

Formation and vortex breakdown of the counter-rotating vortex pair in square and circular laminar jets in crossflow

F. C. Martins * and J. C. F. Pereira 

LAETA, IDMEC, Mechanical Engineering Department/LASEF, *Instituto Superior Técnico*,
Universidade de Lisboa, Av. Rovisco Pais, 1, 1049-001 Lisbon, Portugal



(Received 17 June 2024; accepted 27 June 2025; published 10 September 2025)

Laminar jets in crossflow are analyzed using direct numerical simulation to identify the source of vorticity leading to the formation of the counter-rotating vortex pair. A vorticity transport analysis conducted on steady cases shows that all counter-rotating vortex pair vorticity is generated in the nozzle walls for large velocity ratio jets (R), but that additional boundary layer vorticity is entrained at low R . The transport mechanisms are quantified and discussed in detail, including the influence of having a square jet cross section. The study also examines the effect of varying the velocity ratio in the range $0.25 \leq R \leq 5$ and the jet Reynolds number in the range $100 \leq Re_{jet} \leq 300$. Increasingly complex flow patterns are revealed, even in the steady regime, including the onset of bubble vortex breakdown of the counter-rotating vortex pair. Breakdown onset is quantified and analyzed in detail, revealing that it is a gradual process that depends on the jet vorticity and the adverse pressure gradient formed downstream of the jet inlet. Additionally, breakdown is discovered to precede the onset of unsteady flow for decreasing R and fixed Re_{jet} for jets with both cross sections. A newly discovered asymmetric instability is identified at $Re_{jet} = 300$ and characterized, where each of the counter-rotating vortices undergoes breakdown while oscillating laterally at low frequency. Hairpin vortex shedding, typical of low R laminar jets in crossflow, is captured in our simulations. Breakdown bubbles are identified for $R > 0.4$, where the flow remains approximately steady in the near field. For $R \leq 0.4$, the increasing fluctuation amplitude in the near field results in the rollup of fluid and the destruction of the breakdown bubbles in the instantaneous flow field.

DOI: [10.1103/lp11-vm7k](https://doi.org/10.1103/lp11-vm7k)

I. INTRODUCTION

Jets in crossflow, also known as transverse jets, have been studied for nearly a century (Callaghan [1]) and are present in a very wide range of engineering applications and natural phenomena. These include separation control (Rice *et al.* [2]), cooling heat transfer (Popovac and Hanjalić [3]), gas-turbine air-fuel mixing (Liu *et al.* [4], Zhang and Yang [5]), and wildfire and volcanic plume smoke dispersion (Chung and Koseff [6] and Ernst *et al.* [7]). Due to their relevance, they have been the subject of extensive research (Mahesh [8]), remaining a highly active topic of research to this day (Harris *et al.* [9])

*Contact author: francisco.c.g.martins@ist.utl.pt

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

Jets in crossflow are governed by a host of intricate vortex dynamics phenomena that interact with each other, even in laminar flow (Sau *et al.* [10]). Many of these structures are discussed in the seminal work of Fric and Roshko [11], and some form regardless of the regime of jet to crossflow velocity ratio (R). Horseshoe vortices formed upstream of the orifice may interact with flow inside the pipe and oscillate in phase with other downstream vortical structures, leading complex vortical systems (Kelso and Smits [12]). The counter-rotating vortex pair forms in all jets in crossflow downstream of the injection location, playing a central role in the downstream evolution of the jet (Rivero *et al.* [13]) and its mixing (Wegner *et al.* [14]). Despite this, it may only be identifiable in the time-averaged flow field (Cortelezzi and Karagozian [15]).

Some structures only form at high R . These include the shear layer vortices induced by a Kelvin-Helmholtz instability of the jet (Camussi *et al.* [16], Schlatter *et al.* [17]), the wake vortices shed along the spanwise bisection plane downstream of the jet (Kelso *et al.* [18]), and the upright vortices formed by tilting of boundary layer (Shinohara and Matsushima [19]), or ambient vorticity (Barata *et al.* [20]).

At low R , unsteady flow is known to develop as the result of a Hopf-Bifurcation (Chauvat *et al.* [21]), which is characterized by the periodic shedding of hairpin vortices. These structures have been the subject of several experimental (Cambonie and Aider [22], Klotz *et al.* [23]) and numerical (Bidan and Nikitopoulos [24]) studies. The wavemaker—the flow region where a self-sustained oscillation occurs due to a local absolute instability (Chomaz [25])—has been traced to a small region downstream of the jet, partially adjacent to the backflow region, through global stability analysis (Chauvat *et al.* [21], Ilak *et al.* [26]). Beyond this instability, a string of subsequent symmetric and antisymmetric instabilities is known to occur at low to moderate R values (Ilak *et al.* [26], Schlatter *et al.* [17]), but the resulting flow configurations have not been described, to the authors' knowledge. In general, despite the potential applications to film cooling (Jovanović *et al.* [27]), low R jets in crossflow have been much less thoroughly studied than at high R .

The counter-rotating vortex pair appears as a central element in the study of jets in crossflow, due to its interaction with many of the other vortical structures (Coelho and Hunt [28], New *et al.* [29], Huq and Dhanak [30], Lv *et al.* [31]). Its inception, in particular, has been heavily investigated and several different mechanisms have been proposed: in Kelso *et al.* [18], an inception mechanism was proposed that hinged on the reorientation of large scale vortices in the jet's shear layer toward downstream. Cortelezzi and Karagozian [15] identified the same vortex ring reorientation mechanism in the vortex methods simulations performed. Yuan *et al.* [32] performed LES simulations where quasi-steady hanging vortices were identified near the orifice, in the sides of the jet, from which the counter-rotating vortex pair emerges. It was also concluded that the pair is best visualized by low-pass filtering of the streamwise vorticity field.

Most studies have focused on transverse circular jets in the turbulent flow regime, but many of the previously described vortical structures were also identified in the laminar unsteady regime simulations performed by Sau *et al.* [10]. The study analysed square jets in crossflow for two moderate Reynolds numbers (225 and 300) and for jet to cross-stream velocity ratios 2.5 and 3.5. A different mechanism for the counter-rotating vortex pair inception was described, based on the folding of the lateral shear layers of the square jet, which was shown to be consistent with previous experimental observations for a round jet. Despite the extensive research on this topic, it is clear that the mechanism responsible by the generation of the counter-rotating vortex pair is still not fully understood.

This study employs direct numerical simulations (DNS) to investigate the laminar jet in crossflow at jet Reynolds numbers $100 \leq \text{Re}_{\text{jet}} \leq 300$ and jet to crossflow velocity ratios $0.25 \leq R \leq 5$. The first objective is to better understand and quantify the fundamental mechanisms of vorticity transport that lead to the generation of the counter-rotating vortex pair. This is achieved by performing a vorticity transport analysis to the flow at very low Re_{jet} , where the flow remains steady (this regime was studied in Sgarzi and Leboeuf [33], Shi *et al.* [34]). This approach has become increasingly popular for enhancing the fundamental understanding of the governing mechanisms of various flows (Jia *et al.* [35], Akkala and Buchholz [36], Martins *et al.* [37]). It has been discussed in several

jet in crossflow studies, including Kelso *et al.* [18], Haven and Kurosaka [38], all of which deal with highly chaotic flows where the large scale vortical structures are tightly coupled, disallowing conclusive quantification of the mechanisms proposed. As in Sau *et al.* [10], our analysis is based on a square jet, and we study the role of this cross-section on vorticity transport.

In the intermediate to low R regime, before the steady to unsteady flow transition, the counter-rotating vortex pair was observed to undergo bubble vortex breakdown. In the context of jets in crossflow, vortex breakdown has only been reported very sporadically, and typically as part of studies that deal with large Re_{jet} , where the flow is turbulent, and at relatively large R (Yuan *et al.* [32] and Kelso *et al.* [18]). More broadly, vortex breakdown has historically been a prominent topic in fluid mechanics (Hall [39]), often associated with the onset of unsteady flow (Chan *et al.* [40]). However, research shows that steady-state vortex breakdown can also be obtained (Moise and Mathew [41]). This is consistent with the results of this study, where breakdown of the counter-rotating vortex pair is observed to precede the onset of transition for constant Re_{jet} and decreasing R , and to persist in some of the unsteady flows obtained. To our knowledge, this structure was not reported in any previous studies of laminar jets in crossflow, but most of them study the onset of unsteady flow by increasing R , starting from very low R steady cases. Therefore, the second objective is to provide DNS results of the complex three-dimensional flows where the counter-rotating vortex pair undergoes breakdown, the mechanisms behind the formation of the breakdown bubbles, and the parametric range where they occur, for both square and circular jets. The breakdown bubble dynamics in unsteady flow are also studied. This includes their interaction with the onset of hairpin vortex shedding and with a newly discovered oscillatory instability of the counter-rotating vortex pair. This study was performed at low to moderate R , while keeping a low Re_{jet} value, to prevent the onset of turbulent flow.

In the first section of this work, the numerical methods and the physical setup are described, including the validation and verification studies performed. The results section is divided into three parts. First, the $Re_{jet} = 100$ and $R = 3$ steady laminar jet in crossflow is described, before the vorticity transport analysis is performed along the streamlines that lead to the vortex cores. This section ends with a parametric study on the influence of R on the counter-rotating vortex pair vorticity and its transport, where consecutively more complex flows are observed. The second part analyzes the flow dynamics of vortex breakdown in depth, and the range of parameters where it is observed. The third part of this study deals with the dynamics of the breakdown bubbles after the onset unsteady flow. First, the new low-frequency periodic antisymmetrical instability is described. Then, the interaction of the breakdown bubbles with hairpin vortex shedding displayed by the symmetric instability typical of unsteady low R jets in crossflow is discussed. The main conclusions are listed in the last section.

II. NUMERICAL METHOD

A. Physical setup and numerical methods

Consider the physical setup depicted in Fig. 1, which is commonly used to study jets in crossflow. Here, fluid is injected through an orifice in the lower bounding wall of the main duct via a nozzle. Two inlet devices were considered in this work: one with a square cross section with side L , and a circular pipe with diameter $D = L$. The coordinate system considered in this work is marked in red in the same figure. Its origin is the intersection of the spanwise bisection plane with the inlet boundary and bottom surface planes of the main duct. The distance from the inlet to center of the orifice was iterated on for each regime to ensure a regular development of the boundary layer. It is $7L$ in Sec. III A and $4.5L$ in the remaining results sections.

The governing parameters of this flow are the Reynolds number $Re_{jet} = \frac{u_{jet}L}{\nu}$ based on the average velocity of the jet u_{jet} and nozzle's cross section side length, as well as the jet to crossflow velocity ratio $R = \frac{u_{jet}}{u_{\infty}}$, where u_{∞} is the velocity of the crossflow. At low R , the Reynolds number typically is

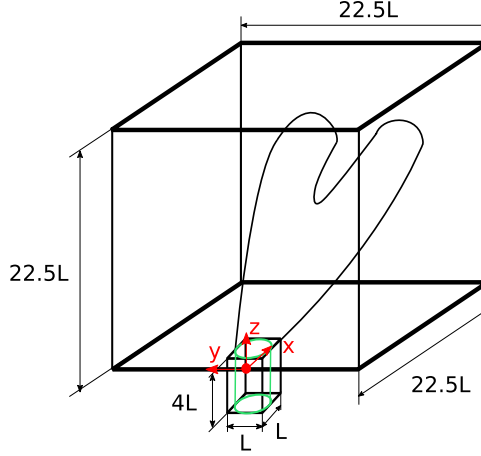


FIG. 1. Domain considered in this work for square and circular (in green) jets, with the origin of the coordinates system marked in red.

based on the crossflow velocity $Re_D = \frac{u_\infty L}{\nu}$, and a correspondence between the two can be made by the relation $Re_{jet} = Re_D R$.

For incompressible and isothermal flow, the governing equations are the Navier-Stokes and continuity equations:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \nu \nabla^2 \mathbf{u} = -\nabla p, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0. \quad (2)$$

The study heavily hinges on the analysis of vorticity and its transport in jets in crossflow, so the vorticity transport equations were considered for postprocessing:

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{u} \cdot \nabla) \boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega}. \quad (3)$$

In steady flow conditions, the material derivative on the left is reduced into convective transport of vorticity. On the right-hand side is the incompressible stretching and tilting term, which indicates regions of the flow where one component of vorticity is being redirected into another, or being amplified due to the local velocity gradients. The second term on the right-hand side is the diffusive term, which is responsible for the diffusive redistribution of vorticity. Thus, it can be noted that in this set of equations, there isn't a true vorticity generation term. The seminal work by Morton [42] proposed that vorticity is generated at walls whenever a tangential pressure gradient is verified. It is then diffused and convected away from the walls, possibly being amplified, tilted or destroyed via cross-cancellation by the flow.

The governing equations were solved iteratively using version 8 of the open source software OpenFOAM (Weller *et al.* [43]). The equations were discretized using the finite volume framework, with a mesh of around 3 to 5 million control volumes (according to the dimensions of the domain), approximately cubic in shape. The unsteady solver *pisoFoam* was used to run the numerical simulations with second order spatial and temporal numerical schemes. Both residuals and velocity probes were utilized, to guarantee convergence of the results and identify if steady state was reached.

