

Unraveling friction forces of droplets on a non-wetting surface

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Understanding the friction forces acting on droplets moving across non-wetting surfaces is essential for advancing applications in fluid transport and microfluidics. In this study we quantify droplet friction on both superhydrophobic and superheated (Leidenfrost) substrates using two complementary experimental approaches: inclined plane dynamics and cantilever force sensing. We identify distinct friction regimes governed by contact-line pinning, viscous dissipation, and aerodynamic drag over a broad range of velocities and viscosities. A force-based scaling law captures the transition between these regimes as a function of the capillary number, integrating contact-line effects, internal dissipation, and aerodynamic contributions. Our model is further validated through white light interferometry and high-speed reflective interference contrast microscopy, which reveal the structure and evolution of the thin air or vapor layers beneath the droplets. These findings offer a comprehensive framework for predicting droplet behavior on non-wetting surfaces and inform the design of surfaces for reduced friction in engineering and microfluidic systems.

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I. INTRODUCTION

Friction has a ubiquitous influence across multiple disciplines—nature, engineering, and industrial processes. An ability to reduce friction has significant technological ramifications, affecting everything from large-scale fluid transport systems [1–3] to precision microfluidic devices [4–7]. This has led to a growing interest in the development of non-wetting surfaces that can minimize friction. Non-wetting arises when a thin film of air is trapped beneath the droplet and substrate, which leads to a substantial reduction in the effective contact area between the substrate and droplet [8,9] and consequently a reduction in friction [10–12]. However, the forces that affect droplet friction change when considering the manner in which they interact with a surface [13,14]. For

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instance, a droplet released on an inclined surface will experience aerodynamic drag, whereas a droplet pulled along a flat surface will not. Thus, unraveling the relevant forces affecting droplet mobility requires multiple experimental approaches and varied wettable surfaces.

One type of non-wetting is formed by the Leidenfrost effect, where a thin vapor film is generated beneath a droplet by gently depositing it onto a superheated surface. This insulating vapor cushion prevents direct contact between the droplet and the substrate, enabling it to hover without wetting the surface [15]. This phenomenon arises when the substrate temperature exceeds the critical temperature ($T > 300^\circ\text{C}$ when the droplet is pure water) for which the evaporation time of the droplet is maximized [16]. The hovering droplet exhibits extreme mobility, as the absence of solid–liquid contact eliminates adhesion and dramatically reduces friction [10,17–19]. Recent studies [18,20] have demonstrated that a Leidenfrost droplet can exhibit characteristics of rolling and slipping, and even self-propulsion if the flow morphology is asymmetric [21].

Extreme mobility and reduced friction are not exclusive to superheated surfaces. Similar effects can also occur at ambient temperatures when a droplet rolls or slides on a superhydrophobic surface [22–25]. Such surfaces are engineered with micro- and nanoscale textures that trap air beneath the droplet, thereby minimizing solid-liquid contact and sustaining a stable air layer [26]. This structure leads to high static contact angles and low contact-angle hysteresis, both of which help reduce contact-line friction [27]. As the droplet moves across the superhydrophobic surface, friction arises from two primary sources: (i) pinning-depinning at the contact line [28] and (ii) viscous dissipation due to internal circulatory flow near the contact disk between the droplet and the substrate [24,26,29,30]. At higher velocities, the droplet can dynamically entrain air beneath itself, giving rise to an aerodynamic Leidenfrost effect [12,31]. While this thin air-film may resemble the classical Leidenfrost vapor layer, it introduces additional dissipative forces which result in an increase in overall friction. Mouterde *et al.* [24] argue that when a droplet approaches terminal velocity while descending along an inclined superhydrophobic surface, aerodynamic drag (distinct from the aerodynamic Leidenfrost effect) becomes the dominant resistive force. Taken together, the dynamics of droplets moving on inclined superhydrophobic or Leidenfrost surfaces are governed by the interplay of three main resistive forces: (i) viscous dissipation from droplet curvature near the substrate [32,33], (ii) energy loss due to pinning-depinning events and hydrodynamic viscous dissipation in the bulk [28,29], and (iii) the aerodynamic resistance of the droplet [24]. These forces are balanced by the external force of gravity.

A number of different approaches have been adopted to examine the kinetic friction forces, including a drop moving on an inclined superhydrophobic surface, where the friction force is computed using the equation of motion [24,34] or direct friction force measurements are made using a cantilever force sensor on a flat surface [25,26,35]. A scaling law for friction has been proposed which shows that the friction scales as $(Ca = \eta U / \sigma)^{2/3}$ [32,33,36], where Ca is the capillary number, η is the viscosity of the film, U is the velocity, and σ is the surface tension of the droplet. However, these studies focused on a droplet moving over a lubricant-impregnated surface, specifically, a thin oil film with much higher viscosity than air. Backholm *et al.* [26] looked at a water droplet moving above a critical velocity on a superhydrophobic surface with a thin air-film trapped underneath the droplet. Due to the low velocities associated with these studies, the aerodynamic drag and viscous dissipation are typically ignored, which must be included at higher velocities. Thus, a comprehensive scaling law that explains friction over a wide range of velocities and capillary numbers remains elusive. To address this gap, we conducted a systematic study examining friction force as a function of velocity over six orders of magnitude (from $10\,\mu\text{m s}^{-1}$ to $2\,\text{ms}^{-1}$). We are able to show transitions between dominant friction forces as the capillary number is increased. For low capillary numbers, friction is constant and depends on contact-angle hysteresis. For intermediate capillary numbers, friction force scales as $Ca^{2/3}$, similar to lubricant-impregnated surfaces. For large capillary numbers, we find that viscous dissipation begins to play a larger role and the aerodynamic drag is small enough to ignore (for the velocities presented). We propose a scaling law that is based on all of the relevant forces rather than just one. Additionally, we investigate how friction correlates with variations in air-film thickness beneath the droplet.

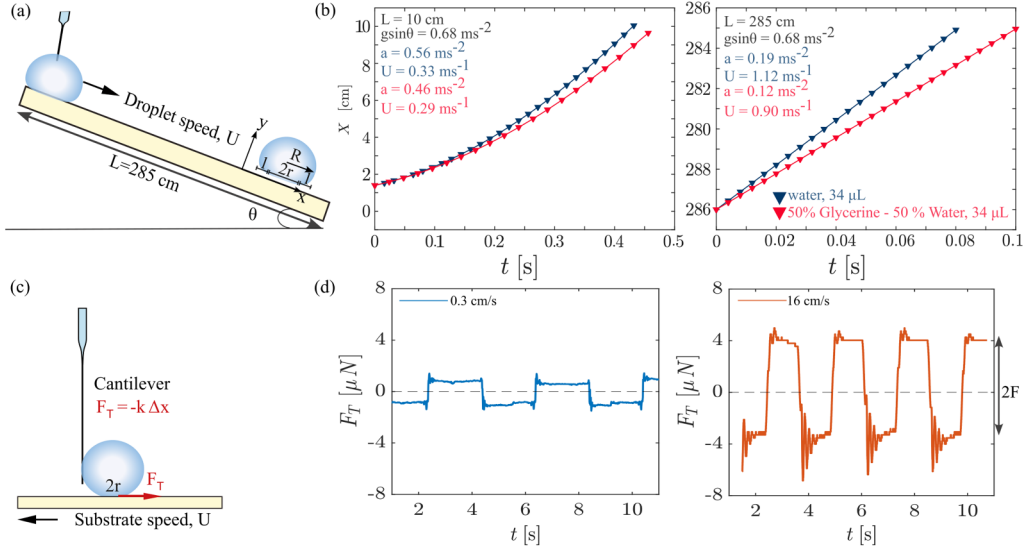


FIG. 1. Schematics of the experimental setup showing two different approaches to estimate the friction force. (a) A force balance approach where a droplet moves on an inclined (θ) stationary non-wetting surface. (b) Droplet x -centroid position vs time for a 10-cm plate still accelerating (left) and a 285-cm-long plate (right) at or near terminal velocity. (c) Friction measurement by a cantilever force sensor where the droplet attached to the cantilever rests on a Glaco-coated, superhydrophobic substrate while the substrate moves back and forth. (d) The total friction force on a water droplet (20 μL) as a function of time, measured by the cantilever force sensor for two different substrate speeds on a 40-cm plate.

