

Transport by waves and turbulence: Dilute suspensions in stably stratified plane Poiseuille flow

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Suspended particle transport is inherent in a wide range of industrial and environmental flows, from energy generation to manufacturing and from marine to atmospheric systems. Such flows are inherently stratified where varying concentration of particulates changes the mean density of the fluid; however, the role of stratification on particle suspension is poorly understood. Here the suspension of sediments in a channel (plane Poiseuille) flow is resolved using numerical simulation where the flow is stably thermally stratified and the particles passive. This set up enables quantification of stratification controls sediment transport processes. Sediment suspension is resolved for settling velocities of $V_s = 0.005$, $V_s = 0.01$, and $V_s = 0.02$, made dimensionless by the friction velocity u_τ . Our results show that stratification generates flow-scale mixing processes, which invalidate extant models of sediment suspension that have been used for over 100 years. For stratified flows, the dimensionless collapse of flow scalars and turbulent statistics is poor near the core of the channel, where large concentration gradients coincide with large-scale mixing events. Here the differences between vertical turbulent diffusivities of sediment and temperature increase with increasing V_s , reaching 15 % for $V_s = 0.02$. Discrepancies arise due to a breakdown of the linear gradient diffusion hypothesis. Predictions using the gradient diffusion hypothesis are expected to worsen with increasing settling velocity, indicating that classical models poorly predict dilute particle transport in strongly stratified flows, which are common in a range of environmental and industrial settings. Further, in stratified flows a feedback loop is established that generates strong buoyancy forces in the core of the duct, having the profound impact of building a self-reinforcing two-layer sediment concentration profile. These results highlight that classical sediment transport models may underpredict the amount of turbulent mixing, and therefore the capability of flows to suspend material, where flow-scale mixing events occur. They further reveal mechanisms for the formation of a barrier to mixing which significantly reduces suspended load capacity in flows with an internal velocity maximum, such as ducts, pipes, gravity currents, and jets.

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I. INTRODUCTION

An improved understanding of particle transport has applications spanning engineering, atmospheric, oceanographic, and earth surface sciences. improved models for particulate transport are critical for our understanding of microplastic transport in the atmosphere and marine/fresh-water

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ecosystems [1,2], sediment and tracer (e.g., organic matter) transport in rivers and oceans [3,4], turbid underwater currents [5], and atmospheric currents in the form of powder snow avalanches [6] and pyroclastic flows [7]. These flows are highly complex and comprised of a wide range of turbulent length and time-scales, and are further complicated by high volume fraction loading, where particles may interact and have a primary control on fluid dynamics. Even in dilute flows density stratification is commonplace, which can have a strong influence on scalar transport [5]. Stable stratification may arise through a compositional change in flow properties (e.g., salinity, temperature) or through suspended particulates. These can lead to regions of strong buoyancy gradient or interfaces which introduce strong anisotropy due to suppressed vertical motion, and subsequently barriers to vertical transport. In addition, it is common for strong buoyancy gradients to introduce large-scale coherent structures in the form of waves and vortices [8]. It has been theorized that large-scale coherent structures play a critical role in sediment transport processes, and may be a ubiquitous feature of many fluid dynamical flows [9], yet the role of coherent structures on sediment transport is poorly understood [10].

Of particular importance for sediment transport predictions is the quantification and modeling of sediment diffusivity due to turbulence, K_c , derived from the gradient diffusion hypothesis [11]. Our ability to predict sediment transport dynamics at large scales is critically dependent on appropriate parametrization of K_c , with simple closures based upon the canonical log-law [11], and more advanced models developed for stratified flows [12]. While these closures are commonly adopted in sediment transport models, it has been noted that they may be inappropriate in regions where density, as determined by sediment concentration, changes rapidly when compared to the scale of mixing events [13]. The gradient diffusion hypothesis assumes that mixing length scales are small compared to the length scales over which concentration varies, and situations may therefore arise when such conditions do not hold.

This study investigates the role of large-scale coherent structures on sediment transport by simulating a weakly stratified plane Poiseuille (or channel) flow. The plane Poiseuille flow is a doubly periodic flow driven by a streamwise pressure gradient between two walls; a canonical fluid dynamics problem that has received considerable interest for studying wall-bounded turbulence [14–18], and to investigate sediment and scalar transport processes in wall bounded flows [19–21]. In the context of sediment transport processes, stable stratification is typically associated with a downward flux of dense settling particles which have a concentration high enough to influence the flow through a buoyancy term in the momentum equations (i.e., the system is one-way coupled). In this case, asymmetry may develop in the flow as the sediment concentration gradients suppress vertical motion; the position of maximum flow velocity moves downwards as a response to reduced turbulence, anisotropy of the Reynolds stresses increases, vertical transport of sediment is suppressed therefore increasing associated concentration gradients, and a turbulent flow may locally re-laminarise if the sediment settling velocity and concentration are sufficiently large [20,21].

Here a channel flow where stratification is driven by a temperature field is investigated, with fixed temperature boundary conditions at the upper and lower walls. Buoyancy forces are dependent on the flow temperature, and the stratification strength is therefore governed by the temperature difference between the upper and lower walls. Subsequently, the flow is parameterised by the friction Reynolds number, the friction Richardson number, and the Prandtl number, respectively, defined as

$$Re_\tau = \frac{u_\tau \delta}{\nu}, \quad Ri_\tau = \frac{\alpha \Delta T g \delta}{u_\tau^2}, \quad Pr = \frac{\kappa_\theta}{\nu}, \quad (1)$$

where u_τ is the friction velocity, δ the channel half-height, ν represents kinematic viscosity, ΔT is the temperature difference between the upper and lower walls, g represents gravitational acceleration, α is the thermal expansion coefficient, and κ_θ represents thermal diffusivity. Under appropriate Reynolds and Richardson numbers, the stratified channel flow possesses a range of coherent structures due to the large buoyancy forces that dominate the core of the channel, and strong shear at the walls. This flow introduces large-scale internal waves in the channel core, driven by hairpin vortex ejections in the outer regions [22,23], which are emerging as a common feature

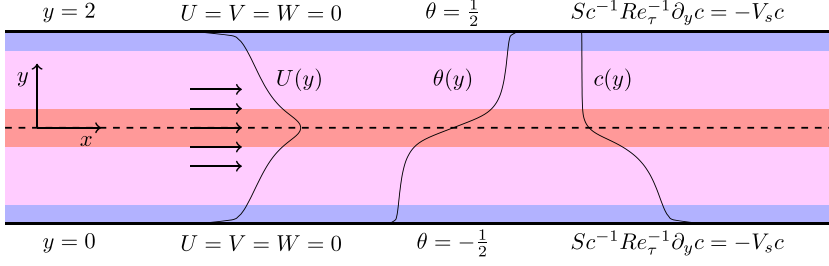


FIG. 1. Numerical domain and simulation boundary conditions. Shaded regions represent the inner regions at $y \leq 0.2$ and $y \geq 1.8$, the outer regions at $0.2 < y \leq 0.8$ and $1.2 \leq y < 1.8$, and the channel core at $0.8 < y < 1.2$. The dashed line represents the channel center center line at $y = 1$.

in stratified shear flows [24,25]. The coexistence of relatively large-scale coherent structures and strong buoyancy gradients makes the thermally stratified channel flow an interesting test-case for investigating sediment transport processes [26,27].

This study quantifies passively transported, negatively (or positively, due to symmetry) buoyant sediment in a thermally stratified channel flow, numerically simulated using spectral element methods. Through this study we develop an understanding of sediment transport processes in stratified shear flows comprising both active turbulence and large-scale coherent structures, free from complexities associated with a fully coupled, nondilute, sediment concentration field, which we reserve for future work. It is shown that while stratification has a significant effect on sediment transport, introducing a strong transport barrier in the channel core, turbulent statistics collapse to near common profiles when appropriately scaled by the friction temperature (θ_τ) for the temperature field and $V_s \bar{c}$ for sediment fields, where V_s is the particle settling velocity and $\bar{c}$ is the temporally and planar-averaged vertically varying sediment concentration. Deviations in statistics only appear in the channel core, where buoyancy forces are strong, sediment concentration gradients are large, and relatively large-scale intermittent mixing events are present. We argue that deviations arise due to a breakdown of the gradient diffusion hypothesis, where higher-order terms containing $\partial_y^3 \bar{c}$ become important, or alternatively, the collapse of scalar statistics requires $\overline{c'v'} \sim \partial_y \bar{c}$ to hold, which is only weakly true in the core of the channel. The breakdown of the gradient diffusion hypothesis results in a 15% maximum deviation in turbulent Prandtl numbers when comparing temperature and sediment scalars, and indicates that higher-order closures are required for accurate predictions of such flows, which are common in a wide range of environmental and industrial settings.

This paper is organized as follows: The methodology is introduced in Sec. II, before reporting our results in Sec. III. Vertically varying turbulent statistics are presented in Sec. III A before discussion multidimensional spectra in Sec. III B and Spectral Proper Orthogonal Decompositions in Sec. III C. Finally we present visualizations of instantaneous flow in Sec. III D, before concluding our study in Sec. IV.

II. METHODOLOGY

A stratified channel flow is simulated (Fig. 1) which is driven by a constant streamwise (negative) pressure gradient with periodic boundary conditions applied in the streamwise (x) and spanwise (z) directions and no-slip conditions at the walls (y -normal boundaries). Stratification is imposed with a fixed temperature (T) difference between the upper and lower walls which affects the density field, ρ , by

$$\rho = \rho_0 - \alpha \rho_0 (T - T_0), \quad (2)$$

where ρ_0 and T_0 represent reference values for density and temperature, respectively, and α is the thermal expansion coefficient. We assume the flow is incompressible and Boussinesq, such that the

governing dimensionless equations are the continuity, momentum, and temperature scalar transport equations:

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{1}{Re_\tau} \frac{\partial^2 U_i}{\partial x_j^2} + f_i + Ri_\tau \theta' \hat{y}_i. \quad (3)$$

$$\frac{\partial U_i}{\partial x_i} = 0, \quad (4)$$

$$\frac{\partial \theta}{\partial t} + U_j \frac{\partial \theta}{\partial x_j} = \frac{1}{Pr Re_\tau} \frac{\partial^2 \theta}{\partial x_j^2}, \quad (5)$$

where $U_i = (U, V, W)'$ represents velocity, $x_i = (x, y, z)'$ represents Cartesian spatial coordinates, t represents time, P represents kinematic pressure, $\theta = (T - T_0)/\Delta T$ is the temperature field, and $\hat{y}_i$ is the y -direction unit vector. The flow is forced by the constant pressure gradient $f_i = (1, 0, 0)'$. Equations are made dimensionless by the channel half height, δ , the friction velocity u_τ , and the temperature difference between the upper and lower walls, ΔT , which is fixed with Dirichlet boundary conditions ($\theta = -0.5$ at $y = 0$ and $\theta = 0.5$ at $y = 2$). Walls are treated as no-slip with $U_i = 0$. The dimensionless parameters governing dynamics are the shear Reynolds number Re_τ , the shear Richardson number, Ri_τ , and the Prandtl number, Pr :

$$Re_\tau = \frac{u_\tau \delta}{\nu}, \quad Ri_\tau = \frac{\alpha \Delta T g \delta}{u_\tau^2}, \quad Pr = \frac{\kappa_\theta}{\nu}, \quad (6)$$

where g represents gravitational acceleration, κ_θ represents thermal diffusivity, and ν represents momentum diffusivity (viscosity). Following Lloyd *et al.* [22], simulations are performed with fixed Reynolds and Prandtl number, $Re_\tau = 550$ and $Pr = 1$, and two Richardson numbers, $Ri_\tau = 0$ and $Ri_\tau = 480$, corresponding to an unstratified channel flow with a passive temperature field (Case U), and a stratified channel flow with shear-dominated regions near the walls and a buoyancy dominated channel core (Case S).

The buoyancy force $Ri_\tau \theta' \hat{y}_i$ is dependent on the temperature fluctuation, $\theta' = \theta - \bar{\theta}$, where $\bar{\theta}$ is the time- and planar-averaged temperature field. In this case the purely wall-normal temperature field $\bar{\theta}$ is absorbed hydrostatically into the pressure term. In practice $\bar{\theta}$ is approximated at each time step by the instantaneous planar-averaged temperature, consistent with previous work [22, 28, 29]. Equations (3)–(5) govern system dynamics, and are analogous to the system of equations solved by Lloyd *et al.* [22], aside from our choice of passive scalar, where we choose to solve for a temperature field rather than a density field.

