

Transition from dripping to jetting of a film flowing down a vertical fiber

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Film flow on vertical fibers has been studied for the past 30 years. However, it is only recently that the influence of the nozzle on the film and bead dynamics has been demonstrated. Consequently, similar to the regimes observed with a faucet, both a dripping and a jetting regime have been reported for film flow on fibers. This study investigates the dynamics of thin liquid films on vertical fibers near the nozzle outlet using both experimental and numerical approaches, drawing insights from dripping faucet studies. This allows for a clear definition of the recently observed dripping and jetting regimes, with a particular focus on gaining a deeper understanding of the flow and formation of beads in the dripping regime, as well as the transition to jetting.

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I. INTRODUCTION

Subjected to gravitational force, a fluid exiting from a vertically aligned nozzle or faucet can exhibit different dynamics depending on the initial velocity of the flow at the nozzle outlet. When the mass flow rate or the flow velocity is low, dripping is observed, characterized by the formation of droplets and subsequent periodic or aperiodic dripping of them from the nozzle. In this regime, depending on factors such as the geometry of the nozzle and fluid properties, the time interval between successive droplets can vary with varying mass flow rate. In the jetting regime, the fluid flows out of the nozzle in a continuous stream, which also eventually breaks up into droplets because of the Rayleigh-Plateau instability mechanism. The transition from dripping to jetting usually occurs as the flow rate increases beyond a certain threshold.

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The study of droplets dripping from an orifice has a long history, beginning with Tate's investigation of droplet size as they drip from a tube [1]. Tate explored how the weight of a drop correlates with the surface tension of the liquid and the diameter of the tube, as described by Tate's law, $M_{\text{drop}} = 2\pi R_o \sigma / g$, with M_{drop} being the mass of a drop and R_o , σ and g being the outer radius of the tube, the surface tension of the fluid and the gravitational acceleration, respectively [1]. Later, Rayleigh modified Tate's equation by replacing 2π with a constant that being a function of the nozzle radius and capillary length l_c , $M_{\text{drop}} = f(R_o/l_c) R_o \sigma / g$, with $l_c = \sqrt{\frac{\sigma}{\rho g}}$ [2,3]. Twenty years later, Harkins and Brown demonstrated that $f(R_o/l_c)$ is correlated by a third-order polynomial, which became known as the Harkins and Brown factor. This correlation was precise enough to suggest an alternative method, the drop-weight method, to measure the surface tension of liquids, which is still listed as one of the standard techniques for measuring surface tension today [4]. Subsequently, further research extended the Harkins factor and the correlation to a wider range of fluid viscosity, surface tension, and different flow conditions [5–7].

In 1985, Shaw [8] discussed the nonlinearity of the dripping faucet problem, emphasizing its dependence on a wide range of influencing factors. When the periodic time between two consecutive drops is monitored over varying mass flow rates, from a certain mass flow rate, the system exhibits an unpredictable and rather chaotic behavior before it transits into the jetting regime. Shaw then approached the topic through the lens of chaos theory and nonlinear dynamical systems, opening another area of research in the study of dripping faucets [8]. Since then, to comprehend the dynamics of drop formation at an orifice or nozzle in the dripping regime, extensive experimental and theoretical research have been conducted to date [9–14]. These studies aimed to investigate the size of the drops and the time interval between the drops on the basis of the fluid properties, nozzle geometry, and the flow rate of the liquid through the nozzle. Several studies have also aimed to identify the transition from the dripping to jetting regime [12,15,16], some of which will be reviewed below.

For a Newtonian liquid of viscosity μ , as the mass flow rate $\dot{m}$ increases, the following dripping subregimes are indicated: at small $\dot{m}$, formation of a primary drop followed by a much smaller satellite drop is observed. With increasing $\dot{m}$, period-1, period-doubling up to period- n and chaotic dripping can be observed for a selected nozzle [8,10,11]. High viscous liquids have been reported to bypass different subregimes of dripping, resulting in a direct transition from a 1-period dripping regime to a jetting regime [12,13]. Later, it will be shown that this is also what is observed through our study, since the viscosity of the working fluid in this work is 50 times higher than that of water. Another important point to note from the previous literature on dripping faucets is that the size of the drops in the dripping regime is not a function of the mass flow rate but is reported to be related only to the surface tension of the liquid and the geometry of the nozzle [1,3,9,11].

All the above-mentioned studies mainly investigated the formation and dynamics of drops at a faucet or nozzle in the dripping regime. Building on the classical Rayleigh model [2,3], numerous studies have also been conducted to analyze and predict the size and speed of droplets produced by jet breakup, utilizing linear and nonlinear stability analyses [17–20]. However, a comprehensive and detailed review of these studies is beyond the scope of this work.

Review of the literature on film flow on vertical fibers also indicates that most studies focus on the breakup of beads on fibers in the jetting regime as the beads form at the end of a stable, unperturbed film length (jet) near the nozzle, which is known as the healing length along which the Nusselt film prevails [21–23]. In the current study, the focus will be on film flow on fibers in the dripping regime, with bead formation at the nozzle (with the fiber at its center) and the transition to the jetting regime.

The study of film flow on fibers began with the pioneering work of Quéré [24], who conducted an experiment by drawing a fishing line out of a silicone bath and identified a critical film thickness at which undulations (beads) begin to form on the fiber. Quéré demonstrated that the formation of beads is governed by Rayleigh-Plateau instability and that the critical thickness below which the instability is suppressed scales as R^3 , with R the radius of the fiber. This implies that films on smaller fibers are less stable, and for fibers with smaller radius, even a very thin film will be prone

to bead formation. Two years later, Frenkel [25] derived a system of approximate equations for film thickness based on the assumption of a very thin film, which confirmed Quéré's observations theoretically. Kalliadasis and Chang [26] also performed an extensive numerical analysis by solving Frenkel's model equation and associated the rapid growth of the amplitude and speed of the solitary wave solutions with the formation of drops in Quéré's experiment. A decade later, Kliakhandler *et al.* [27] introduced a phenomenological classification of the existing regimes for film flow along vertical fibers. In their experiments, they used a container with a hole at its center through which a fiber was stretched, allowing the fluid to flow through the hole along the fiber. The regimes identified by [27] are described as follows: Regime (c) corresponds to the flow with the lowest mass flow rates, characterized by large, widely spaced beads. Despite the lowest mass flow rate, the beads in this regime are reported to be the largest and fastest compared to those in other regimes. Regime (b) belongs to moderate to relatively small flow rates. In this regime, an organized bead train with the same size, space, and speed is observed. Regime (a) corresponds to higher flow rates. The structure of beads in this regime is random and unpredictable. They move rapidly and irregularly, resulting in uneven spacing between the beads and collisions with each other. The influence of the orifice on the flow was not discussed in Ref. [27]. Nevertheless, under the assumption of creeping flow with no inertia terms, and by neglecting the diffusion term in the streamwise direction, a one-equation model (known as the KDB model) was derived. This model follows the assumptions of the lubrication theory, but it retains the full curvature term instead of simplifying it asymptotically.

Since the KDB regimes were established, a number of theoretical studies have been carried out to explore the dynamics of the instabilities linked to these regimes [21,22,28–31]. Craster and Mater [28] developed an equation model similar to the KDB model under the asymptotic assumption that the sum of the film thickness and fiber radius is much smaller than the capillary length. Although their model does not include the full curvature term, their computations were still reasonably consistent with KDB regimes and their own experimental results. Later, Ruyer-Quil *et al.* [22] introduced a two-equation second-order model for film thickness and flow rate based on boundary layer theory and the weighted residual method, known as the Weighted Residual Integral Boundary Layer (WRIBL) model. In contrast to the two earlier models, this model includes inertia and streamwise viscous diffusion terms and was able to capture the three identified KDB regimes with good agreement. Duprat *et al.* [21] took a different approach to describe the flow. They conducted a linear stability analysis using models introduced by Frenkel [25], Kliakhandler *et al.* [27], and Craster and Matar [28], and derived their dispersion relation. On the basis of these dispersion relations, they demonstrated the transition from absolute to convective regimes and their association with regular and irregular wavy patterns. If beads form faster than they are advected over their entire wavelength, regular bead patterns emerge and the instability is said to be absolute. In contrast, at higher flow rates, beads are advected faster than they grow, resulting in a convective instability and leading to an irregular bead pattern.

Although film flows on fibers have been studied for more than two decades, the impact of the nozzle on flow was only recently examined by Sadeghpour *et al.* [23]. In this work, the flow regimes are classified as the isolated droplet regime for small mass flow rates, the Rayleigh-Plateau regime for moderate flow rates, and the convective regime for higher mass flow rates with irregular beads. Sadeghpour *et al.* demonstrated that, for a constant mass flow rate, simply changing the nozzle's outer diameter can alter the bead dynamics (size, frequency, spacing) within the Rayleigh-Plateau regime. Later, Ji *et al.*, from the same research group, included a correction term in the simplified first-order WRIBL model of Ruyer-Quil *et al.* [22] to account for the influence of the nozzle. They also expanded the WRIBL model to include a slip boundary condition along the fiber surface and considered the influence of the nozzle [33]. Gabbard and Bostwick [34] also conducted extensive experimental studies on the influence of the nozzle on the flow. In their setup, however, the fiber does not pass through the nozzle, instead, the nozzle is positioned perpendicular to the fiber, with the fluid injected into the fiber from the side. Despite this different configuration, they observed the same flow regimes as in previous experiments. Their work also further demonstrates that for a constant mass flow rate, only varying the diameter of the nozzle can lead to different flow regimes.

