

Giant Graviton Integrated Correlators at Finite Coupling and All Orders in $1/N$

Augustus Brown¹,[✉] Daniele Dorigoni^{2,*},[✉] and Congkao Wen¹

¹*Centre for Theoretical Physics, Department of Physics and Astronomy, [Queen Mary University of London](#), London, E1 4NS, United Kingdom*

²*Centre for Particle Theory and Department of Mathematical Sciences, [Durham University](#), Durham DH1 3LE, United Kingdom*



(Received 15 April 2026; accepted 10 July 2026; published 3 September 2026)

We study the giant graviton integrated correlator in $SU(N)$ $\mathcal{N} = 4$ super-Yang-Mills at finite complexified coupling τ . Despite the formidable complexity arising from the heavy nature of the operators considered, the large- N expansion simplifies dramatically and exhibits manifest modular invariance. At each order in $1/N$, the expansion coefficients are linear combinations of nonholomorphic Eisenstein series, thus capturing the full spectrum of perturbative and nonperturbative effects in the Yang-Mills coupling. Furthermore, we find additional contributions, which are modular functions exponentially suppressed in N . In the 't Hooft limit, this yields an all-orders result in the $1/N$ expansion at arbitrary coupling λ , extending beyond prior results of leading orders. For the $U(N)$ theory, we obtain a closed-form expression valid for all N and τ and show that the coupling-dependent sector of the large- N expansion is universal between $SU(N)$ and $U(N)$ to all orders. Crucially, we exploit the integrated correlator constraints and determine the giant graviton correlator itself to two-loop order at finite N , previously only accessible in the planar limit.

DOI: [10.1103/15r8-grqb](#)

There has been great interest in the study of correlation functions in $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory. Via the AdS/CFT correspondence, these correlators are holographically dual to superstring scattering amplitudes in AdS and have provided deep insights into superstring theory in various regimes. Traditionally, such quantities have been studied perturbatively in the large 't Hooft coupling expansion [1–8]; see also Refs. [9,10]. The introduction of integrated correlators [11,12] (see Ref. [13] for a review), combined with powerful arguments coming from S duality [14–18] and the mathematics of nonholomorphic modular functions, has since made it possible to access exact, nonperturbative properties of these physical observables [14,15,19–21].

In this Letter, we study correlators in $\mathcal{N} = 4$ SYM with gauge group $SU(N)$ and $U(N)$ involving two light superconformal primary operators in the stress-tensor multiplet and two heavy operators corresponding to sphere giant gravitons, whose conformal dimensions scale as N [22–25]. We refer to these quantities as giant graviton heavy-heavy-light-light (HHLL) correlators. The giant gravitons here considered are holographically dual to the D3-branes wrapping part of the underlying S^5 geometry.

One can also consider the D3-branes wrapping part of the AdS_5 geometry as discussed in Ref. [26]. These HHLL correlators can then be interpreted as scattering amplitudes of two gravitons off D3-branes, thus making these correlators of crucial importance for probing D-brane dynamics in AdS. From a quantum gravity point of view, giant gravitons probe intermediate scales sitting between perturbative (super)graviton states and black hole states. This fact has already been appreciated in the study of the superconformal index of $\mathcal{N} = 4$ SYM which admits an expansion as a sum over sectors with different numbers of giant gravitons [27,28]. The fluctuations in each sector have a semiclassical interpretation on the dual gravity side; see, e.g., Ref. [29]. Furthermore, since giant gravitons are constructed as determinant operators they can be thought as being part of the baryonic sector of $\mathcal{N} = 4$ SYM, closely related to baryons in large- N QCD [30] and their geometric AdS/CFT realization [31].

Compared to correlators of operators with fixed dimensions, much less is known about giant gravitons. Even in free theory, the calculation of giant graviton correlators already exhibits considerable combinatorial complexity due to the large dimension of such operators (see, e.g., Refs. [25,32–37]). Quantum corrections to giant graviton HHLL correlators have so far only been obtained in the leading large- N limit, where results have been derived at weak coupling up to two loops [35,38,39], while at strong coupling only the leading order is known [40,41].

Thanks to a clever use of supersymmetric localization and matrix model techniques, it was shown in Ref. [42] that one can go much further and obtain exact results for integrated giant graviton correlators in the 't Hooft large- N expansion.

*Contact author: daniele.dorigoni@durham.ac.uk

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

However, unlike integrated correlators of light operators which have been solved exactly at finite complexified coupling τ and arbitrary N [16,17,43], the giant graviton integrated correlator has proven to be much harder, and all existing results for this quantity have so far been restricted to the 't Hooft large- N limit [26,42].

In this Letter, we finally obtain exact solutions for giant graviton integrated correlators valid at generic Yang-Mills coupling τ , including all perturbative and nonperturbative effects and manifesting modular properties. Our results hold to all orders in the large- N expansion for the $SU(N)$ theory, while in the $U(N)$ case we provide a particularly simple expression valid for generic N .

What makes the final result more striking is that instanton effects in the presence of higher-dimensional operators are not known in general, and the dimension- N nature of the giant graviton means the localization computation grows rapidly in complexity with N . Since our results are exact in τ , they fix the 't Hooft large- N expansion to arbitrary order, extending the results of Ref. [42]. Our analysis provides exact constraints for the “unintegrated” giant graviton correlator. Utilizing these constraints, we determine the correlator up to two loops for general N , a quantity so far only known at large N .

Giant graviton integrated correlators—We study the HHLL correlator given by

$$T(x_i, Y_i) = \frac{\langle \mathcal{D}(x_1, Y_1) \mathcal{D}(x_2, Y_2) \mathcal{O}_2(x_3, Y_3) \mathcal{O}_2(x_4, Y_4) \rangle}{\langle \mathcal{D}(x_1, Y_1) \mathcal{D}(x_2, Y_2) \rangle}, \quad (1)$$

where x_i are spacetime coordinates and Y_i are $SO(6)_R$ null vectors. The giant-graviton determinant operator $\mathcal{D}$ and single-trace operators $\mathcal{O}_p$ are defined as

$$\mathcal{D}(x, Y) = \det[\Phi_I(x) Y^I], \quad (2)$$

$$\mathcal{O}_p(x, Y) = \text{tr}[\Phi_{I_1}(x) \cdots \Phi_{I_p}(x)] Y^{I_1} \cdots Y^{I_p}. \quad (3)$$