Figure 2(a) shows a typical nested grid utilized in this work, featuring progressively smaller control volumes near the nozzle. The side length of the control volumes is halved in each of the four refinement levels, resulting in a resolution of 32 by 32 control volumes in the cross section of the jet and its near vicinity, where the finest flow features are encountered [Fig. 2(b)]. The suitability

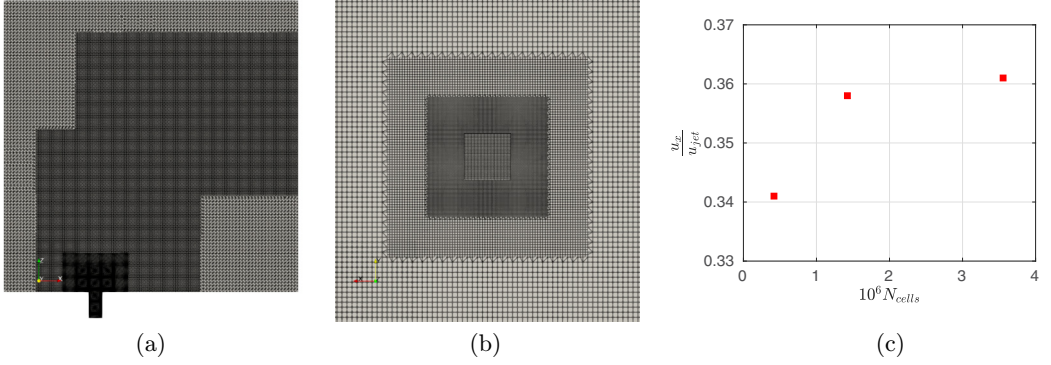


FIG. 2. Slice in the spanwise bisection plane (a) of the mesh utilized mesh in the $R = 5$, $Re_{jet} = 100$ case, and a zoomed-in view of the mesh in the nozzle region viewed from below (b). Convergence analysis of the maximum velocity in the spanwise bisection plane (c).

of the mesh utilized was tested with a mesh convergence analysis, allowing verification of the computations (Roache [44]). The dimensions and refinement ratio of each refinement level were kept constant for every mesh, and all three refinement levels were included in all simulations. Additionally, the dimensions of each refinement region was not altered and the grid was uniformly refined in all three directions. The maximum streamwise velocity value in the spanwise bisection plane [Fig. 2(c)] is well converged in the $N \approx 3 \times 10^6$ mesh, which was utilized in this simulation. Several other variables were tested for convergence, always showing a similar degree of convergence.

The boundary conditions assume the flow rate was imposed at the inlet of the nozzle, instead of specifying a velocity profile (using the *flowRateInletVelocity* OpenFOAM boundary condition and selecting the *extrapolateProfile* option). This way, the velocity field of the control volumes adjacent to the jet inlet is extrapolated to the jet inlet boundary. In the first timestep, the velocity profile was initialized as a top hat profile, after which it converged into a quasi-Poiseuille flow velocity profile. Figure 3(a) illustrates several velocity profiles at the nozzle inlet for $Re_{jet} = 200$ and 300 and $R = 1$ jets, showing that the profiles collapse when nondimensionalized by the jet velocity. The same observation can be made when plotting the velocity profile along the diagonal of the square jets [Fig. 3(b)]. In all cases, the peak to average velocity is around $u_z = 2.30u_{jet}$ in the circular jets and $u_z \approx 2.45u_{jet}$ in the square jets. The walls of the nozzle were considered to be no slip surfaces, to be compatible with the inlet boundary conditions.

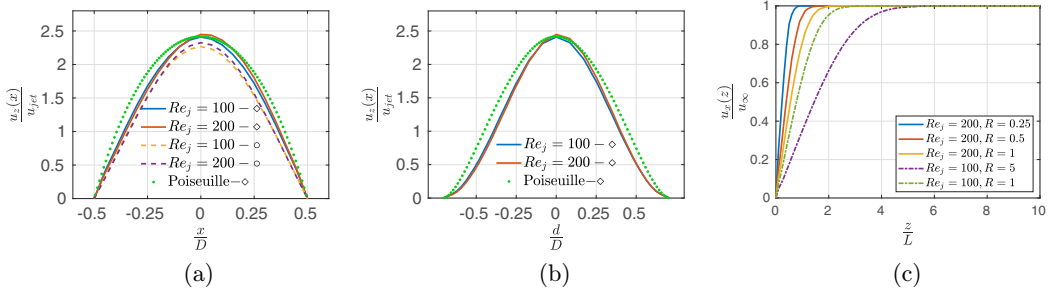


FIG. 3. Inlet vertical velocity profile along the streamwise direction in the jet nozzle for several $R = 1$ jets in crossflow (a), including both square ($\diamond$) and circular ($\circ$) cross sections. The same profile, but along the diagonal of the square duct for the square jets (b). The analytical Poiseuille flow velocity profile in a square duct also is plotted in both figures. The Blasius profiles imposed at the main duct's inlet in several simulations (c).

At the inlet of the main duct, the velocity profile was explicitly imposed. The profiles imposed in several cases are plotted in Fig. 3(c), showing that classical first-order Blasius profiles were used. These were generated from a freestream velocity set to u_∞ , and an origin $x_0 = 31.5l$ upstream of the orifice. The boundary layer Reynolds number $Re_{\delta^*} = \frac{u_\infty \delta^*}{\nu}$ could be based on a characteristic length of the boundary layer, such as the displacement thickness δ^* or momentum thickness θ , could have been considered. However, new Blasius profiles were computed simply by changing the freestream velocity, and keeping the same value of x_0 , with the intent of mimicking a fixed experimental setup. These parameters can be used to compute the displacement and momentum thickness at the inlet, by taking $\delta^* = 1.72 \sqrt{\frac{\nu x}{u_\infty}}$ and $\theta = 0.665 \sqrt{\frac{\nu x}{u_\infty}}$. Furthermore, when testing boundary layers with both larger and smaller values of x_0 (including cases where δ was tripled and where $x_0 = 0$), the trends and physical phenomenon of vortex breakdown described in the following sections were not altered significantly.

At the outlet of the main duct, a constant null pressure value (*fixedValue* in OpenFOAM) was specified, and a zero gradient boundary condition was applied to the velocity. This is standard procedure in incompressible flow simulations, to ensure the stability of the numerical method.

The bottom boundary of the main duct was considered a no slip surface, unlike the top and two spanwise boundaries, which were considered to be slip surfaces. A zero gradient was applied to the pressure in these locations. Also, the distance of these boundaries to the jet was iterated on, to prevent confinement of the jet.

The flow was initialized from rest in all simulations, and a study on different initial solutions showed no measurable effects on either the steady or unsteady configurations discussed in this work.

B. Validation

The numerical method was validated against the available experimental results of circular jets in crossflow from Ref. [23]. This includes the cases where $Re_D = 310$ and $R = 0.29$, and $Re_D = 310$ and $R = 0.63$, which match the two simulations whose results are illustrated in Figs. 4(a) and 4(b). The Instantaneous contours of u_x of these figures show that the flow states obtained in each simulation match the ones observed in the experimental study. In the first case, the flow became steady state, while in the second case, the flow became periodic and characterized by the shedding of hairpin vortices. Additionally, the average jet trajectory and the dimensions of the backflow region show a good agreement with the experimental results. The shedding frequency, f , in the latter simulation was measured to result in a Strouhal number $St_\infty = \frac{fL}{u_\infty} = 0.235$, which matches fairly well the frequency measured experimentally ($St_\infty \approx 0.26$). Moreover, the St_∞ values measured in other validation simulations performed all fall inside the range of $St_\infty \in [0.21, 0.25]$ reported experimentally. This comparison provides a strong validation to our numerical setup.

III. RESULTS

A. Steady-state jet in crossflow $Re_{jet} = 100$ and $R = 5$

To investigate the origin of the counter-rotating vortex pair, we simulated jets in crossflow in the very low Re_{jet} regime ($Re_{jet} = 100$), where both free jets (Danaila *et al.* [45]) and jets in crossflow remain laminar and steady (Klotz *et al.* [23]). This section provides an overview of the flow kinematics and dynamics of a square jet in crossflow, before addressing the origin of the counter-rotating vortex pair, and its dependence on R in the following sections.

The main vortical structures of the $Re_{jet} = 100$, $R = 5$ jet in crossflow, upon reaching steady state, can be visualized using the Q -criterion ($Q = 0.5$) isosurface [46] of Fig. 5(a). This quantity measures the local relative importance of vorticity and strain in the flow. Thus, locations where $Q > 0$ indicate that the effects of vorticity dominate locally over those of strain. In this flow, the counter-rotating vortex pair is skirted by a shear region, a structure which is a common feature in laminar

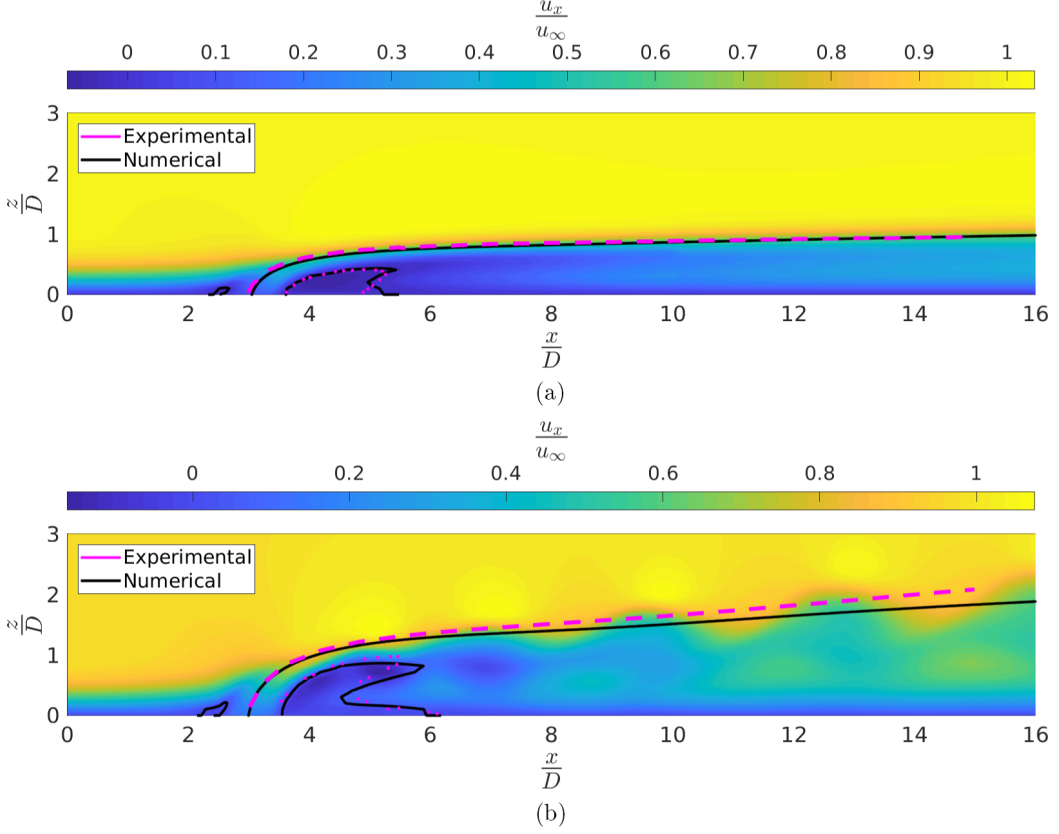


FIG. 4. Instantaneous streamwise velocity contours in the spanwise bisection plane for $\text{Re}_{\text{jet}} = 90$ and $R = 0.29$ (a), and $\text{Re}_{\text{jet}} = 195$ and $R = 0.63$ (b). The average jet trajectory and contour of the backflow region are shown in comparison with the experimental results of Ref. [23].

jets in crossflow (see, e.g., Ilak *et al.* [26]), until around $x/L = 15$. Downstream of this location, the counter-rotating vortex pair becomes clearly distinguishable in the Q isosurface. Although relatively weak in this regime, a horseshoe vortex located just upstream of the orifice is also visible in Fig. 5(a). The jet exhibits increased streamwise momentum [Fig. 5(c)], due to the deflection of its vertical momentum downstream. However, a streamwise momentum deficit region forms between the counter-rotating vortices, extending downwards [Fig. 5(b)]. This momentum deficit region is an extension of the backflow region downstream of the orifice, which is delimited by the white contour in Fig. 5(c), and, in this case, extends approximately $4L$ downstream of the orifice.

In this work, the jet trajectory is defined by the streamline originated in the center of the jet inlet boundary in the spanwise bisection plane. In this case, the jet trajectory streamline shows that the jet penetrates well above the boundary layer, reaching a height of about $18L$ at the downstream boundary of the main duct.

