

Coherent structures driving broadband trailing-edge noise: Spanwise wavenumber selection and low-order modeling

Zhenyang Yuan ^{1,*} Simon Demange ² Kilian Oberleithner ²
André V. G. Cavalieri ³ and Ardeshir Hanifi ¹

¹*FLOW, Department of Engineering Mechanics, KTH Royal Institute of Technology, Stockholm, Sweden*

²*Laboratory for Flow Instability and Dynamics, Technische Universität Berlin, 10623 Berlin, Germany*

³*Divisão de Engenharia Aeronáutica, Instituto Tecnológico de Aeronáutica, São José dos Campos, Brazil*



(Received 11 April 2025; accepted 20 January 2026; published 16 March 2026)

We investigate the mechanisms underlying the generation of broadband trailing-edge noise from an airfoil, focusing on the role of hydrodynamic structures with nonzero spanwise wavenumbers and the development of a corresponding reduced-order model. A wall-resolved compressible large-eddy simulation (LES) of a NACA0012 airfoil at 3° angle of attack, Reynolds number $Re = 200\,000$, and Mach number $M = 0.3$ is performed. The simulation replicates the experimental setup of Demange *et al.* [AIAA Aviation Forum (2023)], including the tripping elements ensuring turbulent boundary layer development. The numerical domain has a large spanwise extent (43.75% of the chord) to allow for the investigation of large spanwise coherent structures. Validation against experimental data shows good agreement. The simulation results show that acoustic radiation from the trailing edge follows the scattering condition, whereby only hydrodynamic waves with spanwise wavenumbers lower than the acoustic wavenumber can generate propagative sound. Notably, the contribution from nonzero spanwise wavenumber modes becomes significant beyond the cut-on frequency at which the first nonzero spanwise acoustic mode becomes propagative. Through spectral proper orthogonal decomposition (SPOD), the associated flow structures are identified as streamwise-travelling wave packets convecting obliquely across the trailing edge. Reduced-order models based on acoustic-domain extended SPOD accurately reproduce the LES sound pressure levels within 2 dB, using only two modes per frequency–wavenumber pair. This compact representation paves the way for targeted noise-reduction strategies based on the control of the identified coherent structures.

DOI: [10.1103/kxbq-ynzd](https://doi.org/10.1103/kxbq-ynzd)

I. INTRODUCTION

Trailing-edge (TE) noise, also referred to as airfoil self-noise, is one of the main sources of noise in both aviation and wind energy applications and contributes significantly to overall noise pollution affecting communities near airports or wind farms. Therefore, accurate modeling of TE noise is essential for developing effective noise-reduction strategies. However, predicting and mitigating it remains challenging due to the complexity of the underlying flow–acoustic interaction. TE noise

*Contact author: zhenyang@kth.se

arises from the interaction between fluctuations of the flow field in the boundary layer and the geometric discontinuity at the airfoil edge [1], and is generally classified as either tonal or broadband. Tonal TE noise, often associated with laminar boundary layers at low to moderate Reynolds numbers, results from an acoustic feedback loop between the edge and upstream boundary-layer instabilities [2–6]. In contrast, broadband TE noise is linked to fully turbulent boundary layers, typically found in high-Reynolds-number flows, and is characterized by a continuous hump in the spectrum rather than discrete peaks. The present work focuses on the latter.

Theoretical prediction of TE noise has been largely based on models derived from acoustic analogies. Curle [7] obtained a solution to Lighthill’s analogy [8] including solid surfaces. In their seminal work, Ffowcs Williams and Hall [9] studied the role of surface edges by introducing a custom Green’s function for a semi-infinite flat plate. Their work demonstrated that turbulent fluctuations convecting past a sharp edge generate significant scattered sound. Alternatively, Amiet proposed a model based on the linearized hydroacoustic equations and the similarity rules of Graham [10], applicable to both leading- [11] and trailing-edge [12] noise. Assuming frozen turbulence composed of parallel gusts, Amiet’s formulation relates the surface pressure spectrum upstream of the edge to the far-field acoustics. Howe [13] reviewed these early works and proposed a unifying theory based on an enthalpy formulation, also employing a Green’s function. Since these foundational studies, numerous extensions have been introduced, for example, to account for finite-chord effects and off-midspan radiation [14] or to model serrated edges [15]. A more recent review by Lee *et al.* [16] provides a comprehensive overview of developments across theoretical, numerical and experimental approaches over the past few decades. The review also highlights the success of these approaches in practical applications.

However, these models quantify the strength of the noise sources, for instance, through spectra or coherence lengths. They do not, however, reveal which turbulent mechanisms or coherent structures generate the sound nor how these could be controlled to reduce it. It therefore remains unclear which of the coherent structures in the turbulent boundary layer are primarily responsible for noise generation. In Amiet’s theory, for instance, the parallel-gust representation is not connected to specific flow structures. While alternative approaches for source identification exist [17,18], we focus here on modal decomposition techniques to extract noise-relevant coherent structures from experimental and numerical datasets. In particular, proper orthogonal decomposition (POD) [19] and its spectral variant SPOD [20] can identify space-time coherent structures from flow realizations with their energetic contributions. The latter technique enabled the correlation of wave packet structures with both tonal [21,22] and broadband [23,24] airfoil noise in previous works. This was achieved under the restrictions of spanwise scales, which will be discussed below. In jet-noise research, such analyses have linked noncompact wave-packet structures to radiated sound, thereby establishing a direct flow mechanism-to-acoustics connection [25–27].

For airfoils, expressing these coherent structures in a spanwise Fourier basis, with wavenumber k_z , is particularly informative because the radiation efficiency of each mode is expected to depend on k_z [11]. Building on Amiet’s formulation, Roger and Moreau [14] derived a criterion for trailing-edge scattering, showing that only modes with k_z smaller than the acoustic wavenumber, k_0 , can efficiently radiate to the far-field. An equivalent relation was later derived by Nogueira *et al.* [28] in the context of installed jet noise, although neglecting mean-flow convection effects. In the present work, we refer to this threshold relation as the scattering condition and use it to interpret the radiation efficiency of the SPOD modes. Note that, according to this condition, only the zeroth spanwise wavenumber is expected to radiate at all frequencies.

Based on the previous observation, several studies have focused on identifying the coherent structures associated with the $k_z = 0$ acoustic radiation. Using large-eddy simulation (LES) datasets from Wolf *et al.* [29] and Tanarro *et al.* [30], Sano *et al.* [23], and Abreu *et al.* [24] applied SPOD to the $k_z = 0$ component of the flow field and identified noncompact wave-packet structures with strong correlations to the radiated acoustics. They further associated these dominant wave packets with the Orr mechanism, leading to Tollmien-Schlichting-like waves embedded within the turbulent boundary layer. However, in the simulations of trailing-edge noise mentioned previously,

the spanwise domain width is much smaller than the acoustic wavelength over a broad frequency range, often on the order of 10% of the chord length [29–31]. Under these conditions, only the $k_z = 0$ mode can radiate, and recovering the sound from an equivalent wider configuration requires corrections to the two-dimensional (2D) acoustic spectrum [32,33] or a three-dimensional acoustic analogy [34]. While such techniques can reconstruct an equivalent three-dimensional (3D) acoustic field for prediction purposes, they do not provide access to the coherent flow structures at nonzero k_z , which requires wider domains. For instance, Ribeiro and Wolf [21] performed a three-dimensional SPOD analysis of the turbulent boundary layer from a LES dataset and recovered significant energetic content at nonzero k_z ; however, with a domain width of only 10% chord, no propagative nonzero modes could be observed.

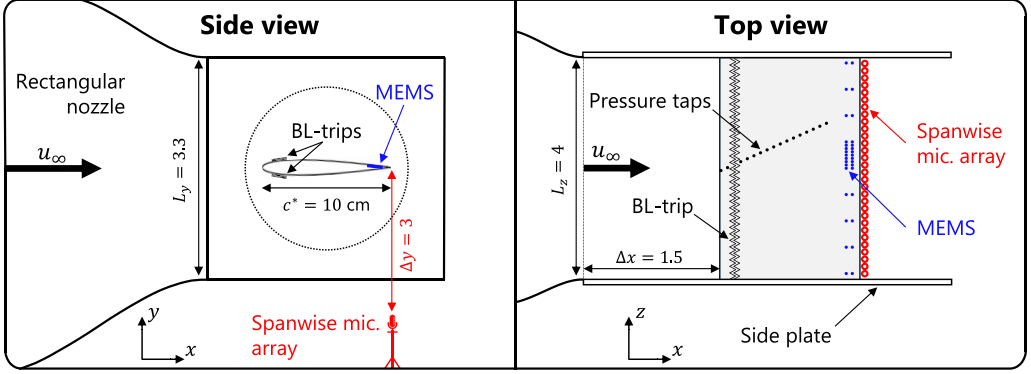
Experimental investigations have also provided insight into the role of nonzero spanwise wavenumbers. In tonal-noise experiments on a controlled-diffusion airfoil, Jaiswal *et al.* [35] used POD of Tomo-PIV data to extract the coherent structures in the turbulent boundary layer associated with the noise. Notably, they observed a modulation between $k_z = 0$ and $k_z > 0$ modes influencing the acoustic radiation, concluding that such wavenumbers should be accounted for in effective noise-reduction strategies. In our recent work with broadband noise [36], a SPOD analysis of the acoustic field measured with a spanwise microphone array revealed dominant spanwise Fourier modes consistent with the scattering condition discussed above, including significant acoustic content at nonzero k_z . However, due to the limited resolution of the surface-pressure sensor array, the corresponding radiating coherent structures in the turbulent boundary layer could not be resolved. Consequently, fundamental questions about the noise generated at higher spanwise remains unanswered.

The objective of the present study is to (i) investigate the contribution of nonzero k_z to acoustics, (ii) find the correlated flow structures for nonzero k_z , and (iii) assess if we can make a reduced order model of these structures and acoustic field. To this end, we perform a wall-resolved, compressible large-eddy simulation of a NACA 0012 airfoil at Mach number $M = 0.3$, chord-based Reynolds number $Re = 2 \times 10^5$, and angle of attack $\alpha = 3^\circ$. Both airfoil sides are subject to a tripped turbulent boundary layer. The computational domain has a large spanwise extent, exceeding the acoustic wavelength over a broad frequency range and thus allowing multiple propagative nonzero k_z modes to be captured. Flow and acoustic fields are sampled at high resolution in space and time, enabling SPOD in a spanwise Fourier basis and frequency–wavenumber spectral analysis. The LES flow field and spanwise modal decomposition of the acoustic field are compared directly to experimental measurements from Demange *et al.* [36], providing a consistent basis for interpretation. The resulting analysis forms the basis for identifying flow features that could be targeted by noise-reduction strategies.

The structure of this paper is as follows: Sec. II describes the numerical setup and its relation to that of the experiments. Section III provides an in-depth comparison between the numerical and experimental datasets and the flow visualization over the airfoil. The comparison includes statistics, spectral analysis of the acoustic and hydrodynamic datasets, and addressing the scattering condition. The low-order modeling is presented in Sec. IV, including acoustic and hydrodynamic based extended SPOD (ESPOD). The analysis focuses on the identification of coherent structures that generate acoustics and the acoustic-based SPOD modeling for the acoustic prediction. Finally, Sec. V presents the conclusions and perspectives of this work.

II. NUMERICAL SIMULATION

The simulation is designed to replicate the experimental setup described in Demange *et al.* [36], enabling direct validation of the LES results against experimental data. This section first summarizes the experimental setup (see Demange *et al.* [36] for full details) and then describes the LES configuration, highlighting the differences and similarities between the two.


 FIG. 1. Sketch of the experimental setup from Demange *et al.* [36] (not to scale).

A. Experimental condition and airfoil model

A sketch of the experimental setup in the large anechoic chamber of TU Berlin is shown in Fig. 1, with the Cartesian coordinate system (x, y, z) defining the streamwise (horizontal), vertical, and spanwise directions, respectively. In the following, the “.” subscript is used to denote dimensional quantities, while quantities with subscript are expressed in nondimensional form. The airfoil chord length, $c^* = 100$ mm, is used as reference length. The freestream is provided by a rectangular nozzle of section $L_y \times L_z = 3.3 \times 4.0$. The airfoil investigated is a modified NACA 0012 profile with a rounded trailing edge of radius $r_{TE} = 0.004$. The model, of spanwise length $L_z = 4$, is held between two vertical side plates, and its leading edge is positioned at a distance $\Delta x = 1.5$ downstream of the nozzle exit section. To investigate broadband trailing-edge noise, a tripping device (a zigzag shaped tape) with a height of $h_{trip} = 0.0042$, an angle of 60° , and a streamwise extend of $l_{trip} = 0.083$ is placed at $x = 0.05$ from the leading edge on both the pressure and suction sides. The experimental campaign considered freestream velocities within $u_\infty^* \in [15, 70]$ ms^{-1} , resulting in freestream Mach numbers $M = u_\infty^*/a_0^* \in [0.044, 0.2]$, where a_0^* is the ambient speed of sound, and to chord-based Reynolds numbers $\text{Re} = u_\infty^* c^*/\nu_\infty^* \in [1, 4.6] \times 10^5$, where ν_∞^* is the kinematic viscosity of air.

Two sets of experimental data are utilized for the present comparisons with LES: acoustic measurements from a horizontal microphone line array (red in Fig. 1) located three chord lengths below the TE, and unsteady surface-pressure measurements from MEMS microphones (blue in Fig. 1) integrated into the airfoil model. The horizontal line array consisted of 40 GRAS 40PL-1 SHORT CCP microphones with a uniform spanwise spacing of $\Delta z = 0.1$ positioned outside the jet flow. The MEMS array comprised two spanwise lines of 16 TDK T4086 microphones each, located at $x = 0.88$ and $x = 0.92$. The outer eight sensors in each line were spaced by $\Delta z = 0.5$ to capture large-scale pressure fluctuations spanning the full airfoil width, while the remaining eight near midspan were spaced by $\Delta z = 0.02$ to resolve fluctuations scaling with the boundary-layer thickness and to estimate spanwise coherence. Both arrays were synchronously sampled over 120 s at 51.2 kHz. In addition, velocity-magnitude profiles, $|\mathbf{u}| = \sqrt{u^2 + v^2 + w^2}$, were measured using a hot-wire probe with a single wire at $x \approx 0.95$ in the suction-side boundary layer and at several streamwise positions in the airfoil wake.

