

Dilepton Production from Moaton Quasiparticles

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The phase diagram of QCD probably exhibits a moat regime over a large region of temperature T and chemical potential $\mu \neq 0$. A moat regime is characterized by quasiparticle *moatons* (pions) whose energy is minimal at nonzero spatial momentum. At $\mu \neq 0$, higher mass dimension operators play a critical role in a moat regime. At dimension six, there are nine possible gauge-invariant couplings between scalars and photons. For back-to-back dilepton production, only one operator contributes, which significantly enhances production near a moat threshold. This enhancement is an experimental signature of moatons.

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The phase diagram of quantum chromodynamics (QCD) at a nonzero temperature T and quark chemical potential μ (or baryon chemical potential $\mu_B = 3\mu$) is important for a variety of physical systems, from neutron stars to heavy ion collisions to the early Universe. We know much about the region of $T \neq 0$ at low $\mu \ll T$ from heavy ion colliders operating at ultrarelativistic energies, such as the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory and the Large Hadron Collider at CERN. We also have a wealth of understanding of $\mu = 0$ behavior from first-principles lattice QCD techniques, which indicate that the chiral transition is a crossover at temperature $T \approx 156.5 \pm 1.5$ MeV [1,2]. However, the lattice cannot reach beyond $\mu \sim T$ on classical computers due to the sign problem. A first-principles understanding of

heavy ion collisions at moderate energies requires studying QCD at $\mu > T$, and neutron stars have large $\mu \gg T$.

As μ increases, it is likely that a critical end point (CEP) develops, where a line of crossover transitions meets a line of first-order transitions [3–7]. From direct computations in QCD using the functional renormalization group (FRG) [8] and Schwinger-Dyson equations [9–11], as well as reconstructions based on lattice data [12,13], this is estimated to occur for $(T_c, \mu_c) \approx (100, 200)$ MeV. This lies in the region $\sqrt{s}/A: 3 \rightarrow 7$ GeV in heavy ion collisions. By perverse coincidence, the beam energy scan by the STAR experiment at RHIC studied collisions down to $\sqrt{s}/A = 7$ GeV and at $\sqrt{s}/A = 3$ GeV in fixed target mode but not in between. While intriguing, their results did not find evidence for a CEP. This region will be studied at the Nuclotron-based Ion Collider Facility at the Joint Institute for Nuclear Research and with the compressed baryon matter (CBM) experiment at the Facility for Antiproton and Ion Research (FAIR) at the GSI Helmholtz Center for Heavy Ion Research.

The CEP, while of fundamental significance, may be difficult to probe directly. In vacuum, the mass of the σ meson is large, perhaps ~ 500 MeV, with a width of similar order, rendering it difficult to extract experimentally. In contrast, at the CEP, the σ is precisely massless [14] but pions remain massive. Because the σ 's singularity is so far from the origin in vacuum, it must travel a long way, suggesting that the basin of attraction to the CEP can be narrow. This naive argument is confirmed by studies of the

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chiral phase transition, where critical scaling is absent even close to the transition; see Refs. [8,16–24].

A compelling target for determining where novel phenomena occur is a “moat” regime, which occurs when the dispersion relations of quark correlations in meson channels have a global minimum at *nonzero* spatial momentum. The FRG [8,24] indicates that the moat regime in QCD is *extensive*, occurring over a *large* region of the phase diagram at finite T and μ , including the vicinity of the CEP. As detailed below, a moat regime gives rise to exotic behavior involving spatial modulations, which may (but need not) include a region with inhomogeneous phases.

In this Letter, we propose a striking signature of moat regimes: new thresholds with a significant enhancement in dilepton production in heavy ion collisions. Experimental observation of a moat regime would confirm the existence of novel phases in QCD and help locate the CEP.

Moat regimes at $\mu \neq 0$ —A moat regime is characterized by a dispersion relation exhibiting a local minimum at nonzero momentum, which can induce exotic phase behaviors. Often, moat regimes onset when the wave function renormalization constant Z for the quadratic spatial momentum term becomes negative. This typically gives rise to a “disorder line” in the phase diagram [25–27], separating a region in which the two-point correlation function has exponential falloff at large distances, from a region in which it behaves as an exponential times an oscillatory function; this “oscillatory regime” is similar to Friedel oscillations, which are screened at nonzero temperature [28]. If the dip in the moat dispersion relation becomes sufficiently deep, then the moat regime may additionally develop a finite-momentum (i.e., spatially nonuniform) condensate, such as, e.g., pion, kaon, or superconducting condensates [29–35]. Direct transitions from a spatially homogeneous broken symmetry phase to an inhomogeneous phase may also arise (see Fig. 1).

