

Accessing Nucleon Transversity with One-Point Energy Correlators

Mei-Sen Gao^{1,*}, Zhong-Bo Kang^{2,3,4,†}, Wanchen Li^{1,‡}, and Ding Yu Shao^{1,5,6,7,§}

¹Department of Physics and Center for Field Theory and Particle Physics, *Fudan University*, Shanghai 200433, China

²Department of Physics and Astronomy, *University of California, Los Angeles*, California 90095, USA

³Mani L. Bhaumik Institute for Theoretical Physics, *University of California, Los Angeles*, California 90095, USA

⁴Center for Frontiers in Nuclear Science, *Stony Brook University*, Stony Brook, New York 11794, USA

⁵Key Laboratory of Nuclear Physics and Ion-beam Application (MOE), *Fudan University*, Shanghai 200433, China

⁶Shanghai Research Center for Theoretical Nuclear Physics, NSFC and *Fudan University*, Shanghai 200438, China

⁷Center for High Energy Physics, *Peking University*, Beijing 100871, China



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We propose a novel probe of the nucleon's transversity distribution h_1^q using the one-point energy correlator (OPEC), an infrared-and-collinear safe jet substructure observable. We demonstrate that in transversely polarized $p^\dagger p$ collisions, the OPEC exhibits a single-spin asymmetry (SSA) with a clean $\sin(\phi_s - \phi_n)$ angular dependence. This method probes SSA over a much wider kinematic range in the angular scale θ_n compared to traditional measurements of hadron transverse momentum $j_\perp$, establishing a complementary and systematically distinct channel to study the nucleon's three-dimensional structure at the RHIC and the future electron-ion collider.

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Introduction—A complete description of nucleon tomography is a central goal of hadron physics. Within the framework of collinear factorization, this structure at leading twist is completely characterized by three parton distribution functions (PDFs): the unpolarized (f_1^q), helicity (g_1^q), and transversity (h_1^q) distributions [1–3]. While f_1^q and g_1^q are well constrained [4], the transversity PDF remains poorly known because of its unique chiral-odd nature [2], which makes it inaccessible in inclusive scattering. Overcoming this experimental challenge is critical, as the first moment of h_1^q defines the nucleon's tensor charge, $\delta q \equiv \int_0^1 dx [h_1^q(x) - h_1^{\bar{q}}(x)]$. This quantity not only provides deep insight into the nucleon spin structure, but also serves as a crucial input for beyond the standard model probes—such as in searches for new tensor interactions in neutron β decay [5].

Phenomenologically, the transversity PDF has been accessed via single-spin asymmetries in semi-inclusive deep-inelastic scattering (SIDIS) and proton-proton collisions, primarily through the Collins effect [6–10] and

dihadron production channels [11–13]. In parallel with these efforts, direct lattice QCD simulations have achieved remarkable precision [14], enabling innovative analyses that use the lattice results as a direct constraint on the phenomenological fits [10,12,13,15] to help resolve historical tensions; see also recent developments in Refs. [9–12,16,17]. Despite this impressive progress, current extractions from experimental data are limited by their reliance on nonperturbative fragmentation functions (FFs), which carry significant modeling uncertainties.

A promising alternative lies in the field of jet substructure. The energy-energy correlator (EEC) [18,19], an infrared and collinear (IRC) safe event shape observable [20,21], provides direct access to jet substructure. EEC has been extensively measured in e^+e^- annihilation [22–35] and at the LHC [36–44]. Precision studies have also been proposed for SIDIS [45–47] at the future Electron-Ion Collider (EIC) [48,49]. Recently, there has been growing interest in employing EEC to explore the nucleon tomography [50–58]. For a recent review on energy correlators, see Ref. [59].

