

Deep-learning-based assessment of skin friction in wall-bounded turbulenceSergio Hoyas ^{*} and Nils Benedikt [†]*Instituto Universitario de Matemática Pura y Aplicada,
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This work investigates the influence of classically coherent structures on wall-shear stress and energy dissipation in turbulent channel flow, utilizing direct numerical simulations (DNS) data and explainable deep learning (XDL). Sweeps, defined as regions of low streamwise velocity moving toward the wall, are found to be the most influential structures for both energy dissipation and drag. Moreover, the volume of these key structures falls within a narrow range, making it easier to identify the most significant ones. Consequently, this work paves the way for the development of novel, highly efficient turbulence-control strategies for the reduction of drag.

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Turbulent flows are an inherent part of our world, governing the movement of rivers, ocean currents, and atmospheric airflow. From an engineering perspective, nearly all flows are turbulent, except for specific cases such as lubrication [1]. Like many other scientific disciplines, the study of turbulence began in the 19th century [2] and has since evolved over 140 years through remarkable ingenuity [3,4]. However, turbulence remains an unsolved problem with profound implications for everyday life. The primary tools for studying turbulent flows have historically been experiments [5], and more recently, direct numerical simulations (DNS) [6,7]. DNS solves the Navier-Stokes equations [1] without additional modeling, but its high computational cost restricts it to small Reynolds numbers. A promising new approach to understanding turbulence is explainable deep learning (XDL) [8], which is the one used here.

Among turbulence's many characteristics, energy dissipation is arguably the most critical in the context of the climate crisis, as turbulence is the dominant mechanism for energy loss in fluid flows. When turbulence interacts with solid boundaries, this dissipation manifests as drag or friction. This energy loss is traditionally quantified using the skin-friction coefficient c_f , which will be discussed later. A substantial fraction of the global energy consumption—up to 15%—is lost due to turbulent drag within the first millimeter of the boundary layer of moving vehicles or flows going through pipelines [9].

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Over the years, numerous approaches have been developed to study frictional losses, beginning with the foundational work of Hagen and Poiseuille in the 19th century. From a practical standpoint, Moody’s diagram has played a crucial role in engineering applications for decades [10]. Today, computational fluid dynamics (CFD) models are widely employed to enhance vehicle design [11]. However, these models are inherently imperfect [1], necessitating the exploration of alternative strategies to reduce drag. One of the most complex yet promising directions involves understanding the nonlinear interactions among turbulent flow structures [9] and how they carry momentum and energy from the boundaries to the outer flow.

In this direction, Guastoni *et al.* [12] showed how convolutional neural networks (CNN) can be used to two-dimensional instantaneous velocity-fluctuation fields at different wall-normal locations. In a follow-up article [13], Guastoni *et al.* demonstrated that highly effective drag control strategies, discovered through deep reinforcement learning, significantly alter the size and shape of these turbulent structures. However, the control model remains a black box, making it crucial to gain a deeper understanding of the underlying mechanisms driving its success. In a recent study, Cremades *et al.* [14] assessed the significance of a specific class of structures, known as Q-events, in shaping the flow’s evolution. By using this approach, the authors revisited and consolidated insights accumulated over the past 30 years, enabling them to characterize the most important structures associated with wall-bounded turbulence. Using this framework, the present work investigates the respective importance of several classically studied coherent structures for predicting wall-shear stress, and consequently, the dissipation losses.

The case studied in this work is the Poiseuille turbulent channel flow (PTC), which consists of two stationary parallel plates separated by a distance of $2h$ and subject to periodic boundary conditions in the streamwise and spanwise directions. The flow is driven by a constant pressure gradient, ensuring a steady mass flow. The periodicity allows for the efficient application of spectral methods in solving the governing equations [15]. The streamwise, wall-normal, and spanwise coordinates are $\mathbf{x} = (x_1, x_2, x_3)$, and the corresponding instantaneous velocity components are denoted by U_i . The main control parameter in this case is the friction Reynolds number $\text{Re}_\tau = u_\tau h / \nu = 125$, where u_τ is the friction velocity and ν is the kinematic viscosity.

The simulation was performed in a box of size $L_{x_1} = 8\pi h$, $L_{x_2} = 2h$, and $L_{x_3} = 3\pi h$. This box is large and wide enough to contain all the structures of the flow [16]. Statistically averaged quantities in x_1, x_3 , and time are denoted by an overbar, whereas fluctuating quantities are denoted by lowercase letters, i.e., $U_1 = \overline{U_1} + u_1$. Primes are reserved for intensities: $u_1' = \overline{u_1 u_1}^{1/2}$.

