

Rayleigh-Plateau instability on an angled and eccentric wireDilip Kumar Maity ^{1,*}, Christopher Wagstaff ², Sandip Dighe ¹ and Tadd Truscott ^{1,†}¹*Department of Mechanical Engineering, Physical Science and Engineering Division, [King Abdullah University of Science and Technology](#), Thuwal 23955, Kingdom of Saudi Arabia*²*Department of Chemical Engineering, [King Abdullah University of Science and Technology](#), Thuwal 23955, Kingdom of Saudi Arabia*

(Received 19 May 2024; accepted 7 October 2025; published 3 November 2025)

This research explores the modulation of Rayleigh-Plateau instability by adjusting the orientation angle and eccentricity of a wire within a nozzle. We demonstrate that both the angle and eccentricity significantly influence the Rayleigh-Plateau instability regimes. They both also influence characteristics, such as bead velocity along the wire, bead spacing (wavelength), and bead volume. Notably, when wires are both angled and eccentric, the effect of angle prevails. Our approach includes an empirical scaling analysis, comparing gravity, and viscosity forces on a single bead, yielding a unified empirical viscous force law, and enhancing understanding of Rayleigh-Plateau regime dynamics. This framework enriches our understanding of the forces at play in Rayleigh-Plateau instability and provides practical insights into the manipulation of fluid dynamics in industrial applications.

DOI: [10.1103/lmmx-fmtb](https://doi.org/10.1103/lmmx-fmtb)**I. INTRODUCTION**

When a wire is placed along the central axis of a nozzle, the liquid flowing through the nozzle gives rise to a hydrodynamic instability [1,2], leading to the formation of a string of droplets on the wire due to the Rayleigh-Plateau instability [3,4] as shown in Fig. 1. Typically, three phenomena can occur: (i) an isolated regime where primary droplets are separated by secondary droplets (dripping from the nozzle), (ii) a periodic Rayleigh-Plateau regime with a constant velocity and bead spacing (jetting from the nozzle), or (iii) an aperiodic convective regime characterized by random coalescence between primary beads (jetting from the nozzle) [2]. Liquid flowing on a wire is commonly employed for heat and mass exchange between liquids and gasses [5] (e.g., distillation [6], absorption, fog harvesting [7,8], and microfluidics [9], etc.). Liquid flowing down a wire offers the prospect of achieving high heat and/or mass transfer by increasing liquid-side circulation [10,11] and decreasing gas-side pressure drop [12–14]. Additional benefits include a reduction in dry area [15], velocity control [2], and surface area control [13,14,16]. Although the manipulation of the Rayleigh-Plateau instability on a wire stands as a crucial aspect of current research, there is little research into how it behaves when the wires are no longer vertical and/or the wire is no longer co-centric with the liquid nozzle.

*Contact author: dilip.maity@kaust.edu.sa†Contact author: taddtruscott@gmail.com

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](#) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Open access publication funded by King Abdullah University of Science and Technology (KAUST).

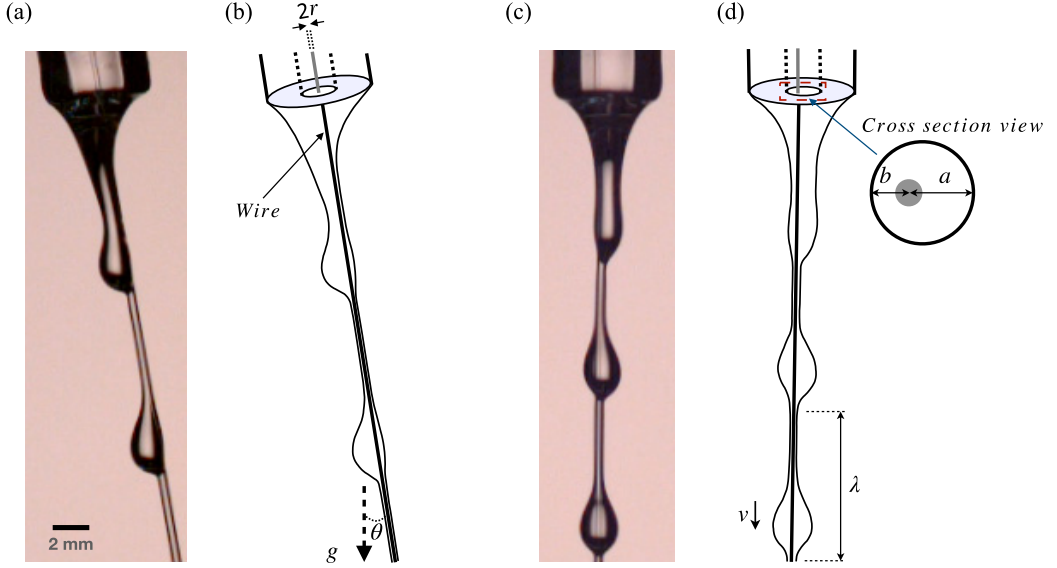


FIG. 1. Rayleigh-Plateau instability on an angled and eccentric wire. (a) Experimental image. (b) Schematic representation depicting the jet and bead shape near the nozzle at a 10° angle (θ) from the vertical with the wire positioned at the center of the nozzle [$e_n = (a - b)/(a + b) = 0$]. (c) Experimental image. (d) Schematic representation illustrating the jet and bead shape near the nozzle at an eccentricity of $e_n = 0.23$ for a vertically oriented wire. The diameter of the nylon wire and nozzle inner diameter is 0.2 mm and 2 mm, respectively.

The Rayleigh-Plateau instability on a vertical wire depends on the fluid properties [17], fluid rheology [18,19], wire diameter [20–22], wire geometry [23], wettability [24], external forcing [21], and nozzle inner/outer diameter [25–27]. Karimi *et al.* [28,29] were the first to investigate the effect of inclination angle on liquid flow along a wire, focusing on dripping, jetting, and drop-off phenomena in thin film flows. Similarly, our study examines angled wires and also observed the drop-off phenomenon. However, unlike their work, we focused on the Rayleigh-Plateau instability and the associated bead properties and scaling relationships. Eghbali *et al.* [30] studied the effect of eccentricity exclusively for vertical configurations. In contrast, our research extends this analysis by exploring the effect of eccentricity even when the wire is inclined. Also, Dejean *et al.* [31] designed a tetrahedron wire packing and observed unique flow behavior related to the angle between legs, and Chinju *et al.* [15] commented that eccentricity could affect droplet spacing. This is interesting because in an industrial setting the wires could easily be angled or eccentric from either design or manufacturing. Furthermore, this approach holds great utility where adjusting the primary parameters are constrained. For instance, if the flow rate, wire diameter, wettability, and wire geometry are fixed, one could still angle the wire to control the Rayleigh-Plateau instability based on our findings.