II. EXPERIMENTAL SETUP

We use two complimentary experimental methodologies to measure the friction force experienced by a droplet on non-wetting surfaces: a force balance approach, where the droplet slides down a stationary inclined non-wetting surface [Figs. 1(a) and 1(b)], and a cantilever force sensor approach, where the droplet remains stationary while the flat and rigid non-wetting surface moves [Figs. 1(c) and 1(d)]. For both of these experimental approaches, we assume that droplet motion is dominated by sliding or slipping rather than rolling. This assumption is supported by the findings of previous studies [22,23,25,30], which have demonstrated that the droplet motion on a superhydrophobic surface is primarily translational, particularly at higher velocities. However, in the velocity regime where droplets can exhibit rolling motion, we adopt the theoretical framework developed by Mahadevan in Ref. [29]. Within this framework, the friction force can be approximated as viscous dissipation occurring in the bulk fluid within a characteristic volume r^3 , where r represents the radius of the contact disk between the droplet and the superhydrophobic surface. This volume represents the spatial extent of the contact region's influence within the droplet near its base.

A. Force balance approach

The inclined plane experimental setup [Fig. 1(a)] consists of three main components: (i) a droplet dispensing system, (ii) a long non-wetting inclined surface on which the droplet moves, and (iii) a high-speed imaging system. A blunt dispensing needle, connected to a syringe pump via surgical tubing, is used to deposit Milli-Q water droplets of 34 and 84 μL onto one end of the inclined non-wetting surface. The non-wetting surface sits on a custom-built platform mounted to a motorized vertical translation stage (ThorLabs MLJ250) to precisely adjust tilt angles (θ), and consequentially,

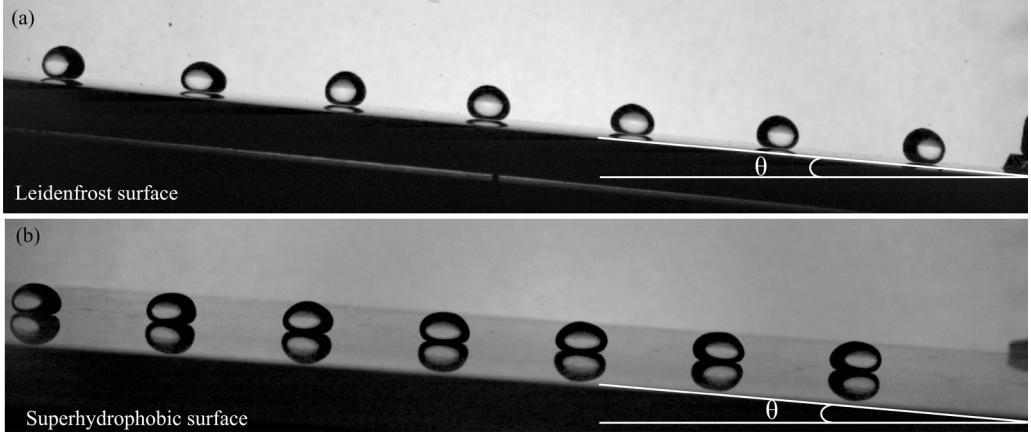


FIG. 2. Seven superimposed images of a 34- μL water droplet moving on an inclined superheated surface with a total length $L = 125$ cm (a) and superhydrophobic surface with a total length $L = 285$ cm (b). Both non-wetting surfaces are inclined at $\theta = 5^\circ$. The field of view of the camera shows only the bottom-most 11 cm of the surfaces. The superimposed droplet images are separated by 1.5 ms. The droplet on the Leidenfrost surface is moving at $U = 1.2 \text{ ms}^{-1}$, while the droplet on the superhydrophobic surface is moving at $U = 1.28 \text{ ms}^{-1}$.

the droplet velocity. The non-wettability characteristics of the surface are achieved by making it either superhydrophobic or superheated. The surface is made superhydrophobic by spray-coating a 2.85-m-long transparent glass plate with Glaco Mirror Coat Zero, a commercially available colloidal suspension of hydrophobic silica nanobeads in isopropanol. For the superhydrophobic case, we also study friction force by depositing a 34- μL droplet of two different viscosities: (i) a mixture of 70% water and 30% glycerine, and (ii) a mixture of 50% water and 50% glycerine mixture. Additionally, a black silicon surface with non-wetting properties was fabricated using cryogenic deep reactive ion etching as described in Koh *et al.* [37]. Glaco and black silicon differ significantly in their surface morphology and wetting behavior. Glaco consists of aggregated porous silica nanoparticles that can introduce more pinning sites, leading to higher friction, whereas black silicon features more uniform, nanoconelike structures that promote smoother droplet motion and lower friction. We also make a copper surface non-wettable by heating it above the Leidenfrost temperature ($T > 300^\circ\text{C}$). The time-resolved images of a droplet sliding down both surfaces (Fig. 2) are acquired using a high-speed camera (Phantom T3610) fitted with a 180-mm lens at 5000 fps with a field of view of 11 cm. A 200-W LED light (Godox 200 SL II) is used as a backlight. Figure 1(c) shows the x -centroid of a droplet sliding on a superhydrophobic and a Leidenfrost surface as a function of time. We compute the velocity and acceleration by fitting a second-order polynomial to the x -centroid data.

B. Cantilever force sensor approach

Figure 1(c) shows the cantilever force sensor setup. A 7-cm-long cantilever made of an acrylic capillary tube with an inner (0.288 mm) and outer radii (0.360 mm) is positioned vertically above the surface. The Glaco-coated superhydrophobic surfaces ($L = 40$ cm) are prepared in the same way as the inclined plate setup. The tip of the cantilever is submerged in a droplet (20 or 30 μL) resting on the substrate (Supplemental Movie 1 in Supplemental Material [38]). The substrate is mounted on a horizontal motorized translation stage (Thorlabs DDS220), allowing it to move at velocities ranging from 10 to $3 \times 10^5 \mu\text{m s}^{-1}$. The deflection of the cantilever (Δx) is captured at frame rates from 100 to 1000 fps using a Kronos camera fitted with a $6.5\times$ zoom lens. We use open-source software (Tracker) to quantify the deflection. Having determined Δx , we directly estimate the total friction as $F_T = k\Delta x$, where k is the stiffness of the cantilever, determined similar to the approach

of Refs. [32,39]. Figure 1(d) shows the friction values as a function of time computed directly from the calibrated cantilever sensors for two different substrate velocities.

