Anyon Bound States and Hybrid Superconductivity

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The interactions of anyonic quasiparticles (vortices) in a Chern-Simons-Landau-Ginzburg theory of the fractional quantum Hall effect is investigated and we show that it manifestly realizes a hybridization of type I or II superconductivity. Through Gauss's law, each vortex simultaneously carries a flux quantum and a proportional Noether charge, thereby realizing an anyonic excitation. The Chern-Simons coupling modifies the screening structure of the gauge fields, producing complex-conjugate masses that yield a common magnetic and electric penetration depth with an oscillatory phase. This altered asymptotic behavior breaks the conventional type-I and type-II dichotomy of the Ginzburg-Landau model, thereby enhancing the superconducting typology within a single-component condensate. As a result, vortex anyons experience short-range repulsion and long-range attraction, enabling the formation of separated multivortex bound states with nonmonotonic interaction energy. This provides a minimal topological route to hybrid superconductivity without multicomponent order parameters or nonlocal interactions.

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Introduction—Topological phases of matter in two spatial dimensions support exotic excitations known as anyons: quasiparticles with fractional statistics interpolating between bosons and fermions. Anyons emerge in diverse condensed matter systems, including the fractional quantum Hall effect (FQHE), where electric charge-magnetic flux attachment generates excitations of fractional charge [1–3], and in spinor Bose-Einstein condensates where vortex braiding realizes anyonic exchange statistics [4]. A particularly striking proposal is anyon superconductivity [5,6], in which anyons condense to form a novel superconducting state.

A powerful mean field-theoretic framework for realizing anyon superconductivity is the Chern-Simons-Landau-Ginzburg (CSLG) model. This theory extends the Ginzburg-Landau (GL) model by the addition of a Chern-Simons (CS) term, which enforces a magnetic flux-charge relation. The resulting vortices not only carry quantized magnetic flux, as in the standard GL model, but also possess electric charge determined by the CS coupling [7]. These charged vortices are anyons, whose statistics are neither bosonic nor fermionic. The CS term is special to $(2+1)$ dimensions, breaks parity and time-reversal symmetry, and implements flux-charge attachment. It therefore provides an effective description of planar topological systems such as quantum Hall states and related chiral superconducting settings.

The presence of the CS term renders the problem of identifying static energy minimizers intrinsically nonlocal and mathematically subtle. In particular, the associated canonical energy functional is not bounded from below, thereby precluding the direct use of standard variational techniques such as gradient descent. Despite these difficulties, the CSLG model provides a natural setting connecting condensed matter physics, field theory, and mathematics. On the condensed matter side, it offers an effective description of anyon condensation in the quantum Hall regime [8–11]. Mathematically, it is linked to rigorous results on vortex condensation in Chern-Simons-Higgs systems [12,13] and to generalized self-dual models [14–19]. The interplay between the Yang-Mills and Chern-Simons actions can give rise to rich and unexpected vortex physics.

Challenges of this type are not unique to the CSLG framework, but arise more broadly across condensed matter and high energy physics. In condensed matter systems, analogous difficulties occur in the treatment of demagnetization effects in chiral magnets [20] and depolarization fields in liquid crystals [21]. Related complications also emerge in nuclear and particle physics—for example, through the Coulomb interaction in the Skyrme model [22] and in the formation of metastable knotted solitons within extensions of the standard model [23].

In this Letter, we revisit the CSLG model with the usual quartic Higgs potential. We first derive the static energy functional and corresponding field equations, emphasizing how Gauss's law enforces the proportionality between electric charge and magnetic flux, thereby making each vortex an anyon. We then investigate vortex interactions

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and demonstrate how the CS level qualitatively alters their long-range behavior. In particular, we show that even at the Bogomolny point $\lambda = 1$, where ordinary Ginzburg-Landau vortices are noninteracting, the presence of a CS term induces repulsion. More generally, the competition between Higgs attraction and electrostatic repulsion allows for the formation of stable multivortex anyon bound states, thereby providing a realization of the hybrid nature of the anyon superconductor. Our central result is that hybrid superconductivity can be realized here in a minimal single-component theory: it arises from topological gauge structure rather than from multiple condensates or competing coherence lengths belonging to different bands.

