

Breakdown of Kolmogorov scaling and modified energy transfer in bubble-laden turbulence

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We investigate the effect of a dispersed bubble phase on forced homogeneous and isotropic turbulence using high-resolution high-performance simulations based on the lattice Boltzmann method. While the classical Kolmogorov energy cascade is largely preserved when considering the system as a whole, a phase-specific analysis reveals striking deviations from the classical turbulence scaling. In particular, the gas phase exhibits significant departures from Kolmogorov's predictions, whereas the continuous liquid phase retains a turbulence structure consistent with classical expectations up to 24% in gas volume fractions. These findings suggest that, despite the presence of a dispersed phase, the global energy transfer remains close to a universal behavior. At the same time, phase-specific interactions are shown to introduce modifications to the turbulent dynamics at small scales. In particular, the gas phase exhibits a nearly flat spectrum at low wave numbers followed by a k^{-3} scaling at intermediate scales pointing to the presence of patterns of localized bursts uniformly distributed between two finite wavelengths. Our results aim at deepening the understanding of multiphase turbulence, particularly in the context of energy transfer mechanisms and phase interactions in bubble-laden flows. This study provides a framework for future investigations into the fundamental properties of multiphase turbulence and its implications for environmental, atmospheric, and industrial flows.

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I. INTRODUCTION

Multiphase turbulent flows are ubiquitous in both natural and industrial settings, playing a crucial role in a broad range of applications, from oceanic and atmospheric processes to chemical reactors and energy systems [1–3]. Among these, bubble-laden turbulence is particularly relevant, as it governs key phenomena such as wave breaking [4,5], gas transfer at the ocean-atmosphere interface [1,6], cavitation [7], and industrial mixing processes [8]. Despite its widespread importance, the fundamental mechanisms underpinning turbulence modulation by dispersed bubbles remain only partially understood. The presence of an additional phase introduces complex interfacial interactions [9], modifies energy transfer across scales [10,11], and alters small-scale dynamics in ways that are challenging to predict using classical turbulence theories. In particular, bubble-induced turbulence involves strongly localized and intermittent events—such as breakup, coalescence recoils, and capillary bursts—that inject and remove energy at scales comparable to or smaller than the Kolmogorov length. These processes couple the dynamics of the gas and liquid phases in a highly nontrivial way, giving rise to phase-dependent spectra, structure functions, and intermittency properties that cannot be captured by mixture-averaged descriptions alone.

One of the most prominent challenges in studying bubble-laden turbulence stems from its intrinsic multiscale and multiphysics nature. The interaction between the continuous phase and the dispersed gas phase leads to subtle but fundamental deviations from classical Kolmogorov turbulence [12,13], requiring novel approaches to accurately characterize energy transfer mechanisms. Understanding these effects is essential for improving predictive models in climate science, where bubble-induced turbulence affects the dispersion of aerosols and microplastics [14], as well as in industrial applications, such as the optimization of bubble column reactors and cavitation mitigation strategies [15,16], to cite some prominent examples. From a fundamental viewpoint, bubble-turbulence interactions challenge the universality of the classical cascade picture: interfacial forces, density, and viscosity contrasts jointly reshape the balance between production, transfer, and dissipation of kinetic energy. A comprehensive description therefore requires access not only to global spectra, but also to phase-resolved statistics and to scale-by-scale measures of the work done by surface tension and pressure across the interface.

Due to the inherent complexity of these flows, direct experimental measurements are often limited, particularly at the small scales where interphase interactions dominate [17]. As a result, numerical simulations play a fundamental role in uncovering the underlying physics. Fully resolved simulations provide access to quantities that are extremely difficult, if not impossible, to measure in the laboratory, including (i) gas- and liquid-specific energy spectra and structure functions; (ii) the scale-dependent work of surface tension and pressure at the interface; and (iii) the detailed dynamics at the Hinze scale, where the competition between turbulent stresses and capillarity controls the transition between breakup and coalescence. However, accurately resolving turbulence in the presence of bubbles requires high-resolution computations, advanced numerical schemes, and high-performance computing (HPC) frameworks capable of handling large density and viscosity contrasts, as well as the complex interactions between phases in turbulent environments. Moreover, numerical experiments offer the unique possibility to vary, in a controlled manner, key parameters such as volume fraction, density and viscosity ratios, and Weber number, thereby enabling systematic exploration of regimes that are difficult to realize experimentally while preserving homogeneous and isotropic conditions.