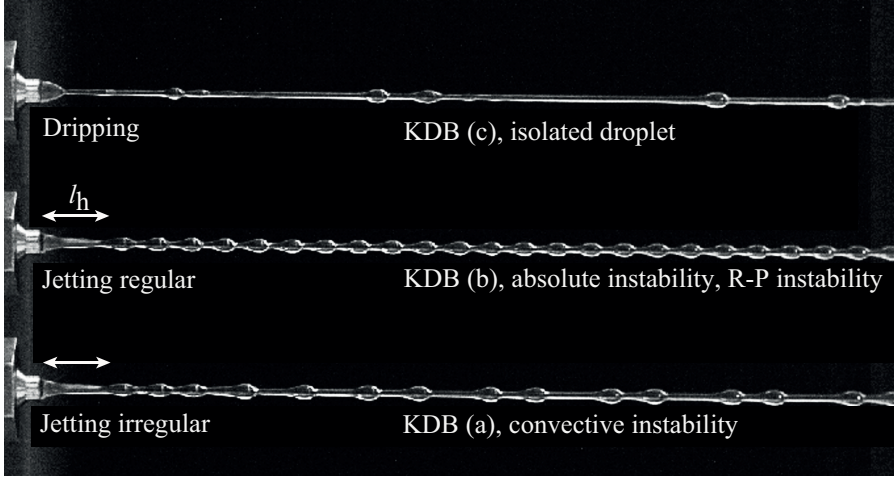


FIG. 1. Film flow regimes on vertical fibers as characterized by the dripping and jetting regimes from [32], by the KDB regimes [(a)–(c)] from [27], by the absolute and convective regimes from [21], and by the isolated droplet, Rayleigh-Plateau and convective instability regimes from [23].

As noted, previous studies on film flows along vertical fibers, including those by Sadeghpour *et al.* [23] and Ji *et al.* [33,35], have primarily investigated flow and bead dynamics from the perspective of nonlinear dynamical systems, with a focus on regimes characterized by beads arising from film instabilities or jet breakup. While most of these works do report isolated bead formations at low flow rates, they do not address the underlying mechanism of bead formation at the nozzle, nor do they distinguish this process from the formation of beads downstream, at the end of a jet. To the best of our knowledge, there is no robust, quantitative indicator in the prior literature for the transition from the isolated-droplet regime (dripping) to the subsequent jetting regime as the mass flow rate increases. This work addresses this gap by introducing robust transition indicators, supported by experiments and a simplified one-dimensional model.

In our recent studies [32,36], we classified flow regimes based on principles similar to those used in dripping faucet studies, identifying a dripping regime for low mass flow rates and a jetting regime for high mass flow rates. This classification of flow regimes not only describes the dynamics of isolated droplets at low flow rates but also enables prediction of the transition to the subsequent regime. Figure 1 illustrates these flow regimes and their equivalent classification in the earlier literature. In the dripping regime, beads form directly at the nozzle, whereas in the jetting regime, they form at the end of the healing length, l_h , the distance over which surface perturbations in the film are negligible. Regular and irregular beads are reported within the dripping and jetting regimes in the previous studies [32,36]. However, in the dripping regime, the irregular bead configuration reported in [32] is attributed to the collision of isolated beads formed at the nozzle with undulations along the fiber induced by the Rayleigh-Plateau instability. This is also evident in the dripping regime shown in Fig. 1. This phenomenon occurs only when the film is sufficiently thick and the fiber is sufficiently thin. Since the irregularity in the dripping regime is not associated with bead formation at the nozzle, it is not regarded as a distinct flow regime in this study. Here, we examine only the bead formation at the nozzle, which remains consistently regular throughout our experiments as a result of the use of a highly viscous liquid.

Furthermore, this study focuses solely on identifying the transition from dripping to jetting, which is not necessarily related to the transition from the dripping regime to the regular jetting regime, illustrated in Fig. 1. As reported by Gabbard and Bostwick [34] and Duprat *et al.* [21], thicker fibers may bypass the regular jetting (R-P or absolute instability) regime and transition

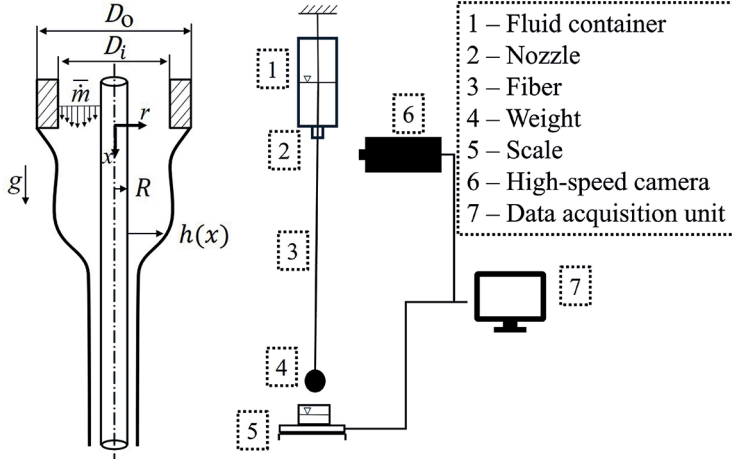


FIG. 2. Experimental setup. A sketch of the entire apparatus is shown on the right, an enlarged view of the nozzle on the left.

directly from the dripping (isolated droplets) regime to the jetting-irregular (convective instability) regime.

In Sec. II, the experiment, the evaluation method, the evaluated parameters, and the model used and its scaling parameters will be presented. Section III presents the results, starting with an overview of the mechanisms responsible for bead formation in the dripping and jetting regimes and then introducing a method for identifying the transition between these two regimes. Conclusions are presented in Sec. IV.

II. MATERIAL AND METHODS

In this study, following previous research on dripping faucets, two characteristic flow parameters in the dripping regime, namely, bead size and bead frequency, will be discussed. The results will then be used to address the transition point from the dripping to the jetting regime. This section briefly presents the experiment and the model used.

A. Experiments

Figure 2 illustrates a schematic overview of the setup and flow. The fluid in the reservoir, silicone oil V50 (with a density of $\rho = 948 \text{ kg/m}^3$, kinematic viscosity $\nu = 50 \text{ mm}^2 \text{ s}^{-1}$, and surface tension $\gamma = 0.021 \text{ Nm}^{-1}$), flows through a nozzle with a nylon fiber in its center and slides down along the fiber. An overview of the varying parameters is given in Table I.

Note that, since the liquid fully wets the nozzle, the outer diameter ($\bar{D}_o$) is the characteristic nozzle parameter influencing the flow [3,15,23]. Therefore, in our measurements, only the outer diameter of the nozzle is varied. The inner diameter ($\bar{D}_i$) is chosen to be as small as possible to ensure controllable low flow rate measurements throughout the entire test campaign.

TABLE I. Range of controlled parameters.

Fiber radius	$\bar{R} = 0.172, 0.225, 0.275 \text{ mm}$
Nozzle diameters	$\bar{D}_i = 1 \text{ mm}; \bar{D}_o = 2, 3, 4 \text{ mm}$
Mass flow rate	$\bar{m} = 5\text{--}50 \text{ mg/s}$

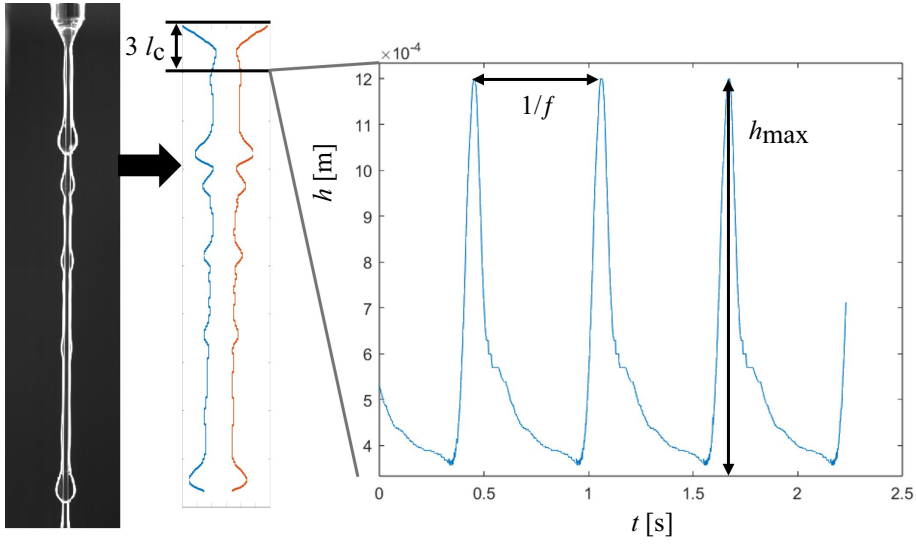


FIG. 3. Film thickness profile over time at the fixed location $x = l_c$ after image processing and the evaluation of the dripping frequency and crest height of the beads.

The flow at different mass flow rates is optically observed by a high-speed camera with a frame rate of 500 images per second. Image processing relies on edge detection techniques to detect the film contours on the fiber in both time and space.