The operator $\mathcal{O}_p$ has scaling dimension $\Delta_p = p$, while the operator $\mathcal{D}$ has $\Delta_{\mathcal{D}} = N$ thus becoming heavy at large N . In this limit, $\mathcal{D}$ can be interpreted holographically as being dual to a D3-brane [22–25]. Superconformal invariance [44,45] constrains (1) to

$$T(x_i, Y_i) = \text{free part} + \mathcal{I}_4(x_i, Y_i) \mathcal{T}_N(U, V; \tau), \quad (4)$$

where the prefactor $\mathcal{I}_4(x_i, Y_i)$ is fixed by kinematics. The reduced correlator $\mathcal{T}_N(U, V; \tau)$ encodes all of the dynamics, depending on the rank of the gauge group N , the cross ratios, and most importantly the Yang-Mills coupling:

$$U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}, \quad (5)$$

$$\tau = \tau_1 + i\tau_2 = \frac{\theta}{2\pi} + i \frac{4\pi}{g_{\text{YM}}^2}. \quad (6)$$

The giant graviton *integrated* correlator, $\mathcal{C}_{\mathcal{D}}(\tau; N)$, is defined by integrating out the coordinate dependence of the reduced correlator against a supersymmetry-preserving measure whose explicit form is found in Refs. [11,12]. Importantly, this integrated correlator can also be computed from the S^4 partition function of $\mathcal{N} = 2^*$ SYM via

$$\mathcal{C}_{\mathcal{D}}(\tau; N) = \frac{\partial_{\mathcal{D}} \partial_{\bar{\mathcal{D}}} \partial_m^2 \log Z(\tau, \tau', m)}{\partial_{\mathcal{D}} \partial_{\bar{\mathcal{D}}} \log Z(\tau, \tau', m)} \Big|_{\tau'=0, m=0}. \quad (7)$$

Here $Z(\tau, \tau', m)$ denotes the $\mathcal{N} = 2^*$ sphere partition function deformed by higher-dimension operators $\mathcal{O}_p$ with $p \neq 2$ defined in (2) with couplings τ'_p , which we write concisely as τ' in $Z(\tau, \tau', m)$. From supersymmetric localization [46], this partition function reduces to the matrix model integral,

$$Z(\tau, \tau', m) = \int d\nu(a_i) |Z_{\text{class}}(a_i; \tau, \tau')|^2 Z_{1\text{-loop}}(a_i; m) \times |Z_{\text{inst}}(a_i; m, \tau, \tau')|^2. \quad (8)$$

The integration variables $a_i \in \mathbb{R}$ with $i = 1, \dots, N$ are unconstrained for the $U(N)$ gauge theory, while they satisfy $\sum_{i=1}^N a_i = 0$ for $SU(N)$. In both cases the integration measure is $d\nu(a_i) = d^N a \prod_{i < j} a_{ij}^2$.

To compute the giant graviton integrated correlator using localization matrix model as in (7) we need to insert the determinant operators via the action of $\partial_{\mathcal{D}}$ (and its conjugate $\partial_{\bar{\mathcal{D}}}$). In the matrix model, this is realized by expressing the determinant operator in terms of trace operators of dimension N , i.e., $(\text{tr } a^N, \text{tr } a^{N-1} \text{tr } a, \dots)$, as well as lower-dimensional operators which must be included via a Gram-Schmidt process [47], due to the fact that on S^4 operators with different dimensions mix. Trace operators can then be expressed in terms of derivatives with respect to τ' and τ . For example, for small values of N in $SU(N)$ we have

$$\begin{aligned} N=2: \partial_{\mathcal{D}} &= -\frac{1}{2} \frac{\partial_{\tau}}{i\pi} - \frac{3}{4}; & N=3: \partial_{\mathcal{D}} &= \frac{1}{3} \frac{\partial_{\tau_3}}{i\pi^{\frac{3}{2}}}; \\ N=4: \partial_{\mathcal{D}} &= -\frac{1}{4} \frac{\partial_{\tau_4}}{i\pi^2} + \frac{1}{8} \frac{\partial_{\tau}^2}{(i\pi)^2} - \frac{5}{16} \frac{\partial_{\tau}}{i\pi} + \frac{75}{64}. \end{aligned} \quad (9)$$

Similar expressions can be obtained for $U(N)$, although we stress that in the $U(N)$ theory one must also include the operator $\mathcal{O}_1$ via $\partial_{\tau'}$, vanishing otherwise in $SU(N)$. The complexity of $\partial_{\mathcal{D}}$ grows extremely rapidly with N , hence imposing further obstacle to obtaining exact results in the large- N expansion.

We now comment on the individual contributions appearing in (8). The one-loop determinant $Z_{1\text{-loop}}$ is not affected by the insertions of higher-dimensional operators

and therefore is independent of τ' ; the classical action $Z_{\text{classical}}$ does depend on the higher-dimensional coupling τ' , but in a trivial manner. Finally, the instanton contribution Z_{inst} [48] may be extracted from [49,50], and it depends on τ' with increasing complexity as the instanton number grows larger. Therefore, the action of the τ'_p -derivatives in $\partial_{\mathcal{D}}$, as in (9), on $Z_{\text{classical}}$ simply induces the insertion of $\text{tr} p$; however, its action on Z_{inst} is highly nontrivial, becoming unyielding at large N . In Supplemental Material [51] the reader can find expressions for $Z_{1\text{-loop}}$, $Z_{\text{classical}}$, and the one-instanton contribution to Z_{inst} .

Despite these difficulties, by exploiting S duality of $\mathcal{N} = 4$ SYM [58], we show that not only can we obtain $\mathcal{C}_{\mathcal{D}}(\tau; N)$ but that the final result takes an unexpectedly simple form to all orders in the large- N expansion at generic τ , containing all instanton contributions.