B. Solver, mesh, and setups

The open-source numerical framework for high-order flux reconstruction PyFR [37] is used for the wall-resolved implicit large eddy simulation (iLES). The compressible Navier-Stokes equations are solved in their conservative format for flow quantities $\mathbf{q} = [\rho, \rho u, \rho v, \rho w, E]$, where ρ is the density, $[u, v, w]$ are the components of the velocity vector $\mathbf{u}$, and $E = \frac{p}{\gamma-1} + 0.5\rho|\mathbf{u}|^2$.

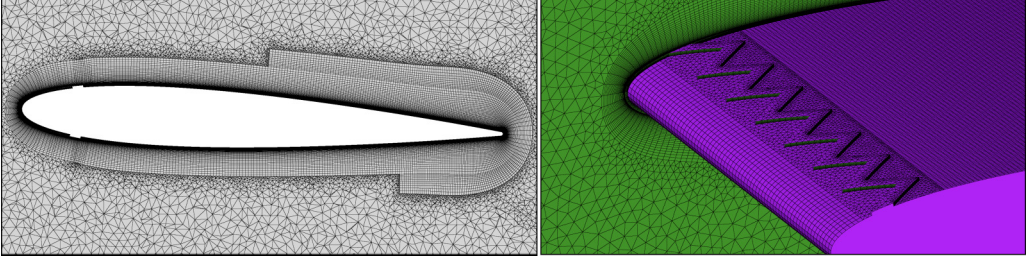


FIG. 2. High order computational mesh elements near the airfoil. Left: The wall-resolved boundary layer mesh and the near field acoustic mesh region. Right: A close look at surface mesh around tripping elements which has been converted to second order to fit the curvature of the geometry. Degrees of freedom inside the elements are skipped for better visualization.

The ideal gas law is also used to complete the set of equations and Sutherland's law is used for the viscosity correction. The discrete set of equations is obtained from the spatial-spectral discretisation, which first decomposes the computational domain into a set of mesh elements and uses high-order polynomials within each element to represent the flow fields. The equations are then integrated in time using an adaptive six-stage Runge-Kutta method.

We use the same airfoil geometry as in the experiments, including the zigzag tripping elements. The advantage of using these physical surface tripping elements is that the position of transition to turbulence is fixed, making the results less sensitive to the low level of inflow turbulence in the experiments (turbulence intensity = 0.2% in the experiments at freestream velocities of 30 m/s [38]), which is ignored in the LES. Further, compared to conventional numerical tripping technique using volume forcing [39], the noise generated by physical tripping elements could be more closely compared to the experiments.

Due to computational cost and practicality, the numerical simulation targets the experimental case with Reynolds number $Re = 2 \times 10^5$, which limits the element count of the mesh, and Mach number $M = 0.3$ to accelerate the convergence of the simulation. Note that the corresponding experimental case has a Mach number of $M = 0.088$. As mentioned in the introduction, the far-field acoustics can be affected by the scattering condition [28]. Therefore, we use a spanwise length of $L_z = 0.4375$, which fits seven periods of the zigzag tripping elements in the LES. This span width ensures that a sufficient number of propagative acoustic waves can appear in the simulation.

A significant difference between LES and experiments is the effect of installation of the model in the wind tunnel. To keep the computational costs reasonable, in the simulation, the airfoil is placed in a uniform clean flow where the installation effect is not considered. We are aware of possible differences this may cause, however, as will be demonstrated in Sec. III, good agreement between experimental and numerical results is achieved.

PyFR supports both structured and unstructured mesh topology. To take advantage of this capability, we adopt the meshing strategy illustrated in Fig. 2. A wall-resolved O-grid with the maximum resolution at the midchord is given in terms of wall units: $\Delta x^+ < 10$, $\Delta z^+ < 5$, and $y^+ < 0.9$. At the trailing edge, resolution in the streamwise direction is kept as $\Delta x^+ < 6$ and $y^+ < 0.6$. A structured mesh is used in the near wall field and wake resolved region until 1.2 chords after the trailing edge. To maintain resolution and avoid grid stretching near the tripping region, an unstructured mesh with refined prism layers is used.

An unstructured mesh is also used in the far field to avoid elements with large aspect ratios and reduce computational cost. The mesh size in the acoustic field above the trailing edge ($x = 1$) is set to $\Delta = [0.02, 0.03, 0.10]$ at the vertical positions $y = [0.25, 1.0, 5.0]$, respectively. The aspect ratio of the elements is kept close to one. Analysis of the simulation data shows that the acoustic waves are well resolved for frequencies up to $St = 30$ within $|y| < 5$. Note that the Strouhal number is defined based on the freestream velocity and chord length $St = f^* c^* / u_\infty^*$. Acoustic data obtained

for $y > 10$ are not considered in the following analysis. The computational domain is extended to 20 chords away from the airfoil surface to reduce the influence of the outflow boundary conditions. The configuration yields a total grid number of approximately 500×10^6 .

A designed sponge zone of seven chords length is added around the outer boundaries to absorb hydrodynamic wake and acoustic waves [40,41]. The sponge is added to the system of equations as a forcing term in the form $-\sigma(q - q_\infty)$. The sponge strength parameter σ is carefully chosen so no significant wave reflection is observed. The outflow boundaries use a nonreflecting Riemann invariant boundary condition to work with sponges, which ensures that a nonreflecting condition is provided. The inflow boundary also uses a Riemann invariant condition to provide a homogeneous potential flow with $M = 0.3$, ambient density, and pressure. A periodic boundary condition is employed in the spanwise direction. Additionally, a nonslip, adiabatic wall condition is applied to the airfoil surface.

C. Data sampling

The simulation is conducted on LUMI-G AMD MI250X 128Gb GPU nodes. A total of 7813 snapshots with a sampling rate of $St = 266.7$ are stored and employed for spectral analysis, ensuring that aliasing from high frequencies has a negligible impact on the spectrum. The full numerical dataset amounts to approximately 200 TB.

To enable direct comparison with the measurements outlined in Sec. II A, additional probe points have been integrated within the LES. Two lines of surface probes, located at the same chordwise positions as the MEMS array in the experiments ($x = 0.88$ and $x = 0.92$) are used, with 187 regularly spaced sensors along the span, resulting in a spanwise resolution of $\Delta z \approx 2.54 \times 10^{-3}$. For the acoustic field, several lines of sensors are positioned at vertical distances $\delta y = [-3.0, -1.0, +1.0, +2.0, +3.0]$ from the TE. Each array contains 60 sensors, resulting in a spanwise resolution of $\Delta z \approx 7.92 \times 10^{-3}$. All probes use the same sampling frequency as the flow snapshots.

III. VALIDATION OF LES WITH EXPERIMENTS

A. LES flow topology

We first examine the LES flow topology and assess the spanwise homogeneity of the turbulent boundary layer near the TE. To characterize the boundary layer, Fig. 3 shows instantaneous Q -criterion isosurfaces together with a map of skin friction of the time-averaged velocity field on the suction side of the airfoil. The skin-friction coefficient is defined as

$$c_f = \frac{\overline{\tau_w}}{0.5\rho_\infty u_\infty^2}, \quad (1)$$

where the overline ($\overline{\cdot}$) indicates time-averaged quantities and $\overline{\tau_w}$ is the time-averaged wall shear stress. The visualizations show that transition to turbulence is not uniform along the spanwise direction downstream of the zigzag trips. Instead, turbulent streaks originate at the downstream corners of the trip elements and are separated by streamwise-elongated regions of laminar flow, which appear as low- c_f bands on the wall map. Further downstream, the turbulent streaks merge, forming a fully turbulent boundary layer that still exhibits a residual spanwise modulation. Owing to the positive angle of attack and the pressure gradient, this streaky organization persists over a longer chordwise distance on the pressure side than on the suction side. Nonetheless, a visual inspection of both the Q criterion and the c_f distribution indicates the disappearance of persistent spanwise organisation and the establishment of an essentially spanwise-homogeneous turbulent boundary layer near the TE.

To further assess the development of spanwise homogeneity, Figs. 4(a)–4(c) present mean boundary-layer profiles extracted at $x = [0.2, 0.5, 0.9]$ and at three representative spanwise positions: a position of the downstream corner of the zigzag trip, a position of the upstream corner,

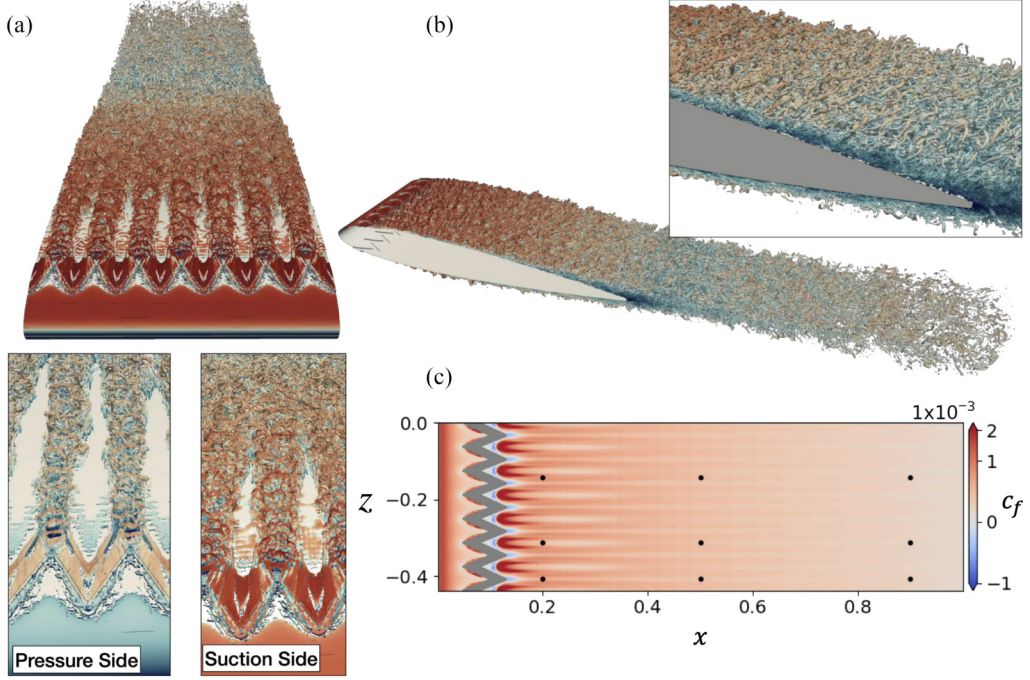


FIG. 3. Instantaneous isosurfaces of the Q criterion ($Q = 0.001$) colored by streamwise u velocity. (a) Tripping elements triggering transition. (b) Fully developed turbulence close to the trailing edge and wake region. (c) Time-averaged skin friction coefficient, c_f , over the suction side of the airfoil. The color map displays the negative and positive values of c_f as blue and red, respectively.

and a location between these two positions. These sampling points correspond to the black dots marked on the c_f map (see Fig. 3). At $x = 0.2$, the three profiles differ noticeably: the streak center exhibits a S-shaped profile with two inflection points, consistent with two shear layers, while the region aligned with the upstream corner shows a thin laminar boundary layer profile. Further downstream, at $x = 0.5$, these differences have already diminished, and by $x = 0.9$ the three profiles collapse onto one another. Figure 4(d) shows the power spectral density (PSD) from the spanwise Fourier transform of the wall pressure fluctuations $\tilde{p}$ at three streamwise locations. At the upstream stations, the spectrum exhibits a distinct peak corresponding to the zigzag tripping wavelength of $L_z/7$. Further downstream, at $x = 0.9$, the hydrodynamic pressure fluctuations associated with the streaks exhibit significantly lower amplitude compared to upstream locations, and the corresponding spectrum becomes increasingly broadband. This progressive convergence demonstrates that the turbulent boundary layer becomes spanwise homogeneous as the trailing edge is approached.

B. Comparison of boundary layer and wake development

To compare the mean-flow development in the LES with experiments, Fig. 5(a) presents time-averaged wall normal velocity-magnitude profiles, $|\overline{\mathbf{u}}|$, extracted from the LES at chordwise positions $x = [0.55, 0.65, 0.85, 0.95]$ in the midspan plane. This plane corresponds to the center of the turbulent streaks discussed in Sec. III A. The velocities are normalized by the local boundary-layer edge value $|\overline{\mathbf{u}}_e|$. Hot-wire measurements were conducted at $x = 0.95$ in the experiment. The LES profile evaluated at the same position agrees closely with these data, indicating that the boundary-layer thickness and shape are well captured by the simulation.

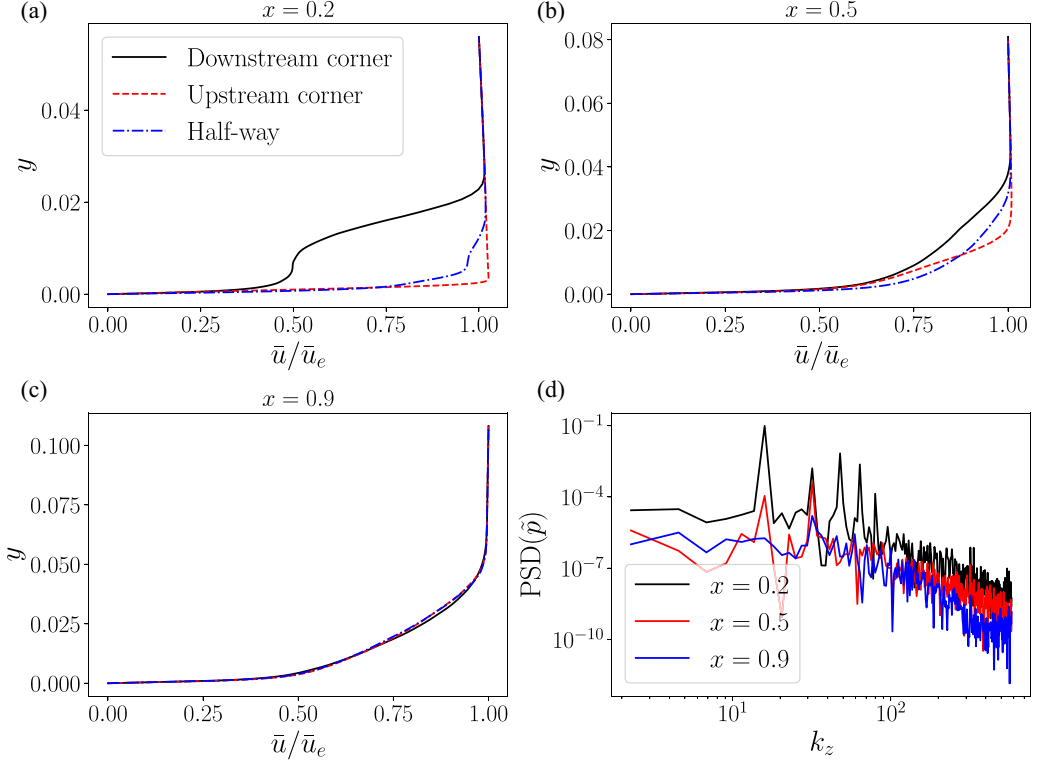


FIG. 4. (a)–(c) Mean velocity profiles obtained at the streamwise/spanwise locations marked as black dots in Fig. 3, corresponding to the position of the downstream corner of the zigzag trip, a position of the upstream corner, and a location halfway in between. The velocity is normalized by the local boundary-layer edge value $\bar{u}_e$. (d) PSD of the spanwise Fourier components of $\bar{u}$ at three streamwise locations.