Dilute sediment is transported passively in this system by solving additional transport equations of the form:

$$\frac{\partial c}{\partial t} + U_j \frac{\partial c}{\partial x_j} = \frac{1}{Sc Re_\tau} \frac{\partial^2 c}{\partial x_j^2} + V_s \frac{\partial c}{\partial y}, \quad (7)$$

where c is the sediment concentration (made dimensionless by the volume averaged concentration c^v), $Sc = \kappa_c/\nu$ is the Schmidt number where κ_c is the nonturbulent sediment diffusivity, and V_s is the sediment settling velocity, made dimensionless by the friction velocity. The sediment concentration scalar transport Eq. (7) is obtained by assuming (i) particles are smaller than the smallest turbulent length scales in the flow, (ii) the aerodynamics response time of the particles is small compared to the smallest time scales of the flow, (iii) sediment concentrations are dilute such that particle-particle interactions and the effects of particle suspension on buoyancy can be neglected, and (iv) the particles are monodisperse. Under these assumptions the velocity field remains divergence-free, fluid density/buoyancy is only affected by the temperature field, and particles settle at a constant velocity, V_s , determined by the balance between the gravitational force and the Stokes drag force acting on individual particles. We solve the sediment transport Eq. (7) for three different sediment species during our simulations, each with a different settling velocity: $V_s = 0.005, 0.01, 0.02$. The

advection term of Eq. (7) captures the mean flow transport and, crucially, the turbulent diffusion of particles across scales from dissipation of energy to flow scale. The diffusion term of Eq. (7) captures the decay of concentration gradients due to hydrodynamic diffusion of particles, Brownian motion, and variations in particle inertia (size/shape) [30–32]. Here we specify $Sc = Pr = 1$ for all simulations and scalars, following previous work where the influence of Schmidt number was found to be negligible so long as $Sc \geq 1$ [33–36]. For a more complete discussion on these assumptions please see the review of Meiburg *et al.* [30] and study of Marshall *et al.* [37].

At the wall, sediment concentration boundary conditions are zero-flux to ensure the volume averaged concentration field is constant in time. This is formally enforced through the condition

$$\left. \frac{1}{ScRe_\tau} \frac{\partial c}{\partial y} \right|_w = -V_s c|_w, \quad (8)$$

where the subscript w represents the wall locations at $y = 0$ and $y = 2$.

The governing equations are discretized and solved using Nek5000, following the methodology of Lloyd *et al.* [22]. In total we solve for eight primitive variables: The three component fluid velocity, pressure, temperature, and three separate sediment species c , obtained by time-evolving the system (3), (4), and (5), with three sediment concentration transport Eq. (7). The domain size is $L_x \times L_y \times L_z = 8\pi \times 2 \times 3\pi$ (normalized by the channel half height), decomposed into $N_x \times N_y \times N_z = 80 \times 44 \times 40$ spectral elements, stretched vertically with a Chebyshev distribution, and further discretized by 8^3 Gauss-Lobatto-Legendre (GLL) nodes (seventh-order polynomials). Third-order time-integration is adopted with a dimensionless time step of 10^{-4} . This choice of grid resolution follows Lloyd *et al.* [22]; while vertical resolution is high enough to fully resolve turbulent scales the spanwise and streamwise directions are underresolved. As a result, subgrid-scale dissipation is addressed using explicit filtering in the modal (spectral) space [38]. A smooth spectral filter with an attenuation amplitude of 0.05 is applied to all primitive variables at each timestep, selectively damping the highest-order polynomial modes to remove unresolved energy and ensure numerical stability. This technique has been successfully adopted in a wide range of high-fidelity wall-bounded turbulent flow simulations [22,39–41]. While this choice of grid resolution under resolves inner region dynamics where shear is largest, Lloyd *et al.* [22] obtained excellent agreement with direct numerical simulation (DNS) in the outer and core regions of the channel ($0.2 < y < 1.8$), which are the regions of interest for this study. Lloyd *et al.* [22] validated velocity and density (equivalent to temperature) statistics via mesh-independence studies and by comparison with DNS [29]. While passive sediments were not included in that study, here we specify $Pr = Sc = 1$, ensuring that the sediment fields share the same dissipative scales as the temperature field. The additional settling velocity introduces a flux in the sediment transport Eq. (7), but this flux acts on much larger temporal and spatial scales than subgrid scales, with a maximum settling velocity of only 2% of the wall friction velocity. This provides confidence that the sediment statistics are resolved with a similar accuracy as the temperature field.

Case U is initialized from a perturbed logarithmic velocity profile with a linear temperature profile and sediment concentrations fixed to $c = 1$. Case S is initialized from Case U data once a statistically steady state has been reached. Both cases are advanced to a statistically steady state in two stages: Firstly, a coarse simulation is performed, using 6^3 GLL points per element; and secondly, the simulation is run with the desired polynomial order (8^3 GLL points). Both steps are advanced to a pseudosteady state which is assessed by monitoring the friction Reynolds numbers Re_τ and wall sediment concentrations. Once pseudosteady, statistics are collected over time T (Table I). Temporal statistics are obtained by collecting instantaneous velocity, temperature, and sediment concentration on two-dimensional (2D) slices every 20 time steps. Several y -normal slices are collected, along with a z -normal slice at $z = L_z/2$. over time T , 22 000 snapshots are collected for Case U and

TABLE I. Cases with time-averaged parameters and case-specific quantities. The bulk Reynolds and Richardson numbers are given by $Re_b = U_b \delta / \nu$ and $Ri_b = \alpha \Delta T g \delta / 2 U_b^2$ where U_b is the bulk velocity. $Nu = 2 \partial_y \bar{\theta}|_w$ is the Nusselt number, and $\Lambda = 2 Re_\tau Pr / \varkappa Ri_\tau Nu$ is the Obukhov length normalized by the channel half height δ with the Kármán constant $\varkappa = 0.41$. Δ_i^+ quantifies the element sizes in wall units (normalization by $\delta_v = \nu / u_\tau$). Note that each element is further discretized by 8^3 GLL points.

Case	Re_τ	Re_b	Ri_τ	Ri_b	Nu	Λ	N_x	N_y	N_z	Δ_x^+	$\Delta_{y,\max}^+$	Δ_z^+	T
U	550	10 507	0	0	21.30	-	80	44	40	172.8	40.2	129.6	44
S	550	14 283	480	0.356	4.13	1.35	80	44	40	172.8	40.2	129.6	75

37 500 snapshots are collected for Case S, enabling convergence of multidimensional spectra and spectral proper orthogonal decomposition (SPOD).

III. RESULTS

A. Time- and planar-averaged statistics

Profiles of time- and planar-averaged statistics are presented in Figs. 2 and 3, where variables are decomposed into their time- and planar-averaged means (represented by overbars) and fluctuations away from the means (represented by primes): $U_i = \bar{U}_i + u'_i$, $\theta = \bar{\theta} + \theta'$, and $c = \bar{c} + c'$. Superscript + represents normalization in wall units, where the appropriate scales for velocity and temperature are u_τ and θ_τ , where $\theta_\tau = \frac{1}{Re_\tau Pr} \frac{\partial \bar{\theta}}{\partial y}|_w$. Case U is partitioned into two regions: The inner region ($y \leq 0.2$ and $y \geq 1.8$) where viscosity and shear are the key controls on dynamics, and the outer region ($0.2 < y < 1.8$) where the direct effects of viscosity are negligible. This classification follows that typically used for boundary layer flows [See, e.g., 42]. Case S differs from Case U by introducing a further region, the channel core ($0.8 > y > 1.2$), where dynamics are dominated by buoyancy forces, indicated by the strong temperature gradients Lloyd *et al.* [22].

Before discussing the influence of density stratification on the sediment scalars, the key features of the stratified channel flow are summarized [22,28,29]. When considering the temperature and

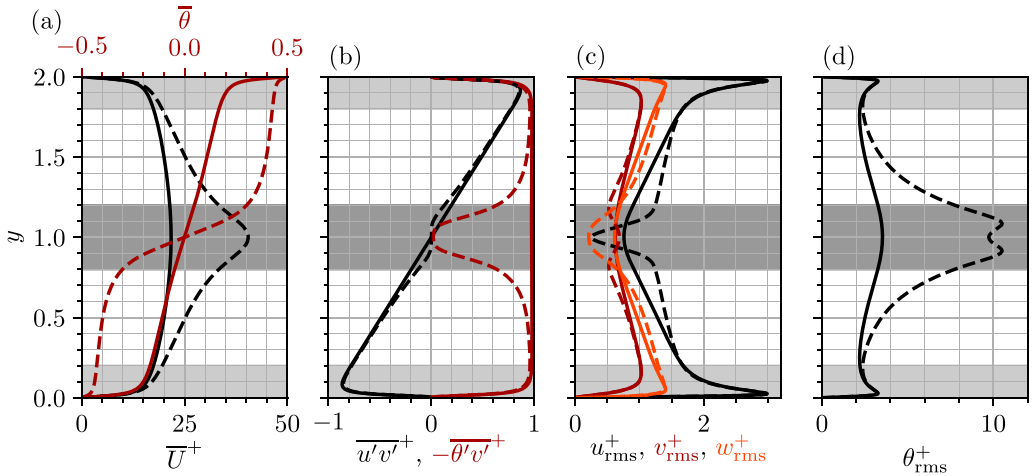


FIG. 2. Time- and planar-averaged velocity and temperature statistics for Case U (solid lines) and Case S (dashed lines). The shaded regions represent the approximate bounds of the inner region ($y \leq 0.2$ and $y \geq 1.8$), the outer region ($0.2 < y \leq 0.8$ and $1.2 \leq y < 1.8$), and the channel core ($0.8 > y > 1.2$).

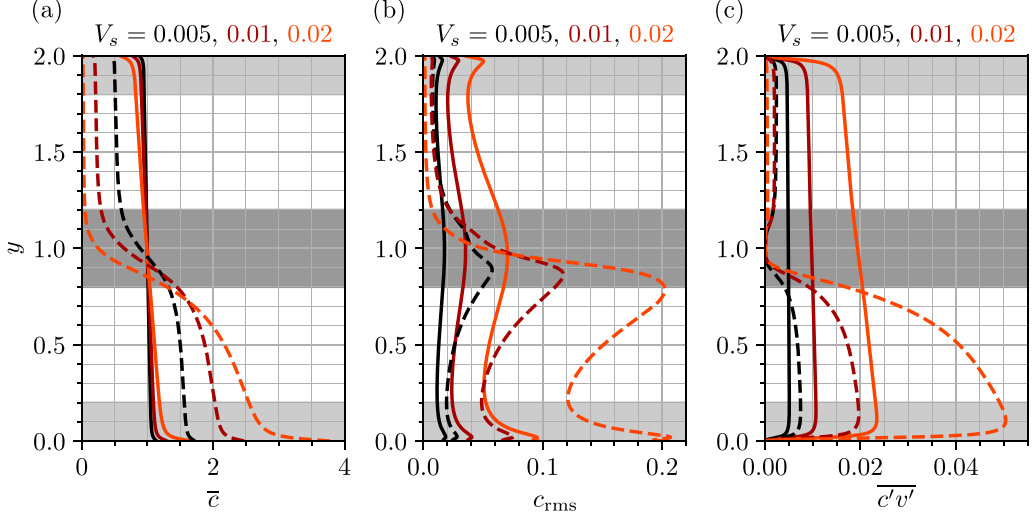


FIG. 3. Time- and planar-averaged sediment concentration statistics for Case U (solid lines) and Case S (dashed lines). Colors represent different variables indicated by subfigure titles, and the shaded regions represent the approximate bounds of the inner region ($y \leq 0.2$ and $y \geq 1.8$), the outer region ($0.2 < y \leq 0.8$ and $1.2 \leq y < 1.8$), and the channel core ($0.8 > y > 1.2$).

velocity fields, key differences between unstratified and stratified cases occur in the outer and core regions. Root-Mean-Square (RMS) temperature ($\theta_{\text{rms}}^2 = \overline{\theta'\theta'}$) and vertical velocity ($v_{\text{rms}}^2 = \overline{v'v'}$) fluctuations peak to local maxima in the channel core, while streamwise ($u_{\text{rms}}^2 = \overline{u'u'}$) and spanwise ($w_{\text{rms}}^2 = \overline{w'w'}$) velocity fluctuations are suppressed. This is due to the internal waves that dominate the core, which suppress turbulence and vertical fluxes of momentum ($\overline{u'v'^+}$) and temperature ($\overline{\theta'v'^+}$). The suppression of turbulence in the core leads to a jetlike peak in mean streamwise velocity, $\overline{U}^+$, approximately twice as high as the unstratified case. The mean temperature $\overline{\theta}$ is approximately constant in much of the inner and outer regions for Case S, aside from in the very-near-wall region. Strong temperature gradients are present in the channel core, responsible for the suppression of turbulence and vertical transport of momentum and temperature, and for enabling the propagation of internal waves.

Sediment concentration (Fig. 3) is approximately linear across most of the channel for Case U, aside from steep gradients near the wall. Local peaks in c_{rms} are observed near the walls and the near the channel center line. The vertical sediment flux is also approximately linear for Case U, unsurprising when noting that the mean sediment concentration is governed by

$$\frac{1}{ScRe_\tau} \frac{\partial \bar{c}}{\partial y} + V_s \bar{c} - \overline{c'v'} = 0. \quad (9)$$

For Case U, diffusive processes are negligible except near the walls (where sediment concentration gradients are large), such that $\overline{c'v'} \approx V_s \bar{c}$. Increasing V_s steepens the sediment concentration gradient, increasing the sediment concentration at the lower wall and reducing it at the upper wall.