Exact solution for the $\text{SU}(N)$ giant graviton integrated correlator—For generic N and τ , as verified in many nontrivial examples, integrated correlators of half-BPS operators can be expressed in terms of a two-dimensional lattice-sum representation [16,17]. This formulation manifests several important properties of these correlators, in particular, their modular structures. A consequence of the lattice sum representation is that the $\text{SL}(2, \mathbb{Z})$ spectral decomposition of the integrated correlators is free of Maass cusp forms and therefore takes the form [59,60]

$$\mathcal{C}_{\mathcal{D}}(\tau; N) = C(N) + \int_{\text{Re } s = \frac{1}{2}} \frac{ds}{2\pi i} g_N(s) (2s-1)^2 E^*(s; \tau), \quad (10)$$

where $C(N) = \lim_{s \rightarrow 1} g_N(s)$ is independent of τ . The entire dependence from the coupling is carried by the non-holomorphic Eisenstein series $E^*(s; \tau) = E^*(1-s; \tau)$,

$$E^*(s; \tau) = \xi(2s)\tau_2^s + \xi(2s-1)\tau_2^{1-s} + \sum_{k \neq 0} e^{2\pi i k \tau_1} 2\sqrt{\tau_2} |k|^{s-\frac{1}{2}} \sigma_{1-2s}(k) K_{s-\frac{1}{2}}(2\pi |k| \tau_2), \quad (11)$$

where $\xi(s) = \pi^{-s/2} \Gamma(s/2) \zeta(s) = \xi(1-s)$ is the completed zeta function, $\sigma_s(n) = \sum_{d|n} d^s$ is the divisor sigma function, and $K_s(z)$ denotes a modified Bessel function of the second kind. The k th Fourier-mode corresponds to a k -instanton contribution. Equation (10) makes manifest the $\text{SL}(2, \mathbb{Z})$ invariance and contains all perturbative and instanton contributions. Importantly, the *spectral overlap* $g_N(s)$ is reflection invariant, i.e., $g_N(s) = g_N(1-s)$, and can be determined by comparing (10) with the purely perturbative expansion of the integrated correlator obtained from the supersymmetric localization matrix model, which takes the form

$$\mathcal{C}_{\mathcal{D}}(\tau; N)|_{\text{pert}} = N \frac{3\zeta(3)\lambda}{2\pi^2} - \left[\frac{3N(N+1)^2 + 2}{N(N+1)} + \frac{(N^2-1)(N+2)}{N(1-(-N)^{N+1})} \right] \frac{15\zeta(5)\lambda^2}{32\pi^4} + \dots, \quad (12)$$

where $\lambda = Ng_{\text{YM}}^2$. We see that the two-loop result naturally splits into two parts, one of which is proportional to $1/[1-(-N)^{N+1}]$ and is therefore exponentially suppressed in the large- N expansion. We have verified that the same structure persists at higher orders [61].

In view of this structure (12) (and its higher-order analogy), we express the spectral overlap as the sum of two contributions,

$$g_N(s) = g_N^{(1)}(s) + g_N^{(2)}(s). \quad (13)$$

Using (11), we evaluate the perturbative expansion of the spectral integral (10) and identify $g_N^{(1)}(s)$ and $g_N^{(2)}(s)$ respectively as the parts of the spectral overlap responsible for the rational contribution in N and the term proportional to $1/[1-(-N)^{N+1}]$ in the expansion (12).

By exploiting the matrix model (8), we obtain an exact result for $g_N^{(1)}(s) = f_N(s) + h_N(s)$, where

$$f_N(s) = \frac{N\pi}{\sin(\pi s)} {}_3F_2(1-N, 1-s, s; 2, 2; 1), \quad (14)$$

$$h_N(s) = \frac{\pi {}_2F_1\left(-N-1, -s; -N-s; -\frac{1}{N}\right) \left(\frac{N+s}{s}\right)}{\sin(\pi s) (1-2s) \left(1+\frac{1}{N}\right)^s} + (s \leftrightarrow 1-s). \quad (15)$$

Note that $f_N(s) = f_N(1-s)$ so that the partial spectral overlap is reflection invariant $g_N^{(1)}(s) = g_N^{(1)}(1-s)$. For $s \in \mathbb{N}$, we find that $g_N^{(2)}(s)$ has the following structure:

$$g_N^{(2)}(s) = \frac{(N-1)(N+2)(N+1)^{4-s}}{1-(-N)^{N+1}} \sum_{i=0}^{3s-9} c_i(s) N^i, \quad (16)$$

where $g_N^{(2)}(2) = 0$. While it is straightforward to compute $g_N^{(2)}(s)$ for a given $s \in \mathbb{N}$, a general closed-form expression remains out of reach. Nevertheless, the key observation is that $g_N^{(2)}(s)$ is responsible for producing the factor $1/[1-(-N)^{N+1}]$ as in (12), rendering it exponentially suppressed at large N —precisely the regime relevant for giant gravitons analyzed in the next section.

We have verified (10) through 12 loops in perturbation theory using $Z_{1\text{-loop}}$ via the “full algebra method” [62], and at the one-instanton level for all $N \leq 10$ using the one-instanton partition function, which can be found in Supplemental Material [51].

As emphasized, the result imposes exact integral constraints on the giant graviton correlator. Applying these using the perturbative expansion (12), together with additional OPE constraints coming from the LL and HH channels, we determine the reduced correlator to two-loop order for general N [63]:

$$\begin{aligned} \mathcal{T}_N(U, V; \tau)|_{\text{pert}} = & N x_{13}^4 x_{24}^4 \left[\frac{\lambda}{4\pi^2} f^{(1)}(x_i) - \frac{1}{8} \left(\frac{\lambda}{4\pi^2} \right)^2 \right. \\ & \times \left(f_2^{(2)}(x_i) - f_1^{(2)}(x_i) \right) \\ & \left. + c(N) f_1^{(2)}(x_i) \right] + O(\lambda^3), \end{aligned} \quad (17)$$

where the color factor $c(N)$ contains the nonplanar and exponentially suppressed contributions

$$c(N) = \frac{3N(N+1)+2}{N^2(N+1)} + \frac{(N^2-1)(N+2)}{N^2[1-(-N)^{N+1}]}. \quad (18)$$

The $f_a^{(\ell)}(x_i)$ are f -graph functions introduced in Refs. [65,66]. In Supplemental Material [51], the reader can find explicit expressions for the one-loop box integral $f^{(1)}(x_i)$ and the two-loop double-box and squared one-loop box integrals contained in $f_1^{(2)}(x_i)$ and $f_2^{(2)}(x_i)$. At large N , $c(N)$ is suppressed and (17) agrees with Refs. [35,39]. Furthermore, for $N=2$ and $N=3$ the result reproduces the known correlators $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$ for SU(2) and $\langle \mathcal{O}_3 \mathcal{O}_3 \mathcal{O}_2 \mathcal{O}_2 \rangle$ for SU(3) [66,67]. We refer to Supplemental Material [51] for more details.