Figure 5(b) presents the mean velocity profiles in inner scaling. The velocity is normalized by the friction velocity, $u_\tau^* = \sqrt{\tau_w^*/\rho^*}$, and the wall-normal coordinate is scaled by the viscous length scale, $y^+ = y u_\tau^*/\nu^*$. In these units, the velocity distributions collapse onto the canonical structure of a turbulent boundary layer, with a linear region close to the wall and a logarithmic region farther out. The good agreement with the log-law reference, $u^+ = 1/\kappa \ln y^+ + B$, with $\kappa = 0.385$ and $B = 5.2$ [42,43],

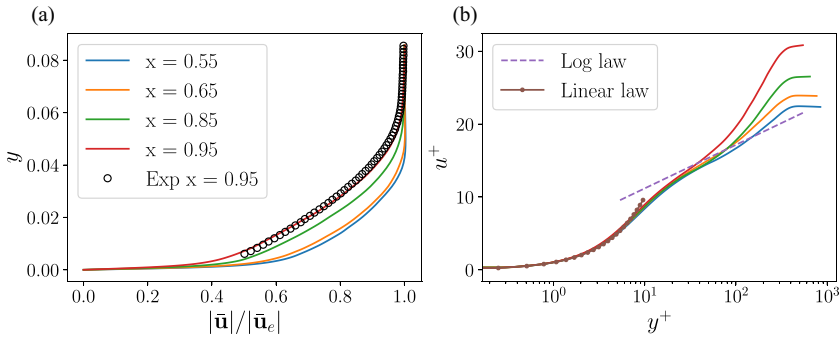


FIG. 5. Mean velocity profile scaled with (a) outer and (b) inner units at the streamwise stations $x = [0.55, 0.65, 0.85, 0.95]$. The log law is defined as $u^+ = 1/\kappa \ln y^+ + B$, where $\kappa = 0.385$ and $B = 5.2$.

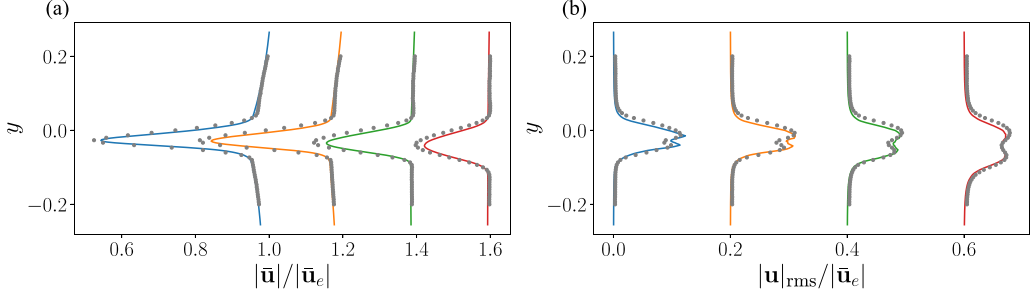


FIG. 6. (a) Mean and (b) RMS wake velocity profiles measured at $x = [1.1, 1.2, 1.5, 2.0]$, corresponding to the blue, yellow, green, and red solid lines. The experimental data is showed in grey dots. The velocity profiles are shifted by $\Delta u = 0.2$ for downstream locations.

indicates that the LES mesh resolution is sufficient to yield a physically consistent wall stress and to reproduce the expected mean near-wall structure. A mismatch is observed in the velocity profiles at the downstream locations, which can be attributed to pressure gradient effects. Similar observations have been reported and discussed in the numerical study by Wolf *et al.* [29] and the experimental work by Lee and Kang [44]. Experimental profiles cannot be shown in inner scaling due to the absence of wall-stress data.

Finally, Figs. 6(a) and 6(b) compare mean and turbulence-intensity profiles in the near wake between LES and hot-wire measurements. All profiles are normalised by the freestream velocity, and for clarity are horizontally offset by $\Delta u = 0.2$. The turbulence intensity is given by the root mean square (RMS) of the velocity fluctuations measured by the single-wire probe. The LES reproduces the experimental wake width and turbulence levels with good fidelity. A minor underprediction of the wake-center velocity is observed and attributed to numerical diffusion introduced by mesh coarsening downstream. Given the limited acoustic relevance of the wake compared with the TE boundary layer, further refinement in this region was not pursued. Overall, the similarity between LES and experiment demonstrates that the wake is represented with sufficient accuracy to support the subsequent surface-pressure and acoustic analyses.

C. Comparison of surface pressure fluctuations

To enable a fair comparison between LES and experimental datasets, several differences in sensor configuration and data processing must be addressed [45]. In the experiments, the MEMS microphones have a diameter of $\phi_{\text{MEMS}} = 0.002$ and a spanwise spacing of $\Delta z = 0.02$, whereas in the LES the probes are strictly pointwise with a much finer spacing of $\Delta z = 0.00024$. To mimic the experimental conditions, the LES signals are reprocessed by spatially averaging over the equivalent sensor area and by matching the experimental sensor positions when computing the CSD. Results are presented both for the unprocessed pointwise LES signals, termed pointwise, and for the postprocessed signals, termed sensor-averaged.

We begin with the comparison of turbulent wall pressure spectra from single locations near the TE. Figure 7(a) shows the PSD of surface-pressure fluctuations obtained at $x = 0.88$ and $x = 0.92$ on the suction side. Sensor-averaged LES data is presented. The PSDs are normalized by the dynamic pressure, $0.5\rho^*u_\infty^{*2}$. The LES spectra collapse well with the experimental data up to $St \approx 30$, which covers the broadband-noise frequency range. The high-frequency mismatch is attributed to the limited LES grid resolution and the restricted frequency range of the sensors.

Since the spanwise coherence of surface-pressure fluctuations is a key parameter in trailing-edge noise generation [11,13,16], we evaluate it from the cross-spectral density (CSD) matrix of the surface-pressure signals. Entries of the CSD for pressure fluctuations $\hat{p}$ in the frequency domain,

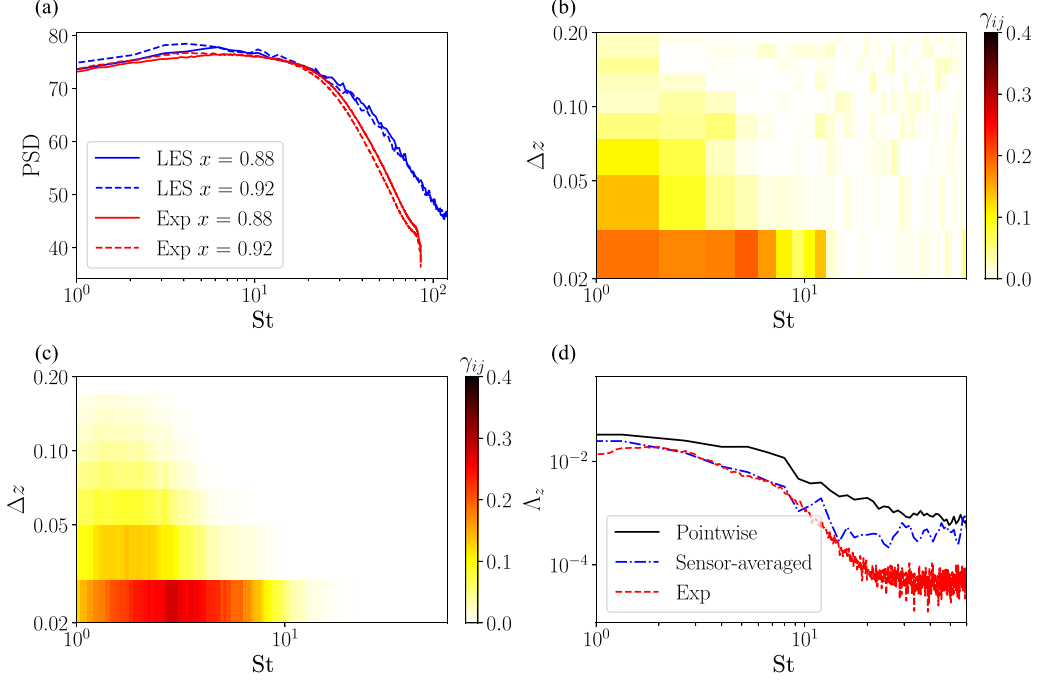


FIG. 7. Comparison of SPFs: (a) PSD of the SPFs scaled by $0.5\rho_\infty^* u_\infty^{*2}$ at $x = 0.88$ and $x = 0.92$ and in the midplane from LES and experiments. Spanwise coherence (γ_{ij}) at $x = 0.88$ from (b) experiments and (c) sensor-averaged LES. (d) Spanwise coherence length (Λ_z) from experiments and pointwise, sensor-averaged LES data, also at $x = 0.88$.

corresponding to the i th and j th sensors, are defined as

$$C_{ij} = E\{\hat{p}_i(\omega), \hat{p}_j(\omega)^H\}, \quad (2)$$

where the superscript H denotes the complex conjugate. The CSD is approximated using the Welch method [46], in which the signal is divided into N overlapping segments, a Hanning windowing function is applied to each segment, and the segments are then transformed into the frequency domain using the fast Fourier transform. For each discrete frequency, the complex-valued pressure values are then averaged over all segments:

$$C_{ij}(\omega) = \frac{1}{N} \sum_{k=1}^N \hat{p}_{i,k}(\omega) \hat{p}_{j,k}(\omega)^H. \quad (3)$$

From the CSD, two derived quantities are obtained: the spanwise coherence between sensor pairs and the associated coherence length. The spanwise coherence between two sensors separated by Δz is defined as

$$\gamma_{ij}(\omega) = \frac{|C_{ij}(\omega)|}{\sqrt{P_{ii}(\omega) \times P_{jj}(\omega)}}, \quad (4)$$

where P_{ii} and P_{jj} are the power spectral density estimates at each sensor. Finally, the frequency-dependent coherence length is obtained by integration over the spanwise separations resulting from all possible sensor pairs, employing a trapezoidal rule [47]:

$$\Lambda_z(\omega) = \int_0^\infty \gamma(\omega, \Delta z) d\Delta z. \quad (5)$$

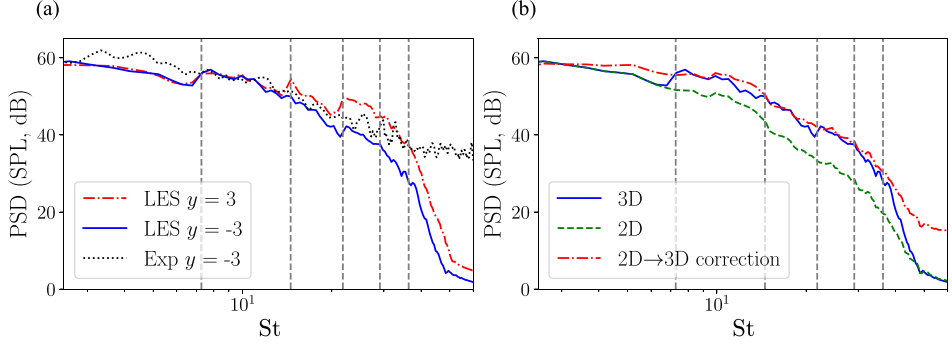


FIG. 8. (a) PSD from the midspan sensor in the acoustic line array positioned at $x = 1$ and $y = [+3, -3]$ in the LES and $y = -3$ in the experiment. The pressure fluctuations are normalized by M^5 . The vertical dashed grey line indicates scattering condition limited by airfoil spanwise length in LES. (b) Comparison of two-dimensional (2D), three-dimensional (3D), and 2D-to-3D corrected acoustic spectra obtained from LES at $y = -3$. The 2D spectrum is computed using spanwise-averaged acoustic signals, and the 2D-to-3D correction is applied following Eq. (7).

Because the LES employs periodic spanwise boundary conditions, the spanwise integration for the coherence length is restricted to half the numerical domain width, while the full span is considered in the experiments. Furthermore, the limited duration of the LES time series complicates the estimation of low-frequency content when using a standard Welch method with fixed bin sizes. To overcome this, each frequency is treated independently by selecting a bin size that includes at least four periods of the inspected frequency. This adaptive strategy enhances the resolution of low frequencies while reducing excessive averaging at high frequencies.

Figures 7(b) and 7(c) show the spanwise coherence obtained from the experiment and from the sensor-averaged LES data. In both cases, the coherence is concentrated at low frequencies ($St \leq 10$) and decays with increasing spanwise separation. The LES and experimental results are in close agreement up to $St \approx 15$, with the experimental curves appearing smoother due to sensor-averaging effects, which suppress the high-frequency fluctuations resolved in the LES. The corresponding coherence lengths, shown in Fig. 7(d), exhibit a maximum of about $\Lambda_z \approx 2 \times 10^{-2}$ and a decay for $St > 1$. At the lowest frequencies ($St \rightarrow 1$), the experiment reveals a decrease in coherence length that is less apparent in the LES because of its limited frequency resolution. Overall, the comparison demonstrates that the LES reproduces the spanwise correlation features of surface-pressure fluctuations consistently with the experimental data.

D. Acoustic field comparison and spanwise wavenumber selection

We begin by comparing the far-field acoustic spectra obtained from LES and experiments. Figure 8(a) presents the PSD of pressure fluctuations, expressed in sound pressure level (SPL), measured by the midplane microphones of the spanwise array. These microphones are located at $x = 1, y = -3$ in the experiment (see Fig. 1), and the LES data is extracted from the corresponding location. The SPL is expressed in decibels (dB) as

$$\text{PSD}(\text{SPL, dB}) = 10 \log_{10} \left(\frac{P_{xx}}{p_0^2} \right), \quad (6)$$

where $p_0 = 2 \times 10^{-5}$ Pa is the reference sound pressure, and P_{xx} denotes the PSD of the pressure fluctuations. The spectra are normalized using the classical trailing-edge noise Mach-number scaling, M^5 [9,48], to account for differences in freestream Mach numbers between datasets. Previous experimental work by Demange *et al.* [49] confirmed that this scaling effectively collapses the spectra in the broadband regime $3 \leq St \leq 13$, across different freestream velocities. Overall, good

agreement is observed between the LES and experimental results up to $St \approx 30$, which corresponds to the mesh cutoff frequency in the simulation.

