. In the weakly interacting case, exact diagonalization (ED) on a 4-rung ladder shows that the spectrum has well-separated number of particle sectors. Even though $\hat{H}_{\text{SOC}}$ does not conserve the triplon number $N_t = -\frac{3}{4}JL + J\sum_i \langle t_i^\dagger t_i \rangle$, the expectation values are close to integer for this choice of parameters. Because $\hat{H}_{\text{SOC}}$ contains only even powers of $t_i^\dagger$, single-triplon states—such as the in-gap mode $|\psi_{\text{top}}\rangle$ —scatter mainly off states from the three-triplon sector $|\psi_N^3\rangle$. This is confirmed by evaluating the matrix elements $\langle \psi_N^3 | \hat{H}_{\text{SOC}} | \psi_{\text{top}} \rangle$ which are in general finite, because there is no (hidden) symmetry that prevents the states from interacting. Instead, the boundary modes are expected to enjoy topological protection in the form of a geometrically enforced matrix element suppression, due to their localization at the terminations of the ladder. The remainder of this Letter investigates this localization and how robust the protection remains under increasingly strong interactions.

To confirm that the in-gap mode is really localized at the boundaries of the system, we first calculate the local density of states of the in-gap mode ρ_i^{top} for $L = 8$, and then compute the associated particle number by integrating it for the system's left termination (see Supplemental Material [89])

$$n_t = \int_1^{L/2} d\rho_i^{\text{top}}. \quad (10)$$

We find two localized peaks at the x boundaries of the ladder with fractional particle number per termination. Our numerical result $n_t \sim 0.43$ deviates slightly from the predicted value of 0.5 [21] due to spectral leakage caused by the aforementioned finite-size and time effects. For larger values of field, n_t becomes smaller and vanishes at the phase transition at $h_y/D_y = 1$. Details are found in Supplemental Material [89].

The noninteracting triplon winding number is a topological band property [21], so the edge modes are naively expected to be stable only as long as there is a notion of “bands,” or in other words as long as triplons provide a suitable quasiparticle description of the ladder's excitation spectrum. We consider three limits of the model's parameter space: (i) large magnetic field h_y , (ii) large rail coupling K and (iii) large spin-orbit interaction D_y and Γ_y . In case (i), the field-polarized phase, the lower triplon modes condense at $h_y \sim J$ and the system becomes ferromagnetic with magnons as its low-lying excitations. The band structures derived from linear spin wave theory are given in Supplemental Material [89] and are topologically trivial. Case (ii) is the limit of two weakly coupled anti-ferromagnetic Heisenberg chains. The system does not order, even for strong rail coupling ($K \sim 2J$). The antiferromagnetic order parameter is finite also for small values of K , but approaches zero as the system size L is increased, as confirmed by finite size scaling [89]. Case (iii) is the most relevant benchmark of the edge modes' stability that we can provide with the available handles in the model, because here interactions become important [see Eq. (9)]. In this limit the eigenstates cannot straightforwardly be assigned to a particle number sector, and ED yields noninteger values of N_t as shown in Fig. 3(a). States with a triplon number close to $N_t = 3$ (which we have previously shown to mix with single-triplon states through $\hat{H}_{\text{SOC}}$) now occupy the same energy range as the edge mode, the quasiparticle