It is well accepted that the Rayleigh-Plateau regime shifts to the convective regime with increasing flow rates when all other parameters remain the same [2]. The phenomenon requires that the fluid be sufficiently viscous and the four main forces in competition are gravity, inertia, surface tension, and viscosity. Theoretical investigations [2,17,20,21,26,32] begin by estimating Nusselt thickness. Nusselt thickness is defined by the undisturbed film thickness provides a base state, derived by neglecting the surface tension and inertia and determining a velocity profile on the thickness. The undisturbed jet is assumed to be a cylinder because of analytical constraints, yet, in most experiments the jet is conical. Therefore, the theory often doesn't capture the instability quantitatively. Here, we derive empirical laws for the dominant forces by performing

a force balance on a single droplet specifically within the Rayleigh-Plateau regime, below the preinstability region. While this approach is more effective in quantifying the governing forces, which were difficult to predict in previous theoretical studies, it is also unable to capture the regime transition.

The current investigation involves a deliberate alteration of the Rayleigh-Plateau instability through two main adjustments: (i) manipulating the wire angle, and (ii) varying the eccentricity. We also present a scaling analysis within the Rayleigh-Plateau regime that juxtaposes the impacts of gravity, and viscous forces.

II. EXPERIMENTAL METHODS

In our experimental setup, we employ silicone oil with a density of 963 kg/m^3 , kinematic viscosity of 50 cSt and 100 cSt, and surface tension of 20.8 mN/m at 22°C . This silicone oil flows down an angled/eccentric nylon wire, with four nylon wires of radii $r = 0.1, 0.15, 0.2$, and 0.25 mm . Figure 1 and Fig. S1 in the Supplemental Material illustrate the experimental configuration. In the first configuration of this study, we rotated the whole experimental configuration to understand the effect of the angle on the Rayleigh-Plateau instability. We rotate the setup from the vertical at an angle of θ that varies from 0° – 40° at intervals of 5° as above around 40° no jet follows the wire. The setup includes a cylindrical container connected to a nozzle, continuously filled by a syringe pump to maintain a constant flow rate (Q). The fluid height in the cylindrical container initially rises and eventually stabilizes at a fixed level when the inflow and outflow rates become equal. Measurements were taken after this stabilization period and we vary the flow rate from 100 ml/h to 500 ml/h at an interval of 25 ml/h. The nozzle has inner and outer diameters of 2 mm and 6 mm, respectively. A weight is attached to ensure a stretched wire condition. Precision control of the wire angle and eccentricity is maintained using X-Y stages. Bead dynamics are recorded at 1500 fps using a Phantom T3610 camera, and an LED backlight illuminates the bead-wire system. Each experiment was observed for a few minutes to accurately characterize the distinctive regimes. Note, experiments were repeated three times for each data point in all figures, and resulting error bars are smaller than the marker size. Bead dynamics were recorded at a resolution of 1280×800 pixels, with droplet velocity determined using TRACKER software [33]. MATLAB is used to determine various properties, including bead spacing (wavelength), volume, and bead size. From the images captured by the camera, the bead volume is calculated by approximating the droplet as a series of small cross-sectional disks with a thickness equal to the pixel size. Numerical integration is then performed across the entire bead to compute its total volume, accounting for and subtracting the volume of the wire (for detailed information, refer to Sec. II in the Supplemental Material).

III. RESULTS

A. The effect of wire angle on the Rayleigh-Plateau instability

Our first objective is to scrutinize the variations in wire orientation to manipulate the Rayleigh-Plateau instability without altering any other parameters. In our experiment, we found similar regimes to Kliakhandler *et al.* [2] who classified the beading patterns resulting from thin film flow on a wire into three main categories: isolated regime, Rayleigh-Plateau regime, and convective regime. The primary focus of our study is to examine the transition boundary from the Rayleigh-Plateau regime to the convective regime and gain insights into bead properties within the Rayleigh-Plateau regime. Such insights are crucial for enhancing efficiency in practical applications [34].

When the wire is placed vertically (i.e., $\theta = 0^\circ$) the Rayleigh-Plateau regime is observed within the flow rate range of 250–425 ml/h for wire radius $r = 0.1 \text{ mm}$ and 50 cSt silicone oil as shown in Fig. 2(a), with isolated and convective regimes below and above this range of flow rates, respectively. As the angle of the wire is increased the required flow rate for the Rayleigh-Plateau

