

Erratum: Drag force on spherical particles trapped at a liquid interface **[Phys. Rev. Fluids 7, 124001 (2022)]**

Zhi Zhou , Petia M. Vlahovska *, and Michael J. Miksis [†]

Department of Engineering Sciences and Applied Mathematics, Northwestern University, Evanston, Illinois 60208, USA



(Received 21 June 2023; published 28 August 2023)

DOI: [10.1103/PhysRevFluids.8.089901](https://doi.org/10.1103/PhysRevFluids.8.089901)

In Secs. II and III, we noted a missing term in the interface conditions for the disturbance fields at orders Ca and δ . This error does not affect any of the qualitative results or conclusions of this paper so there are minimal changes to the text of the paper. But it does impact figures and a few equations. The specific quantitative changes in results are given below.

The interface conditions in Eqs. (42) and (B23) in the original paper should read

$$\mathbf{u}_D^{(1,0)} \cdot \hat{\mathbf{e}}_z = -(\mathbf{u}_D^{(0,0)} + \mathbf{u}^\infty) \cdot \hat{\mathbf{n}}^{(1,0)} - \frac{\partial \mathbf{u}_D^{(0,0)}}{\partial z} h^{(1,0)} \cdot \hat{\mathbf{e}}_z \quad (42)$$

and

$$\mathbf{u}_{\perp,D}^{(1,0)} \cdot \hat{\mathbf{e}}_z = -(\mathbf{u}_{\perp,D}^{(0,0)} + \mathbf{u}_\perp^\infty) \cdot \hat{\mathbf{n}}^{(1,0)} - \frac{\partial \mathbf{u}_{\perp,D}^{(0,0)}}{\partial z} h_\perp^{(1,0)} \cdot \hat{\mathbf{e}}_z, \quad (\text{B23})$$

where the terms containing $\mathbf{u}^\infty$ and $\mathbf{u}_\perp^\infty$ were missing in the original equations. This correction also impacts Eqs. (49), (51), (B29), and (B30), Figs. 5–7, 11, 13–15, 17, and 21, and the asymptotic coefficients obtained by fitting to numerical solutions [below Eqs. (81) and (83)], which are revised as follows.

Equations (49), (51), (B29), and (B30) should read

$$F_D^{(0,1)} = \pi(\lambda - 1) \int_1^\infty \frac{9}{8r^7} \left\{ (r^2 - 1) \left[(6r^2 - 4)h^{(0,1)} + \frac{r(-9 + 27r^2 - 4r^3 - 18r^4)}{3(r^2 - 1)} \frac{dh^{(0,1)}}{dr} \right] \right\} dr \\ + \frac{27}{16} \pi(\lambda - 1) \tilde{b} - 3\pi(\lambda - 1)(-\tilde{\theta}_s + \tilde{b})C_0K_0(\sqrt{\text{Bo}}), \quad (49)$$

$$f^{(1)} = \frac{3\pi}{2\sqrt{\text{Bo}}} - \frac{9\pi}{2\text{Bo}} + \frac{75\pi}{8\text{Bo}^{3/2}} + O\left(\frac{1}{\text{Bo}^2}\right), \quad (51)$$

$$[\sigma_\perp^{(0,0)}] \cdot (-\hat{\mathbf{e}}_z) \cdot \mathbf{u}_{\perp,D}^{(1,0)} = [\sigma_{\perp zz}^{(0,0)}] \left((\mathbf{u}_{\perp,D}^{(0,0)} + \hat{\mathbf{e}}_y) \cdot \hat{\mathbf{n}}^{(1,0)} + \frac{\partial \mathbf{u}_{\perp,D}^{(0,0)}}{\partial z} h_\perp^{(1,0)} \cdot \hat{\mathbf{e}}_z \right) \quad (\text{B29})$$

$$= [\sigma_{\perp zz}^{(0,0)}] \left(-u_{\perp x}^{(0,0)} \frac{\partial h_\perp^{(1,0)}}{\partial x} - u_{\perp y}^{(0,0)} \frac{\partial h_\perp^{(1,0)}}{\partial y} + \frac{\partial u_{\perp z}^{(0,0)}}{\partial z} h^{(1,0)} \right). \quad (\text{B30})$$