C. Interferometry measurement

We performed reflective interference contrast microscopy (RICM) experiments to visualize the changes in the air-film thickness between the droplet and the superhydrophobic surface. The setup includes an LED light-emitting 530-nm green light focused onto the surface through a microscope objective ($2\times$ magnification and numerical aperture $NA = 0.055$). Interferometric recordings of the thin air-film interference patterns are captured at 3000 fps using a Kronos camera attached to an infinity-corrected tube lens. Separately, we obtain the absolute thickness of the air-film (point measurement) with white-light interferometry in which a coaxial illumination fiber-optic cable (RP23 Thorlabs) shines white light onto the surface from below. The fiber-optic cable also consists of multimode read fiber, which directs the reflected light into a spectrometer (CCS100 Thorlabs). The absolute local film thickness is then deduced by analyzing the peaks and valleys of the normalized spectroscopic signal acquired from the spectrometer.

III. RESULTS AND DISCUSSIONS

We begin by showing the sequential motion of a 34- μ L water droplet moving on two different non-wetting surfaces: a superheated surface and superhydrophobic surface. This sequential motion is visualized by overlaying experimentally captured side-view images of the droplet as it moves down the $\theta = 5^\circ$ inclined surface (Fig. 2). Seven superimposed images of a water droplet moving down a 125-cm-long superheated surface is shown in Fig. 2(a) and a 285-cm-long superhydrophobic surface in Fig. 2(b) (camera field of view is 10 cm from the end of the plate for both cases), both tilted at $\theta = 5^\circ$. Images are spaced by 1.5 ms. We use a fine 25-gauge needle with a superhydrophobic tip to gently dispense droplets and minimize impulsive pinch-off. By slowly retracting the needle after dispensing the desired volume, the droplet detaches smoothly and settles on the substrate with minimal deformation or bouncing. Nevertheless, a small initial bounce can still occur due to pinch-off dynamics during detachment (Supplemental Movie 2 [38]). This transient motion quickly subsides as the droplet transitions to smooth sliding along the inclined superhydrophobic surface (Supplemental Movie 3 [38]). At higher velocities, however, a mild bouncing behavior reappears (Supplemental Movie 4 [38]), likely driven by aerodynamic levitation or local pinning-depinning events. These interactions can cause a gradual buildup of energy, resulting in shape undulations and intermittent bouncing [13]. We quantitatively examine the resulting kinematics of a droplet as it moves down an inclined superheated or superhydrophobic surface. The total friction force is then explored by comparing both inclined plate and cantilever measurements.

A. Droplet kinematics

In Fig. 3(a), we compare the experimentally measured velocities of droplets moving on inclined superheated (Leidenfrost) and superhydrophobic surfaces against the theoretical upper velocity limit for a perfectly sliding droplet with no energy loss, given by $U_{\text{th,max}} = \sqrt{2gL \sin \theta}$. We have performed the experiments on inclined plates of three different lengths ($L = 73, 125$, and 285 cm) and tilt angles ranging from $\theta = 1.5^\circ$ to 8.5° . For a short plate length ($L = 73$ cm), the velocity of the Leidenfrost droplet remains below the theoretical upper limit but consistently exceeds that of a droplet on a superhydrophobic surface at the same inclination and length. This implies that droplets on superhydrophobic surfaces experience greater energy dissipation compared to those in the Leidenfrost state. When the inclined length is increased to $L = 125$ cm—over 70% longer—the Leidenfrost droplet continues to accelerate without significant additional energy loss. This is also evidenced by Fig. 3(b), where the droplet acceleration approaches $g \sin \theta$, indicating minimal resistance and low-energy dissipation. Remaining losses in the Leidenfrost regime likely stem from translational and internal rotational dynamics [21]. Figure 3(a) also shows measurements for

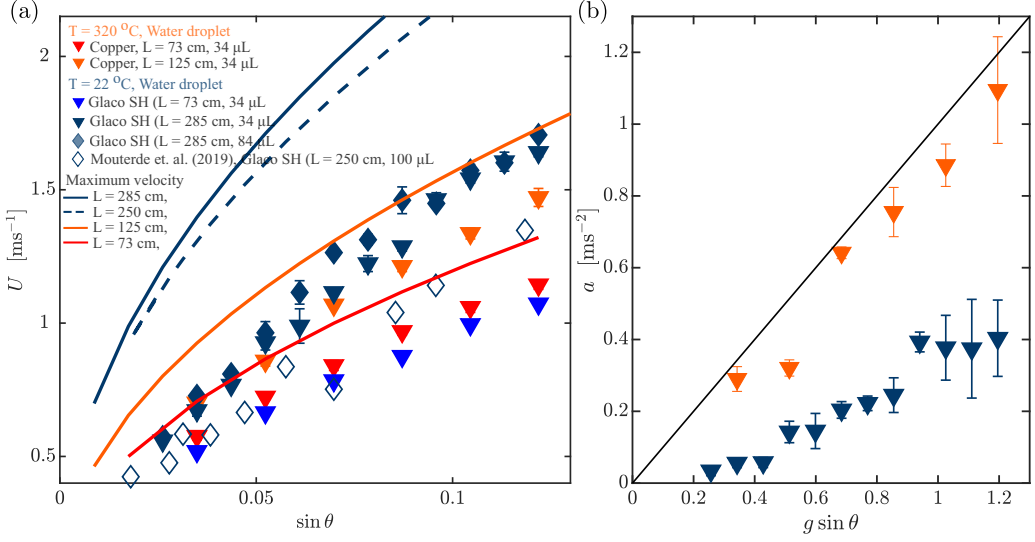


FIG. 3. (a) Measured velocity comparisons of a water droplet (34 and 84 μL) moving on inclined superheated and superhydrophobic surfaces vs the maximum potential energy velocity. A comparison is made against the upper velocity limit ($U_{\text{th,max}} = \sqrt{2gL \sin \theta}$) for a perfectly sliding droplet with no energy loss for varying L , as marked. Data extracted from [24] as marked. (b) Measured acceleration vs gravitational acceleration (line) for two cases in (a).

water droplets on a superhydrophobic surface of length $L = 285$ cm for 13 different tilt angles: $1^\circ < \theta < 8^\circ$ and two droplet volumes (34 and 84 μL). The velocities for both volumes are nearly identical at each angle but remain well below the theoretical maximum limit. This suggests that, in addition to gravitational driving, the kinematics are counteracted by frictional forces arising from viscous dissipation within the droplet, flow within the thin air layer beneath it, and aerodynamic drag. Consistent with this observation, the estimated accelerations are significantly lower than $g \sin \theta$ and lower than those for Leidenfrost droplets, as indicated in Fig. 3(b). Nevertheless, since the measured acceleration remains small but nonzero, we include it in our friction force calculation for completeness. Finally, we compare our data with results from Mouterde *et al.* [24], who used a superhydrophobic, Glaco-coated surface of length $L = 250$ cm. Their reported terminal velocities are comparable to those we observed for droplets on an $L = 285$ cm inclined superhydrophobic surface, indicating that droplet motion in our experiments is similarly friction-limited.