Giant graviton integrated correlator at large N —We now derive the large- N expansion of the integrated correlator at fixed coupling τ . As discussed, $g_N^{(2)}(s)$ in (13) is exponentially suppressed, and we omit its contribution in this limit. Hence we focus our attention solely toward $g_N^{(1)}(s)$. Using Eqs. (14) and (15) we compute the large- N expansions for the building blocks,

$$f_N(s) = \sum_{\ell=0}^{\infty} N^{1-s-\ell} f^{(\ell)}(s) + (s \rightarrow 1-s), \quad (19)$$

$$h_N(s) = \sum_{\ell=0}^{\infty} N^{1-s-\ell} h^{(\ell)}(s) + (s \rightarrow 1-s), \quad (20)$$

where we find

$$f^{(\ell)}(s) = \frac{\tan(\pi s) \Gamma(s-1)^2}{2^{2s+\left\lfloor \frac{\ell}{2} \right\rfloor} \sqrt{\pi} \Gamma\left(s + \left\lfloor \frac{\ell}{2} \right\rfloor + \frac{1}{2}\right)} P_1^{(\ell)}(s), \quad (21)$$

$$h^{(\ell)}(s) = \frac{\Gamma(s-1)}{(1-2s)} P_2^{(\ell)}(s), \quad (22)$$

with $P_1^{(\ell)}(s), P_2^{(\ell)}(s)$ being simple polynomials in s , e.g.,

$$P_1^{(0)}(s) = 1, \quad P_1^{(1)}(s) = \frac{1-s}{2}, \quad (23)$$

$$P_2^{(0)}(s) = 1, \quad P_2^{(1)}(s) = \frac{(s-1)(s-2)}{2}. \quad (24)$$

Note that, since the spectral integral (10) and the Eisenstein series (11) are invariant under $s \leftrightarrow 1-s$, we omit the reflected part ($s \rightarrow 1-s$) in (19) and (20) and multiply the terms presented by a factor of 2.

Substituting the large- N expansions (19) and (20) in (10), we evaluate the spectral integral by closing the contour of integration toward $\text{Res} \rightarrow +\infty$ while collecting residues using Eqs. (21) and (22). We obtain in this way the modular invariant large- N expansion for the giant graviton integrated correlator,

$$\mathcal{C}_D(\tau; N) \sim \tilde{C}(N) - \hat{E}(1; \tau) - \sum_{\ell=1}^{\infty} N^{\frac{1}{2}-\ell} \mathcal{C}_D^{(\ell)}(\tau). \quad (25)$$

Some further comments on the derivation of the above result are in order. First, at any given order in large N we find that the spectral overlap has finitely many poles located at half-integer values of s . This leads to expansion coefficients which are linear combinations of Eisenstein series with half-integer indices,

$$\mathcal{C}_D^{(\ell)}(\tau) = \sum_{k=1}^{\ell} c_k^{(\ell)} E\left(k + \frac{1}{2}; \tau\right), \quad (26)$$

where $E(s; \tau) = 2E^*(s; \tau)/\Gamma(s)$, with $E^*(s; \tau)$ given in (11). The coefficients $c_k^{(\ell)}$ are rational numbers, with examples given in Supplemental Material [51].

The residue at $s=1$ has to be treated carefully. For $\ell=0$, we find a double pole at $s=1$ which leads to the term $\hat{E}(1; \tau)$ in (25), which is a regularized version of the singular Eisenstein series $E(1; \tau)$, i.e.,

$$\begin{aligned} \hat{E}(1; \tau) &= \lim_{s \rightarrow 1} \left[E(s; \tau) - \frac{1}{s-1} \right] \\ &= \frac{\pi \tau_2}{3} + 2\gamma - \log(4\pi \tau_2) + 2 \sum_{k \neq 0} e^{2\pi i k \tau_1} \sum_{m|k} \frac{1}{m}, \end{aligned} \quad (27)$$

where γ is the Euler-Mascheroni constant. For $\ell \neq 0$, the residue at $s=1$ yields a coupling-independent constant which, when combined with $C(N)$ in (10), leads to the large- N expansion of the coupling-independent part,

$$\tilde{C}(N) = 2N - \log(N) - 2 + \sum_{k=1}^{\infty} \frac{2}{N^k} \left[\frac{B_k}{2k} - (-1)^k \right], \quad (28)$$

where B_k denotes the k th Bernoulli number.

These results have important holographic implications. At large N and fixed τ , since $1/\sqrt{N} \sim \alpha'$, each term appearing in (25) corresponds to stringy corrections to two-graviton scattering in the presence of D3-branes in $\text{AdS}_5 \times S^5$ and provides constraints on the amplitudes. In particular, the functions $\hat{E}(1; \tau)$ and $E(\frac{3}{2}; \tau)$ are associated with the BPS higher-derivative terms R^2 and $D^2 R^2$ and are in agreement with EFT expectations due to supersymmetry [68–70]. The leading term $2N$ of the coupling independent part $\tilde{C}(N)$ reproduces the tree-level supergravity contribution [40], while the subleading terms warrant further holographic study.

We stress that the spectral integral (10) receives additional modular invariant contributions from the large- s region, which are however exponentially suppressed in N [71]. We find two different types of such nonperturbative corrections, which at leading order are

$$\mathcal{C}_{\mathcal{D}}(\tau; N)|_{\text{NP}} = -2N^{\frac{3}{4}}\pi^{\frac{1}{2}}D_N\left(\frac{1}{4}; \tau\right) \pm i4N^{\frac{1}{2}}D_N\left(\frac{1}{2}; \tau\right) + \dots \quad (29)$$

The real-analytic modular function $D_N(s; \tau)$ first introduced in Ref. [43] (see also Ref. [72]) is defined as

$$D_N(s; \tau) = \sum_{(n,m) \neq (0,0)} e^{-4\sqrt{N\pi\frac{|n\tau+m|^2}{\tau_2}} \frac{\tau_2^s/(16\pi)^s}{|n\tau+m|^{2s}}}. \quad (30)$$

The nonperturbative terms in (29) proportional to $D_{N/4}(s; \tau)$ give rise to the leading corrections, and they originate from the large- s asymptotic of the spectral integral for $h_N(s)$ as in (22). The subleading corrections in (29) proportional to $D_N(s; \tau)$ instead come from $f_N(s)$ as in (21). In Supplemental Material [51] we give higher-order corrections to (29). The seeming $\pm i$ ambiguity in (29) reflects the non-Borel summability of the asymptotic expansion (25), and the nonperturbative corrections (29) are precisely the ambiguity-cancelling contributions required for a well-defined resurgent completion [71].