Below Eq. (81), the correct asymptotic coefficients are $C_1 \approx 7.32$ and $C_2 \approx -12.37$ for $f_\perp^{(1)}$ and $C_1 \approx 5.82$ and $C_2 \approx -8.25$ for $f_\parallel^{(1)}$ with $L = 4$ and $15 \leq \sqrt{\text{Bo}} \leq 30$ with Bo as the Bond number;

*petia.vlahovska@northwestern.edu

[†]miksis@northwestern.edu

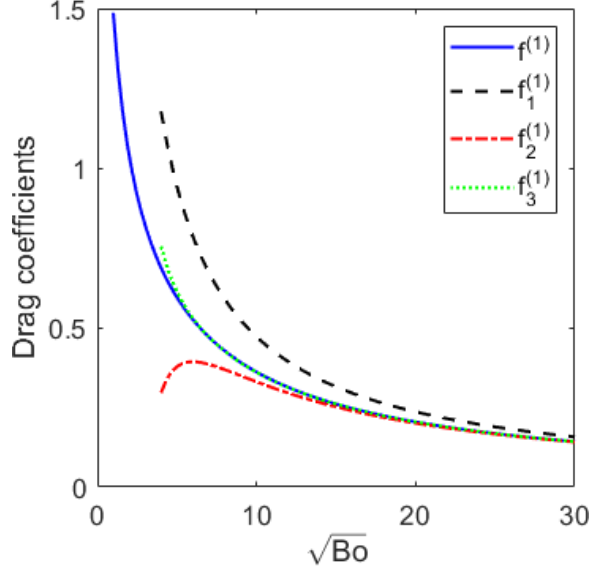


FIG. 5. The drag coefficient $f^{(1)}$ and its approximations $f_n^{(1)}$ for large Bond number plotted as functions of $\sqrt{\text{Bo}}$.

below Eq. (83), the correct asymptotic coefficients are $D \approx 1.06$ for $f_{\perp}^{*(1)}$ and $D \approx 2.14$ for $f_{\parallel}^{*(1)}$ with $\text{Bo} = 1$ and $20 \leq L \leq 40$. In addition, for larger Bo and L , $D \approx 0.81$ for $f_{\perp}^{*(1)}$ and $D \approx 1.57$ for $f_{\parallel}^{*(1)}$ with $100 \leq \text{Bo} \leq 400$ and $50 \leq L \leq 200$.

Additionally, we correct a typographical error in Eq. (40), which should read “ $\mathbf{u}_D^{(0,0)} = -\mathbf{u}^{\infty}, \dots$ ”. The corrected figures and captions are presented below.