In this Letter, we propose that one simple jet substructure observable—the one-point energy correlator (OPEC) [18,19]—provides a novel and direct probe of the transversity PDF. This observable measures the angular distribution of energy flux $\langle \mathcal{E}(\hat{n}) \rangle$. The energy flow operator $\mathcal{E}(\hat{n})$

$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r \rightarrow \infty} r^2 n^i T_{0i}(t, r\hat{n}) \quad (1)$$

*Contact author: msgao@fudan.edu.cn

†Contact author: zkang@physics.ucla.edu

‡Contact author: wanchenli@fudan.edu.cn

§Contact author: dyshao@fudan.edu.cn

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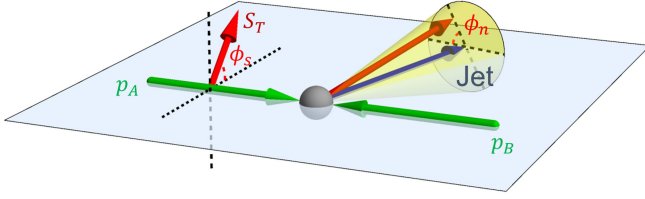


FIG. 1. Kinematics of hadron production inside a jet in transversely polarized $p^\uparrow p$ collisions. The reaction plane is defined by the incoming proton momenta p_A , p_B , and the jet axis. The azimuthal angles ϕ_s and ϕ_n specify the orientations of the proton transverse spin vector S_T and the energy detector with respect to this plane.

deposits the energy of final-state particles in a specific direction $\hat{n}$. While OPEC has garnered recent attention [60–63], its connection to spin physics, particularly to the transversity PDF, has remained unexplored.

We demonstrate that in transversely polarized $p^\uparrow p$ collisions, this observable exhibits a clean single-spin asymmetry (SSA) with a characteristic $\sin(\phi_s - \phi_n)$ angular modulation, where ϕ_n and ϕ_s are the azimuthal angles of the energy flux detector and the nucleon spin relative to the reaction plane spanned by the incoming protons and the jet axis, as shown in Fig. 1. This measurement provides a new channel to access nucleon transversity with different dynamical weighting than existing methods. Compared with unweighted transverse momentum dependent (TMD) [64] studies [8,65–69], the OPEC provides a cleaner input from a broader range of the jet substructure kinematics, while reducing the model dependence of nonperturbative fragmentation. This in turn enables a more robust determination of the transversity PDF. It offers a crucial, independent test of the universality of spin-dependent hadronization and establishes a powerful connection between the formal theory of energy flow and the nonperturbative partonic structure of the nucleon. In particular, this observable provides a timely avenue for phenomenological studies using Solenoidal Tracker at RHIC (STAR) data [70–73], and more broadly, this Letter opens a new frontier in the study of spin-dependent jet substructure and establishes a synergistic path between the ongoing Relativistic Heavy Ion Collider (RHIC) spin program [74] and the precision measurements envisioned at the future EIC [48,49].

OPEC—In the collinear limit, the OPEC is encoded within general light cone correlators that define a new class of fragmenting jet functions (FJFs) [75,76]. For a jet initiated by a quark, this correlator is given by

$$\Delta^q(z, \hat{n}) = \sum_X \sum_{i \in J} \langle \Omega | \bar{\chi}_n \delta_{Q, p_n} \delta^{(2)}(\hat{n} - \hat{n}_i) | J(h_i) X \rangle \times \frac{E_i}{E_J} \langle J(h_i) X | \chi_n | \Omega \rangle, \quad (2)$$

where χ_n is the gauge invariant quark field in soft-collinear effective theory [77–80] that creates collinear particles from

the vacuum. E_J and E_i represent the energy of the jet and the measured hadron, respectively. $z \equiv E_J/Q$ represents the energy fraction of the jet that is initiated from the quark with energy Q . The measurement is performed on the energy flux inside the jet J , and the state $|J(h_i)X\rangle$ represents the final-state unobserved particles X and the jet J with the hadron h_i inside. This definition implicitly depends on the jet algorithms (e.g., the jet radius R), which we suppress for notational simplicity. Taking into account the unpolarized and transversely polarized quark initiating the jet, we can parametrize the above correlators at the leading power as [64]