The Navier-Stokes equations were solved using the LISO code [17,18], a highly scalable parallel solver that includes optimized input/output (I/O) operations. This feature enables both high-resolution simulations [19] and computations requiring significant computational resources [7]. The code has been instrumental in advancing turbulence research, providing the basis for new theoretical developments [20]. Consequently, the results obtained from this database can be considered reliable and validated.

During the simulation, the three-dimensional velocity field was saved at regular intervals of $\Delta t^+ = 5$ viscous time units ($t^+ = t u_\tau^2 / \nu$). The database obtained database comprises 18 000 three-dimensional (3D) fields. This database will be used for training, testing, and the explainability analysis of structures.

The definition of a structure in turbulent flows is not straightforward, a problem that some of the authors of this work have addressed in a different study [21]. Several definitions exist, each one focusing on a particular characteristic of turbulence. Streamwise streaks [22] and Reynolds-stress events [23] were the first ones described in the literature. The streamwise streaks are regions of either low ($u < 0$) or high ($u > 0$) streamwise velocity, typically elongated in the direction of the mean flow. They are thought to play an important role in turbulence production near the wall and the transport of turbulent kinetic energy to the outer region of the flow. They are defined as

$$\sqrt{u_1^2(\mathbf{x}, t) + u_3^2(\mathbf{x}, t)} > \alpha u_\tau, \quad (1)$$

where α is the percolation index [24], and in this case, for both kind of streaks $\alpha = 4$.

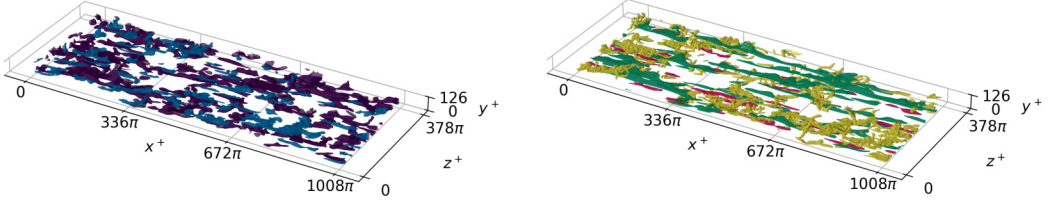


FIG. 1. Instantaneous visualization of the different types of coherent structures at the same time, but separated in two panels to facilitate their visualization. Ejections (purple), sweeps (blue), low-velocity streaks (green), high-velocity streaks (pink), and vortices (yellow).

Intense Reynolds stress structures or Q-events [9] are defined by

$$|\tau(\mathbf{x}, t)| > \beta u'_1(x_2)u'_2(x_2), \quad (2)$$

where $\tau(\mathbf{x}, t) = -u_1(\mathbf{x}, t)u_2(\mathbf{x}, t)$ is the instantaneous point-wise tangential Reynolds stress and β is the percolation index for uv -structure identification. In this case, $\beta = 1.40$. Depending on the sign of u and v , there are four possible Q-events. As shown in Ref. [14], the most important ones are ejections ($u < 0, v > 0$) and sweeps ($u > 0, v < 0$).

The definition of vortices [25] is based on the analysis of the velocity-gradient tensor. The key point is finding regions of the flow where rotation is larger than shear. Vortices appear close to ejections [24] and are part of the proposed cycle to sustain turbulence involving the streaks. Since they carry high levels of enstrophy, vortices can be considered dissipative structures [26]. Instantaneous visualizations of ejections, sweeps, both types of streaks, and vortices are shown in Fig. 1.

The skin-friction coefficient is defined using the shear stress $\tau_w = 1/2c_f\rho U_b^2$, where U_b is the bulk velocity, defined as the average velocity in the three directions and time. On the other hand, $\tau_w = \mu d\bar{U}_1/dx_2$, where μ is the dynamic viscosity. This implies

$$c_f = \frac{2\nu}{U_b^2} \frac{d\bar{U}_1}{dx_2}, \quad (3)$$

relating the changes in $d\bar{U}_1/dx_2$ to the skin-friction coefficient.

