

Fluid structure interaction in pulsatile flow through an elastic pipe segment

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Fluid-structure interaction (FSI) in elastic vessels occurs in a wide range of systems in nature and technology. A difficulty in the development and validation of FSI models is the lack of quantitative measurements of vessel response to unsteady flow forcing. We here investigate the canonical case of an elastic tube subject to laminar pulsatile flow. For this purpose, we develop an experimental setup comprising a high-precision piston, a three-dimensional (3D)-printed elastic pipe, and a camera equipped with a microscope objective to determine the wall displacement. We find that for weak forcing the response is similar to a forced damped harmonic oscillator. Near resonance and for moderate forcing amplitudes, the wall motion exhibits complex dynamics, including multiple frequencies and nonaxisymmetric tube deformation. When the amplitude of the response exceeds a threshold, the tube collapses nonaxisymmetrically during parts of the cycle, thereby drastically reducing the cross-sectional area of the tube. We additionally perform fully coupled axisymmetric numerical simulations based on the solution of the Navier-Stokes equations and a thin-shell model for the tube. Our simulations agree with experimental results, except near resonance and under large amplitude forcing, where a 3D model accounting for structural damping would be necessary. Our results shed light on the rich nonlinear dynamics of this canonical system and serve as a basis to validate future models.

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I. INTRODUCTION

Fluid-structure interactions (FSIs) occur in many systems where flow-induced stresses and pressure fluctuations act upon an elastic vessel, whose deformation feeds back to the fluid flow. FSI in pipelines conveying fluids can increase drag and thus energy expenditure [1]. In more extreme

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circumstances, FSI may lead to failure of the pipe system [1]. In physiology, FSI plays a central role in the cardiovascular system. Dilatation of large arteries such as the aorta permits the storage of blood during systole and release during diastole, the so-called Windkessel effect, which is crucial for the perfusion of the coronary and the peripheral arteries [2]. The compliance of the aortic walls decreases with age, leading to higher peak pressure and irregular flow patterns in the aortic blood flow [3]. This may ultimately cause hypertension and hypertrophy [4]. Increased wall movement due to flow may also trigger and sustain early turbulence [5]. Thus, understanding FSI is crucial for modeling blood flow in arteries and understanding possible relationships between blood flow and cardiovascular diseases. To this end, quantitative experimental data of the FSI is essential for model validation.

While FSI of fluid-conveying elastic vessels has been studied analytically and numerically for many decades, the focus was predominantly on cases where the flow is driven by a stationary pressure drop or flow rate. In such cases, higher flow velocities tend to induce instabilities in the pipe, including self-induced oscillations, high-speed flutter or divergence [1,6]. FSI under unsteady flow, however, is more complex. It is not only governed by the Reynolds number (a dimensionless number comparing inertial and viscous forces) and the characteristics of the conduit wall, but also by the Womersley number (a dimensionless expression of the pulsatile flow frequency in relation to viscous diffusion of momentum) and the pulsation amplitude. In such conditions, the coupling of unsteady forcing and elastic walls can give rise to resonance phenomena, which occur when the frequency of the forcing matches the natural frequency of the system. The relevant time scales of the coupled fluid-structure system are governed by both the fluid properties and the mechanical characteristics of the wall material. Depending on the regime, resonance may result in maximum wall dilatation or flow velocity, indicating the intricate coupling between fluid flow and structural response in pulsatile systems [7]. In this work, we specifically investigate resonance in relation to maximum radial wall displacement. The full equations describing the coupled fluid-flexible vessel system are in general not tractable analytically. Theoretical solutions exist only under strongly simplified scenarios [8]. Womersley [9] developed a well-known 1D model for basic FSI of oscillatory and pulsatile flows through an isotropic elastic tube. His wave propagation model is based on the linearized, axisymmetric Navier-Stokes equations for the viscous fluid motion and on the assumption of a linear, isotropic, and homogeneous tube for the wall behavior. Other authors extended this model or independently developed similar linearized wave propagation models. In general, these formulations tend to converge to the same result for ratios of pipe radius to wall thickness larger than ten [8]. These early models provide a reasonable description of axisymmetric deformations of flexible pipes, but their applicability is restricted to laminar flow and small deformations. Additional refinements of 1D models include nonlinear treatment of the flexible pipe, utilizing a large-deformation stress-strain law and empirical closures for the wall friction term [10]. The latter enabled prediction of wall motion of small amplitude across laminar and fully turbulent flow conditions. Other analytical approaches include models based on the generalized Darcy's law which relates average flow linearly to the pressure gradient [7,11]. These 1D models capture rich dynamics and can be applied to solve entire networks. Sazonov and Nithiarasu [12] further refined this approach using nonlinear correction steps obtaining a more accurate and robust model. Their model showed a good agreement with experimental observations of single pulse propagation in aneurysm models. However, a comprehensive experimental validation of such models remains lacking.

A number of numerical studies investigated the FSI of unsteady flows in flexible tubes [e.g., 13,14]. These studies primarily focused on method development and validation based on wave propagation in flexible vessels, finding good agreement with experiments in straight and tapered vessels. Xu *et al.* [15] explored the nonlinear interaction between laminar pulsatile flow and a flexible wall in channel flow numerically and analytically over different Womersley and Reynolds numbers. Their numerical model was fully coupled and considered the flexible wall as a thin elastic membrane subject to prestress. They observed that the movement of the membrane behaved similarly to a damped harmonic oscillator, and they developed an analytical model capable of predicting

the simulated resonant frequency and amplitude of the wall deformation. Their application was limited to a 2D, thin-walled channel and the approach still awaits comparison to experiments.

There is a limited number of experimental investigations on the behavior of flexible pipes under unsteady flow forcing [16]. Previous studies on the wall movement due to FSI used oscillatory flows, i.e., zero mean flow, to show that the wall motion follows rather faithfully the transmural pressure difference and induces a delayed volume flux [17]. Interestingly, nonlinear effects in terms of skewed response of the wall motion with respect to sinusoidal forcing may appear depending on the peak transmural pressure difference [18]. Olsen and Shapiro [10] performed experimental measurements of the wall motion for sinusoidal pulsating flow through a flexible pipe and compared it to a semiempirical model. They also found nonlinear wall displacement depending on the forcing conditions. That is, wall motion in the first segment of the distensible tube strongly deviated from a sinusoidal form for large pulsation amplitudes and Reynolds numbers. An increasing mismatch in terms of wall deformation between the experiment and their semiempirical model near resonance was evident.

In general, quantitative studies experimentally investigating the fundamental FSI of a flexible pipe under pulsatile flow are scarce. More importantly, there is no quantitative comparison between experiments and simulations. Moreover, the movement of the flexible pipe wall around resonance conditions has not been investigated experimentally yet. In this work, we present a canonical experimental setup that allows for precise measurement of conduit deformations caused by pulsatile flow leveraging high-magnification imaging. We complement the experiments with fully coupled axisymmetric numerical simulations based on a thin-shell model. Our goal is to quantitatively characterize the response of harmonically forced pulsatile flows through an elastic tube near the natural (resonant) frequency of the system. Our results provide benchmark measurements for validating FSI models, particularly those designed to capture the transition from linear to nonlinear response regimes.

II. METHODS

A. Experimental setup

1. Test apparatus

A sketch of the experimental apparatus is shown in Fig. 1. The flow circuit was oriented vertically to prevent the flexible pipe segment from sagging nonaxisymmetrically due to gravity. The air pressure in the chamber surrounding the elastic segment was continuously monitored with a differential pressure sensor (HSC, Honeywell, USA) and regulated using a system of valves (VUVG, Festo, Germany) and a microcontroller (Arduino Uno), so that the chamber pressure matched the hydrostatic pressure at the outlet of the flexible pipe segment within an error of 30 Pa. The dynamic transmural pressure differences (0–150 Pa) were estimated using another pressure sensor with higher measurement resolution (DP103, Validyne, USA). Before starting an experiment, we set the constant chamber pressure with a maximal error of 30 Pa. This may result in a constant offset of up to 30 Pa in the static transmural pressure difference, which however does not affect the accuracy of the dynamic transmural pressure difference. The flexible pipe was clamped to adapters machined out of aluminum. The adapters slide onto the glass pipes creating a stepless joint at the inner surface. The flexible pipe and the adapters were jointly designed to minimize flow disturbances at the transition [see inset (a) in Fig. 1]. The inner diameter of the elastic pipe at the clamping sections was offset to precisely fit the adapters.