Figure 3 illustrates the assessment of the film thickness profile over time using image analysis, as well as the evaluation of the maximum film thickness $h_{\max}$, and the bead frequency f of the dripping beads from the nozzle in the dripping regime. The evaluation is performed at a fixed location, $x = 3 l_c$, corresponding to three times the capillary length. This distance is selected based on experience, ensuring that the beads have already fully evolved, though they have not yet undergone any secondary instability or change in size as they flow along the fiber. The maximum film thickness refers to the film thickness at the crest height of the beads and will later be used as a parameter for comparing the bead size. The frequency of the beads is calculated as the inverse of the time interval between two beads at the selected location and for a constant mass flow rate.

Another parameter discussed later in the Results section is the point of pinching (POP). This point is assessed in a similar way to the Clanet and Lasheras study on the transition from dripping to jetting at nozzles (without fiber) [15]. In their study, the length z_{tip} represents the distance from the nozzle to the tip of the hanging droplet as it grows over time until detachment. In our study, the presence of a fiber at the center of the nozzle causes the film to flow continuously along the fiber, preventing the formation of a distinct droplet tip. Consequently, we defined POP as an equivalent assessment parameter.

POP refers to a point in the spatial coordinates where the accumulating fluid on the nozzle exhibits a neck for the first time. This can be detected by measuring the slope of the film profile at the interface. Before a bead is fully formed, the accumulating fluid at the nozzle always shows a negative interfacial slope, as the film profile gradually decreases from the outer diameter of the nozzle to meet the lubricating film on the fiber. Once the fluid reaches a critical size, a neck is formed, signaled by a sudden positive slope at the interface. At this point, the interface collapses on a timescale much shorter than the bead formation time, causing the bead to detach from the nozzle. Figure 4 shows the position of the points where a positive slope is detected at each frame rate. The diagram at the bottom of the figure shows the trajectory of all the points, which is assessed at each frame with a temporal resolution of 2 ms. In the top diagram, these points are illustrated in the flow

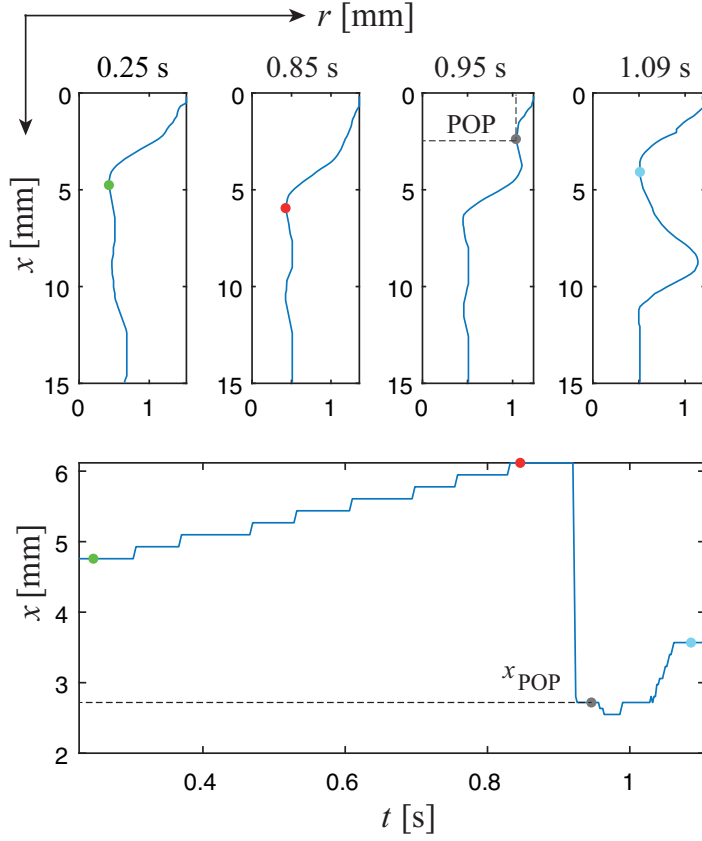


FIG. 4. At the top, the film profile along the fiber of radius $\bar{R} = 0.225$ mm, starting from the nozzle, is shown at different times. At the bottom, the diagram shows the trajectory of points along the x axis, starting from the nozzle, where the slope of the film interface first becomes positive. The circular markers indicate these axial positions over time. The gray circle denotes the point of pinching (POP), marking the onset of necking and its position along the axes.

at exemplary time instances. To reduce noise, a moving average filter is applied to the detected film profile at the interface.

As fluid continues to accumulate at the nozzle, the point with positive slope shifts further downstream, moving away from the nozzle (see the first two profiles at the top of Fig. 4). This leads to a gradual increase in the trajectory shown in the bottom diagram of Fig. 4. Once the neck is formed, the point with positive slope suddenly jumps to a position close to the nozzle, and within a few milliseconds, the pinch-off occurs. This point is referred to as the point of pinching (POP). The x and r coordinates of the POP can be determined through image processing. As illustrated in Fig. 2, x_{POP} is measured starting from the lower end of the nozzle and r_{POP} from the center of the fiber.

B. Model formulation

In this study, the KDB model [27] is used for theoretical analysis. To account for the influence of the nozzle on the flow, a correction term is incorporated into the model equation. The derivation of the model is described in the following. The dimensional quantities are represented with a bar over the variable when necessary to distinguish them from the dimensionless counterpart.

The dimensionless governing equations for a Newtonian fluid with constant viscosity and density, flowing axisymmetrically under the influence of gravity down a vertical fiber, as illustrated in Fig. 2, are defined as follows:

$$\frac{1}{r}(rv)_r + u_x = 0, \quad (1)$$

$$\text{Re}(u_t + uu_x + vu_r) = -p_x + \frac{(ru_r)_r}{r} + u_{xx} + 1, \quad (2)$$

$$\text{Re}(v_t + uv_x + vv_r) = -p_r + \left(\frac{(rv)_r}{r}\right)_r + v_{xx}, \quad (3)$$

where the subscript denotes the partial derivative. Coordinates x and r are scaled by the unperturbed Nusselt film thickness $\bar{h}_N$. The axial and radial velocity components, u and v , respectively, are scaled by $\bar{U} = g\bar{h}_N^2/\nu$. The time t is normalized by $\bar{h}_N/\bar{U}$. The pressure p is in units of $\rho g\bar{h}_N$ and the Reynolds number is defined as $\text{Re} = \bar{U}\bar{h}_N/\nu = g\bar{h}_N^3/\nu^2$.

At fiber surface, $r = R = \bar{R}/\bar{h}_N$, a no-slip and no penetration boundary condition is imposed:

$$u(r = R) = v(r = R) = 0. \quad (4)$$

At the free surface, $r = R + h(x, t)$, the tangential stress is set to zero:

$$(1 - (h_x)^2)(u_r + v_x) + 2h_x(v_r - u_x) = 0, \quad (5)$$

and the normal stress is balanced by the full curvature, which accounts for the surface tension with consideration of both axial and radial curvatures of the film at the free interface:

$$p - p_0 + \frac{2}{(1 + h_x^2)}[(u_r + v_x)h_x - v_r - h_x^2 u_x] = \frac{1}{Bo} \left[\frac{h_{xx}}{(1 + h_x^2)^{3/2}} - \frac{1}{(R + h)(1 + h_x^2)^{1/2}} \right], \quad (6)$$

where p_0 is the pressure of the ambient gas and $Bo = \rho g\bar{h}_N^2/\sigma$ denotes the Bond number. Note that in this formulation, the curvature term on the right-hand side is expressed with a positive sign, so that the pressure jump is positive when the interface is convex toward the air. The kinematic boundary condition at the free surface is:

$$h_t + uh_x - v = 0. \quad (7)$$

Integrating the continuity equation (1) along the film and applying noslip (4) and kinematic (7) boundary conditions, the conservative form of the kinematic condition is derived as

$$h_t + \frac{1}{R + h} \left(\int_R^{R+h} ur dr \right)_x = 0, \quad (8)$$

where $\int_R^{R+h} ur dr$ represents the volume flow rate q per unit circumference in the axial direction.

1. Nusselt flow

To determine the dimensional flow rate $\bar{Q}_N$ as a function of the Nusselt film thickness $\bar{h}_N$, assuming a stationary ($\partial_t \rightarrow 0$) and unidirectional ($v \rightarrow 0$) flow in the x direction with negligible gradient in the streamwise direction ($\partial_x \rightarrow 0$), the dimensional x -momentum equation can be simplified to

$$g + \nu \frac{1}{\bar{r}} (\bar{r} \bar{u}_r)_r = 0. \quad (9)$$

By solving Eq. (9), using a no-slip condition at the fiber and zero friction at free surface ($\partial u / \partial r = 0$), the Nusselt velocity u_N is derived as

$$\bar{u}_N(r) = \frac{g}{4\nu} \left(\bar{R}^2 - \bar{r}^2 + 2(\bar{h}_N + \bar{R})^2 \ln \frac{\bar{r}}{\bar{R}} \right). \quad (10)$$

The volume flow rate as a function of Nusselt thickness within the healing length can then be derived by integrating the flow velocity across the film:

$$\begin{aligned}\bar{Q}_N &= 2\pi \int_{\bar{R}}^{\bar{R}+\bar{h}_N} \bar{r} \bar{u}_N(\bar{r}) d\bar{r} \\ &= \frac{\pi g}{2\nu} (\bar{h}_N + \bar{R})^4 \ln \frac{\bar{h}_N + \bar{R}}{\bar{R}} \\ &\quad - \frac{\pi g \bar{h}_N}{8\nu} (3\bar{h}_N^3 + 12\bar{h}_N^2 \bar{R} + 14\bar{h}_N \bar{R}^2 + 4\bar{R}^3).\end{aligned}\quad (11)$$

In dimensionless form, the expression for the Nusselt volume flow rate per unit circumference becomes

$$\begin{aligned}q_N(h) &= \int_R^{R+h} r u_N(r) dr \\ &= \frac{(h+R)^4}{4} \ln \frac{h+R}{R} \\ &\quad - \frac{h}{16} (3h^3 + 12h^2 R + 14hR^2 + 4R^3).\end{aligned}\quad (12)$$

2. KDB model

The derivation of the KDB model is closely aligned with the boundary layer approximation, which assumes that the velocity gradient in the streamwise direction and the radial velocity itself are negligible. However, since the pressure gradient in the axial direction p_x is retained in the momentum equation, the model cannot be considered an asymptotic model. Below is a brief review of the model derivation and the assumptions made.