Two different U-net architectures based on convolutional neural networks (CNN) [27] were used for two different error-based loss functions. In the first case, as in Ref. [14], the idea is to reconstruct the whole velocity-fluctuation field from that field five viscous time units prior. In the second case, we focus on the reconstruction of $dU_1/dx_2|_{y=0}$ from the velocity-fluctuation field five viscous time units prior to obtain the global friction at the wall. For the first network, the neural network comprises a total of four levels with two three-dimensional convolutions in the encoder and one in the decoder. The connections between the levels use mean pooling of size 2 to transfer to a deeper level and a transpose convolution otherwise. Finally, the last field of each level of the encoder is connected through a concatenation with the output of the transposed convolution. The levels exhibit a total of 32, 64, 128, and 256 filters, each reaching a total of 6 million parameters. Concerning the second network, the encoder is three-dimensional and the decoder is two-dimensional. The dimensional compression of the data is applied through a convolution along the wall-normal axis before entering the decoder. This model uses 8, 16, 32, and 64 filters in each layer, reaching a total of 1.5 million parameters.

In both cases, 55.5% of the 18 000 available flow fields, are used for the training-and-validation process (out of which 80% are used for training and 20% for validation). The remaining 44.5% of fields are reserved for testing and explainability analysis and are not seen by the network during training. The models are trained until reaching a mean relative error of 1% with respect to the maximum value of the velocity and skin-friction coefficient, respectively.

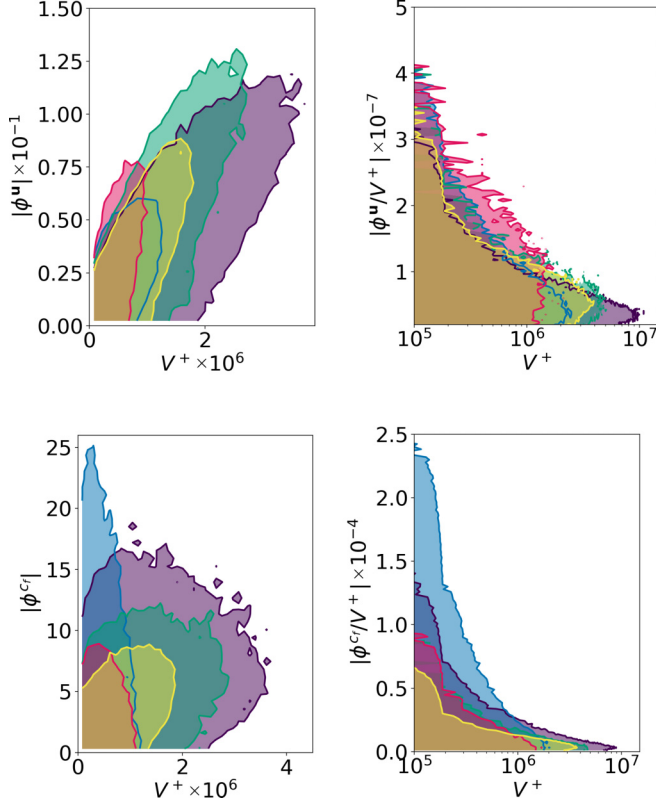


FIG. 2. Joint probability density function (JPDF) of the different SHAP values as a function of the volume. (Top) and (bottom) show the SHAP value for the velocity fluctuations and the skin-friction coefficient, respectively, while (left) and (right) represent the SHAP distributions and the SHAP per unit volume, respectively. The cutoff value for the corresponding JPDF is chosen as 10^{-4} of the maximum of the corresponding JPDF. Colors as in Fig. 1.

To study the importance of the various types of coherent structures to predict the skin-friction coefficient, we will use explainability methods based on feature attribution [8]. In particular, we use the kernel-SHAP algorithm [28], which combines two techniques: LIME [29] and Shapley values [30].

Although the Shapley values can quantify the marginal contribution of each structure, their exact computation is challenging. This problem is solved by approximating their solution using the so-called kernel-SHAP methodology [28] mentioned above. We refer interested readers seeking more technical details to our previous works [8,14].