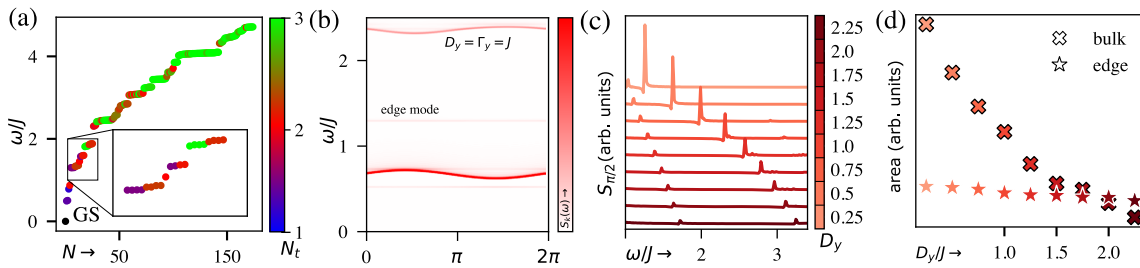


FIG. 3. (a) Many-body excitation spectrum with $K = 0.01J$, $h_y = 0.05J$ and strong SOC $D_y = \Gamma_y = J$ on a ladder with $L = 4$ rungs. The color indicates the triplon number N_t , which is not conserved as indicated by the noninteger values (purple and brown) of many eigenstates, also within the energy range close to the in-gap mode as shown in the inset. (b) DSF for the same parameters with $L = 32$. The upper quasiparticle peak has moved up in energy and lost intensity. The edge mode remains flat and is marked by text. (c) DSF at the gap closing point $k = \pi/2$ as a function of energy for different values of $D_y = \Gamma_y$. As a function of SOC, the in-gap mode (left peak) and the bulk band (right peak) move up in energy with a growing separation in energy. (d) Area of the bulk (cross) and the in-gap (star) mode for increasing $D_y = \Gamma_y$. The intensity of the bulk band decreases as it moves up in energy, while the intensity of the in-gap mode is roughly constant.

peak of which remains discernable in the DSF and is marked by text in Fig. 3(b). The in-gap mode remains flat (localized), which rules out avoided quasiparticle decay through level repulsion that has been shown to enhance quasiparticle lifetimes in some strongly interacting systems [97,98]. Clearly, the harmonic approximation is no longer valid. In fact, we find that the harmonic Bogoliubov-de-Gennes Hamiltonian ceases to be positive definite already at just below $D_y = \Gamma_y = J/2$. The intensity of what previously was the upper single-triplon mode is strongly reduced, and we track its evolution together with that of the topological edge mode as a function of SOC at $k = \pi/2$ in Fig. 3(c). The upper peak moves up in energy faster than the in-gap mode, and in contrast to case (i), increasing D_y and Γ_y does not lead to condensation of the lower mode. We did not find any evidence for a phase transition up to values of $D_y = \Gamma_y = 2.6J$. Analogous to the quasiparticle residue in Fermi liquid theory, which is typically interpreted to be a measure of quasiparticle coherence [99–101], we plot the areas under the two peaks shown in Fig. 3(c) as a function of D_y/J in Fig. 3(d). We find that the topological and bulk modes show qualitatively different behaviors: The bulk-triplon peak falls off quickly, whereas the area under the in-gap mode is roughly constant (up to some spectral leakage, which becomes more severe as D_y is increased) with a crossover at $D_y/J \approx 2$.

Discussion—We have demonstrated that, contrary to what the absence of experimental evidence for many predicted materials might suggest, topological bosonic edge modes can persist in the presence of the full many-body interaction. Even though correlations have been identified as the reason for the absence of topological responses in some cases, our Letter clearly shows that they do not generically suppress bosonic edge modes, and it prompts the question of what other mechanism is responsible. A natural extension of this Letter is to apply our method to two-dimensional systems wrapped onto a cylinder. Multirail ladders [102,103], a ferromagnet on a Lieb lattice [104], or the Shastry-Sutherland model [105] are obvious candidates, but also topological magnon systems that are potentially more fragile due to their larger decay phase space. In two dimensions, the edge modes transport the intrinsic part of the thermal Hall response, implying that suppressing edge modes results in an attenuated response.

The absence of signatures of topological edge modes even in systems with good agreement between theory and experiment for bulk properties suggests that the responsible decay channel might be a surface effect, i.e., it lies at the boundary. In a quasi-two-dimensional model the edge modes are not completely localized, but propagate along the boundary of the system. This setup would further allow the implementation of a defect at the boundary and to investigate the stability of the topological bosons in its presence.

Thermal effects might be another candidate to explain the missing Hall response in Refs. [41,42], and indeed strong thermal damping of triplon modes has been observed in a Shastry-Sutherland compound [106]. However, the experiments that report an absence of thermal Hall current still find sharp single-triplon quasiparticle peaks, which suggests that it is unlikely that the broadening due to damping would suppress the signal almost entirely. Thermally activated renormalization effects, as reported in CrBr_3 [107], cannot be ruled out by our Letter. However, the zero-temperature DSF does not suggest enhanced renormalization at the gap closing point (for $|h_y| = |D_y|$) compared to other values of k , and a temperature-induced topological phase transition, as predicted in CrI_3 , seems unlikely for low temperatures and no external magnetic field [108].