Assuming a unidirectional flow ($v = 0$) and a creeping flow in the x direction, the convective terms in the x -momentum equation are neglected. Additionally, because of the small velocity gradient, the diffusion term in the streamwise direction is also neglected. Consequently, the momentum equations in streamwise and radial directions are reduced to

$$p_x = \frac{(ru_r)_r}{r} + 1, \quad (13)$$

$$p_r = 0. \quad (14)$$

For flow interface boundary conditions, simplifications are made, including neglecting viscous terms in the normal force balance (6), as the pressure jump at the interface is assumed to dominate over viscous forces. In addition, the influence of the ambient gas is neglected. Consequently, the pressure from (14) is constant across the film and expressed as

$$p = \frac{1}{Bo} \left[\frac{h_{xx}}{(1 + (h_x)^2)^{3/2}} - \frac{1}{(R+h)(1 + (h_x)^2)^{1/2}} \right]. \quad (15)$$

Assuming that the gradients in the streamwise direction are small ($\partial_x \rightarrow 0$), the tangential stress balance at the film interface (5) reduces to

$$u_r|_{R+h} = 0. \quad (16)$$

Solving (13) with the given boundary conditions [(4) and (16)], substituting (15) and inserting the solution for u in (8), leads to the following one equation model:

$$\begin{aligned}h_t + \frac{1}{R+h} q_x &= 0, \\ q &= q_N(1 + p_x),\end{aligned}\quad (17)$$

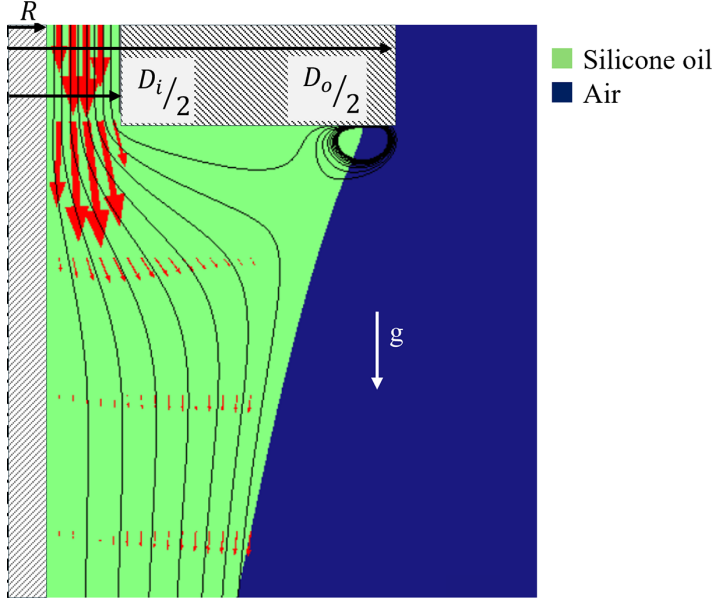


FIG. 5. Illustration of the velocity field and streamlines of the film beneath a nozzle, with the fiber positioned at its center, generated using Comsol Multiphysics for silicone oil V50.

where q_N and p are given in Eqs. (12) and (15), respectively, and q represents the local volume flow rate per unit circumference.

Equation (17) was developed by Kliakhandler *et al.* [27] to model film flow on vertical fibers and has demonstrated reasonable agreement with their experimental results for flow along the fiber, away from the nozzle and without accounting for nozzle effects. However, since our study focuses on the flow near the nozzle, considering the influence of the nozzle wall is crucial to accurately predict the transition point from dripping to jetting. To evaluate the impact of the nozzle on the flow, a detailed analysis of the flow dynamics directly beneath the nozzle is essential.

The presence of the nozzle wall near the outlet imposes a no-slip boundary condition at the nozzle wall, resulting in damping effects on the flow velocity close to the nozzle. This leads to a thicker film near the nozzle, which gradually thins with increasing distance from nozzle (see Fig. 4).

To illustrate the effect of the nozzle on the flow, a direct numerical simulation performed with COMSOL Multiphysics is presented in Fig. 5. The figure shows the velocity field and streamlines as the fluid flows out of the nozzle, with the fiber positioned at the center. As depicted, the zero velocity on the nozzle wall creates a wake surrounding the fluid exiting the nozzle. This wake induces a large velocity gradient and therefore friction within the film close to the nozzle, leading to a reduction in velocity u in the x direction. According to the continuity principle, this reduction in the axial velocity u generates flow in the radial direction, resulting in a thicker film.

As the fluid flows down the fiber, the large velocity gradients are gradually smoothed out by momentum diffusion. Gravity then reestablishes the x direction as the primary flow direction, resulting in a steady unidirectional lubricating film along the fiber. This film is characterized by the Nusselt film thickness and, for a given mass flow rate is thinner than the actual film near the nozzle.

Since the wake region beneath the nozzle wall at the outlet is small and since beyond this area, the flow dynamics is in accordance with the KDB model, a correction term γ is introduced and incorporated into the model equation without increasing its complexity. This correction increases

the film thickness near the nozzle by applying a damping effect to the flow rate terms in Eq. (17):

$$\begin{aligned} h_t + \frac{1}{R+h} q_x &= 0, \\ q &= q_N(1 + p_x) - \gamma q_x; \quad \gamma = aRe \exp(-bx). \end{aligned} \quad (18)$$

The parameters a and b in γ are determined based on the experimental results. In this work $a = 4$ and $b = \frac{\bar{h}_N}{2 \text{ [mm]}}$. The damping factor γ is selected based on the following criteria: first, it should decrease rapidly enough with x , ensuring that it has no effect below the wake region. Second, it should increase with the mean flow rate (Re), as a higher velocity at the nozzle outlet results in larger velocity gradients and friction beneath the nozzle and therefore a locally thicker film. The exponential function ensures that the influence of γ becomes negligible for the flow farther away from the nozzle. The rate of decay is controlled by the characteristic length factor b .

For the numerical implementations, we primarily followed the steps described by Ji *et al.* [35]. The fourth-order partial differential equation (18) is solved by decomposition into a system of first-order differential equations. The boundary conditions are selected based on a similar principle described by [35], where the film is pinched at the nozzle outlet to account for the nozzle's influence on the flow dynamics. Consequently, two Dirichlet boundary conditions are imposed at the nozzle for both the film thickness h and the flow rate q :

$$\begin{aligned} h(0, t) &= h_0 = \frac{\bar{D}_o - 2\bar{R}}{2\bar{h}_N}, \\ q(0, t) &= \frac{\bar{m}}{2\pi\rho\left(\frac{g\bar{h}_N^2}{v}\right)\bar{h}_N^2}, \end{aligned} \quad (19)$$

with $\bar{m}$ representing the measured average mass flow rate from the measurements. The outer diameter, $\bar{D}_o$, is selected as the nozzle parameter, since in the experiments the silicon oil wets the nozzle. Similar to [35], we impose a soft boundary condition at the outlet by replacing Eq. (18) by $h_t + h_x = 0$ at the last two nodes of the domain. This condition allows the liquid to exit the computational domain without reflection and prevents liquid accumulation at the fiber (domain) end [22,35]. Enforcing $h_t + h_x = 0$ at the last two grid nodes provides the two downstream boundary conditions needed to close Eq. (18), which is fourth order in space.

As an initial condition, a constant flow rate and a piecewise linear profile are used:

$$q(x, 0) = \frac{1}{3}, \quad (20)$$

$$h(x, 0) = \begin{cases} h_0 + (1 - h_0)\frac{x}{10}, & \text{for } 0 \leq x \leq \frac{2}{3}L, \\ 1, & \text{for } x \geq \frac{2}{3}L, \end{cases}$$

where $L = \bar{L}/\bar{h}_N$ is the dimensionless domain length. In this work, L was chosen such as to keep the dimensional length at 15mm, i.e., long enough to ensure that the outlet boundary condition does not influence the flow near the nozzle.

The PDEs are discretized using an implicit central difference scheme. The implicit equation system is solved with the Levenberg-Marquardt algorithm as implemented in MINPACK and wrapped in Scipy [37]. The domain length is divided into 80 segments, the independence of the solutions for higher numbers of segments having been checked. To speed up the computation, adaptive time stepping is used. The time step is shortened only at the moment of pinching. This is done by using the following approach to determine the size for the next time step:

$$\begin{aligned} \Delta t^{n+1} &= \Delta t_{\max} \cdot \left(0.1 + 0.9 \exp\left(\frac{-\Delta h^n}{c}\right) \right), \\ \Delta h^n &= \left| \min_i \{h_i^n - h_i^{n-1}\} \right|, \end{aligned} \quad (21)$$

where indices n and i refer to the temporal and the spatial steps, respectively; $\Delta t_{\max}$ and c are constants that have to be adjusted to the problem. In this work, simulations are performed using $\Delta t_{\max} = 0.9\Delta x$ and $c = 0.1$.