Traditionally, superconductivity has been classified as type I or type II, with type 1.5 emerging in multicomponent systems that support nonmonotonic intervortex forces. It is known that the typology in quantum Hall liquids (described by the CSLG model without the Yang-Mills term) is richer than that in single-component superconductors, owing to the lack of time-reversal symmetry [24]. We demonstrate that the Chern-Simons extension of the Ginzburg-Landau model realizes the same dual nature in a single-component condensate. This establishes a new route to hybrid superconductivity, extending the typology beyond the dichotomy of type I and II and multicomponent type 1.5, by showing that topological gauge couplings can generate nonmonotonic vortex matter.

The Ginzburg-Landau model of anyon superconductivity—The CSLG model is described by the superconducting order parameter $\psi : \mathbb{R}^{2+1} \to \mathbb{C}$, also known as the Higgs field, and an Abelian gauge field $\vec{A} = (A_0, A_1, A_2) \in \mathbb{R}^{2+1}$. Associated to the Abelian gauge field is the gauge covariant derivative $D_\mu = \partial_\mu + iqA_\mu$, where q is the gauge charge. The gauge field strength is given by the curvature $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. From the field strength we define the magnetic field $B = F_{12}$ and the electric field $E_i = F_{0i}$. We will consider the model defined on Minkowski spacetime $\mathbb{R}^{2+1}$, which is endowed with the Minkowski metric η and metric signature $(+--)$.

This GL model of anyon superconductivity is a gauge field theory that exhibits spontaneous symmetry breaking. The local U(1) invariance is realized by the gauge transformation $A_\mu \mapsto A_\mu + \partial_\mu \alpha(x)$ and $\psi \mapsto \psi e^{i\alpha(x)}$. Although the local U(1) symmetry is broken in the Higgs sense, the phase mode is absorbed by the gauge field, so no physical Goldstone mode appears. The action of the anyonic theory is $S = \int \mathrm{d}^3x \mathcal{L}$, where the Lagrangian is given by [8–11]

$$\mathcal{L} = \frac{1}{2} D_\mu \psi \overline{D^\mu \psi} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - V(|\psi|) + \frac{\kappa}{4} \epsilon^{\alpha\beta\gamma} A_\alpha F_{\beta\gamma}. \quad (1)$$

The first three terms describe the GL model, also known as the Abelian Higgs model in this context, where the second term is the Yang-Mills, or Maxwell, contribution. The last term is the topological Chern-Simons term,

$$\mathcal{L}_{\mathrm{CS}} = \frac{\kappa}{4} \epsilon^{\alpha\beta\gamma} A_\alpha F_{\beta\gamma} = \frac{\kappa}{2} (A_0 B - \epsilon_{ij} A_i E_j). \quad (2)$$

This term is characteristic of $(2+1)$-dimensional gauge theory: it is gauge invariant up to a boundary term and breaks parity and time-reversal symmetry.

In this Letter we are only interested in static vortex anyons. These are defined as minimizers of the static energy with nontrivial topological degree. In the GL model, the electric field is absent when considering statics. However, due to the presence of the CS term (2), we see that the electric field does not vanish $E_i = F_{0i} = -\partial_i A_0 \neq 0$ and neither does the kinetic term $D_0 \psi \overline{D_0 \psi} = q^2 A_0^2 |\psi|^2 \neq 0$. After an integration by parts, the static Lagrangian can be expressed as

$$\mathcal{L}_{\mathrm{static}} = \frac{1}{2} (\partial_i A_0)^2 + \kappa A_0 B + \frac{1}{2} q^2 A_0^2 |\psi|^2 - \left[\frac{1}{2} D_i \psi \overline{D_i \psi} + \frac{1}{2} B^2 + V(|\psi|) \right]. \quad (3)$$

Varying the static Lagrangian with respect to the electric potential A_0 reveals Gauss's law as an elliptic partial differential equation,

$$(-\nabla^2 + q^2 |\psi|^2) A_0 = -\kappa B. \quad (4)$$

Since static vortex anyons are minimizers of the static energy functional, we introduce the static energy of the theory by defining

$$\begin{aligned} E &= -\int_{\mathbb{R}^2} \mathrm{d}^2x \mathcal{L}_{\mathrm{static}} \\ &= \int_{\mathbb{R}^2} \mathrm{d}^2x \left\{ \frac{1}{2} D_i \psi \overline{D_i \psi} + \frac{1}{2} B^2 + V(|\psi|) \right. \\ &\quad \left. - \frac{1}{2} (\partial_i A_0)^2 - \kappa A_0 B - \frac{1}{2} q^2 A_0^2 |\psi|^2 \right\}. \end{aligned} \quad (5)$$