In this work, a recently developed HPC implementation [18–20] of the lattice Boltzmann (LB) method [21,22] is employed to investigate the effect of dispersed bubbles on homogeneous isotropic turbulence (HIT). The LB framework is particularly well suited for this task: its locality and explicit time integration make it highly efficient on GPU-based architectures, while its kinetic formulation facilitates a robust coupling with diffuse-interface phase-field models at high density and viscosity ratios. The resulting thread-safe, multi-GPU implementation enables long, fully resolved simulations on large three-dimensional domains, which are needed to attain statistically steady bubbly turbulence and to perform converged scale-by-scale analyses. By performing

numerical experiments across different bubble volume fractions, density ratios, and viscosity ratios, we provide a detailed analysis of energy transfer mechanisms at both global and phase-specific levels. Our results reveal that, while the overall energy spectrum maintains a Kolmogorov-like behavior, the gas and liquid phases exhibit distinct turbulence characteristics, with the gas phase deviating significantly from classical scaling laws. In particular, we observe that bubble-induced momentum transfer redistributes turbulent energy, leading to the emergence of a nonclassical cascade within the dispersed phase. We further show that these phase-specific cascades are tightly linked to interfacial processes at the Hinze scale, where both the third-order structure functions and the surface-tension work change sign, signaling a switch in the dominant direction of energy transfer between the interface and the bulk flow.

These findings offer unique insights into the role of gas-liquid interactions in turbulent energy transfer and highlight the necessity of phase-specific analyses in multiphase turbulence studies, which are also fundamental to improving two-fluid models. In particular, the phase-resolved statistics and scale-dependent budgets reported here provide quantitative benchmarks for the development and calibration of subgrid-scale closures and reduced-order models for bubbly flows, where interfacial effects are typically parametrized rather than explicitly resolved. The results presented here confirm that a robust computational framework is crucial for future investigations into bubble-laden turbulence and open the way for improved models in environmental and industrial multiphase flow applications. More broadly, they underscore how fully resolved numerical experiments, combined with advanced analysis tools, can bridge the gap between idealized theories of turbulence and the complex, multiphysics reality of bubbly flows encountered in nature and technology.

II. METHODS

A. Navier-Stokes equations for multiphase flows with interfaces

The multiphase system considered in this study is modeled by solving the continuity and momentum equations using the LB method. These equations are given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (1)$$

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot (\rho \nu [\nabla \mathbf{u} + (\nabla \mathbf{u})^\top]) + \mathbf{F}^s, \quad (2)$$

where $\mathbf{u}$ represents the velocity field, ρ the density, p the macroscopic pressure, ν the kinematic viscosity, and $\mathbf{F}^s$ the surface-tension force. Greek indices indicate Cartesian components of vectors and tensors.

The surface-tension force, $\mathbf{F}^s$, can generally be written as the divergence of a capillary stress tensor:

$$\mathbf{F}^s = \nabla \cdot [\sigma (\mathbf{I} - \mathbf{n} \otimes \mathbf{n})], \quad (3)$$

where $\mathbf{n}$ is the local normal to the interface and σ is the surface-tension coefficient, here assumed constant (zero tangential stresses).

1. Allen-Cahn equation for interface tracking

The interface dynamics is captured through the evolution of a phase field, ϕ , governed by the conservative Allen-Cahn equation:

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = D \nabla^2 \phi - \kappa \nabla \cdot [\phi(1 - \phi)\mathbf{n}], \quad (4)$$

where D is the interface diffusivity, and $\kappa = 4D/\delta$, with δ denoting the interface width. In this model, ϕ takes values within the range $\{0, 1\}$, such that the interface between the two immiscible fluids is located at $\phi_0 = 0.5$. It has been demonstrated [23] that the equilibrium profile for an

interface positioned at x_0 follows:

$$\phi(x, y, z) = \phi_0 \pm \frac{\phi_H - \phi_L}{2} \tanh \left(\frac{|x - x_0| + |y - y_0| + |z - z_0|}{\delta} \right). \quad (5)$$

The interface tracking equation is dynamically coupled to the Navier-Stokes equations through the advection term.