The flow was driven at a pulsatile rate by a piston pump consisting of a piston (CRDNG-80-1500-PPV-A, Festo, Germany), a linear actuator (ESBF-BS-80-1500-15P, Festo, Germany) and a motor (EMMS-AS-70-M-LS-RS, Festo, Germany). The actuated piston had an active volume of 7.5 l and a stroke length of 1.5 m. The piston pump was connected to glass pipes with an internal diameter of $D_o = 20$ mm (Duran, DWK Life Sciences, Germany). The inflow section length is $L_{in} = 1.7$ m ($85 D_o$) and the outflow section downstream of the flexible segment consisted of the same glass

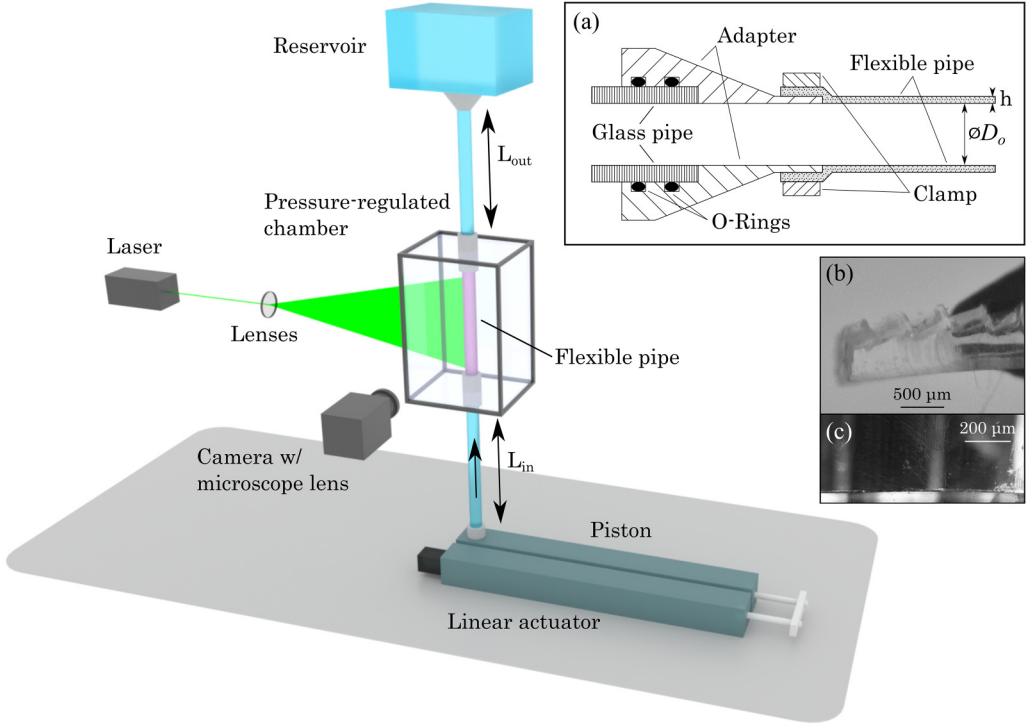


FIG. 1. Sketch of the experimental setup, consisting of a piston pump, glass pipes of length $L_{in} = 85D_o$ and $L_{out} = 55D_o$, a pressure-regulated chamber, the flexible pipe segment, pipe adapter pieces, an overflow reservoir, a laser, lenses and a camera equipped with a microscope lens. Inset (a) connection of the flexible pipe to the glass pipes with custom designed adapter pieces. Inset (b) profile view of a cut section of the flexible pipe showing the smooth inner surface and the textured outer surface due to the 3D printing. Inset (c) close-up microscopy image of a plan view of the flexible pipe's inner surface.

pipes with a length of $L_{out} = 1.1$ m ($55D_o$), connecting to a final reservoir. The glass pipes were carefully aligned to ensure minimal disturbances to the flow, resulting in a maximum deviation from the vertical axis of 2 mm over the total length of the setup. To ensure a constant hydraulic head, the reservoir featured an overflow and the discharged working fluid was continuously cycled back. The working fluid was water kept at a constant temperature of 22 °C.

2. Flexible pipe

The flexible pipe segment had an inner diameter D_o of 20 ± 0.02 mm, a length of $L_{el} = 150 \pm 0.1$ mm ($7.5D_o$) and a nominal wall thickness h of 0.5 mm ($0.025D_o$). The inner diameter at the clamping positions was increased by 1 ± 0.02 mm to ensure a smooth and continuous transition between rigid and flexible pipe segments, see the inset (a) in Fig. 1. As precise casting of such thin-walled pipes was challenging, the flexible pipe was instead 3D-printed in silicone (MED-4920, Avantor-NuSil, USA) onto a rotating cylinder which was machined out of aluminum. Thus, the inner diameter of the elastic pipe matched the outer diameter of the aluminum form. The surface of the aluminum cylinder was carefully polished to guarantee a smooth inner surface of the flexible tube. The concentricity of the aluminum form was within 0.01 mm over the whole length and straightness deviation was less than 0.04° . The flexible pipe's wall is printed spirally in a single layer onto the aluminium cylinder form, meaning it consists of only one layer radially. After deposition of the silicone ink, the vessel model on the cylinder form was cured for one hour at 120 °C in a chamber

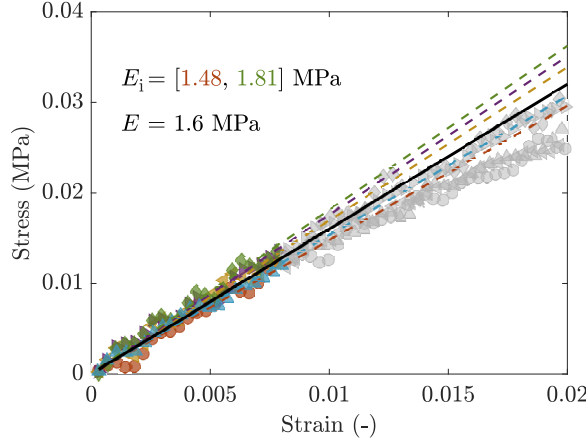


FIG. 2. Stress-strain curves and Young’s modulus measured for five dog-bone shaped samples of the silicone used for the fabrication of the flexible pipe. The Young’s modulus of each sample E_i is shown by the linear fit (dashed lines). The range of Young’s moduli is $[1.48 - 1.81]$ MPa. Colored markers are considered for the calculation of the respective Young’s modulus (strain threshold of 0.0075). The solid line represents the average Young’s modulus, $E = 1.6$ MPa, computed from all five samples.

pressurized to 2 bar which serves to eliminate any porosity stemming from the 3D-printing. No shrinkage of the flexible pipe could be measured after curing. The outer surface of the flexible tube was textured due to the 3D-printing layer size, which resulted in a small variance in effective wall thickness, 0.51 ± 0.05 mm, which was deemed satisfactory. The inner surface is smooth, matching the surface finish of the polished aluminium cylinder (see insets b and c in Fig. 1). Once fully cured, the 3D-printed silicone forms a flexible solid that acts as a solid–fluid boundary with the water. In the numerical model, it is therefore treated as a classical no-slip boundary, as detailed below.

We carried out stress-strain tests on five dog-bone shaped samples of the silicone used for the fabrication of the flexible pipe, to determine the Young’s modulus of the 3D-printed pipe. The test samples were cured following the same procedure as the 3D-printed pipe. The stress-strain measurements were performed on a precision tensile tester (AGS-X10kN, Shimadzu, Japan) and are shown in Fig. 2. The numerical value of the Young’s modulus ranged from 1.41 MPa to 1.97 MPa, depending on the sample and the threshold of the strain over which the Young’s modulus was determined. For large strain, the stress increases slower than linearly with strain, indicating a softening behavior of the material. In addition, we performed quasi static inflation tests of the flexible pipe, which showed a hysteresis, however, at significantly larger pressure amplitudes and associated wall displacements than considered in the present study. For small wall displacements, we did not observe hysteretic behavior.

Since the flexible pipe segment is surrounded by air, refractive index matching with the external medium is not possible —a requirement for optical velocimetry techniques such as particle image velocimetry. As a result, velocity measurements inside the tube, such as capturing the inlet velocity profile, were not performed. However, the influence of the inflow profile on the observed wall dynamics was evaluated numerically, as detailed below.