At a first glance this appears not to be bounded from below. However, the static energy can be transformed into a form that is bounded below and positive (semi)definite as follows. Let us take the inner product of Gauss's law (4) with the potential A_0 and then integrate by parts to obtain

$$\begin{aligned} -\int_{\mathbb{R}^2} \mathrm{d}^2x \kappa A_0 B &= \int_{\mathbb{R}^2} \mathrm{d}^2x [-A_0 \partial_i \partial_i A_0 + q^2 |\psi|^2 A_0^2] \\ &= \int_{\mathbb{R}^2} \mathrm{d}^2x [(\partial_i A_0)^2 + q^2 |\psi|^2 A_0^2]. \end{aligned} \quad (6)$$

Substituting this into the static energy (5) gives an expression for the static energy that is clearly bounded below by 0,

$$E = \int_{\mathbb{R}^2} \mathrm{d}^2x \left\{ \frac{1}{2}|\vec{D}\psi|^2 + \frac{1}{2}B^2 + V(|\psi|) + \frac{1}{2}|\overrightarrow{\nabla} A_0|^2 + \frac{1}{2}q^2 A_0^2 |\psi|^2 \right\}. \tag{7}$$

This energy is positive definite and bounded below, therefore it is amenable to gradient descent methods.

To summarize, vortices in this Ginzburg-Landau model of anyon superconductivity are minimizers of the positive definite static energy (7), such that the electromagnetic scalar potential A_0 satisfies the nonlinear elliptic Gauss constraint (4).

Anyon superconductivity—In $(2+1)$ dimensions, adding a Chern-Simons term to the action endows particles with fractional statistics. This means that when you adiabatically braid one particle around another, the many-body wave function Ψ picks up a phase,

$$\Psi \to e^{i(\theta_i+\phi)}\Psi, \tag{8}$$

where θ_i is the intrinsic statistics angle associated with the charge [6]. For bosons $\theta_i = 0 (\mathrm{mod}\ 2\pi)$ and $\theta_i = \pi (\mathrm{mod}\ 2\pi)$ for fermions. The other contribution, ϕ, is a Berry, or Aharonov-Bohm, phase coming from the adiabatic braiding. Since the Berry phase is added directly to the intrinsic statistics angle θ_i, it fundamentally alters the statistics of the particles. For example, if we consider fermionic charged particles and $\phi = \pi$, then the resulting statistics of the composite particles is bosonic. In general, since the Berry phase ϕ need not be an integer multiple of π, it can lead to particles endowed with statistics lying between that of bosons and fermions. These composite particles are known precisely as anyons.

The braiding phase of the anyons is the Berry phase and is determined by the level (coupling κ) of the CS term. For the CS term (2), the Berry phase of the charge-flux composite particles is proportional to

$$\phi = \frac{2\pi q^2}{\kappa}. \tag{9}$$

This phase originates from flux-charge attachment: when one vortex encircles another, the bound charge accumulates an Aharonov-Bohm phase from the enclosed flux, so κ fixes the statistical angle. So, we see that the CS level κ determines the exchange statistics by setting how much charge binds to flux, and that directly fixes the braiding phase. This is exactly the same topological mechanism used in the Zhang-Hansson-Kivelson model of the FQHE [1]. We will now derive this charge-flux relationship explicitly.

In the presence of a CS term the relationship between magnetic flux and electric charge requires special care. The starting point is the Gauss constraint (4), which ultimately relates the flux to the charge. This equation encapsulates the mixing between electric and magnetic fields: a localized magnetic flux distribution acts as a source for the electrostatic potential A_0.