Stratification has a profound impact on sediment transport [5]. The region of strong buoyancy gradient acts as a transport barrier, leading to a two-layer sediment concentration profile with high concentration in the lower region of the channel, and very low sediment concentrations in the upper regions of the channel. This is better visualized in Fig. 4 where the sediment statistics (for $V_s = 0.02$) are scaled logarithmically. Here, the sediment concentration varies by two orders of magnitude between the upper and lower regions of the channel. These regions are separated by a strong vertical concentration gradient, with a local maximum (negative) value near the lower edge of the channel

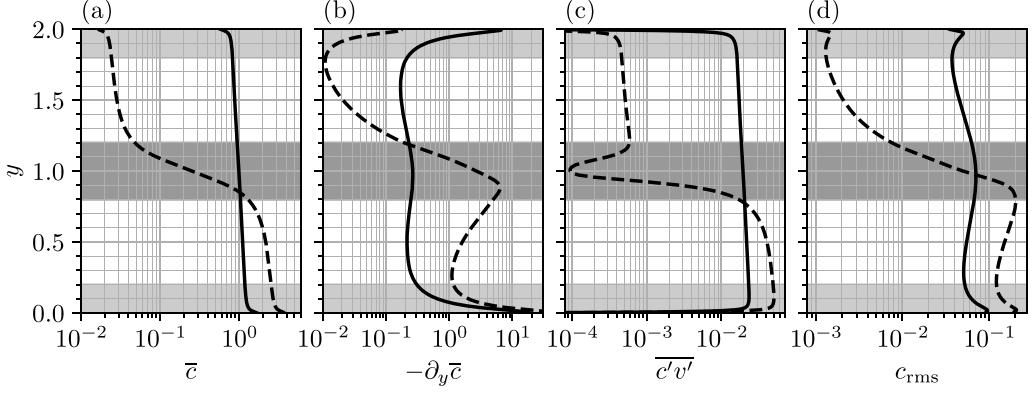


FIG. 4. Time- and planar-averaged sediment statistics for Case U (solid lines) and Case S (dashed lines) with $V_s = 0.02$, logarithmically scaled.

core, coinciding with a peak in c_{rms} . The sediment vertical flux is unsurprisingly similar in profile to the sediment concentration, but has a local minimum at the channel center line. V_s appears to have a much stronger influence on sediment statistics than for unstratified flow (Fig. 3); at higher V_s there is a much larger difference between sediment concentrations below/above the channel core, and much larger fluctuations in sediment concentrations. Increasing V_s also shifts the local maxima for sediment concentration gradient and c_{rms} downwards, further from the channel center line.

A key difference between Case U and Case S scalar transport processes is the dominance of diffusive processes, as opposed to turbulent mixing processes, in the quasilaminar core of the stratified flow. This is evident when assessing the vertical transport budgets of sediment concentration (9) and temperature:

$$\frac{1}{PrRe_\tau} \frac{\partial \bar{\theta}}{\partial y} - \overline{\theta'v'} = \text{const} = \theta_\tau, \quad (10)$$

or alternatively

$$\frac{1}{PrRe_\tau} \frac{\partial \bar{\theta}^+}{\partial y} - \overline{\theta'v'}^+ = 1. \quad (11)$$

Budgets of vertical sediment concentration (9) and temperature (10) transport are presented in Fig. 5. The terms $\overline{c'v'}$ and $\overline{\theta'v'}$ are suppressed in the channel core for Case S, where the vertical transport of both sediment and temperature is governed by respective mean vertical gradients. Sediment and temperature scalar transport equations differ, clearly, due to the settling term, $V_s \bar{c}$. Interestingly, when scaling the vertical transport equation for sediment concentration by $V_s \bar{c}$ one obtains essentially the same balance between turbulent and diffusive processes as the temperature transport equation throughout the full channel height, aside from a change in sign (due to the increase in temperature with y , as opposed to the decrease in sediment concentration) as observed in Fig. 5. When written explicitly we see

$$\frac{1}{V_s \bar{c}} \frac{1}{ScRe_\tau} \frac{\partial \bar{c}}{\partial y} - \frac{1}{V_s \bar{c}} \overline{c'v'} = -1, \quad (12)$$

which clearly has similarities to Eq. (11), which balances diffusive and turbulent components. Functionally these equations are similar, given that the sediment scalars are dictated by the velocity field which is coupled to the temperature field. Further, the symmetry about the channel center line for Eq. (12) budgets is well explained when noting that at both walls, we must have $\frac{1}{V_s \bar{c}} \frac{1}{ScRe_\tau} \partial_y \bar{c} \approx -1$, and Case U must have $-\frac{1}{V_s \bar{c}} \overline{c'v'} \approx -1$ everywhere else. Case S differs from Case U in the outer

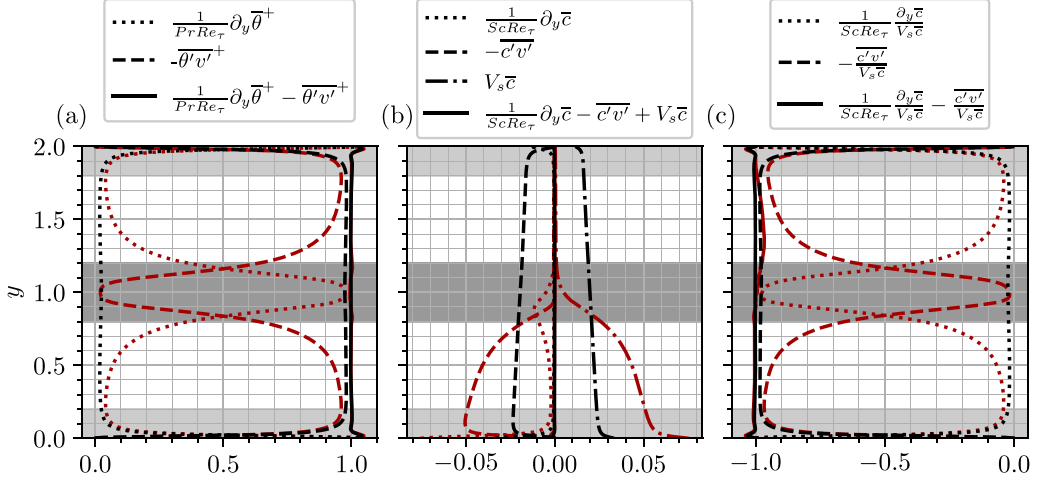


FIG. 5. Budgets of transport equations for (a) temperature, (b) sediment with $V_s = 0.02$, and (c) sediment with $V_s = 0.02$ and scaled by $V_s \bar{c}$. Transport terms correspond to different linestyles (see legends) and are as per Eqs. (11) for (a), (9) for (b), and (12) for (c). Colors represent Case U data (black) and Case S data (red).

and core regions of the channel, particularly in the core where diffusive processes dominate with $\frac{1}{V_s \bar{c}} \frac{1}{ScRe_\tau} \partial_y \bar{c} \approx -1$. Under the scalings of θ_τ and $V_s \bar{c}$ there are obvious similarities between scalar transport processes in the cases where either turbulent or diffusive components dominate. For example, when turbulent processes dominate $\overline{\theta'v'}^+ \approx -\frac{1}{V_s \bar{c}} \overline{c'v'} \approx -1$. It is less obvious how scalars should compare when both of the two contributions are relevant.

It should be noted that, near the walls ($y \lesssim 0.05$ and $y \gtrsim 1.95$), there is an approximate 3% error in the balance of computed budgets. This is due to the unresolved subgrid-scale energy that leads to an underprediction of near-wall turbulent kinetic energy (TKE) [22]. Despite this underprediction of near-wall TKE, outer and core region dynamics are well predicted [22], which are the regions of interest in this study. There is also an additional discrepancy for Case S that is revealed when scaling budgets according to Eq. (12), which shows that in the upper region of the channel the pseudosteady-state vertical transport equation for passive sediment is not entirely satisfied (up to a 3% error). This discrepancy is only apparent in the budgets upon scaling by $V_s \bar{c}$, observed when comparing Figs. 5(b) and 5(c). Since $\bar{c}$ is two orders of magnitude smaller in the upper regions when compared to the lower regions (Fig. 4), all unscaled budgets are negligible in upper regions ($y \gtrsim 1.1$) when compared against the lower regions. When scaled by $V_s \bar{c}$, slight differences ($\lesssim 3\%$) emerge in the region $1.3 \lesssim y \lesssim 1.5$, which we attribute to the long time scales required to obtain statistically steady solutions for the sediment scalars, which are of the order δ/V_s . In other words, there is a small time dependence to the sediment transport budgets for Case S, in the upper regions of the flow where the sediment concentrations are small. This is further complicated by the transport barrier in the channel core which restricts the vertical sediment flux, therefore slowing convergence further. We expect that the 3% errors would be reduced further if simulations were allowed to evolve further, but given the steady-state transport budgets are well satisfied in the regions where sediment concentrations are non-negligible (i.e. $y \lesssim 1.2$) we deem convergence adequate for the present study.

The scalings θ_τ and $V_s \bar{c}$ appear to collapse statistics for θ_{rms} , $\overline{\theta'u'}$, and $\overline{\theta'v'}$, and equivalent sediment concentration statistics, as shown in Fig. 6. Aside from some slight deviations in the channel core (particularly at the maxima of θ_{rms}) and the upper region for Case S where $\bar{c}$ is small, the collapse between different scalars is remarkable. Note that the deviations in statistics for $y > 1.2$ are most likely due to the convergence issues discussed, particularly when noting that the sediment field statistics tend closer to the temperature field statistics as V_s increases. One would expect

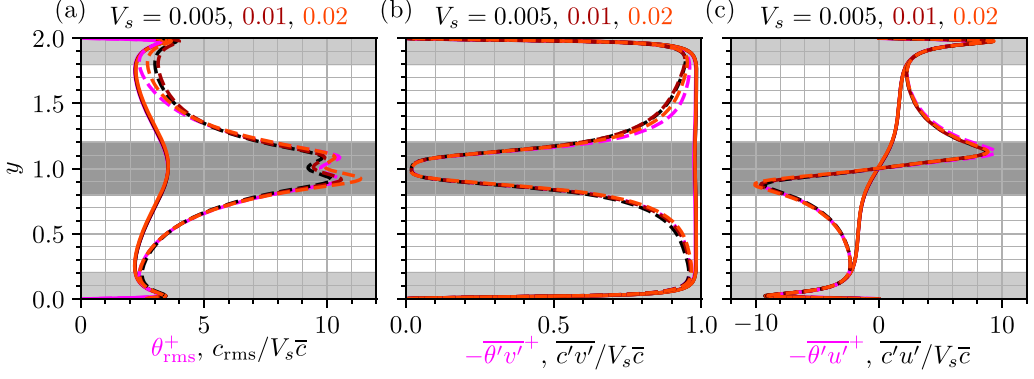


FIG. 6. Temperature and sediment concentration statistics, with temperature statistics scaled by θ_τ and concentration statistics scaled by $V_s \bar{c}$. Solid lines represent Case U data, dashed lines represent Case S data.

discrepancies between statistics to increase as V_s increases, given this is parameter that controls the difference between temperature and sediment transport, but this result can be justified by noting that as V_s increases the time scales required for convergence (δ/V_s) decrease, and therefore cases with higher V_s are better converged, and in better agreement with the temperature field statistics.

The collapse of the appropriately scaled vertical scalar fluxes can be explained by the balance of respective scalar transport equations, (11) and (12), and previous discussions. What is less clear is why the other statistics appear to collapse, aside from deviations in the channel core. Some insight can be gained by assessing the production terms for the transport of each of these statistics:

$$\mathcal{P}_{cu} = -\overline{c'v'} \frac{\partial \bar{U}}{\partial y} - \overline{u'v'} \frac{\partial \bar{c}}{\partial y}, \quad \mathcal{P}_{cv} = -\overline{v'v'} \frac{\partial \bar{c}}{\partial y}, \quad \mathcal{P}_{cc} = -2\overline{c'v'} \frac{\partial \bar{c}}{\partial y}, \quad (13)$$

with equivalent production terms for θ statistics obtained by substituting c' and $\bar{c}$ for θ' and $\bar{\theta}$. The collapse of statistics in Fig. 6 implies that production terms of respective statistics should also scale similarly, such that

$$\mathcal{P}_{\theta u}^+ \approx -\frac{1}{V_s \bar{c}} \mathcal{P}_{cu}, \quad \mathcal{P}_{\theta v}^+ \approx -\frac{1}{V_s \bar{c}} \mathcal{P}_{cv}, \quad \mathcal{P}_{\theta \theta}^+ \approx \frac{1}{(V_s \bar{c})^2} \mathcal{P}_{cc}, \quad (14)$$

noting that $c_{\text{rms}}^2 = \overline{c'c'}$ and $\theta_{\text{rms}}^2 = \overline{\theta'\theta'}$. Indeed, Fig. 7 demonstrates that the production terms do collapse subject to this scaling, aside from some small deviations in the channel core.