Circulation, $\Gamma = \int_z \int_y \omega_x dy dz$, provides an integral measure of the intensity of the vortices. However, when integrated over any streamwise-facing slice, the total circulation must always be null, since the velocity at the duct walls is null. The circulation plot of Fig. 6(a) confirms that this condition holds true in the current simulation, indicating an accurate circulation calculation. To track the evolution of vortex intensity along x , it is more useful to measure the circulation in a half-slice ($\Gamma_h = \int_z \int_{y=0}^{y=11.25L} \omega_x dy dz$). This measure is particularly insightful, when analyzed alongside the vorticity contour plots of Figs. 6(b) to 6(d). The peak in Γ_h at the orifice is linked to the injection

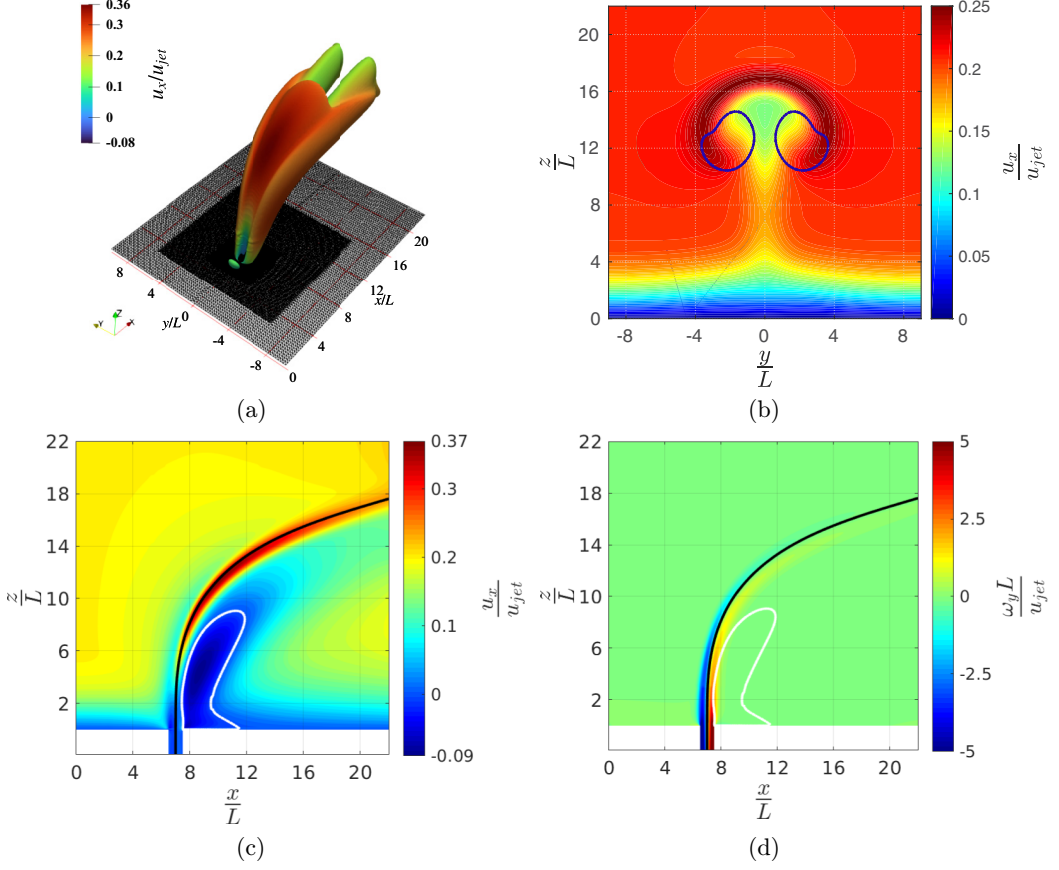


FIG. 5. Steady state $Q = 0.5$ isosurface of the $R = 5$, $Re_{jet} = 100$ jet in crossflow shaded with the streamwise velocity contours. The contours of streamwise velocity at $\frac{x}{L} = 20$ in panel (b), where the $Q = 0.5$ contours are plotted in blue. The streamwise velocity (c) and spanwise vorticity (d) contours in the spanwise bisection plane, where the backflow region ($u_x = 0$ contour) is in white, and the jet trajectory is in black.

of jet vorticity into the main duct through the orifice, as illustrated by Fig. 6(b). Downstream of this location, Γ_h declines sharply due to strong cross-cancellation of vorticity injected into the main duct and oppositely signed (negative) vorticity entering the half-slice.

There are three main sources of negative vorticity in the half-slice:

(I) Diffusive transport across the spanwise bisection plane. Negative vorticity from the opposite half-slice is diffused across the plane, progressively cross-cancelling the two shear regions and counter-rotating vortex pairs. The diffusion and cancellation rates are proportional to the intensity of these structures.

(II) Circulation-induced lateral entrainment. The secondary flow field drives fluid toward the spanwise bisection plane, creating a pressure gradient that produces negative vorticity on the bottom surface of the main duct [Fig. 6(e)].

(III) Tilting of spanwise vorticity. Spanwise vorticity from the jet [see Fig. 5(d)] is tilted downstream [Fig. 6(c)] due to the spanwise gradient of streamwise velocity ($\frac{\partial u_x}{\partial y}$), generating negative vorticity below the jet (which is bounded by the $u_x = u_\infty$ velocity contour).

The intensity of the negative vorticity patches in the half-slice is quantifiable by $\Gamma_{h-} = \int_z \int_{y=0}^{11.25L} \min(\omega_x, 0) dy dz$. However, this negative vorticity is eventually canceled by positive streamwise vorticity in the vortex and in the shear region, as well as positive vorticity tilted from

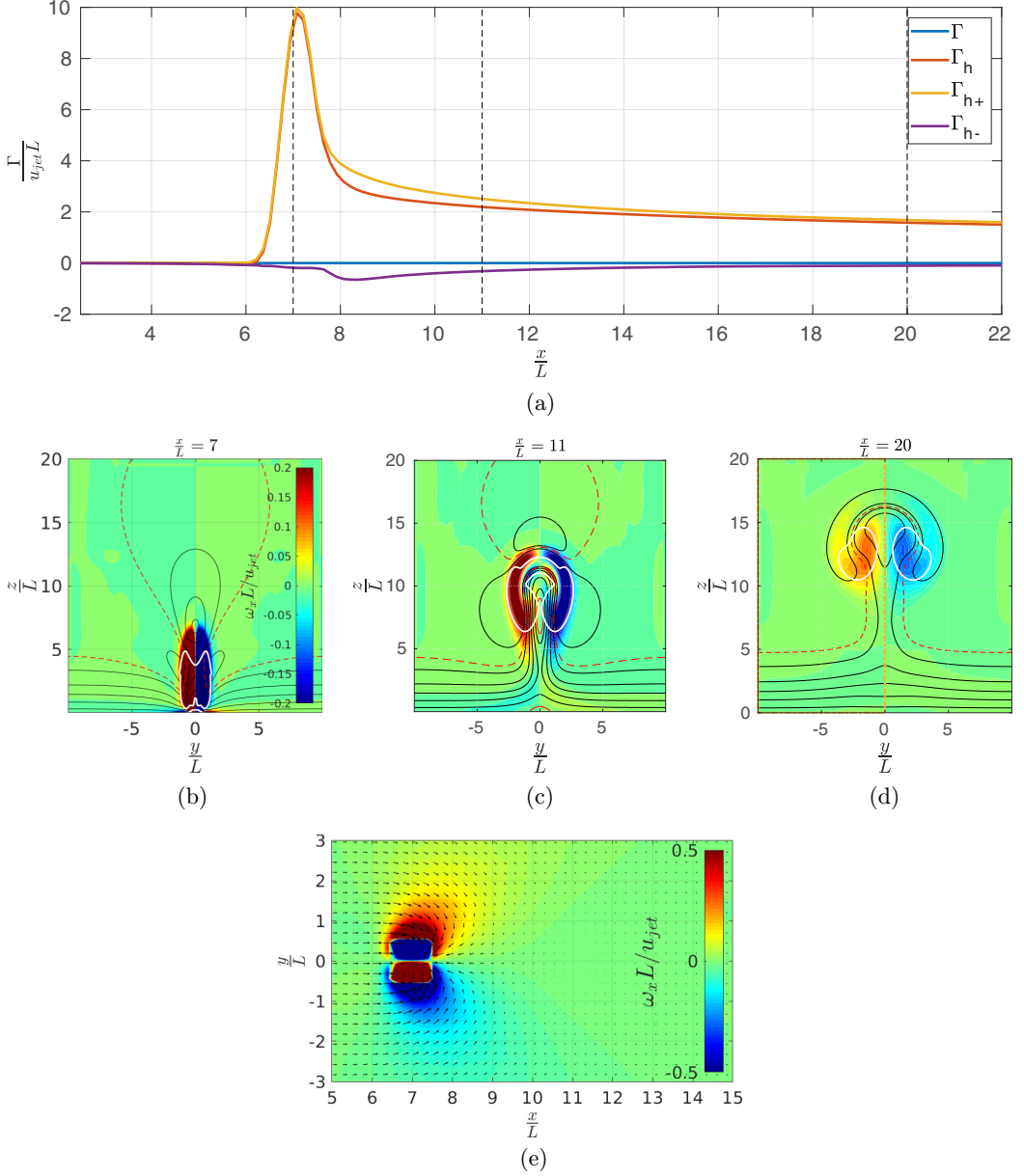


FIG. 6. Circulation (a) in a full streamwise-facing slice (Γ), in the left half slice (Γ_h) as delineated in panel (d), and its positive and negative components (Γ_{h+} and Γ_{h-}). Streamwise vorticity contours in slices downstream of the orifice $x/L = 5.5$ (b), $x/L = 9.5$ (c) and $x/L = 20$ (d). In white is the $Q = 0.5$ contour, and in black are six equally spaced contours of $u_x \in [0.12, 1.2]/u_\infty$. The dashed and full red lines show the $u_x = u_\infty$ and $u_x = 0$ contours, respectively. Streamwise vorticity on the bottom surface (e).

the spanwise vorticity above the jet. The intensity of the positive signed vorticity in the half slice is given by $\Gamma_{h+} = \int_z \int_{y=0}^{y=11.25L} \max(\omega_x, 0) dy dz$.

As Fig. 6(d) shows, the rapid circulation decay leads to the dissipation of the shear region surrounding the counter-rotating vortex pair. Further downstream, the circulation remains nearly

constant, due to the increasing separation between the vortices and the reduced radial vorticity gradients, which significantly weaken cross-cancellation of the vortex pair.

The current integral analysis provides valuable insights into vorticity dynamics in jets in cross-flow, and provides a way to quantify the intensity of the counter-rotating vortex pair, particularly far downstream of the orifice. However, a local vorticity transport analysis is necessary to study the formation of the counter-rotating vortex pair, due to the large contribution of the shear region to the vorticity integral near the orifice. This is the focus of Sec. III B. Before that, next subsection examines the flow inside the square nozzle.

1. Flow inside the square nozzle

Since most of the vorticity is generated within the nozzle and injected through the orifice into the main duct, it is essential to analyze of the flow field inside the nozzle. In particular, the impact of the injection nozzle's square cross section on the generation of vorticity becomes relevant, as four pairs of corner vortices are known to be produced by the secondary flow field in fluid flow inside square ducts. Figure 7(a) shows that these vortices are well captured in the current numerical setup.

Comparison of the ω_z field at the inlet and at the orifice [Figs. 7(a) and 7(b)], reveals that the signal of the corner vortices reverses inside the nozzle. This reversal is caused by the “opening up” of the jet as the fluid reaches the orifice, and the transition from a square cross-section into a circular one, upon leaving the duct. This process was also observed in simulations of square free jets performed in a preliminary validation study, and is well known to occur in all jets with nonround cross section [47]. As shown by the secondary velocity field of Fig. 7(b), the pressure gradients driving this transition cause the corner vortices to change in sign. However, the streamwise vorticity field remains largely unaltered [compare Figs. 7(c) and 7(d)]. Note that its magnitude is at least one order of magnitude larger than the vertical vorticity throughout the nozzle. This is because the primary source of streamwise vorticity, the vertical pressure gradient along the nozzle walls, is large enough to remain mostly unaffected by the secondary flow.

From this analysis of the flow field in the nozzle, the contribution of the corner vortices to the generation of the counter-rotating vortex is expected to be minimal in this case. However, a more conclusive analysis of the interaction of the two vorticity components inside the nozzle is provided in the following section.

B. Vorticity transport in the jet in crossflow

The focus is now shifted to the origin of the counter-rotating vortex pair. This was investigated through a vorticity transport analysis along streamlines that become part of the vortices, leveraging the steady flow regime. The pink streamlines of Fig. 8(a) indicate that the location of the counter-rotating vortex coincides with the region where streamlines originating in the crossflow (in green) and in the jet (in white) converge and whirl around the vortex core. To better understand the formation of the counter-rotating vortex pair, it is important to analyze the generation and transport of vorticity along representative streamlines originating from each of these locations.

The yellow streamline of Fig. 8(b) originates in the jet, and its coordinates are plotted in Fig. 8(c), where s is the distance along each streamline, and its maximum value corresponds to the streamline's total length. Before being injected into the main duct, the streamline remains vertical and adjacent to the lateral wall of the nozzle. In this region, the streamwise vorticity [plotted in Fig. 8(d)] is generated by the local pressure gradient on the nozzle's wall (and the orifice), reaching its maximum value. Notably, no tilting from the two other components was detected in this region. Partly due to their low comparative value, as discussed in Sec. III A 1, but also because velocity gradients that could favor this are absent. Figure 8(d) shows that, downstream of the orifice, the streamwise vorticity declines sharply due to the significant amount of viscous diffusion in this region [which is plotted in the budgets of Fig. 8(e)]. This occurs without significant vorticity stretching or

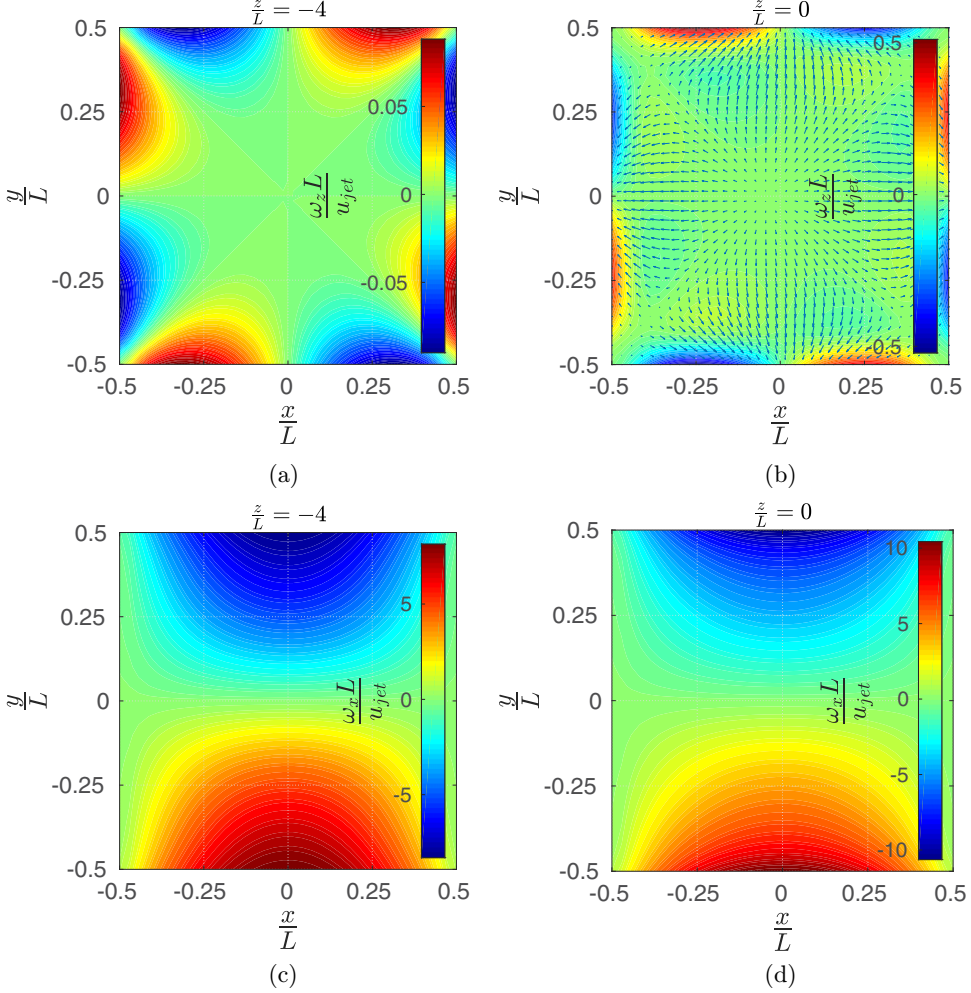


FIG. 7. Vertical vorticity contours at the inlet (a) and outlet (b) of the square nozzle for the $R = 5$, $\text{Re}_{\text{jet}} = 100$ case. The streamwise vorticity in the same locations and for the same case is respectively shown in panels (c) and (d). The arrows illustrate the secondary flow at the outlet. The crossflow direction is from left to right.

tilting from other components. As a result, the streamwise vorticity far downstream becomes largely residual, and the counter-rotating vortices, though weak, are very large in radius.