The physical electric field is $\vec{E} = -\overrightarrow{\nabla} A_0$, so we can compute the electric charge density via the Maxwell equation

$$\rho_e = \overrightarrow{\nabla} \cdot \vec{E} = -\nabla^2 A_0 = -\kappa B - q^2|\psi|^2 A_0, \tag{10}$$

where we have used Gauss's law (4) to express the charge in terms of the matter and magnetic fields. Hence, the total physical electric charge Q_e is

$$Q_e = \int_{\mathbb{R}^2} \mathrm{d}^2x \rho_e = -\kappa\Phi - q^2 \int_{\mathbb{R}^2} \mathrm{d}^2x A_0 |\psi|^2, \tag{11}$$

where $\Phi = \int_{\mathbb{R}^2} \mathrm{d}^2x B$ is the total magnetic flux. A single magnetic flux quantum is $\Phi_0 = 2\pi/q$, so the total quantized magnetic flux is $\Phi = N\Phi_0$. For localized static solutions, A_0 decays exponentially and $E_i \to 0$ at spatial infinity. Therefore, the total electric charge should be zero for localized solutions, $Q_e = 0$, which gives the relation

$$\int_{\mathbb{R}^2} \mathrm{d}^2x q^2 A_0 |\psi|^2 = -\kappa\Phi. \tag{12}$$

Although the total Maxwell charge Q_e vanishes, the condensate carries a nontrivial internal U(1) charge Q_m. Under a global phase rotation of the scalar, $\psi \mapsto e^{i\alpha}\psi$, the associated Noether current is

$$J_\mu = \frac{iq}{2}(\psi\partial_\mu\bar{\psi} - \bar{\psi}\partial_\mu\psi) + q^2 A_\mu |\psi|^2. \tag{13}$$

From the supercurrent (13), we obtain the Noether electric charge density

$$\rho_m = J_0 = q^2 A_0 |\psi|^2. \tag{14}$$

Therefore, we find that the magnetic flux Φ and the Noether electric charge Q_m are related by [25]

$$Q_m = \int_{\mathbb{R}^2} \mathrm{d}^2x \rho_m = -\kappa\Phi. \tag{15}$$

It is important to distinguish the physical Maxwell charge Q_e, which vanishes for localized configurations, from the internal U(1) Noether charge Q_m. The latter is tied directly to magnetic flux through Gauss's law and controls the flux-charge attachment responsible for anyonic statistics. Importantly, as we have shown, the physical Maxwell charge Q_e vanishes for localized anyons, such that the system remains globally electrically neutral. In addition to this, both the magnetic flux Φ and internal electric charge Q_m are conserved in time. This follows naturally from the definition of a Noether charge.

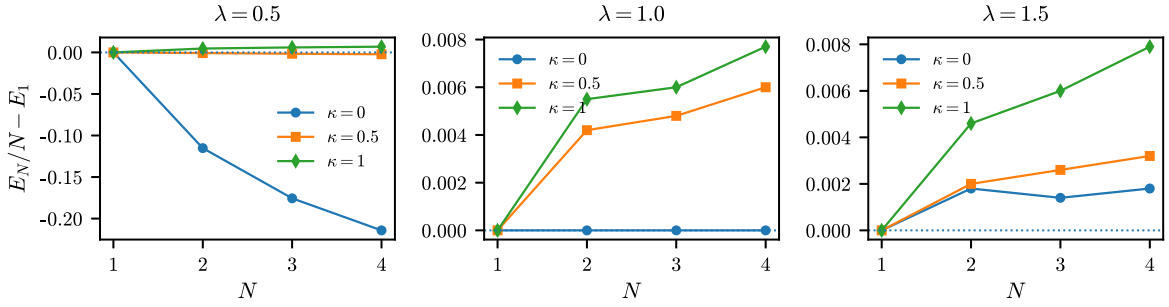


FIG. 1. The binding energy per vortex anyon $E_{\text{bind}} = E_N/N - E_1$ shown for three parameter regimes, corresponding to the value of the Ginzburg-Landau parameter λ. In the regular Ginzburg-Landau model, the Ginzburg-Landau parameter λ controls the type of superconductivity and there is a dichotomy between these types. The first is the type I regime ($\lambda < 1$), where the forces between vortices are attractive. Secondly, we have the type II regime ($\lambda > 1$), where the intervortex forces are repulsive. At the Bogomolny point ($\lambda = 1$), there are no forces between the vortices. This dichotomy is broken in the Chern-Simons extension of the model by introducing an electrostatic repulsion, altering the interaction landscape. Vortex anyons at the Ginzburg-Landau Bogomolny point ($\lambda = 1$) now feel a repulsive force. For larger enough Chern-Simons coupling κ, the type I attractive regime can switch to a repulsive type II regime. Furthermore, a hybrid superconducting state can exist where the vortex anyons experience short-range repulsion and long-range attraction, forming stable bound multivortex anyons.