The collapse of production terms implies that the scalar gradients and fluxes are intricately connected and scale in the same way, which can be readily explained by the gradient diffusion hypothesis. Following Nielsen and Teakle [13], the gradient diffusion hypothesis can be derived via a Taylor series expansion of a concentration field $c(y \pm l_m/2)$ undergoing some vertical mixing process of length scale l_m (assumed small) and velocity scale v_m . The subsequent vertical sediment flux can be approximated by

$$-c'v' \approx v_m [c(y + l_m/2) - c(y - l_m/2)] \approx v_m l_m \frac{\partial c}{\partial y} \left(1 + \frac{l_m^2}{24} \frac{\partial^3 c}{\partial y^3} + \dots \right). \quad (15)$$

To leading order,

$$-c'v' = v_m l_m \frac{\partial c}{\partial z} = K_c \frac{\partial c}{\partial z}, \quad (16)$$

where $K_c = v_m l_m$ represents (Fickian) turbulent diffusion of the sediment field. The vertical sediment (and indeed temperature) flux is therefore intricately linked to the vertical sediment

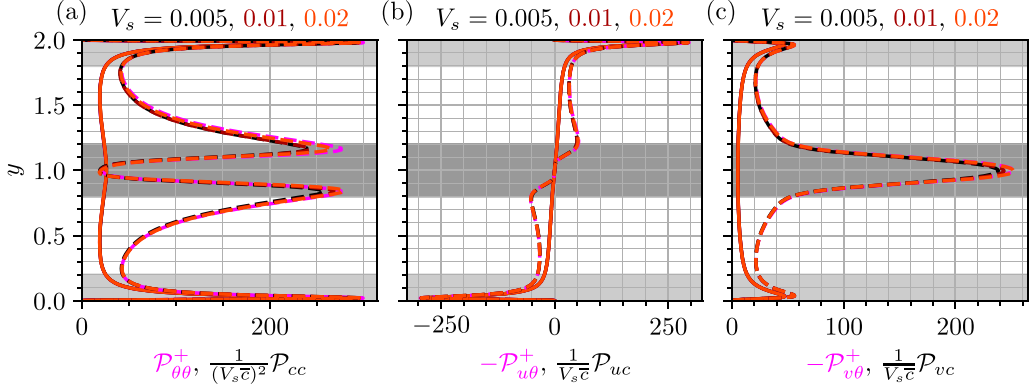


FIG. 7. Production of passive (sediment concentration) and active (temperature) scalar fluxes and fluctuations, scaled by θ_τ for temperature and $V_s \bar{c}$ for sediment. Solid lines represent Case U data, dashed lines represent Case S data.

concentration gradient, if gradient diffusion holds. We should therefore expect to see the mean concentration gradients scale like the mean sediment fluxes, assuming that K_c has no dependence on concentration or V_s . In fact, we observe that K_c and K_θ (equivalent turbulent diffusion for temperature) collapse to approximately the same curve for all scalars, for both Case U and Case S, to leading order [Fig. 8(a)]. Here we have used the definitions $K_c = -\overline{c'v'}/\partial_y \bar{c}$, and $K_\theta = -\overline{\theta'v'}/\partial_y \bar{\theta}$. The leading order collapse of all scalar turbulent diffusivities implies that the mean scalar gradients are strongly related to the mean vertical sediment flux, such that $\partial_y \bar{\theta}^+ \approx \partial_y \bar{c}/V_s \bar{c}$, given that $\overline{\theta'v'}^+ \approx \overline{c'v'}/V_s \bar{c}$. The collapse of the scalar turbulent diffusivities explains the close agreement between their respective turbulent statistics, when appropriately scaled (Figs. 6 and 7).

However, deviations between scalar transport statistics are present in the core of the channel, particularly for $c_{\text{rms}}/V_s \bar{c}$ and θ_{rms}^+ at $y \approx 0.9$, and for $\overline{c'u'}/V_s \bar{c}$ and $\overline{\theta'u'}$ at respective maxima/minima. Note that all statistics collapse to single profiles for unstratified flow; strong buoyancy gradients are the most likely cause for the separation between the different scalars for Case S. A possible cause for this can be revealed by noting that despite seeming similar [Fig. 8(a)], the K_c and K_θ profiles actually harbor large differences in the channel core, particularly in the region where both diffusive and turbulent components of Eqs. (11) and (12) are important (at $y \approx 0.9$, turbulent fluxes contribute approximately 20% to the total budget, shown in Fig. 5).

Deviations between diffusivities are shown explicitly by reporting the turbulent Prandtl number $Pr_t = \nu_t/K$ for the different scalars, where $\nu_t = -\overline{u'v'}/\partial_y \bar{U}$ represents the eddy viscosity [Fig. 8(d)]. First note that the Case U scalars all approximately collapse to $Pr_t \approx 0.8$ aside from strong deviations at the center line where $\overline{u'v'}$ and $\partial_y \bar{U}$ vanish. In contrast, Pr_t varies considerably with both vertical location and scalar species, for Case S. Near the center line Pr_t varies wildly for Case S due to the small momentum flux and velocity gradient [Fig. 2(b)]. More interestingly, the deviation between scalars increases as they approach the core, and reach a maximum deviation of approximately 10–15% at $y \approx 0.9$ and $y \approx 1.1$. The difference between the scalars increases with increasing V_s , consistent with the appropriately scaled turbulent statistics of Figs. 6 and 7. It is also interesting that while ν_t/K_θ is symmetric across the center line, ν_t/K_c is not: Turbulent diffusion is larger (smaller Pr_t) for $y < 1$, and smaller (larger Pr_t) for $y > 1$, for the sediment scalars, and this effect increases with V_s .

Without strong buoyancy gradients (Case U), differences in scalar turbulent diffusivities are negligible. We hypothesize that the differences observed in appropriately scaled sediment and temperature statistics for Case S (Figs. 6 and 7) emerge due to these differences in the turbulent diffusivities, which arise due to a violation of the gradient diffusion hypothesis. In particular, the

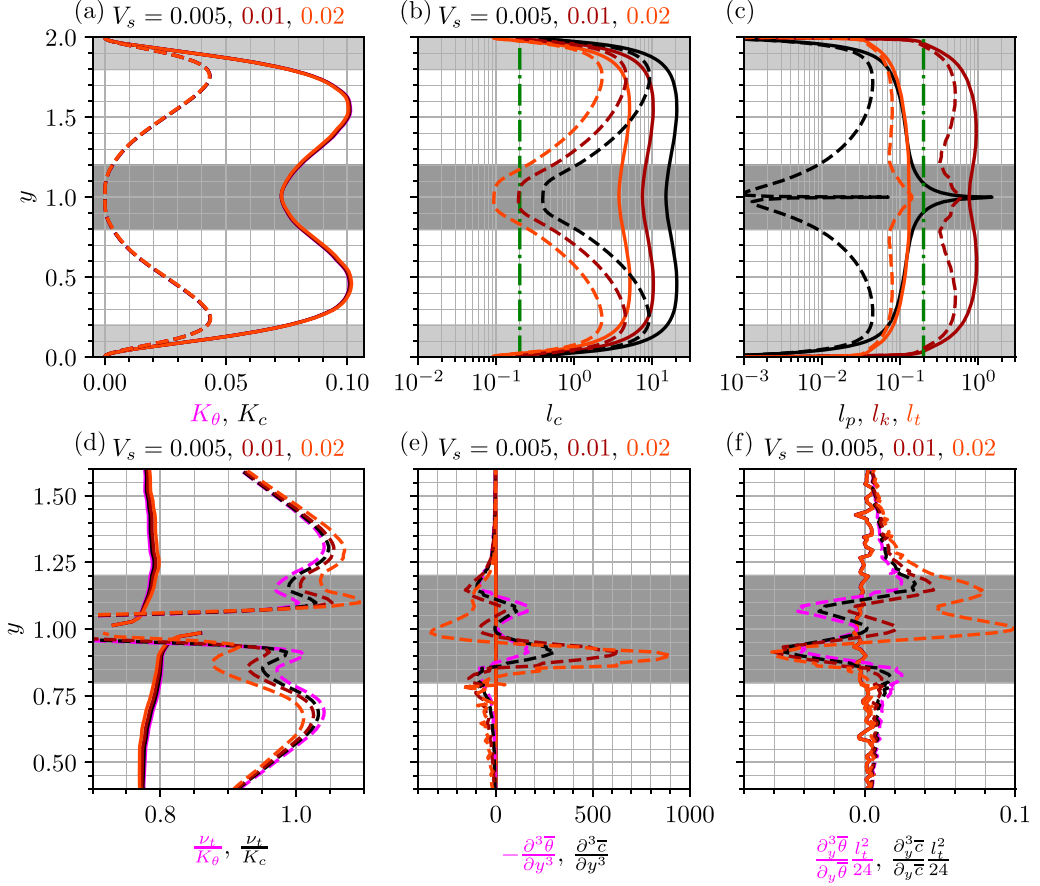


FIG. 8. Turbulent diffusivities of passive (sediment concentration) and active (temperature) diffusivities (a). (b) shows estimations of the concentration length scale l_c , while (c) shows three estimates the mixing length scale (l_m). The vertical dashed line of (b) and (c) marks a length scale of 0.2, equal to the half-width of the channel core. (d) shows the turbulent Prandtl numbers of the different scalar fields. (e) reports the third-order vertical gradients of mean temperature and sediment concentration. (f) estimates the correction term of Eq. (15), using the Taylor length scale as an approximation for l_m . In all subfigures, solid lines represent Case U data and dashed lines represent Case S data.

higher-order terms of the Taylor series expansion (15) can only be neglected if the mixing length l_m is much smaller than the indicative length scale over which the concentration field varies (here termed l_c). This was formally demonstrated by Nielsen and Teakle [13] for the idealised case of an exponentially decaying concentration field, where $l_c \gg l_m$ is required for the higher-order terms of Eq. (15) to be neglected. When stratification is present, and at higher values of V_s , the concentration field varies greatly over small distances at the edge of the channel core, yet the length scales over which turbulent mixing occurs are relatively large.

To provide evidence for this we attempt to estimate the next correction term of Eq. (15). However, this requires a choice to be made regarding the mixing length l_m , which is implicitly connected to the mixing velocity v_m and the scalar turbulent diffusivities. The mixing length is an empirical quantity with several choices available for its estimation. We first consider the Prandtl mixing length (l_p), where $l_p^2 = -\overline{u'v'}/\partial_y \overline{U}$, which implicitly assumes $v_m = l_p |\partial_y \overline{U}|$. While this is a good approximation for near-wall flows, it is a poor choice near the turbulent channel center line due to

the vanishing momentum flux and velocity gradient. Instead, an alternative choice for l_m could be that implied by two-equation turbulence models [42]: $l_k = k^{3/2}/\varepsilon$, where $k = \frac{1}{2}\overline{u_i' u_i'}$ represents TKE, and ε represents the TKE dissipation rate. Finally, we estimate the Taylor microscale l_t , where $l_t^2 = 10\nu k/\varepsilon$. The Taylor microscale marks the transition from energy-containing eddies and the scales where viscous dissipation becomes important. We also define $l_c = \bar{c}/(\partial_y \bar{c})$, which represents the length scale over which the mean sediment concentration varies.

These various length scales are reported in Figs. 8(b) and 8(c). l_c is very large across much of the channel, particularly for Case U scalars, indicating very slow variation in concentration. For both Case U and Case S, l_c decreases with V_s , as would be expected with the resultant increase in concentration gradient. While l_c is approximately constant across the full channel height for Case U, it reduces by over an order of magnitude in the channel core for Case S, relative to the outer regions. For Case S, l_c decreases by over an order of magnitude in the channel core for all three sediment species, and the sediment with the highest settling velocity ($V_s = 0.02$) leads to $l_c \lesssim 0.3$ in the channel core bounds.

The mixing length estimates vary strongly depending on their construction. For Case U, l_p and l_t approximately collapse for the majority of the channel, except for the near-wall region and near the center line where l_p increases dramatically due to its dependence on $\partial_y \bar{U}$. l_p is therefore a poor choice of length scale in the present study where we are interested in channel core/outer region dynamics. l_k is the largest of the three length scale choices, reaching $\mathcal{O}(1)$ away from the walls. For Case U, l_c is at least an order of magnitude larger than l_p and l_t . While $l_c > l_k$ everywhere, the ratio l_k/l_c cannot be considered small for $V_s = 0.02$. Note however, if the model coefficient $C_\mu = 0.09$ [42] were included in this construction of the mixing length ($l_m \sim C_\mu k^{3/2}/\varepsilon$) it would be reduced to the same order of magnitude as l_t and l_p .