The usual 't Hooft large- N expansion at fixed λ is readily extracted from (25) by using $\tau_2 = 4\pi N/\lambda$ and keeping only the zero mode of the nonholomorphic Eisenstein series (11). Explicitly, in the large- N 't Hooft regime the giant graviton integrated correlator is written as

$$\mathcal{C}_{\mathcal{D}}(\tau; N) \sim \mathcal{I}(\lambda) = \sum_{\ell=0}^{\infty} N^{1-\ell} \mathcal{I}^{(\ell)}(\lambda), \quad (31)$$

where the genus- ℓ contribution is given by

$$\mathcal{I}^{(\ell)}(\lambda) = \int_{\mathcal{C}} \frac{ds}{\pi i} (2s-1)^2 [f^{(\ell)}(s) + h^{(\ell)}(s)] \xi(2s) \left(\frac{\lambda}{4\pi^2} \right)^{-s}. \quad (32)$$

The contour of integration $\mathcal{C}$ is chosen as to separate the poles located at $-s \in \mathbb{N}^{\geq 1}$, whose contributions correspond to the weak coupling expansion, from all remaining poles, located at $s = 0$ and $s = k + 1/2$ with $k \in \mathbb{Z}$ and $k \geq -[\ell/2]$, which instead contribute to the strong coupling expansion. The expression (32) provides the 't Hooft large- N expansion of the giant graviton integrated correlator to all orders. In particular, we reproduce the first two orders obtained in [42], namely terms with $\ell = 0, 1$. In Supplemental Material [51] we present several new higher-order terms.

Importantly, from (22) we note that the integrand of (32) proportional to $h^{(\ell)}(s)$ is analytic for $\text{Re}(s) > 2$. Hence for strong 't Hooft coupling, the corresponding contribution to the Mellin representation (32) will produce only a finite number of $1/\lambda$ corrections, and it will be otherwise responsible for the leading nonperturbative corrections of the form $O(e^{-\sqrt{\lambda}})$. These corrections have been studied at genus-zero in [26] for a more general class of giant graviton correlators.

Such nonperturbative effects arise precisely by considering the 't Hooft limit of the modular invariant functions $D_{N/4}(s; \tau)$ appearing in (29). The integrand of (32) proportional to $f^{(\ell)}(s)$ is instead responsible for smaller nonperturbative corrections of the form $O(e^{-2\sqrt{\lambda}})$ which coincide with the 't Hooft limit of the modular invariant functions $D_N(s; \tau)$ appearing in (29). Holographically, these contributions appear to be related to (p, q) -string world-sheet instantons [43].

Exact solution for the $U(N)$ giant graviton integrated correlator—We now consider the $U(N)$ theory. Unlike what we found for the $SU(N)$ theory, the $U(N)$ giant graviton integrated correlator does not receive $O(N^{-N})$ -type contributions. This absence is what allows us to obtain an exact closed-form solution valid for all N and τ . Following the same procedure carried out for $SU(N)$, we derive a spectral representation identical to (10), with spectral overlap given by

$$\tilde{g}_N(s) = f_N(s) + \frac{\pi \left[\binom{N+1-s}{1-s} - \binom{N+s}{s} \right]}{(2s-1) \sin(\pi s)}, \quad (33)$$

where $f_N(s)$ was given in (14), and we note that the second factor is again symmetric in $s \leftrightarrow 1-s$. This simple expression provides the complete result for the giant graviton integrated correlator in the $U(N)$ theory, and we have explicitly verified the expression perturbatively and at the one-instanton level.

The large- N expansion for the second contribution to (33) is easy to derive:

$$\frac{\pi \binom{N+1-s}{1-s}}{(2s-1) \sin(\pi s)} = \sum_{\ell=0}^{\infty} N^{1-s-\ell} \frac{\Gamma(s-1)}{(1-2s)} \tilde{P}^{(\ell)}(s), \quad (34)$$

and it is precisely of the same form as that of $h_N(s)$ given in (20)–(22). The polynomials $\tilde{P}^{(\ell)}(s)$ with $\ell = 0, 1$ in fact coincide with the corresponding $SU(N)$ results (24), and furthermore, just as for the expansion of $h_N(s)$ in $SU(N)$, we note that at large N each order of (34) is analytic for $\text{Re}(s) > 1$. As explained below (25), from the large- N expansion of (33) we evaluate the analogous spectral overlap integral (10) for the $U(N)$ giant graviton correlator by closing the s contour of integration to $\text{Re}(s) \rightarrow +\infty$. As a consequence of these facts, we find that at large N the leading and subleading orders of the $SU(N)$ and $U(N)$ theories are identical, in agreement with the results of [42] in the large- N 't Hooft expansion. At higher orders, the polynomials $\tilde{P}^{(\ell)}(s)$ generally differ between the two theories; however, these differences affect only the coupling-independent constants and exponentially suppressed terms.

In terms of 't Hooft expansions, this means that at any given genus the $SU(N)$ and $U(N)$ giant graviton integrated correlators are identical to all orders in the strong-coupling expansion, up to λ -independent constants. This suggests that certain coupling-dependent dynamics of giant graviton correlators are insensitive to the difference between $SU(N)$ and $U(N)$ gauge groups, which constitutes another universal property of giant graviton integrated correlators, analogous to those uncovered in [26,73].

Conclusion and discussion—In this Letter, we obtain exact solutions in the Yang-Mills coupling τ for the giant graviton integrated correlator in $\mathcal{N} = 4$ SYM, to all orders in the large- N expansion for gauge group $SU(N)$ and for arbitrary N in $U(N)$, via the integral representation (10) with spectral overlaps given in Eqs. (14) and (15) for $SU(N)$ and in Eq. (33) for $U(N)$. Despite the formidable complexity of the underlying computation, the result takes a remarkably simple form, making $SL(2, \mathbb{Z})$ invariance manifest. From our results, we determine at weak coupling the giant graviton HHLL correlator to the two-loop order for arbitrary N . We also reveal that the coupling-dependent part of the large- N strong 't Hooft coupling expansion is universal between $SU(N)$ and $U(N)$ theories to all orders. Holographically, our large- N analysis at finite τ provides exact constraints on two-graviton scattering from a D3-brane in $AdS_5 \times S^5$ at finite string coupling.