The typical “workflow” for finding topological edge modes usually consists of classifying the topology of a translationally invariant model, and then inferring their existence by bulk-boundary correspondence. Even if the symmetry that protects the band topology is broken weakly, the edge states are still expected to be present, as long as the perturbation is smaller than the gap they live in (these approximate symmetries are sometimes called quasisymmetries [109,110]). The chiral symmetry that allows for the definition of a winding number in the noninteracting case is broken by the rail coupling K [21], and is also clearly broken in the limit of strong SOC. It is a central question of the field how quantum geometric quantities can be defined in the presence of strong interactions, e.g., when bands are substantially broadened [44]. Our Letter circumvents this ambiguity and approaches the problem from “the other end of the bulk-boundary correspondence” by including the boundary explicitly. Nonetheless, the extension of topological (bulk) classifications to interacting systems is a standing problem, and in recent years considerable effort has been made to find alternative invariants [96,111–115]. These schemes account for the many-body nature of the problem using either a matrix product state or Green’s function representation, but are not applicable for the classification of bosonic excitation spectra. Detection of phase transitions using entanglement have been brought forward, but are—even though directly related to the dynamical response of a spin system—not sensitive to topological phase transitions [116]. It is essential to extend existing or find new methods that are able to capture the topological properties of, e.g., dynamical spin responses, and we hope our Letter inspires fruitful research in that direction.

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N. H. performed all numerical calculations and wrote the manuscript. H. K. identified and computed the Hilbert space fractionalization. All authors contributed to discussions, provided feedback, and revised the manuscript.

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End Matter

Appendix on bond operator theory—We give an expression for the Hamiltonian in terms of triplon operators, following the Supplemental Material of Ref. [21].

As in the main text, we consider strong antiferromagnetic rung coupling and denote the singlet state as $|t^0\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ [see Eq. (6)]. Then, the triplon operators are defined as

$$t^{x\dagger}|t^0\rangle = -(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)/\sqrt{2}, \quad (\text{A1})$$

$$t^{y\dagger}|t^0\rangle = i(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)/\sqrt{2}, \quad (\text{A2})$$

$$t^{z\dagger}|t^0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}, \quad (\text{A3})$$

and the spin operators can be written as

$$\hat{S}_{ai}^\gamma = \frac{1}{2} \left(\pm i t_i^{\gamma\dagger} P_i \mp i P_i t_i^\gamma - i \epsilon_{\gamma\gamma'\bar{\gamma}} t_i^{\gamma'\dagger} t_i^{\bar{\gamma}} \right), \quad (\text{A4})$$

with $\alpha = l, r$, and $P_i = 1 - \sum_\gamma t_i^{\gamma\dagger} t_i^\gamma$. Then

$$\hat{S}_{li} \cdot \hat{S}_{ri} = -\frac{3}{4} + \sum_\gamma t_i^{\gamma\dagger} t_i^\gamma, \quad (\text{A5})$$

and inserting this into the Hamiltonian in the main text [see Eq. (1)] yields

$$\begin{aligned}
 \hat{H} = & J \sum_{\gamma} \sum_i t_i^{\gamma\dagger} t_i^{\gamma} - \frac{3}{4} JL + i h_y \sum_i \left[t_i^{x\dagger} t_i^z - t_i^{z\dagger} t_i^x \right] + \frac{K}{2} \sum_i \sum_{\gamma} \left[t_i^{\gamma\dagger} P_i P_{i+1} t_{i+1}^{\gamma} - t_i^{\gamma\dagger} P_i t_{i+1}^{\gamma\dagger} P_{i+1} + \text{H.c.} \right] \\
 & + \frac{D_y}{2} \sum_i \left[t_i^{z\dagger} P_i P_{i+1} t_{i+1}^x - t_i^{z\dagger} P_i t_{i+1}^{x\dagger} P_{i+1} - t_i^{x\dagger} P_i P_{i+1} t_{i+1}^z + t_i^{x\dagger} P_i t_{i+1}^{z\dagger} P_{i+1} + \text{H.c.} \right] \\
 & + \frac{\Gamma_y}{2} \sum_i \left[t_i^{z\dagger} P_i P_{i+1} t_{i+1}^x - t_i^{z\dagger} P_i t_{i+1}^{x\dagger} P_{i+1} + t_i^{x\dagger} P_i P_{i+1} t_{i+1}^z - t_i^{x\dagger} P_i t_{i+1}^{z\dagger} P_{i+1} + \text{H.c.} \right].
 \end{aligned} \tag{A6}$$