$$\Delta_q^{[\gamma^+]} = \mathcal{J}^q, \quad \Delta_q^{[i\sigma^{\alpha+}\gamma_s]} = \epsilon_T^{\alpha\beta} \hat{n}_{T,\beta} \frac{P_J^-}{2} \theta_n \mathcal{J}_{1,\perp}^q, \quad (3)$$

where we have $p_J^- = \bar{n} \cdot p_J$, $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$, $n \cdot \bar{n} = 2$, and $\epsilon_T^{\alpha\beta} = \epsilon^{\alpha\beta\rho\sigma} \bar{n}_\rho n_\sigma / (n \cdot \bar{n})$. $\hat{n}_T$ is the transverse direction of the energy flow relative to the jet axis. $\mathcal{J}^q$ is the unpolarized OPEC FJF, while $\mathcal{J}_{1,\perp}^q$ denotes the transversely polarized OPEC FJF, which captures the fragmentation of a transversely polarized quark into an azimuthally asymmetric energy flux within the jet.

Factorization formula—We consider the OPEC in inclusive jet production for the polarized proton and unpolarized proton scattering process, $p^\uparrow + p \rightarrow J + X$. The OPEC observable is defined as

$$\frac{d\Sigma}{d\theta_n d\phi_n d\eta dp_T} = \sum_{h \in J} \int_0^1 dz_h \int d^2\Omega_h \delta(\phi_n - \phi_h) \times \delta(\theta_n - \theta_h) z_h \frac{d\sigma}{dz_h d^2\Omega_h d\eta dp_T}, \quad (4)$$

where the jet is characterized by rapidity η and transverse momentum p_T , and the sum runs over hadrons inside the jet. For each hadron h , the solid angle $d^2\Omega_h \equiv \sin\theta_h d\theta_h d\phi_h$ specifies its direction relative to the jet axis, with θ_h denoting the polar angle and ϕ_h the azimuthal angle. The weight factor z_h represents the energy fraction of the fragmenting quark jet carried by the hadron.

The OPEC observable offers two key advantages over traditional hadron-in-jet distributions. First, the built-in integral over z_h in Eq. (4) projects the full z_h dependence of the hadron distribution onto a single moment. As a result, a TMD fit involving OPEC requires only a parametrization in the Fourier variable b , rather than in the two-dimensional space (b, z_h) , substantially reducing the model dependence associated with final-state fragmentation [56,57]. Second, OPEC benefits from angular measurements at far higher precision than hadron-in-jet studies [8,65–73], where substructure is accessed through the hadron’s transverse momentum $j_\perp \simeq p_T z_h \theta_n$ in the collinear limit. For STAR kinematics at $\sqrt{s} = 510$ GeV and $p_T = 32.3$ GeV with a typical cut $z_h = 0.1$, the accessible

$j_\perp$ range (0.1,1) GeV corresponds to $\theta_n \simeq (0.03, 0.3)$ rad. By contrast, modern detectors achieve resolutions of order 10^{-3} rad: CMS has measured energy correlators below this scale [40], while the EIC is expected to reach down to 10^{-4} rad for both polar and azimuthal angles at high p_T [49,81,82]. The OPEC framework thus enables jet substructure studies at angular scales more than an order of magnitude finer than previously probed, while remaining IRC safe and robust in the small-angle regime.

The azimuthal dependence of the OPEC distribution can be decomposed as

$$\frac{d\Sigma}{d\theta_n d\phi_n d\eta dp_T} = Z_{UU} + \sin(\phi_s - \phi_n) Z_{UT}, \quad (5)$$

where the unpolarized (Z_{UU}) and spin-dependent (Z_{UT}) structure functions take the factorized forms

$$Z_{UU} = \frac{\alpha_s^2}{s} p_T^2 \theta_n \sum_{a,b,c} \int \frac{dx_1}{x_1} f_{a/A}(x_1, \mu) \int \frac{dx_2}{x_2} f_{b/B}(x_2, \mu) \times \mathcal{J}^c(\theta_n, Q) H_{ab \rightarrow c}^U(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}), \quad (6)$$

$$Z_{UT} = \frac{\alpha_s^2}{s} p_T^2 \theta_n \sum_{a,b,c} \int \frac{dx_1}{x_1} h_1^a(x_1, \mu) \int \frac{dx_2}{x_2} f_{b/B}(x_2, \mu) \times p_T \theta_n \mathcal{J}_{1,\perp}^c(\theta_n, Q) H_{ab \rightarrow c}^{\text{Collins}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}). \quad (7)$$