This analysis ultimately demonstrates that, for streamlines originated in the jet in this case, the streamwise vorticity does not undergo a complex generation or redirection mechanism. It is generated on the walls of the nozzle, and is diffused and convected downstream, forming the counter-rotating vortex pair.

For streamlines generated in the crossflow, the transport of vorticity occurs differently. Figure 8(f) shows that the red streamline remains almost horizontal and is inside the boundary layer until it reaches the orifice. The positive-signed spanwise vorticity originated in the boundary layer is visible in Fig. 8(g), including a peak that coincides with the location of the horseshoe vortex, however, it does not appear to take a significant role in the generation of the streamwise vorticity. It is the negative-signed spanwise vorticity injected in the duct by the jet that is tilted toward the streamwise direction [Fig. 8(h)]. This mechanism of streamwise vorticity generation, reminiscent of the one described by Kelso *et al.* [18], is thus shown to also be present in steady laminar jets in crossflow.

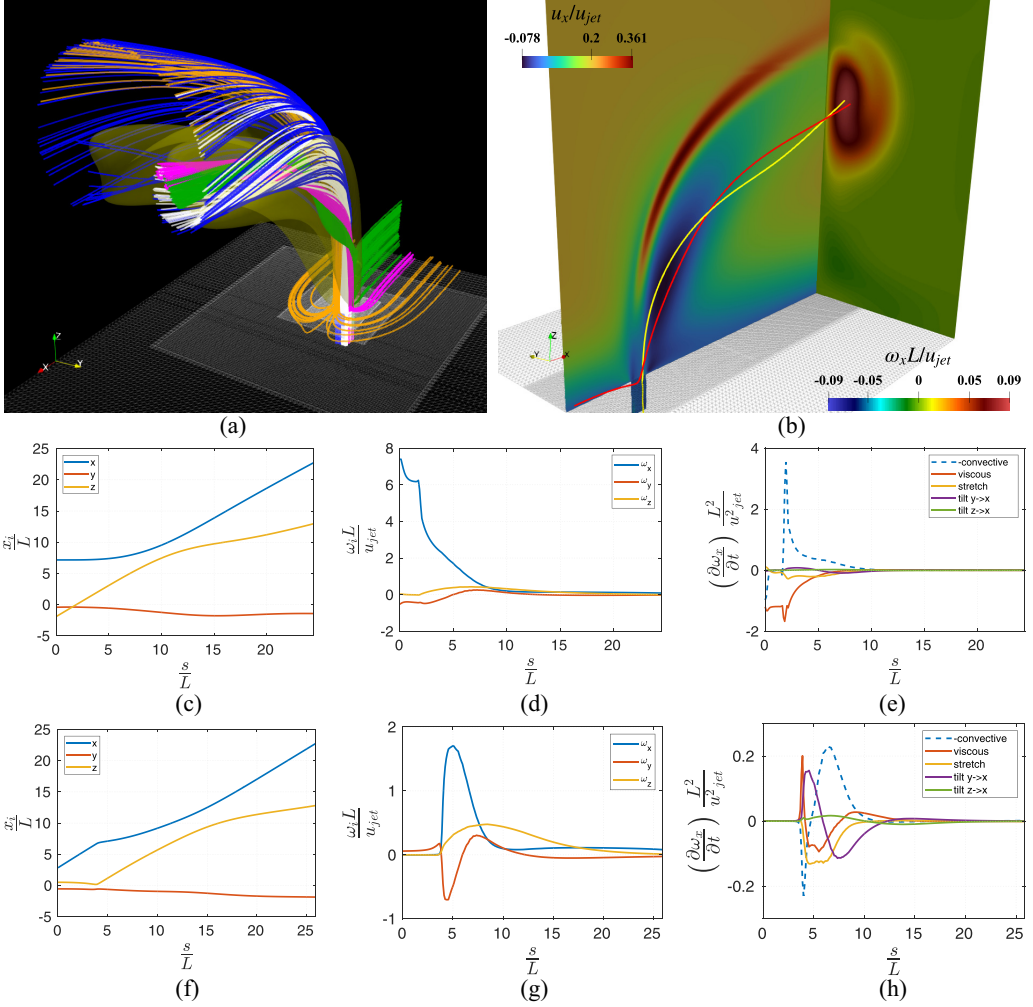


FIG. 8. $Q = 0.5$ isosurface (in yellow) and the streamlines of the $R = 5$, $Re_{jet} = 100$ jet in crossflow color coded according to their origins (a). The representative streamlines considered (b) and the streamwise velocity and vorticity contours on the spanwise bisecton and downstream planes, respectively. Position of each point along the streamlines (c), (f), vorticity components (d), (g), and streamwise vorticity transport budgets (e), (h) of the yellow and red representative streamlines, respectively.

Spanwise vorticity is generated in the same way as the streamwise vorticity is along the yellow streamline, although, in this case, the wall in question is the upstream wall of the nozzle.

From this analysis it is evident that, regardless of the origin of the streamline, the generation of streamwise vorticity is highly dependent on the vorticity generated in the nozzle. This is reflected in the generally larger values of streamwise vorticity, particularly near the orifice, in the streamlines originated in the nozzle. Additionally, it complements the integral analysis of circulation in Sec. III A by isolating the counter-rotating vortex pair streamlines from the surrounding shear layer and quantifying each streamwise vorticity source.

1. Effect of R on vorticity transport

In the current section, we consider the same $Re_{jet} = 100$ jet, but with an increased crossflow velocity (Re_D), such that $R = 1$. This adjustment allows to assess whether and how the previously

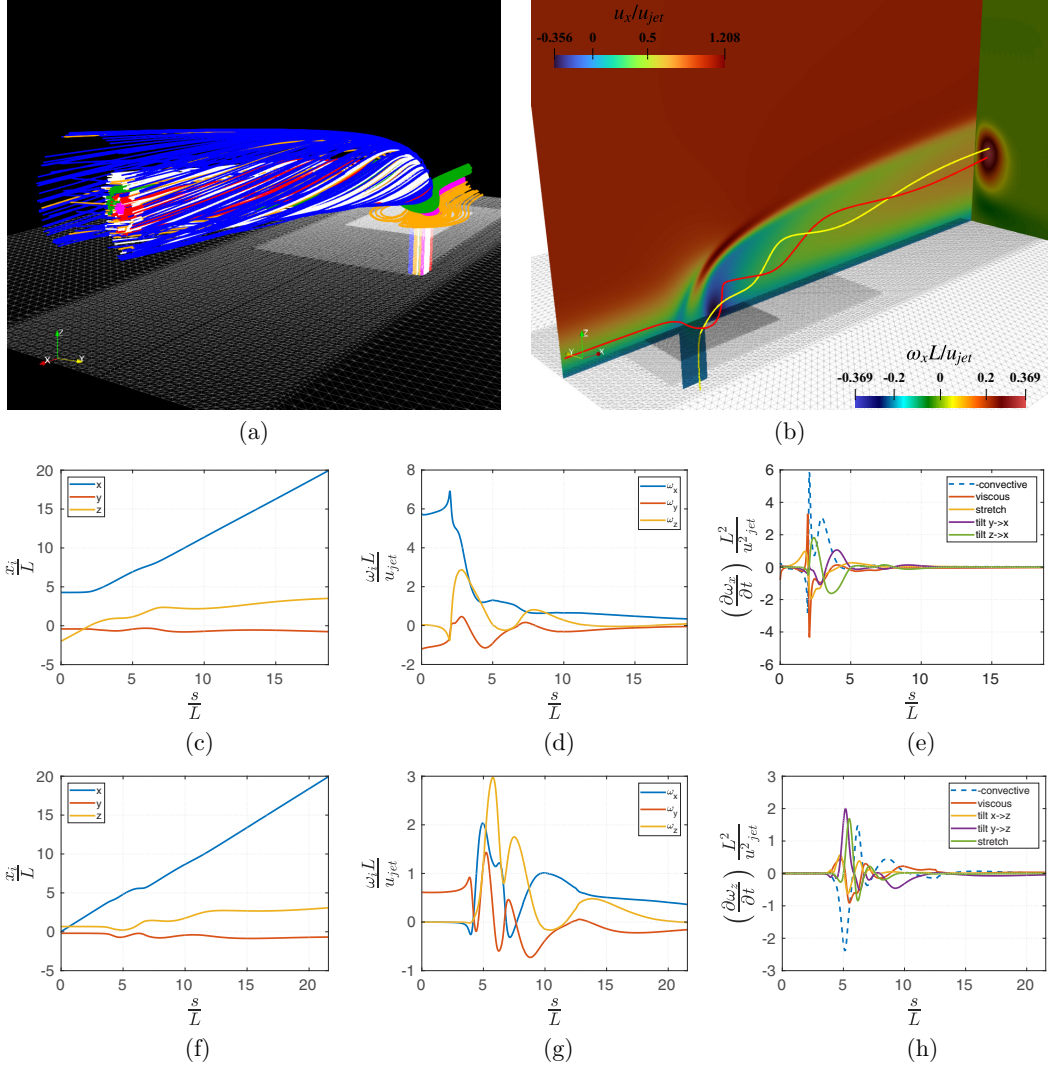


FIG. 9. Streamlines of the $R = 1$, $Re_{jet} = 100$ jet in crossflow color coded according to their origins (a). The representative streamlines considered (b) and the streamwise velocity and vorticity contours on the spanwise bisection and downstream planes, respectively. Position of each point along the streamlines (c), (f), vorticity components (d), (g) of the yellow and red representative streamlines, respectively. Streamwise vorticity transport budget along the yellow streamline (e) and vertical vorticity transport budget along the red streamline (h).

identified vorticity generation and transport mechanisms translate to a lower R regime. Despite the increase in Re_D , this simulation also reached steady state.

The streamlines of Fig. 9(a) illustrate the flow. Due to the increased crossflow momentum, the jet penetrates significantly less into the crossflow, to a height of about $4.6L$ at the downstream surface of the main duct. The same color code of Sec. III B was utilized: the pink streamlines lead to the core of the counter-rotating vortex pair, while the yellow streamline in Fig. 9(b) represents the jet originated streamlines that become part of the counter-rotating vortex pair. Figures 9(c), 9(d), and 9(e) show the streamline's coordinates, the evolution of each vorticity component, and the evolution of the streamwise vorticity transport equation terms, respectively.

Inside the nozzle, apart from some stretching due to the streamwise acceleration of the fluid near the orifice, the same vorticity generation mechanisms described in the previous sections are present. Downstream of the orifice, due to the whirling of the streamlines causes the vorticity to be cyclically tilted between each of the three components. This mechanism is responsible for the alternating peaks of streamwise, spanwise and vertical peaks of vorticity, starting with tilting of vertical vorticity toward the streamwise component. The origin of this vertical vorticity is more clearly identified by inspecting the vertical vorticity budget of the red streamline, plotted in Fig. 9(h), which reveals significant tilting from the spanwise to the vertical component.

Due to the increased deflection of the jet, the low pressure associated with the backflow region decreases further. Thus, as in the flow around an object, the crossflow originated streamlines are no longer directly entrained into the jet, but through the backflow region, as illustrated by the red streamline of Fig. 9(b). In this process, the spanwise vorticity generated in the boundary layer, which is mostly negligible at large R , is tilted upwards and immediately stretched upon reaching the backflow region, due to the vertical acceleration of the fluid. This results in the large vertical vorticity peak of Fig. 9(g), which is subsequently tilted into streamwise vorticity, when the streamline is reoriented downstream.

The combined effect of this additional vorticity entering the counter-rotating vortex pair, the cyclic tilting of vorticity between its three components, and the increased convective streamwise velocity lead to a pair of significantly narrower counter-rotating vortices, but with a larger vorticity at their core, upon reaching the downstream boundary of the main duct. Finally, note that the boundary layer vorticity was also redirected through the backflow region in the $R = 5$, $Re_{jet} = 100$ case. However, it mostly remained part the high spanwise vorticity shear layer linking the vortex pair, and was not directly entrained into the counter-rotating vortex pair itself.