III. RESULTS

In this section, we first take a closer look at bead formation on a vertical fiber in the dripping and jetting regimes. This analysis provides insight into the processes that drive bead formation in each regime and help distinguish their underlying mechanisms. Next, we introduce the parameters used to characterize the two regimes. We then show how these parameters and the transition region are defined, first for the experimental results and then for the theoretical results. Finally, we present exemplary results for different fiber and nozzle geometries, comparing experimental measurements with simulation data.

A. Bead formation mechanism

Figure 6 illustrates the film within the evolution time of a bead at the nozzle in the dripping regime for three consecutive time instances. Figure 7 provides the same illustration for the formation of beads at the end of a jet stream in the jetting regime. In these figures, the blue axis and the corresponding curves represent the film contour, evaluated from experiments and numerical results. The black axis and the corresponding curves compare the pressures along the fiber axis, calculated using a simple linear hydrostatic pressure approximation, $p_x = \rho g$, and (15) that account for the capillary pressure by considering the two principal curvatures of the interface. The values of the maximum film thickness at the crest of the beads and the axial and radial POPs are provided on the right side of the figures for comparison.

By comparing the two figures across all time instances for both the dripping and jetting regimes, two main observations can be made. First, in the dripping regime, when comparing the maximum film thickness across all time instances, from time zero to the final diagram at time 60 ms when the bead is fully developed, the maximum thickness shows little to no change. In other words, in the dripping regime, the film thickness exhibits little to no positive growth. This indicates that in the dripping regime the bead formation mechanism is primarily driven by the neck formation and the collapse of the film interface, as observed at the POP and not a growing instability mechanism.

In contrast, as illustrated in Fig. 7, the jetting regime follows a completely different mechanism. Here, a clearly observable growth in the film thickness is evident in both the experimental and numerical results. Thus, in the jetting regime, a visible growth mechanism of film perturbations drives local film thickening, leading to bead formation. In contrast, in the dripping regime, the film thickness remains almost constant and beads form solely through periodic collapse of the interface or pinching. The second key difference in bead formation mechanisms between the dripping and jetting regimes can be observed by examining the pressure distribution at the film interface, shown as black curves in Figs. 6 and 7. Assuming that the pressure along the film can be approximated by (15) and comparing its variation with the hydrostatic pressure along the fiber axis (dashed linear black lines), clear differences in pressure behavior at the bead formation locations in the two flow regimes become apparent.

In the dripping regime (Fig. 6), from the nozzle outlet up to the pinching locations (where POP is observed in both experiment and model), the pressure within the film follows the hydrostatic pressure with a good approximation at all times. Consequently, it can be speculated that the flow in the dripping regime follows a quasistatic equilibrium between gravity and surface tension, with pinching occurring because of instant imbalance in this equilibrium as the mass increases by the constant film flow rate at the nozzle outlet. This is consistent with previous studies on dripping faucets, where the mass of droplets remains constant and is estimated using the force balance equation of gravity and surface tension [1,3,14].

At higher mass flow rates, in the jetting regime (Fig. 7), the pressure at the flow interface at the point of pinching shows deviations from the hydrostatic pressure and is, in fact, lower. This indicates

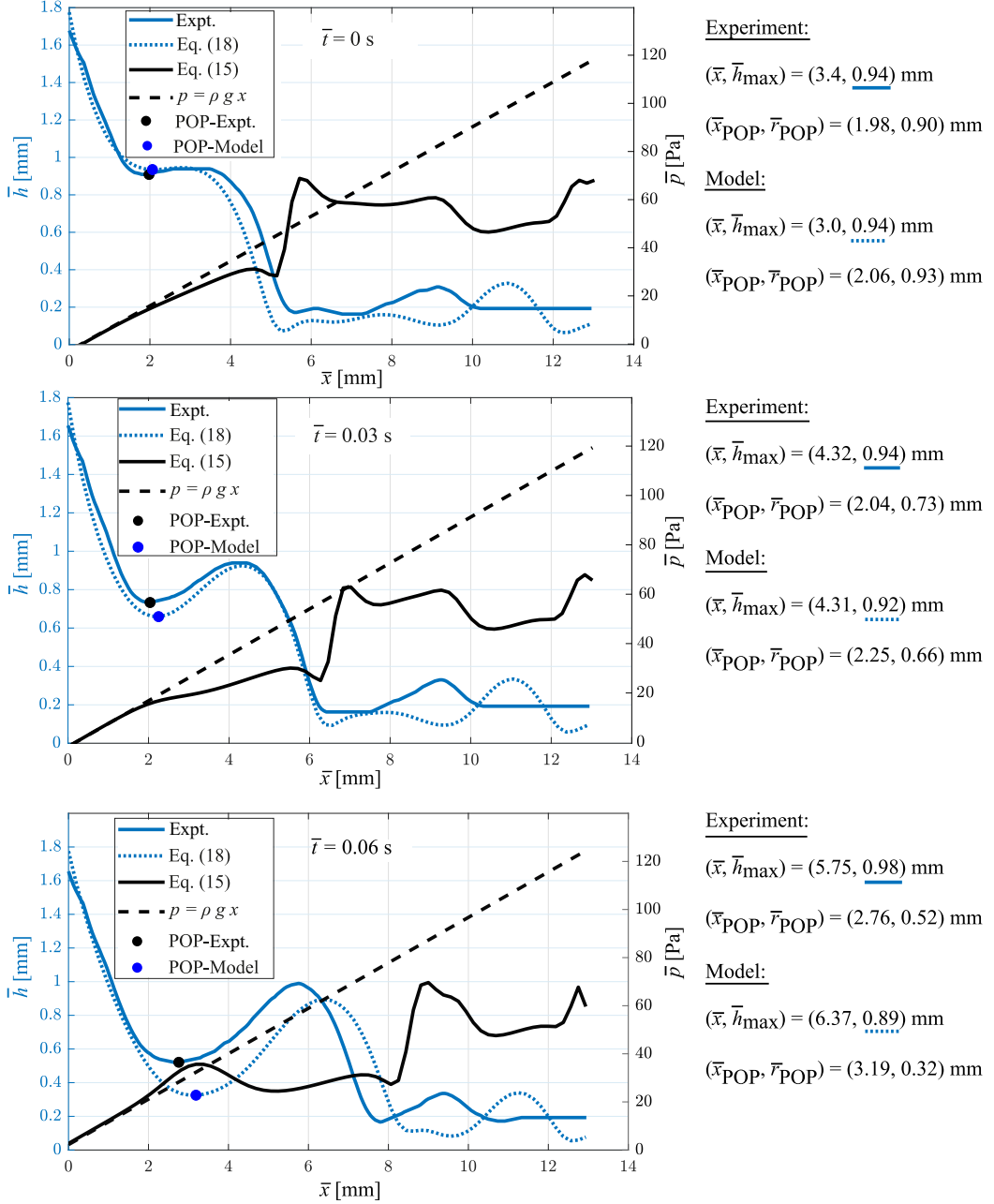
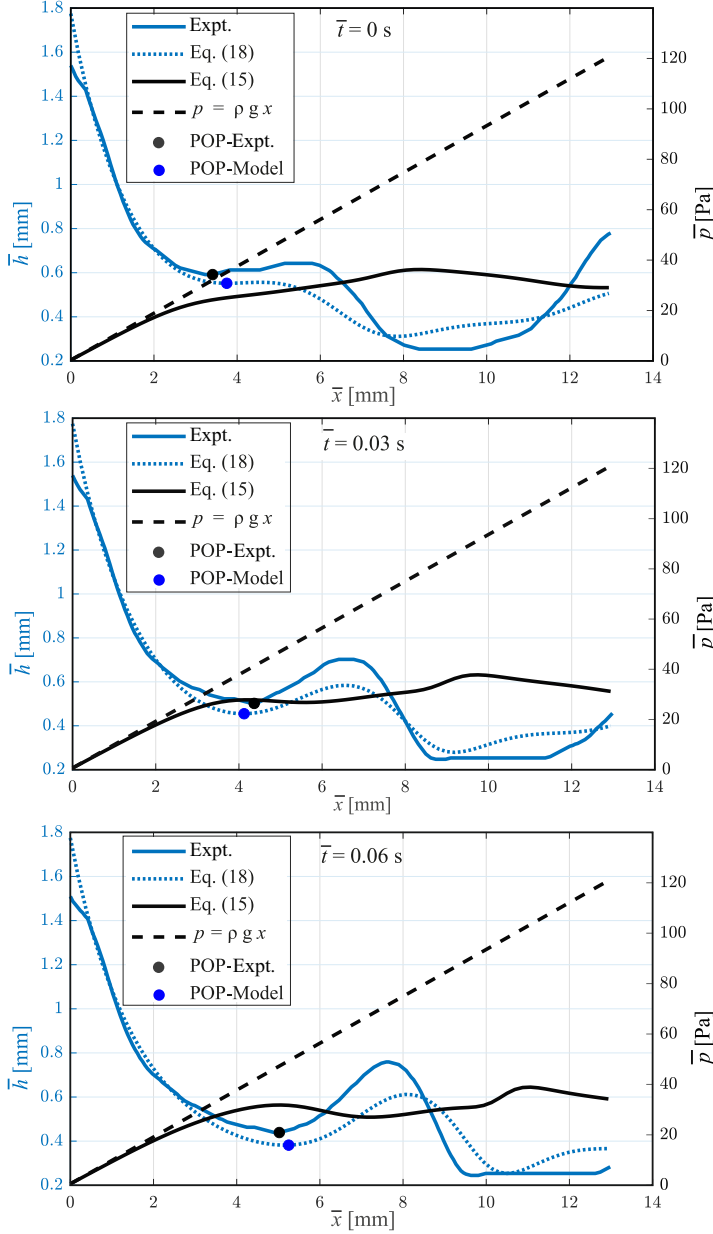