Additional LES spectra are obtained in the acoustic field on the suction side at $y = 3$. Differences between the spectra on the suction and pressure sides are clearly observed. As discussed in Appendix E, these discrepancies are attributed to non-trailing-edge sound sources, which increase the sensitivity to spurious contributions in the acoustic field, such as pressure fluctuations originating from the tripping device. It is also worth noting that below $St \leq 14$, the spectra from the two sides collapse, indicating the same PSDs on both sides of the airfoil.

Figure 8(b) presents a comparison between the 2D, 3D, and 2D-to-3D corrected acoustic spectra obtained from the LES at $y = -3$. The 2D spectrum is computed by spanwise averaging of the acoustic pressure signals, whereas the 3D spectrum corresponds to the signal recorded at a single midspan sensor. The 2D-to-3D spectral correction follows the formulation by Oberai *et al.* [50], given by

$$\hat{p}_{3D} = \hat{p}_{2D} \frac{1-i}{2} \sqrt{\frac{k_0}{\pi r}}, \quad (7)$$

where $\hat{p}_{2D}$ and $\hat{p}_{3D}$ represent the 2D and 3D spectral pressure fluctuations, respectively. $k_0 = k_0^* c^*$ is the acoustic wavenumber and r is the distance to the TE. This correction has been successfully applied in previous studies involving simulations with narrow spanwise domains [32,33,51], where only 2D pressure waves are permitted to propagate. In the present case, the corrected spectrum agrees well with the directly computed 3D spectrum, confirming that the simulation captures a rich set of acoustic waves with non-zero spanwise wavenumbers, whose contributions to the acoustic field are significant.

To interpret these spectra, we recall the trailing-edge scattering condition [11,14,28], which predicts the spanwise wavenumbers that radiate to the far field. In the formulation given by Roger and Moreau [14], efficient radiation occurs only if the spanwise wavenumber is smaller than the acoustic wavenumber,

$$k_z < \frac{k_0}{\beta} \quad (8)$$

where $\beta = \sqrt{1-M^2}$ accounts for the convection effect associated with a uniform flow of Mach number M . For the LES Mach number, $\beta = 0.954$. Under the convention used in this paper, the acoustic wavenumber is directly related to the Helmholtz number by $k_0 = He = 2\pi StM$, so the inequality can equivalently be expressed in terms of He . The detailed derivation of the scattering condition is given in Appendix A.

Note that the Helmholtz number at which non-zero spanwise wavenumbers start to radiate depends directly on the spanwise extent of the domain. For a finite span L_z , the admissible wavenumbers are discrete, $k_z = 2\pi n_z/L_z$ with $n_z = 0, 1, 2, \dots$, and the first nonzero mode ($n_z = 1$) becomes propagative when $He \geq 2\pi/(\beta L_z)$ according to the scattering condition (8). For the present LES span ($L_z = 0.4375$), this corresponds to $He \geq 14.36$, while the experimental span ($L_z = 4$) gives a much lower threshold of $He \gtrsim 1.6$. By contrast, the narrower spans often used in the literature ($L_z \approx 0.1$) [17,29,30,52] yield $He \gtrsim 65$, such that TE noise is effectively two-dimensional, with only the $n_z = 0$ mode contributing over the practically relevant frequency range.

In practice, the spanwise decomposition of the acoustic field is performed with a spanwise Fourier transform of the CSD matrix. For the LES, the use of a spanwise Fourier transform is directly supported by the periodic boundary conditions, and further justified by the spanwise homogeneity of the turbulent boundary layer near the trailing edge discussed in Sec. III A. Despite the lack of spanwise periodicity in the experimental setup due to the solid side plates, the eigen decomposition of the CSD from the spanwise array sensors indicates that Fourier modes provide an adequate representation of the experimental acoustic field. The corresponding analysis is shown in Appendix B.

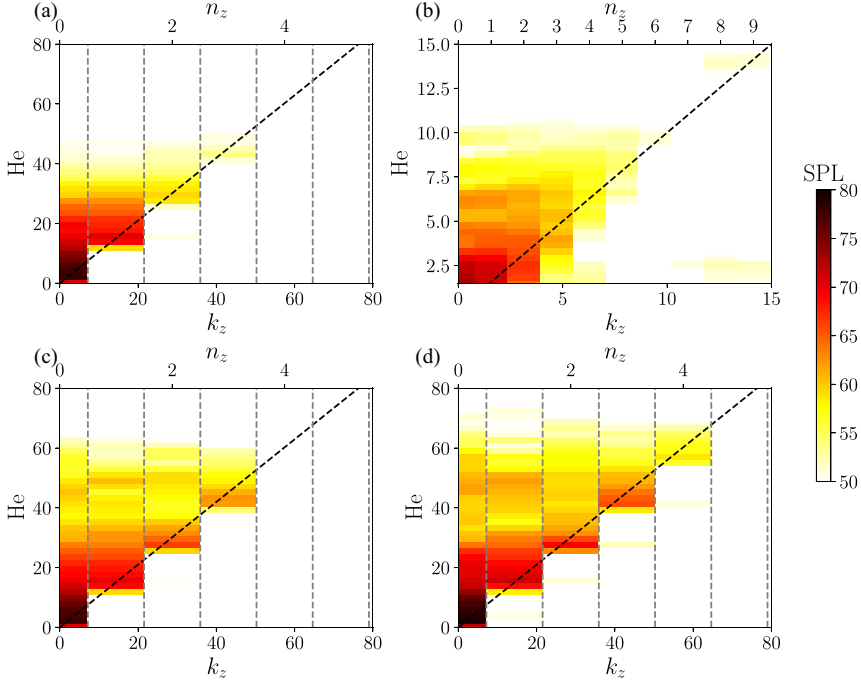


FIG. 9. Frequency-wavenumber spectrum (presented in SPL with dB) based on spanwise Fourier transform of CSD matrix (with respect to the middle sensor) of acoustic line array. (a) LES, $y = -3$, (b) experiment, $y = -3$, (c), (d) LES, $y = -1$ and 1. Vertical dashed line indicates scattering condition for each spanwise wavenumber. The dual x axis indicates spanwise wavenumber $k_z = 2\pi n_z/L_z$, $n_z = 0, 1, 2, \dots$. The black dashed line indicates acoustic wavenumber $k_0 = \text{He}$.

Figure 9 presents the frequency-wavenumber spectra obtained from the acoustic line arrays, displayed in the (He, k_z) plane. The spectra are computed from the CSD between the mid-span microphone and all other sensors in the array. Figures 9(a) and 9(b) show the results at $y = -3$ from the LES and experiment, respectively, while Figs. 9(c) and 9(d) present the LES results in the nearfield ($y \pm 1$). In each figure, the dashed black line indicates the scattering condition (8), and the vertical dashed lines mark the cut-on Helmholtz numbers for the successive non-zero spanwise modes ($n_z = 0, 1, 2, \dots$).

The comparison between experiment and LES at $y = -3$ reveals good agreement overall. In both cases, the acoustic energy is concentrated above the scattering line, consistent with the theoretical condition, and successive bands of energy appear at Helmholtz numbers corresponding to the cut-on thresholds of the discrete spanwise modes. In the experimental spectra, however, elevated levels are also observed at low Helmholtz numbers below the scattering line. These contributions do not fit the trailing-edge scattering condition and are associated with other acoustic sources in the facility, such as shear-layer interactions with the nozzle lip or diffraction at the side plates and leading edge.

The LES results at $y = \pm 1$ provide additional insight into the acoustic nearfield. On the suction side ($y = +1$), the spectra show stronger contributions than on the pressure side, and the two sides are similar only when $n_z = 0$ or 1 are radiating. This indicates that the asymmetry of the radiated sound field observed earlier is associated with higher-order spanwise modes ($n_z \geq 2$).

On the suction side of the LES nearfield ($y = +1$), sharp peaks appear at these thresholds, matching the features observed in the (He, k_z) spectra. As discussed by Soni *et al.* [53], such peaks are not physical tones but artefacts of the spanwise discretisation: sound waves with a spanwise wavenumber component can become trapped in the periodic duct, mimicking perfect

spanwise correlation of their sources. The physically relevant acoustic energy lies in the neighboring frequency bands around these thresholds. Importantly, the LES does feature radiating modes with $n_z \geq 1$ that contribute significantly to the acoustic field, which is precisely the condition required to investigate the associated flow structures in the boundary layer.

IV. DATA-DRIVEN MODELING OF WAVE PACKETS DRIVING TRAILING-EDGE NOISE

In this section, we further investigate the hydrodynamic coherent structures associated with acoustic radiation and establish their correlation by extending our previous analysis using extended spectral proper orthogonal decomposition (ESPOD).

A. Spectral proper orthogonal decomposition

Proper orthogonal decomposition (POD) is a data-driven technique that provides a set of optimal orthogonal basis from flow realizations, such as LES snapshots, to describe spatially coherent structures within the flow [19,54]. In its spectral form, known as SPOD, this basis is established through the eigenvalue decomposition of the cross-spectral density (CSD) matrix of the Fourier-transformed realizations at each frequency, thereby revealing the spatial-temporal coherence of these structures. Due to the large number of spatial points in the numerical simulations, it is not feasible to construct the CSD matrix everywhere in the domain. Following the so-called snapshot method [26,55], the decomposition is based on the inner product between different flow realizations q_i , given by

$$\langle q_i, q_j \rangle = \int_{\Omega} q_j^H \mathcal{W} q_i d\mathbf{x} = q_j^H \mathbf{W} q_i, \quad (9)$$

where Ω is the region of interest, and the discretized weighting operator, $\mathbf{W}$, is chosen such that the sum of the eigenvalues defines an energy norm. The SPOD mode shapes have a unit norm, so that the square root of each SPOD eigenvalue represents the amplitude that each mode has in the flow.

In this work, we rely on the extended version of SPOD (ESPOD) to reveal the correlation between boundary layer hydrodynamics and far-field acoustics. The concept of ESPOD is derived from extended POD (EPOD) [56], which enables the analysis to datasets with correlated dynamics and captures the relationship between them. Its spectral version, ESPOD, examines the spatial correlation under the same discrete frequency. This is done by carefully choosing the weighting applied in (9), which can be expressed as

$$\mathbf{W}_E = M_{\Omega} \mathbf{W}, \quad (10)$$

where $\mathbf{W}_E$ is the weight matrix used by the ESPODs and M_{Ω} describes a mapping coefficient which sets the entries of the quadrature weight matrix $\mathbf{W}$ to zero for the points outside a region of interest. Thus, the energy calculated by the inner product 9 will only be evaluated in the region of interest. However, SPOD modes are reconstructed over the whole domain as linear combinations of flow realisations.

In the present work, the flow realizations are defined as the complex Fourier modes of the flow fluctuations $\hat{\mathbf{q}}(\mathbf{x}, t) = [\hat{\rho}, \hat{u}, \hat{v}, \hat{w}, \hat{T}]^T$ around the stationary span-time-averaged flow field $\bar{\mathbf{q}}(\mathbf{x}) = [\bar{\rho}, \bar{u}, \bar{v}, \bar{w}, \bar{T}]^T$. The norm used to define the optimal orthogonal basis is the compressible energy [57–59] imposed through the $\mathcal{W}$ operator (9), defined as a positive definite weighting tensor:

$$\mathcal{W} = \text{diag} \left(\frac{\bar{T}}{\gamma \bar{\rho} M^2}, \bar{\rho}, \bar{\rho}, \bar{\rho}, \frac{\bar{\rho}}{\gamma(\gamma - 1) \bar{T} M^2} \right). \quad (11)$$

where γ is the specific heat ratio.

We consider two sub-domains to conduct the ESPOD analysis in this work, illustrated in Fig. 10. Their definition is intended to facilitate the extraction of specific physical phenomena whose contribution dominates the production of compressible energy in different flow regions.

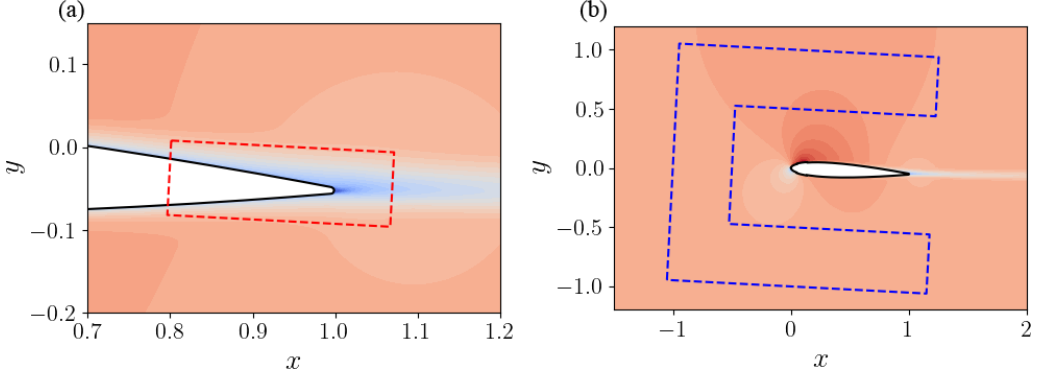


FIG. 10. The chosen subdomains for SPOD weighting matrix $\mathcal{W}$. (a) H-SPOD, (b) A-SPOD. The contour plot illustrates the streamwise mean flow $\bar{u}$.

The first subdomain contains mainly the turbulent boundary layer and wake around the trailing edge, illustrated by the red box in the Fig. 10(a), where the contribution of (turbulent) hydrodynamics is significantly greater than that of acoustics. Consequently, the basis of SPOD modes obtained from fluctuations in this region will be optimal in terms of turbulent kinetic energy, similar to Sano *et al.* [23], Abreu *et al.* [24]. The ESPOD based on this region, referred to as *hydrodynamic ESPOD* (abbreviated to H-SPOD), identifies the hydrodynamic structures that correlate with the farfield acoustics.

The second sub-domain is located in the freestream, illustrated by the blue box in the Fig. 10(b), where the contribution of acoustics dominates in the energy norm. This approach is similar to that of Bugeat *et al.* [60] for jet noise and Ribeiro [61] for airfoil tonal noise. It results in an optimal basis in terms of the acoustic energy, akin to sound power $\bar{p}^2$, thereby providing comprehensive validation of the correlation between hydrodynamics and acoustics. The ESPOD based on this region will be termed *acoustic ESPOD* in the remaining (abbreviated to A-SPOD). In both the H/A-SPOD, the resulting mode shapes are limited to a square region extending three chord lengths away from the airfoil surface, in order to reduce computational costs.