A natural question is whether a moat regime exists in QCD-like theories or is an artifact of a given approximation method. Moat regimes are *ubiquitous* in condensed matter, which like finite-density QCD breaks Lorentz invariance; see Refs. [37–42]. Indeed, the term “moat” hails from condensed matter [59,60]. Furthermore, moat regimes naturally arise in theories with combined charge and complex conjugation (CK) symmetry [26]. Because the Dirac operator is non-Hermitian at $\mu \neq 0$, QCD has CK symmetry [27,61]. This is a particular case of broad non-Hermitian theories with generalized PT symmetries [62–64].

Moat regimes and their associated phases appear in many models sharing features with QCD. A canonical example is the Gross-Neveu model in $(1+1)$ dimensions. This theory is asymptotically free, spontaneously breaks chiral symmetry through dynamical generation of mass, and is tractable when the number of fermions $N \rightarrow \infty$. The phase diagram at low $\mu < T$ is similar to what we suspect occurs in QCD in the chiral limit: A line of second-order

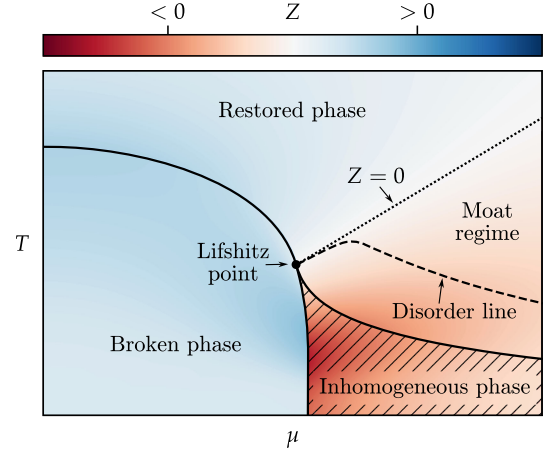


FIG. 1. A phase diagram sketch based on the Gross-Neveu model in Ref. [36], which exhibits *all* three characteristic behaviors of moat regimes: a disorder line, oscillatory regime, and inhomogeneous phase. Red shading indicates a moat regime, where $Z < 0$. Solid lines separate different phases. Dashed lines indicate qualitative changes in system correlations, related to spatial modulations; the oscillatory regime lies between the disorder line and the inhomogeneous phase. The exact location and nature of each transition is model dependent. However, an extensive moat regime is a common feature of models and is also seen in FRG analyses of QCD [8]. In the Gross-Neveu model, the CEP is replaced by a Lifshitz point due to the emergence of an inhomogeneous phase.

transitions extends from $\mu = 0$ and $T \neq 0$. Because of the emergence of the Peierls transition, this line ends not in a CEP but in a Lifshitz point; for lower T , there is a region with spatially inhomogeneous condensates [65–73]. Crucially, the Gross-Neveu model has a moat regime in a broad region of the T - μ plane [36] (Fig. 1).

Numerous studies of moat regimes have been carried out in $(3+1)$ -dimensional models sharing features of QCD. In the Nambu-Jona Lasinio model [39,74–79], the presence of an inhomogeneous phase depends strongly upon the regularization scheme, while a moat regime is robust [80]. Schwinger-Dyson analysis shows that the symmetric solution is unstable to an inhomogeneous phase at low T and high μ [81,82]. Disorder lines appear in Polyakov-Nambu-Jona Lasinio models [83,84], Polyakov-quark-meson models [15], quark-meson models [85], static quark models at strong coupling [86], liquid-gas models [87], and four-fermion models in $(2+1)$ dimensions [88,89]. For scalar models with global $O(N)$ symmetry, Goldstone bosons can disorder the system into a quantum pion liquid [91–93]. Moat regimes are more prevalent in models of hadronic physics at nonzero density than inhomogeneous phases.