3. Measurement procedure

The dynamic transmural pressure differences on the order of 10 – 150 Pa, resulted in a pipe deformation of up to 100 μ m. In order to measure such small displacements, the wall movement of the flexible pipe was recorded with a camera (Fastcam SA5, Photron, Japan) equipped with a microscope objective (SMZ-2T, Nikon, Japan). The resulting optical resolution was between 500

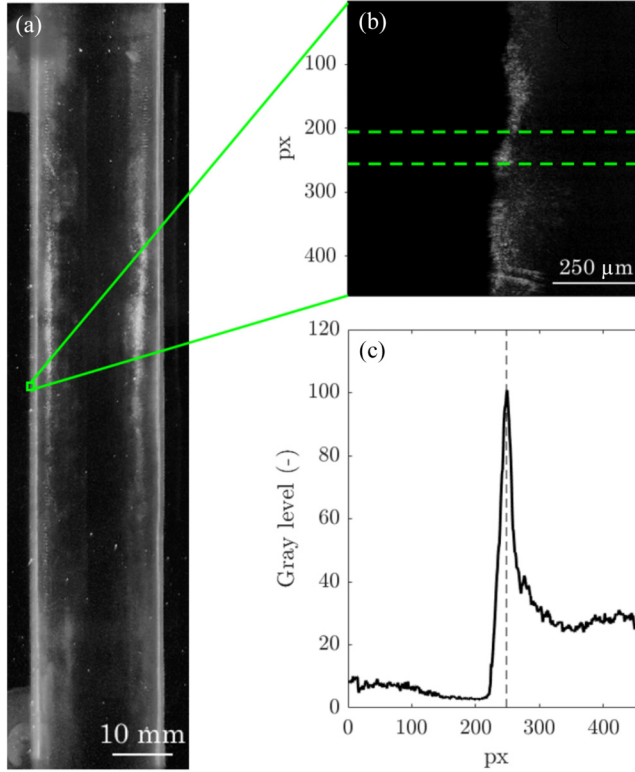


FIG. 3. (a) Photograph of the full flexible pipe. (b) Example grayscale recording of the flexible pipe's outer wall used to evaluate the wall deformation. Dashed green lines represent the area over which the gray levels were evaluated. (c) Corresponding gray level distribution. The dashed line indicates the location of the outer contour.

and 550 px/mm resulting in a magnification of approximately 10 : 1. Images were recorded at a frame rate of 125 fps corresponding to at least 45 images per period.

Due to the high optical magnification of the lens to measure the radial displacement, the measurements were limited to a short axial section of the entire elastic pipe segment. Specifically, in all cases we measured the displacement of the axial midpoint. For this purpose the region of interest at the longitudinal pipe midpoint was illuminated with a thin laser sheet axially aligned to the flexible pipe and perpendicular to the camera's optical axis. At forcing frequencies sufficiently below and above the natural frequency, we observed the first eigenmode of the system, in which the entire pipe deforms axisymmetrically and in phase. In this regime, the dynamics of the system are therefore well described by the motion of a single point. For nonaxisymmetric deformations, such point measurements are unable to capture the complexity of the motion, but are still useful for validation purposes.

The recorded raw images were post-processed using contrast limited adaptive histogram equalization and a median filter with a kernel size of 9×9 px to remove pixel noise. Figure 3(b) shows an example raw image. Following Ref. [17], the location of the outer boundary of the elastic pipe was determined by the profile of gray levels across the pipe wall averaged over 50 px (roughly 0.1 mm) in the streamwise direction, as indicated by the dashed green lines in Fig. 3(b). An example of the gray level distribution is shown in Fig. 3(c). The outer contour is clearly distinguishable by the sharp peak. We tested different locations and sizes of regions for evaluating the gray level distribution within the recorded images. The relative position of the peak remains virtually

unchanged for different axial locations of the evaluation window. A larger window size leads to higher signal-to-noise ratio, however the window size had to be smaller than the 3D-printing layer size ($\approx 0,35$ mm) to prevent ambiguous signals.

The location of the outer boundary of the pipe was determined with the fluid at rest for a period of 10 s to characterize the robustness of the measurements and to estimate the accuracy of determining the wall dilatation of the flexible pipe. The scatter was approximately ± 1 px ($1.9 \mu\text{m}$) and the root mean square (rms) of the deviation from the mean is of the order of 0.5 px ($0.9 \mu\text{m}$). For each case, after an initialization of 10 periods, $N = 30$ periods were recorded and the measured instantaneous wall motion ζ was phase-averaged to obtain the average wall displacement over one period $\bar{\zeta}(t) = \frac{1}{N} \sum_{i=1}^N \zeta(t + iT)$. Pressure drop measurements downstream of the flexible pipe confirmed that 10 pulsation cycles are sufficient to realize fully periodic flow for all Wo considered here. We performed a convergence study where we varied the number of periods considered for the phase averaging. This showed that the phase averaged wall dilatation converges when considering at least 25 periods, that is, the variation is within $\pm 4\%$.

B. Dimensionless parameters

Pulsatile flows are governed by the Reynolds number, $Re = \bar{U}D_o/\nu$, the forcing amplitude, $A = U_{\max}/\bar{U} - 1$ and the Womersley number, $Wo = D_o/2(\omega/\nu)^{0.5}$, where $\bar{U}$ is the mean bulk flow velocity, D_o the pipe diameter, ν the kinematic viscosity of the fluid and $\omega = 2\pi f$ the angular frequency with the forcing frequency f . In this study, we kept the Reynolds number at $Re = 1000$ and imposed sinusoidal pulsations with $Wo \in [10, 60]$. The forcing amplitude was varied between 0.25 and 1.25. Even though, for some flow parameters ($A > 0.5$, $Wo = 10$), the flow in a straight rigid pipe may be susceptible to transition to phase-locked turbulence due to helical instabilities [19,20], turbulence is not able to survive the whole pulsation period at this Re , and for all the flow parameters considered here [21]. In the experiments, no disordered flow was observed in the elastic segment.

The elastic tube stiffness is characterized by the dimensionless parameter $\mathcal{I} = \frac{\mu\bar{U}}{D_o E_{\text{eff}}}$, which relates solid and fluid stress, where $E_{\text{eff}} = E/(1 - \nu_p^2)$ is the effective young's modulus with Poisson ratio ν_p . With the experimentally determined average Young's modulus of $E = 1.6$ MPa we obtain $\mathcal{I} = 1.032 \times 10^{-9}$. With these parameters the wall motion is large enough to elicit measurable wall displacements, while precluding the appearance of sustained turbulence. Finally, due to the vertical setup, the numerical model presented in the next section must account for the hydrostatic force due to the water column. This is characterized by the Froude number, $Fr = \bar{U}^2/(gD_o) = 0.01111$, where g is the gravitational acceleration.

C. Numerical simulations

For the numerical simulations, we consider the flow of an incompressible, Newtonian fluid with constant fluid properties in a vertical collapsible pipe. As in the experiments, the pipe is divided in three sections: a rigid inflow segment, an elastic tube and a rigid outflow segment (with lengths $L_{\text{in}} = 5D_0$, $L_{\text{out}} = 55D_0$ and $L_{\text{el}} = 7.25D_0$, respectively). Scaling all length on the undeformed pipe diameter D_0 , velocities on the mean bulk velocity $\bar{U}$ and the pressure on $\mu\bar{U}/D_0$, the fluid motion is governed by the dimensionless Navier-Stokes equations,

$$\text{Re} \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = -\nabla p + \nabla^2 \mathbf{u} - \frac{\text{Re}}{\text{Fr}} \mathbf{e}_z,$$

$$\nabla \cdot \mathbf{u} = 0,$$

where $\mathbf{u}$ is the velocity field, p the pressure, $Re = \frac{D_0 \bar{U}}{\nu}$ the Reynolds number and $Fr = \frac{\bar{U}^2}{gD_0}$ the Froude number. We model the flow in cylindrical coordinates and assume it to be axisymmetric $\mathbf{u} = u(r, z, t)\mathbf{e}_r + w(r, z, t)\mathbf{e}_z$. A sketch of the numerical setup is shown in Fig. 4.

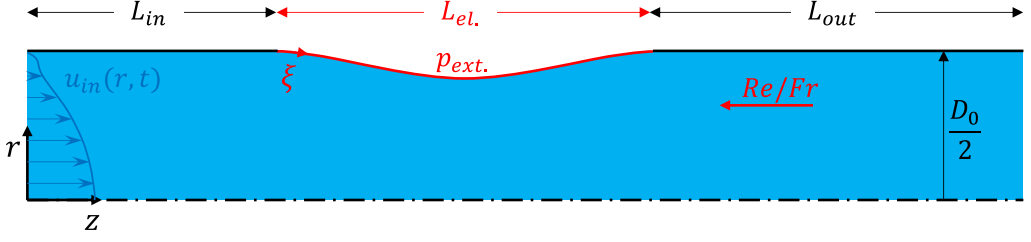


FIG. 4. Sketch of the fluid flow (from left to right) in an axisymmetric pipe with a rigid inflow (outflow) of length L_{in} (L_{out}) and an elastic tube of length L_{el} clamped in between. At the inflow boundary the laminar Sexl-Womersley profile $u_{in}(r, t)$ is imposed to drive the pulsatile flow corresponding to the assumption of fully developed flow at the inlet. A constant pressure and parallel flow are assumed at the outlet. The Lagrangian parameter ξ denotes the position on the elastic pipe segment.