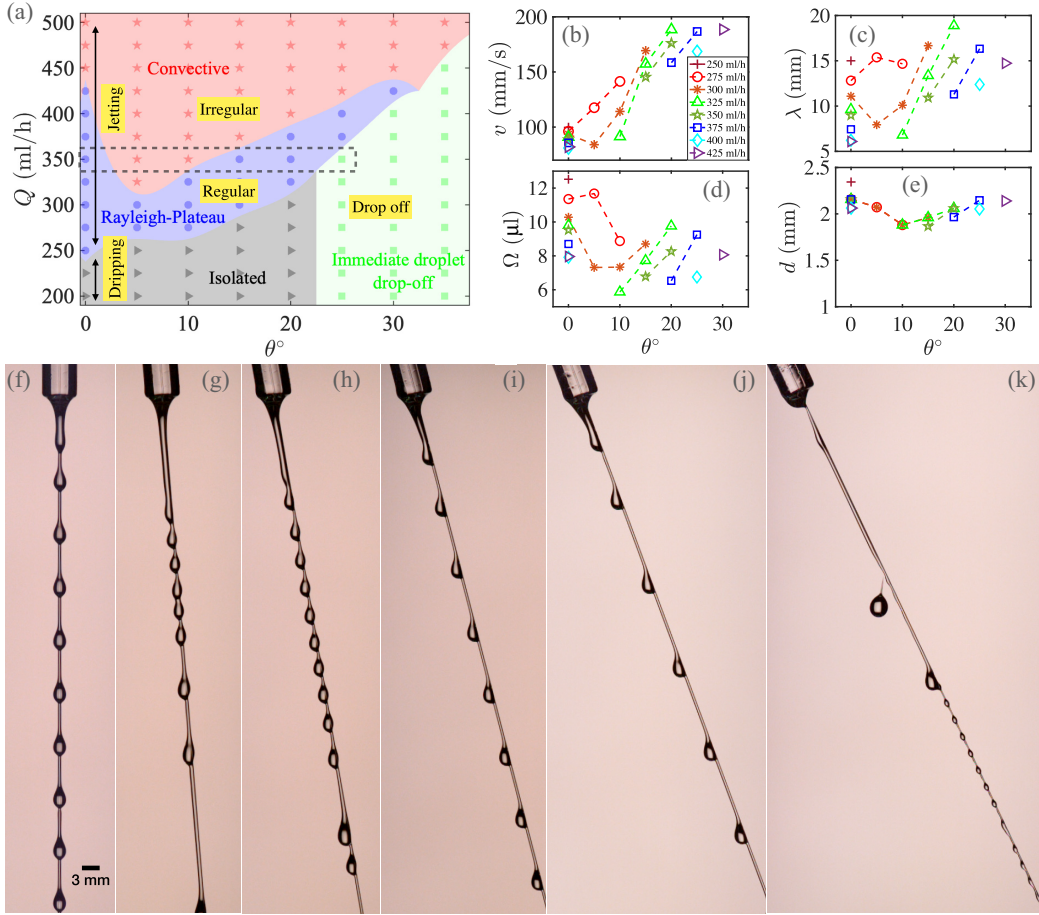


FIG. 2. Influence of angle on the Rayleigh-Plateau instability. (a) Map illustrating different beading regimes within the parameter space of flow rate and angle of orientation for wire radius $r = 0.1$ mm and 50 cSt silicone oil. Shaded regions approximate boundaries, and data points highlight domains where isolated, Rayleigh-Plateau, convective, and immediate droplet drop-off events are observed. Variation of bead (b) velocity, (c) wavelength, (d) volume, and (e) diameter with angle of orientation for the data of the Rayleigh-Plateau regimes for various flow rates. Disconnected dashed lines in (b)–(e) indicate the absence of Rayleigh-Plateau between data points. Images from cases marked by the rectangular dashed box in (a) depicting beads at (f) $\theta = 0^\circ$, (g) $\theta = 5^\circ$, (h) $\theta = 10^\circ$, (i) $\theta = 15^\circ$, (j) $\theta = 20^\circ$, and (k) $\theta = 25^\circ$.

regime increases and the range of flow rates is reduced. The isolated regime switches to immediate droplet dropoff when $\theta > 20^\circ$ as the weight of the drop overcomes surface tension forces that maintain the drop on the wire [28,29]. Convective regime cases also lead to droplet dropoff at larger θ but not immediately, also observed in [28,29]. This happens as an interaction of irregular droplets at a larger flow rate and larger angle. An intriguing observation is made where the Rayleigh-Plateau regime switches to the convective regime and back again when considering increasing angles at flow rates between 325–425 ml/h. For example, images taken from the dotted rectangular region of Fig. 2 revealing the transition from Rayleigh-Plateau to convective regime (f), (g), convective to Rayleigh-Plateau regime (h), (i), and Rayleigh-Plateau regime to immediate dropoff happens (j), (k); (see Videos 1–6 in the Supplemental Material for the dynamics [35]).

At $\theta = 0$ a symmetric undisturbed jet (preinstability jet) is converted to a series of symmetric droplets as shown in Fig. 2(f). The spacing between successive beads, denoted as wavelength (λ), decreases with increasing flow rate while the downward bead velocity along the wire (v) also decreases with flow rate but less prominently, eventually increasing flow rate transforms into the convective regime. This transition is typically associated with experimental conditions where the bead spacing and the droplet velocity decrease. When the wire is angled, Figs. 2(i) and 2(j) show that the symmetry of the jet and droplets are disrupted. This asymmetry results in changes in the bead properties we can measure in the Rayleigh-Plateau regime. At $Q = 300$ ml/h the bead velocity (v) along the wire exhibits a distinctive pattern of initial decrease followed by an increase for a specific flow rate (Q), eventually transitioning into the isolated regime or immediate droplet dropoff, as depicted in Fig. 2(b). In Fig. 2(c), the wavelength (λ) undergoes a similar behavior for the same flow rate, decreasing and then increasing up to the onset of the isolated regime or immediate droplet dropoff. In the vertical configuration at $Q = 350$ ml/h, the Rayleigh-Plateau regime is observed. As the angle increases to $\theta = 5^\circ$ and 10° , the Rayleigh-Plateau regime transitions to the convective regime but reappears at $\theta = 15^\circ$ and 20° . Figures 2(g) and 2(h) and the corresponding [35] (1–5) for 350 ml/h reveal that bead spacing and velocity are smaller near the preinstability region (even within the convective regime) compared to Figs. 2(f) and 2(i), which correspond to the Rayleigh-Plateau regime. This again confirms that the regime shift occurs when both wavelength and velocity drop below a certain threshold for a given flow rate. Furthermore, the initial decrease followed by an increase in wavelength and velocity with the angle appears to be a general trend for a given flow rate (see Sec. III in the Supplemental Material). However, for certain flow rates, this trend is not observed, likely due to the transition from the Rayleigh-Plateau regime to the convective regime occurring midway through the angle range. The fluid volume (Ω) within one wavelength (see Fig. 1 for the portion of one single wavelength) exhibits analogous behaviors concerning angle and flow rate, resembling the trends observed in v and λ , as illustrated in Fig. 2(d). The bead diameter remains nearly constant, Fig. 2(e).

B. The effect of eccentricity on the Rayleigh-Plateau instability in vertical orientation

For the second experimental configuration, we implemented a vertical wire setup, manipulating the eccentricity of the wire at the nozzle to explore its influence on the Rayleigh-Plateau instability under subtle adjustments, as depicted in Fig. 1(c). Nozzle eccentricity is quantified as $e_n = (a - b)/(a + b)$, where a and b represent the distances of the wire from the nozzle wall to the center of the wire on the right and left sides of the wire, respectively.