B. Total friction force

The total frictional force on a droplet depends on both the shape and size of the droplet. A droplet of volume V with density ρ_w , viscosity η_w , and surface tension σ can deform due to gravity when placed gently on a non-wetting surface. The degree of deformation depends on whether the nominal radius [$R_o = (3V/4\pi)^{1/3}$] is smaller or greater than the capillary length [$l_c = (\sigma/\rho g)^{1/2}$] and is governed by the Bond number [$\text{Bo} = (R_o/l_c)^2$] [29]. A sufficiently small droplet with $\text{Bo} \ll 1$ retains a quasispherical shape. A droplet becomes more flattened when the droplet radius approaches the capillary length ($\text{Bo} \approx 1$). The droplet deformation at the bottom forms an assumed radius (r) near the substrate [shown in Fig. 1(a)]. The viscous dissipation for such a droplet occurs at the length scale of $\delta = (\eta r/\rho U)^{1/2}$, which is the viscous boundary layer within the droplet [29]. Note, the maximum radius of the flattened droplet is given by $R \approx (V/\pi l_c)^{1/2}$.

The friction forces in the cantilever vs inclined plane experiments are not the same. In the case of our cantilever measurements, a superhydrophobic surface moves underneath a droplet, where

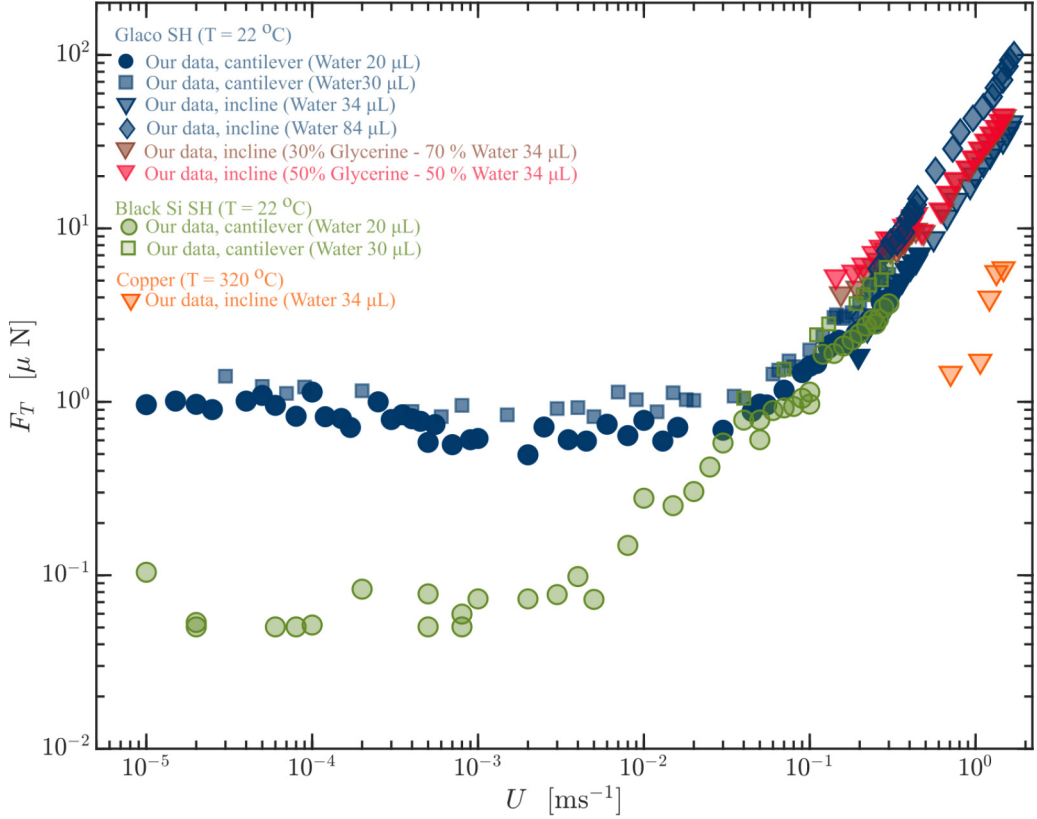


FIG. 4. Total friction force plotted vs velocity, measured on various non-wetting surfaces: Glaco-coated superhydrophobic surface, black-silicon surface, and a Leidenfrost surface. The velocity corresponds to the speed of a droplet moving down an inclined plane when the friction force is estimated using the force balance approach. For measurements with a cantilever force sensor, velocity refers to the speed of the substrate.

its motion is opposed by friction forces arising from hydrodynamic viscous dissipation in the bulk which arises from interaction of the air-film, viscous dissipation from droplet curvature near the substrate, and energy loss due to pinning–depinning processes in the absence of an air layer [28,29,32,33,40]:

$$F_T = \begin{cases} 2r\sigma(\cos\theta_r - \cos\theta_a), & \text{if not levitated} \\ \alpha \frac{\eta_w U}{\delta} \pi r^2 + \beta_1 \left(\frac{\eta_a U}{\sigma} \right)^{2/3} 2\pi\sigma r, & \text{otherwise} \end{cases} \quad (1)$$

We find $\alpha = 0.09$ and $\beta_1 = 6$ as a best fit for all cantilever cases, including those from Daniel *et al.* [41]. In the case of an inclined superhydrophobic and superheated surface, external gravitational forces drive the motion and aerodynamic drag [24] forces are also considered such that

$$F_T = mg \sin\theta - ma = \alpha \frac{\eta_w U}{\delta} \pi r^2 + \beta_2 \left(\frac{\eta_a U}{\sigma} \right)^{2/3} 2\pi\sigma r + \zeta \frac{\rho_a U^2}{\text{Re}^{1/2}} \pi R^2, \quad (2)$$

where $\alpha = 0.09$, $\beta_2 = 1$, and $\zeta = 1$ for all cases presented here, including Ref. [24]. These prefactors are determined by fitting U as a function of $\sin\theta$ in Fig. 3.

Figure 4 illustrates the variation in total friction as a function of velocity for two different types of superhydrophobic surfaces in the cantilever experiment: (i) a Glaco-coated surface and (ii) a black silicon surface. For these cases, the total friction does not significantly increase, despite a

three-order magnitude rise in velocity from $U = 10^{-5}$ to 10^{-3} ms^{-1} . This consistent friction force across a broad range of velocities suggests that the friction primarily arises from contact-angle hysteresis similar to Refs. [25,26] and is velocity independent [Eq. (1)]. With a further increase in the velocity $U > 10^{-2} \text{ ms}^{-1}$, the friction forces steadily increase for the black-silicon surface at lower velocities than for the Glaco-coated superhydrophobic surface. This marks a transition from a regime dominated by contact-line pinning to one where a thin air layer forms beneath the droplet, reducing pinning across most of the contact area of the droplet, as will be shown in Sec. V.

The functional dependence of friction on velocity at these increased speeds implies that the contact-line friction is no longer the dominant friction force driving the dynamics, and the black-silicon surface supports droplet levitation more readily than the Glaco-coated superhydrophobic surface. In the cantilever experiments, the substrate moves while the droplet remains stationary, ruling out any aerodynamic resistance. Thus, the increase in friction likely stems from viscous dissipation, both in the bulk and along the rim. Additionally, we also observe a small increase in the friction force associated with an increase in droplet volume.