Here, α_s denotes the strong coupling constant and s is the square of the center-of-mass energy of the proton-proton system. The hard functions $H_{ab \rightarrow c}^U$ and $H_{ab \rightarrow c}^{\text{Collins}}$ represent the unpolarized and spin-dependent partonic matrix elements, respectively, for the subprocess $ab \rightarrow c$ [65,68,83,84]. The variables $\hat{s}$, $\hat{t}$, and $\hat{u}$ are the Mandelstam variables of the partonic interaction. The collinear PDF f_a and transversity PDF h_1^a depend on the factorization scale μ , while the energy-weighted jet functions $\mathcal{J}^c$ and $\mathcal{J}_{1,\perp}^c$ are evaluated at the hard scale Q .

Relations to the Collins function—A powerful application of our formalism is to perform direct tests of the universality of polarized fragmentation. The Collins function [6,7], which encodes the information of the azimuthal distribution of hadrons fragmenting from a transversely polarized quark, has been a cornerstone for accessing transversity, with its universality across different scattering processes being a crucial, yet challenging, assumption to verify [85–89].

Our polarized FJF, $\mathcal{J}_{1,\perp}^q$, provides a direct probe of the same underlying physics but in terms of energy flow rather than a specific hadron species. This distinction is paramount. As an IRC safe observable, the OPEC FJF is theoretically cleaner and less susceptible to the complexities of hadronization models. Universality can therefore be tested by comparing measurements of $\mathcal{J}_{1,\perp}^q$ in distinctly different environments: (1) e^+e^- annihilation, the cleanest

environment where jets are produced from a quark-antiquark pair; (2) SIDIS, probing the fragmentation of a quark struck from within a nucleon; and (3) polarized proton-proton collisions, the most complex hadronic environment. A comparison of the extracted $\mathcal{J}_{1,\perp}^q$ from these channels, after accounting for their respective QCD evolution, would constitute a rigorous and independent test of the factorization and universality of the polarized fragmentation mechanism.

In the limit $\theta_n \gg \Lambda_{\text{QCD}}/Q$, the unpolarized and transversely polarized OPEC FJFs can be systematically matched onto their collinear counterparts using the operator product expansion (OPE)

$$\mathcal{J}^q(\theta_n, Q) = \sum_h \int_0^1 dz_h z_h \int_0^\infty \frac{dbb}{2\pi} J_0(p_T \theta_n b) \times \hat{C}_{i \leftarrow q}^D \otimes D_{h/i}(z_h, \mu_b) e^{-\frac{1}{2} S_{\text{pert}}(Q, b)}, \quad (8)$$

$$p_T \theta_n \mathcal{J}_{1,\perp}^q(\theta_n, Q) = \sum_h \int_0^1 dz_h z_h \int_0^\infty \frac{dbb^2}{2\pi} J_1(p_T \theta_n b) \times \delta \hat{C}_{i \leftarrow q}^{\text{Collins}} \otimes \hat{H}_{h/i}^{(3)}(z_h, \mu_b) e^{-\frac{1}{2} S_{\text{pert}}(Q, b)}, \quad (9)$$

where we have included QCD evolution in the perturbative Sudakov factor S_{pert} [64]. Here J_0 and J_1 are Bessel functions, $\mu_b = 2e^{-\gamma_E}/b$ is the OPE scale of the collinear FFs. $D_{h/i}$ is the unpolarized collinear FF, and $\hat{H}_{h/i}^{(3)}$ is the first moment of the Collins function in the convention of Refs. [8,88]. In the expressions above, we neglect the resummation of jet radius logarithms, and the complete factorized structure and renormalization group evolution of the OPEC FJFs are detailed in Supplemental Material [90]. The usual convolution $\otimes$ is defined as