References [94,95] showed that a moat regime affects particle production through two-particle correlations generated by thermodynamic fluctuations [95] and Hanbury Brown–Twiss interferometry for pions [96,97]. In this Letter, we compute how quasiparticle pions in a moat

regime, which we dub “moatons,” affect dilepton production. For technical reasons, we limit ourselves to dilepton pairs with zero spatial momentum and leave the analysis of nonzero spatial momentum to a later publication.

Related analyses of dilepton production have been carried out in a color superconductor [98], near a critical end point [99,100], and in a chiral spiral [101,102].

Effective Lagrangian for the moat regime—A simple model of a moat regime is given by the Lagrangian [91,92]

$$\mathcal{L} = \frac{1}{2}(\partial_0 \vec{\phi})^2 + \frac{Z}{2}(\partial_i \vec{\phi})^2 + \frac{1}{2M^2}(\partial_i^2 \vec{\phi})^2 + \frac{m^2}{2}\vec{\phi}^2 + \frac{\lambda}{8}(\vec{\phi}^2)^2, \quad (1)$$

where $\vec{\phi} = (\sigma, \vec{\pi})$. This Lagrangian is written in the preferred frame provided by the thermal bath, using a Euclidean metric. To maintain causality, we insist that time derivatives enter only quadratically.

A moat regime arises in Eq. (1) when Z is negative, so the energy has a minimum at nonzero momentum. This was first observed in QCD at finite T and μ in an FRG analysis [8]. Consequently, we must include a nonrenormalizable term quartic in spatial derivatives in Eq. (1). While this term may look unfamiliar, it is natural in derivative expansions of effective Lagrangians. Examples include systems near second-order phase transitions [110] and condensed matter systems [37–41] (where such derivatives may be replaced by their discrete lattice counterparts [111]).

A priori, we cannot drop higher mass dimension terms when constructing a full effective field theory (EFT) of a moat regime. Thus, our model in Eq. (1) only caricatures an EFT about the global minimum of the dispersion relation [112], valid when $\langle (\partial_i \vec{\phi})^2 \rangle \ll M^4$ and $\langle (\vec{\phi}^2)^2 \rangle \ll M^2$. We cannot rule out additional local minima [113]; however, the dominant effects stem from field configurations at the global minimum, as in Refs. [94–96], which our model captures. For simplicity, we work in the symmetric ($\langle \vec{\phi} \rangle = 0$) regime.

Dilepton production—At leading order in $\alpha = e^2/4\pi$, the dilepton production rate is proportional to the photon self-energy $\Pi^{\mu\nu}$ [114]:

$$\frac{d^4 R}{dp^4} = \frac{\alpha}{12\pi^4} \frac{n(\omega)}{-P^2} \text{Im} \Pi^{\mu\mu}(p_0, \vec{p}). \quad (2)$$

Here, $\omega > 0$ with the Bose function $n(\omega) = 1/[\exp(\omega/T) - 1]$. We analytically continued the Euclidean energy p_0 to Minkowski space, $p_0 = -i(\omega + i\epsilon)$, where $\epsilon \rightarrow 0^+$ so $-P^2 = P_M^2 = \omega^2 - \vec{p}^2$, where P_M is a Minkowski four-vector.

In vacuum, vector meson dominance implies that pion production proceeds entirely through virtual ρ_μ mesons [115]. In heavy ion collisions, successful fits to the dilepton excess below the ρ_μ meson peak are usually obtained by a

broadened ρ_μ in nuclear matter [116–118]. In contrast, our analysis is valid in the chirally restored phase; cf. Fig. 1. Here, we study additional contributions from moatonic pions.

We consider the possible dimension-6 terms that can contribute to dilepton production in a QCD moat regime in four dimensions, where a spin zero field $\vec{\phi}$ carries dimensions of mass. We compare this to the chiral perturbation theory Lagrangian in Supplemental Material [103]. To obey gauge invariance of couplings to the photon field (A_μ), we replace $\partial_\mu \rightarrow D_\mu$ in Eq. (1):

$$\frac{1}{2}(D_0 \vec{\phi})^2 + \frac{Z}{2}(D_i \vec{\phi})^2 + \frac{1}{2M^2}(D_i^2 \vec{\phi})^2, \quad (3)$$

where the covariant derivative acts as $D_\mu \vec{\phi} = (\partial_\mu \sigma, \partial_\mu \pi^1 - eA_\mu \pi^2, \partial_\mu \pi^2 + eA_\mu \pi^1, \partial_\mu \pi^3)$. This can be written as $D_\mu \cdot \vec{\phi} = (\partial_\mu - iet_2 A_\mu) \vec{\phi}$; t_2 is the corresponding Pauli matrix, which acts only on the π^1 and π^2 directions and is zero otherwise; these form charged pions as $\pi^\pm = (\pi^1 \pm i\pi^2)/\sqrt{2}$ [119].