For droplets sliding on an inclined plate, Fig. 4 reveals a monotonic increase in friction with velocity, consistent with trends observed in the cantilever experiments. This trend suggests that contact-line pinning is no longer the dominant source of friction once the droplet enters the aerodynamic Leidenfrost regime, where a dynamically entrained air layer lifts the droplet and minimizes solid-liquid contact (see Sec. V). Figure 4 also shows an increase in friction force as droplet volume is increased from 34 to 84 μL , in agreement with cantilever-based observations. This increase is attributed to the additional viscous dissipation, which scales with the volume of fluid influenced by the expanded contact region [29]. Crucially, a more pronounced difference emerges when comparing water droplets to those with higher viscosities. At lower velocities, viscous droplets experience higher friction. This increase is likely due to the localized contact with tall surface asperities present on the substrate, leading to pinning–depinning events, which in turn result in energy dissipation near the contact regions. However, this disparity diminishes at higher velocities, suggesting that the air layer becomes sufficiently thick to further minimize the local contact even for more viscous fluids, thereby reducing the influence of surface roughness on dissipation. Additionally, we also notice a small difference in friction force between inclined plane and cantilever measurements for water droplets at comparable velocities ($U = 190\text{--}300 \text{ cm s}^{-1}$). In contrast, droplets on superheated surfaces (Leidenfrost) experience friction forces that are an order of magnitude lower than those on superhydrophobic surfaces at similar velocities. This significant reduction arises from the fundamentally different levitation mechanisms and the nature of the supporting vapor layers. In the Leidenfrost state, the droplet is supported by a vapor layer generated through continuous evaporation, with the vapor escaping radially through a narrow annular gap near the substrate. This behavior differs markedly from the air entrainment mechanism that supports aerodynamically levitated droplets. The resulting vapor flow in Leidenfrost droplets acts as an efficient lubricant, substantially reducing viscous shear and minimizing friction, even at higher droplet velocities.

Figure 5 shows the variation of nondimensional friction force as a function of capillary numbers, revealing three distinct regimes, each characterized by different scaling behaviors, suggesting that no single universal law governs the droplet motion across the full parameter space explored. These regimes are observed consistently across a variety of droplet volumes and surface types, including both superhydrophobic and super-slippery (black silicon) surfaces. At low capillary numbers (black silicon: $\text{Ca} < 5 \times 10^{-7}$ and Glaco-coated surface: $\text{Ca} < 5 \times 10^{-6}$), the normalized friction remains constant and independent of Ca , indicating that this regime is dominated by contact-line friction. This behavior is well described by the relations $F_T/2r\sigma \propto \cos \theta_r - \cos \theta_a$. As the capillary number increases, the nondimensional friction force begins to scale as $\text{Ca}^{2/3}$ up to $\text{Ca} \approx 5 \times 10^{-5}$ for both superhydrophobic and black-silicon surfaces. This scaling is characteristic of a regime in which the droplet levitates on a thin air-film, consistent with the Landau-Levich-Derjaguin law. Similar $\text{Ca}^{2/3}$ scaling has been observed in a system where oil droplets slide over water layers (see Daniel

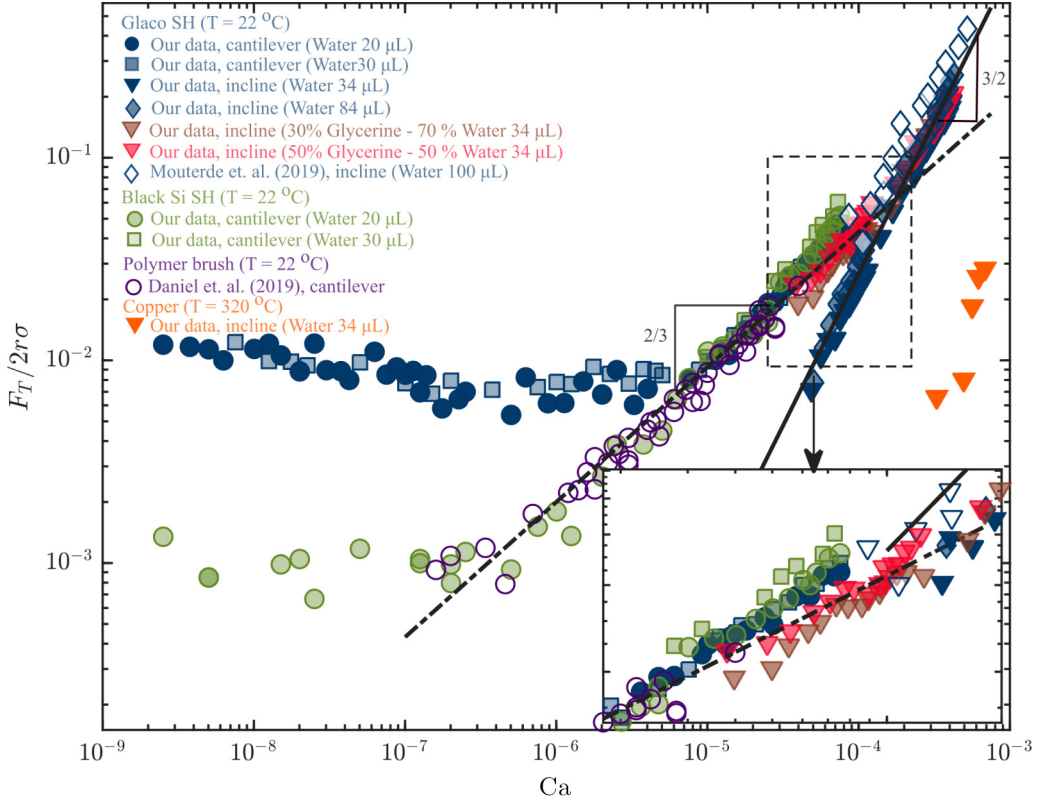


FIG. 5. (a) The nondimensional friction force as a function of capillary number for cantilever and inclined plate experiments. Low $Ca \lesssim 10^{-6}$ have constant friction forces associated with contact-line friction. At larger capillary numbers ($Ca \gtrsim 10^{-4}$), the friction deviates from $Ca^{2/3}$ scaling, indicating the onset of additional dissipation mechanisms. For viscous droplets (30% and 50% glycerin mixture), the normalized friction force follows $Ca^{2/3}$ scaling under accelerating conditions, while pure water droplets exhibit a $Ca^{3/2}$ scaling under the same conditions. However, once droplets—regardless of viscosity—reach terminal or near-terminal velocity, their friction force consistently follows the $Ca^{3/2}$ scaling. Other studies as marked respectively [24,41].

et al. [41]). Beyond $Ca > 5 \times 10^{-5}$, the nondimensional friction forces computed directly from cantilever-based measurements begin to deviate from the $Ca^{2/3}$ trend (see inset of Fig. 5). This deviation is attributed to increased internal viscous dissipation within droplets, as described by the first term in Eq. (1), which becomes increasingly significant at higher capillary numbers. Crucially, the nondimensional friction forces from water droplets (both 34- and 84- μL droplets) racing down inclined surface experiments show a pronounced deviation from the $Ca^{2/3}$ scaling and instead follow a steeper $Ca^{3/2}$ trend over a similar range of capillary numbers. At higher capillary numbers ($Ca > 9 \times 10^{-5}$), this scaling persists for both water (34 and 84 μL) and higher-viscosity droplets (34- μL water-glycerine mixture droplets with 30% and 50% glycerine by volume). In this regime, the droplet moving on an inclined surface approaches at or near terminal velocity, where aerodynamic drag is an additional source of friction, as suggested by Mouterde *et al.* [24]. By contrast, for viscous droplets at moderate capillary numbers ($5 \times 10^{-5} < Ca < 7.5 \times 10^{-5}$), the deviation from $Ca^{2/3}$ to $Ca^{3/2}$ is suppressed, and the nondimensional friction forces revert to the $Ca^{2/3}$ scaling (Fig. 5). This suggests that increased viscosity delays transition from $Ca^{2/3}$ to $Ca^{3/2}$ scaling.