Thus, we have two different SHAP values. The first one, ϕ^u , represents the importance of each structure in the reconstruction of the velocity fluctuations. The second one, ϕ^{cf} , represents the impact of the same structures on the reconstruction of the instantaneous value of the wall shear. $|\phi^u|$ is roughly linearly correlated to the inner-scale volume of the structures computed as the sum of their voxels [31], V^+ , as shown in Fig. 2 (top left). The slope of this linear correlation is slightly higher for the streaks. It is also clear that sweeps and vortices are the less important structures, with sweeps being smaller in volume. However, the correlation between the importance and volume disappears if we study the density of SHAP, $|\phi^u/V^+|$ [14] (Fig. 2, top right). Large streaks reach slightly higher values of $|\phi^u/V^+|$ than the larger structures of the other types for the same volume. Vortices and sweeps follow with midranged values of $|\phi^u/V^+|$ for the larger structures. The larger

ejections have the least importance per unit volume, suggesting that the definition of the ejections and in extension of the Q-events does not match the high-importance regions detected by the present data-driven methods [14,21]. Note that for all the structures, the higher and lower SHAP densities are obtained for small structures.

Extending the results of Ref. [14], the picture emerging from these findings is that streaks are the most important type of structure for the evolution of the velocity field since they capture a high amount of volume with a high amount of importance per volume. This fact is also consistent with the results in Ref. [21].

In contrast to $|\phi^u|$, the wall-shear-stress-based SHAP $|\phi^{c_f}|$ is not linearly correlated with the volume, as shown in the bottom part of Fig. 2. The most important structures to predict the skin-friction coefficient $|\phi^{c_f}|$ are sweeps, which are smaller than the other structures. Notice that most of the small sweeps are not important. This agrees with Ref. [32], in which was found that, in an interventional manner, some sweeps are the most important structures for the flow, even if they did not relate explicitly to the wall-shear stress. In fact, SHAP values are associated with causal links in the system, as they detect the most influential regions for the evolution of the flow [33]. Further, some small ejections are also important. This agrees with the authors of Ref. [34], who found (in a noninterventional manner) that both sweeps and ejections are the most important structures for the wall-shear stress. This difference between sweeps, on the one hand, and the rest of the structures, on the other, may be rooted in the fact that sweeps are, by definition, moving towards the wall while the other structures move away from the wall (ejections) or roughly move parallel to the wall (streaks, vortices).

Regarding the importance per unit of volume $|\phi^{c_f}/V^+|$, shown in Fig. 2 (bottom-right), there is an inverse correlation. All important structures are of small volume, and all large structures have low $|\phi^{c_f}/V^+|$. However, most small structures also have low importance per volume. Again, sweeps are by far the most important structure. The effect is even more pronounced than when studying $|\phi^{c_f}|$.

To better understand the influence of these small structures on the friction coefficient, Fig. 3 shows the same values plotted against the enstrophy they carry

$$\Omega_e = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2}. \quad (4)$$

Here, ω_i are the components of the vorticity vector $\omega_i = (\nabla \times \mathbf{u})_i$. The enstrophy, which clusters in regions of high shear, can be seen as a measure of the dissipation of the flow [26], and is thus related to smaller structures [35].

Three different situations appear with $|\phi^u|$ and the enstrophy contribution, Fig. 3 (top). First, low-velocity streaks present the largest contribution to $|\phi^u|$. However, they do not carry vorticity. Their contribution by volume is small. High-velocity streaks with medium levels of enstrophy per volume unit are the most important in terms of reconstruction. Finally, sweeps have different behavior, carrying a large amount of enstrophy and with relatively small $|\phi^u|$ and $|\phi^u/V^+|$. This disparity may arise from the fact that the structures carry similar $|\phi^u/V^+|$ values, as shown in Fig. 2, but differ greatly in their enstrophy per volume, as indicated in Fig. 3 (top right). Consequently, enstrophy contribution alone does not adequately explain $|\phi^u|$.

This observation changes when considering $|\phi^{c_f}|$. In this case, the slopes of $|\phi^{c_f}|$ concerning enstrophy contribution have a similar shape, Fig. 3 (bottom left).

Nevertheless, the structures that contribute most to enstrophy generally exhibit the highest importance for c_f within each category. This suggests that high $|\phi^{c_f}|$ levels can be attributed to substantial enstrophy contributions. Furthermore, larger structures exert a greater effect, supporting the hypothesis that the most important structures are those transporting fluctuation velocities—and thus turbulent kinetic energy—to the wall. Given that this energy eventually dissipates, these structures likely also contribute to higher enstrophy levels.