FIG. 6. Formation of beads in the dripping regime at three times: $t = 0 - 0.03 - 0.06 \text{ s}$. The blue curves represent the film thickness from both experiment and model. The black curves compare the pressure inside the film, computed using the linearized hydrostatic approximation ($p = \rho g x$) and Eq. (15); the linearized approximation is referenced to $p(x = 0)$ obtained from (15). On the right side, the maximum film thickness and its position along the x axis as well as the POP coordinates from experiment and model are provided for comparison. Mass flow rate $\dot{m} = 12.25 \text{ mg/s}$, fiber radius $\bar{R} = 0.225 \text{ mm}$ and nozzle outer diameter $\bar{D}_o = 4 \text{ mm}$.



Experiment:

$$(\bar{x}, \bar{h}_{\max}) = (5.56, \underline{0.64}) \text{ mm}$$

$$(\bar{x}_{\text{POP}}, \bar{r}_{\text{POP}}) = (3.41, 0.59) \text{ mm}$$

Model:

$$(\bar{x}, \bar{h}_{\max}) = (4.69, \underline{0.56}) \text{ mm}$$

$$(\bar{x}_{\text{POP}}, \bar{r}_{\text{POP}}) = (3.75, 0.55) \text{ mm}$$

Experiment:

$$(\bar{x}, \bar{h}_{\max}) = (6.64, \underline{0.70}) \text{ mm}$$

$$(\bar{x}_{\text{POP}}, \bar{r}_{\text{POP}}) = (4.37, 0.50) \text{ mm}$$

Model:

$$(\bar{x}, \bar{h}_{\max}) = (6.56, \underline{0.58}) \text{ mm}$$

$$(\bar{x}_{\text{POP}}, \bar{r}_{\text{POP}}) = (4.12, 0.45) \text{ mm}$$

Experiment:

$$(\bar{x}, \bar{h}_{\max}) = (7.75, \underline{0.76}) \text{ mm}$$

$$(\bar{x}_{\text{POP}}, \bar{r}_{\text{POP}}) = (5.02, 0.44) \text{ mm}$$

Model:

$$(\bar{x}, \bar{h}_{\max}) = (8.06, \underline{0.61}) \text{ mm}$$

$$(\bar{x}_{\text{POP}}, \bar{r}_{\text{POP}}) = (5.25, 0.38) \text{ mm}$$

FIG. 7. Same as in Fig. 6 for the jetting regime with $\bar{m} = 26 \text{ mg/s}$, $\bar{R} = 0.225 \text{ mm}$, and $\bar{D}_o = 4 \text{ mm}$.

that the force balance in the jetting regime no longer follows a quasistatic equilibrium. Additional effects such as viscous diffusion influence the flow distribution and the local pressure variations that develop along the film.

B. Regime map: Experimental results

In order to quantify the transition between dripping and jetting regime, two quantifying parameters are used: the bead frequency and the point of pinching (POP). Previous studies on dripping

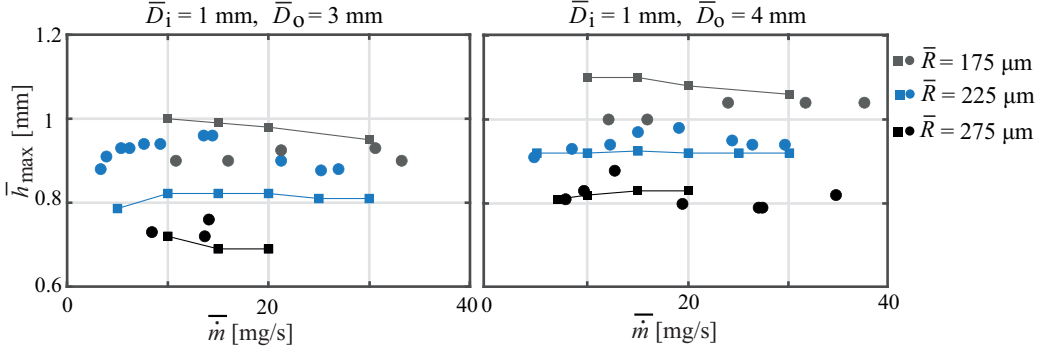


FIG. 8. Maximum film thickness, $h_{\max}$, representing bead crest height, versus mass flow rate in the dripping regime as obtained from the model (18) (squares) and from experiments (circles), for three different fiber radii.

faucets have shown that within the dripping regime, the size of detaching droplets remains constant and is not affected by variations in the mass flow rate for constant nozzle geometry and fluid properties [1,3,9,11]. This has been explained by the fact that, at low flow rates in the dripping regime, where inertia has minimal influence, droplet detachment is primarily governed by the balance between surface tension and gravitational forces.

To examine the size of beads in the dripping regime on vertical fibers, we evaluated the maximum film thickness, which indicates the crest height of the beads, for various mass flow rates using both experiments and numerical simulations. The results are presented for three different fiber radii and two nozzle configurations in Fig. 8. The diagrams indicate that the experimental results for the larger nozzle align more closely with the numerical predictions. Nonetheless, the results of both diagrams demonstrate that the crest height of the beads appears to be less affected by the mass flow rate for all investigated fibers, since the crest heights evaluated for each fiber are consistent, with deviations of less than 5% from their respective averages.

This observation leads to the conclusion that, as the mass flow rate increases, the frequency of bead formation at the nozzle must also increase. If the bead size is considered constant, a linear relationship exists between the frequency and the mass flow rate.

The total mass flow rate, $\bar{m}$, is decomposed as the sum of the mass of the detached beads over time (calculated as the product of the mass of the beads m_b and the frequency of the beads $\bar{f}$) and the “lubricating mass flow rate”, $\bar{m}_l$ defined as the mass transported by the lubricating film between two consecutive beads.

$$\bar{m} = \bar{m}_b \cdot \bar{f} + \bar{m}_l \quad \text{with} \quad \bar{m} = \rho \bar{Q}_N. \quad (22)$$

If we assume that the mass transported by the lubricating film is negligibly small, the bead frequency will become a linear function of the mass flow rate,

$$\bar{f} = \frac{\bar{m}}{\bar{m}_b}. \quad (23)$$

Figure 9 presents the measured frequencies for a set of experiments conducted on a fiber with a radius of $\bar{R} = 0.225$ mm and nozzle dimensions $\bar{D}_i = 1$ mm and $\bar{D}_o = 3$ mm. As discussed previously, the mass flow rate in the experiments is controlled by adjusting the height of the liquid in the fluid container (see the setup in Fig. 2). The arrows in Fig. 9(a) indicate the direction of incrementally increasing fluid height in the fluid reservoir. The results show that, although the fluid height in the reservoir increases at each consecutive measurement point, the mass flow rate unexpectedly decreases at a certain point, accompanied by a sudden jump to higher frequencies (see the measurements marked by red circles in Fig. 9). Next, it will be shown that this increase in frequency and the reduction in mass flow rate, despite an increase in fluid height within the

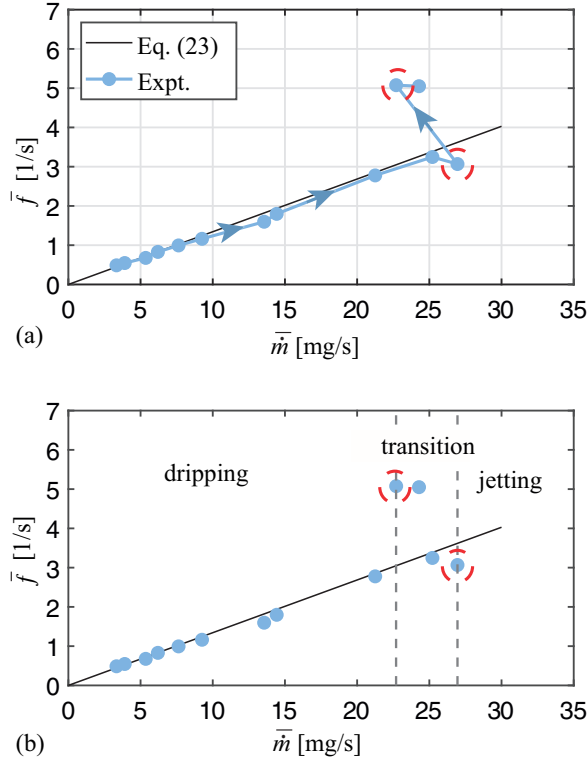


FIG. 9. (a) Experimentally measured bead frequency as a function of mass flow rate. The blue arrows indicate the direction of increasing fluid height in the reservoir during the measurements. Despite the increasing reservoir level, a decrease in the measured mass flow rate is observed at the onset of the jetting regime, accompanied by a sudden increase in bead frequency. This leads to the existence of low (dripping) and high (jetting) bead frequencies over a limited range of flow rates. (b) Definition of flow regimes based on the bead-frequency behavior. The transition region indicates the range of flow rates over which both dripping and jetting may be observed.

container, are indicators of the transition to the jetting regime. Figure 9 shows that in the dripping regime, while beads continue to form at the nozzle, a linear increase in frequency with constant slope (due to constant bead size) is observed for increasing mass flow rate. The experimental results coincide with the linear correlation given by (23).