In the experiments, the fluid motion is driven by the displacement of a piston, resulting in a prescribed time-dependent volume flux at the rigid inlet pipe segment. Analogously, we impose the same pulsatile volume flux

$$\dot{V}(t) = (1 + A) \cos\left(\frac{Wo^2}{Re} t\right) \quad (1)$$

in the numerical simulations. The experimental inlet is very long ($85D_0$) and allows the velocity profile to fully develop before the elastic segment at the Reynolds number considered ($Re = 1000$) [22]. Accordingly, to drive the flow in the numerical simulations, we impose a (fully-developed) Sexl-Womersley profile [9] with pulsatile volume flux (1) at the inlet. A zero pressure, parallel outflow is enforced at the downstream end of the rigid outlet segment and no-slip boundary conditions are applied at the rigid walls as well as the instantaneous position of the elastic walls of the pipe.

The elastic pipe segment is modeled as an axisymmetric, massless thin shell of nominal thickness h and viscoelastic behavior is incorporated as a Kelvin-Voigt model. Again, all lengths are scaled on the undeformed pipe diameter D_0 , whereas stresses are scaled with the effective Youngs modulus $E_{eff} = E/(1 - \nu_p^2)$, where ν_p is the Poisson ratio. The shape of the shell is parameterized by the dimensionless Lagrangian coordinate ξ , so the position vector of a material point at the shell is given by $R(\xi, t) = R_0(\xi) + d(\xi, t)$ where $R_0(\xi) = (L_{in} + \xi)\mathbf{e}_z + \frac{1}{2}\mathbf{e}_r$ is the undeformed shape and $\mathbf{d}(\xi, t)$ the time dependent displacement. We solve a free-boundary problem and apply boundary conditions at the instantaneous position of the moving wall. The deformation of the elastic shell is governed by the principle of virtual displacement [23], [24], which states that the work done by external forces $\mathbf{f}$ under a virtual displacement (or variation $\delta\mathbf{R}$) must balance the work done by internal (elastic and viscous) stresses $\int_V \sigma^{ij} \delta\epsilon_{ij} dV$. The Kelvin-Voigt model assumes a linear viscoelastic constitutive law $\sigma^{ij} = E^{ijkl} \epsilon^{kl} + D^{ijkl} \dot{\epsilon}^{kl}$, where the elasticity tensor for cylindrical shells is given by

$$E^{ijkl} = \frac{E}{2(1 + \nu_p)} \left(\delta^{ik} \delta^{jl} + \delta^{il} \delta^{jk} + \frac{2\nu_p}{1 - \nu_p} \delta^{ij} \delta^{kl} \right), \quad (2)$$

and the damping tensor by

$$D^{ijkl} = \frac{\mathcal{D}}{2(1 + \nu_p)} \left(\delta^{ik} \delta^{jl} + \delta^{il} \delta^{jk} + \frac{2\nu_p}{1 - \nu_p} \delta^{ij} \delta^{kl} \right), \quad (3)$$

where $\delta^{\alpha\beta}$ is the Kronecker-Delta. Using the summation convention and expressing the strain tensor ϵ (strain rate tensor $\dot{\epsilon}$) in terms of the midplane strain tensor γ_{ij} (midplane strain rate tensor $\dot{\gamma}_{ij}$) and bending tensor κ_{ij} (bending rate tensor $\dot{\kappa}_{ij}$), the principle of virtual displacement for thin shells is

given by

$$\int_0^{L_{el}} \left[E^{ijkl} \left(\gamma_{ij} \delta \gamma_{kl} + \frac{h^2}{12D_0^2} \kappa_{ij} \delta \kappa_{kl} \right) + D^{ijkl} \left(\dot{\gamma}_{ij} \delta \gamma_{kl} + \frac{h^2}{12D_0^2} \dot{\kappa}_{ij} \delta \kappa_{kl} \right) - \left(\frac{D_0}{h} \mathbf{f} \cdot \delta \mathbf{R} \right) \right] d\xi = 0. \quad (4)$$

The load vector is

$$\mathbf{f} = -p_{\text{ext}} \mathbf{n} + \mathcal{I}(p\mathbf{n} - (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \cdot \mathbf{n})$$

and takes into account an external pressure and the fluid stresses, whereas the dimensionless parameter $\mathcal{I} = \frac{\mu \bar{U}}{D_0 E_{\text{eff}}}$ arises from the different scaling of fluid and solid stresses and measures the strength of the fluid-structure interaction.

The fully coupled system of partial differential equations for the fluid and solid are solved numerically using the open-source code `oomph-lib`, which employs a finite element method and monolithic approach to the fluid structure interaction problem [25]. The equations are discretized using Taylor-Hood elements on a moving, body-fitted mesh and integrated in time using an implicit second-order backward-difference method (BDF2). At each time step, the fully coupled nonlinear system is solved using the Newton-Raphson method. All simulations are carried out with $E = 1.6$ MPa in accordance to the experimentally determined average Young's modulus. Additional simulations with a Young's modulus of $E = 1.8$ MPa were carried out to assess the uncertainty in the experimental determination of E .

D. Verification of inflow boundary conditions

To verify the assumption of a fully developed Sexl-Womersley profile at the rigid inlet, we studied the effect of different inflow conditions on the wall motion, by performing two additional simulations in which we drove the flow with either a spatially uniform (plug) inlet profile or with a prescribed pressure gradient that produces the same pulsatile volume flux as in Eq. (1). The centerline velocity is shown for each of the three cases in Fig. 5(a). For the case of the imposed pressure gradient, the velocity is initially zero, but converges to the Sexl-Womersley profile after about 55 periods. The plug profile has a significantly lower centerline velocity, as expected. In Fig. 5(b), we compare the motion of the midpoint of the elastic segment.

We find that driving the system with a fully developed Sexl-Womersley profile or a plug profile at the rigid inlet of the numerical simulations, does not significantly affect the wall motion. After an initial transient behavior, the midpoint of the elastic pipe segment settles to a harmonic oscillation with the same frequency as the forcing frequency showing only a very small deviation in amplitude for the two cases. Specifically, the difference in the response amplitude is about 1% for amplitude $A = 0.25$ and $Wo = 20$ and about 0.035% for $Wo = 45$. This suggests that the exact inflow velocity profile does not have a substantial effect on the wall motion for the parameters considered throughout this study. When the pressure gradient is prescribed, the elastic wall oscillates with the same amplitude after an initial transient behavior. The deviation of the response amplitude, compared to the case with an imposed Sexl-Womersley profile, is about 1% for $Wo = 20$ and 0.01% for $Wo = 45$. Overall, the tube deformation (the quantity of interest throughout this study) already reaches a statistically stationary state after about $t = 5T$ periods, much faster than it takes to develop the profile.

In summary, the assumption of a Sexl-Womersley profile at the rigid inflow has a negligible effect on the dynamics of the wall deformation and reflects the experimental conditions accurately. Thus, in all subsequent simulations we drive the flow by imposing the fully developed Sexl-Womersley profile at the inlet.