In relation to the FQHE, we can identify the filling fraction ν from the charge-flux relation (15), that is, $\nu = (|Q_m|/q) = (2\pi\kappa/q^2)$. Then, we see that the Noether electric charge in terms of the filling fraction is $Q_m = -qN\nu$, which can be fractional as the filling fraction ν can be fractional. Therefore, the charge carried by each anyon is $-q\nu$, which can also be fractional. In addition, the spatial part of the Chern-Simons current is given by $J^i = \kappa\epsilon^{ij}E_j$, from which we can identify the Hall conductance σ_{xy} with the CS level, $\sigma_{xy} = \kappa = (q^2/2\pi)\nu$.

Multivortex anyon bound states—The inclusion of the CS term has a profound effect on the binding properties of vortices. Consider the conventional GL model with quartic potential,

$$V(|\psi|) = \frac{\lambda}{8}(m^2 - |\psi|^2)^2, \tag{16}$$

where the Higgs mass is $m_H = \sqrt{\lambda}m$ and λ is the GL parameter dictating the type of superconductivity. The interaction between vortices in the GL model is controlled by the relative size of the Higgs coherence length $\xi_H = 1/m_H$ and the magnetic penetration depth $\lambda_H = 1/m_A$, with gauge mass $m_A = qm$. For $\lambda < 1$ the Higgs mode is lighter than the gauge mode and vortices attract (type I). For $\lambda > 1$, vortices repel (type II). At the Bogomolny point $\lambda = 1$, the two modes are degenerate and the intervortex forces cancel.

In the CSLG model, this dichotomy is altered. Each vortex carries not only magnetic flux but also a Noether electric charge tied to the flux by Gauss's law (4). The resulting electrostatic interaction contributes an additional repulsive channel. Numerical minimization of the static energy (7) is carried out using the SOLITON_SOLVER package [26] with the constrained Newton flow algorithm. Constrained Newton flow reveals that for $\kappa > 0$ the interaction behavior of vortices can change qualitatively. For small CS coupling κ, vortices in the $\lambda < 1$ regime still experience overall attraction and tend to coalesce, resembling type I behavior. At intermediate κ, the long-range Higgs-mediated attraction remains dominant, but the electrostatic repulsion becomes significant at short distances. This frustrates collapse into a single axially symmetric multivortex and instead favors the formation of spatially separated bound states with negative binding energy. At larger κ, the repulsion dominates at all scales, even in the nominal type I regime, driving vortices apart and mimicking type II superconductivity. The resulting binding energies of multivortex anyons in the three GL parameter λ regimes, for various CS couplings κ, is shown in Fig. 1.

The intermediate hybrid regime, where vortex anyons repel at short range but attract at longer range, is similar to that of type 1.5 superconductivity [27–29]. This phenomenon is illustrated in Fig. 2, where an $N = 4$ configuration relaxes into a stable nonaxially symmetric bound state: vortex anyons remain separated due to short-range repulsion yet are held together by the residual long-range attraction. Each constituent vortex carries magnetic flux $\Phi_0 = 2\pi/q$ and a compensating Noether charge $-\kappa\Phi_0$, and the multivortex anyon cluster retains overall electric charge neutrality in the Maxwell sense.

The stability of such bound states has important implications. Unlike type I or II, where vortices either collapse into a single giant core or form an Abrikosov lattice, anyon systems can support molecularlike vortex clusters. These bound states could underpin new collective phases of anyon superconductors, distinct from both the homogeneous condensates of conventional superconductivity and from the crystalline order of type II. Their existence also strengthens the analogy with multicomponent superconductors, where competing length scales naturally give rise to hybrid physics [30,31]. Experimentally, such bound states could be sought through vortex clustering or