We plot the contributions of each term in Eqs. (1) and (2) as a function of the capillary number in Fig. 6 to analyze the contributions of each friction component, when levitated. Specifically, at

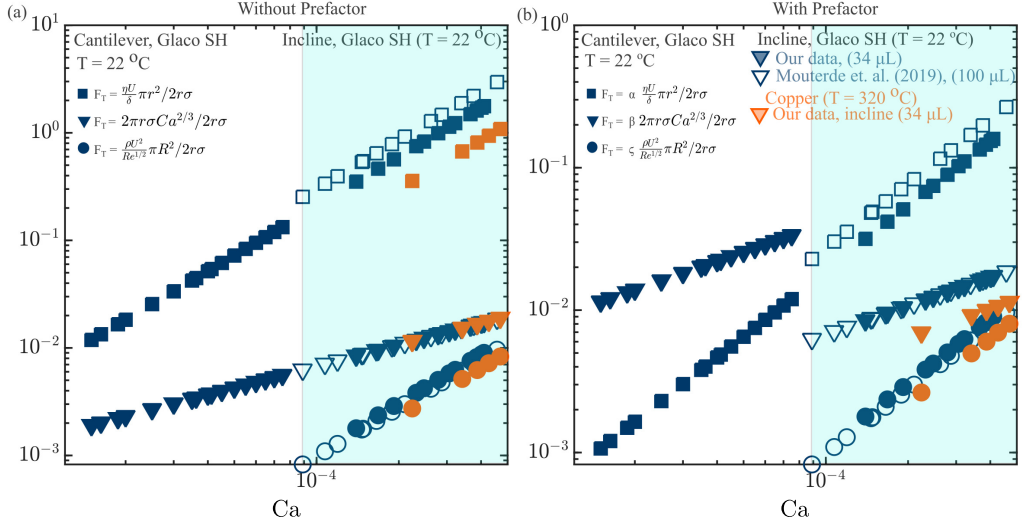


FIG. 6. Forces that make up the total force of friction [Eqs. (1) and (2)] as a function of capillary number, (a) without a prefactor and (b) with prefactors from Table I. Viscous dissipation in the bulk (squares), viscous dissipation from the droplet curvature (triangles), and aerodynamic drag (circles) as marked. Other data extracted from [24]. Cantilever experiments are dominated by viscous dissipation in the rim when $Ca < 2 \times 10^{-4}$, and viscous dissipation in the bulk plays a larger role as Ca , which is easier to see in (b). On the inclined plate, viscous dissipation in the bulk dominates the total friction forces; however, aerodynamic drag does have a higher slope, indicating that at higher Ca , aerodynamic drag may dominate. Viscous dissipation in the bulk is ignored for Leidenfrost cases because of the very thick vapor layer.

higher droplet velocity (large capillary number), we find that there is significant contribution of the viscous dissipation in the droplet to the total friction, regardless of whether the droplet or the superhydrophobic surface is moving [Fig. 6(a)]. In Fig. 6(b) the prefactors (Table I) are added, and it becomes more evident how the viscous dissipation term becomes more important as the capillary number is increased (increased velocity). In the cantilever region the viscous dissipation term becomes more important $Ca^{2/3} > 1 \times 10^{-5}$, and in the inclined plate region, the viscous dissipation and $Ca^{2/3}$ terms are much larger than the aerodynamic drag term.

To address the observed discrepancies in data, we propose a simple yet robust scaling based on Eqs. (1) and (2), which accounts for all relevant forces contributing to the droplet motion. When these contributions, including the internal viscous dissipation and aerodynamic drag, are incorporated, the deviation seen in Fig. 5 disappears, resulting in excellent agreement between the experimental data and proposed scaling model, as evidenced by Fig. 7. Notably, the discrepancies observed for viscous droplets at moderate to high capillary numbers are also resolved when internal

TABLE I. Prefactors used for estimating the forces at play in the droplet data presented. We apply the same prefactors for a given type of experiment to all cases, including those extracted from other studies. However, we also present the prefactors other studies used as a comparison.

Force prefactor From Eqs. (1) and (2)	Cantilever Herein Others	Incline superhydrophobic Herein Others	Incline Leidenfrost Herein
α	0.09 0 [41]	0.09 0.85 [24]	0
β	$\beta_1 = 6$ 3.81 [41]	$\beta_2 = 1$ NA	0.6
ζ	0	1 34 [24]	1

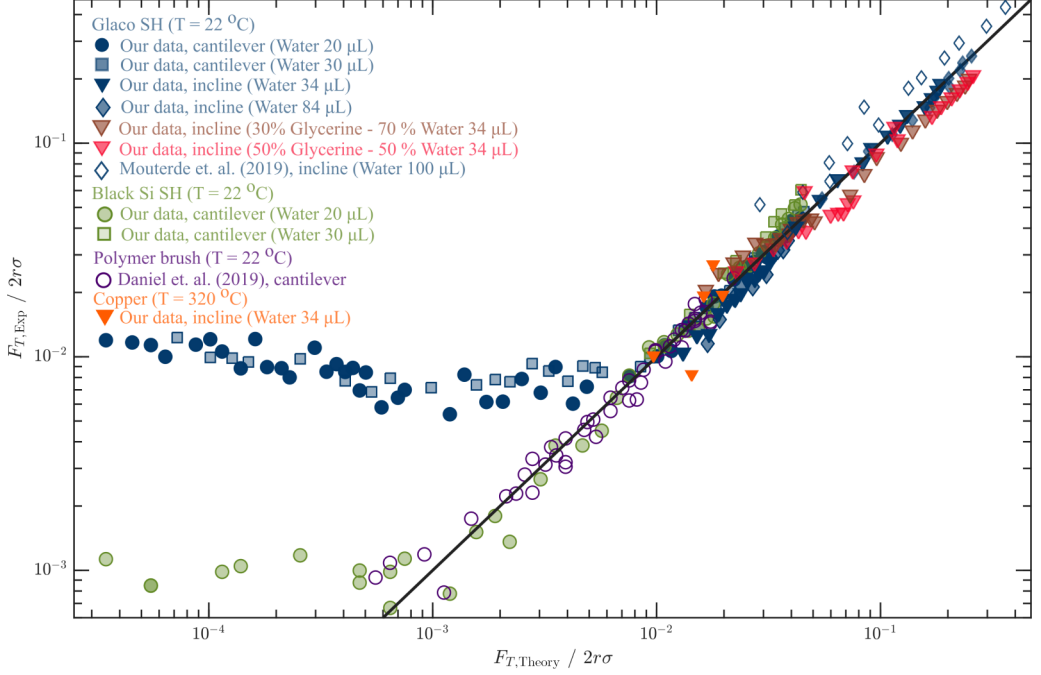


FIG. 7. Comparison of normalized experimental ($F_{T,\text{Exp}}$) and theoretical friction forces ($F_{T,\text{Theory}}$) based on Eqs. (1) and (2). Other data extracted from Refs. [24,41]. The black line represents a slope of 1.

viscous dissipation is included in the analysis. Furthermore, data from our superhydrophobic cases and those reported by Mouterde *et al.* [24] collapse onto the same scaling curve when all the forces are considered for the same prefactors as shown in Table I. It is important to highlight that the prefactors used here differ from those used by [24], where the second term in the force balance is neglected, and from those in Ref. [41], where the first (viscous) term is omitted. These variations in prefactor also depend on the specifics of contact-line distortion [42]. We can evaluate the contact-line distortion to observe the overall state of the droplet (e.g., levitating, contact-line distortion, etc.) by looking at the interference pattern underneath the droplet in Sec. IV.