This configuration also yields three regimes similar to those identified by Kliakhandler *et al.* [2]. Figure 3(a) illustrates a regime map in the parameter space of Q and nozzle eccentricity (e_n) for $r = 0.1$ mm and 50 cSt silicone oil. This configuration enables us to comprehend the transitions between these regimes and their impact on bead properties. Specifically, when the wire is vertically oriented ($\theta = 0^\circ$), we observe Rayleigh-Plateau regimes within the flow rate range of 250–425 ml/h in Fig. 3(a). Isolated regimes and convective regimes are observed below and above this flow rate range, respectively. The Rayleigh-Plateau regime diminishes and disappears up to $e_n = 0.42$, beyond which only the transition from isolated to convective regime is observed. Experimental images at $Q = 300$ ml/h depicting the three regimes are presented in Figs. 3(f)–3(j) (see Supplemental Videos 7–11 for the dynamics [35]). When the wire is eccentric ($e_n \neq 0$), the pre-instability jet and the droplets just below the instability position exhibit asymmetry, but droplets below that level become symmetric (see Fig. 3(h) for most obvious Rayleigh-Plateau asymmetry near the nozzle). Both the asymmetrical undisturbed jet and transition from asymmetric to symmetric droplets arise due to the eccentricity of the wire altering the magnitude of the upward acting viscous force, leading to significant changes in the observed regimes and bead properties.

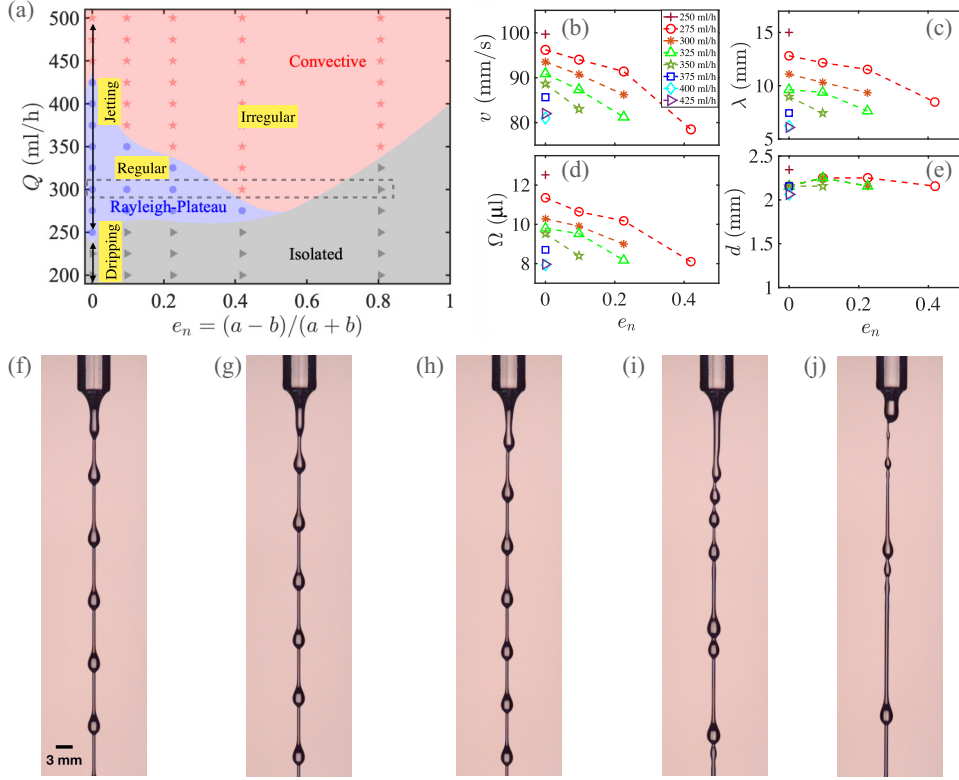


FIG. 3. Influence of eccentricity on the Rayleigh-Plateau instability for a vertically oriented wire. (a) Regime map within flow rate and eccentricity at $\theta = 0^\circ$ for wire radius $r = 0.1$ mm and 50 cSt silicone oil. Shaded colors depict the approximate regions, while discrete data points indicate the domains where the isolated, Rayleigh-Plateau, and convective regimes are observed. Variation of bead (b) velocity, (c) wavelength, (d) volume, and (e) diameter with eccentricity for different flow rates in the Rayleigh-Plateau regime. v . Images of the beads at $Q = 300$ ml/h for e_n : (f) 0, (g) 0.1, (h) 0.23, (i) 0.42, and (j) 0.81.

Focusing again on the bead properties within the Rayleigh-Plateau regimes we get straightforward relationships. In Figs. 3(b)–3(d), velocity, wavelength, and volume, all decrease monotonically for increasing e_n and Q . In addition, droplet diameter remains nearly constant as shown in Fig. 3(e).

C. The effect of eccentricity on the Rayleigh-Plateau instability in angled orientation

We continue by combining angle and eccentricity. The eccentricity can take on two directions within the angled nozzle due to the disruption in symmetry. These directions are designated as x and y at the nozzle-tip tilted reference frame with respect to gravity (g) as shown in the insets of Figs. 4(a) and 4(f). The eccentricity is adjusted from -1 to 1 in the x direction and 0 to 1 in the y direction due to symmetry. Figures 4(a) and 4(f) display a regime map for the x and y eccentricities at $\theta = 10^\circ$ for $r = 0.1$ mm and 50 cSt silicone oil and show that the isolated and Rayleigh-Plateau regimes are affected only at the extremes. The velocity, wavelength, and volume exhibit mild dependence on eccentricity as depicted in Figs. 4(b)–4(d) and 4(g)–4(i). Note that when θ is introduced, the effect of eccentricity becomes less significant. This is because gravity dominates the direction of the jet and droplet asymmetry over the eccentricity for higher angles ($\theta \gtrsim 10^\circ$).