Our results open up several interesting directions for future work. An immediate question is to determine the $O(N^{-N})$ contributions in the $SU(N)$ case and, perhaps more importantly, to understand their holographic interpretation. This may be achievable by finding certain recursion relations like Laplace-difference equations of Refs. [16,17], which would also provide a pathway to rigorously prove the results for any number of instantons following the ideas of Ref. [43]. Another direction is to extend our analysis to

subdeterminant operators and dual giant gravitons for which presently only the leading large- N 't Hooft expansion is known [26], and the associated modular functions have yet to be understood. These results would also allow for studying the intriguing relation between sphere giant gravitons and AdS giant gravitons via the map $N \leftrightarrow -N$, as found in Ref. [26], beyond the planar limit [74]. Moreover, our all-orders results make it possible to investigate whether the universality between the giant graviton integrated correlators here discussed and those in certain $\mathcal{N} = 2$ SCFTs [73,75], currently established at the first two orders at large N , extends to higher orders. Finally, it would be interesting to generalize our results to other gauge groups, where the action of S duality on the correlators is even more nontrivial [76]. In these cases, analogous lattice-sum representations have been identified only for integrated correlators of light operators [77,78], and it would be valuable to extend this analysis to include giant gravitons.

Acknowledgments—We would like to thank Mikhail Bershtein, Giulio Bonelli, Frank Coronado, Stefano Cremonesi, Francesco Galvagno, Song He, Masazumi Honda, Sanjaye Ramgoolam, Canxin Shi, Yichao Tang, Alessandro Tanzini, and Mitchell Woolley for insightful discussions. D. D. is supported by the Royal Society Grants No. ICA\R2\242058 and No. IEC\R3\243103. C. W. is supported by a Royal Society University Research Fellowship No. UF160350 and an STFC Consolidated Grant No. ST/T000686/1 “Amplitudes, strings and duality.”

Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

-
- [1] Daniel Z. Freedman, Samir D. Mathur, Alec Matusis, and Leonardo Rastelli, Correlation functions in the $CFT(d)/AdS(d+1)$ correspondence, *Nucl. Phys.* **B546**, 96 (1999).
 - [2] Leonardo Rastelli and Xinan Zhou, Mellin amplitudes for $AdS_5 \times S^5$, *Phys. Rev. Lett.* **118**, 091602 (2017).
 - [3] F. Aprile, J.M. Drummond, P. Heslop, and H. Paul, Quantum gravity from conformal field theory, *J. High Energy Phys.* **01** (2018) 035.
 - [4] Simon Caron-Huot and Anh-Khoi Trinh, All tree-level correlators in $AdS_5 \times S_5$ supergravity: Hidden ten-dimensional conformal symmetry, *J. High Energy Phys.* **01** (2019) 196.
 - [5] Luis F. Alday and Xinan Zhou, Simplicity of AdS supergravity at one loop, *J. High Energy Phys.* **09** (2020) 008.
 - [6] Luis F. Alday and Tobias Hansen, The AdS Virasoro-Shapiro amplitude, *J. High Energy Phys.* **10** (2023) 023.
 - [7] Francesco Aprile, Stefano Giusto, and Rodolfo Russo, Holographic correlators with BPS bound states in $\mathcal{N} = 4$ SYM, *Phys. Rev. Lett.* **134**, 091602 (2025).