In the present work, the conservative Allen-Cahn equation is advanced in time using an explicit first-order Euler scheme with a time-staggered advection velocity, namely:

$$\phi^{n+1} = \phi^n + \Delta t [-\mathbf{u}^{n+\frac{1}{2}} \cdot \nabla \phi^n + D \nabla^2 \phi^n - \kappa \nabla \cdot (\phi^n (1 - \phi^n) \mathbf{n}^n)]. \quad (6)$$

The velocity field $\mathbf{u}^{n+\frac{1}{2}}$ is provided by the LB Navier-Stokes solver and yields a midpoint evaluation of the advecting field, improving the consistency of the coupling between the fluid dynamics and the phase-field transport. To enhance the accuracy of the model in the presence of strong advection, the advection term in Eq. (4) is discretized using a fifth-order weighted essentially nonoscillatory (WENO-5) scheme [24]. In one spatial direction, the conservative form of the advective contribution reads

$$\frac{\partial \phi}{\partial t} + \frac{\partial g}{\partial x} = 0, \quad g = u_x \phi, \quad (7)$$

and its semidiscrete finite-difference approximation is

$$\left. \frac{\partial g}{\partial x} \right|_i \simeq \frac{\hat{g}_{i+\frac{1}{2}} - \hat{g}_{i-\frac{1}{2}}}{\Delta x}, \quad (8)$$

where $\hat{g}_{i\pm\frac{1}{2}}$ are high-order numerical fluxes reconstructed by the WENO-5. For $u_x > 0$, the left-biased candidate fluxes at $x_{i+\frac{1}{2}}$ are

$$\hat{g}_{i+\frac{1}{2}}^{(0)} = \frac{1}{3}g_{i-2} - \frac{7}{6}g_{i-1} + \frac{11}{6}g_i, \quad (9)$$

$$\hat{g}_{i+\frac{1}{2}}^{(1)} = -\frac{1}{6}g_{i-1} + \frac{5}{6}g_i + \frac{1}{3}g_{i+1}, \quad (10)$$

$$\hat{g}_{i+\frac{1}{2}}^{(2)} = \frac{1}{3}g_i + \frac{5}{6}g_{i+1} - \frac{1}{6}g_{i+2}, \quad (11)$$

with $g_j = u_{x,j} \phi_j$. The smoothness indicators of Jiang and Shu are

$$\beta_0 = \frac{13}{12}(g_{i-2} - 2g_{i-1} + g_i)^2 + \frac{1}{4}(g_{i-2} - 4g_{i-1} + 3g_i)^2, \quad (12)$$

$$\beta_1 = \frac{13}{12}(g_{i-1} - 2g_i + g_{i+1})^2 + \frac{1}{4}(g_{i-1} - g_{i+1})^2, \quad (13)$$

$$\beta_2 = \frac{13}{12}(g_i - 2g_{i+1} + g_{i+2})^2 + \frac{1}{4}(3g_i - 4g_{i+1} + g_{i+2})^2. \quad (14)$$

The nonlinear WENO weights are computed as

$$\alpha_r = \frac{d_r}{(\varepsilon + \beta_r)^2}, \quad \omega_r = \frac{\alpha_r}{\sum_{s=0}^2 \alpha_s}, \quad r = 0, 1, 2, \quad (15)$$

where $(d_0, d_1, d_2) = (1/10, 6/10, 3/10)$ are the optimal linear weights and ε is a small parameter preventing division by zero. The final fifth-order numerical flux is then

$$\hat{g}_{i+\frac{1}{2}} = \sum_{r=0}^2 \omega_r \hat{g}_{i+\frac{1}{2}}^{(r)}. \quad (16)$$

For $u_x < 0$, a symmetric right-biased reconstruction is adopted. The multidimensional operator is obtained by applying the same one-dimensional WENO-5 procedure along each Cartesian direction. This scheme provides high-order accuracy in smooth regions while suppressing spurious oscillations across sharp interfaces, thereby ensuring a robust and low-dissipative transport of the phase field. See [19,25] for validations of the present two-phase flow solver at high density ratio.

In all the simulations, the conservative Allen-Cahn interface thickness is set to $\delta = 5 - 6\Delta x$, corresponding to approximately five to six grid points across the hyperbolic-tangent profile. All bubbles in the simulation remain significantly larger than the diffuse-interface thickness, ensuring that interface curvature and breakup and coalescence dynamics are resolved. Mass conservation was monitored during all runs: in a fully periodic domain, the total phase-field mass is conserved to within 1% – 2% over several hundred thousand LB time steps. Furthermore, as shown below, the correct Hinze-scale transition in bubble-size probability density function (PDF) and the sign change in the surface-tension work spectrum further confirm that the interface dynamics is adequately resolved.