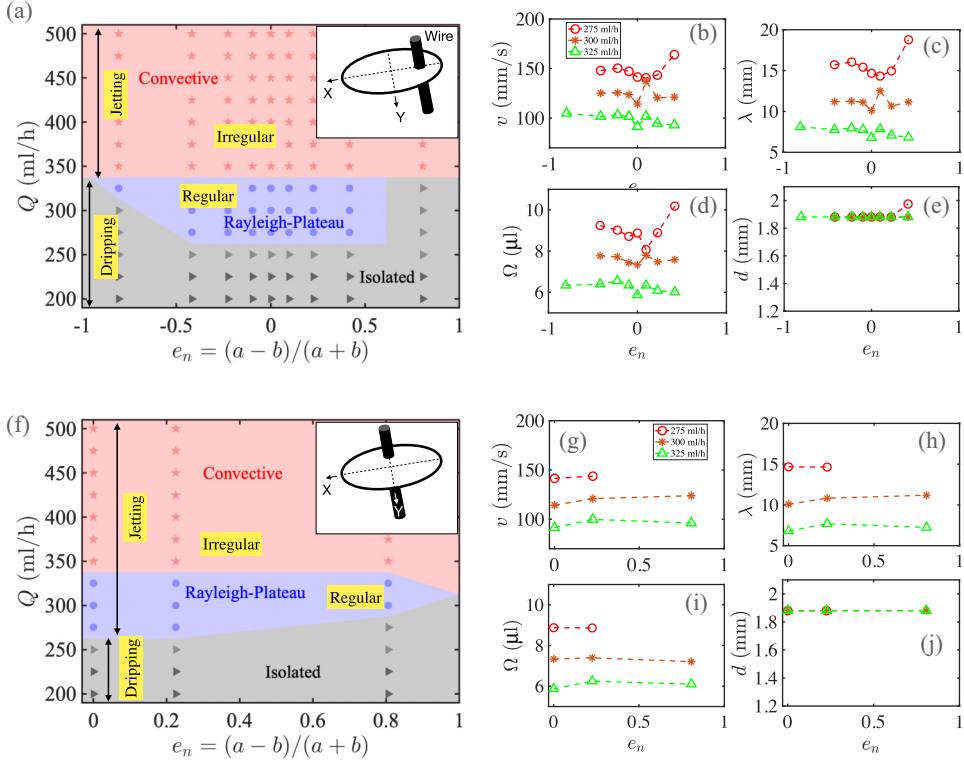


FIG. 4. Influence of eccentricity on the Rayleigh-Plateau instability for an angled wire. Regime map illustrating the beading regimes in the parameter space of flow rate and eccentricity at $\theta = 10^\circ$ for the shift of the (a) x direction and (f) y direction for wire radius $r = 0.1$ mm and 50 cSt silicone oil. Shaded colors depict the approximate regions, while discrete data points indicate the domains where the isolated, Rayleigh-Plateau, and convective regime are observed. Variation of (b), (g) velocity, (c), (h) wavelength, (d), (i) volume, and (e), (j) diameter with the nozzle eccentricity (e_n) for different flow rates (x direction, y direction) in the Rayleigh-Plateau regime.

D. Scaling law on the Rayleigh-Plateau instability on wire

In earlier investigations [2,17,20,21,26,32], researchers established a transition boundary between Rayleigh-Plateau and convective regimes. This was accomplished by delineating the Nusselt thickness by focusing solely on the interplay between viscous and gravity forces through the Navier-Stokes equations and solving for velocity to yield the thickness of the film on the wire. Notably, they accomplish a solution to the velocity profile by ignoring surface tension, which is later added to address the instability. Consequently, the theoretical model implies that an undisturbed jet (preinstability jet) maintains a purely cylindrical shape, in contrast to the observed conical form in most experiments [Fig. 3(f)]. This discrepancy leads us to question the realism and practicality of these theoretical approaches. Here, we propose a perspective by examining the interplay of three forces in the Rayleigh-Plateau regime: gravity, and viscosity acting on a single bead that enables us to derive an empirical law for viscous, which are otherwise highly challenging to address through theoretical methods. However, we acknowledge that our approach is unable to predict the transition.

We adopt a single-bead approach (control volume) to elucidate the empirical law, where the bead comprises two segments: a slender cylindrical part and a substantial droplet section [see Figs. 5(a) and 5(b)]. In our model, the control volume is taken over a length corresponding to one wavelength,

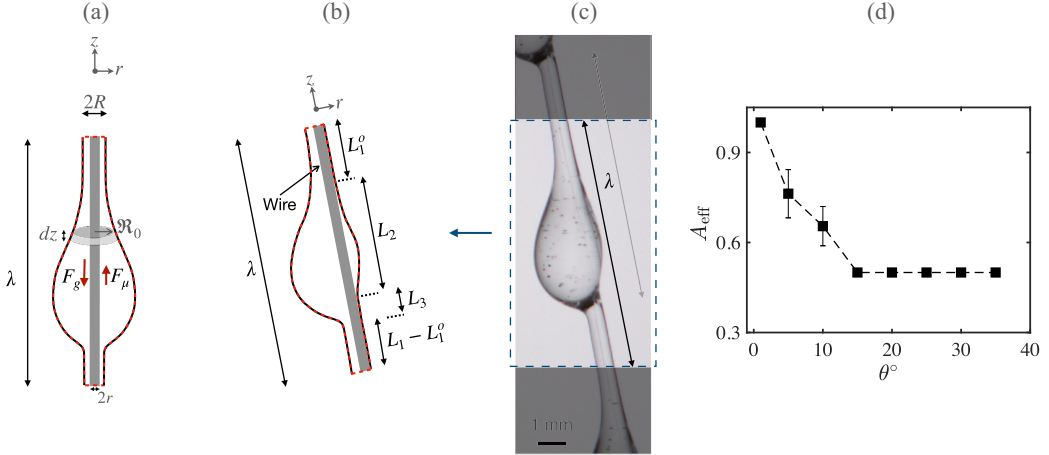


FIG. 5. Viscous force for Rayleigh-Plateau instability on a wire. (a) Schematic representation of a bead for $r = 0.1$ mm, $e_n = 0$, and 50 cSt silicone oil at (a) $\theta = 0^\circ$ and (b) $\theta = 10^\circ$. (c) Experimental image of a bead shown in (b). (d) Variation of A_{eff} with θ , including error bars as indicated.