After describing the effect of decreasing R for the $Re_{jet} = 100$ jet in crossflow, the next section will expand parametric study to $Re_{jet} = 200$ jets and further increasing Re_D , where vortex breakdown may occur.

C. Vortex breakdown and unsteady flow

1. Steady vortex breakdown

One of this study's main findings is that the counter-rotating vortices may undergo bubble vortex breakdown. The definition of vortex breakdown considered here is the classical definition by Leibovich [48]: “a disturbance characterized by the formation of an internal stagnation point on the vortex axis, followed by reversed flow in a region of limited axial extent.” Figure 10(a) [and the

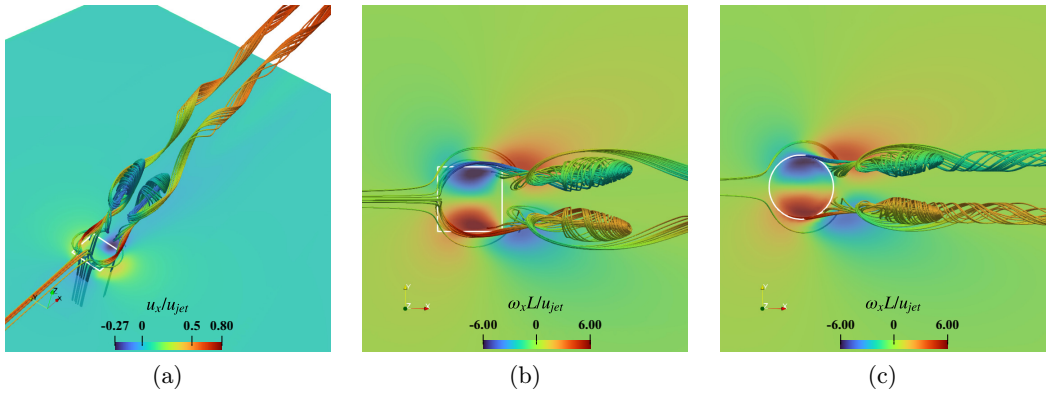


FIG. 10. Streamlines of the $R = 1$, $Re_j = 200$ case after reaching steady state, showing vortex breakdown of the counter-rotating vortex pair (a). A zoomed-in top view of the same figure shaded by the ω_x contours (b). The same zoomed-in view for a circular jet with the same parameters (c).

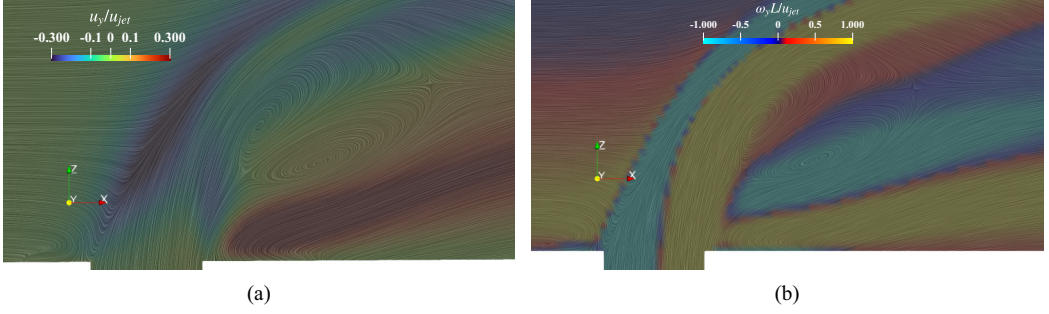


FIG. 11. Surface LIC illustrating the streamlines on a plane ($y = -0.25L$) that bisects one of the breakdown bubbles for $R = 1$, $Re_{jet} = 200$ overlaid on the spanwise velocity (a) and vorticity (b) contours. y direction is positive into the page.

close-up top view of Fig. 10(b)] clearly illustrate the recirculation bubbles that form along the axis of the counter-rotating vortices in a $R = 1$ and $Re_{jet} = 200$ square jet. Notably, the same structures were obtained in a simulation of a circular jet with the same governing parameters, as shown in Fig. 10(c).

According to the definition adopted, the streamlines in a plane that bisects the vortex in the spanwise direction “open up” at the stagnation point and converge inward, toward the vortex core, at the end of the recirculation region. This matches the structure depicted in Figs. 11(a) and 11(b), where the streamlines are overlaid on the spanwise velocity and vorticity contours on a plane that bisects one of the breakdown bubbles. This flow pattern is characteristic of vortex breakdown. The results show that the cores of the vortex and the breakdown bubble are well aligned, and that the streamwise velocity inside the bubble is negative. Additionally, the breakdown bubble is slightly distorted, following the curvature of the jet trajectory, but the bubble remains single-celled.

To visualize the onset of the breakdown for a constant velocity ratio value ($R = 1$), a series of simulations were performed in the range $100 < Re_{jet} < 200$, in increments of $\delta(Re_{jet}) = 5$, and increments of $\delta(Re_{jet}) = 1$ in the subrange of $130 \leq Re_{jet} \leq 135$. As illustrated in Figs. 12(a) to 12(d), the onset of vortex breakdown was tracked using both the $u_x = 0$ isosurface and the streamlines that lead to the counter-rotating vortex pair. In cases where the breakdown occurred ($Re_{jet} \geq 135$), the red streamline illustrates the flow inside the bubble, while the black streamline illustrates the flow around the bubble, which ends up part of one of the counter-rotating vortex cores. In contrast, in cases where breakdown does not occur [Figs. 12(a) and 12(b)], both streamlines are integrated into the vortex core. These provide a reference for how the streamlines of the remaining cases ($Re_{jet} \geq 135$) would look like in the absence of breakdown.

The $u_x = 0$ isosurface usually delimits the backflow region. However, when breakdown occurs, two ellipsoidal structure protrude from its downstream end, as Figs. 12(c) and 12(d) show. In cases where breakdown is imminent (e.g., $Re_{jet} = 134$) these structures also form, due to the surrounding (black) streamlines spiralling around the bubble and the slight misalignment of the vortex core with the streamwise direction. The core is slightly slanted upwards, as seen in Fig. 13(a). Thus, the streamwise velocity u_x includes a small contribution from the azimuthal velocity around the vortex core. Despite this, tracking the $u_x = 0$ isosurface remained the most efficient method for detecting the possibility of breakdown in simulations. In every case where this isosurface exhibited the protrusions indicative of breakdown, its occurrence was confirmed by computing the streamlines inside the bubbles.

The vortex core, within a breakdown bubble, was defined as the location of the minimum streamwise velocity in any given streamwise-facing plane. This definition was necessary, because, although the breakdown bubbles are clearly visible in streamline plots, they form relatively close to

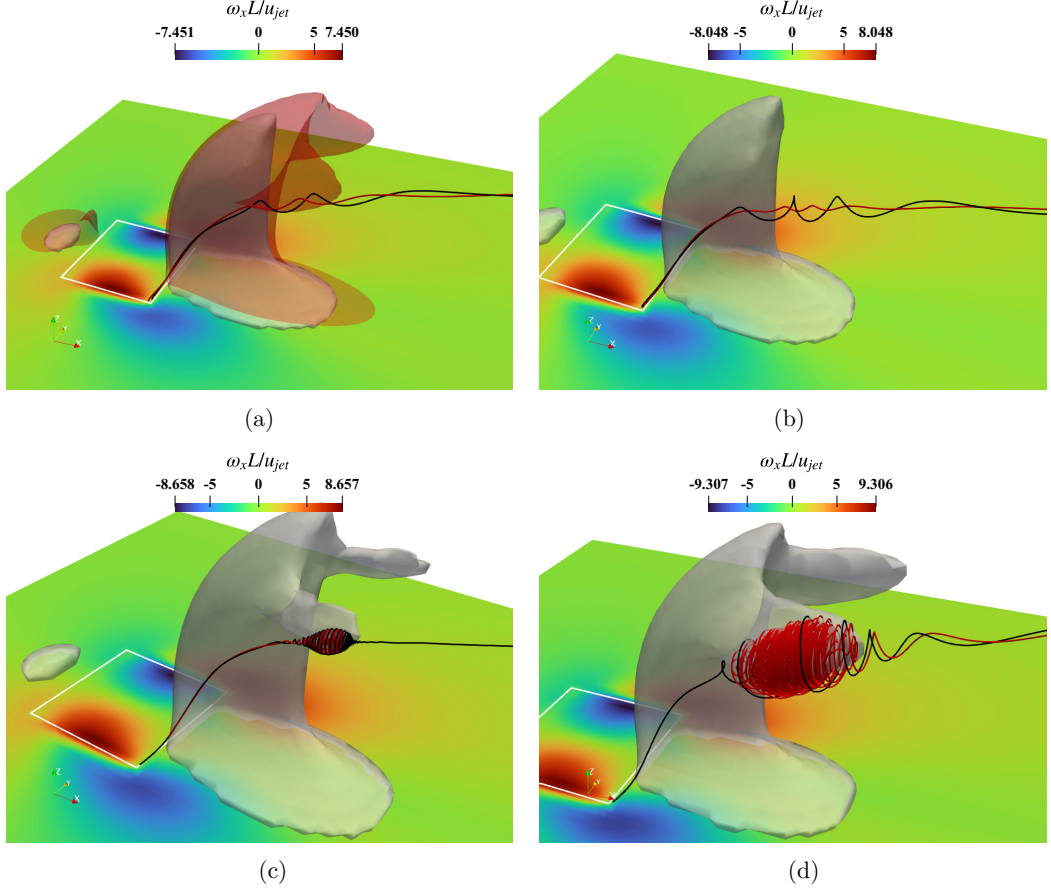


FIG. 12. Streamlines illustrating the onset of breakdown in one of the counter-rotating vortices for $R = 1$ and $\text{Re}_{\text{jet}} = 125$ (a), $\text{Re}_{\text{jet}} = 130$ (b), $\text{Re}_{\text{jet}} = 135$ (c), and $\text{Re}_{\text{jet}} = 140$ (d). In gray and red are the $u_x = 0$ and $\frac{u_x}{u_{\text{jet}}} = 0.04$ isosurfaces, respectively.

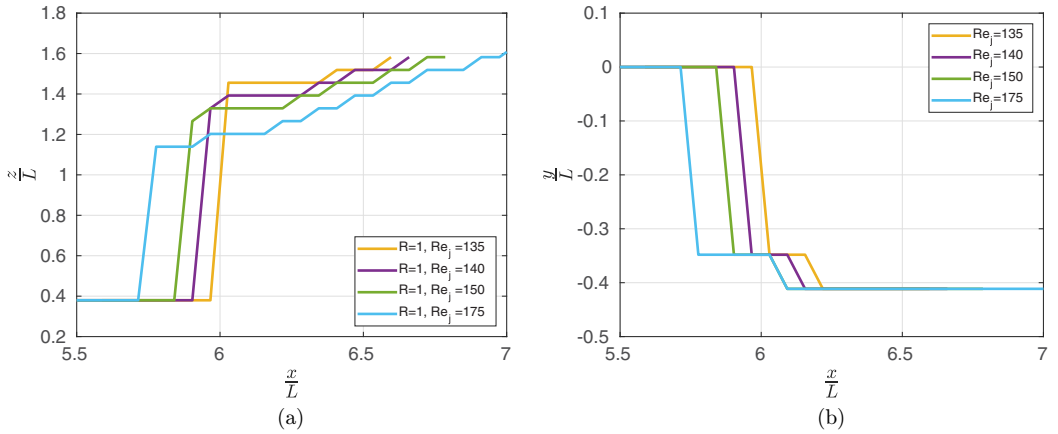


FIG. 13. Height (a) and distance to the spanwise bisection plane (b) of the bubble cores in cases where vortex breakdown occurs.

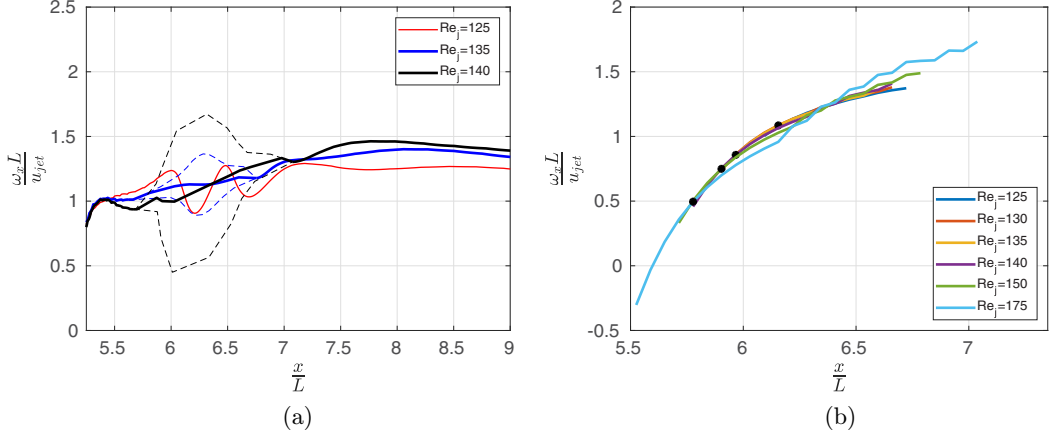


FIG. 14. Streamwise vorticity along the black streamlines for three cases (a) and in the vortex cores (b). In (a), the dashed lines show the range of ω_x in the whirling region around the bubble, and the full line the average value.

the orifice (within $1L$ to $2L$ downstream of it), where the shear layer remains strong. Consequently, identifying vortex cores based on a peak in streamwise vorticity (ω_x) was not possible.