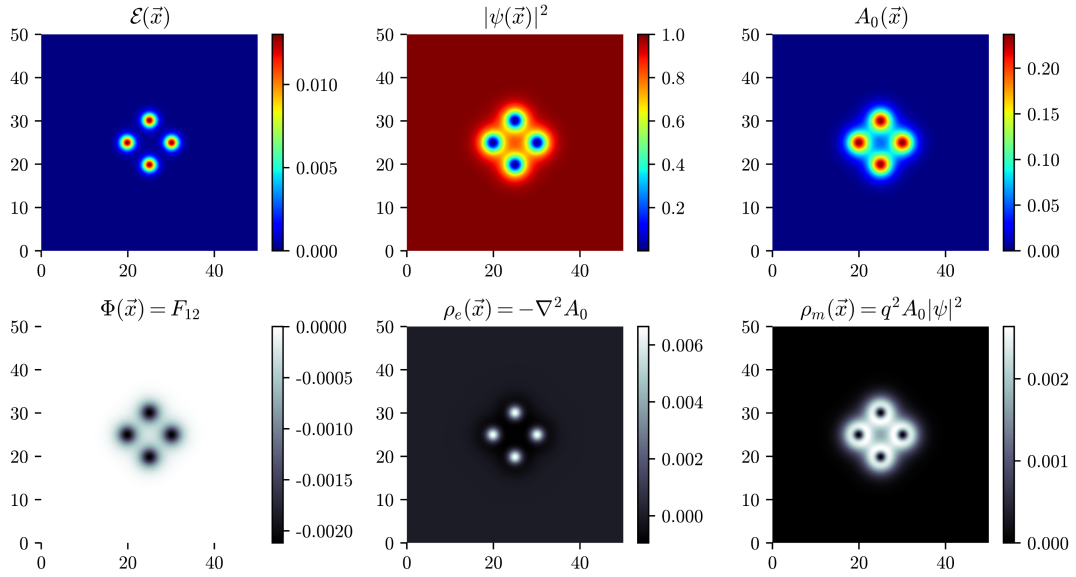


FIG. 2. Density plots of an $N = 4$ vortex anyon exhibiting hybrid anyon superconductivity. Each individual vortex carries a magnetic flux $\Phi_0 = -2\pi$ and an electric charge $-\kappa\Phi_0$, with $\kappa = \frac{1}{2}$. It can be seen that the magnetic flux is centered on the cores of the vortices (bottom left panel), and the electric Noether charge forms a ring of charge around each vortex core (bottom right panel). These vortex anyons are obtained for the Higgs potential (16), with parameters $m = 1$ and $\lambda = \frac{1}{2}$. The binding energy of this four-vortex is negative and, so, the vortices form a bound state.

transport signatures of nonmonotonic intervortex forces in planar topological superconducting systems.

Hybrid superconductivity—To explain the observation of the hybrid superconductivity in our single-component model, we consider the asymptotic tails of the vortex anyons and their long-range interactions. That is, we expand about the ground state configuration $\{\psi, A_\mu\} = \{m + \phi, 0 + a_\mu\}$ and consider field equations linear in the perturbations $\{\phi, a_\mu\}$. We find that the magnetic field and electric field must satisfy the same linearized field equations, $\Delta_\kappa B = 0$ and $\Delta_\kappa E_i = 0$, where [32]

$$\Delta_\kappa = \left[(-\nabla^2 + m_A^2)^2 + \kappa^2\nabla^2\right]. \tag{17}$$

This linearized equation takes the same form as the linearized field equation for the order parameter in superfluids with fermionic imbalance [45]. The operator Δ_κ can be factorized into complex-conjugate eigenmodes $m_\pm$, which are the static screening masses, given by

$$m_\pm = \sqrt{m_A^2 - \frac{\kappa^2}{4}} \pm i\frac{\kappa}{2}. \tag{18}$$

This agrees with the results of Stålhammar *et al.* [46]. Therefore, we see that the magnetic and electric fields share a common penetration depth λ_{gauge} but differ by an oscillation frequency $1/\lambda_{\text{osc}}$, with

$$\lambda_{\text{gauge}} = \frac{1}{\sqrt{m_A^2 - \kappa^2/4}}, \qquad \lambda_{\text{osc}} = \frac{4\pi}{\kappa}. \tag{19}$$

That is, asymptotically, the magnetic and electric fields have a damped oscillatory profile and share the same decay rate. This shared penetration depth should not be interpreted as ordinary plasmon or purely electrostatic screening. Rather, it is a direct consequence of the CS Gauss constraint, which ties charge and flux together so that the electric and magnetic sectors are not independent. The distinction between the two sectors therefore appears only through an oscillatory phase, not through separate real screening lengths. The real dynamical gauge masses of the theory are obtained from the static screening masses by a Wick rotation $\kappa \mapsto i\kappa$.