λ as shown in Figs. 5(a) and 5(b). We can define the forces in the z -direction on the control volume as

$$\rho\Omega \frac{dv}{dt} = -F_g + F_\mu. \quad (1)$$

If we only consider droplets within the Rayleigh-Plateau regime where the droplets are evenly spaced with constant velocity, then $dv/dt = 0$. Thus, our hypothesis posits that the downward force from gravity (F_g) is equal to the upward force from viscous dissipation due to wire-fluid interaction (F_μ) along the wire axis (z).

Calculating the force (F_g) due to gravity is straightforward: $F_g = \rho\Omega g \cos \theta$, with ρ and g denoting liquid density and gravitational acceleration.

The viscous force (F_μ) is quantified as proportional to dynamic viscosity, area (dA), and the velocity gradient (dv/dr):

$$F_\mu = \int_A \mu dA \frac{dv}{dr}. \quad (2)$$

The force equation is derived by integrating the differential viscous force contributions over a control volume of length λ , as illustrated in Fig. 5(a). Here, $\Re_0(z)$ denotes the local radius of the liquid-air interface at axial position z . The elementary surface area of the wire over an infinitesimal segment dz is given by $dA = 2\pi r dz$, where r is the radius of the wire. Although the local fluid thickness at position z is $\Re_0(z) - r$, the internal flow visualization using tracer microparticles (see Supplemental Movie 12 [35]) suggests that the velocity gradient near the wire develops over a much smaller distance (r), confined close to the wire surface. Therefore, the velocity gradient can be taken as $dv/dr \sim v/r$ near the wire. Following these assumptions, and starting from first principles, Eq. (1) can be simplified for the vertical configuration as

$$F_\mu \propto \mu \int_{z=0}^{\lambda} 2\pi r \left(\frac{v}{r} \right) dz = T\mu v \lambda, \quad (3)$$

where $T/2\pi$ is the proportionality constant.

As the inclination angle deviates from zero, the bead becomes increasingly asymmetric, resulting in a reduction of the viscous force. Due to this asymmetry, integration of Eq. (2) is analytically intractable from first principles. To address this complexity, we introduce an effective area parameter,

A_{eff} , defined phenomenologically as a function of the inclination angle to capture the influence of asymmetry on the viscous force [defined in Eq. (4)]. We assume that the viscous force scales with the contact area between the droplet and the wire; however, under inclination, the asymmetric shape and motion of the droplet effectively reduce this area. This empirical derivation mirrors Stokes law [36], a frictional force model for spherical objects in a low Reynolds number implying a high viscous force setting. We experimentally measure A_{eff} by witnessing the shape of the bead as depicted in Fig. 5(c) and Supplemental Movie 12 [35]. We notice that the shape of the droplet is no longer symmetric for $\theta > 0$. Consequently, We track the interface of a droplet on both sides of the wire [Fig. 5(b)]. On the left side of the wire within the regions of lengths L_1 , L_2 , and L_3 ($\lambda = L_1 + L_2 + L_3$), the air-liquid interface is separated from the left wire surface by some amount of fluid. Whereas, on the right side of the wire, the air-liquid interface is only separated from the wire along L_2 . Thus, we assume that viscous drag acts on the left half of the wire of length λ and right half of the wire of length L_2 . Consequently, A_{eff} is defined as the ratio of the affected area for viscous drag to the total surface area as

$$A_{\text{eff}} = \frac{\pi r L_1 + 2\pi r L_2 + \pi r L_3}{2\pi r \lambda} = \frac{1}{2} \left(1 + \frac{L_2}{\lambda} \right). \quad (4)$$

For all vertical cases, we observe $L_2 = \lambda$ as the shape of the bead is symmetric and the liquid is present on both sides of the wire. For $\theta \geq 15^\circ$ we found $L_2 = 0$ as there is no fluid present on the right side due to the vertical component of gravity concerning the wire axis. Therefore, $A_{\text{eff}} = 1$ for all vertical cases and $A_{\text{eff}} = 0.5$ for the cases where $\theta \geq 15^\circ$, and is unaffected by variations in wire diameter and flow rate [Fig. 5(d)] given our experimental conditions. Notably, there is a pronounced variation in A_{eff} (between 0.5 and 1) observed at lower wire angles $\theta = 5^\circ$ and 10° with the specified error bar as shown in Fig. 5(d).

To account for the effect of inclination, we assume that the viscous force scales with the effective area as $F_\mu = T\mu v \lambda A_{\text{eff}}^n$. The exponent n is determined by fitting the data to the relation $\rho \Omega g \cos \theta = T\mu v \lambda A_{\text{eff}}^n + 2\pi R \sigma$, where $2R$ is the diameter of the thin cylinder. The fitted exponent (n) of A_{eff} was found to be 2 ± 0.2 . This outcome stems from the fact that the formulation of A_{eff} is based on a two-dimensional visualization, where we approximate that only half of the circumference contributes or not. In reality, this contribution is not exactly one-half and cannot be determined precisely, as only two-dimensional information is available. The term $F_c = 2\pi R \sigma$ is introduced so that the upward force balances the downward force on the control volume (see Fig. S7 in the Supplemental Material where F_g vs F_μ is plotted and reveals an offset). This term is employed here as a correction that aids in reconciling the data, particularly within the Rayleigh-Plateau regime. The physical origin of this contribution remains unclear. Nevertheless, the fact that such a correction is required even under complete wetting suggests the presence of subtle interfacial mechanisms not accounted for in this model. A rigorous theoretical foundation for this additional force remains an open problem for future work. However, $2R$ expands with the wire diameter, yet remains nearly unaffected by the flow rate, as illustrated in Fig. 6.

This is further validated by the observed increase in velocity and wavelength when the system transitions back from the convective regime to the Rayleigh-Plateau regime. This transition suggests a force imbalance acting on the droplet. In the convective regime, inertial effects become significant, leading to a net reduction in the upward force on the droplet. The implication is that the Galilei number is important, defined as $Ge = F_g/F_\mu = \rho \Omega g \cos \theta / (T\mu v \lambda A_{\text{eff}}^2)$, which represents the ratio of gravitational to viscous forces. Here, increasing the flow rate leads to a substantial decrease in the wavelength λ , while other parameters remain approximately constant. We attribute this reduction to a substantial decrease in the upward viscous force, while the downward gravitational force remains largely unchanged. This results in a significant increase in the Galilei number, indicating a growing dominance of inertial effects in the convective regime.