IV. THIN FILM INTERFEROMETRY

Figure 8 highlights the difference in the interference pattern for a 34- μL water droplet aerodynamically levitated at comparable capillary number but under different experimental conditions—(i) cantilever-based measurements [Fig. 8(a)], (ii) drop racing on inclined plane [Fig. 8(b)], and (iii) Leidenfrost droplet [Fig. 8(c)]. The interference patterns are obtained from time-resolved RCM imaging. The interference pattern in the superhydrophobic cases [Figs. 8(a) and 8(b)] is qualitatively similar, showing closely spaced fringes at the top and bottom (in the lateral direction, perpendicular to the droplet motion) and flatter fringes near the center, indicating that the air-film is thinnest at the top and bottom (in lateral direction) and relatively thicker at the center. However, notable differences emerge: the interference patterns for cantilever-based experiments appear distorted, whereas they appear smoother and symmetric for the inclined plane case. These variations reflect localized differences in the air layer thickness, as schematically illustrated in Figs. 8(d) and 8(e). Despite comparable capillary numbers, different experimental setups lead to distinct interference patterns and air-film geometries, contributing to variations in the prefactors.

Furthermore, the Leidenfrost cases also scale with the forces outlined in Eq. (2), though with slightly different prefactors. As shown in Fig. 8(c), the interference pattern for the Leidenfrost

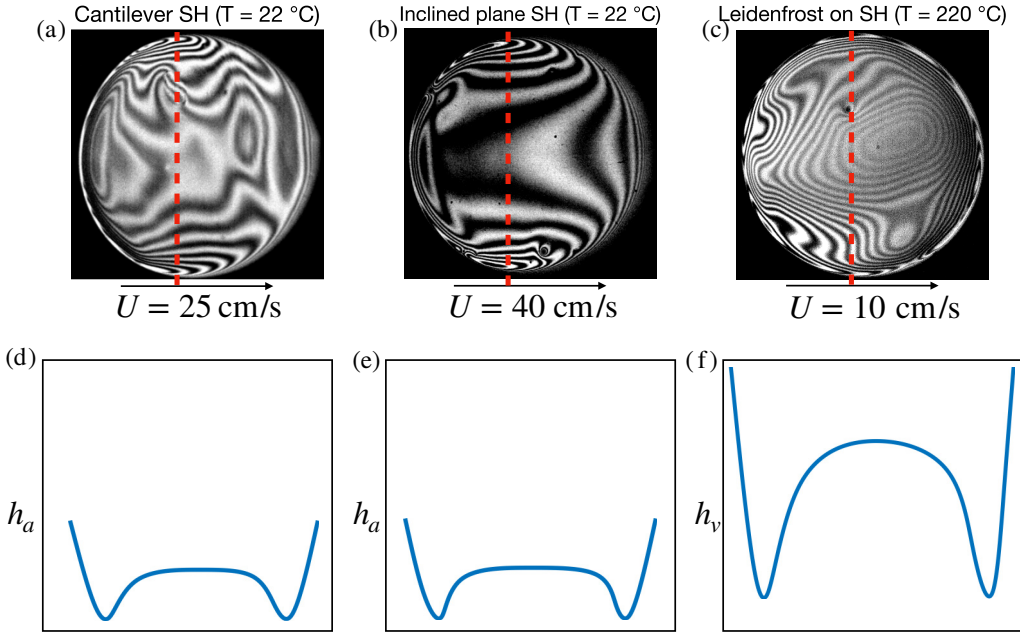


FIG. 8. Reflection interference contrast microscopy (RICM) images showing the air-film profile beneath a levitated droplet moving from left to right on (a), (b) Glaco-coated superhydrophobic surfaces at room temperature ($T = 22^\circ\text{C}$) and (c) Glaco-coated superhydrophobic surfaces heated to a temperature $T = 220^\circ\text{C}$. (a), (b) The interference fringe pattern for a $34\text{-}\mu\text{L}$ water droplet aerodynamically levitated in two experimental configurations: (a) cantilever-based experiments, where a droplet is dragged on a Glaco-coated superhydrophobic surface using a capillary tube, and (b) inclined plane setup ($\theta = 4^\circ$). In both cases, a characteristic horseshoe-shaped fringe pattern is observed, where closely packed fringes appear at the top and bottom of the droplet in the lateral direction (perpendicular to the droplet motion) and a flat fringe pattern around the droplet center, indicating the thinnest air-film at top and bottom (in lateral direction) and a thicker film at the center. (c) A Leidenfrost droplet on a heated Glaco-coated surface ($T = 220^\circ\text{C}$) exhibits a different interference pattern, more curvature in the middle, and a larger offset from the surface. Instead, densely packed fringes appear around the droplet perimeter, indicating vapor film thickness varies rapidly near its perimeter. However, an equally spaced fringe pattern appears away from the perimeter, indicating that vapor layer thickness is continuously varying beneath the droplet, with the thickest around the center. (d)–(f) Schematic illustrations of the corresponding 2D air/vapor layer profiles (a and b, respectively) beneath levitated droplets in each case along the dashed red line. (d)–(f) Not to scale.

droplet differs markedly from that of droplets on superhydrophobic surfaces. Specifically, they exhibit pronounced curvature near the center and a larger offset from the substrate, indicating a substantially thicker vapor layer as shown in Fig. 8(f) (similar to [19]). The fringes are densely packed near the droplet perimeter, reflecting rapid spatial variation in vapor thickness, while more uniformly spaced fringes toward the center suggest a gradually varying, thicker vapor cushion beneath the droplet. This is in sharp contrast to the thinner air-films supporting aerodynamically levitated droplets. Owing to the significantly thicker vapor layer in the Leidenfrost regime, viscous dissipation within the supporting layer is nearly negligible and is thus excluded from the friction analysis in these cases.