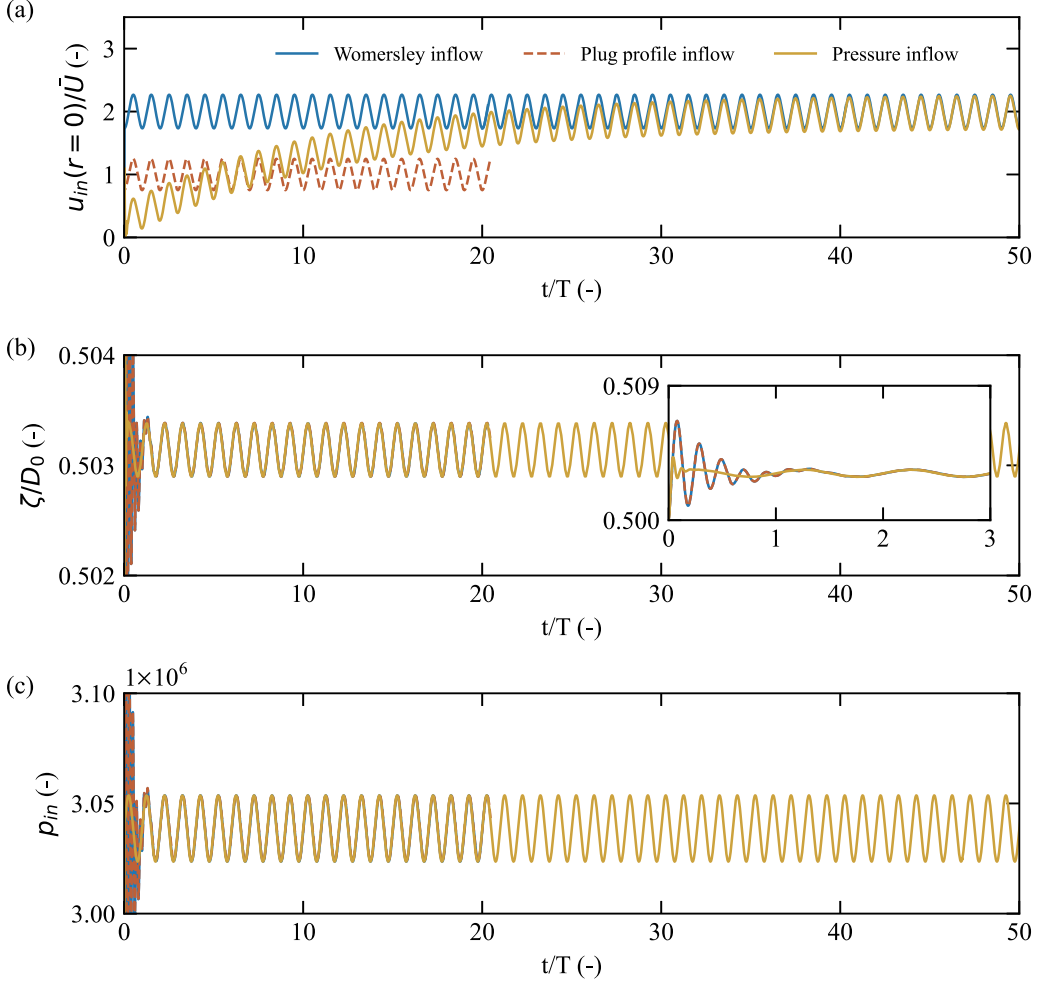


FIG. 5. (a) The dimensionless centerline velocity of the inflow profile, (b) the dimensionless deformation of the elastic pipe's midpoint and (c) the dimensionless inlet pressure for $Wo = 20$, $A = 0.25$ and $\mathcal{D} = 72 \text{ kPa s}$. The inset shows the dimensionless midpoint deformation for the first three periods. The flow is driven by imposing the analytical Sexl-Womersley profile, a spatially constant pulsating profile or by prescribing the pressure gradient at the inlet.

III. RESULTS

A. Natural frequency of the system

We determined the natural (resonance) frequency of the system by measuring the response to a single impulsive movement of the piston starting from rest. For this, the pressure immediately downstream of the flexible pipe was measured with a highly sensitive pressure sensor (DP103, Validyne, USA) at 1 kHz. The measured pressure response is shown in Fig. 6. Since the initial impulse to the system is very strong and sharp, a large number of fundamental modes is excited. Each mode decays with its own rates and in order to capture the dominant (least damped) mode of the system, we wait for the other modes to decay before performing the fit. The resonance frequency, $f^* = 3.30 \text{ Hz}$, was determined with a least-squares fit of the analytical solution for a damped harmonic oscillator for $t > 4 \text{ s}$ as shown in the inset of Fig. 6. The corresponding resonant

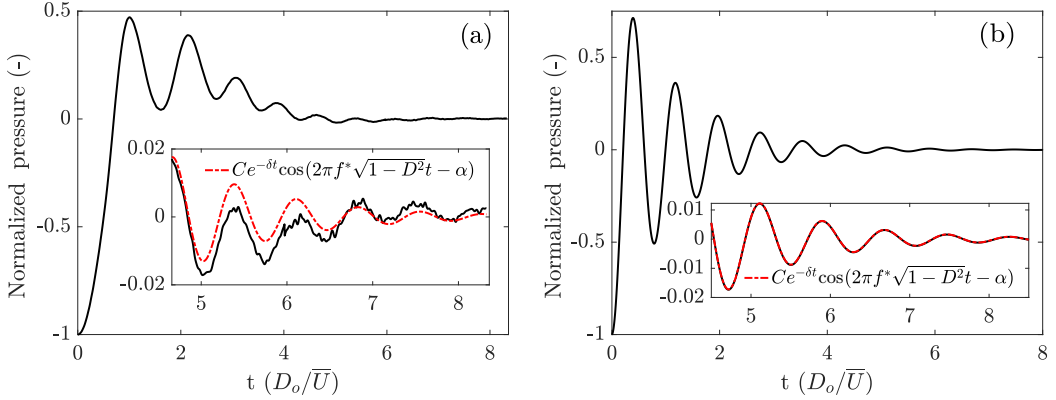


FIG. 6. (a) Measured pressure response of the system to a sharp singular pulse. The frequency of the oscillations corresponds to the resonance frequency f^* of the system. The inset shows the pressure response for $t > 4$ s and a least-squares fit of the analytical solution for a damped harmonic oscillator (red dashed line), with $f^* = 3.30$ Hz ($Wo^* = 46.6$), $\delta = 2.0$ Hz, and $D = \delta/(2\pi f^*) = 0.1$. (b) Response of the numerically computed pressure to a sharp singular pulse. The pressure response is fitted with the same function (scaled by D_o and $\bar{U}$) with $Wo^* = 44.6$ and $D = 0.106$

Womersley number is $Wo^* = 46.6$. The estimated decay rate is $\delta = 2.0$ Hz, resulting in a damping factor of approximately $D = \delta/(2\pi f^*) = 0.1$. While the resonance frequency estimation is robust, as substantiated by the good match of the peak locations in Fig. 6(a), the estimation of the damping is more uncertain as evidenced by the difference in the peak amplitudes. In order to determine the damping coefficient $\mathcal{D}$ in Eq. (3), we perform a simulation of the numerical system's response to the same impulse. We monitor the decaying oscillation of the radial displacement, shown in Fig. 6(b), and again fit the response with the solution of a damped harmonic oscillator. The numerical resonance Womersley number of $Wo^* = 44.6$ is in good agreement with the resonance Womersley number determined in the experiments, showing a deviation of about 4%. This indicates a reasonably accurate representation of restoring forces in the numerical simulations and supports the *a priori* determination of the Young modulus. The damping coefficient in the numerical model $\mathcal{D}$ is then adapted, so that the damping factor D in the fit of the numerical response agrees with the fit of the experimental response, where $D = 0.1$. For a damping coefficient of $\mathcal{D} = 72 \text{ kPa s}$ in the numerical simulations, we obtain $D = 0.106$, which is deemed sufficiently accurate considering all uncertainties. Note that only the damping coefficient is fitted to the experiments, whereas the resonance frequency of the numerical system is obtained independently from the experiments. To demonstrate the significance of structural damping, we perform the same simulation solely taking the fluid's viscous damping into account ($\mathcal{D} = 0$). This leads to a damping factor of $D = 0.0176$ and highlights the dominant role of structural damping in this system. In the following, results from the numerical simulations are shown with a structural damping coefficient fixed to $\mathcal{D} = 72 \text{ kPa s}$.