Focusing on the SHAP density $|\phi^{c_f}/V^+|$, sweeps are the dominant structures, as shown in Fig. 3 (bottom left), reaching significantly high values. However, there are numerous instances of sweeps with high enstrophy per volume and low importance and vice versa. This was also reported in

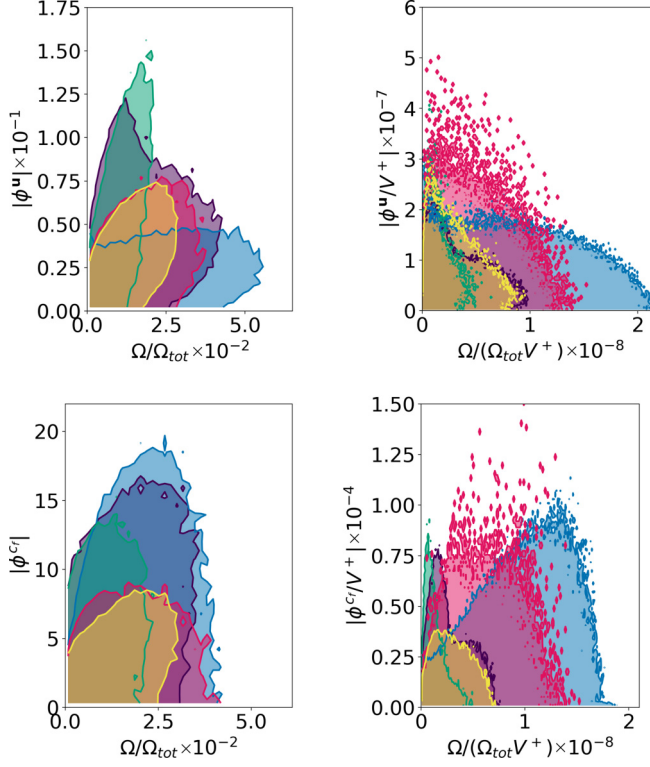


FIG. 3. Joint probability density function (JPDF) of the SHAP of the different SHAP values as a function of the enstrophy. (Top) and (bottom) show the SHAP value for the velocity fluctuations and the skin-friction coefficient, respectively, while (left) and (right) represent the SHAP distributions and the SHAP per unit volume, respectively. Colors as in Fig. 1.

Ref. [21], raising doubts about the dynamic importance of sweeps everywhere. These findings can be summarized in Fig. 4, where we present the joint probability density function (JPDF) of the two SHAP indicators. Clearly, the most important structures for the skin-friction coefficient are the

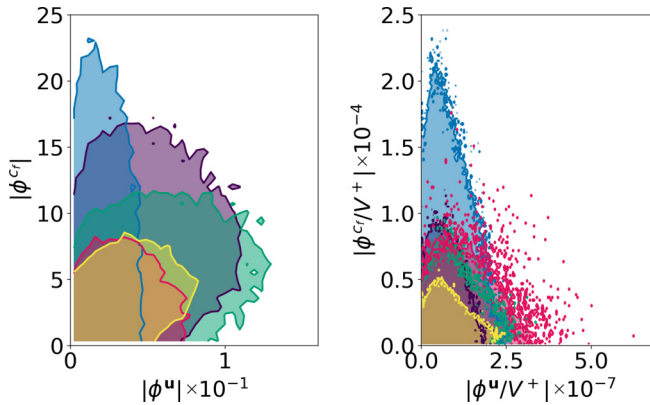


FIG. 4. JPDF of the SHAP values $|\phi^u|$ and $|\phi^{cf}|$ (left), and divided by volume (right). Colors as in Fig. 1.

sweeps, but they are of limited importance for the velocity-fluctuation reconstruction. On the other hand, some ejections are important for both.

In conclusion, extending the methods and database of Ref. [14], we identified the most informative structures to predict the losses due to friction in turbulent channel flow. Two different SHAP values were used, one accounting for the reconstruction of the velocity-fluctuation field $|\phi^u|$ and the second one for the skin-friction coefficient $|\phi^{c_f}|$. While $|\phi^u|$ strongly correlates with enstrophy contribution, the slope varies significantly across structures, indicating that enstrophy alone does not fully explain $|\phi^u|$. However, for $|\phi^{c_f}|$, the slopes are more consistent, suggesting that structures with high enstrophy contributions generally exhibit higher importance. Sweeps, in particular, dominate in terms of $|\phi^{c_f}/V^+|$, although their importance per volume can vary. The present results evidence the capacity of the SHAP values to detect high-importance patterns for understanding turbulent flows [21]. Therefore, a new method to identify the high-SHAP sweeps would help to completely characterize the most important structures for the skin-friction coefficient, an application of great importance in the context of flow control, as shown in the work by Beneitez *et al.* [36].

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Data availability. The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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