The black linear curve in the figure is plotted by calculating the bead mass using the evaluated bead frequency of the first measurement at the lowest mass flow rate. Once the bead mass, which determines the slope in (23), is established, the frequency can be interpolated across the entire range of measured mass flow rates.

As the mass flow rate increases, a discontinuity and a sharp increase of frequency are observed in Fig. 9. This discontinuity is associated with the transition from dripping to the jetting regime. Once a jet is formed and beads are no longer generated at the nozzle, a sudden change (reduction) in bead size occurs, leading to the sharp increase in frequency.

In this study, for the experimental results, we define the dripping regime as the range in which the measured frequency matches or falls below the predicted values from the linear model (23) (black line in Fig. 9). Measurements showing frequencies higher than those predicted by the linear model are classified as belonging to the jetting regime. Accordingly, the boundaries of the experimental transition region are defined as the range between the highest mass flow rate at which the flow still

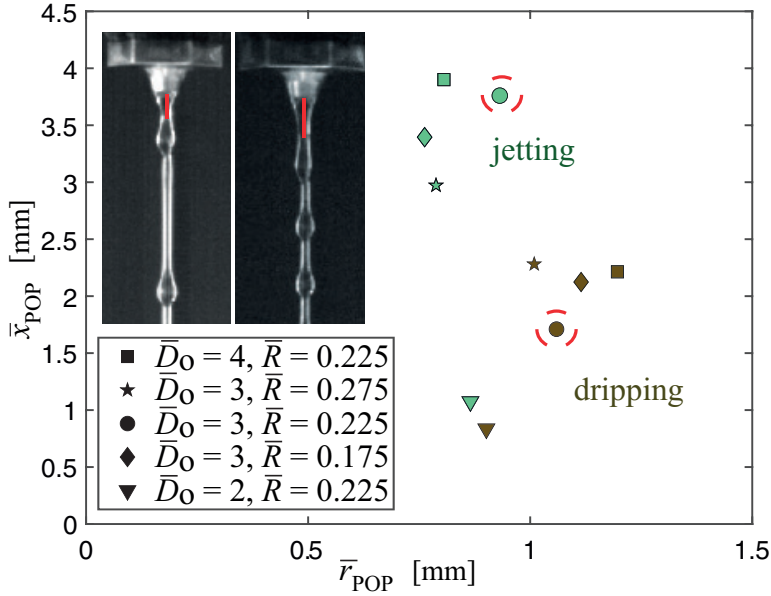


FIG. 10. POP coordinates at regime transition from dripping to jetting. The measurements corresponding to the presented results, for the specified radius and nozzle geometry in Fig. 9, are highlighted with red circles. The red lines in the pictures represent x_{POP} .

satisfies the dripping criterion and the lowest mass flow rate at which the jetting regime is observed for the first time. This transition is accompanied by an abrupt frequency jump occurring within this interval (see Fig. 9). In this range, the relationship between bead frequency and mass flow rate becomes nonlinear and discontinuous, indicating the presence of a transition region. To validate the observations in Fig. 9 and, consequently, to characterize the proposed regimes, the POP parameter is used.

Figure 10 shows the axial and radial components of the evaluated POPs for two consecutive measurements during the transition from dripping to jetting, using different fibers and nozzles. As also intuitively expected, the results in Fig. 10 show that the axial point of pinching x_{POP} increases in the jetting regime, as bead formation and pinching of the beads occur farther downstream, i.e., away from the nozzle. The results also indicate that once the flow transitions to jetting and beads begin to form along the jet stream instead of forming at the nozzle, the point of pinching in the radial direction, r_{POP} , decreases. This difference in r_{POP} between the dripping and jetting regimes becomes more pronounced when the outer diameter of the nozzle is significantly larger than the diameter of the fiber. The reduction in r_{POP} helps to explain why, despite the increasing height of the fluid in the container and therefore the increase in hydrostatic pressure at the nozzle outlet, the mass flow rate drops.

To explain this behavior, we begin with a simple approximation of the pressure inside the film, given by the Laplace pressure $p = 2\sigma\kappa$, where κ is the mean curvature of the interface. Here, the positive sign is used because the pressure jump from the liquid to the air is defined as positive. Neglecting the in-plane curvature, we can approximate $2\kappa \approx 1/r_{POP}$, and therefore the pressure inside the film near the nozzle can be written as $p \approx \sigma/r_{POP}$.

In Fig. 10, we observe that as the flow transitions to the jetting regime and beads form along the jet stream, r_{POP} becomes smaller. This occurs because the point of pinching of the beads, where the fluid pinches off, shifts away from the meniscus formed at the nozzle. As a result, a smaller r_{POP} is observed in the jetting regime, where the beads are formed along the jet. A reduced r_{POP} indicates

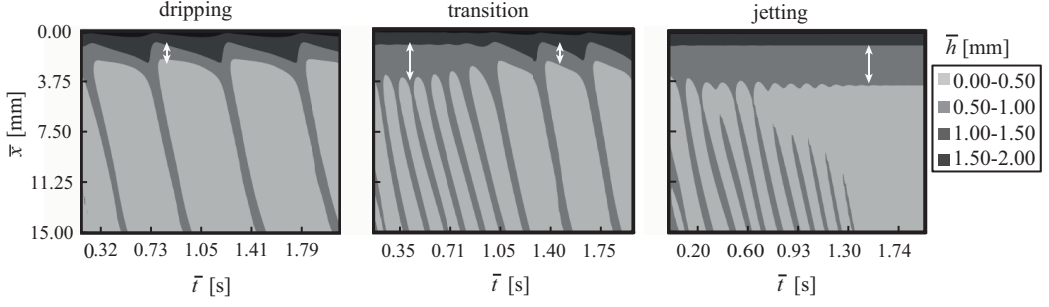


FIG. 11. Spatiotemporal diagrams from simulation results of (18), illustrating the flow behavior in the dripping, transition, and jetting regimes. Fiber radius $\bar{R} = 0.225$ mm and outer nozzle diameter $D_o = 3$ mm. White arrows mark the axial point of pinching, qualitatively.

a higher pressure within the fluid stream, suggesting that an increase in hydrostatic pressure at the outlet is necessary to drive the flow out of the nozzle with the same mass flow rate.

In conclusion, during the transition to the jetting regime, the diameter of the stream, which is represented here by r_{POP} , decreases and with it the pressure inside the film close to the nozzle increases. This leads to a reduction in the mass flow rate, despite an increase in hydrostatic pressure at the nozzle outlet due to a higher liquid level in the container compared to its previous measurement.

In Fig. 10, snapshots of the two consecutive measurement points in dripping and jetting regime, marked in red in both this figure and Fig. 9 are provided for better clarification. In these snapshots, in addition to the increase in x_{POP} , indicated by the red lines, the jump in frequency between the dripping and jetting regimes can be qualitatively detected by the decreasing space between the beads.

C. Regime map: Theoretical results

So far, the transition from dripping to jetting in the experiments has been addressed. Next, we will explain how the transition region from dripping to jetting in the numerical results is defined and identified. For ease of comparison with the experimental data, we refer to this as the numerical transition region.

Figure 11 shows spatiotemporal diagrams of three simulation results at increasing flow rates, illustrating examples of flow in the dripping, transition, and jetting regimes. Each diagram displays the film thickness on the fiber over a length of 15 mm from the nozzle and a period of approximately 1.8s. Each of the parallel dark gray lines in the top diagrams represents a single bead and its movement along the fiber over time and space. Similar to the experiments, the bead frequency in the simulations is evaluated by measuring the time interval between two consecutive beads.

In the first diagram on the left in Fig. 11, the fluid exhibits the characteristics of the dripping regime. This is easily detectable by the white double arrows, which provide a qualitative estimate of the axial pinching point of the beads x_{POP} . In this diagram, the white arrow has a minimal length, indicating that the beads are formed at the nozzle. Additionally, the equal spacing between beads in the spatiotemporal diagram indicates a single dominant bead frequency.

With increasing mass flow rate, the middle diagram in Fig. 11 shows the coexistence of the dripping and jetting regimes, reflected in two distinct bead frequencies. This is evident from the bead spacing in the diagram. Moreover, the white arrows indicate that regions of higher bead frequency correspond to substantially larger x_{POP} values, i.e., bead formation farther downstream, along a jet. In contrast, lower frequencies are associated with beads forming close to the nozzle, characterized by smaller x_{POP} . The mass flow rate interval over which both dripping and jetting can be observed is defined as the numerical transition region.

In the jetting regime in Fig. 11, a consistently longer x_{POP} is observed throughout the simulation time. Additionally, a simple qualitative analysis of the bead spacing in the regions where beads form in the diagram reveals a single dominant frequency that significantly exceeds the range observed in the dripping regime.