In Fig. 4 we give a schematic of the (bilinear) terms in Eq. (A6). As long as SOC points along the y direction, it only couples the x - and z -triplon modes. In a realistic material, disorder can break the crystallographic symmetries that enforce $D_x = D_z = 0$, however the resulting broadening would be parametrically small and—for sufficiently large D_y and Γ_y —much smaller than the topological gap. As described in the main text, the triplon number N_t is not conserved. If $D_y = \Gamma_y$, the terms with opposite signs cancel and one obtains an expression for $\hat{H}_{\text{SOC}}$ as given in Eq. (9) in the main text.

In contrast to the case of topological magnons of ordered magnets [39,117–121], triplon excitations are gapped. As a consequence of this bulk gap the triplons, including the topological edge modes, do not have any low-energy states to decay into. Therefore their lifetime is not reduced significantly even in the presence of weak interactions. By applying an external magnetic field or through spin-anisotropy exchanges, magnon systems can become gapless too. However, a bulk gap induced by these effects is typically much smaller than the rung coupling J , which is the relevant spin exchange energy scale that determines the size of the gap in the ladder system.

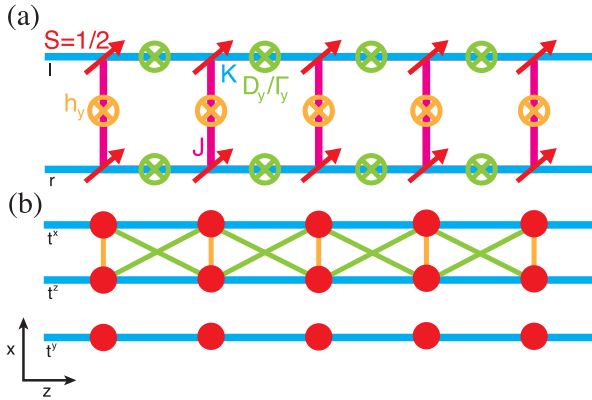


FIG. 4. (a) Schematic of the model of the quantum paramagnet from Eq. (1). (b) The same model in the basis of triplon operators t^x up to bilinear terms. D_y, Γ_y (green) and h_y (orange) couple the t^x and t^z chains along the correspondingly colored bonds. J is a chemical potential. The full Hamiltonian is given in Eq. (A6).

Appendix on Hilbert space fragmentation—Systems that are useful for storing and processing quantum information are highly sought after. Typically, the quantum information of a many-body state is destroyed rapidly. Overcoming fast decoherence of quantum states is one of the main challenges of quantum computing, and has driven extensive theoretical interest in finding ways to increase their coherence times using many-body localization [122–126], prethermalization [127–129],