There are additional terms involving two derivatives and four $\vec{\phi}$'s [92]:

$$\frac{C_1}{M^2}(D_0 \vec{\phi})^2 \vec{\phi}^2 + \frac{C_2}{M^2}(D_i \vec{\phi})^2 \vec{\phi}^2 + \frac{C_3}{M^2}(\vec{\phi} \cdot D_0 \vec{\phi})^2 + \frac{C_4}{M^2}(\vec{\phi} \cdot D_i \vec{\phi})^2. \quad (4)$$

Since the thermal bath provides a preferred frame of reference, the coefficients of terms involving temporal and spatial derivatives may differ. Gauge invariance requires that photons couple only through the field strength, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, giving

$$\left(\frac{C_5}{M^2} F_{0i}^2 + \frac{C_6}{M^2} F_{ij}^2 \right) \vec{\phi}^2. \quad (5)$$

The above terms are mundane, but there is also

$$\frac{eC_7}{M^2} F_{0i} \left(D_0 \vec{\phi} \cdot it_2 \cdot D_i \vec{\phi} \right) + \frac{eC_8}{M^2} F_{ij} \left(D_i \vec{\phi} \cdot it_2 \cdot D_j \vec{\phi} \right). \quad (6)$$

In terms of charged pions,

$$\frac{ieC_7}{M^2} F_{0i} (D_0 \pi^+ D_i \pi^- - D_i \pi^+ D_0 \pi^-) + \frac{ieC_8}{M^2} F_{ij} (D_i \pi^+ D_j \pi^-). \quad (7)$$

In this form, their invariance under charge conjugation is clear, as then $A_\mu \rightarrow -A_\mu$ and $\pi^+ \leftrightarrow \pi^-$. These terms are invariant under time reversal, as they have an even number of powers of ∂_0 and A_0 [120].

Equations (3)–(6) represent the complete set of operators to sixth order in the mass dimension. Other terms, e.g., $(D_i D_j \vec{\phi}) \cdot (D_i D_j \vec{\phi})$ and $(D_i D_j \vec{\phi}) \cdot (D_j D_i \vec{\phi})$, are linear combinations of the above. We emphasize that all these new operators, including the ones in Eq. (3), are dictated by gauge symmetry.

Next, we examine which operators in Eqs. (3)–(6) contribute to Eq. (2). Only the imaginary part of $\Pi^{\mu\nu}$ enters Eq. (2), so we can drop the terms in Eqs. (4) and (5), as they affect only tadpolelike diagrams. There is a contribution to dilepton production from the $(D_i^2 \vec{\phi})^2$ term in Eq. (3), through the trilinear coupling between $A_i(P)$, $\pi^+(K)$, and $\pi^-(-P-K)$. This vertex is

$$\Gamma^j(P, K) = ie(2k + p)^j \left[Z + \frac{1}{M^2} (\vec{k}^2 + (\vec{k} + \vec{p})^2) \right]. \quad (8)$$

From Eq. (1), the (effective) pion propagator in the moat regime is

$$\begin{aligned} \Delta^{-1}(K) &= k_0^2 + Z\vec{k}^2 + \frac{(\vec{k}^2)^2}{M^2} + m^2 \\ &= k_0^2 + \frac{(\vec{k}^2 - k_M^2)^2}{M^2} + m_{\text{eff}}^2, \end{aligned} \quad (9)$$

where the moat momentum k_M satisfies $k_M^2 = -ZM^2/2$ and the effective mass is $m_{\text{eff}}^2 = m^2 - Z^2 M^2/4$. A moat arises when Z is negative and k_M^2 is positive.