The analysis is performed with the numerical code from Rogowski *et al.* [62], using $N_{\text{FFT}} = 512$ frequency bins per block with an overlap of 75%, resulting in $N_{\text{blocks}} = 57$ blocks. The two-sided system is considered since the dataset is complex-valued, which results in the frequency resolution $\Delta\text{He} = 0.97$. A Hanning window is applied to the blocks of flow realizations for the Welch method [46] in all the analyses considered here. A validation of the convergence of SPOD is presented in the Appendix C.

B. Wave packets in the turbulent boundary layer

1. Energy spectrum of A/H-SPOD

The energy spectra of the A/H-SPOD are presented in Fig. 11, illustrating the distribution of energy between coherent structures at each discrete frequency and wavenumber. Figures 11(a)–11(d) show the H-SPOD spectra for the first four spanwise wavenumbers. Overall, the spectra exhibit a similar shape to the surface pressure spectra shown in Fig. 7(a). For all wavenumbers, the spectra display a plateau up to $\text{He} = 11$, followed by a sharp decay at higher frequencies. This steep decay indicates that the contribution of unresolved high-frequency fluctuations near the trailing edge is negligible.

Additionally, the ratio of H-SPOD mode energy to total energy ($\lambda_i / \sum_j \lambda_j$) is presented in Figs. 11(e)–11(h) to show the contribution of each H-SPOD mode. Within the low-frequency regime ($1 \leq \text{He} \leq 20$), the leading H-SPOD mode constitutes a significant portion of the total energy. For

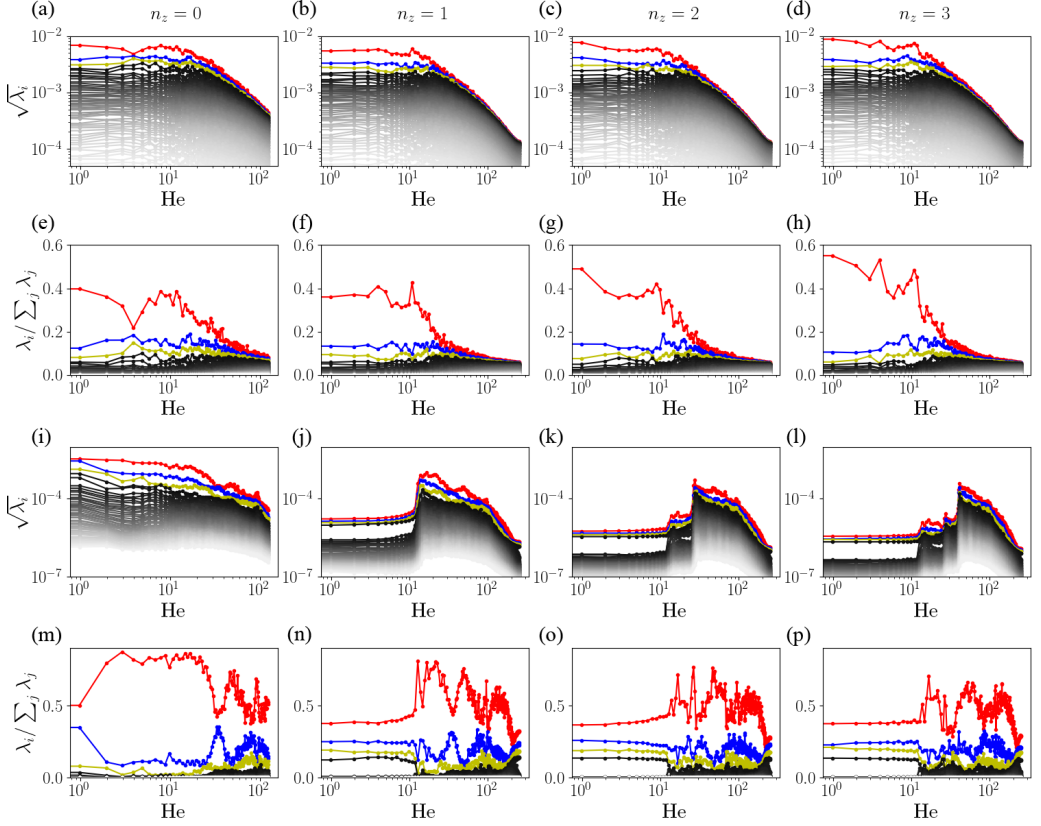


FIG. 11. SPOD eigenvalues corresponding to the first four spanwise wavenumbers ($n_z = 0, 1, 2, 3$, from left to right). (a)–(d) the square root of H-SPOD eigenvalues, (e)–(h) the energy ratio of each H-SPOD mode to the sum of all modes, (i)–(l) the square root of A-SPOD eigenvalues, (m)–(p) the energy ratio of each A-SPOD mode to the sum of all modes. The red, blue and yellow colours are corresponding to the leading three eigenvalues.

$n_z = 0$, the leading H-SPOD mode accounts for approximately 40% of the energy, whereas the second mode contributes less than 20% of the total energy. As the wavenumber increases, the leading H-SPOD mode gains a greater proportion of energy. In the case of $n_z = 3$, the leading H-SPOD mode takes up to 54% of the total energy. Such proportions in the energy distribution indicate that the trailing edge hydrodynamics are mostly low rank for frequencies up to $\text{He} = 20$. In the higher-frequency regime, the contributions of the leading and sub-leading modes become less distinguishable, i.e. the system has high-rank dynamics, and thus we restrict our analyses within the low-frequency regime.

Next, we focus on the A-SPOD spectra presented in Figs. 11(i)–11(l) and the relative energy ratio is presented in Figs. 11(m)–11(p) for the first four wavenumbers. In the case of $n_z = 0$, the spectrum is broadband. The energy contained in the leading mode exceeds 80% in the frequency range $3 \leq \text{He} \leq 25$. The ratio between the energy contained in the leading SPOD mode and the total energy is more significant in the A-SPOD than in the H-SPOD, which is expected given that turbulent dynamics are a much more complex phenomenon than acoustic dynamics. In the case of $n_z = 1$ [see Fig. 11(j)] the shape of the A-SPOD spectrum significantly differs from that of the H-SPOD. A low Helmholtz numbers, ($\text{He} \lesssim 12$), the energy remains nearly constant and low, corresponding to the evanescent regime below the cut-on frequency. At the threshold frequency, a sharp increase in energy occurs as the $n_z = 1$ acoustic mode becomes propagative. For frequencies

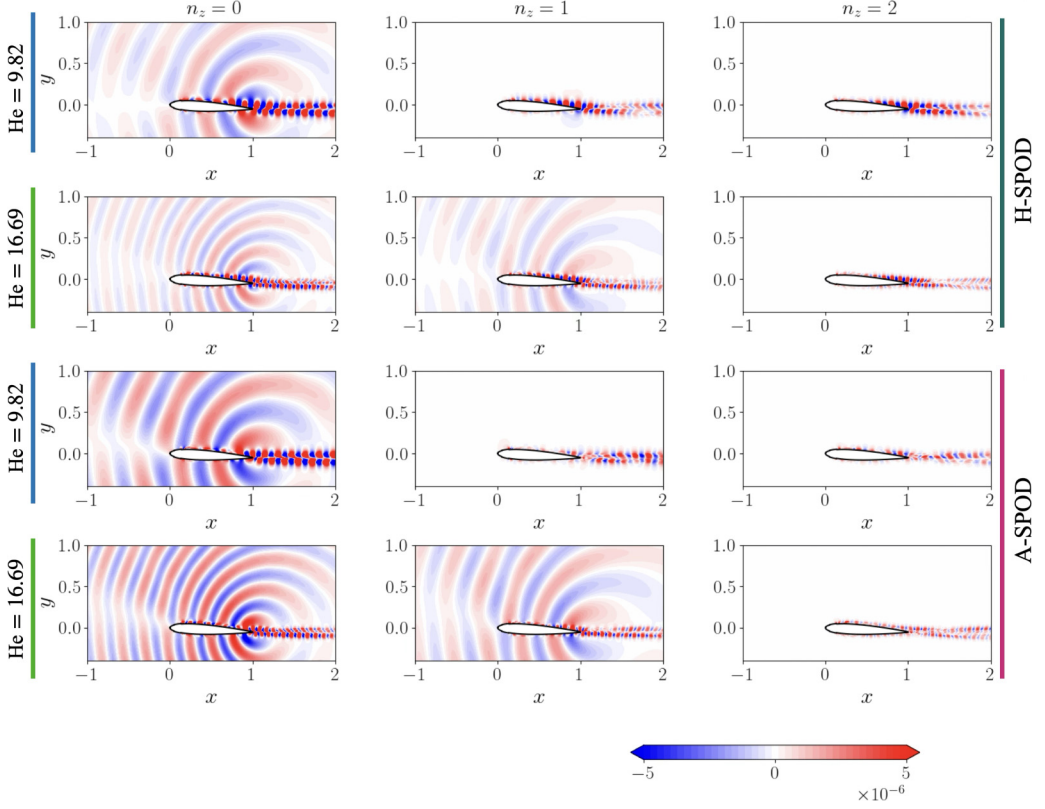


FIG. 12. The *leading* H-SPOD mode shape $\tilde{p}$ for the first three spanwise wavenumbers ($n_z = 0, 1, 2$) at the frequency $\text{He} = 9.82$ and $\text{He} = 16.69$, respectively. Both A-SPOD and H-SPOD mode shapes are presented, as labeled in the figure. The $\tilde{p}$ is premultiplied by the square root of its eigenvalue to give the real amplitude.

above this threshold, the overall shape of the spectrum is similar to that observed for $n_z = 0$. As the wavenumbers are increased further, the spectrum exhibits the appearance of multiple steps, which is consistent with the increased number of propagative waves present at the higher frequencies. The frequency thresholds follow the scattering conditions, as illustrated in Fig. 9.

2. Mode shapes of A/H-SPOD

Figure 12 illustrates the pressure fluctuations associated with the leading A/H-SPOD mode for the first three azimuthal wavenumbers, $n_z = 0, 1, 2$, at $\text{He} = 9.82$ and 16.69 . According to the scattering condition, these two frequencies fall within the propagative regimes for $n_z = 0$ and $n_z = 1$, respectively, indicating that acoustic radiation is expected for these wavenumbers at the selected frequencies. For the wavenumbers and frequencies considered here, the H-SPOD mode shapes reveal a wave packet extending from the trip location on both the suction and pressure sides, continuing into the wake of the airfoil. Due to the adverse pressure gradient, the fluctuations on the suction side exhibit significantly higher amplitudes compared to those on the pressure side. For the $n_z = 2$ case, the wave-packet structures appear more concentrated near the TE region. The A-SPOD mode shapes, similar to those obtained from H-SPOD, also exhibit wave-packet-like behavior. However, slightly different to H-SPOD, the A-SPOD modes show reduced fluctuation amplitudes near the trip location but increased concentration of energy in the vicinity of the TE and wake regions.

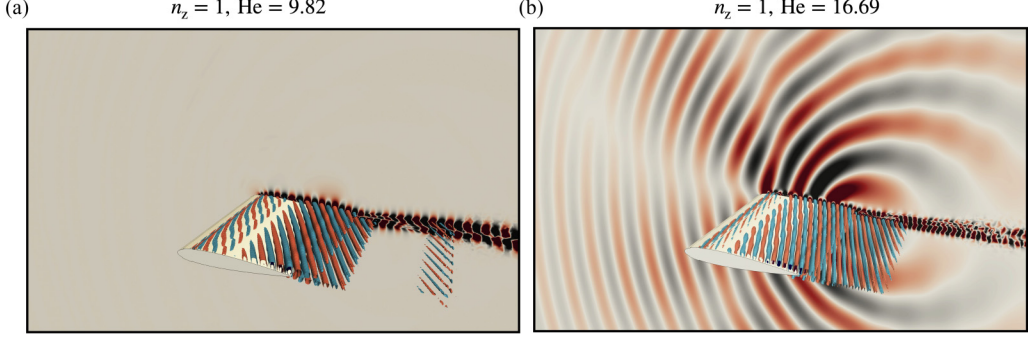


FIG. 13. Pressure isosurface of three-dimensional SPOD mode shapes $\tilde{p}$ for (a) evanescent and (b) propagative conditions. The color scale for the acoustic pressure fluctuation is saturated in the range $[-5 \times 10^{-6}, 5 \times 10^{-6}]$ from black to red, and the hydrodynamic pressure fluctuation is in the range $[-0.003, 0.003]$ from blue to orange. The airfoil is extended along z axis for the visualization purpose.

In the freestream region, strong acoustic waves radiated from the TE can be identified for the frequency-wavenumber combinations $[n_z = 0, \text{He} = 9.82]$ and $[n_z = 0, 1, \text{He} = 16.69]$. The acoustic waves are evanescent for the cases of $[n_z = 1, \text{He} = 9.82]$ and $n_z = 2$ for both frequencies. This is in agreement with the scattering condition (and as observed in Sec. III D). Note that the amplitudes of evanescent waves decrease exponentially with increasing distance to the airfoil, and are not visible for the used color-scale. Compared to H-SPOD, A-SPOD exhibits stronger pressure fluctuations in the acoustic field. This observation is consistent with the low-rank nature of A-SPOD, as seen in the energy spectra, where a major portion of the acoustic energy is captured by the leading A-SPOD mode. Further comparisons between A-SPOD and H-SPOD are provided in Appendix D, where the mode shapes of the velocity components $\tilde{u}$ and $\tilde{v}$ are examined. The results show good agreement between the two formulations, particularly in the hydrodynamic regions.

It should be noted that due to the non-zero wavenumbers, the SPOD modes are three-dimensional, whereby the wave packets and scattered acoustic waves are no longer parallel to the trailing edge. To enhance our understanding of these dynamics, the three-dimensional pressure field of the SPOD modes is reconstructed, following

$$\tilde{p}_{3D}(\omega) = \tilde{p}_{2D}(\omega)e^{ik_z z}, \quad (12)$$

where ω is the angular frequency of the H-SPOD modes. The results associated with $n_z = 1$ at the evanescent and propagative frequencies are visualized in Fig. 13. In both cases, the hydrodynamic wave packets are oblique with respect to the trailing edge. The amplitude of the wave-packet structures decay shortly downstream of the tripping element and then grow again close to the trailing edge. To better visualize the three-dimensional spatiotemporal structures of the acoustic and hydrodynamic fluctuations, animations corresponding to the three-dimensional reconstructions of the SPOD modes at additional wavenumbers are provided as Supplemental Material [63].