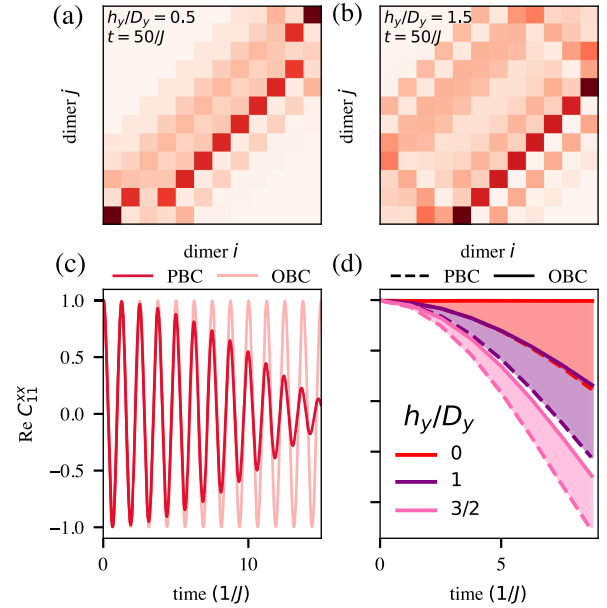


FIG. 5. Real part of the spin-spin correlation function at time $t = 50/J$ in the (a) trivial and (b) the topological case with $K = 0.01J$ and field strengths $h_y/D_y = 0.5$ and $h_y/D_y = 1.5$, respectively, on a ladder with $L = 12$ rungs. The topological autocorrelation at this time step has peaks at the boundaries, which are absent in the trivial case. (c) Time dependence of the boundary autocorrelation $\text{Re} C_{11}^{xx}$ for $K = 0.01J$, $D_y = \Gamma_y = 0.1J$ and no magnetic field on a ladder with $L = 8$ rungs for different boundary conditions. For PBC (no in-gap mode), it decays rapidly whereas for OBC (in-gap mode) it does not decay. (d) The decay envelopes of (c) for different values of magnetic field. For finite field strengths the correlator decays also for OBC, but more slowly than for PBC.

quantum many-body scarring [130–132] or the states’ topological properties [133–136]. Investigating time evolution of topological states—specifically in the presence of defects or nonlinearities [137–139]—as well as imaging and detection techniques [140–142] are thriving fields of research and crucial for bringing real-world applications on their way. In the following, we show that the boundary mode in the topological quantum paramagnet has enhanced time coherence, which we then argue to be linked to Hilbert space fragmentation.

In Figs. 5(a) and 5(b) the real part of $C_{ij}^{yy'}(t)$ for the topological and trivial case at a fixed time step are plotted. The former shows strong correlations that are pinned to the edges of the system, whereas in the latter they are absent. In Fig. 5(c), we plot the time dependence of $\text{Re}C_{11}^{xx}$ at zero magnetic field for periodic (PBC) and open boundary conditions (OBC). For PBC there is no topologically protected edge mode at $i = j = 1$, and the excitation decays over time rapidly. In the open system, the edge mode has strongly enhanced time coherence, and we cannot resolve any decay during the time it takes the excitation to hit the boundary at the other end of the ladder. Figure 5(d) shows the decay envelope of the excitation for increasing magnetic field for periodic (dashed line) and open (solid line) boundaries. For finite magnetic fields, the boundary correlation starts to decay, and for stronger fields the difference in lifetime for PBC and OBC—as indicated by the shaded area—becomes less pronounced.

There is no abrupt change in behavior at the phase transition $D_y = \Gamma_y = h_y$, which makes it unlikely that the topological properties of the system alone are responsible for the observed behavior. Rather, it seems to imply that the $h_y/D_y = 0$ is a special point in the phase diagram. Consider the case of $K = h_y = 0$ and $D_y = \Gamma_y$. Then, Eq. (1) simplifies to $\hat{H} = \hat{H}_{\text{rung}} + \hat{H}_{\text{SOC}}$ and $\hat{H}_{\text{SOC}}$ is given by

$$\hat{H}_{\text{SOC}} = 2D_y \sum_i \left(\hat{S}_{li}^z \hat{S}_{li+1}^x + \hat{S}_{ri}^z \hat{S}_{ri+1}^x \right). \quad (\text{B1})$$

We can now define a flux operator on each plaquette as

$$\hat{W}_i = 16 \hat{S}_{li}^x \hat{S}_{ri}^x \hat{S}_{li+1}^z \hat{S}_{li+1}^z, \quad (\text{B2})$$

with eigenvalues ± 1 . Because $\hat{W}_i$ commutes with $\hat{H}$, the total Hilbert space splits into exponentially many pieces labeled by the eigenvalues of $\hat{W}_i$. This effect is called local Hilbert space fragmentation [143] and is linked to non-ergodic behavior of many-body states, i.e., “slower than usual” thermalization [132]. For a finite system of size L , there are additional conserved charges given by the operators $\hat{W}_{\text{top}} = 4 \hat{S}_{l1}^z \hat{S}_{r1}^z$ and $\hat{W}_{\text{bottom}} = 4 \hat{S}_{lL}^x \hat{S}_{rL}^x$.

The presence of these conserved charges is a property of the model, and while it certainly deserves further investigation, it is not the focus of this Letter and we leave it to future research to understand its unusual time evolution.