It is readily verified that the vertex in Eq. (8) satisfies the Ward identity

$$P^\mu \Gamma^\mu(P, K) = ie[\Delta^{-1}(K + P) - \Delta^{-1}(K)]. \quad (10)$$

The Ward identity implies that the photon self-energy is always transverse, $P^\mu \Pi^{\mu\nu}(P) = 0$. Note that this would not be the case without the new contribution $\sim 1/M^2$.

In this Letter, we compute dilepton pair production at rest in the frame of the thermal medium, with $\vec{p} = 0$, where only spatial components $\text{Im} \Pi^{ii}$ enter. To illustrate the effect, we include only the coupling to photons from Eq. (3) and set the couplings $C_7 = C_8 = 0$. The case of $\vec{p} \neq 0$, with these new couplings, will be analyzed in future work.

The leading contributions to $\text{Im} \Pi^{\mu\nu}$ come from the moaton-antimoaton production diagram:

$$\Pi^{ii}(p_0) = e^2 T \sum_{n'} \int \frac{d^3 k}{(2\pi)^3} (\Gamma^i)^2 \Delta(P + K) \Delta(P), \quad (11)$$

where $K = (2\pi n' T, \vec{k})$. Evaluating the bosonic Matsubara frequency sum over n' ,

$$\begin{aligned} \text{Im} \Pi_{jj}(\omega) &= 4\alpha\theta(\omega^2 - 4m_{\text{eff}}^2) \left[1 + 2n\left(\frac{\omega}{2}\right) \right] \\ &\times \frac{\sqrt{\omega^2 - 4m_{\text{eff}}^2}}{M\omega} \sum_{k_\pm} k_\pm^3 \theta(k_\pm), \end{aligned} \quad (12)$$

where

$$k_\pm^2 = k_M^2 \pm \frac{M\sqrt{\omega^2 - 4m_{\text{eff}}^2}}{2} \quad (13)$$

is the spatial momentum of the moatons. As the moatons are back to back, their momenta have the same magnitude. Near threshold, where $\omega^2 \approx 4m_{\text{eff}}^2 - i\epsilon$, we can approximate $\vec{k} \approx k_M \hat{k} + \delta\vec{k}$, where $\hat{k}^2 = 1$. About this point, the energy is quadratic in $\delta\vec{k}$:

$$E_\pm(\vec{k}) \approx m_{\text{eff}} + \frac{2k_M^2}{m_{\text{eff}} M^2} (\hat{k} \cdot \delta\vec{k})^2 + \dots \quad (14)$$

For pions with a normal dispersion relation $E^2 = k^2 + m^2$, the photon self-energy is

$$\text{Im} \Pi_{jj} = \frac{\alpha}{2} \theta(\omega^2 - 4m^2) \left[1 + 2n\left(\frac{\omega}{2}\right) \right] \frac{(\omega^2 - 4m^2)^{3/2}}{\omega}. \quad (15)$$

The result for dilepton production from moatons [Eq. (12)] obscures an interesting cancellation. For $\vec{p} = 0$, for the imaginary part of the loop integral, energy momentum conservation imposes

$$\delta(\omega^2 - 4m_{\text{eff}}^2) = \frac{M^2}{2k_M^2} \frac{\delta(k - k_\pm)}{|\hat{k} \cdot \delta\vec{k}|}, \quad (16)$$

where the numerator on the right-hand side arises from the Jacobian going from a δ function in ω to one in k . This factor would produce a Van Hove singularity, as arises for dilepton production from plasminos in the hard thermal loop approximation [121]. For moatons near threshold, however,

$$\Gamma^j(P, K) \approx 2iek^j \left[\frac{4k_M}{M^2} (\hat{k} \cdot \delta\vec{k}) \right] \quad (17)$$

vanishes as $\delta\vec{k} \rightarrow 0$, and there is no Van Hove singularity. Consequently, the dilepton rate turns on smoothly at threshold. Preliminary numerical computations of Eq. (2) at $\vec{p} \neq 0$ indicate that the threshold behavior with contributions from timelike moatons remains qualitatively similar. We will report on these investigations in a future publication.