$$\hat{C}_{i \leftarrow q} \otimes F_{h/i} = \sum_i \int_{z_h}^1 \frac{d\xi}{\xi} \hat{C}_{i \leftarrow q}(\xi, \mu_b) F_{h/i}\left(\frac{z_h}{\xi}, \mu_b\right), \quad (10)$$

where $F \in \{D, \hat{H}^{(3)}\}$ and we have ignored the two-variable twist-3 FFs that appear in the collinear factorization region [88]. At LO, both matching coefficients $\hat{C}_{i \leftarrow q}^D$ and $\delta \hat{C}_{i \leftarrow q}^{\text{Collins}}$ reduce to $\delta_{qi} \delta(1 - \xi)$, while beyond LO, the matching coefficients generally depend on the jet algorithm [69,91–93]. The complete one-loop expressions for the anti- k_T algorithm [94] are provided in Supplemental Material [90].

Single spin asymmetry—The transverse single-spin asymmetry (SSA) of the OPEC is given by

$$A_{UT}^{\sin(\phi_s - \phi_n)} = \frac{Z_{UT}}{Z_{UU}}. \quad (11)$$

In our numerical calculations, we have included contributions from the gluon-initiated jet channel, which enter the

unpolarized structure function Z_{UU} and thus influence the normalization of the SSA. We perform numerical evaluations at next-to-leading logarithmic (NLL) accuracy, setting $\mu = Q = p_T$ and employing the LO matching coefficients in Eqs. (8) and (9). The perturbative Sudakov factors are detailed in Supplemental Material [90]. A more complete treatment, including the resummation of jet radius, which is necessary for achieving higher theoretical precision, will be presented in a future work. In addition, the full Sudakov exponential is modeled as $e^{-S_{\text{pert}}(Q, b_*)/2 - S_{\text{NP}}(Q, b)}$. In S_{pert} , we separate perturbative and nonperturbative domains using the standard b_* -prescription, where $b_* = b/\sqrt{1 + b^2/b_{\text{max}}^2}$ freezes the impact parameter at $b_{\text{max}} = 1.5 \text{ GeV}^{-1}$ [95–97]. This prevents the evolution from entering a region where perturbation theory is unreliable. For the unpolarized FJF, the nonperturbative component $S_{\text{NP}}^{D_1}(Q, b)$ is adapted from the well-established form used for Drell-Yan and SIDIS processes [98]. For the transversely polarized FJF, the corresponding nonperturbative factor $S_{\text{NP}}^{\text{Collins}}(Q, b)$ is taken from global analyses of the Collins effect [8].

Furthermore, since only charged $\pi^\pm$ Collins FFs are known, we apply a less inclusive version of our formula, replacing the sum over all hadrons $\sum_h$ in Eqs. (8) and (9) with a subset of hadrons sharing the same quantum numbers, $\sum_{h \in \mathbb{S}}$ [99–103]. In this Letter, we only consider charged pions $\pi^\pm$ and present the SSAs $A_{UT}^{\sin(\phi_s - \phi_n)}$ in two approaches: (i) TMD evolution framework: The full TMD evolution formalism given in Eqs. (8) and (9) at NLL accuracy, where we parametrize the transversity PDF and Collins FF according to [8], and retain the same collinear inputs as in that analysis: CT10nlo PDF [104] and NLO DEHSS FF [105]. (ii) JAM3D framework: The approach of the JAM3D-22 global analysis [10]. Here, the Sudakov