When stratification is present, l_t and l_k predict qualitatively similar behavior, with a general reduction in mixing length when compared to Case U aside from the inner regions, and a local increase in mixing length in the core. These two estimates of l_m vary in terms of magnitude, but are in much better agreement when compared to l_p which is very small in the core, and begins to vanish rapidly across the majority of the outer regions. As we shall show in Sec. III D, this is in qualitative disagreement with the large-scale hairpin vortices that dominate the outer regions of the stratified channel flow [22]. For reference, we have included $l_m = 0.2$ in the visualizations of l_c and l_m (Fig. 8), which is equal to the half-height of the approximate channel core region. This approximation of l_m , which falls between the Case S estimates l_t and l_k , is motivated by the approximate vertical amplitude of internal waves in the channel core, which will be discussed in Secs. III B and III D. It is clear that l_m predictions vary strongly depending on their construction. However, aside from l_p which is ill-posed in the channel core, estimates of l_m are comparable to l_c in much the channel core for Case S. This is certainly true for l_k which is greater than l_c for $V_s = 0.02$. l_t is a more conservative estimate but still reaches at least 40% of l_c for $V_s = 0.02$ within the channel core bounds. This flow therefore does not meet the condition that $l_m \ll l_c$ for gradient diffusion to hold. Given that the correction term of Eq. (15) scales like l_m^2 we choose to adopt l_t as a conservative approximation of the mixing length, due to its close agreement with l_p in the outer regions of Case U, and its qualitative agreement with l_k for Case S, despite the lower magnitude.

The approximate high-order correction term of Eq. (15) is reported in Fig. 8(f). For reference, the third-order mean scalar vertical derivatives are reported in 8(e), which show that, while third-order derivatives vanish for Case U, they are very large for Case S in and near the channel core, reaching their maximum value at $y \approx 0.9$, and increasing with increasing V_s . This location coincides with that of the maximum deviation in Pr_t between the different scalars [Fig. 8(d)]. As expected, the high-order correction is approximately zero for Case U [Fig. 8(f)], but is appreciable for Case S, reaching a maximum of 10% relative to the leading order term of Eq. (15). The term increases with increasing V_s , and at $y = 0.9$ is approximately 6% for $V_s = 0.02$ and 2.5% for the temperature field. The correction is larger at and above the center line, and there is a transition across the core from large negative corrections at $y \approx 0.9$ to large positive corrections at $y \approx 1.1$. This sign-change

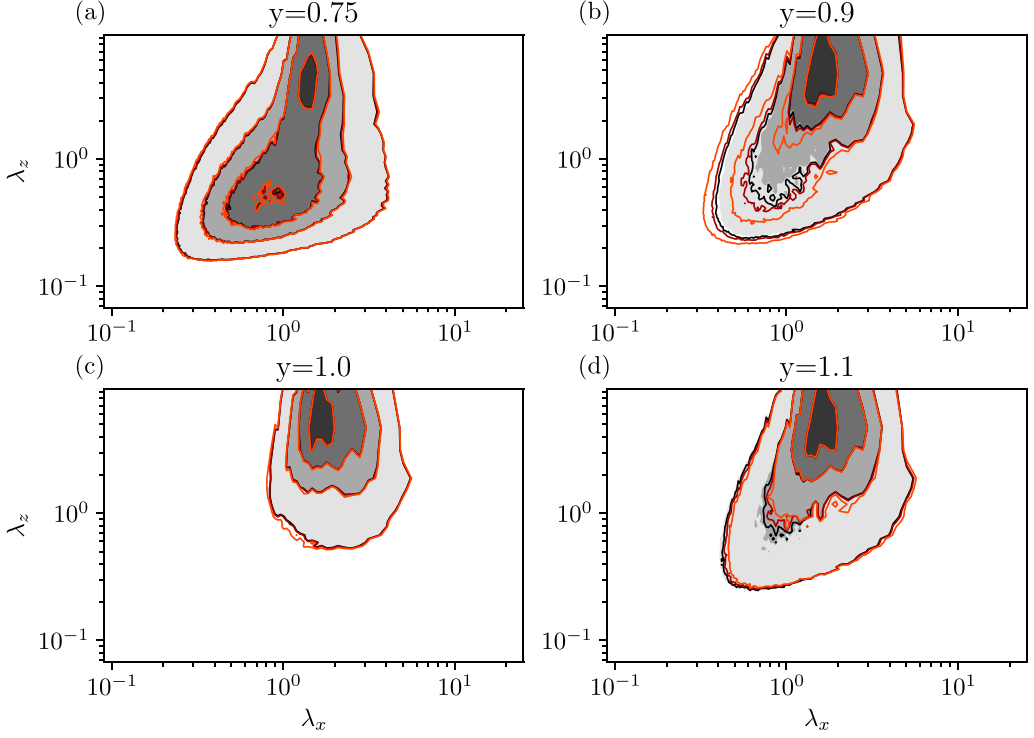


FIG. 9. Premultiplied 2D energy spectra of scalar fluctuations as a function of streamwise and spanwise wavelengths at four y positions. Grey filled contours represent temperature spectra, $k_x k_z E_{\theta\theta}^{2D}$, and colored contour lines represent sediment concentration spectra, $k_x k_z E_{cc}^{2D}$, for $V_s = 0.005$ (black), $V_s = 0.01$ (red), and $V_s = 0.02$ (orange). Contour values represent 20 %, 40 %, 60 %, and 80 % of respective maxima.

is in agreement with the sign-change of the relative differences between sediment concentration turbulent Prandtl numbers and the temperature turbulent Prandtl number [Fig. 8(d)]. Qualitatively, the correction term is similar when l_k is adopted instead of l_t , but reaches $\mathcal{O}(1)$ in magnitude. The comparisons between the concentration and mixing length scales, the behavior of the mean third-order scalar derivatives, and the estimations of the high-order correction term, provide evidence that the linear gradient diffusion hypothesis fails in the channel core region where concentration gradients become very large. This, therefore, provides an explanation as to why turbulent statistics deviate in these regions: The correlation between vertical turbulent fluxes and mean scalar gradients is less strong due to the effects of high-order terms on the turbulent diffusivities, and therefore the turbulent Prandtl numbers. Of course, at the center of the channel turbulent mixing processes have a small influence on dynamics due to their suppression by buoyancy forces, but closer to the edges of the core (e.g., $y \approx 0.9$) turbulence has a leading order effect. These results therefore suggest that the coincidence of large-scale mixing events and strong concentration gradients leads to a deviation from first-order gradient diffusion.

B. Spectra

2D energy spectra for the scalar fluctuations are presented in Fig. 9. Here we calculate energy spectra as a function of streamwise and spanwise wavelength, λ_x and λ_z , on several y -normal slices. The contours presented represent 20 %, 40 %, 60 %, and 80 % of respective maxima. First note that we only report spectra in and near the channel core, finding that all scalars collapse to the same spectra in the majority of the channel. An example of the clear collapse of all scalars

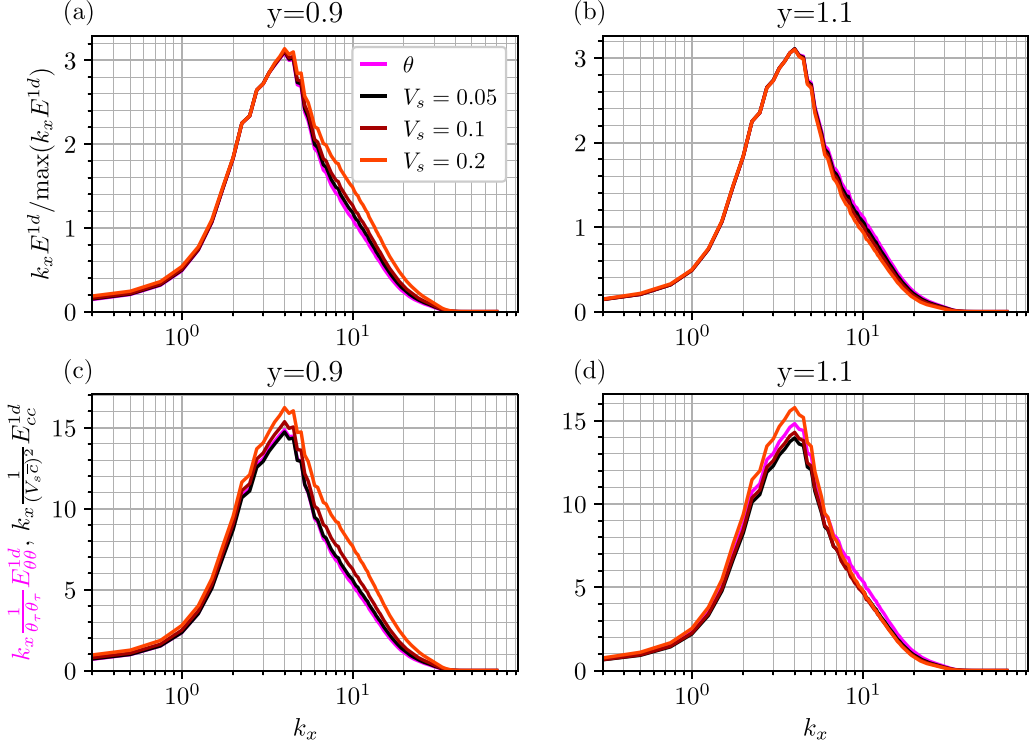


FIG. 10. Premultiplied 1D energy spectra of scalar fluctuations at $y = 0.9$ in (a) and (c), and $y = 1.1$ in (b) and (d), as a function of streamwise wave number. Spectra are normalized by respective maxima in (a) and (b), and θ_τ^2 for temperature spectra and $(V_s \bar{c})^2$ for sediment concentration spectra in (c) and (d). Lines are as per the legend in (a).

is in Fig. 9(a), where all four scalars show two distinct peaks in spectral energy. The higher λ_z peak is associated with the large-scale internal waves in the channel core, while the lower λ_z peak is associated with turbulent activity [22,29]. The peak associated with turbulence subsides in the channel core, until only the peak associated with internal waves is present at $y = 1$. At $y = 1$ we see all scalars collapse onto essentially the same energy spectra contours, demonstrating that the effect of internal waves is essentially the same for all scalar transport, relative to respective spectral maxima. However, deviations between the scalars are clear closer to the edges of the core, at $y = 0.9$ and $y = 1.1$. While the peak associated with the internal waves shows reasonable collapse between the scalars, the tail of the spectra deviates significantly, with larger differences associated with larger V_s . This indicates that it is the turbulent structures that are the cause of deviations between scalar fluctuations, not the largely reversible internal wave motions. Deviations of E_{cc}^{2D} from $E_{\theta\theta}^{2D}$ manifest in a broadening spectral peak for $y = 0.9$ as V_s increases, and a sharpening spectral peak at $y = 1.1$. In other words, the contour lines shift outwards as V_s increases at $y = 0.9$, but inwards at $y = 1.1$.

1D spectra at $y = 0.9$ and $y = 1.1$ are reported in Fig. 10, as a function of streamwise wave number $k_x = 2\pi/\lambda_x$. In addition to normalization by respective maxima, spectra are shown scaled by θ_τ for temperature and $V_s \bar{c}$ for sediment concentration. Both scalings lead to qualitatively similar behavior. At $y = 0.9$ higher V_s leads to a broader range of energetic scales, particularly at the higher wave numbers associated with turbulence. At $y = 1.1$ higher V_s leads to a sharper peak associated with the internal waves, with a reduced spectral energy associated with turbulence. We speculate that this difference leads to the asymmetry in the turbulent Prandtl numbers [Fig. 8(d)] for the sediment concentrations, where, relative to ν_t/K_θ , we see ν_t/K_c increase in the upper regions of the

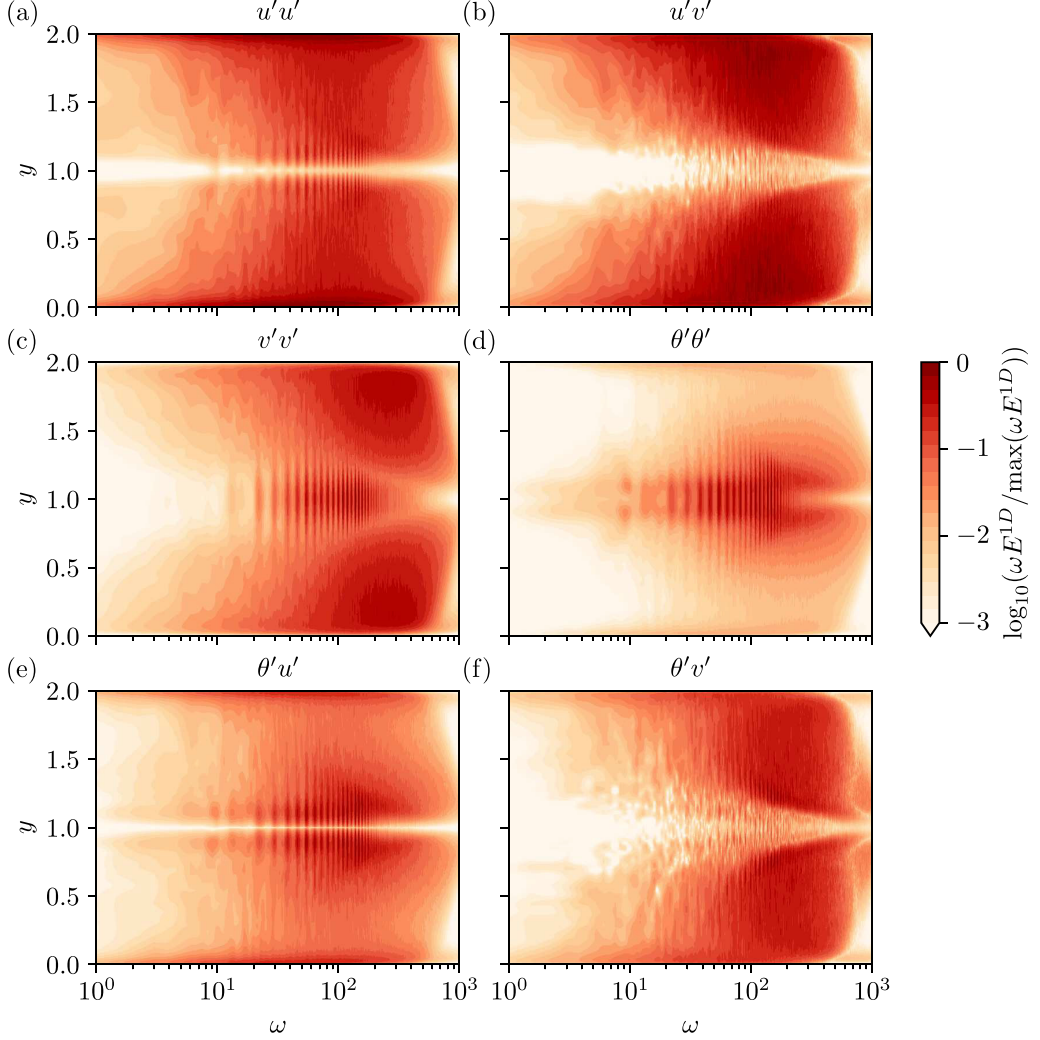


FIG. 11. Premultiplied 1D energy spectral magnitude as a function of temporal frequency and vertical coordinate for Case S. (a)–(f) represent different statistics for fluid velocity, temperature, and associated fluxes. All spectra are normalized by respective maxima.