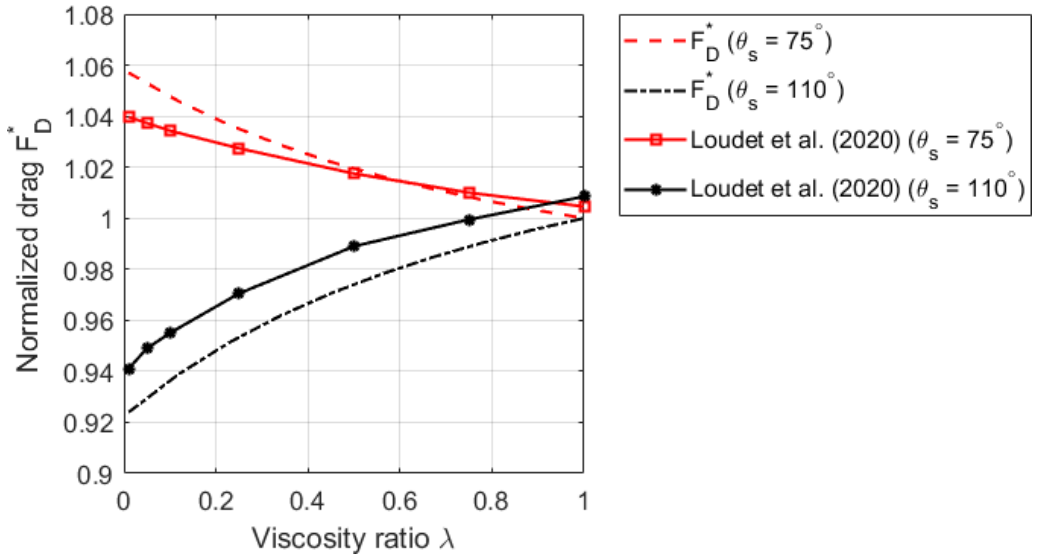


FIG. 6. Comparison of the normalized drag [F_D^* ; Eq. (52)] with numerical results from Loudet *et al.* (2020), for contact angles $\theta_s = 75^\circ$ and 110° , $\text{Bo} \approx 0.2$, $\tilde{b} = 0$.

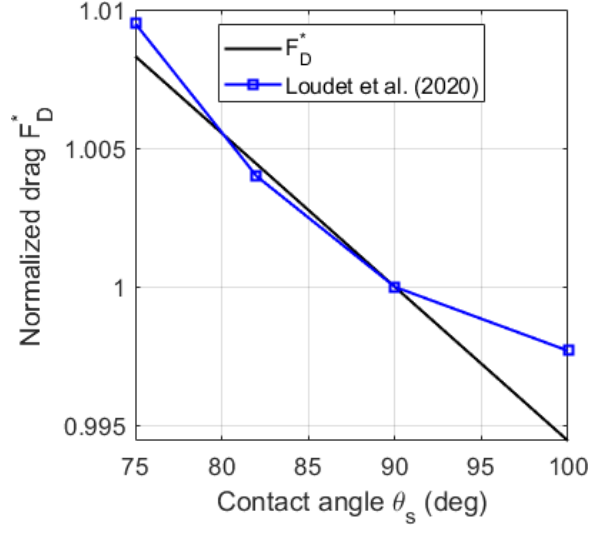


FIG. 7. Comparison of the normalized drag [F_D^* : Eq. (52)] with numerical results from Loudet *et al.* (2020) for viscosity ratio $\lambda = 0.75$, $Bo \approx 0.2$, and $\tilde{b} = 0$.

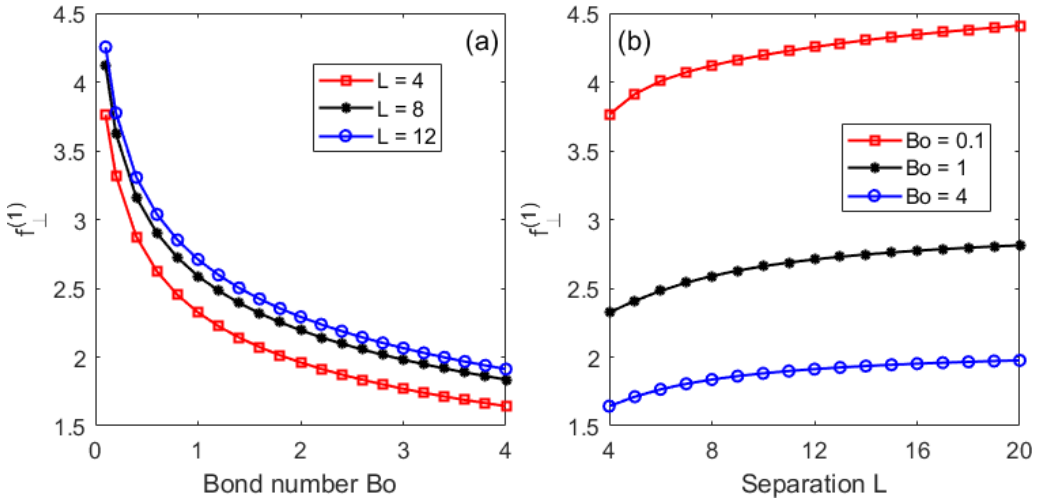


FIG. 11. The drag coefficient plotted as functions of (a) Bond number Bo and (b) separation L .