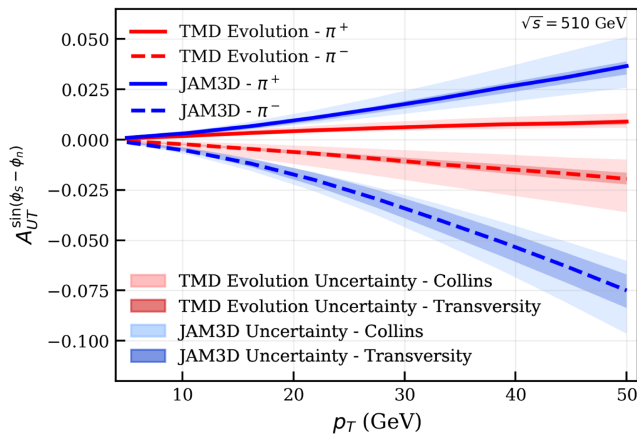


FIG. 2. SSA $A_{UT}^{\sin(\phi_s - \phi_n)}$ as a function of jet transverse momentum p_T for $\pi^\pm$ at $\sqrt{s} = 510 \text{ GeV}$, evaluated using two approaches: the JAM3D-22 and the TMD evolution framework. Error bands represent uncertainties from the transversity PDF and the Collins FF parametrizations, respectively. The p_T distribution is obtained by integrating over $\theta_n \in [0, 0.1]$.

exponentiation (with Fourier transform) in Eqs. (8) and (9) is replaced by a Gaussian modeling of the transverse momentum dependence, and the corresponding collinear components are evolved using Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equations.

In Fig. 2, we present SSA $A_{UT}^{\sin(\phi_s - \phi_n)}$ as a function of jet transverse momentum p_T at $\sqrt{s} = 510 \text{ GeV}$, computed using both the JAM3D-22 and full evolution frameworks. The evolved prediction that is based on the global fit of [8] yields systematically smaller asymmetries compared to the JAM3D-22 result. We also analyze the uncertainties originating from two key sources: the transversity PDFs and the Collins FFs. In both approaches, we observe that the dominant source of uncertainty arises from the Collins function, while the transversity contribution plays a comparatively smaller role. Additional theoretical scale uncertainties are discussed in Supplemental Material [90].

In Fig. 3, we show the SSA $A_{UT}^{\sin(\phi_s - \phi_n)}$ as a function of the angle θ_n of the energy detectors at two kinematic settings extensively studied by the STAR Collaboration [70–73]: (a) $\sqrt{s} = 510 \text{ GeV}$, $p_T = 32.3 \text{ GeV}$, (b) $\sqrt{s} = 200 \text{ GeV}$, $p_T = 13.3 \text{ GeV}$, respectively. The error bands reflect combined uncertainties from the transversity PDFs and Collins FFs. The energy-weighted asymmetries exhibit similar magnitudes across the two collision energies,

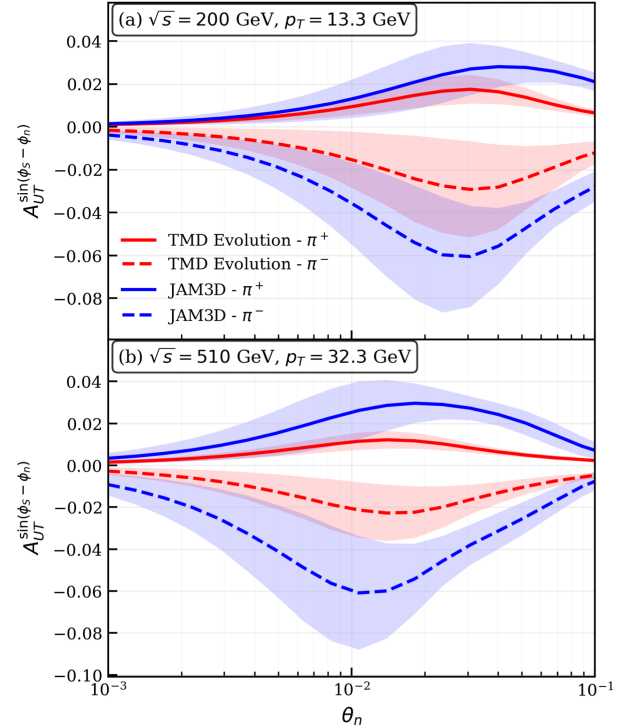


FIG. 3. SSA $A_{UT}^{\sin(\phi_s - \phi_n)}$ as a function of θ_n at (a) $\sqrt{s} = 510 \text{ GeV}$, $p_T = 32.3 \text{ GeV}$, (b) $\sqrt{s} = 200 \text{ GeV}$, $p_T = 13.3 \text{ GeV}$, computed using the JAM3D-22 and TMD evolution frameworks. Error bands represent combined uncertainties from transversity PDFs and Collins FFs.