C. Trailing-edge noise reduced-order models

To assess whether the acoustic field represented by the leading H-SPOD modes captures the overall sound radiation quantitatively, we consider the integrated acoustic spectrum. This quantity is obtained by integrating the acoustic pressure spectrum along an arc in the freestream, which removes the influence of directivity and provides a measure of the total radiated sound. The arc has a radius of $1.5c$ centered at the trailing edge, and the integration excludes the wake region to avoid contamination by hydrodynamic fluctuations. Furthermore, to highlight the contribution of each wavenumber, the acoustic field is reconstructed by accumulating the contributions from the first five

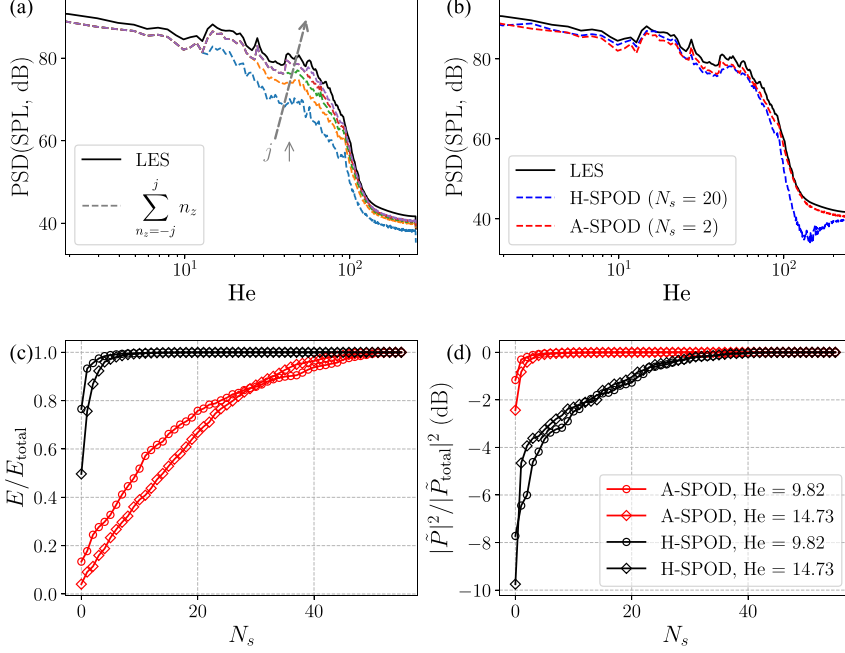


FIG. 14. (a) Contribution of the accumulated spanwise wavenumbers n_z to the radiated pressure obtained from the H-SPOD modes. The PSD from the LES is computed directly from the three-dimensional field, and the spectra are integrated along the arc. (b) Comparison of the integrated PSD obtained from reconstructed H-SPOD, A-SPOD, and the original dataset. Note that 20 H-SPOD modes and two A-SPOD modes are used for the reconstructions. (c) Ratio of acoustic energy between accumulated SPOD modes (N_s) to total acoustic energy integrated in the freestream [see Fig. 10(b)] (d) Ratio of compressible energy between accumulated SPOD modes to total energy integrated near the trailing edge [see Fig. 10(a)].

spanwise wavenumbers. In this reconstruction, the leading twenty H-SPOD modes are used for each wavenumber. The resulting reconstruction is shown in Fig. 14(a).

Figure 14(a) illustrates the dominant contribution of each wavenumber within its corresponding frequency range. As the frequency increases, higher-wavenumber modes become increasingly important. The cut-on frequency associated with each wavenumber agrees well with the theoretical scattering condition. Unlike the spectral correction formulation in (7), which is derived for two-dimensional pressure waves, the accumulated spectrum offers a physical interpretation of the role of nonzero spanwise wavenumbers in representing the three-dimensional acoustic field.

However, it requires 20 H-SPOD modes per wavenumber to ensure that the maximum deviation from the LES-based PSD remains below 2 dB. This high-rank requirement arises from the nature of H-SPOD, which is designed to identify acoustic fluctuations correlated with the most energetic hydrodynamic structures. In contrast, A-SPOD identifies hydrodynamic fluctuations that are most strongly correlated with the energetic acoustic pressure field. In this sense, A-SPOD is a more suitable candidate for low-order reconstruction of far-field acoustics.

To assess this potential, the same acoustic integration along the circular arc as in Fig. 14(a) is performed, now using the leading two A-SPOD modes for each wavenumber. For comparison, the reconstructed spectrum based on the leading twenty H-SPOD modes is reused. The results are shown in Fig. 14(b), where contributions from all wavenumbers are included. As seen in the figure, the reconstruction using only two A-SPOD modes outperforms that using 20 H-SPOD modes for frequencies up to $\text{He} \approx 25$. At higher frequencies, additional A-SPOD modes are needed to further improve the reconstruction. Nonetheless, even with only two A-SPOD modes, the reconstructed

acoustic levels remain comparable to those obtained using twenty H-SPOD modes. This favorable comparison highlights the efficiency of A-SPOD in producing a low-rank, reduced-order model of the acoustic field.

The convergence rate of A/H-SPOD with respect to acoustic energy is evaluated by computing the sound power, $|\tilde{p}|^2$, in a freestream subdomain as illustrated in Fig. 10(b). The corresponding results are shown in Fig. 14(c). For both frequencies considered, the difference in SPL between the leading H-SPOD mode and the total acoustic energy exceeds 7.8 dB. In sharp contrast, the leading A-SPOD mode captures the acoustic energy within 2.4 dB of the total. A-SPOD exhibits significantly faster convergence with increasing mode number. Specifically, only two A-SPOD modes are required to reconstruct the acoustic field within 1 dB of the full energy content, whereas H-SPOD requires 24 modes to achieve the same level of accuracy. These results clearly show that the A-SPOD approach is a more appropriate candidate for the reduced-order modeling of acoustics.

The convergence rate with respect to turbulent kinetic energy is assessed by integrating the weighted SPOD energy in the near-trailing-edge region, as marked by the red box in Fig. 10(a). The results are presented in Fig. 14(d), where E_{total} denotes the total energy associated with a given frequency and wavenumber. In contrast to the convergence behavior observed for acoustic energy, H-SPOD exhibits a significantly faster accumulation of turbulent kinetic energy with increasing mode number. The leading H-SPOD mode captures a substantial portion of the total hydrodynamic energy ($\geq 50\%$), whereas the leading A-SPOD mode accounts for only 5–15%, with a much slower convergence rate as additional modes are included.

In summary, both A-SPOD and H-SPOD identify the hydrodynamic wave packets that drive the trailing-edge noise. However, the two methods differ significantly in their efficiency for low-order modeling of the acoustics. A-SPOD selectively captures the hydrodynamic structures that effectively radiate sound, as filtered by the trailing-edge scattering condition. In contrast, H-SPOD includes a wider range of coherent structures, both propagative and nonpropagative, that may not all contribute to acoustic radiation. Consequently, A-SPOD provides a more focused and efficient basis for reduced-order modeling of the acoustic field.

V. CONCLUSIONS

Numerical simulations have been conducted to investigate the broadband trailing-edge noise problem, using the same geometric configuration as in the experiments. A moderate Reynolds number of $\text{Re} = 2 \times 10^5$ and a Mach number $M = 0.3$ are considered. The computational domain includes a large spanwise extent of $L_z = 43.75\%$ of the chord length, which allows for the resolution and analysis of large-scale spanwise-coherent structures. Validation against experimental data demonstrates good agreement in terms of mean flow, turbulence statistics, acoustic spectra, and hydrodynamic–acoustic coherence.

Spanwise Fourier decomposition of the acoustic spectra reveals that acoustic radiation follows the scattering condition originally proposed by Amiet [11]. The results show that only modes with spanwise wavenumber $k_z < k_0$ can propagate, while modes with $k_z > k_0$ are evanescent. This phenomena indicates that the trailing edge with finite span width acts as a spatial low-pass filter, allowing only selective wavenumbers to be radiative in the trailing-edge noise frequency range.

Key to this work is the investigation of nonzero spanwise wavenumber ($k_z > 0$) hydrodynamic structures and their contribution to the acoustic radiation. The analysis is done through the spatially weighted SPOD, namely, acoustic-based (A-SPOD) and hydrodynamic-based (H-SPOD). The results demonstrate that their contribution to the acoustics is significant and that the frequency range associated with each wavenumber aligns with the scattering condition. SPOD analysis reveals that $k_z > 0$ modes arise from wave packets convecting obliquely across the trailing edge, yet they share remarkably similar structures and scattering mechanisms with the $k_z = 0$ modes. This suggests that a common physical mechanism underlies the amplification and radiation of all radiating coherent structures.

In the reduced-order modeling analysis, the acoustic field is reconstructed using 20 H-SPOD modes. However, a more efficient reconstruction can be achieved using only two A-SPOD modes. Energy convergence analysis confirms this advantage: A-SPOD is more effective in capturing the acoustic energy, while H-SPOD provides a better representation of the underlying hydrodynamic structures. This distinction arises from the design of A-SPOD, which extracts hydrodynamic structures that are filtered by the trailing edge and are strongly correlated with the radiated acoustic field. The low-rank nature of A-SPOD, at frequencies relevant to TE noise, suggests its potential for implementation in control strategies targeting the dominant wave-packet structures.

ACKNOWLEDGMENTS

Z.Y. acknowledges support from the Swedish Research Council under Grant No. 2020-04084 and from Swedish innovation agency VINNOVA Project No. 2023-00924. The computations were performed on resources by the National Academic Infrastructure for Supercomputing in Sweden (NAISS) at the LUMI supercomputer cluster in Finland and the Dardel supercomputer at PDC KTH, Sweden. S.D. acknowledges Deutsche Forschungsgemeinschaft for supporting current work under Grant No. 458062719.

DATA AVAILABILITY

The data that support the findings of this article are available upon reasonable request.

APPENDIX A: DERIVATION OF THE SCATTERING CONDITION

The goal of this Appendix is to show that the scattering condition used in the present paper is not an *ad hoc* constraint, but follows directly from the classical derivations of Amiet [11] and their extension by Roger and Moreau [14]. We follow the steps of Roger and Moreau, while omitting the additional leading-edge back-scattering correction since it does not affect the wavenumber threshold relation we seek to highlight.

Consider a frozen three-dimensional incident pressure gust p' , convected by a uniform stream-wise flow of velocity U past a flat plate of zero thickness and zero angle of attack, with a trailing edge located at $x = 0$. We recall that our coordinate system (x, y, z) denotes the streamwise, vertical, and spanwise directions, respectively (this differs from the notation of Roger and Moreau [14]). For clarity, we first write the relations in dimensional form and indicate the corresponding nondimensional relation using the same conventions as in the main text at the end. Let a_∞ be the ambient sound speed, $M = U/a_\infty$ the Mach number, and $\beta^2 = 1 - M^2$. We also define the acoustic wavenumber $k_0 = \omega/a_\infty$.

The governing equation is the convected wave equation in a uniform mean flow,

$$\frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial y^2} + \frac{\partial^2 p'}{\partial z^2} - \frac{1}{a_\infty^2} \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 p' = 0, \quad (\text{A1})$$

where a_∞ is the ambient sound speed.

We seek time-harmonic solutions of the form

$$p'(x, y, z, t) = \hat{p}(x, y, z) e^{i\omega t}. \quad (\text{A2})$$

Following Roger and Moreau [14], the incident gust is modeled as a frozen oblique Fourier component with spanwise wavenumber k_z and a prescribed streamwise oscillation. We make explicit the propagation assumption that the streamwise wavenumber is proportional to the acoustic wavenumber,

$$k_x = \alpha k_0, \quad \alpha > 0, \quad (\text{A3})$$

so the gust convects at phase speed $u_c = \omega/k_x = a_\infty/\alpha$ (the choice $\alpha = 1/M$ recovers $u_c = U$, similar to Amiet [11]). With this, a single spanwise Fourier component of the gust can be written

$$p'(x, y, z, t) = \hat{p}(x, y) \exp[i\omega t + i\alpha k_0 x - ik_z z]. \quad (\text{A4})$$

In the notation of Roger and Moreau [14], the streamwise factor may appear as $e^{i(k_0 M/\beta^2)}$ or, equivalently, as $e^{i\alpha K X}$ with $X = x$. In our notation, taking $k_x = \alpha k_0$ covers both representations; while the particular choice $\alpha = M/\beta^2$ reproduces their first form.

Substituting this ansatz into (A1) and collecting terms yields an equation for $\hat{p}(x, y)$. With the particular choice $\alpha = M/\beta^2$ made by Roger and Moreau [14], the equation takes a simpler form. Introducing the scaled coordinate $Y = \beta y$ (as in) brings the operator to the canonical two-dimensional Helmholtz form required to apply Schwarzschild's/Wiener-Hopf edge solution:

$$\frac{\partial^2 \hat{p}}{\partial x^2} + \frac{\partial^2 \hat{p}}{\partial Y^2} + \bar{\mu}^2 \hat{p} = 0, \quad (\text{A5})$$

with the effective (squared) wavenumber

$$\bar{\mu}^2 = \left(\frac{k_0}{\beta^2}\right)^2 - \frac{k_z^2}{\beta^2}. \quad (\text{A6})$$

The nature of the solution depends on the sign of $\bar{\mu}^2$. If $\bar{\mu}^2 > 0$, the solutions of the Helmholtz problem are oscillatory in Y and radiate to the acoustic far field. If $\bar{\mu}^2 < 0$, the solutions are exponentially decaying in Y : such subcritical gusts do not radiate. The radiation (cut-on) condition is therefore

$$\boxed{|k_z| < \frac{k_0}{\beta}}, \quad (\text{A7})$$

which is the scattering condition used in the main text.

In our nondimensional convention, based on the chord as length scale, both k_0 and k_z scale consistently with $1/c$. The scattering condition (A7) therefore retains the same form in dimensional and nondimensional variables. For a finite span L_z , the spanwise Fourier integral yields a sinc function [14], which effectively restricts radiation to discrete values $k_z = 2\pi n_z/L_z$, leading to the discrete cut-on values used in our (He, k_z) maps.