To differentiate breakdown bubbles from the backflow region to which they are often connected, we defined breakdown onset as the point where the minimum negative streamwise velocity is no longer located in the spanwise bisection plane. As shown in Fig. 13(b), this transition is abrupt in all cases, confirming that the backflow region and breakdown bubbles are distinct flow features. This is further supported by cases where the bubbles fully detach from the main $u_x = 0$ isosurface, such as in the alternating breakdown regime discussed in Sec. III C 2. Although the true onset of the bubbles may occur slightly upstream, this criterion provided the most objective way to distinguish the two flow features in cases where the $u_x = 0$ encloses both. Thus, additional data was measured upstream of the onset point for the same transverse coordinates of the onset location.

Despite its name, bubble-type vortex breakdown does not destroy the original vortex downstream of the breakdown bubble. Figures 12(c) and 12(d) confirm that the counter-rotating vortex pair persists beyond the breakdown bubbles in our simulations. This is further corroborated by streamwise vorticity measurements along the black streamlines in Fig. 14(a), and along the vortex core (inside the bubbles) of Fig. 14(b). In all cases, vorticity in the counter-rotating vortex pair increases approximately linearly with Re_{jet} . Additionally, no sudden reversal of vorticity sign is detected, which is consistent with previous experimental measurements (e.g., Leibovich [48]). Instead, the azimuthal vorticity ω_θ becomes negative inside the breakdown bubbles, aligning with the negative streamwise velocity, as noted by (Brown and Lopez [49]). In our simulations, this component roughly aligns with ω_y , and, as shown in Fig. 11(b), our results are consistent with these observations.

Although breakdown introduces a striking change in streamline topology, its onset occurs gradually as Re_{jet} increases. The $\frac{u_x}{u_{jet}} = 0.04$ isosurface illustrates this process, revealing that even when breakdown does not occur, a streamwise momentum deficit region extends downstream of the backflow region, forming two protrusions. This contributes to the whirling motion seen in the counter-rotating vortex streamlines (also noted in Sec. III B 1).

In jets in crossflow, a pressure minimum forms just downstream of the orifice, due to the jet's curvature and its obstruction of the crossflow (Muppidi and Mahesh [50]). This pressure minimum is followed by an adverse pressure gradient, which extends to the vortex cores [Fig. 15(a)]. In cases where breakdown doesn't occur, the location of the vortex cores was also identified from the local minimum streamwise velocity, but allowing $u_x > 0$. In the same figure, black dots mark the onset

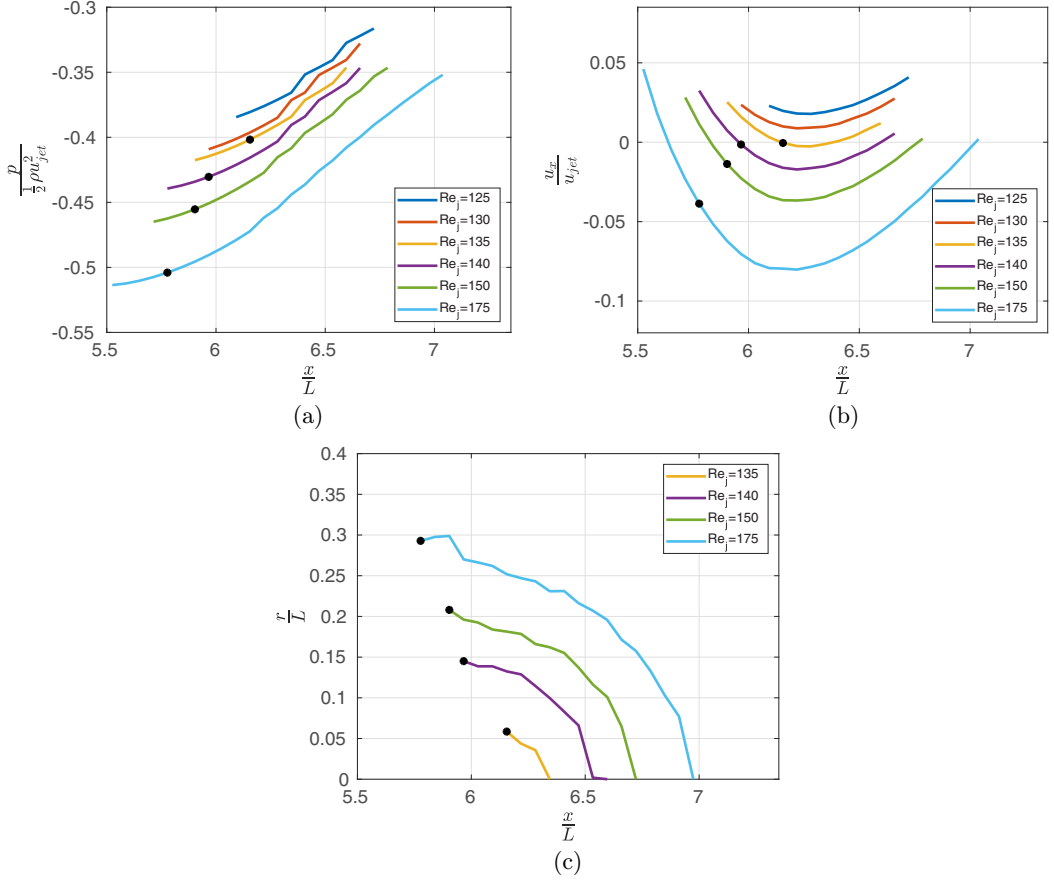


FIG. 15. Pressure (a) and streamwise velocity (b) in the breakdown bubble cores, and the radius of the bubbles (c).

of the bubbles. As vorticity increases, pressure in the bubbles continues to drop, eventually causing the streamwise velocity in the vortex cores to become negative [Fig. 15(b)], leading to breakdown. The length of the breakdown bubbles, which can be estimated from Fig. 15(b), also depends on this mechanism. The sharp pressure drop with Re_{jet} [visible in Fig. 15(a)] results in longer bubbles with onset locations farther upstream.

The bubble radii plotted in Fig. 15(c) were measured as the average distance from the bubble core to the surrounding $u_x = 0$ contour, in a given streamwise-facing plane. This definition is only valid in regions where the bubble extends past the backflow region, so data from upstream locations was excluded. The measured radius is linked to the radial velocity away the stagnation point at the bubble onset. Therefore, the sharp increase in bubble radius with Re_{jet} is driven by the increasing upstream velocity inside of the bubbles, due to continuity. Interestingly, despite the radius increasing, the distance between the vortices remains constant [Fig. 13(b)], indicating that the latter is not sensitive to the two governing parameters studied. Instead, it is likely dependent on the characteristic length of the orifice or other parameters.

Figures 16(a) and 16(b) show the full parametric range explored in this work. Compared to most other studies on laminar jets in crossflow, the range of R investigated is broader and includes several intermediate values. The threshold value of R below which breakdown occurs increases

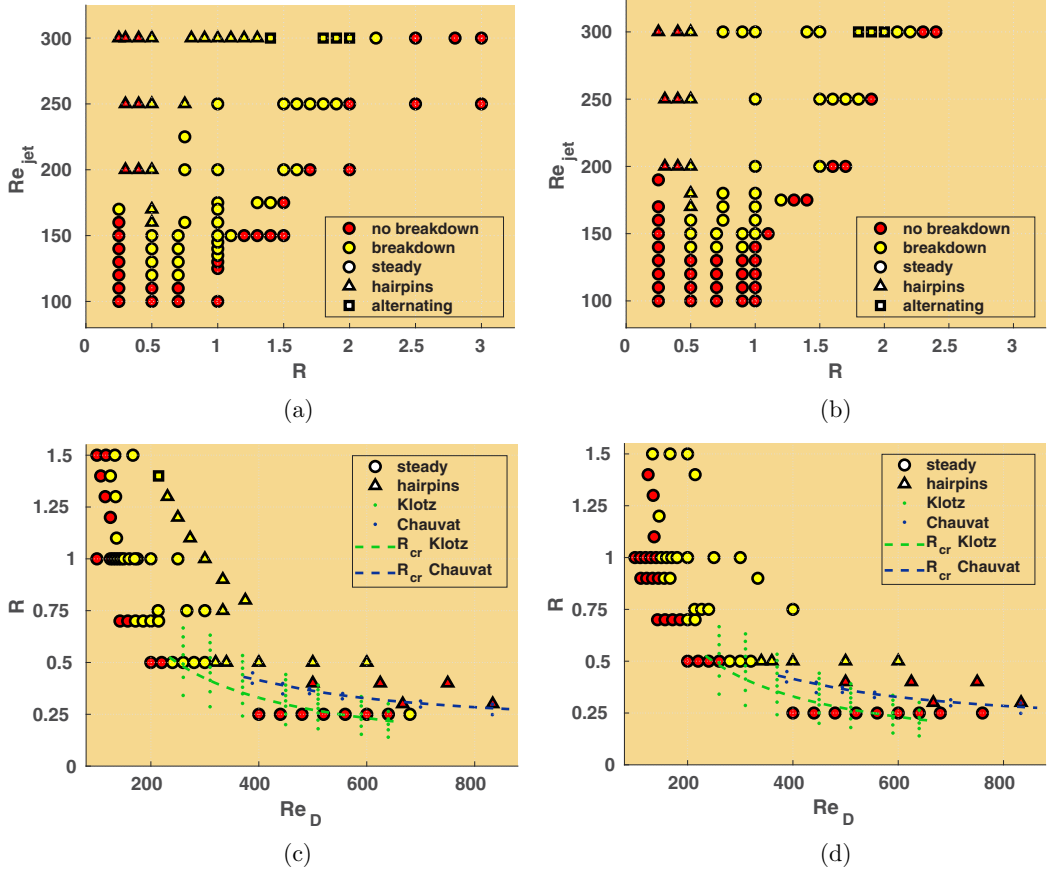


FIG. 16. Diagram of the simulations performed during this work, showing the onset of vortex breakdown for square (a) and round (b) jets in crossflow. The same data is plotted in panels (c) and (d) in $Re_D - R$ coordinates for $R \leq 1.5$. The data points and stability curves by Klotz *et al.* [23] and Chauvat *et al.* [21] are included for reference.

approximately linearly with Re_{jet} , consistently with the physical mechanisms analyzed in this section.

Similar trends were obtained for both cross sections studied. However, at $R = 0.25$, breakdown was identified in several square jets, but not in circular jets. This is due to the lower vertical momentum of circular jets, which penetrate less into the crossflow, when compared with square jets with the same average velocity. As a result, the adverse pressure gradient downstream of circular jets is weaker, preventing breakdown in these cases.

While all previously discussed cases reached steady state, the diagrams show that several unsteady cases were also obtained. The $Re_D - R$ diagrams of Figs. 16(c) and 16(d) are zoomed in on the low R regime, where transition to unsteady flow is characterized by hairpin vortex shedding. Datapoints from Klotz *et al.* [23] and Chauvat *et al.* [21] are included in the diagrams for reference. Unlike the current work, which focuses on studying a broader range of parameters, the previous works focused on finding a critical velocity ratio (R_{cr}) curve above which the flow is unsteady, so their datapoint density in this region is significantly larger. Despite this, our data fits fairly well with the stability curves plotted, particularly with that of Chauvat *et al.* [21]. The differences between our data and the curve of Klotz *et al.* [23] likely result from differences in the jet profile at the orifice. The profiles reported from their experimental work are almost top-hat, while we have a

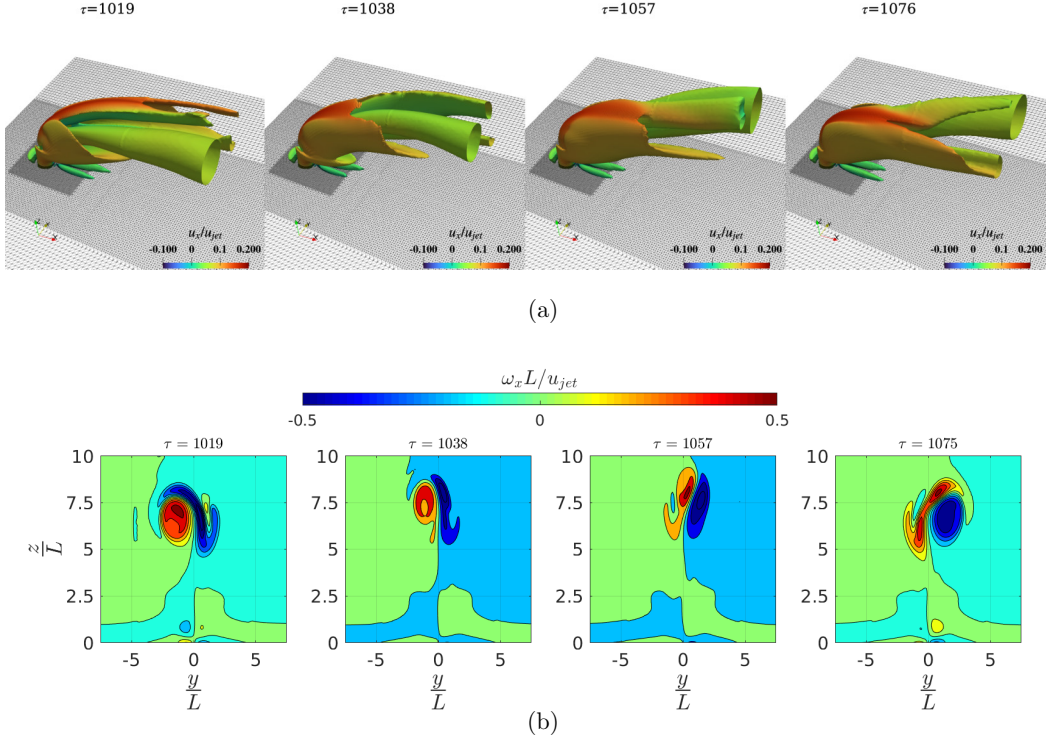


FIG. 17. $Q = 100$ isosurface throughout half of one oscillation period (a) and the streamwise vorticity contours at $x = 17.5L$ (b) for the same instants.

fully developed profile. Our data also indicates that the R_{cr} threshold for circular and square jets falls in the same $R \in [0.25, 0.5]$ interval, but a significant number of additional datapoints would be required to compute the stability curves needed for an accurate comparison.