B. Regime diagram

The ranges of forcing amplitudes and Womersley numbers studied in this work are summarized in the regime diagram of Fig. 7(a). At low and high forcing frequencies the deformation of the tube was axisymmetric, whereas near resonance the tube deformed nonaxisymmetrically. Nonaxisymmetric deformation was observed visually at the downstream end of the flexible pipe segment for cases specified in Fig. 7. When the total nonaxisymmetric deformation ($\bar{\zeta}_{\max} - \bar{\zeta}_{\min}$) of the tube exceeded roughly $300 \mu\text{m}$ (3% of the tube's radius), the tube suddenly collapsed, meaning that opposite sides of the tube touched each other. This collapse manifested itself during diastole and occurred in the form of an $n = 2$ circumferential mode [1,26] at the downstream end of the flexible pipe, as shown

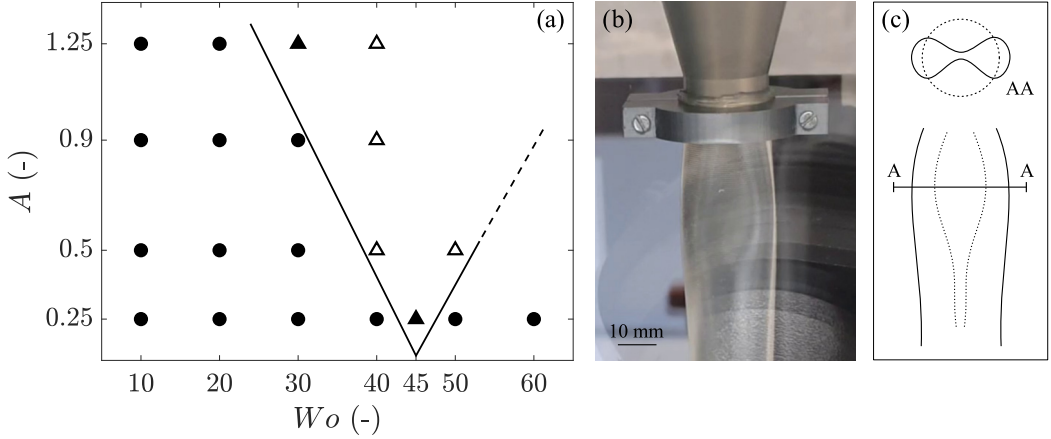


FIG. 7. (a) Regime diagram of amplitude vs Womersley number summarizing the results of all experimental runs. Circles denote runs with axisymmetric tube deformation, full triangles denote cases with nonaxisymmetric deformation and empty triangles indicate nonaxisymmetric collapse of the tube. The line is to guide the reader's eyes and illustrates the transition to nonaxisymmetric deformation. (b) Photograph of the downstream part of the flexible pipe collapsing at $Wo = 40$ and $A = 0.5$. (c) Schematics of the collapsed tube where the solid line depicts the deformed tube wall. The lower schematic depicts the same view as in (b) while the upper schematic shows the cross section AA with the undeformed tube also shown (dashed line).

for $Wo = 40$ and $A = 0.5$ in Fig. 7(b), and illustrated schematically in Fig. 7(c). The downstream part of the elastic pipe was most prone to collapse, since the surrounding air pressure was matched to the hydrostatic pressure at the elastic pipe's outlet, resulting in a decrease of the transmural pressure ratio from the up- to downstream end of the flexible segment.

The frequency range in which nonaxisymmetric deformation and collapse was observed widens as the forcing amplitude increases, thereby giving rise to a wedge shaped region in (Wo, A) -parameter space [Fig. 7(a)]. In what follows, the motion of the flexible tube's midpoint is analyzed quantitatively for the cases without collapse; cases with collapse featured deformations of several millimeters and could not be captured with our optical setup.

C. Dynamics of the wall displacement

Figure 8 shows time series of the displacement of the center of the elastic tube for $Wo \in [10, 30]$ at $A = 0.5$ (a) and $A = 1.25$ (b). The high frequency fluctuations present in all time series are due to optical measurement errors (see Fig. 3). The average relative rms deviation from the mean response shape is less than 10 % for all studied cases except for $Wo = 10$ where displacements are the smallest. However, for the $Wo = 10$ cases the average absolute rms deviation from the mean is less than $1.8 \mu\text{m}$, which is comparable to the measurement accuracy determined for the system at rest (see Sec. II A 3). This confirms that our experimental setup enables accurate and repeatable wall displacement measurements over a range of approximately $2 \mu\text{m}$ up to $300 \mu\text{m}$.

Leveraging the periodicity of the wall displacement time series, we compute the phase averaged wall deformation $\bar{\zeta}$ which results in a smoother signal, as measurement noise is averaged out. The solid lines in Fig. 9 show the phase-averaged wall displacement for $A \in [0.25, 1.25]$ and $Wo \in [10, 30]$. In each case, the experimental uncertainty represented by the rms pulse to pulse variation is indicated by an error bar. Because of the sinusoidal wave form of the imposed pulsatile flows, the resulting temporal evolution of the wall displacement is expected to also follow a sinusoidal curve of the same period as long as the response is linear [10]. Indeed, cases in Figs. 9(a)–9(c) exhibit a predominantly sinusoidal wave form and wall motion for flows of $A \leq 0.9$ and $Wo \leq 30$ may be considered linear. Further, it can be seen that wall deformation increases with A

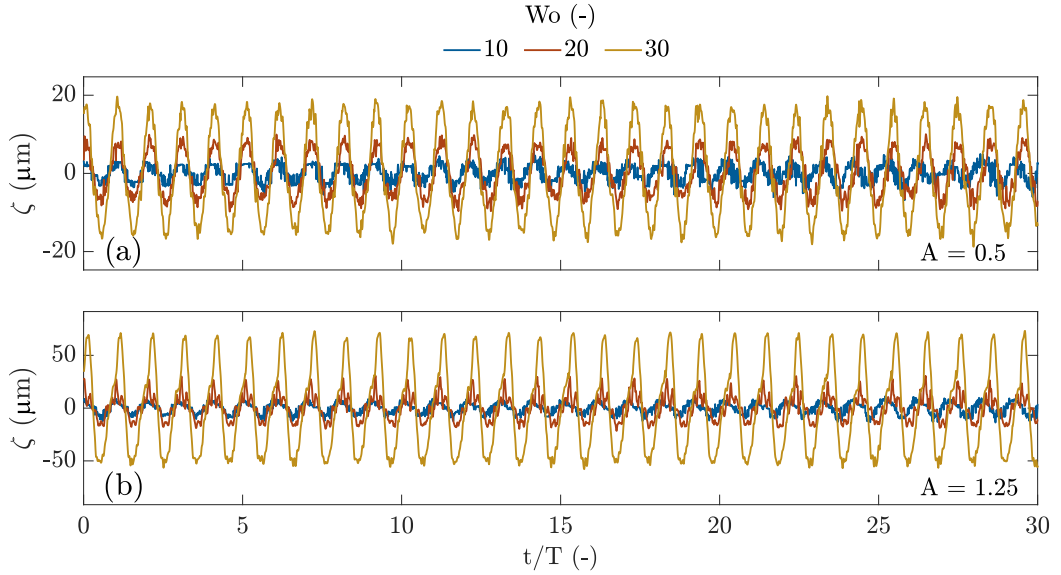


FIG. 8. Instantaneous wall displacement ζ at the tube midpoint over 30 pulsation periods for $Wo \in [10, 30]$ corresponding to $f = (0.149, 0.594, 1.337)$ Hz at pulsation amplitude $A = 0.5$ (a) and $A = 1.25$ (b).

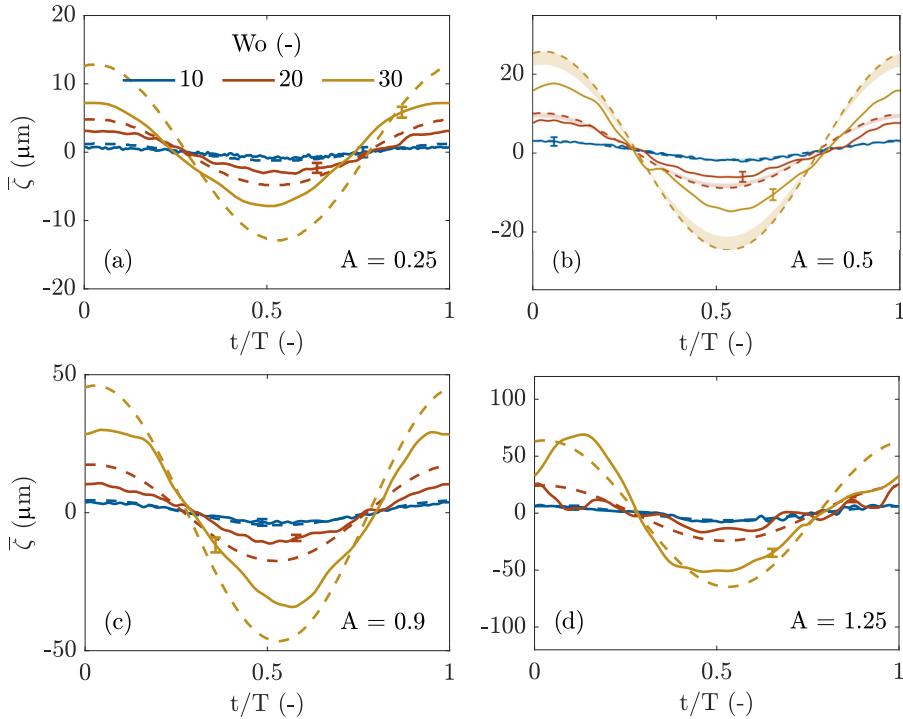


FIG. 9. Phase averaged wall displacement $\bar{\zeta}$ at the tube midpoint for $Wo \in [10, 30]$ and $A = 0.25$ (a), $A = 0.5$ (b), $A = 0.9$ (c) and (d) $A = 1.25$. Experimental results are shown as solid lines, whereas dashed lines depict the numerical results. Experimental uncertainty is indicated with error bars, numerical uncertainties due to the determination of Young's modulus E is shown in (b) as shaded areas.