B. Thread-safe lattice Boltzmann model

The Navier-Stokes equations for single-phase flows can be efficiently solved using the LB method. This section provides a brief overview of a thread-safe LB framework, which integrates an efficient LB strategy with a regularized LB approach [18,20,26].

The Boltzmann equation, discretized over a velocity space of q discrete vectors, is given by

$$f_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) = f_i(\mathbf{x}, t) + \omega(f_i^{\text{eq}} - f_i) + \left(1 - \frac{\omega}{2}\right) S_i, \quad (17)$$

where f_i are the distribution functions, f_i^{eq} their equilibrium counterpart, and S_i represents an external forcing term. The relaxation rate is controlled by ω , which is linked to the kinematic viscosity of the fluid via the relation $\nu = c_s^2(1/\omega - 0.5)$ (see [21,22] for details on the LB method). By decomposing $f_i = f_i^{\text{eq}} + f_i^{\text{neq}}$, the postcollision distribution can be rewritten as

$$f_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) = f_i^{\text{eq}} + (1 - \omega)f_i^{\text{neq}} + \left(1 - \frac{\omega}{2}\right) S_i. \quad (18)$$

In the thread-safe approach, both the equilibrium and nonequilibrium parts are reconstructed from the macroscopic hydrodynamic fields, eliminating the need to read the full set of lattice populations. This prevents race conditions arising from nonlocal memory access [19,26]. The equilibrium distributions take the form

$$f_i^{\text{eq}} = w_i \left(p^* + \frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^2} + \frac{(\mathbf{c}_i \mathbf{c}_i - c_s^2 \mathbf{I}) : \mathbf{u} \mathbf{u}}{2c_s^4} \right), \quad (19)$$

where $p^* = p/(\rho c_s^2)$ is a dimensionless pressure. The nonequilibrium part is reconstructed up to second order as

$$f_i^{\text{neq}} = \frac{\mathbf{c}_i \mathbf{c}_i : \mathbf{a}_{\text{neq}}}{2c_s^4}. \quad (20)$$

$\mathbf{a}_{\text{neq}}$ is the nonequilibrium stress tensor.

After the collision step, the macroscopic pressure and velocity fields are retrieved using

$$p^*(\mathbf{x}, t) = \sum_i f_i(\mathbf{x}, t), \quad (21)$$

$$\mathbf{u}(\mathbf{x}, t) = \sum_i f_i(\mathbf{x}, t) \mathbf{c}_i + \frac{1}{2} \sum_i S_i \mathbf{c}_i. \quad (22)$$

The forcing term S_i follows the Guo forcing scheme [27]:

$$S_i = w_i \left(\frac{\mathbf{c}_i - \mathbf{u}}{c_s^2} + \frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^4} \mathbf{c}_i \right) \cdot \mathbf{F}, \quad (23)$$

where $\mathbf{F}$ represents the external force field per unit mass.

C. Extension for high-density and viscosity contrasts

Additional forces are introduced to capture surface-tension effects and inhomogeneous density contributions to model multiphase flows. The total force is given by

$$\mathbf{F} = \mathbf{F}^s + \mathbf{F}^p + \mathbf{F}^v, \quad (24)$$

where the surface-tension force is computed as follows [23]:

$$\mathbf{F}^s = -\gamma \kappa \nabla \phi |\nabla \phi|. \quad (25)$$

The pressure force is [28]

$$\mathbf{F}^p = -p^* c_s^2 \nabla \rho, \quad (26)$$

where $\rho = \rho_l \phi + (1 - \phi) \rho_g$. Since the local pressure is $p = p^* \rho c_s^2$, its gradient includes both embedded and additional terms, requiring the explicit inclusion of $\mathbf{F}^p$.

The viscous force is given by [28]

$$\mathbf{F}^v = -\frac{\nu \omega}{c_s^2 \Delta t} \left[\sum_i (f_i - f_i^{\text{eq}}) \mathbf{c}_i \mathbf{c}_i \right] \nabla \rho. \quad (27)$$

This framework enables stable and efficient simulation of multiphase flows with high-density and viscosity contrasts while maintaining thread safety through regularization [19].