The long-range oscillation behavior of the gauge modes leads to more interesting long-range interactions. We find that the interaction energy of a pair of separated vortex anyons is determined by

$$V_{\text{int}}(R) = 2\pi|c_B|^2\sqrt{\frac{2\pi}{m_A R}}e^{-\sqrt{m_A^2 - \frac{\kappa^2}{4}}R}\cos\left(\frac{\kappa}{2}R - \gamma\right) - 2\pi c_H^2 K_0(m_H R). \tag{20}$$

The standard scalar superconducting order parameter contribution remains monotone attractive, as in the regular GL model. However, the gauge term becomes a damped oscillator with envelope $e^{-\alpha R}/\sqrt{R}$, decay rate $\alpha = \frac{1}{2}\sqrt{4m_A^2 - \kappa^2}$, and oscillation frequency $\beta = \kappa/2$, alternating between attractive and repulsive behavior. If the gauge term dominates at a shorter range than the Higgs term, it can provide a repulsive force initially

$[R < (\pi/\kappa) + (2\gamma/\kappa)]$ before switching to an attractive force at longer range $[R > (\pi/\kappa) + (2\gamma/\kappa)]$, and repeating this behavior in a decaying oscillatory fashion as the separation distance R increases. In contrast to type 1.5 superconductivity in multiband superconductors, where the hybrid behavior arises to due competing length scales with $\xi_1 < \lambda < \xi_2$, this long-range oscillatory behavior can give rise to the shorter-range repulsion and long-range attraction essential for hybrid superconductivity.

Conclusion—In this Letter we have examined vortex physics in the CSLG model, highlighting how the addition of a CS term profoundly modifies the landscape of superconductivity in two spatial dimensions. Gauss's law ties each magnetic flux quantum to a compensating Noether electric charge, rendering vortices intrinsically anyonic. This charge-flux attachment mechanism introduces an electrostatic contribution to the vortex interaction and leads to nontrivial screening behavior. Unlike the Ginzburg-Landau model, where vortices exhibit a simple type I and II dichotomy determined by the GL parameter, the CSLG model yields complex-conjugate gauge screening masses. Consequently, the electric and magnetic fields decay with a common penetration depth but acquire an oscillatory phase shift.

Our numerical simulations, carried out via the constrained Newton flow algorithm, demonstrate that this altered screening structure allows for stable multivortex anyon bound states. These bound states exhibit shortrzange repulsion and long-range attraction, the defining characteristic of hybrid superconductivity. We have thereby provided a theoretical realization of hybrid behavior in a single-component anyon superconductor.

We note that the dual superconducting nature of the anyon superconductor is not unique to this single-component model, that is, other single-component superconducting systems can give rise to nonmonotonic intervortex forces. For example, in noncentrosymmetric superconductors, broken inversion symmetry produces spiral magnetic field decay [47] and parity-breaking terms can cause magnetic field inversion [48]. These both lead to nonmonotonic intervortex forces that stabilize vortex bound states. This is in close analogy to how the Chern-Simons term in our model generates oscillatory gauge tails and dual-nature vortex interactions.

The emergence of molecularlike vortex anyon clusters suggests that anyon superconductivity may host a much richer set of collective states than previously appreciated. Recent works have also explored related anyonic and topological mechanisms in two-dimensional systems: anyonic molecules in fractional quantum Hall liquids [49], doped fractional quantum anomalous Hall insulators exhibiting itinerant anyon superconductivity [50], and chiral pseudospin liquids in moiré heterostructures [51]. These studies underscore the growing interest in anyon-mediated condensation and topological gauge responses, providing complementary microscopic contexts for the effective Chern-Simons mechanism and hybrid vortex behavior examined here. Experimentally, our results suggest looking for direct signatures of vortex clustering and nonmonotonic intervortex interactions in planar superconductors with broken time-reversal symmetry and in superconducting topological heterostructures. In addition to extending the classification of superconducting phases, these results highlight deep connections between topological gauge theories, fractional statistics, and nonstandard superconductivity.

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Data availability—The data that support the findings of this article are openly available [52].

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