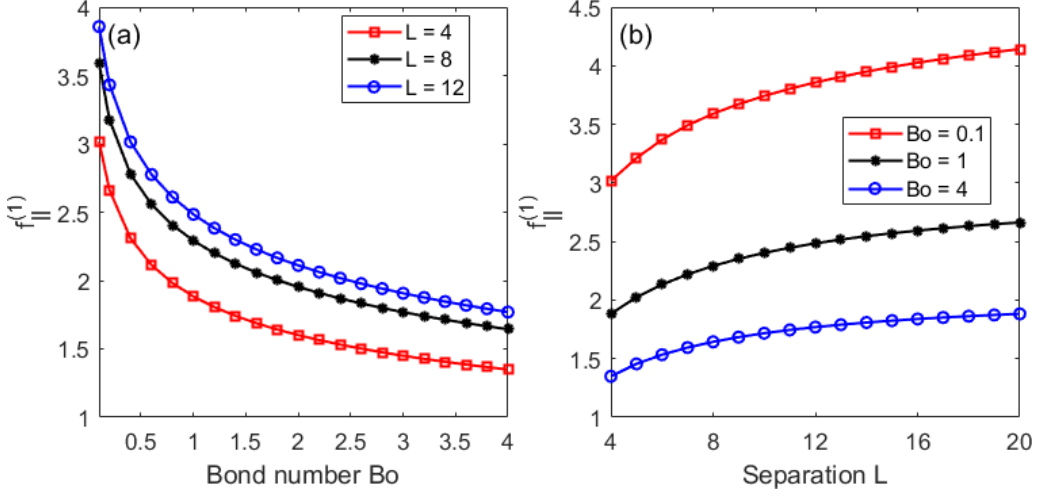


FIG. 13. The drag coefficient plotted as functions of (a) Bond number Bo and (b) separation L .

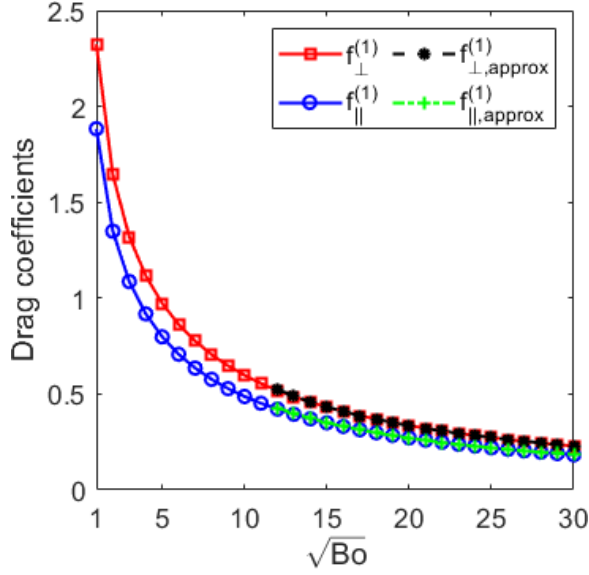


FIG. 14. Comparison of the drag coefficients, $f_{\perp}^{(1)}$ and $f_{\parallel}^{(1)}$, and their corresponding asymptotic approximations, $f_{\perp}^{(1),\text{approx}}$ and $f_{\parallel}^{(1),\text{approx}}$, for large Bond number with $L = 4$, where $C_1 \approx 7.32$ and $C_2 \approx -12.37$ for $f_{\perp}^{(1)}$ and $C_1 \approx 5.82$ and $C_2 \approx -8.25$ for $f_{\parallel}^{(1)}$.