APPENDIX B: EIGENVALUE DECOMPOSITION TO THE CSD MATRIX

The scattering condition presented in frequency-wavenumber spectrum (Fig. 9) requires a Fourier transform in both the spanwise and temporal directions. However, applying this transformation is not straightforward due to the presence of nonperiodic boundary conditions imposed by the side plates. To investigate the spanwise wave structures in the acoustics, the SPOD analysis is performed on the signals obtained from a spanwise line array. Note that SPOD analysis in this section is the direct eigenvalue decomposition to the cross spectral density matrix built with respect the sensor in the middle of the domain. This analysis is applied to both the numerical and experimental datasets for consistent comparison.

The results at frequency He = 2 for the experimental signals and He = 16 for the numerical signals are shown in Fig. 15. The frequencies are chosen to have two propagative waves according to the scattering condition. In both cases, the eigenvalues (shown in the left column) show that the leading modes have significant energy compared to the others. There are three eigenvalues with significant amplitudes compared to the others. A rapid decrease in amplitude can be seen after $n_{\text{eig}} = 2$.

The shape of the dominant SPOD modes are shown in the middle column of Fig. 15. The real part of the four modes are presented, corresponding to the numbers labeled in the spectra. The results show that the mode shape is similar to the Fourier modes. Further insight into the similarity

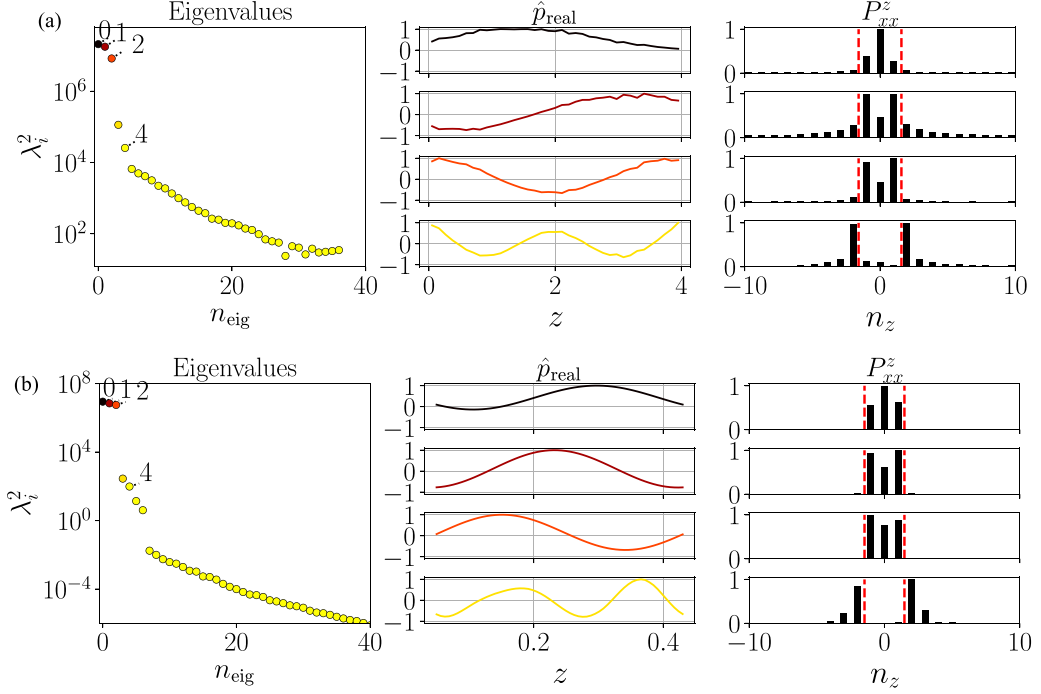


FIG. 15. Eigenvalue analysis on the CSD matrices constructed from the acoustic line array signals from the (a) simulation and (b) experimental datasets. Eigenvalues (left), eigenmodes (mode) and the spanwise Fourier decomposition of the eigenmodes (right) are shown. The scattering condition ($-k_0 < k_z < k_0$), indicated by the vertical dashed red line, is shown to illustrate the wavenumbers that generate the radiated sound.

of the mode shapes compared to the Fourier modes can be obtained by performing a spatial Fourier transform of the SPOD modes, as shown in the right column. It can be seen that the SPOD modes are dominated by very few spanwise Fourier modes. The first SPOD mode consists mainly of a mode with zero spanwise wavenumber, while the next lower modes consists of two modes with same wavenumber but opposite sign. This observation suggests that the SPOD modes represent a superposition of left and right traveling acoustic waves identified by positive and negative n_z . These observations indicate that the periodic boundary condition has less influence on the mode shapes. Thus, a fair comparison can be made for both datasets.

In Fig. 15, the scattering condition (given by the acoustic wavenumber k_0) is indicated as vertical dashed lines. The region where $-k_0 < k_z < k_0$ indicates the propagative waves, while the rest are evanescent waves. As can be seen, only the first three SPOD modes are propagative, which explains their significant energy shown in the eigenvalue spectrum. Overall, the SPOD of the CSD matrix reveals modes that are very similar to spanwise Fourier modes, which justifies the Fourier decomposition of the acoustic data.

APPENDIX C: SPOD CONVERGENCE ANALYSIS

To illustrate the convergence and robustness of the computed SPOD modes, a convergence analysis was conducted. The methodology employed is similar to that described by Abreu *et al.* [24], where the entire time signal is split into two equal parts, each comprising 75% of the original dataset. Subsequently, SPOD was performed on each part separately. Similar to (4), a correlation

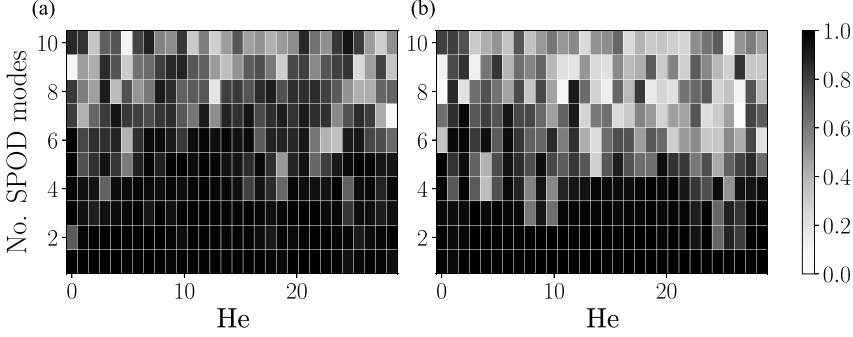


FIG. 16. Correlation coefficient $\sigma_{1,k}$ for the leading 10 SPOD modes with $\text{He} \leq 30$. (a) H-SPOD and (b) A-SPOD. Here we show the dataset $i = 1, 2$ with 75% overlapping with the original LES dataset.

relation between the modes from each dataset can be defined to determine the convergence:

$$\sigma_{i,k} = \frac{\langle \Psi_k, \Psi_{i,k} \rangle}{\sqrt{\langle \Psi_k, \Psi_{i,k} \rangle} \sqrt{\langle \Psi_{i,k}, \Psi_{i,k} \rangle}}, \quad (\text{C1})$$

where σ stands for the correlation coefficient, $i \in \{1, 2\}$ are two subsets of the original LES data, and $k \in \{1, 2, 3, \dots, N_b\}$ are the indices of the SPOD modes. Note that $\langle \cdot, \cdot \rangle$ indicates the nonweighted inner product, which represents the projection of the partial dataset $\Psi_{i,k}$ to the full dataset Ψ_k . It is anticipated that in the event of a sufficiently long time series, the correlation coefficient will approach a value of one.

Figure 16 illustrates the correlation coefficient $\sigma_{1,k}$ for the leading ten SPOD modes within the frequency range $\text{He} < 30$ for both H-SPOD and A-SPOD. Accordingly, the leading SPODs are very well converged, with all leading SPOD modes having $\sigma_{1,k} \geq 0.98$ for $\text{He} < 25$. For the majority of analyses, the leading SPOD modes are presented, and therefore, it can be concluded that the SPOD analysis is sufficiently well converged.

APPENDIX D: SPOD MODE SHAPES OF THE VELOCITY COMPONENTS

Figure 17 shows velocity components $\tilde{u}$ and $\tilde{v}$ of the leading SPOD mode for the first two spanwise wavenumbers ($n_z = 0, 1$) at frequency $\text{He} = 9.82$, where the relative energy share of the leading SPOD mode is maximum. For the wavenumbers and frequency considered here, the mode shapes reveal a wave packet extending from the tripping elements on both the suction and pressure sides to the wake of the airfoil. The maximum amplitude of the wave packet is found in the region close to the trailing edge. Similar to the pressure mode presented in Fig. 12, the fluctuations on the suction side have a significantly larger amplitude than those on the pressure side. Furthermore, over the suction side of the airfoil, the streamwise velocity fluctuations $\tilde{u}$ show a phase change in the wall normal direction. Similar phase change phenomena have also been reported in adverse-pressure-gradient turbulent boundary layers over airfoils by Abreu *et al.* [24] and in turbulent jet shear layers by Pickering *et al.* [64]. Consistent with these studies, we identify the Orr mechanism as the primary driver of wave-packet formation near the trailing edge of the airfoil, which ultimately leads to the generation of broadband trailing-edge noise in the present configuration. This mechanism is known to optimally promote Tollmien-Schlichting instabilities in boundary layers [65] and is further amplified by adverse pressure gradients—explaining the dominant wave-packet growth observed on the suction side of the airfoil.

A comparison between A-SPOD and H-SPOD shows that the overall mode shapes are generally consistent. However, a clear difference is observed for the case of $n_z = 1$. At the frequency $\text{He} = 12.21$, the acoustic waves are evanescent according to the scattering condition. In this regime, the near-field hydrodynamic structures reconstructed by A-SPOD appear less coherent and significantly

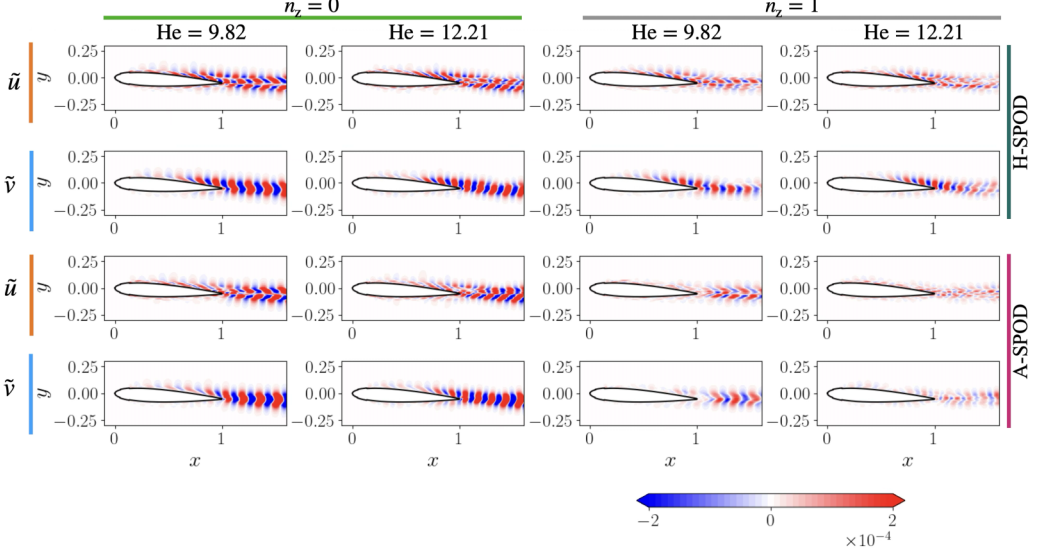


FIG. 17. The leading SPOD mode shapes ($\tilde{u}$, $\tilde{v}$) for the first two spanwise wavenumbers ($n_z = 0, 1$) at the frequency $He = 9.82$ and 12.21 . Both A-SPOD and H-SPOD mode shapes are presented.

weaker than those obtained from H-SPOD. This is consistent with the properties of A-SPOD, where TE filters hydrodynamic content, making it less effective at capturing nonpropagative components.

Overall, both A-SPOD and H-SPOD yield comparable velocity reconstructions for propagative wavenumbers and frequencies. This agreement reinforces the conclusion that turbulent boundary layer wave packets serve as the primary drivers of the TE noise scattering.

APPENDIX E: NOISE DUE TO THE TRIPPING DEVICES

To investigate the noise generation from the tripping devices, we first present the spanwise Fourier-transformed acoustic spectra at the locations $x = 1$, $|y| = 3$, as shown in Fig. 18. Acoustic signals are acquired from both the pressure and suction sides of the airfoil. As shown in the spectra, the difference between the suction and pressure sides primarily occurs in the higher frequency range ($St > 17$) for all n_z , which indicates that this discrepancy appears to be independent of the spanwise wavenumber. For the cases of $n_z > 0$, the difference between the suction and pressure side spectra is more significant.

Figure 19 presents the A-SPOD mode shapes at the frequency $St = 20.8$ for spanwise wavenumbers $n_z = 1$ and 2 . For the $n_z = 1$ case, both trailing-edge noise and trip-induced noise are visible,

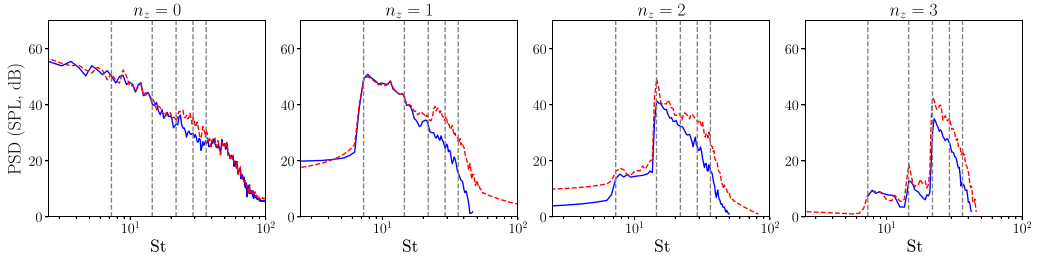


FIG. 18. Acoustic spectrum of the leading four wavenumbers ($n_z = 0, 1, 2, 3$) at two spatial locations $x = 1$, $|y| = 3$. Red line indicates the suction side and blue is the pressure side.