core ($y \approx 1.1$), and decrease in the lower regions ($y \approx 0.9$), as V_s increases. This is also consistent with the changes in the high-order correction term of Eq. (15) [Fig. 8(f)], supporting the argument that it is a breakdown of the leading order gradient diffusion hypothesis that causes deviations between scalar transport statistics. This argument is also further supported when noting that the deviations occur at relatively large turbulent scales with $\lambda_x \sim 1$ and $\lambda_z \sim 0.6$ (Fig. 9); while vertical mixing scales will be smaller than the horizontal scales, particularly due to suppression by buoyancy forces, the magnitudes of l_t and l_k appear reasonable compared to these streamwise and spanwise wavelengths.

1D Temporal spectra are presented in Figs. 11 and 12, presented as a function of vertical coordinate y and frequency ω , for Case S. Figure 11 presents temperature and velocity statistics, clearly showing the extent and influence of the internal waves in the channel core. Spectral energy for $u'u'$, and for streamwise and vertical momentum and temperature fluxes all show local minima

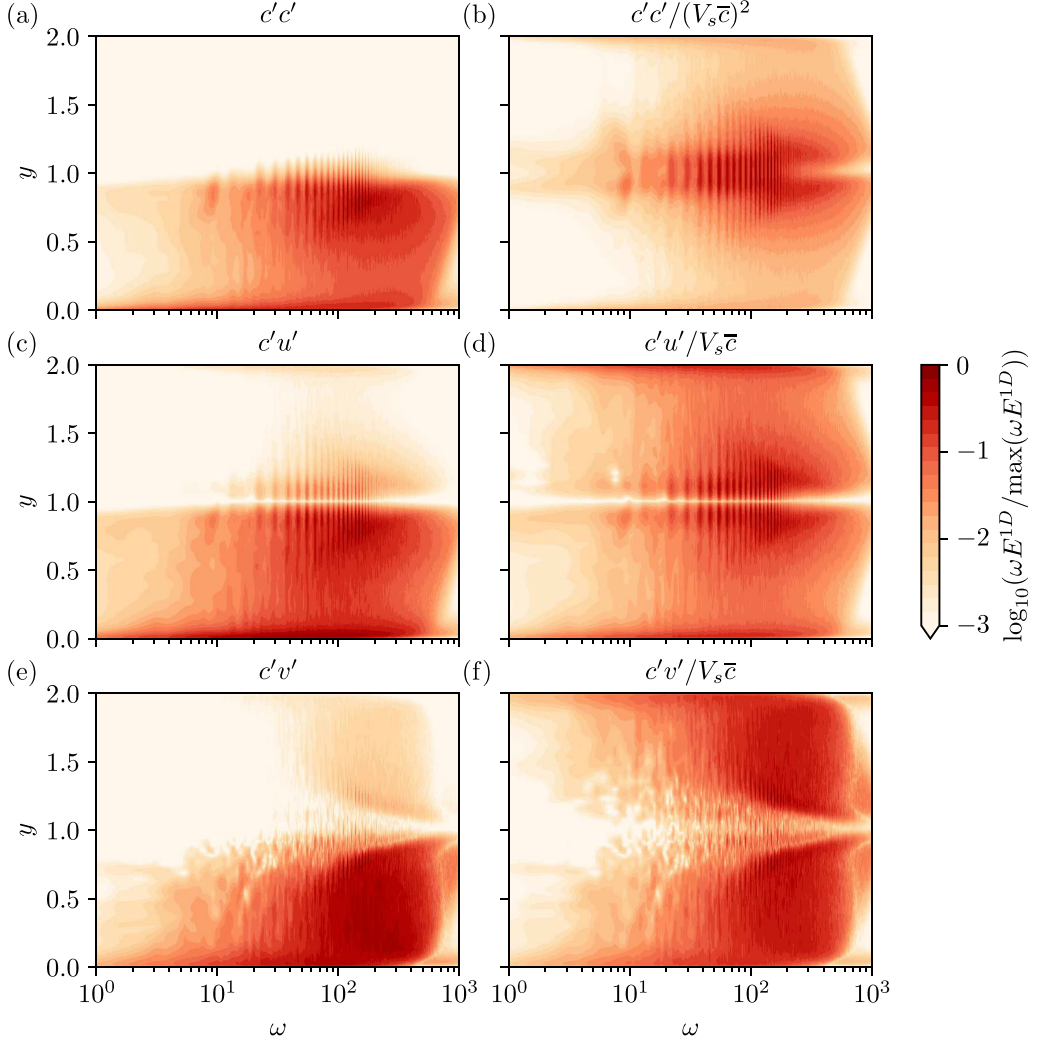


FIG. 12. Premultiplied 1D energy spectral magnitude as a function of temporal frequency and vertical coordinate for Case S. (a), (c), and (e) present spectra associated with sediment concentration fluctuations, streamwise concentration flux, and vertical concentration flux, respectively, for $V_s = 0.02$. (b), (d), and (f) present the same spectra as (a), (c), and (e) but scaled by $V_s \bar{c}$. All spectra are normalized by respective maxima.

at $y = 1$. The streamwise flux of momentum and temperature is zero at $y = 1$ for both Case S and Case U, owing to the symmetry of the flow and the local velocity maxima. In contrast, the minima for vertical momentum and temperature flux, and streamwise velocity fluctuations arises due to the presence of strong buoyancy gradients and subsequent internal waves and suppression of turbulent irreversible motion. Internal waves manifest as coherent and distinct peaks in spectral energy at discrete temporal frequencies, focused in the channel core. The discrete temporal frequencies ω arise due to the finite spatial bounds of the periodic computational domain. Specifically, these peaks correspond to the convection of streamwise wave numbers k_x in multiples of $2\pi/L_x$ Lloyd *et al.* [22]. An interesting aspect of Fig. 11 is the clear vertical bounds of the distinct and discrete peaks associated with internal waves, complementing the spectral and dynamic mode decomposition data of Lloyd *et al.* [22]. Figure 11 also shows that buoyancy gradients in the channel core suppress the

large-scale momentum and vertical fluxes more than the small-scale (or high frequency) structures. The suppression of large-scale vertical fluxes arises due to the buoyancy and concentration fluctuations being out-of-phase with the vertical velocity fluctuations, due to the internal waves (this will be demonstrated using SPOD in Sec. III C).

Temporal spectra for the sediment fluctuations and fluxes are presented in Fig. 12, for Case S and $V_s = 0.02$. The imprint of the internal waves on sediment statistics is clear, with spectral amplitudes dramatically decreasing above the channel core, particularly for $c'c'$. When scaled by $V_s \bar{c}$ sediment spectra are in clear agreement with the temperature statistics of Fig. 11, further demonstrating the similarities between the transport of the different scalars.

C. Spectral Proper Orthogonal Decomposition (SPOD)

To further establish the role internal waves have on sediment transport we adopt SPOD to identify the coherent motion in the channel core. SPOD is performed on z -normal slice data using the method of Schmidt and Colonius [43]. Data are organized into snapshots $\mathbf{q}(\mathbf{x}, t)$ consisting of streamwise and vertical velocity, temperature, and sediment concentration for $V_s = 0.02$. Following the method of Welch, snapshots are split into 35 consecutive and overlapping (with a 50 % overlap) blocks ($N_{\text{blk}} = 35$), each containing snapshots ($N_{\text{fft}} = 2048$). Fourier transforms are subsequently performed on each block containing N_{fft} snapshots, using a Hamming window, to obtain $\hat{\mathbf{q}}(\mathbf{x}, \omega)$.

After obtaining $\hat{\mathbf{q}}$, analysis is performed one frequency at a time. For each frequency the data are arranged into a matrix $\hat{\mathbf{Q}}$,

$$\hat{\mathbf{Q}} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ \hat{q}^1 & \hat{q}^2 & \dots & \hat{q}^N \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}, \hat{\mathbf{Q}} \in \mathbf{C}^{M \times N}, \quad (17)$$

where M is the total number of degrees of freedom and N is the number of realizations (equal to the number of data blocks, N_{blk}) at the given frequency. The SPOD modes, $\hat{\Phi}$, are eigenvectors of the sample cross-spectral density, given by

$$\hat{\mathbf{C}} = \frac{1}{N-1} \hat{\mathbf{Q}} \hat{\mathbf{Q}}^H, \quad (18)$$

which are obtained by solving either

$$\hat{\mathbf{C}} \mathbf{W} \hat{\Phi} = \hat{\Phi} \hat{\Lambda}, \quad (19)$$

or

$$\hat{\mathbf{Q}}^H \mathbf{W} \hat{\mathbf{Q}} \hat{\Psi} = \hat{\Psi} \hat{\Lambda}, \quad \hat{\Phi} = \hat{\mathbf{Q}} \hat{\Psi}, \quad (20)$$

where $\mathbf{W}$ is a weighting matrix, and $\hat{\Lambda}$ represents the eigenvalues or energies associated with the SPOD modes $\hat{\Phi}$. In our case $M \gg N$ making the second formulation favorable, which computes the intermediate left singular vectors, $\hat{\Psi}$, before obtaining the SPOD modes. This approach avoids computation of the $M \times M$ cross-spectral density matrix $\hat{\mathbf{C}}$ and the subsequently high-dimensional eigenvalue problem based on $\hat{\Phi}$. The SPOD modes for a given frequency are orthonormal, with $\hat{\Phi}^H \mathbf{W} \hat{\Phi} = \mathbf{I}$. The weighting matrix $\mathbf{W}$ is chosen to favorably identify coherent structures in the core of the channel, taking the value of 1 for $0.8 \leq y \leq 1.2$, and 0 elsewhere. Tests with and without this weighting function revealed that while the SPOD modes are qualitatively similar, low rank behavior is more clear in the SPOD energies when modes are no longer polluted by near-wall data. For more information, see Schmidt and Colonius [43].