The left panel in Fig. 2 illustrates three dispersion relations: normal, a shallow moat, and a deep moat; the

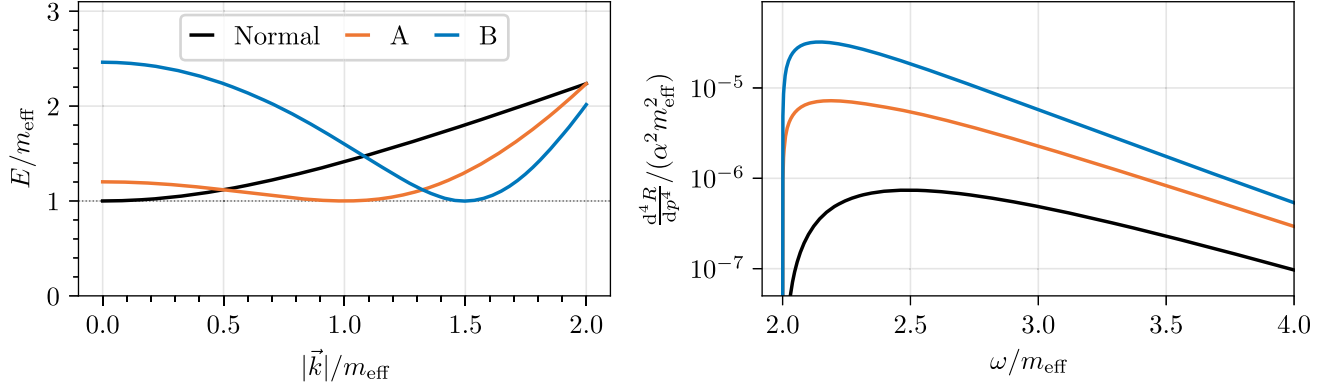


FIG. 2. Left: dispersion relations for various choices of Z and M in Eq. (9) and right: their corresponding dilepton production rate in Eq. (2) as a function of ω at $T = 0.5m_{\text{eff}}$. The normal dispersion relation has $Z = 1$, $M = \infty$, and $m_{\text{eff}} = m$. Both pions with a shallow moat (curve A, with $k_M/m_{\text{eff}} = 1.0$ and $M/m_{\text{eff}} = 1.5$) and pions with a deep moat (curve B, with $k_M/m_{\text{eff}} = 1.5$ and $M/m_{\text{eff}} = 1.0$) dramatically enhances dilepton production. Note, however, that the value of m_{eff} does change as T and μ vary. As discussed in the text, the terms C_{1-6} never contribute, while we set $C_7 = C_8 = 0$.

right panel illustrates their corresponding dilepton production rates. These curves have qualitatively similar shapes, but the dilepton rate is *significantly* enhanced for moatons, regardless of the moat depth. The moaton threshold occurs at a nonzero spatial momentum k_M with phase space $\sim k_M^2 dk$, whereas the normal pion threshold is at $k = 0$.

The detailed contribution of moatons to dilepton production is dependent upon effective models. Nevertheless, surely m_{eff} is of the order of m_π . Thus, moaton production should appear as an excess starting at energies $\sim 2m_\pi$, above the usual contributions from a broadened ρ_μ meson [116,117,122]. This should be a distinctive experimental signature. We note that there is also dilepton production near second-order transitions, which generates an excess starting from zero energy [98–100]. This is kinematically distinct; also, for the reasons discussed above, that from moatons is enhanced over such contributions.

Outlook—A large body of literature suggests that QCD probably exhibits a moat regime in a broad swath of the T - μ plane. In this Letter, we showed that enhancement of dilepton production is an important signature of a moat regime. Our principal shortcoming is that we cannot predict values for parameters like M and Z and, thus, k_M and m_{eff} . Importantly, the value of m_{eff} depends *strongly* upon T and μ and can be computed only in an effective model. We expect that M is a few times m_π , and m_{eff} is moderately less than m_π . Work is in progress on extracting these parameters using the FRG [123]. This shows that the moat regime is in the spacelike regime, with a significant width for the spectral density. In future work, we will extend our results to $p \neq 0$. It will be interesting to see if a Van Hove singularity, which is washed out at $\vec{p} = 0$ by the vertices in Eq. (17), emerges when $p \neq 0$. Inevitably, this singularity is smoothed out by the finite width of moatons.

Assuming that freeze-out occurs in the moat regime, moat signals from dilepton production and Hanbury Brown–Twiss interferometry [95,96] should be observable at the Beam Energy Scan Program at RHIC and CBM at FAIR.

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