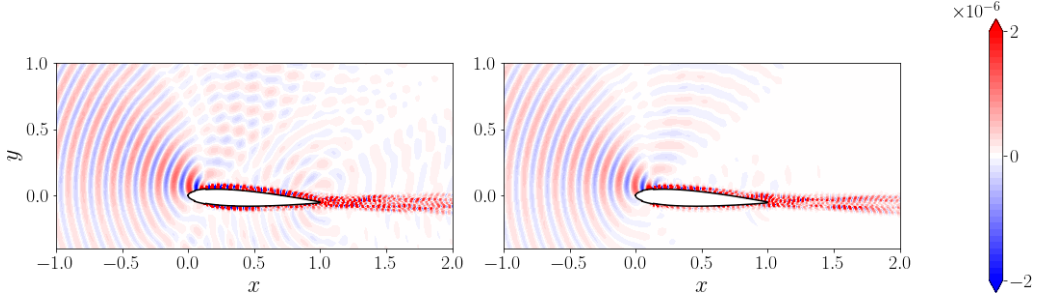


FIG. 19. A-SPOD mode shapes at $St = 20.8$, (left) $n_z = 1$ and (right) $n_z = 2$. Pressure fluctuation component is presented.

with the latter being more pronounced on the suction side. In the $n_z = 2$ case, the trip noise on the suction side exhibits a larger amplitude than the trailing-edge contribution. Overall, at high frequencies, the suction-side trip noise plays a significant role, whereas the pressure-side contribution is much weaker. This suggests that the nonsymmetric behavior observed in the spectrum likely originates from the trip noise. A more quantitative assessment of the trip-induced acoustic contribution could be obtained by performing an ESPOD analysis with a weighting region focused near the trip location. However, such an analysis would involve very high frequencies, where the SPOD is not low rank, and may be of limited relevance to the trailing-edge noise problem.

-
- [1] T. F. Brooks, D. S. Pope, and M. A. Marcolini, Airfoil self-noise and prediction, NASA Reference Publication (RP) No. NASA-RP-1218 (1989), <https://ntrs.nasa.gov/citations/19890016302>.
 - [2] R. Longhouse, Vortex shedding noise of low tip speed, axial flow fans, *J. Sound Vib.* **53**, 25 (1977).
 - [3] H. Arbey and J. Bataille, Noise generated by airfoil profiles placed in a uniform laminar flow, *J. Fluid Mech.* **134**, 33 (1983).
 - [4] S. Pröbsting, F. Scarano, and S. C. Morris, Regimes of tonal noise on an airfoil at moderate Reynolds number, *J. Fluid Mech.* **780**, 407 (2015).
 - [5] V. Golubev, Recent advances in acoustics of transitional airfoils with feedback-loop interactions: A review, *Appl. Sci.* **11**, 1057 (2021).
 - [6] T. R. Ricciardi, W. R. Wolf, and K. Taira, Transition, intermittency and phase interference effects in airfoil secondary tones and acoustic feedback loop, *J. Fluid Mech.* **937**, A23 (2022).
 - [7] N. Curle, The influence of solid boundaries upon aerodynamic sound, *Proc. R. Soc. London Ser. A* **231**, 505 (1955).
 - [8] M. J. Lighthill, On sound generated aerodynamically I. General theory, *Proc. R. Soc. London Ser. A* **211**, 564 (1952).
 - [9] J. E. Ffowcs Williams and L. H. Hall, Aerodynamic sound generation by turbulent flow in the vicinity of a scattering half plane, *J. Fluid Mech.* **40**, 657 (1970).
 - [10] J. M. R. Graham, Similarity rules for thin aerofoils in non-stationary subsonic flows, *J. Fluid Mech.* **43**, 753 (1970).
 - [11] R. Amiet, Acoustic radiation from an airfoil in a turbulent stream, *J. Sound Vib.* **41**, 407 (1975).
 - [12] R. Amiet, Noise due to turbulent flow past a trailing edge, *J. Sound Vib.* **47**, 387 (1976).
 - [13] M. Howe, A review of the theory of trailing edge noise, *J. Sound Vib.* **61**, 437 (1978).
 - [14] M. Roger and S. Moreau, Back-scattering correction and further extensions of Amiet's trailing-edge noise model. Part 1: Theory, *J. Sound Vib.* **286**, 477 (2005).
 - [15] B. Lyu, M. Azarpeyvand, and S. Sinayoko, Prediction of noise from serrated trailing edges, *J. Fluid Mech.* **793**, 556 (2016).

- [16] S. Lee, L. Ayton, F. Bertagnolio, S. Moreau, T. P. Chong, and P. Joseph, Turbulent boundary layer trailing-edge noise: Theory, computation, experiment, and application, *Prog. Aerosp. Sci.* **126**, 100737 (2021).
- [17] D. Kang, S. Lee, D. Brouzet, and S. K. Lele, Wavelet-based pressure decomposition for airfoil noise in low-Mach number flows, *Phys. Fluids* **35**, 075112 (2023).
- [18] J. W. Delfs and R. Ruck, A quantity to identify turbulence related sound generation on surfaces, *J. Sound Vib.* **586**, 118490 (2024).
- [19] J. L. Lumley, *Stochastic Tools in Turbulence* (Academic Press, New York, 1970), p. 209.
- [20] K. Taira, S. L. Brunton, S. T. M. Dawson, C. W. Rowley, T. Colonius, B. J. McKeon, O. T. Schmidt, S. Gordeyev, V. Theofilis, and L. S. Ukeiley, Modal analysis of fluid flows: An overview, *AIAA J.* **55**, 4013 (2017).
- [21] J. H. M. Ribeiro and W. R. Wolf, Identification of coherent structures in the flow past a NACA0012 airfoil via proper orthogonal decomposition, *Phys. Fluids* **29**, 085104 (2017).
- [22] M. Sanjose, A. Towne, P. Jaiswal, S. Moreau, S. Lele, and A. Mann, Modal analysis of the laminar boundary layer instability and tonal noise of an airfoil at Reynolds number 150,000, *Int. J. Aeroacoust.* **18**, 317 (2019).
- [23] A. Sano, L. I. Abreu, A. V. Cavalieri, and W. R. Wolf, Trailing-edge noise from the scattering of spanwise-coherent structures, *Phys. Rev. Fluids* **4**, 094602 (2019).
- [24] L. I. Abreu, A. Tanarro, A. V. Cavalieri, P. Schlatter, R. Vinuesa, A. Hanifi, and D. S. Henningson, Spanwise coherent hydrodynamic waves around flat plates and airfoils, *J. Fluid Mech.* **927**, A1 (2021).
- [25] A. V. G. Cavalieri, P. Jordan, and L. Lesshafft, Wave-packet models for jet dynamics and sound radiation, *Appl. Mech. Rev.* **71**, 020802 (2019).
- [26] O. T. Schmidt and T. Colonius, Guide to spectral proper orthogonal decomposition, *AIAA J.* **58**, 1023 (2020).
- [27] E. Pickering, G. Rigas, O. T. Schmidt, D. Sipp, and T. Colonius, Optimal eddy viscosity for resolvent-based models of coherent structures in turbulent jets, *J. Fluid Mech.* **917**, A29 (2021).
- [28] P. A. Nogueira, A. V. Cavalieri, and P. Jordan, A model problem for sound radiation by an installed jet, *J. Sound Vib.* **391**, 95 (2017).
- [29] W. R. Wolf, J. L. F. Azevedo, and S. K. Lele, Convective effects and the role of quadrupole sources for aerofoil aeroacoustics, *J. Fluid Mech.* **708**, 502 (2012).
- [30] A. Tanarro, R. Vinuesa, and P. Schlatter, Effect of adverse pressure gradients on turbulent wing boundary layers, *J. Fluid Mech.* **883**, A8 (2020).
- [31] M. Wang and P. Moin, Computation of trailing-edge flow and noise using large-eddy simulation, *AIAA J.* **38**, 2201 (2000).
- [32] R. Ewert and W. Schröder, On the simulation of trailing edge noise with a hybrid LES/APE method, *J. Sound Vib.* **270**, 509 (2004).
- [33] R. Ewert, C. Appel, J. Dierke, and M. Herr, RANS/CAA based prediction of NACA 0012 broadband trailing edge noise and experimental validation, in *15th AIAA/CEAS Aeroacoustics Conference (30th AIAA Aeroacoustics Conference)* (American Institute of Aeronautics and Astronautics, Reston, VA, 2009).
- [34] R. D. Sandberg and N. D. Sandham, Direct numerical simulation of turbulent flow past a trailing edge and the associated noise generation, *J. Fluid Mech.* **596**, 353 (2008).
- [35] P. Jaiswal, Y. Pasco, G. Yakhina, and S. Moreau, Experimental investigation of aerofoil tonal noise at low Mach number, *J. Fluid Mech.* **932**, A37 (2022).
- [36] S. Demange, S. Jekosch, B. Church, E. Sarradj, K. Oberleithner, and A. V. Cavalieri, Experimental investigation of tonal and broadband trailing-edge noise for the flow around a NACA0012 profile with rounded trailing edge, in *AIAA AVIATION 2023 Forum* (American Institute of Aeronautics and Astronautics, San Diego, CA, 2023), p. 1.
- [37] F. Witherden, A. Farrington, and P. Vincent, PyFR: An open source framework for solving advection–diffusion type problems on streaming architectures using the flux reconstruction approach, *Comput. Phys. Commun.* **185**, 3028 (2014).
- [38] E. W. Schneehagen, E. Sarradj, and D. Gründemann, Design and characterization of an aeroacoustic wind tunnel, in *Proceedings of the 47th Annual Acoustics Conference* (DEGA, Wien, 2021), pp. 1414–1417.

- [39] P. Schlatter and R. Örlü, Turbulent boundary layers at moderate Reynolds numbers: Inflow length and tripping effects, *J. Fluid Mech.* **710**, 5 (2012).
- [40] J. B. Freund, Proposed inflow/outflow boundary condition for direct computation of aerodynamic sound, *AIAA J.* **35**, 740 (1997).
- [41] D. J. Bodony, Analysis of sponge zones for computational fluid mechanics, *J. Comput. Phys.* **212**, 681 (2006).
- [42] M. Hultmark, M. Vallikivi, S. C. C. Bailey, and A. J. Smits, Turbulent pipe flow at extreme Reynolds numbers, *Phys. Rev. Lett.* **108**, 094501 (2012).
- [43] I. Marusic, J. P. Monty, M. Hultmark, and A. J. Smits, On the logarithmic region in wall turbulence, *J. Fluid Mech.* **716**, R3 (2013).
- [44] H. Lee and S.-H. Kang, Flow characteristics of transitional boundary layers on an airfoil in wakes, *J. Fluids Eng.* **122**, 522 (2000).
- [45] G. M. Corcos, The resolution of turbulent pressures at the wall of a boundary layer, *J. Sound Vib.* **6**, 59 (1967).
- [46] P. Welch, The use of fast fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms, *IEEE Trans. Audio Electroacoust.* **15**, 70 (1967).
- [47] A. Herrig, M. Kamruzzaman, W. Würz, and S. Wagner, Broadband airfoil trailing-edge noise prediction from measured surface pressures and spanwise length scales, *Int. J. Aeroacoust.* **12**, 53 (2013).
- [48] T. F. Brooks and M. A. Marcolini, Scaling of airfoil self-noise using measured flow parameters, *AIAA J.* **23**, 207 (1985).
- [49] S. Demange, Z. Yuan, S. Jekosch, E. Sarradj, A. Hanifi, A. V. Cavalieri, and K. Oberleithner, Identification of structures driving trailing-edge noise. Part I—Experimental investigation, [arXiv:2412.09536](https://arxiv.org/abs/2412.09536).
- [50] A. A. Oberai, F. Roknaldin, and T. J. R. Hughes, Computation of trailing-edge noise due to turbulent flow over an airfoil, *AIAA J.* **40**, 2206 (2002).
- [51] M. Soni, R. Ewert, J. Delfs, and K. Masilamani, Towards wall-modeled LES with lattice Boltzmann method for aeroacoustics: Application and understanding, in *28th AIAA/CEAS Aeroacoustics 2022 Conference, Aeroacoustics Conferences* (American Institute of Aeronautics and Astronautics, Southampton, UK, 2022).
- [52] J. Winkler, H. Wu, S. Moreau, T. Carolus, and R. D. Sandberg, Trailing-edge broadband noise prediction of an airfoil with boundary-layer tripping, *J. Sound Vib.* **482**, 115450 (2020).
- [53] M. Soni, R. Ewert, J. Dierke, and J. Delfs, Evaluation of the Noise Reduction Potential of a Krueger Flap High-Lift device via the lattice Boltzmann method, in *AIAA AVIATION 2023 Forum* (American Institute of Aeronautics and Astronautics, San Diego, CA, 2023).
- [54] G. Berkooz, P. Holmes, and J. L. Lumley, The proper orthogonal decomposition in the analysis of turbulent flows, *Annu. Rev. Fluid Mech.* **25**, 539 (1993).
- [55] L. Sirovich, Turbulence and the dynamics of coherent structures, *Q. Appl. Math.* **45**, 561 (1987).
- [56] J. Borée, Extended proper orthogonal decomposition: A tool to analyze correlated events in turbulent flows, *Exp. Fluids* **35**, 188 (2003).
- [57] B.-T. Chu, On the energy transfer to small disturbances in fluid flow (Part I), *Acta Mech.* **1**, 215 (1965).
- [58] L. M. Mack, Boundary layer stability theory, in *AGARD Conference Proceedings* (NATO, Pasadena, CA, 1984), Vol. 1, pp. 1–151.
- [59] A. Hanifi, P. J. Schmid, and D. S. Henningson, Transient growth in compressible boundary layer flow, *Phys. Fluids* **8**, 826 (1996).
- [60] B. Bugeat, U. Karban, A. Agarwal, L. Lesshafft, and P. Jordan, Acoustic resolvent analysis of turbulent jets, *Theor. Comput. Fluid Dyn.* **38**, 687 (2024).
- [61] J. H. M. Ribeiro, Analysis of coherent structures in wall-bounded turbulent flows using proper orthogonal decomposition, Tech. Rep., Dissertação (mestrado), Universidade Estadual de Campinas, Faculdade de Engenharia Mecânica, SP : [s.n.], 2017, <https://repositorio.unicamp.br/acervo/detalhe/983511>.
- [62] M. Rogowski, B. C. Y. Yeung, O. T. Schmidt, R. Maulik, L. Dalcin, M. Parsani, and G. Mengaldo, Unlocking massively parallel spectral proper orthogonal decompositions in the PySPOD package, *Comput. Phys. Commun.* **302**, 109246 (2024).

- [63] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/kxbq-ynzd> for 3D SPOD mode animation.
- [64] E. Pickering, G. Rigas, P. A. S. Nogueira, A. V. G. Cavalieri, O. T. Schmidt, and T. Colonius, Lift-up, Kelvin–Helmholtz and Orr mechanisms in turbulent jets, *J. Fluid Mech.* **896**, A2 (2020).
- [65] D. G. Crighton and M. Gaster, Stability of slowly diverging jet flow, *J. Fluid Mech.* **77**, 397 (1976).