III. RESULTS

A. Single-phase homogeneous isotropic turbulence: Spectra and single-point statistics

We first proceed to analyze single-phase HIT energy and pressure spectra and selected single-point statistics so to assess the accuracy of ACCLB [19,20] to perform fully developed turbulence simulations.

For both single-phase and bubble-laden simulations the homogeneous isotropic turbulent state is maintained through large-scale energy injection via the ABC (Arnold-Beltrami-Childress) forcing. This deterministic, divergence-free forcing scheme introduces a time-independent velocity field that acts as a steady source of energy at the largest scales. Specifically, the forcing takes the form

$$\mathbf{f}(\mathbf{x}) = A \begin{pmatrix} \sin(k_f z) + \cos(k_f y) \\ \sin(k_f x) + \cos(k_f z) \\ \sin(k_f y) + \cos(k_f x) \end{pmatrix},$$

where A is the forcing amplitude and k_f the forcing wave number. In this work, the forcing amplitude and the forcing wave number have been set to 2×10^{-7} (expressed in units of lattice) and 2, respectively. The ABC forcing guarantees a sustained turbulent regime by balancing viscous dissipation with energy input, without introducing significant anisotropy, thereby providing an efficient and widely adopted method to achieve statistically stationary HIT in periodic domains [29].

We note here that the forcing is applied only in the bulk liquid phase, as in [30].

Two cases with increasing Taylor-scale Reynolds number ($\text{Re}_\lambda = u_{\text{rms}} \lambda / \nu$, $\lambda = \sqrt{15\nu} u_{\text{rms}}$), $\text{Re}_\lambda = 150$ and 230, have been considered. The simulations were conducted on a 512^3 Cartesian uniform

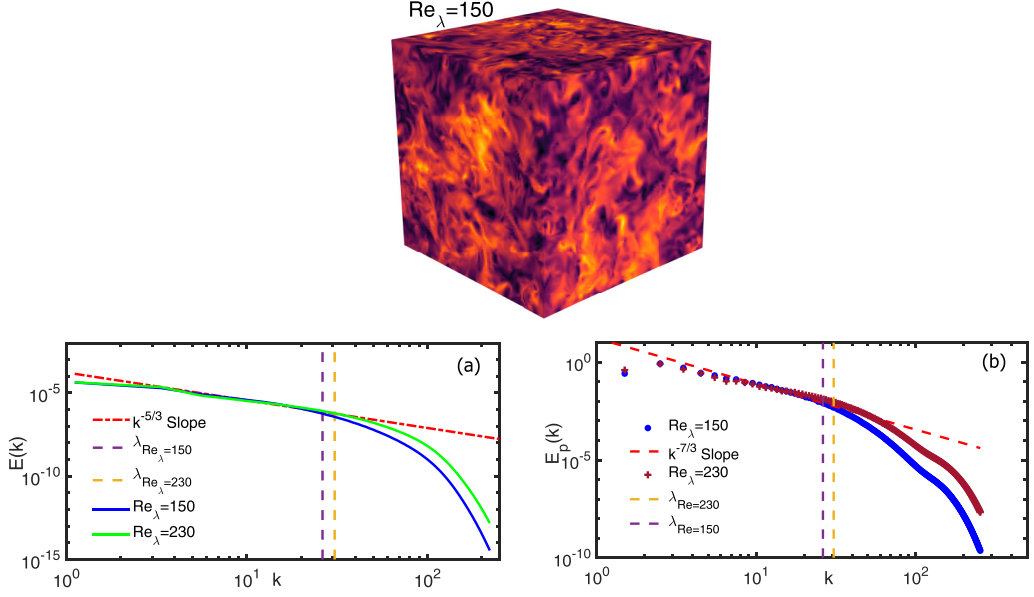


FIG. 1. Volume representation of the instantaneous velocity field in a homogeneous single-phase turbulent flow at $Re_\lambda = 150$. The domain size is 512^3 . (a) The energy spectrum for two different Taylor Reynolds lambdas, namely, 150 and 230. The expected $-5/3$ law in the inertial range is well reproduced across a wide range of wave numbers. (b) The pressure spectra for the same Taylor Reynolds number are shown to follow the $-7/3$ scaling law within the inertial range.

grid. The $Re_\lambda = 150$ case qualifies as a direct numerical simulation (DNS), with the Kolmogorov length scale approximately Δx , ensuring that the smallest turbulent scales are resolved. On the other hand, the $Re_\lambda = 230$ case operates slightly outside the strict DNS regime, with a Kolmogorov length scale of $0.4\Delta x$.