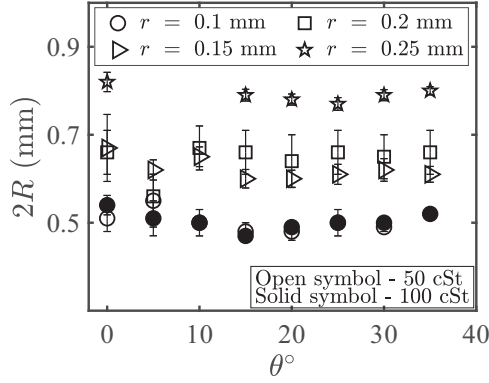


FIG. 6. Variation of the slender cylinder diameter ($2R$) is shown as a function of θ for all cases within the Rayleigh-Plateau regime.

Consequently, the force balance equation [Eq. (1)] for the Rayleigh-Plateau regime can be formulated as

$$\rho\Omega g \cos \theta = T\mu v \lambda A_{\text{eff}}^2 + 2\pi R\sigma. \quad (5)$$

In Fig. 7, we illustrate the force balance relationship (i.e., F_μ vs $[F_g - F_c]$) for four wire diameters and two different viscous fluids. The force balance equation [Eq. (5)] is indicated by the black line with $T = 1.4$. All data conform to the force balance equation and are even confirmed by analyzing the Grünig *et al.* [14] data where surface tension and viscosity of the fluids are different than

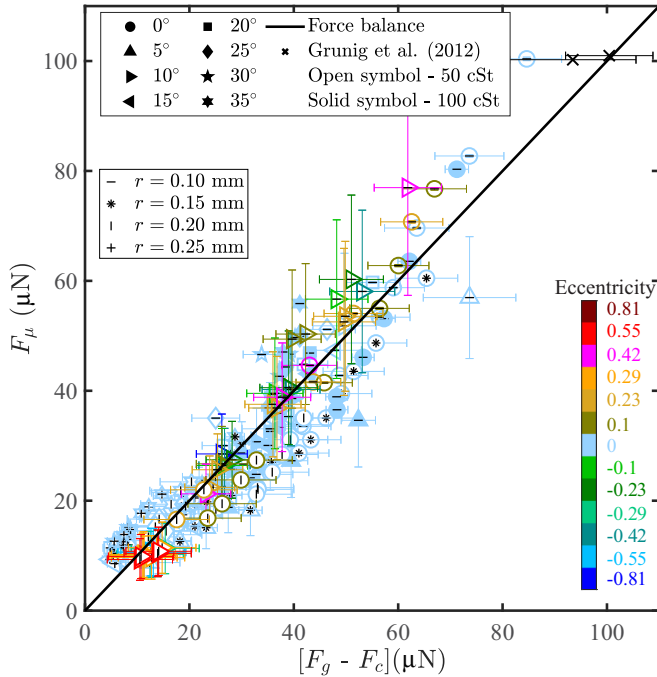


FIG. 7. Unified scaling law of Rayleigh-Plateau instability on a wire. Unified scaling for the force balance equation [Eq. (5)] across different wire radii and viscosities in the Rayleigh-Plateau regime.

current fluids. Unfortunately, the model is limited to the Rayleigh-Plateau regime, as it relies on constant values of λ and ν to derive empirical laws. In convective regimes, these parameters are not constant, and in the isolated regime, the entire wire is not wetted due to the presence of dry regions.

IV. CONCLUSION

In conclusion, our study has successfully demonstrated the significant impact of manipulating the Rayleigh-Plateau instability by altering both the angle of the nozzle-wire system and the eccentricity of the wire within the nozzle. The investigation revealed a strong dependence of the Rayleigh-Plateau instability regimes on angle and eccentricity. In general, angling the wire decreases the range and increases the average of the range of volumetric flow rate. Eccentricity affects mainly vertical wires where the Rayleigh-Plateau regime shrinks with increasing eccentricity. The effect of angle dominates over eccentricity when the wire is both angled and eccentric. Within the Rayleigh-Plateau regime noteworthy changes occur in velocity, wavelength, and bead volume.

Furthermore, to provide a comprehensive understanding, we conducted an empirical scaling analysis, comparing the forces associated with gravity, and viscosity on a single bead in the Rayleigh-Plateau regime. The outcome led to the formulation of a unified empirical viscous force law, which serves as a valuable contribution to the understanding of the intricate dynamics within only the Rayleigh-Plateau regime.

ACKNOWLEDGMENTS

We wish to thank Prof. William L. Roberts for the initial idea and inquiry, and Prof. Sigurdur Thoroddsen and his laboratory for letting us use their equipment. We also thank Atefeh Pour Karimi, Benoit Scheid, Spencer Truman, Abhijit Kumar Kushwaha, Nilamani Sahoo, John Bush, Jacco Snoeijer, and the referees for their fruitful discussions.

DATA AVAILABILITY

The raw and processed datasets generated during the study are not publicly archived due to their large size and storage constraints. Nevertheless, the data that support the findings of this article are available from the corresponding author upon reasonable request.

-
- [1] D. Quéré, Thin films flowing on vertical fibers, [Europhys. Lett. **13**, 721 \(1990\)](#).
 - [2] I. Kliakhandler, S. H. Davis, and S. Bankoff, Viscous beads on vertical fibre, [J. Fluid Mech. **429**, 381 \(2001\)](#).
 - [3] J. A. F. Plateau, *Statique Expérimentale et Théorique des Liquides Soumis aux Seules Forces Moléculaires*, (Gauthier-Villars, Paris, 1873), Vol. 2.
 - [4] L. Rayleigh, Xvi. on the instability of a cylinder of viscous liquid under capillary force, [Lond. Edinb. Dubl. Phil. Mag. J. Sci. **34**, 145 \(1892\)](#).
 - [5] F. Kreith and R. Boehm, *Direct-Contact Heat Transfer* (Springer-Verlag, Berlin, Heidelberg GmbH, 1988).
 - [6] A. Sadeghpour, Z. Zeng, H. Ji, N. Dehdari Ebrahimi, A. Bertozzi, and Y. Ju, Water vapor capturing using an array of traveling liquid beads for desalination and water treatment, [Sci. Adv. **5**, eaav7662 \(2019\)](#).
 - [7] H. Bai, J. Ju, R. Sun, Y. Chen, Y. Zheng, and L. Jiang, Controlled fabrication and water collection ability of bioinspired artificial spider silks, [Adv. Mater. **23**, 3708 \(2011\)](#).