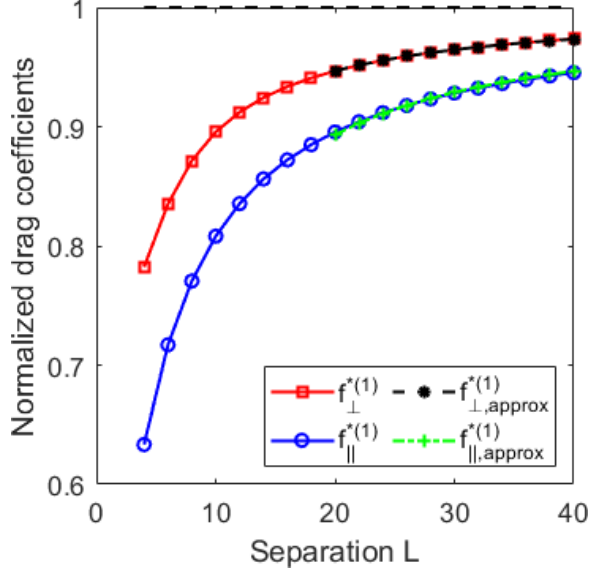


FIG. 15. Comparison of the normalized drag coefficients, $f_{\perp}^{*(1)}$ and $f_{\parallel}^{*(1)}$, and their corresponding asymptotic approximations, $f_{\perp}^{*(1)}$ and $f_{\parallel}^{*(1)}$, for large L with $Bo = 1$, where $D \approx 1.06$ for $f_{\perp}^{*(1)}$ and $D \approx 2.14$ for $f_{\parallel}^{*(1)}$.

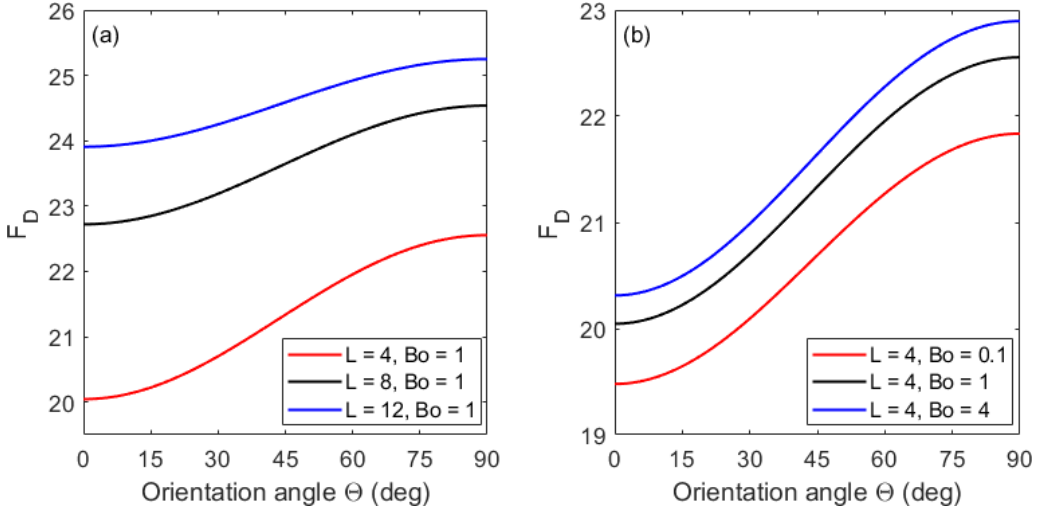


FIG. 17. The magnitude of the drag force plotted as a function of the orientation angle Θ with $\delta = \theta_s = 1$, $\lambda = 1/2$, and varying values of (a) separation L and (b) Bond number Bo .

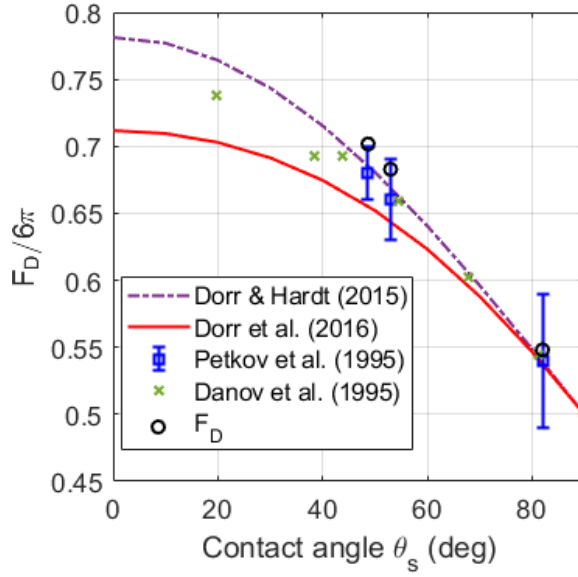


FIG. 21. Comparison of the normalized drag computed from F_D [Eq. (50)] with experimental data from Petkov *et al.* [25] and theoretical predictions from the literature [7–9].