- [8] Bo Wang, Di Wu, and Ellis Ye Yuan, Kaluza-Klein AdS Virasoro-Shapiro amplitude near flat space, *Phys. Rev. Lett.* **135**, 041603 (2025).
- [9] Paul Heslop, The SAGEX review on scattering amplitudes, chapter 8: Half BPS correlators, *J. Phys. A* **55**, 443009 (2022).
- [10] Agnese Bissi, Aninda Sinha, and Xinan Zhou, Selected topics in analytic conformal bootstrap: A guided journey, *Phys. Rep.* **991**, 1 (2022).
- [11] Damon J. Binder, Shai M. Chester, Silviu S. Pufu, and Yifan Wang, $\mathcal{N} = 4$ Super-Yang-Mills correlators at strong coupling from string theory and localization, *J. High Energy Phys.* **12** (2019) 119.
- [12] Shai M. Chester and Silviu S. Pufu, Far beyond the planar limit in strongly-coupled $\mathcal{N} = 4$ SYM, *J. High Energy Phys.* **01** (2021) 103.
- [13] Daniele Dorigoni, Michael B. Green, and Congkao Wen, The SAGEX review on scattering amplitudes chapter 10: Selected topics on modular covariance of type IIB string amplitudes and their supersymmetric Yang-Mills duals, *J. Phys. A* **55**, 443011 (2022).
- [14] Shai M. Chester, Michael B. Green, Silviu S. Pufu, Yifan Wang, and Congkao Wen, Modular invariance in superstring theory from $\mathcal{N} = 4$ super-Yang-Mills, *J. High Energy Phys.* **11** (2020) 016.
- [15] Shai M. Chester, Michael B. Green, Silviu S. Pufu, Yifan Wang, and Congkao Wen, New modular invariants in $\mathcal{N} = 4$ Super-Yang-Mills theory, *J. High Energy Phys.* **04** (2021) 212.
- [16] Daniele Dorigoni, Michael B. Green, and Congkao Wen, Exact properties of an integrated correlator in $\mathcal{N} = 4$ SU(N) SYM, *J. High Energy Phys.* **05** (2021) 089.
- [17] Daniele Dorigoni, Michael B. Green, and Congkao Wen, Novel representation of an integrated correlator in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, *Phys. Rev. Lett.* **126**, 161601 (2021).
- [18] Luis F. Alday, Shai M. Chester, Daniele Dorigoni, Michael B. Green, and Congkao Wen, Relations between integrated correlators in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, *J. High Energy Phys.* **05** (2024) 044.
- [19] Shai M. Chester, Ross Dempsey, and Silviu S. Pufu, Bootstrapping $\mathcal{N} = 4$ super-Yang-Mills on the conformal manifold, *J. High Energy Phys.* **01** (2023) 038.
- [20] Simon Caron-Huot, Frank Coronado, and Zahra Zahraee, Bootstrapping $\mathcal{N} = 4$ sYM correlators using integrability and localization, *J. High Energy Phys.* **05** (2025) 220.
- [21] Ross Dempsey, Robin Karlsson, Silviu S. Pufu, Zahra Zahraee, and Alexander Zhiboedov, Conformal collider bootstrap in $\mathcal{N} = 4$ SYM, [arXiv:2512.10796](https://arxiv.org/abs/2512.10796).
- [22] John McGreevy, Leonard Susskind, and Nicolaos Toumbas, Invasion of the giant gravitons from anti-de Sitter space, *J. High Energy Phys.* **06** (2000) 008.
- [23] Akikazu Hashimoto, Shinji Hirano, and N. Izhaki, Large branes in AdS and their field theory dual, *J. High Energy Phys.* **08** (2000) 051.
- [24] Vijay Balasubramanian, Micha Berkooz, Asad Naqvi, and Matthew J. Strassler, Giant gravitons in conformal field theory, *J. High Energy Phys.* **04** (2002) 034.
- [25] Steve Corley, Antal Jevicki, and Sanjaye Ramgoolam, Exact correlators of giant gravitons from dual $N = 4$ SYM theory, *Adv. Theor. Math. Phys.* **5**, 809 (2002).
- [26] Augustus Brown, Daniele Dorigoni, Francesco Galvagno, and Congkao Wen, Universality of giant graviton correlators, *J. High Energy Phys.* **11** (2025) 034.
- [27] Yosuke Imamura, Finite-N superconformal index via the AdS/CFT correspondence, *Prog. Theor. Exp. Phys.* **2021**, 123B05 (2021).
- [28] Davide Gaiotto and Ji Hoon Lee, The giant graviton expansion, *J. High Energy Phys.* **08** (2024) 025.
- [29] Giorgos Eleftheriou, Sameer Murthy, and Martí Rosselló, The giant graviton expansion in $\text{AdS}_5 \times S^5$, *SciPost Phys.* **17**, 098 (2024).
- [30] Edward Witten, Baryons in the $1/N$ expansion, *Nucl. Phys.* **B160**, 57 (1979).
- [31] Edward Witten, Baryons and branes in anti-de Sitter space, *J. High Energy Phys.* **07** (1998) 006.
- [32] Robert de Mello Koch and Rhiannon Gwyn, Giant graviton correlators from dual SU(N) super Yang-Mills theory, *J. High Energy Phys.* **11** (2004) 081.
- [33] Yusuke Kimura and Sanjaye Ramgoolam, Branes, anti-branes and Brauer algebras in gauge-gravity duality, *J. High Energy Phys.* **11** (2007) 078.
- [34] A. Bissi, C. Kristjansen, D. Young, and K. Zoubos, Holographic three-point functions of giant gravitons, *J. High Energy Phys.* **06** (2011) 085.
- [35] Yunfeng Jiang, Shota Komatsu, and Edoardo Vescovi, Structure constants in $\mathcal{N} = 4$ SYM at finite coupling as worldsheet g-function, *J. High Energy Phys.* **07** (2020) 037.
- [36] Yunfeng Jiang, Shota Komatsu, and Edoardo Vescovi, Exact three-point functions of determinant operators in planar $N = 4$ supersymmetric Yang-Mills theory, *Phys. Rev. Lett.* **123**, 191601 (2019).
- [37] Adolfo Holguin and Wayne W. Weng, Orbit averaging coherent states: Holographic three-point functions of AdS giant gravitons, *J. High Energy Phys.* **05** (2023) 167.
- [38] Yu Wu, Yunfeng Jiang, Chang Liu, and Yang Zhang, All giant graviton two-point functions at two-loops, *J. High Energy Phys.* **03** (2026) 097.
- [39] Yunfeng Jiang, Yu Wu, and Yang Zhang, Giant correlators at quantum level, *J. High Energy Phys.* **05** (2024) 345.
- [40] Junding Chen, Yunfeng Jiang, and Xinan Zhou, Giant graviton correlators as defect systems, *Phys. Rev. Lett.* **135**, 081602 (2025).
- [41] Junding Chen, Yunfeng Jiang, and Xinan Zhou, Defect approach to giant graviton dynamics, *Rep. Prog. Phys.* **89**, 067801 (2026).
- [42] Augustus Brown, Francesco Galvagno, and Congkao Wen, Exact results for giant graviton four-point correlators, *J. High Energy Phys.* **07** (2024) 049.
- [43] Daniele Dorigoni, Michael B. Green, Congkao Wen, and Haitian Xie, Modular-invariant large- N completion of an integrated correlator in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, *J. High Energy Phys.* **04** (2023) 114.
- [44] Burkhard Eden, Anastasios C. Petkou, Christian Schubert, and Emery Sokatchev, Partial nonrenormalization of the stress tensor four point function in $N = 4$ SYM and AdS/CFT, *Nucl. Phys.* **B607**, 191 (2001).
- [45] M. Nirschl and H. Osborn, Superconformal Ward identities and their solution, *Nucl. Phys.* **B711**, 409 (2005).