- [8] J. Ju, H. Bai, Y. Zheng, T. Zhao, R. Fang, and L. Jiang, A multi-structural and multi-functional integrated fog collection system in cactus, *Nat. Commun.* **3**, 1247 (2012).
- [9] T. Gilet, D. Terwagne, and N. Vandewalle, Digital microfluidics on a wire, *Appl. Phys. Lett.* **95**, 014106 (2009).
- [10] T. Nozaki, N. Kaji, and Y. H. Mori, Heat transfer to a liquid flowing down vertical wires hanging in a hot gas stream: An experimental study of a new means of thermal energy recovery, in *International Heat Transfer Conference Digital Library* (Begell House Inc., Korea, 1998).
- [11] G. D. Wehinger, J. Peeters, S. Muzaferija, T. Eppinger, and M. Kraume, Numerical simulation of vertical liquid-film wave dynamics, *Chem. Eng. Sci.* **104**, 934 (2013).
- [12] K. Hattori, M. Ishikawa, and Y. H. Mori, Strings of liquid beads for gas-liquid contact operations, *AIChE J.* **40**, 1983 (1994).
- [13] J. Grünig, T. Skale, and M. Kraume, Liquid flow on a vertical wire in a countercurrent gas flow, *Chem. Eng. J.* **164**, 121 (2010).
- [14] J. Grünig, E. Lyagin, S. Horn, T. Skale, and M. Kraume, Mass transfer characteristics of liquid films flowing down a vertical wire in a counter current gas flow, *Chem. Eng. Sci.* **69**, 329 (2012).
- [15] H. Chinju, K. Uchiyama, and Y. H. Mori, “String-of-beads” flow of liquids on vertical wires for gas absorption, *AIChE J.* **46**, 937 (2000).
- [16] C. Wagstaff, M. Al-Juaied, D. Prabhudharwadkar, and W. L. Roberts, Wetted-wire columns: A potential alternative to packed or spray columns, *Rev. Chem. Eng.* **40**, 601 (2023).
- [17] L. B. Smolka, J. North, and B. K. Guerra, Dynamics of free surface perturbations along an annular viscous film, *Phys. Rev. E* **77**, 036301 (2008).
- [18] F. Boulogne, L. Pauchard, and F. Giorgiutti-Dauphiné, Instability and morphology of polymer solutions coating a fibre, *J. Fluid Mech.* **704**, 232 (2012).
- [19] C. T. Gabbard and J. B. Bostwick, Bead-on-fibre morphology in shear-thinning flow, *J. Fluid Mech.* **961**, A14 (2023).
- [20] C. Duprat, C. Ruyer-Quil, S. Kalliadasis, and F. Giorgiutti-Dauphiné, Absolute and convective instabilities of a viscous film flowing down a vertical fiber, *Phys. Rev. Lett.* **98**, 244502 (2007).
- [21] C. Duprat, C. Ruyer-Quil, and F. Giorgiutti-Dauphiné, Spatial evolution of a film flowing down a fiber, *Phys. Fluids* **21**, 042109 (2009).
- [22] C. T. Gabbard and J. B. Bostwick, Asymmetric instability in thin-film flow down a fiber, *Phys. Rev. Fluids* **6**, 034005 (2021).
- [23] Q. Xie, R. Liu, X. Wang, and X. Chen, Investigation of flow dynamics of thin viscous films down differently shaped fibers, *Appl. Phys. Lett.* **119**, 201601 (2021).
- [24] S. Haefner, M. Benzaquen, O. Bäumchen, T. Salez, R. Peters, J. D. McGraw, K. Jacobs, E. Raphaël, and K. Dalnoki-Veress, Influence of slip on the Plateau–Rayleigh instability on a fibre, *Nat. Commun.* **6**, 7409 (2015).
- [25] A. Sadeghpour, Z. Zeng, and Y. S. Ju, Effects of nozzle geometry on the fluid dynamics of thin liquid films flowing down vertical strings in the Rayleigh–Plateau regime, *Langmuir* **33**, 6292 (2017).
- [26] H. Ji, C. Falcon, A. Sadeghpour, Z. Zeng, Y. Ju, and A. Bertozzi, Dynamics of thin liquid films on vertical cylindrical fibres, *J. Fluid Mech.* **865**, 303 (2019).
- [27] H. Ji, A. Sadeghpour, Y. S. Ju, and A. L. Bertozzi, Modelling film flows down a fibre influenced by nozzle geometry, *J. Fluid Mech.* **901**, R6 (2020).
- [28] A. Pour Karimi, M. Rietz, W. Rohlf, B. Scheid, and R. Kneer, Experimental study of dripping, jetting and drop-off from thin film flows on inclined fibers, *Eur. Phys. J.: Spec. Top.* **232**, 443 (2023).
- [29] A. Pour Karimi, B. Hormann, H. Askarizadeh, W. Rohlf, B. Scheid, and R. Kneer, Drop-off characteristics of film flow on inclined fibers, *Eur. Phys. J.: Spec. Top.* **234**, 3153 (2025).
- [30] S. Eghbali, L. Keiser, E. Boujo, and F. Gallaire, Whirling instability of an eccentric coated fibre, *J. Fluid Mech.* **952**, A33 (2022).
- [31] B. Dejean, M. Meyer, and D. Rouzineau, Design and conception of an innovative packing for separation column—part I: Hydrodynamic study on wire intersections, *Chem. Eng. Res. Des.* **160**, 11 (2020).
- [32] R. Craster and O. Matar, On viscous beads flowing down a vertical fibre, *J. Fluid Mech.* **553**, 85 (2006).
- [33] More details are available on the webpage of the free software tracker (2016), <https://physlets.org/tracker/>.

- [34] Z. Zeng, A. Sadeghpour, and Y. S. Ju, Thermohydraulic characteristics of a multi-string direct-contact heat exchanger, *Int. J. Heat Mass Transf.* **126**, 536 (2018).
- [35] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/lmmx-fmtb> for videos..
- [36] G. G. Stokes, On the effect of the internal friction of fluids on the motion of pendulums, *Transactions of the Cambridge Philosophical Society* **9**, 8-106 (1851).