In the current setup, vortex breakdown consistently precedes the onset of unsteady flow, when Re_{jet} is fixed and R is decreased [Figs. 16(a) and 16(b)]. This approach to varying the control parameter corresponds to the one used by the Ann Karagozian group (Harris *et al.* [9]). The unsteady cases were breakdown was identified include some characterized by hairpin vortex shedding, and others where the flow loses its symmetry, leading to the alternating breakdown of each vortex. This was observed for both jet cross sections studied and will be analyzed in the following sections.

2. Unsteady alternating breakdown

Both square and circular jets at $Re_{jet} = 300$ are unsteady at intermediate velocity ratios ($R = 2$). The simulations initially display a brief transient phase, with shedding of shear-layer vortices, as is typical of unsteady laminar jets in crossflow at moderate R (Sau *et al.* [10]). Afterwards, the system transitions into a new periodic flow state, where neither hairpin vortices nor shear layer vortices are shed. Instead, the flow is characterized by an oscillatory behavior of the counter-rotating vortices, which alternatingly grow and shrink, as shown in Fig. 17(a).

The self-sustained oscillatory behavior is further illustrated by the streamwise vorticity contours of Fig. 17(b). Each oscillation period begins with a counter-rotating vortex moving laterally after growing, and crossing the spanwise bisection plane. At this point, it splits into two smaller same signed vortices against the oppositely signed counter-rotating vortex. The upper vortex is dissipated, but the lower vortex remains inside its half plane. In the next phase, the opposite counter-rotating vortex follows the same process, concluding one period. Note that the whole counter-rotating pair

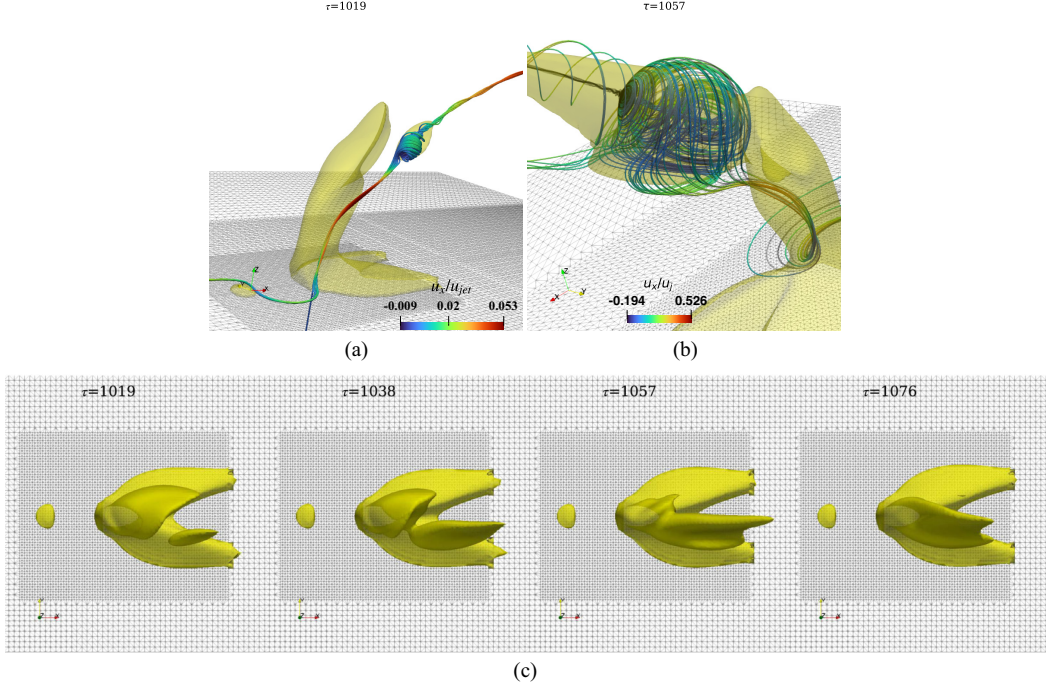


FIG. 18. Streamlines and $u_x = 0$ isosurface (in yellow) demonstrating the temporal evolution of the breakdown bubbles. Breakdown bubble at $\tau = 1019$, where the bubble is initially detached from the backflow region (a), and after coalescing with it (b) at $\tau = 1057$. Top view of the $u_x = 0$ isosurface in different instants of a half-period (c).

moves relatively uniformly and this is phenomenon is not limited to the far downstream region, as the Q isosurfaces of Fig. 17(a) confirm.

The counter-rotating vortex pair undergoes vortex breakdown in this regime, as indicated in the simulations diagram. However, in this case, only one vortex undergoes breakdown at a time, in phase with the oscillatory dynamic of the vortices. Figures 18(a) and 18(b) illustrate that the breakdown bubble initially forms detached from the backflow region before coalescing with it, forming a single $u_x = 0$ isosurface. This process coincides with the growth of the vortex in radius, before it starts to move toward the spanwise bisection plane.

A complementary view of the breakdown process is presented in Fig. 18(c), which displays the $u_x = 0$ isosurface from above. Two peaks are visible in the breakdown bubble protrusion at times $\tau = 1059$ and $\tau = 1076$, which coincide with the splitting of the traversing vortex into two smaller corotating vortices. These peaks help conveying the movement of the top vortex toward the opposite side of the jet in the spanwise direction), further demonstrating the alternating breakdown dynamics.

The streamwise velocity in two probes is plotted in Fig. 19(a), one inside each of the two breakdown bubbles. The oscillatory state is established at around $\tau = 450$, and its frequency, $St_\infty = 0.016$ ($St_{jet} = 0.0082$), corresponds to the first peak frequency in the power spectral density plot of Fig. 19(b). This Strouhal is more than one order of magnitude lower than the typical characteristic shedding frequency of jets in crossflow at low R .

The state described in this section is reminiscent of one of the POD modes discussed in Schlatter *et al.* [17]. There, the jet is said to oscillate laterally, also with a frequency about one order of magnitude lower than the hairpin shedding frequency, but the flow is highly complex. Another asymmetric flow state was briefly reported in Ilak *et al.* [26] for $R = 3$. It is possible that hairpin vortex shedding coexists with the current instability in that flow state, as described in Martins and

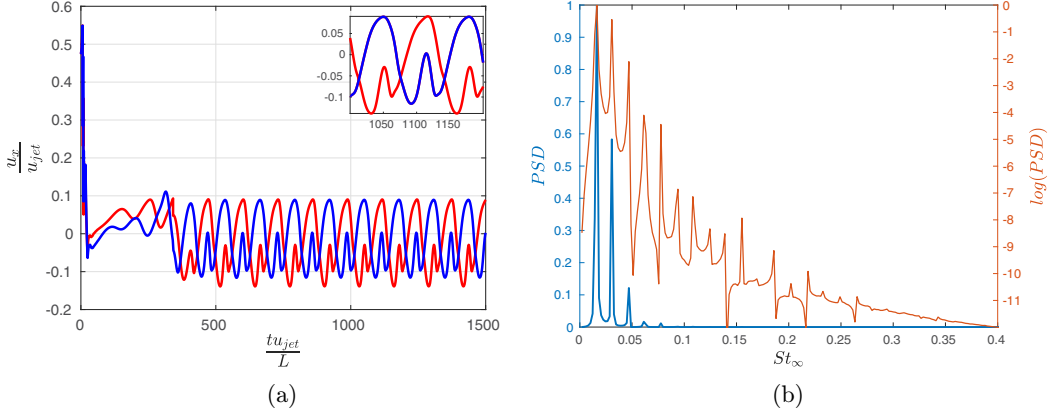


FIG. 19. Probes of the streamwise velocity taken from the breakdown bubbles (a), and the correspondent power spectrum after the flow settled into its final oscillatory state ($\tau > 500$).

Pereira [51]. However, to the authors' knowledge, no prior study has presented results where the current instability was isolated and characterized.

3. Vortex breakdown and hairpin shedding

The onset of unsteady flow in low velocity ratio jets in crossflow is characterized by periodic shedding of hairpin vortices, due to a Hopf bifurcation (Ilak *et al.* [26], Chauvat *et al.* [21]). This flow state was obtained in simulations with both jet cross sections, as indicated in the diagrams of Figs. 16(a) and 16(b). To maintain consistency with the rest of the simulations, these cases were plotted in terms of $\text{Re}_{\text{jet}} - R$ in the diagrams. However, in this regime, the governing Reynolds number is typically Re_D , since the jet behaves as a perturbation to the crossflow. Table I shows the conversion between the two Reynolds number definitions for the range $\text{Re}_{\text{jet}} \in [200, 300]$ and $R \in [0.3, 0.5]$ discussed in this section.

Vortex breakdown was identified in the hairpin vortex shedding cases with a velocity ratio exceeding the critical threshold $R > 0.4$ for both square and circular jets. The breakdown bubbles are illustrated by the streamlines of Fig. 20(a), which follow the same color code of Fig. 12(a). The vertical velocity fluctuation $\frac{u_z - \bar{u}_z}{u_{\text{jet}}}$ in the spanwise bisection plane for this case, plotted in Fig. 20(b), shows that the fluctuation amplitude remains small near the orifice. This was observed in all $R = 0.5$ cases simulated, and shows that the breakdown bubbles remain largely unaffected by the unsteadiness. Thus, upon shedding, weak rollup is observed in the hairpins, where streamlines undulate vertically coinciding with the regions of high spanwise vorticity.

For velocity ratios below $R \leq 0.4$, no breakdown bubbles were identified in the instantaneous velocity field. Instead, Fig. 20(c) shows that hairpin vortices exhibit strong shear layer rollup, where streamlines fully loop around the hairpin vortex axis. Thus, the fluctuation amplitude near the orifice grows significantly larger [Fig. 20(d)], than in cases where $R > 0.4$.

TABLE I. Conversion between Re_D and Re_{jet} in the simulations where hairpin vortex shedding was obtained.

	$R = 0.3$			$R = 0.4$			$R = 0.5$		
Re_D	670	833	900	500	625	750	400	500	600
Re_{jet}	200	250	300	200	250	300	200	250	300

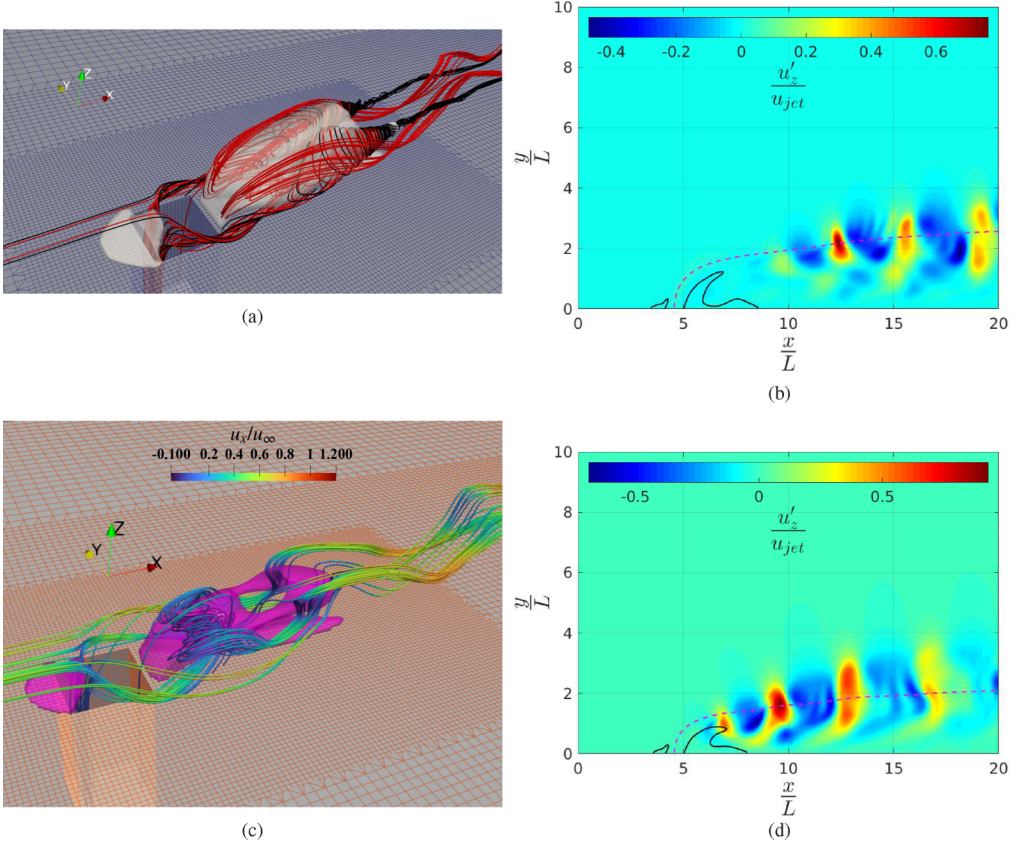


FIG. 20. Streamlines of two $Re_D = 500$ simulations, illustrating vortex breakdown (a) for $R = 0.5$, and shear layer rollup (c) for $R = 0.4$. Vertical velocity fluctuation $\frac{u'_z}{u_{jet}}$ the two corresponding cases in panels (b) and (d), respectively. The jet trajectory streamline is depicted by the dashed line and the $\overline{u_x} = 0$ isocontour by the full line.

Vortex breakdown was not identified in previous studies on this regime. This is likely because the onset of unsteady flow has traditionally been studied by considering low (or very low) R circular jets as a starting point. According to our results, this is where breakdown is least likely to occur, particularly for circular jets. It is also possible that the bubbles were masked by the shear region near the orifice, especially in experimental studies. Additionally, most studies were focused on characterizing hairpin shedding frequencies and flow behavior in the spanwise bisection plane. Our results indicate that vortex breakdown is not directly identifiable in this plane.