Note that in the jetting regime, bead formation at the end of the jet originates from the initial development of small perturbations, which grow into beads that slide along the fiber. In the provided spatiotemporal diagram, bead formation is observable at an early stage, when large amplitude perturbations arise due to the initial transient phase as the system adjusts to the imposed conditions. However, at later times, when perturbations reduce to numerical noise, the bead formation is no longer visible. Therefore, we identified the jetting regime by the presence of a stable and extended x_{POP} throughout the simulation period, as indicated by the white arrows on the right-hand diagram in Fig. 11.

In conclusion, the numerical transition region is marked by the presence of more than one bead frequencies. This observation is consistent with the study by Clanet and Lasheras [15] and Ambravaneswaran *et al.* [12] on the transition from dripping to jetting in dripping faucets (without a fiber), where the transition region is identified by a period-doubling bifurcation. Furthermore, in those studies, this transition was also associated with bifurcations in a length which represents the length from the nozzle to the tip of the droplet at the moment of droplet breakup. In this study, x_{POP} can be considered as an equivalent characteristic length. However, it should be noted that in this study we did not investigate the existence of a period doubling or bifurcation of the point of pinching. Rather, two distinct frequencies and POPs were observed, as shown in Fig. 11 for the transition case. Within this mass flow rate range, the flow initially undergoes a transient jetting regime. A jet forms (indicated by a longer x_{POP}), and beads appear at a higher frequency. With time, the flow returns to the dripping regime, characterized by a shorter x_{POP} and a lower bead frequency, and remains in this state thereafter. The end of the numerical transition region and the onset of the jetting regime are characterized by the formation of a consistently large x_{POP} , which persists throughout the simulation time.

D. Regime map across geometries

Figure 12 presents the bead frequency as a function of mass flow rate for various fiber diameters and nozzle geometries based on experimental and simulation results. This allows us to map the transition from dripping to jetting, as described in the previous two subsections. In this figure, the red data points represent the theoretical results, while the blue data points correspond to the experimental results. The experimental and numerical transition regions are indicated by vertical lines in the corresponding colors. Overall, both results show that, in the dripping regime, the bead frequency increases approximately linearly with mass flow rate, while this trend breaks down in the transition region, where discontinuities appear.

As explained earlier (see Fig. 9), the experimental transition region (blue vertical lines in Fig. 12) is defined as the interval between the last frequency associated with the dripping regime and the first observed frequency jump, which marks the onset of the jetting regime. It can be observed that, for some fiber-nozzle combinations, the transition to jetting is accompanied by a backward shift toward lower mass flow rates, whereas other combinations exhibit a very narrow transition region characterized by an abrupt jump to higher frequencies without any backward shift in mass flow rate. This behavior is evident, for example, for the 0.175 mm fiber. In this case, the transition occurs smoothly, with a clearly observable increase in frequency as the mass flow rate increases continuously. While the mechanism responsible for the backward shift is discussed above, the absence of this effect for certain nozzle-fiber combinations is not yet fully understood and should be addressed in future work.

In the simulation results (red data in Fig. 12), the sudden appearance of two distinct bead frequencies with increasing mass flow rate indicates the onset of a numerical transition region, as previously illustrated and described in the middle diagram of Fig. 11. In Fig. 12, this region lies

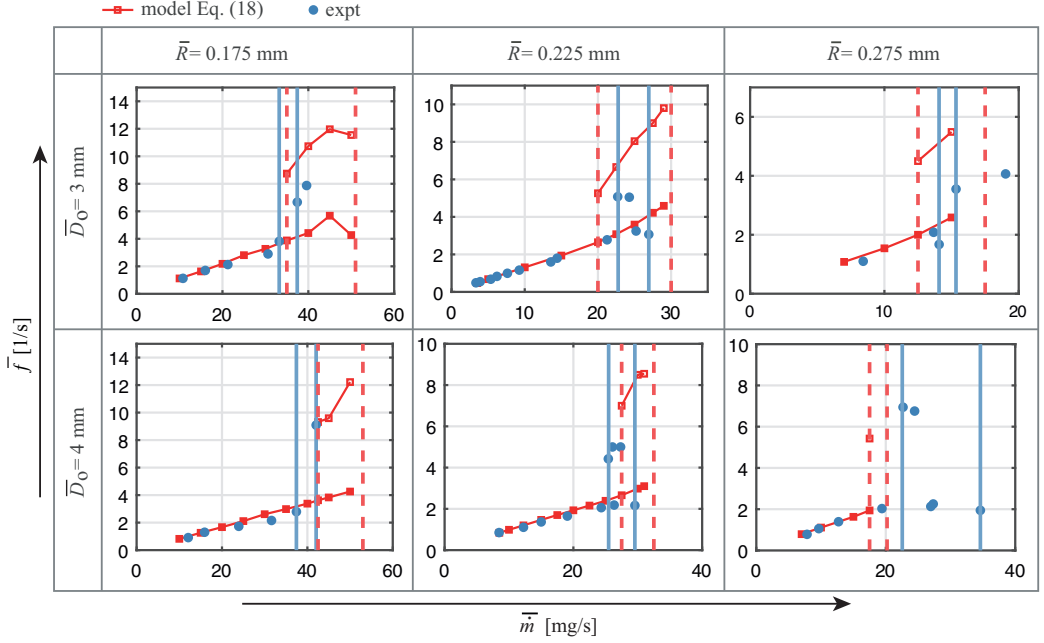


FIG. 12. Regime maps based on bead frequency as a function of mass flow rate for film flow on different fibers and with different nozzle geometries. The blue and red vertical lines indicate the beginning and end of the transition region from dripping to jetting regime for the experimental and simulation data, respectively. The regions before and after the transition correspond to the dripping and jetting regimes, respectively. The second red vertical line marks the onset of a continuous jet, for which no bead frequency can be evaluated (see the right panel in Fig. 11).

between the red vertical lines. At higher mass flow rates, as earlier shown in the right diagram of Fig. 11, a continuous and elongated x_{POP} is observed. This case is marked by the second red vertical line in Fig. 12 as the end of the numerical transition region and the onset of the jetting regime.

A comparison of results shows that while the bead frequencies in the dripping regime exhibit strong agreement, the frequency jump in the transition region is significantly larger in the model than in the experimental data. This discrepancy may suggest limitations of the model in accurately capturing flow behavior in the jetting regime. Despite this, the estimated transition region demonstrates reasonable agreement across all the fiber and nozzle configurations, as shown in Fig. 12.

Note that investigating the flow and bead frequencies in the jetting regime is beyond the scope of this study. Our measurements were discontinued once the flow transitioned into the jetting regime, therefore, no results are provided beyond the transition region. However, a previous study by Duprat *et al.* [30] on film flow along vertical fibers has shown that in the jetting regime, the bead frequency increases with flow rate and eventually tends to saturate at higher values.

IV. CONCLUSION AND OUTLOOK

In conclusion, this study aimed to identify quantifiable parameters that characterize the flow in the dripping regime and ultimately predict the transition from dripping to jetting. First, a comparative analysis of bead formation in the dripping and jetting regimes was conducted. This analysis revealed that bead formation in the dripping regime does not result from a growth instability but instead occurs via a local pinching mechanism near the nozzle. In this regime, the pressure in the film at point of pinching (bead formation location), governed by capillary forces, closely

approximates the hydrostatic pressure along the fiber. In contrast, bead formation in the jetting regime is driven by a spatiotemporal growth process, where the pressure at the bead formation point deviates significantly from hydrostatic conditions.

Second, it was shown that in the dripping regime, the bead size remains nearly independent of the mass flow rate. As a result, the bead frequency increases linearly with flow rate, a relationship that breaks down at the transition to jetting. The resulting loss of linearity and the associated frequency discontinuity were used to identify the transition from the dripping to the jetting regime. In both experiments and simulations, a range of flow rates exists over which both dripping and jetting can be observed; this range is defined as the transition region. To further characterize the regimes, we introduced a second key parameter: the point of pinch-off (POP). This parameter provides an additional means of identifying the transition between regimes. In the dripping regime, the axial location x_{POP} is close to the nozzle, and the radial pinch-off diameter r_{POP} is larger than in the jetting regime.

A. Model strengths and limitations

The one-dimensional model employed in this study captures the essential features of the dripping regime and the transition to jetting with reasonable accuracy. It provides physical insight into the mechanism governing bead formation near the nozzle by successfully reproducing both the frequency behavior and the spatial evolution of the film in the dripping regime. This offers a step toward understanding the criteria that bridges the gap between low-flow-rate dripping and high-flow-rate jetting regimes. However, the model assumes negligible inertia and does not account for the dynamic flow behavior characteristic of the jetting regime. Additionally, the nozzle-fiber coupling is modeled in a simplified, *ad hoc* manner, which may limit the predictive accuracy.

B. Outlook

Future work should extend both the experimental dataset and the model beyond the transition point to fully capture the jetting regime. This will require incorporating inertial effects and investigating the long-term evolution and stability of jets along the fiber. In particular, whether, and under which circumstances, the bead frequency saturates or varies irregularly in the jetting regime remains an open question, since bead formation in jetting follows a different mechanism than in dripping, as demonstrated in this study. Moreover, additional jetting-specific parameters (e.g., the healing length) may vary with mass flow rate and thereby influence bead dynamics.

In addition, exploring liquids with different physical properties, such as lower viscosity or altered wetting characteristics, could offer further insight into regime boundaries. This may help clarify the fluid conditions under which dripping or jetting behavior dominates and assess whether, at low viscosity, the dripping regime exhibits distinct sub-regimes (e.g., period-doubling or, more generally, period- n dripping), as reported in classical dripping-faucet studies.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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