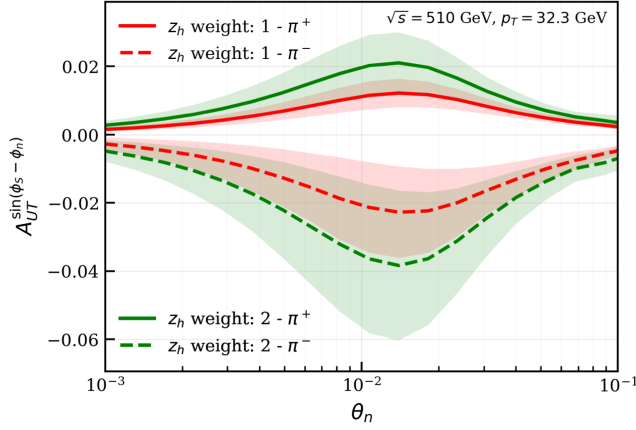


FIG. 4. SSA $A_{UT}^{\sin(\phi_s - \phi_n)}$ as a function of θ_n at $\sqrt{s} = 510$ GeV and fixed jet transverse momentum $p_T = 32.3$ GeV, computed for two different $z_h^{(n)}$ weights $n = 1, 2$ in the TMD evolution framework. The error bars reflect combined uncertainties from both the transversity parton distribution functions and the Collins FF parametrizations.

consistent with recent observations of energy independence of the SSA in $p^\uparrow p$ collisions [73]. However, their peak positions in θ_n differ, reflecting a shift in the angular structure of the fragmentation dynamics. Moreover, the partial overlap between the error bands from the TMD evolution and JAM3D approaches suggests that current parametrizations do not yet allow us to resolve the effects of TMD evolution. Nonetheless, the reduced overlap observed at higher p_T highlights the potential for future data to provide clearer resolution.

Generalization—In the standard definition of OPEC in Eq. (4), the energy weight z_h enters linearly. A natural generalization is to raise z_h to an arbitrary power, thereby probing higher Mellin moments of the Collins FFs. In Fig. 4, we compare the resulting SSA as functions of θ_n for different choices of the z_h weight. We observe that increasing the z_h weight, i.e., higher Mellin moment, leads to a remarkable enhancement in the magnitude of the SSA.

Conclusions and outlook—We introduced the OPEC in jets for transversely polarized $p^\uparrow p$ collisions, sensitive to the transverse polarization effect through a $\sin(\phi_s - \phi_n)$ modulation. Defined via an energy-weighted angular distribution of energy flux detectors inside jets, this observable provides a clean probe of the transversity PDF and Collins FF at high momentum scales. Compared with prior studies, we estimate that OPEC can access a much broader kinematic range of the jet substructure.

We also present a phenomenology study of SSA from OPEC via two frameworks: (a) a TMD evolution framework using global fits from [8], (b) the JAM3D-22 global analysis [10], which omits TMD evolution. Both approaches reveal similar qualitative behavior, with partially overlapping uncertainty bands between the two approaches. The OPEC SSA offers a complementary

channel to constrain the transversity PDF and Collins function at RHIC. Looking forward, implementing this observable at the future EIC would enable a pristine test of the fundamental principle of universality for the Collins FF by allowing a direct comparison between hadronic ($p^\uparrow p$) and deep-inelastic scattering ($ep^\uparrow$) processes. This establishes a vital new tool for precision studies of spin-dependent fragmentation and the rich internal landscape of the nucleon.

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Data availability—The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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