Simulations of $Re_{jet} > 300$ jets consistently resulted in unsteady flows regardless of the velocity ratio. These cases exhibited a more chaotic flow behavior, so they are beyond the scope of this study, which focuses on vortex breakdown of the counter-rotating vortex pair and its dynamics in steady and unsteady flow, rather than in the transition to turbulence.

IV. CONCLUSIONS

This study numerically investigated square and circular laminar jets in crossflow within $100 \leq Re_{jet} \leq 300$ and $0.25 \leq R \leq 5$, focusing on both the steady flow regime and the onset of unsteady flow in the moderate to low R . Several key findings emerged from this analysis:

In the steady flow regime, streamlines originating from both the jet and the crossflow converge, forming the counter-rotating vortex pair. Analysis of the circulation and vorticity transport at large R revealed that the streamwise vorticity in the counter-rotating vortex pair is originated along the nozzle walls. Streamwise jet vorticity is directly convected downstream, but spanwise vorticity is tilted by the spanwise gradients of streamwise velocity. The latter is in agreement with the mechanism described in Kelso *et al.* [18], but in the absence of unsteady shear layer vortices.

At low R , jet penetration is significantly reduced, forcing entrainment into the counter-rotating vortex pair through the backflow region. This leads to injection of boundary layer vorticity into the counter-rotating vortex pair through vertical tilting of spanwise vorticity, followed by vertical stretching and streamwise tilting. As a result, for a constant Re_{jet} , decreasing R leads to a stronger counter-rotating vortex pair.

The counter-rotating vortex pair may undergo bubble-type vortex breakdown when its vorticity and the adverse pressure gradient downstream of the jet are large enough. This makes breakdown more likely in square jets than in circular jets, but breakdown with both cross sections was observed. At very low R , the streamwise momentum deficit is not strong enough to induce breakdown of the counter-rotating vortex pair in circular jets. Breakdown causes a sudden change in streamline topology, but quantification of streamwise velocity and pressure in the bubbles shows that breakdown onset is a gradual process. It also clarifies that the counter-rotating vortex pair persists downstream of the breakdown bubbles.

The range of R and Re_{jet} where breakdown occurs was investigated, revealing that it precedes the onset of unsteady flow as R is lowered for a fixed Re_{jet} . The simulation diagram shows an approximately linear increase in the upper limit of R with Re_{jet} for breakdown to occur for jets with both cross sections. Additionally, breakdown was found to persist in several cases where unsteady flow was obtained.

A new instability was identified in both square and circular jets for $Re_{jet} = 300$ and intermediate $1.4 \leq R \leq 2$. In this regime, the counter-rotating vortices oscillate laterally at a low frequency without shedding hairpin vortices. One oscillation period is characterized by one of the vortices crossing the spanwise bisection plane and splitting into a pair of corotating vortices against the opposite-signed vortex, which then undergoes the same process. Vortex breakdown of each counter-rotating vortex occurs in phase with this oscillation.

Hairpin vortex shedding, typically observed in low R laminar jets in crossflow was captured in our simulations, and a hard limit of $R > 0.4$ was found for obtaining vortex breakdown in this regime. In these cases, the near field remains approximately steady state, conserving the breakdown bubbles. At lower R , the velocity field fluctuation amplitude destroys the breakdown bubbles in the instantaneous flow field, by inducing the roll up of fluid typically described in the literature.

Further experimental and theoretical research is needed to explore the onset of vortex breakdown and the asymmetric instability in jets in crossflow. In particular, the potential of these structures for enhancing mixing in flows where they may develop can be of strong scientific and practical interest.

ACKNOWLEDGMENTS

This work was supported by Fundação para a Ciência e Tecnologia (FCT) through the SmokeStorm Project No. PCIF/MPG/0147/2019 and through IDMEC. The authors acknowledge Fundação para a Ciência e a Tecnologia (FCT) for its financial support via the project LAETA Base Funding (DOI: 10.54499/UIDB/50022/2020). F.C.M. acknowledges Fundação para a Ciência e Tecnologia (FCT) for Scholarship No. 2020.07104.BD with DOI: 10.54499/2020.07104.BD.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

- [1] E. E. Callaghan, *Investigation of the Penetration of an AirJet Directed Perpendicularly to an Air Stream* (National Advisory Committee for Aeronautics, Hampton, VA, 1948), p. 1615.
- [2] T. T. Rice, K. Taylor, and M. Amitay, Pulse modulation of synthetic jet actuators for control of separation, *Phys. Rev. Fluids* **6**, 093902 (2021).
- [3] M. Popovac and K. Hanjalić, Vortices and heat flux around a wall-mounted cube cooled simultaneously by a jet and a crossflow, *Int. J. Heat Mass Transf.* **52**, 4047 (2009), special Issue Honoring Professor D. Brian Spalding.
- [4] E. Liu, X. Liu, M. Zhao, H. Zheng, J. Lu, and Z. Zhang, Turbulent fuel-air mixing study of jet in crossflow at different velocity ratios using les, *Int. J. Heat Fluid Flow* **85**, 108633 (2020).
- [5] L. Zhang and V. Yang, Flow dynamics and mixing of a transverse jet in crossflow—Part I: Steady crossflow, *J. Eng. Gas Turbines Power* **139**, 082601 (2017).
- [6] H. Chung and J. R. Koseff, Interaction of a buoyant plume with a turbulent canopy mixing layer, *Phys. Rev. Fluids* **8**, 064501 (2023).
- [7] G. G. J. Ernst, J. P. Davis, and R. S. J. Sparks, Bifurcation of volcanic plumes in a crosswind, *Bull. Volcanol.* **56**, 159 (1994).
- [8] K. Mahesh, The interaction of jets with crossflow, *Annu. Rev. Fluid Mech.* **45**, 379 (2013).
- [9] E. W. Harris, T. Shoji, A. Besnard, S. G. Schein, R. T. M'Closkey, L. Cortelezzi, and A. R. Karagozian, Effects of controlled vortex generation and interactions in transverse jets, *Phys. Rev. Fluids* **7**, 013902 (2022).
- [10] A. Sau, T. W. H. Sheu, R. R. Hwang, and W. C. Yang, Three-dimensional simulation of square jets in cross-flow, *Phys. Rev. E* **69**, 066302 (2004).
- [11] T. F. Fric and A. Roshko, Vortical structure in the wake of a transverse jet, *J. Fluid Mech.* **279**, 1 (1994).
- [12] R. M. Kelso and A. J. Smits, Horseshoe vortex systems resulting from the interaction between a laminar boundary layer and a transverse jet, *Phys. Fluids* **7**, 153 (1995).
- [13] A. Rivero, J. A. Ferre, and F. Giralt, Organized motions in a jet in crossflow, *J. Fluid Mech.* **444**, 117 (2001).
- [14] B. Wegner, Y. Huai, and A. Sadiki, Comparative study of turbulent mixing in jet in cross-flow configurations using les, *Int. J. Heat Fluid Flow* **25**, 767 (2004).
- [15] L. Cortelezzi and A. R. Karagozian, On the formation of the counter-rotating vortex pair in transverse jets, *J. Fluid Mech.* **446**, 347 (2001).
- [16] R. Camussi, G. Guj, and A. Stella, Experimental study of a jet in a crossflow at very low Reynolds number, *J. Fluid Mech.* **454**, 113 (2002).
- [17] P. Schlatter, S. Bagheri, and D. S. Henningson, Self-sustained global oscillations in a jet in crossflow, *Theor. Comput. Fluid Dyn.* **25**, 129 (2011).
- [18] R. M. Kelso, T. T. Lim, and A. E. Perry, An experimental study of round jets in cross-flow, *J. Fluid Mech.* **306**, 111 (1996).
- [19] M. Shinohara and S. Matsushima, Formation of fire whirls: Experimental verification that a counter-rotating vortex pair is a possible origin of fire whirls, *Fire Safety J.* **54**, 144 (2012).
- [20] B. A. C. Barata, F. C. Martins, and J. C. F. Pereira, Les of fire plumes subjected to crosswind inducing vertical vorticity, *Fire Safety J.* **144**, 104112 (2024).
- [21] G. Chauvat, A. Peplinski, D. S. Henningson, and A. Hanifi, Global linear analysis of a jet in cross-flow at low velocity ratios, *J. Fluid Mech.* **889**, A12 (2020).
- [22] T. Cambonie and J.-L. Aider, Transition scenario of the round jet in crossflow topology at low velocity ratios, *Phys. Fluids* **26**, 084101 (2014).
- [23] L. Klotz, K. Gumowski, and J. E. Wesfreid, Experiments on a jet in a crossflow in the low-velocity-ratio regime, *J. Fluid Mech.* **863**, 386 (2019).
- [24] G. Bidan and D. E. Nikitopoulos, On steady and pulsed low-blowing-ratio transverse jets, *J. Fluid Mech.* **714**, 393 (2013).
- [25] J.-M. Chomaz, Global instabilities in spatially developing flows: Non-normality and nonlinearity, *Annu. Rev. Fluid Mech.* **37**, 357 (2005).
- [26] M. Ilak, P. Schlatter, S. Bagheri, and D. S. Henningson, Bifurcation and stability analysis of a jet in cross-flow: Onset of global instability at a low velocity ratio, *J. Fluid Mech.* **696**, 94 (2012).

- [27] M. B. Jovanović, H. C. Lange, and A. Steenhoven, Effect of hole imperfection on adiabatic film cooling effectiveness, *Int. J. Heat Fluid Flow* **29**, 377 (2008).
- [28] S. L. V. Coelho and J. C. R. Hunt, The dynamics of the near field of strong jets in crossflows, *J. Fluid Mech.* **200**, 95 (1989).
- [29] T. H. New, T. T. Lim, and S. C. Luo, Elliptic jets in cross-flow, *J. Fluid Mech.* **494**, 119 (2003).
- [30] P. Huq and M. R. Dhanak, The bifurcation of circular jets in crossflow, *Phys. Fluids* **8**, 754 (1996).
- [31] Y. Lv, H. Wei, T. Liu, X. Zhao, Y. Liu, B. Huang, and G. Wang, Numerical investigation of the round jet in crossflow at high velocity ratios with special emphasis on the evolution of vortex structures, *Phys. Fluids* **34**, 034116 (2022).
- [32] L. L. Yuan, R. L. Street, and J. H. Ferziger, Large-eddy simulations of a round jet in crossflow, *J. Fluid Mech.* **379**, 71 (1999).
- [33] O. Sgarzi and F. Leboeuf, *Analysis of Vortices in Three-Dimensional Jets Introduced in a Cross-Flow Boundary-layer*, 78682 (ASME, New York, NY, 1997).
- [34] Z. Shi, J. WU, and J. WU, Symmetric and asymmetric jets in a uniform crossflow, in *Proceedings of the 29th Aerospace Sciences Meeting* (AIAA, Reno, 1991), p. 722.
- [35] K. Jia, T. Scofield, M. Wei, and S. Bhattacharya, Vorticity transfer in a leading-edge vortex due to controlled spanwise bending, *Phys. Rev. Fluids* **6**, 024703 (2021).
- [36] J. M. Akkala and J. H. J. Buchholz, Vorticity transport mechanisms governing the development of leading-edge vortices, *J. Fluid Mech.* **829**, 512 (2017).
- [37] F. C. Martins, J. M. C. Pereira, and J. C. F. Pereira, Vorticity transport in laminar steady rotating plumes, *Phys. Fluids* **32**, 043604 (2020).
- [38] B. A. Haven and M. Kurosaka, Kidney and anti-kidney vortices in crossflow jets, *J. Fluid Mech.* **352**, 27 (1997).
- [39] M. G. Hall, Vortex breakdown, *Annu. Rev. Fluid Mech.* **4**, 195 (1972).
- [40] S. T. Chan, J. T. Ault, S. J. Haward, E. Meiburg, and A. Q. Shen, Coupling of vortex breakdown and stability in a swirling flow, *Phys. Rev. Fluids* **4**, 084701 (2019).
- [41] P. Moise and J. Mathew, Bubble and conical forms of vortex breakdown in swirling jets, *J. Fluid Mech.* **873**, 322 (2019).
- [42] B. R. Morton, The generation and decay of vorticity, *Geophys. Astrophys. Fluid Dyn.* **28**, 277 (1984).
- [43] H. G. Weller, G. Tabor, H. Jasak, and C. Fureby, A tensorial approach to computational continuum mechanics using object-oriented techniques, *Comput. Phys.* **12**, 620 (1998).
- [44] P. J. Roache, *Verification and Validation in Computational Science and Engineering*, Vol. 895 (Hermosa, Albuquerque, NM, 1998).
- [45] I. Danaila, J. Dušek, and F. Anselmetti, Coherent structures in a round, spatially evolving, unforced, homogeneous jet at low Reynolds numbers, *Phys. Fluids* **9**, 3323 (1997).
- [46] M. S. Chong, A. E. Perry, and B. J. Cantwell, A general classification of three-dimensional flow fields, *Phys. Fluids* **2**, 765 (1990).
- [47] A. Bejan, S. Ziaei, and S. Lorente, Evolution: Why all plumes and jets evolve to round cross sections, *Sci. Rep.* **4**, 4730 (2014).
- [48] S. Leibovich, The structure of vortex breakdown, *Annu. Rev. Fluid Mech.* **10**, 221 (1978).
- [49] G. L. Brown and J. M. Lopez, Axisymmetric vortex breakdown: Part 2. Physical mechanisms, *J. Fluid Mech.* **221**, 553 (1990).
- [50] S. Muppidi and K. Mahesh, Direct numerical simulation of round turbulent jets in crossflow, *J. Fluid Mech.* **574**, 59 (2007).
- [51] F. C. Martins and J. C. F. Pereira, Numerical study of laminar unsteady circular and square jets in crossflow in the low velocity ratio regime, *Fluids* **9**, 292 (2024).