The velocity of the Leidenfrost droplet near the theoretical upper velocity limit in Fig. 3(a) also implies that the viscous dissipation of the droplet is near zero (i.e., $\alpha = 0$). The viscous dissipation from the droplet curvature is overestimated in the Leidenfrost case as the thickness of the vapor layer is large, and thus β is decreased. Further, Landau-Levich-Derjaguin estimation can also be written

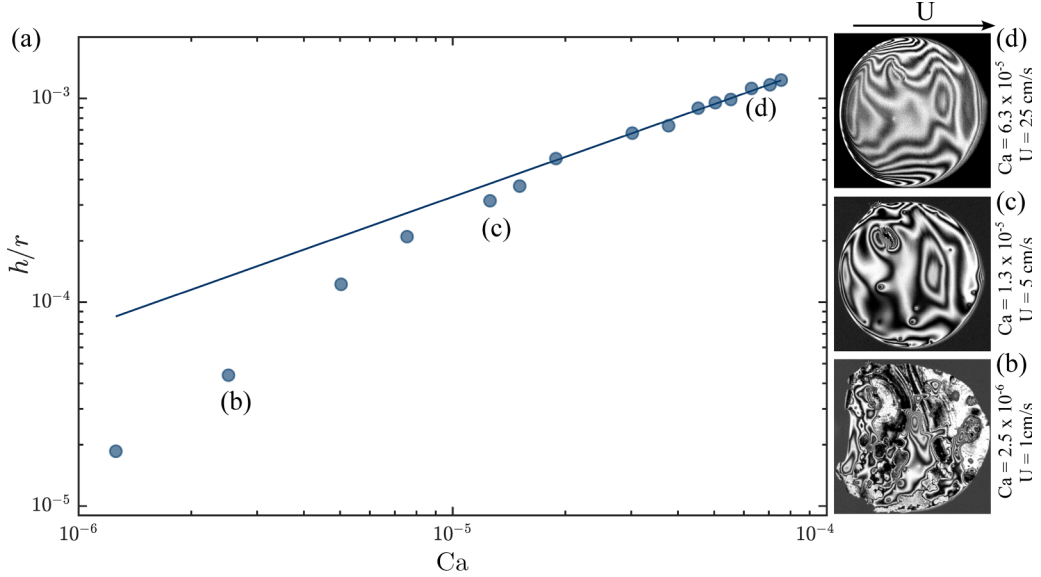


FIG. 9. (a) The air-film thickness in the central region of a 34- μL water droplet, plotted as a function of the capillary number. The droplet moves on a Glaco-coated superhydrophobic surface ($T = 22^\circ\text{C}$) at a controlled speed on a cantilever, and the air-film thickness is measured using white-light interferometry. The solid line represents the thickness predicted by the Landau-Levich-Derjaguin model, expressed as $h \approx r\text{Ca}^{2/3}$, where Ca is the capillary number and r is the contact radius. (b)–(d) Reflection interference contrast microscopy (RICM) is used to visualize changes in the air-film thickness when a droplet moves from left to right on a superhydrophobic surface at progressively increased controlled velocity (Ca as marked). RICM images confirm that the thin air-film thicknesses adhere to LLD law. (b) has a very small film that is in contact with the surface in many places, whereas (d) shows a larger air-film that is thicker in the center with thinner portions top and bottom and more evenly distributed than the other two cases.

as a function of the vapor thickness ($\eta_a U/h$) $2\pi r l \approx \gamma(\eta_a U/\sigma)^{2/3} 2\pi \sigma r$, where a thicker vapor layer will lower the frictional force. This can be accomplished by setting $\beta = 0.6$.

Our approach reveals that the dominant forces responsible for relative droplet motion between a substrate change from (i) contact-angle hysteresis to (ii) viscous dissipation from droplet curvature to (iii) viscous dissipation in the bulk, and potentially (iv) aerodynamic drag at much higher velocities not reached in this study.

V. LANDAU-LEVICH-DERJAGUIN SCALING

Next, we show that the air-film thickness beneath a droplet moving on a superhydrophobic surface increases nonlinearly with droplet velocity and exhibits Landau-Levich-Derjaguin (LLD) scaling when the droplet becomes levitated. We employ white-light interferometry to measure the air-film thickness around the flattest region of the droplet, which falls along the midsection of the droplet [shown in Fig. 9(a)]. We also show time-resolved RCM imaging from below to establish correlations with friction forces stemming either from the contact-line pinning or viscous dissipation [Figs. 9(b)–9(d), Supplemental Movies 6 and 7 [38]]. We find that for a specific range of capillary numbers ($10^{-5} < \text{Ca} < 10^{-4}$), where friction scales as $(\text{Ca}^{2/3})$, the air-film thickness beneath the droplet varies in accordance with $h \approx \text{Ca}^{2/3}$, as seen in Fig. 9(a). This observation suggests that the air-film thickness adheres to the Landau-Levich-Derjaguin (LLD) law [43], which correlates with the absence of contact-line pinning [32], typically observed when a moving droplet entrains air from its surroundings. The entrained air generates a lift force, causing the droplet to levitate above the sur-

face [32] (aka aerodynamic Leidenfrost). The LLD theory breaks down when Ca drops below 10^{-5} . This is evidenced in Fig. 9(a), where the thickness of the air-film does not seem to scale with $Ca^{2/3}$.

We also perform RCM imaging for a broad range of capillary numbers to find the origin of the friction for a droplet moving over a tilted superhydrophobic surface. Figures 9(b)–9(d) shows RCM images for three different capillary numbers, each representing distinct dynamics. We find that a droplet moving on a superhydrophobic surface at a low velocity, $U = 0.01 \text{ ms}^{-1}$ ($Ca = 2.5 \times 10^{-6}$) gives rise to heterogeneous wetting, as seen in Fig. 9(b) (Supplemental Movie 7c [38]). This suggests that the friction force affecting the droplet motion primarily stems from contact-line pinning [44,45]. This is consistent with the observation made in Fig. 4 at $Ca = 2.5 \times 10^{-6}$, where friction forces are independent of velocity. Increasing the velocity beyond $Ca > 1.3 \times 10^{-5}$ increases the air-film thickness as shown in the RCM image [Fig. 9(c), Supplemental Movie 7a [38]]. However, formation of capillary bridges at the receding contact line and the presence of discrete pinning/depinning points suggests that friction originates from both contact-line friction and viscous dissipation around the rim [42]. Further increasing the droplet velocity reveals a smooth and continuous receding and advancing front with no presence of capillary bridges as seen in Fig. 9(d) (Supplemental Movie 7b [38]). Furthermore, the absence of visible discrete pinning points provides corroborative evidence that the droplet has levitated. The horseshoe-shaped fringe pattern seen in Fig. 9(d) is a characteristic of an aerodynamically levitated droplet [also shown in Figs. 8(a) and 8(b)], where the levitated droplet forms a thin air-film beneath the droplet characterized by a flat region near the center and two local minima in thickness located near the downstream rim in the lateral direction (perpendicular to the direction of motion).

VI. CONCLUSION

We systematically investigated the friction forces acting on droplets moving over non-wetting surfaces, using both inclined plane and cantilever-based experiments across a wide range of velocities and fluid viscosities. Our results reveal distinct friction regimes governed by contact-angle hysteresis, viscous dissipation from the droplet curvature near the substrate, viscous dissipation in the bulk, and aerodynamic drag. Each mechanism dominates under different capillary number ranges, with transitions clearly observed and characterized.

We propose a scaling model that incorporates all these force contributions, which accurately collapses diverse experimental data—including both our measurements and those from prior studies—onto a single predictive framework. The agreement is further supported by direct interferometric measurements of the air and vapor films beneath the droplets, which reveal the dynamic formation and scaling of these layers consistent with Landau-Levich-Derjaguin theory.

These findings not only clarify the origins of friction on non-wetting surfaces, but also offer practical guidance for designing surfaces and systems—such as in microfluidics, drag reduction, or heat transfer—where droplet mobility is critical. Our framework provides a foundation for optimizing such technologies by enabling precise prediction and control of droplet-surface interactions across regimes.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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