SPOD energies as a function of frequency can be observed in Fig. 13(a). For a given frequency SPOD modes are ranked based upon their energy, with mode 1 corresponding to the highest modal energy. The lines of Fig. 13(a) and markers of Fig. 13(b) are colored by the mode number i . Distinct peaks in Mode 1 and Mode 2 are observed for $20 \lesssim \omega \lesssim 200$ corresponding to the internal waves

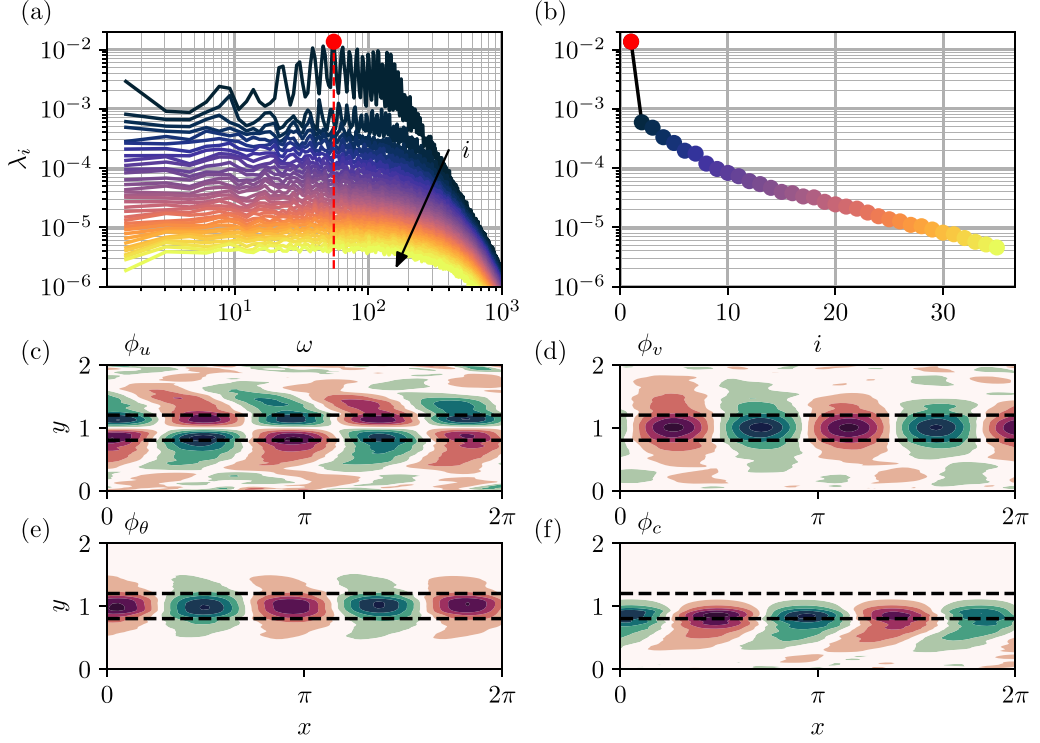


FIG. 13. SPOD of Case S with $V_s = 0.02$, computed using z -normal slice data. Panel (a) reports modal energies $\hat{\lambda}$ as a function of temporal frequency. Lines and Colors represent the mode number i , increasing from 1 to 35, with mode number 1 representing the modes with the highest energy for a given frequency. Modal energies at $\omega = 55.2$, indicated by the vertical dashed line in (a), are plotted against mode number in (b). The spatial structure of the highest energy mode at $\omega = 55.2$ [identified by the red marker in (a) and (b)] is reported in (c)–(f), where horizontal dashed lines represent the approximate bounds of the channel core region, $y = 0.8$ and $y = 1.2$. The streamwise velocity, vertical velocity, temperature, and sediment concentration components of $\hat{\Phi}$ are reported in (c) to (f), respectively.

in the channel core. These distinct peaks correspond to those observed in the temporal spectra of Figs. 11 and 12, and appear low-rank with mode 1 energies at least an order of magnitude higher than mode 2 energies. Figure 13(b) shows the modal energies associated with the SPOD for the frequency corresponding to the vertical dotted line in Fig. 13(a), where the low-dimensional nature of the SPOD is more clear. Interestingly, the peaks and troughs associated to the internal waves for mode 1 energies are reversed for mode 2 energies, with mode 2 energies typically decreasing when peaks in mode 1 are observed. This further demonstrates the low-rank nature of the internal waves in the channel core. Figures 13(c)–13(f) visualize the spatial structure of the SPOD mode corresponding to the modal energy identified by the red marker in Figs. 13(a) and 13(b). The u , v , θ , and c components of $\hat{\Phi}$ are shown, in clear agreement with the dynamic mode decomposition data of Lloyd *et al.* [22]. The peaks and troughs in vertical velocity and temperature components are out of phase and concentrated in the core region $0.8 \lesssim y \lesssim 1.2$. The influence of the waves extends into the outer regions but the magnitude of the mode is significantly decreased. The c component of the SPOD modes is negatively correlated to the temperature field; peaks in Φ_θ correspond to troughs in Φ_c , owing to the difference in sign between respective wall-normal gradients. The key difference between the different scalars is the downward offset of Φ_c peaks, which have their maximum value near the lower bound of the core, $y \approx 0.8$. This offset is consistent with the local

maxima of sediment RMS fluctuations and vertical sediment concentration gradient (Fig. 4). The tails of Φ_c extend further into the turbulent regions of the flow than Φ_θ , and peaks/troughs do not extend above the bounds of the core. The influence of channel core waves extends deeper into outer regions of the flow for high settling velocity sediments. This effect is due to the large concentration gradient over the channel core; while not shown here, scaling the SPOD modes appropriately by $V_s \bar{c}$ unsurprisingly leads to a more symmetric mode, further demonstrating the importance of this scaling on concentration statistics.

SPOD clearly demonstrates that while the internal waves are prolific in the channel core, their influence on sediment statistics is not limited to that region, and that the coincidence of large-scale structures and steep concentration gradients may lead to nontrivial dynamics not captured by leading order gradient diffusion models.

Note that the spatial structure for all internal wave modes are qualitatively similar and so are not shown here, differing only in the number of structures that fit in the domain (i.e., their streamwise wave number k_x). Sensitivity of the decomposition to the parameters N_{blk} and N_{fft} was assessed by also computing SPOD for $N_{\text{fft}} = 4098$, reducing the number of blocks to $N_{\text{blk}} = 18$. While results were qualitatively similar, some sensitivity was observed for the low frequency peaks in energy, where the higher N_{fft} better resolved individual peaks at the expense of higher levels of noise.

D. Instantaneous structure

The instantaneous structure of Case S is shown in Fig. 14, where a snapshot of the flow is visualized on a z -normal plane. Temperature contours overlay all the figures with negative θ marked with dashed lines and positive θ with solid lines, clearly indicating the approximate bounds of the core region, where contours are approximately parallel and tightly packed (indicating the strong buoyancy gradient). The internal wave field is most easily observed through the contours of v' , which show minima/maxima occurring throughout the channel core, approximately out of phase with the temperature contours. The general shape of the sediment concentration contours are in close agreement with the temperature field, particularly in the core where turbulence is suppressed, and wave elasticity can form a barrier to vertical scalar transport [5]. Of particular note is the clear downward offset of the c' minima/maxima, in agreement with the statistics presented in the previous sections.

The hairpin structures identified by Lloyd *et al.* [22] can be identified in the sediment streamwise and vertical fluxes, which act to transport sediment into the channel core from the outer regions. This is most clear from the $c'u'$ fluxes which indicate large peaks in negative $c'u'$, where relatively slow high concentration fluid has been lifted up from the outer portion of the flow. In the core $c'v'$ has local minima/maxima corresponding to these ejection events, although these upward/downward vertical fluxes are in approximate balance owing to the mostly reversible motions of the waves in the channel core. The vertical fluxes are however large in the outer regions of the flow and at the core edge, where ejections of hairpins contribute most to vertical sediment flux. The intermittency of these structures is worth considering in regard to mean sediment transport. Sediment fluxes within the core are comprised of relatively large-scale and energetic ejection events into the quiescent channel core, but these occur intermittently [22], and therefore the time-averaged scalar flux is relatively small. However, the fact that these structures are relatively large (of a similar order to the channel-core half-height when observing Fig. 14), results in the situation where strong concentration gradients intersect with large mixing lengths, violating the linear gradient diffusion hypothesis.

IV. DISCUSSION AND CONCLUSIONS

We have simulated the passive transport of buoyant particles in a channel flow stratified by thermal gradients. Stratification has a profound impact on sediment transport in the core of the channel, where buoyancy forces dominate. We find that stratification introduces a strong transport

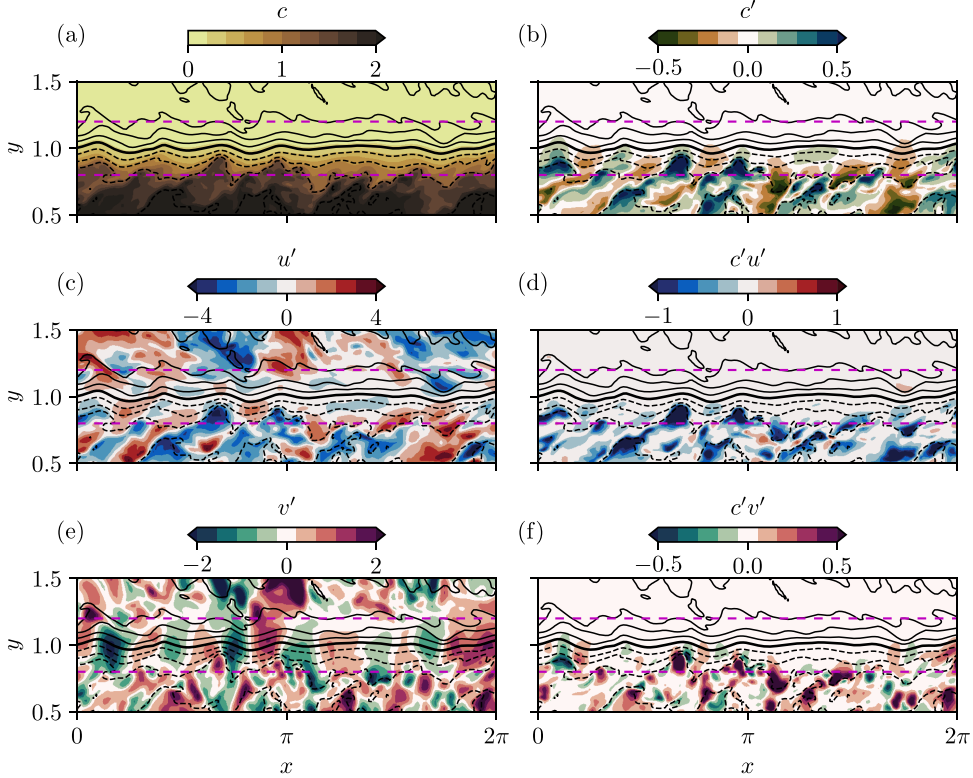


FIG. 14. Instantaneous structure of Case S with $V_s = 0.02$ plotted using z -normal slice data. Data are sediment concentration (a), sediment concentration fluctuations (b), streamwise velocity fluctuations (c), streamwise sediment flux (d), vertical velocity (e), and vertical sediment flux (f). Solid lines are temperature contours with negative values indicated by dashed lines, and the $\theta = 0$ contour represented by a thicker line. Dashed horizontal lines represent the approximate channel core bounds, $y = 0.8$ and $y = 1.2$.

barrier in the core, leading to a two-layer sediment concentration profile with sediment trapped in the lower regions of the channel.

The strong sediment concentration gradients are inherently coupled to the temperature profiles, due to their passive transport. In the parameter regime investigated ($Re_\tau = 550$, $Ri_\tau = 480$), thermal gradients are largest at the channel center line due to the local low turbulence, which is insufficiently diffusive to mix the temperature profile. The subsequent strong buoyancy forces arising at the center line lead to suppressed turbulence across the channel core region [22]. The channel core is coupled to the outer region turbulence; hairpin vortices, originating in the outer region of the flow, perturb the channel core and lead to large-scale internal waves and wave-breaking events [22]. Here, we have found that when passively transported sediment is introduced into this system, the channel core acts as a barrier to vertical mixing of the sediment, potentially due to both the local low levels of intermittent turbulence, akin to the slow-diffusion zone hypothesized by Luchi *et al.* [44], and due to internal wave elasticity [5], where mixing across the barrier only occurs during extreme wave-breaking events.

Concentration gradients and fluctuations are enhanced by stratification, but are offset from the core center line due to the enhanced concentration beneath the core, an effect that increases with the particle settling velocity, V_s . In addition, the influence of V_s on sediment statistics is enhanced by stratification due to the large differences in sediment concentrations above and below the channel core. SPOD and temporal spectra show that the influence of waves on sediment is offset downwards

compared to temperature, due to the increased sediment concentration in the lower region of the channel; sediment fluctuations are negatively correlated to the temperature fluctuations, and as V_s increases the peak in fluctuations due to waves shifts further downwards towards the core-edge, and the influence of waves is felt through more of the channel outer region.

However, we find that the differences between scalar transport of the three sediments and the temperature field are approximately the same when appropriately scaled by θ_τ for temperature and $V_s \bar{c}$ for the sediment concentrations. These scalings collapse all statistics to approximately the same vertical profiles and spectra. Given that only the temperature field is two-way coupled to the momentum equation, it follows that the sediments are indirectly coupled to temperature transport. The approximate collapse of statistics for the different scalars can be explained by assessing their vertical transport budgets which have functionally the same form when appropriately scaled; a turbulent scalar flux balances a term proportional to the mean vertical gradient of the scalar. In the case that either term dominates over the other it is clear that either $-\partial_y \bar{c} / Sc V_s \bar{c} \approx \partial_y \bar{\theta}^+ / Pr \approx 1$ or $\overline{v'c'} / V_s \bar{c} \approx -\overline{v'\theta'}^+ \approx 1$. Further, turbulent statistics for the different scalars approximately collapse using the same scaling if the gradient diffusion hypothesis holds, given that production of turbulent fluctuations and fluxes are dependent on not only the scalar fluxes but also the scalar mean vertical gradients.