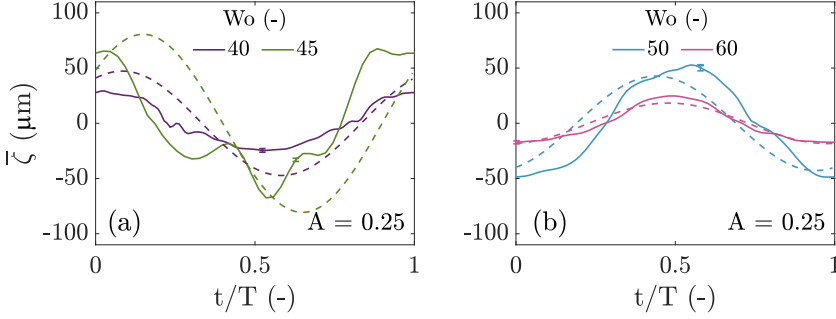


FIG. 10. As Fig. 9 for $A = 0.25$ and Wo around resonance ($Wo^* = 46.6$): (a) $Wo = (40, 45)$ and (b) $Wo = (50, 60)$. Solid lines show the experimental and dashed lines the numerical results. Experimental uncertainty is indicated by the error bars.

and Wo in this regime. Our numerical model predicts a sinusoidal wall displacement for all investigated cases. Qualitatively, the numerical results (shown as dashed lines) are in good agreement with the experiments for $A \in [0.25, 0.9]$. However, the maximal tube deformation is increasingly overestimated in the simulations as Wo increases. For a larger forcing amplitude [$A = 1.25$ in Fig. 9(d)], which leads to an increased response amplitude at a constant Wo , the experimental results deviate from a sinusoidal shape already at moderate Wo ($Wo = 20$ and 30). As the frequency increases towards the systems natural frequency of $Wo^* = 46.6$, the response amplitude increases and the wall motion becomes complex already at low forcing amplitudes, as shown for $A = 0.25$ in Fig. 10(a). At $Wo = 45$, for which the deformation is nonaxisymmetric, superposed frequencies are clearly visible in the phase-averaged wall displacement. These oscillations are substantially larger than the measurement uncertainty and are in phase every period. They are not reproduced in the numerical simulations, because the equations solved are axisymmetric and thus unable to capture non axisymmetric deformation by construction. While maximum wall displacement amplitudes are reasonably well captured by the numerical results, a phase shift is noticeable. As the frequency is further increased past resonance [Fig. 10(b)], the wall motion decreases in amplitude and becomes axisymmetric again, resulting in good agreement between simulation and experiment. At higher forcing amplitudes ($A > 0.25$), nonaxisymmetric effects and additional frequencies arise in the response already at lower Womersley number due to the increased displacement [e.g., at $Wo = 20$ for $A = 1.25$ in Fig. 9(d)].

For a more detailed study of the non-sinusoidal wall displacements, we performed Fourier transformations of the instantaneous wall displacement signals. The corresponding spectra are shown in Fig. 11, where the frequency f_{fit} (horizontal axis) is normalized by the corresponding forcing frequency f . The average level of the broadband noise is readily discernible at $O(10^{-4})$ mm/Hz, whereas the main peaks of the forcing frequency at $f_{\text{fit}}/f = 1$ are on the order of $O(10^{-3} - 10^{-2})$ mm/Hz. Close to resonance, the spectra exhibit peaks at the forcing frequency and at its harmonics (integer multiples of the forcing frequency). These peaks are at least one order of magnitude larger than the noise level. They are a consequence of the peaks present in the signal of the phase-averaged wall motion, clearly visible for, e.g., $A = 0.25$ and $Wo = 45$ in Fig. 10(a). Similarly, superposed frequencies occur in the wall motion for smaller Wo , but are only discernible at high amplitudes, e.g. for $A = 1.25$ in Figs. 11(a)–11(c).

In general, higher harmonics are observed for all cases where the wall displacement amplitude is larger than approximately $20 \mu\text{m}$ (corresponding to $(\bar{\zeta}_{\text{max}} - \bar{\zeta}_{\text{min}})/D_o \geq 3 \times 10^{-3}$) as well as for all cases of the highest pulsation amplitude. In Fig. 11(h) we show exemplarily the spectrum for the numerical simulations at $Wo = 20$, $A = 1.25$. This contains a single peak at the forcing frequency, characteristic of a linear response. Thus, as noted before, qualitative agreement of our model with

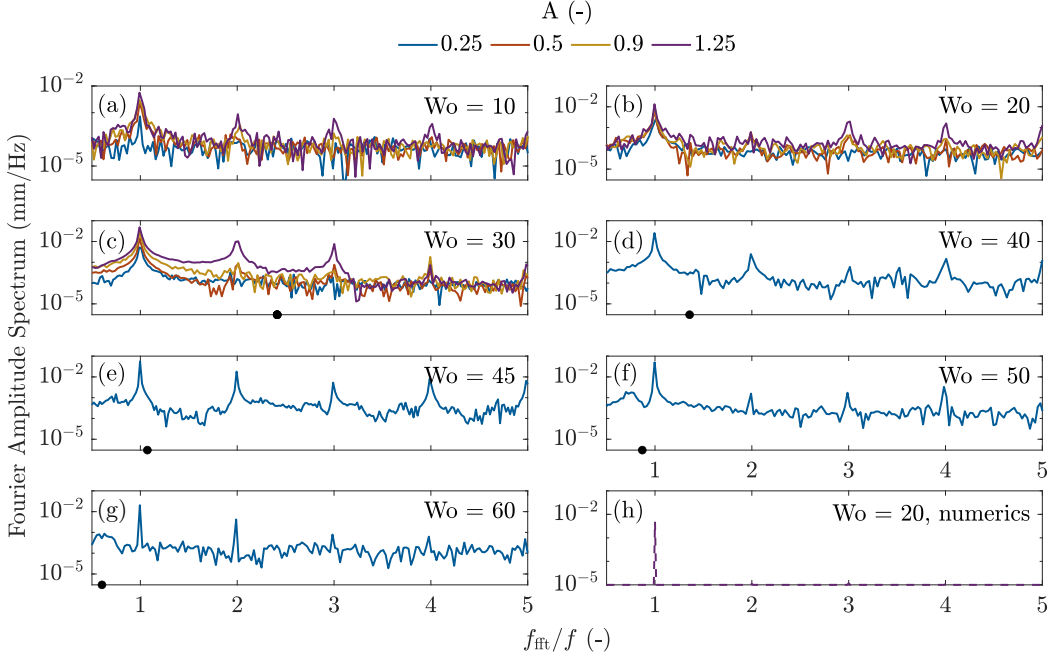


FIG. 11. Fourier amplitude spectra of the wall displacement signals (see Figs. 9 and 10) shown on a logarithmic y-axis. Colors indicate the pulsation amplitude A , each panel corresponds to a different Wo , as indicated in the legend. Frequencies are normalized by the respective forcing frequency, f . (a)–(g) show the Fourier amplitude spectra of the experimentally measured wall displacements while (h) shows the Fourier amplitude spectrum of the numerically simulated wall displacement at $Wo = 20$, $A = 1.25$.

the experiments is good as long as A is small and Wo is sufficiently far from resonances, resulting in wall displacements of less than $20\ \mu\text{m}$.

D. Response curves

The amplitude of the response to the pulsatile forcing, defined as the maximum wall displacement relative to the average, $(\bar{\zeta}_{\max} - \bar{\zeta}_{\min})/D_0$, is shown in Fig. 12 for all cases in which no collapse was observed. For the lowest forcing amplitude, $A = 0.25$, the amplitude of the response increases as Wo increases from 10, until it reaches a distinct peak at $Wo = 45$ (near resonance, $Wo^* = 46.6$), thereafter it decreases as Wo is further increased [see the blue solid line in Fig. 12(a)]. The numerical simulations capture qualitatively this trend, but they overestimate the response. In Fig. 12(b), the amplitude of the response is shown as a function of the amplitude of the forcing for several Womersley numbers. At $Wo = 10$, the response is linear, exactly as in a forced damped harmonic oscillator. This is consistent with the sinusoidal phase averaged wall displacements shown in Figs. 9(a)–9(c). As the Womersley number is increased, the amplitude of the response increases faster than linear in the experiments, whereas it remains linear in the simulations. This nonlinear increase in the response amplitude at large deformations is consistent with the softening behavior of the silicone measured in the stress-strain tests of Fig. 2.