- [46] Vasily Pestun, Localization of gauge theory on a four-sphere and supersymmetric Wilson loops, *Commun. Math. Phys.* **313**, 71 (2012).
- [47] Efrat Gerchkovitz, Jaume Gomis, Nafiz Ishtiaque, Avner Karasik, Zohar Komargodski, and Silviu S. Pufu, Correlation functions of Coulomb branch operators, *J. High Energy Phys.* **01** (2017) 103.
- [48] Nikita A. Nekrasov, Seiberg-Witten prepotential from instanton counting, *Adv. Theor. Math. Phys.* **7**, 831 (2003).
- [49] Lothar Gottsche, Hiraku Nakajima, and Kota Yoshioka, Instanton counting and Donaldson invariants, *J. Diff. Geom.* **80**, 343 (2008).
- [50] Giulio Bonelli, Antonio Sciarappa, Alessandro Tanzini, and Petr Vasko, Six-dimensional supersymmetric gauge theories, quantum cohomology of instanton moduli spaces and $\mathfrak{gl}(N)$ quantum intermediate long wave hydrodynamics, *J. High Energy Phys.* **07** (2014) 141.
- [51] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/15r8-grqb> for details of the sphere partition function for $\mathcal{N} = 2^*$ SYM deformed by higher-dimensional operators; further results for the large- N expansion of the giant graviton integrated correlator, which includes Ref. [52]; and details of the giant graviton correlator at two loops for generic number of colors N , which includes Refs. [53–57].
- [52] Daniele Dorigoni, An introduction to resurgence, trans-series and alien calculus, *Ann. Phys. (Amsterdam)* **409**, 167914 (2019).
- [53] N. I. Usyukina and Andrei I. Davydychev, Exact results for three and four point ladder diagrams with an arbitrary number of rungs, *Phys. Lett. B* **305**, 136 (1993).
- [54] Congkao Wen and Shun-Qing Zhang, Integrated correlators in $\mathcal{N} = 4$ super Yang-Mills and periods, *J. High Energy Phys.* **05** (2022) 126.
- [55] Augustus Brown, Paul Heslop, Congkao Wen, and Haitian Xie, Integrated correlators in $\mathcal{N} = 4$ SYM beyond localisation, *Phys. Rev. Lett.* **132**, 101602 (2024).
- [56] V. V. Belokurov and N. I. Usyukina, Calculation of ladder diagrams in arbitrary order, *J. Phys. A* **16**, 2811 (1983).
- [57] F. Fiamberti, A. Santambrogio, C. Sieg, and D. Zanon, Wrapping at four loops in $N = 4$ SYM, *Phys. Lett. B* **666**, 100 (2008).
- [58] C. Montonen and David I. Olive, Magnetic monopoles as Gauge particles?, *Phys. Lett.* **72B**, 117 (1977).
- [59] Scott Collier and Eric Perlmutter, Harnessing S-duality in $\mathcal{N} = 4$ SYM & supergravity as $SL(2, \mathbb{Z})$ -averaged strings, *J. High Energy Phys.* **08** (2022) 195.
- [60] Hynek Paul, Eric Perlmutter, and Himanshu Raj, Integrated correlators in $\mathcal{N} = 4$ SYM via $SL(2, \mathbb{Z})$ spectral theory, *J. High Energy Phys.* **01** (2023) 149.
- [61] It can be seen from the matrix model through the small coupling expansion of $Z_{1\text{-loop}}$ that the $O(\lambda^1)$ term contains only $\text{tr} a^2$, which implies that this contribution can be obtained by $\tau_2 \partial_{\tau_2}$ acting on the two-point function of giant gravitons (then normalized by the two-point function itself), so the complicated color factors arising from giant gravitons cancel out. Therefore there is no exponentially suppressed contribution for the $\zeta(3)$ term in (12).
- [62] M. Billo, F. Fucito, A. Lerda, J. F. Morales, Ya. S. Stanev, and Congkao Wen, Two-point correlators in $N = 2$ gauge theories, *Nucl. Phys.* **B926**, 427 (2018).
- [63] See Ref. [64] for the study of determinant operators related to the one here considered at finite N from a twistor space approach.
- [64] Simon Caron-Huot, Frank Coronado, and Beatrix Mühlmann, Determinants in self-dual $\mathcal{N} = 4$ SYM and twistor space, *J. High Energy Phys.* **08** (2023) 008.
- [65] Burkhard Eden, Paul Heslop, Gregory P. Korchemsky, and Emery Sokatchev, Hidden symmetry of four-point correlation functions and amplitudes in $N = 4$ SYM, *Nucl. Phys.* **B862**, 193 (2012).
- [66] Burkhard Eden, Paul Heslop, Gregory P. Korchemsky, and Emery Sokatchev, Constructing the correlation function of four stress-tensor multiplets and the four-particle amplitude in $N = 4$ SYM, *Nucl. Phys.* **B862**, 450 (2012).
- [67] Dmitry Chicherin, James Drummond, Paul Heslop, and Emery Sokatchev, All three-loop four-point correlators of half-BPS operators in planar $\mathcal{N} = 4$ SYM, *J. High Energy Phys.* **08** (2016) 053.
- [68] Constantin P. Bachas, Pascal Bain, and Michael B. Green, Curvature terms in D-brane actions and their M theory origin, *J. High Energy Phys.* **05** (1999) 011.
- [69] Anirban Basu, Constraining the D3-brane effective action, *J. High Energy Phys.* **09** (2008) 124.
- [70] Ying-Hsuan Lin, Shu-Heng Shao, Xi Yin, and Yifan Wang, Higher derivative couplings in theories with sixteen supersymmetries, *Phys. Rev. D* **92**, 125017 (2015).
- [71] Daniele Dorigoni and Rudolfs Treilis, Large- N integrated correlators in $\mathcal{N} = 4$ SYM: When resurgence meets modularity, *J. High Energy Phys.* **07** (2024) 235.
- [72] Conghuan Luo and Yifan Wang, Casimir energy and modularity in higher-dimensional conformal field theories, *J. High Energy Phys.* **07** (2023) 028.
- [73] L. De Lillo, Z. Duan, M. Frau, F. Galvagno, A. Lerda, P. Vallarino, and C. Wen, $\mathcal{N} = 2$ universality at strong coupling, *J. High Energy Phys.* **02** (2026) 019.
- [74] Preliminary calculations for general sphere giant gravitons with dimension αN and AdS giant gravitons with dimension βN suggest that the spectral overlap can again be separated into two parts for $SU(N)$ theory. The first part, valid at all orders in $1/N$, relates the sphere giant gravitons and AdS giant gravitons by $N \leftrightarrow -N$ at a given s , while the exponentially suppressed parts are not related to each other in this simple manner. For $U(N)$ theory, there are no $O(N^{-N})$ contributions, and the spectral overlap for sphere giant gravitons is fully related to the AdS giant graviton spectral overlap by $N \leftrightarrow -N$.
- [75] Shai M. Chester, Pietro Ferrero, and Daniele R. Pavarini, Modular invariant gluon-graviton scattering in AdS at one loop, *J. High Energy Phys.* **08** (2025) 208.
- [76] P. Goddard, J. Nuyts, and David I. Olive, Gauge theories and magnetic charge, *Nucl. Phys.* **B125**, 1 (1977).
- [77] Daniele Dorigoni, Michael B. Green, and Congkao Wen, Exact results for duality-covariant integrated correlators in $\mathcal{N} = 4$ SYM with general classical gauge groups, *SciPost Phys.* **13**, 092 (2022).
- [78] Daniele Dorigoni and Paolo Vallarino, Exceptionally simple integrated correlators in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, *J. High Energy Phys.* **09** (2023) 203.