However, when stratification is strong these scalings are unable to collapse statistics in and at the edges of the channel core. Here, differences between scalars increase with increasing V_s ; deviations in the scalar turbulent diffusivities reach 10 to 15 % when $V_s = 0.02$, in the channel core. We argue that these deviations arise due to the violation of the gradient diffusion hypothesis, where concentration length scales are small compared to mixing length scales. This is demonstrated by estimating length scales in Fig. 8, and inspection of instantaneous data in Fig. 14, where intermittent large-scale coherent structures, ejected from the outer regions of the flow, are the main source of turbulent mixing in the core of the channel. In addition, deviations between $\partial_y^3 \bar{\theta}$ and $\partial_y^3 \bar{c}$ become very large in the channel core, which are the first neglected derivatives in the Taylor Series expansion for $\overline{\theta'v'}$ and $\overline{c'v'}$ if the linear gradient diffusion hypothesis were to hold. Through estimation of the mixing length, we approximate the next correction term of this Taylor Series expansion, finding that it peaks at approximately 8% of the leading order term in the locations where the scalar turbulent statistics and diffusivities deviate the strongest ($y \approx 0.9$ and $y \approx 1.1$). When inspecting spatial spectra at these locations we see that differences between the scalars arise due to turbulent scales rather than at the wave numbers associated with internal waves, supporting the conclusion that it is turbulent mixing that lead to the differences between scalar transport.

However, it is important to discuss the challenges associated with estimating the mixing length. Here we have chosen to adopt the Taylor microscale as a conservative estimate of l_m , although note its good agreement with the key wave numbers of spatial spectra and the instantaneous flow visualizations. These estimates do not consider flow anisotropy which is of course important, particularly for stratified flows. Perhaps then, the RMS vertical velocity would be a more appropriate velocity scale in the Taylor microscale estimates than the square-root of the TKE, although we note that the dominance of internal waves in the core-region results in vertical velocities dominating over other components [Fig. 2(c)]. As a result, l_m estimates in the channel-core change little when replacing the velocity scale of l_t . In addition, estimations of l_m do not account for flow intermittency, which is important to consider since turbulent mixing in the otherwise quiescent channel core is driven by intermittent ejection events of hairpin vortices. We, therefore, argue that mixing lengths based on time-averaged data will underpredict the true mixing lengths in such flows. Despite these issues, there is clear evidence that it is the breakdown of the gradient diffusion hypothesis in the channel core, or the importance of higher-order terms in the Taylor series expansion (15), that is the primary cause of the separation between the different scalars.

This study therefore supports the conclusions of Nielsen and Teakle [13]; to adequately model flows with strong concentration gradients that coincide with large-scale mixing events a higher-order diffusivity coefficient is required. While this study presents an idealised sediment transport problem

the dynamics explored are commonplace in natural and industrial flows. Stratification, whether imposed through background variations in composition or suspended sediments, dominate dynamics in regions of strong buoyancy gradients, leading to scalar transport barriers where concentration gradients are large [5]. Relatively large-scale intermittent mixing events will therefore lead to deviations in vertical scalar turbulent diffusivities, and leading order gradient diffusion models will be unable to accurately predict dynamics [13]. This study was limited to small V_s but we demonstrate that increasing V_s leads to larger concentration gradients and therefore smaller concentration length scales compared to turbulent mixing length scales. We therefore speculate that further increase of V_s , through an increase of the particle size or density, will have a larger control on sediment transport, where classical models may greatly underpredict turbulent mixing. In addition, future work should investigate the sensitivity of dynamics to incorporation of erosion/deposition mechanics through the sediment concentration boundary conditions. Interactions between the channel-core transport barrier and a nonzero vertical sediment flux across the boundaries would be highly relevant to natural flows. We further speculate that Sc will have a key control on dynamics in the core region, given the dominance of (molecular) diffusive processes. While it is often argued that Sc has a small effect on transport dynamics [See, e.g., Refs. 33–36], it will have a larger influence on regions where turbulence is suppressed [37]. Like V_s , an increase in Sc will sharpen sediment concentration gradients in the channel core, and therefore likely cause further deviations between scalar transport and leading order gradient diffusion models. Future work should also explore the effects of higher concentration particulates that feed back on the fluid momentum via an additional buoyancy forcing term. In this way, stratification could be imposed by the sediment concentration field [20], or a double-diffusive problem could be formulated.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [45].

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- [1] E. Van Sebille, C. Wilcox, L. Lebreton, N. Maximenko, B. D. Hardesty, J. A. Van Franeker, M. Eriksen, D. Siegel, F. Galgani, and K. L. Law, A global inventory of small floating plastic debris, *Environ. Res. Lett.* **10**, 124006 (2015).
 - [2] N. Evangeliou, H. Grythe, Z. Klimont, C. Heyes, S. Eckhardt, Susana Lopez-Aparicio, and A. Stohl, Atmospheric transport is a major pathway of microplastics to remote regions, *Nat. Commun.* **11**, 3381 (2020).
 - [3] W. Wu, S. S. Y. Wang, and Y. Jia, Nonuniform sediment transport in alluvial rivers, *J. Hydraul. Res.* **38**, 427 (2000).
 - [4] A. K. Morrison, D. W. Waugh, A. McC. Hogg, D. C. Jones, and R. P. Abernathy, Ventilation of the southern ocean pycnocline, *Annu. Rev. Mar. Sci.* **14**, 405 (2022).
 - [5] R. M. Dorrell, J. Peakall, S. E. Darby, D. R. Parsons, J. Johnson, E. J. Sumner, R. B. Wynn, E. Özsoy, and D. Tezcan, Self-sharpening induces jetlike structure in seafloor gravity currents, *Nat. Commun.* **10**, 1381 (2019).
 - [6] P. Bartelt, O. Buser, C. V. Valero, and Y. Bühler, Configurational energy and the formation of mixed flowing/powder snow and ice avalanches, *Ann. Glaciol.* **57**, 179 (2016).

- [7] R. Sulpizio, P. Dellino, D. M. Doronzo, and D. Sarocchi, Pyroclastic density currents: State of the art and perspectives, *J. Volcanol. Geotherm. Res.* **283**, 36 (2014).
- [8] O. Bühler, *Waves and Mean Flows* (Cambridge University Press, Cambridge, 2014).
- [9] C. P. Caulfield, Layering, instabilities, and mixing in turbulent stratified flows, *Annu. Rev. Fluid Mech.* **53**, 113 (2021).
- [10] M. G. Wells and R. M. Dorrell, Turbulence processes within turbidity currents, *Annu. Rev. Fluid Mech.* **53**, 59 (2021).
- [11] H. Rouse, Modern conceptions of the mechanics of fluid turbulence, *Trans. Am. Soc. Civ. Eng.* **102**, 463 (1937).
- [12] S. R. McLean, On the calculation of suspended load for noncohesive sediments, *J. Geophys. Res.* **97**, 5759 (1992).
- [13] P. Nielsen and I. A. L. Teakle, Turbulent diffusion of momentum and suspended particles: A finite-mixing-length theory, *Phys. Fluids* **16**, 2342 (2004).
- [14] J. Kim, P. Moin, and R. Moser, Turbulence statistics in fully developed channel flow at low Reynolds number, *J. Fluid Mech.* **177**, 133 (1987).
- [15] R. D. Moser, J. Kim, and N. N. Mansour, Direct numerical simulation of turbulent channel flow up to $Re_\tau = 590$, *Phys. Fluids* **11**, 943 (1999).
- [16] J. C. Del Alamo and J. Jiménez, Spectra of the very large anisotropic scales in turbulent channels, *Phys. Fluids* **15**, L41 (2003).
- [17] A. W. Vreman and J. G. M. Kuerten, Comparison of direct numerical simulation databases of turbulent channel flow at $Re_\tau = 180$, *Phys. Fluids* **26**, 015102 (2014).
- [18] J. Jiménez, The streaks of wall-bounded turbulence need not be long, *J. Fluid Mech.* **945**, R3 (2022).
- [19] M. I. Cantero, S. Balachandar, A. Cantelli, C. Pirmez, and G. Parker, Turbidity current with a roof: Direct numerical simulation of self-stratified turbulent channel flow driven by suspended sediment, *J. Geophys. Res.: Oceans* **114**, 2008JC004978 (2009).
- [20] M. I. Cantero, S. Balachandar, and G. Parker, Direct numerical simulation of stratification effects in a sediment-laden turbulent channel flow, *J. Turbul.* **10**, N27 (2009).
- [21] S. Dutta, M. I. Cantero, and M. H. García, Effect of self-stratification on sediment diffusivity in channel flows and boundary layers: A study using direct numerical simulations, *Earth Surf. Dyn.* **2**, 419 (2014).
- [22] C. J. Lloyd, R. M. Dorrell, and C. P. Caulfield, The coupled dynamics of internal waves and hairpin vortices in stratified plane poiseuille flow, *J. Fluid Mech.* **934**, A10 (2022).
- [23] C. J. Lloyd and R. M. Dorrell, Linear dynamics in turbulent stratified plane poiseuille flow, *J. Fluid Mech.* **999**, A104 (2024).
- [24] T. Watanabe, J. J. Riley, K. Nagata, K. Matsuda, and R. Onishi, Hairpin vortices and highly elongated flow structures in a stably stratified shear layer, *J. Fluid Mech.* **878**, 37 (2019).
- [25] J. S. Salinas, S. Balachandar, M. Shringarpure, J. Fedele, D. Hoyal, and S. Zuñiga, Anatomy of subcritical submarine flows with a lutocline and an intermediate destruction layer, *Nat. Commun.* **12**, 1649 (2021).
- [26] X. Zhang, F. Zonta, Z. F. Tian, G. J. Nathan, R. C. Chin, and A. Soldati, Dynamics of semi-and neutrally-buoyant particles in thermally stratified turbulent channel flow, *Int. J. Multiphase Flow* **139**, 103595 (2021).
- [27] M. Boetti, Pair dispersion of inertial particles crossing stably stratified turbulent/nonturbulent interfaces, *Int. J. Multiphase Flow* **166**, 104502 (2023).
- [28] V. Armenio and S. Sarkar, An investigation of stably stratified turbulent channel flow using large-eddy simulation, *J. Fluid Mech.* **459**, 1 (2002).
- [29] M. Garcia-Villalba and J. C. del Alamo, Turbulence modification by stable stratification in channel flow, *Phys. Fluids* **23**, 045104 (2011).
- [30] E. Meiburg, S. Radhakrishnan, and Mohamad Nasr-Azadani, Modeling gravity and turbidity currents: Computational approaches and challenges, *Appl. Mech. Rev.* **67**, 040802 (2015).
- [31] J. M. Ham and G. M. Homsy, Hindered settling and hydrodynamic dispersion in quiescent sedimenting suspensions, *Int. J. Multiphase Flow* **14**, 533 (1988).
- [32] R. H. Davis and M. A. Hassen, Spreading of the interface at the top of a slightly polydisperse sedimenting suspension, *J. Fluid Mech.* **196**, 107 (1988).

- [33] F. Necker, C. Härtel, L. Kleiser, and E. Meiburg, Mixing and dissipation in particle-driven gravity currents, *J. Fluid Mech.* **545**, 339 (2005).
- [34] M. M. Nasr-Azadani and E. Meiburg, TURBINS: An immersed boundary, Navier-Stokes code for the simulation of gravity and turbidity currents interacting with complex topographies, *Comput. Fluids* **45**, 14 (2011).
- [35] M. M. Nasr-Azadani and E. Meiburg, Turbidity currents interacting with three-dimensional seafloor topography, *J. Fluid Mech.* **745**, 409 (2014).
- [36] J. Pelmaud, S. Norris, and H. Friedrich, LES grid resolution requirements for the of gravity currents, *Comput. Fluids* **174**, 256 (2018).
- [37] C. R. Marshall, R. M. Dorrell, S. Dutta, G. M. Keevil, J. Peakall, and S. M. Tobias, The effect of schmidt number on gravity current flows: The formation of large-scale three-dimensional structures, *Phys. Fluids* **33**, 106601 (2021).
- [38] P. Fischer and J. Mullen, Filter-based stabilization of spectral element methods, *C. R. Acad. Sci. Math.* **332**, 265 (2001).
- [39] T. Chatterjee and Y. T. Peet, Regularization for large-eddy simulation in wall-bounded turbulence: An explicit filtering-based approach, *Int. J. Numer. Methods Fluids* **88**, 1 (2018).
- [40] J. K. Lai, E. Merzari, and Y. A. Hassan, Sensitivity analyses in a buoyancy-driven closed system with high resolution CFD using boussinesq approximation and variable density models, *Int. J. Heat Fluid Flow* **75**, 1 (2019).
- [41] E. Merzari, A. Obabko, P. Fischer, and M. Aufiero, Wall resolved large eddy simulation of reactor core flows with the spectral element method, *Nucl. Eng. Des.* **364**, 110657 (2020).
- [42] S. B. Pope, *Turbulent Flows* (Cambridge University Press, Cambridge, 2001).
- [43] O. T. Schmidt and T. Colonius, Guide to spectral proper orthogonal decomposition, *AIAA J.* **58**, 1023 (2020).
- [44] R. Luchi, S. Balachandar, G. Seminara, and G. Parker, Turbidity currents with equilibrium basal driving layers: A mechanism for long runout, *Geophys. Res. Lett.* **45**, 1518 (2018).
- [45] C. Lloyd and R. Dorrell, Dataset for "Transport by waves and turbulence: Dilute suspensions in stably stratified plane Poiseuille flow" [Data set], Zenodo (2025), <https://doi.org/10.5281/zenodo.17258873>.