In Fig. 13, we report the phase shift of the wall displacement relative to the flow rate $\dot{V}$. To estimate the experimental phase shifts, least-squares sine fits to the respective phase-averaged wall displacement signals were computed. We find that the measured wall displacement leads the inlet flow rate by approximately 0.5π until resonance is reached, after which an abrupt phase shift of π occurs. In Womersley flows of high Wo , pressure leads the flow rate by 0.5π . As such, the

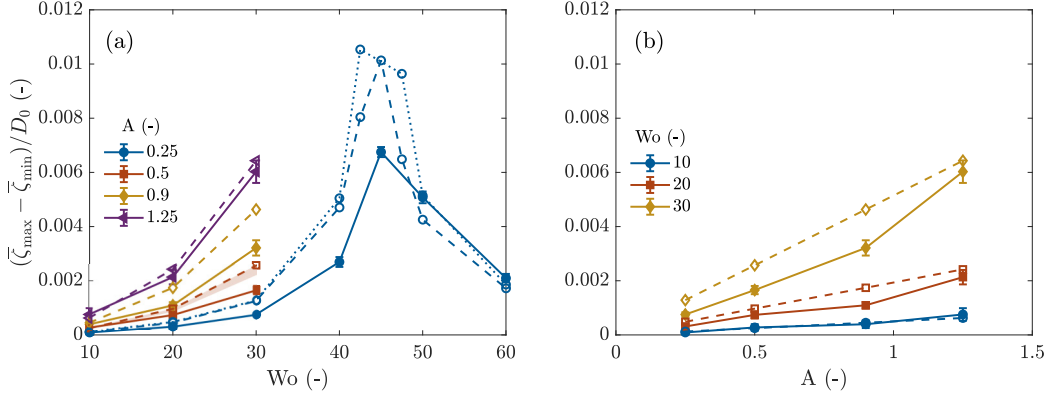


FIG. 12. Maximum (relative) phase-averaged wall displacement at the tube's midpoint as a function of Wo for different forcing amplitudes, A , (a) and as a function of A for the different Wo (b). Experimental results are shown in solid lines, whereas dashed lines show the numerical results. The dotted line in (a) depicts the numerical results without structural damping for $A = 0.25$.

measured wall displacement follows the inlet pressure rather faithfully for Womersley numbers sufficiently below the natural frequency. Further, the observed phase shift of π at resonance is again reminiscent of the behavior of a forced damped harmonic oscillator [27]. The phase shift in the numerical model shows an overall good agreement with the experiments, though the transition at resonance is considerably smoother. Further, the resonant frequency is slightly underestimated, evident from the phase change observed at a lower Wo with respect to the experiments, which is consistent with the peak position of the maximum wall displacement [Fig. 12(a)] and the estimation of the natural frequency presented in Sec. III A.

To assess the effect of structural damping in our model, we carry out additional simulations in which structural damping is neglected ($\mathcal{D} = 0$). For most cases, the omission of structural damping is negligible, with the exception of Wo around resonance [as an example, the $A = 0.25$ case is

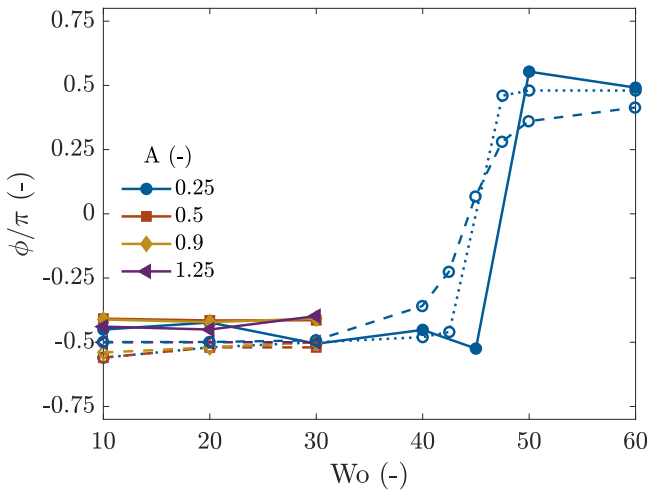


FIG. 13. Phase shift of the wall displacement relative to the flow rate $\hat{V}$ as a function of Wo for different pulsation amplitudes A . Experimental results are shown in solid lines, whereas dashed lines show the numerical results. The dotted line depicts the numerical results without structural damping for $A = 0.25$.

plotted as a dotted line in Fig. 12(a)]. Here, structural damping lowers the amplitude of the response and improves the agreement with the experimental results. However, neglecting structural damping results in a better agreement with the phase delay measured in the experiments (compare blue solid line with dotted line in Fig. 13). Together, these results suggest that the procedure used to determine damping in Fig. 6 may overestimate the damping factor and that the experimentally measured smaller deformation close to resonance may be due to nonlinear damping or nonaxisymmetric effects.

IV. DISCUSSION

In this work, we presented an experimental setup for investigating FSI of pulsatile flows through an elastic pipe segment. We applied high magnification imaging to accurately record wall motions on the order of one μm . We found that for small forcing amplitudes and small Wo , the response is predominantly linear and axisymmetric. At larger forcing amplitudes, the response is no longer linear, contains superposed higher harmonics and becomes nonaxisymmetric. Similarly, the wall displacement is nonaxisymmetric and strongly damped for Wo close to resonance. For even stronger forcing, we observed a sudden collapse of the flexible pipe. Our results provide a valuable initial benchmark for future modeling efforts aimed at capturing the transition to nonlinear regimes, particularly in cases where single-point measurements at the pipe midpoint are relevant. While this study focuses on single-point measurements, future experiments incorporating multipoint or 3D measurements would be critical for describing the spatial structure of the response for nonaxisymmetric deformations. Such advancements would also enable the development of more comprehensive benchmarks for 3D FSI models.

To complement the experiments, we performed numerical simulations based on the solution of the Navier-Stokes equations, coupled to the thin-shell theory, analogous to the study of Xu *et al.* [15] for channel flow with a compliant wall. In cases where the tube deformation is linear and axisymmetric, the numerical model is able to reproduce the measured wall displacements and captures the dynamics of the system. Near resonance and at high forcing amplitudes, the amplitude of the response is overestimated. If structural damping is disregarded, the amplitude is even further overestimated [Fig. 12(a)], but the phase delay of the response agrees better with the experimental results (Fig. 13). This suggests the need for a more complex viscoelastic constitutive law of the silicon segment including nonlinear damping. This hypothesis is supported by the experimental impulse response shown in Fig. 6, which initially shows a decaying mean value of the oscillation. The constitutive law could also implement the softening behavior exhibited by the material (Fig. 2). However, this effect is first noticeable at larger deformations and is therefore not expected to cause the observed discrepancies between experiment and simulation.

Clearly, the main limitation of the model is that it precludes nonaxisymmetric motions by construction, whereas in the experiments these can be observed for sufficiently high response amplitudes. Because of the single-point measurement technique used in the experiments, there is great uncertainty in the measured deformation in the nonaxisymmetric regime. This could partly explain why the deformation close to resonance is smaller in the experiments than in the axisymmetric simulations. Further discrepancies could arise from neglecting any prestress of the elastic tube that may be introduced while mounting the tube. According to the analytic model of Ref. [15] for channel flow, including the prestress in the numerical simulations would result in a higher resonance frequency and a smaller amplitude of the response. However, we expect the prestress is presumably much smaller than the hoop stress, and thus likely of little relevance. Similarly, the neglected viscous damping due to the air surrounding the elastic tube is expected to be negligible.

In summary, this work quantitatively investigates the fundamental problem of FSI in pulsating flow through a flexible pipe segment around resonance. We show that the system responds as a forced damped harmonic oscillator for sufficiently small deformations. In this regime, our simple axisymmetric numerical model is in good agreement with the experimental results. However, for significant deformations the experimental response is nonaxisymmetric and exhibits rich nonlinear

dynamics, eventually leading to a collapse of the tube at moderate deformations already. Our results open avenues for advancing the accuracy and reliability of numerical simulations of FSI in elastic tubes and other systems. Specifically, we suggest that to improve the agreement between experiments and numerical simulations, future numerical models should be extended to three dimensional motions and possibly include a nonlinear viscoelastic constitutive law.

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