However, comparison with the fully DNS-resolved $Re_\lambda = 150$ case shows excellent agreement in energy and pressure spectra, velocity-gradient statistics, and structure functions. Moreover, as also reported in the remainder of the paper, all bubble-laden trends are found to be consistent between the two Reynolds numbers and the physical mechanisms discussed in the paper (Hinze-scale transition, k^{-3} gas-phase scaling, intermittency modification) occur at scales well above the grid cutoff. Hence the results of the slightly under-resolved case at $Re_\lambda = 230$ do not undermine, but rather confirm, the findings of this study.

All the simulations have been run on the Leonardo supercomputer [31] at CINECA, Italy. The details of the implementation of ACCLB and the related performances on both single and multi-GPU can be found in [20].

Figure 1(a) compares the energy spectra for both cases with the classical $-5/3$ slope expected in the inertial subrange [12,32]. The agreement is evident and this stands as a fundamental proof of the ability of ACCLB to capture energy transfer across scales in single-phase turbulent flows accurately.

Moreover, the pressure fluctuations spectra are displayed and compared against the expected $-7/3$ power law [33] in Fig. 1(b). For both the pressure and energy spectra, the simulation at the highest Reynolds number exhibits the theoretical scaling over a broader range than the $Re_\lambda = 150$ case, reflecting the extended inertial range and increased scale separation. As expected, the spectra start to deviate from the $-5/3$ and $-7/3$ power laws at the larger scales, in proximity of the Taylor length scale, which is denoted by the vertical dashed lines in the figure.

Single-point statistics of the two HIT flows in scrutiny are shown in Fig. 2. In particular, the left panel of Fig. 2 reports the probability density function (PDF) of the longitudinal velocity

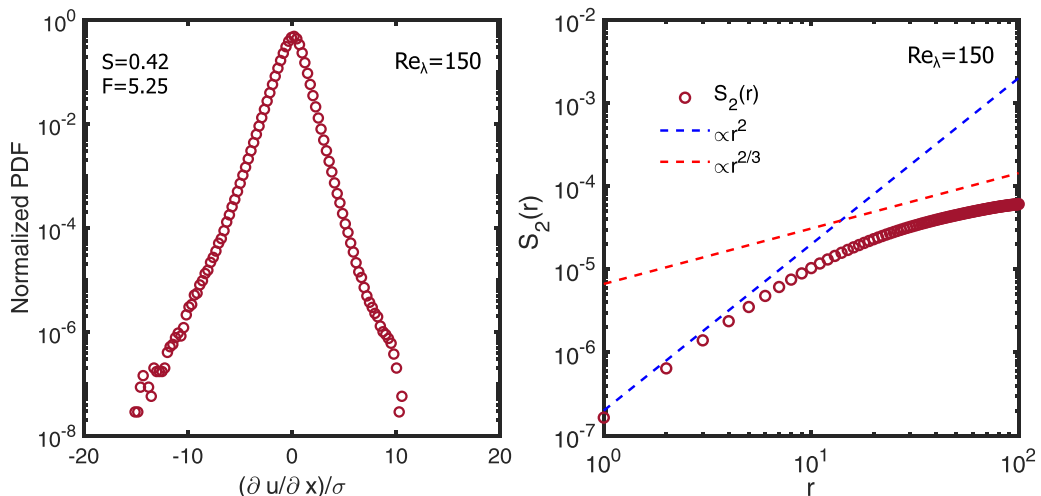


FIG. 2. Left panel reports the normalized probability density function (PDF) of the longitudinal velocity gradients at $Re_\lambda = 150$, with the values of skewness and kurtosis in the legend. Right panel: second-order structure function compared with two theoretical scaling laws, namely, the r^2 scaling in the dissipation range and the Kolmogorov scaling, $r^{2/3}$, in the inertial range.

gradients for the flow at $Re_\lambda = 150$. As expected, the PDF is far from Gaussian and slightly skewed, with a skewness factor, $S = 0.42$ ($S = -\langle (\partial u / \partial x)^3 \rangle / \langle (\partial u / \partial x)^2 \rangle^{3/2}$), and a flatness factor $F = 5.25$ ($F = \langle (\partial u / \partial x)^4 \rangle / \langle (\partial u / \partial x)^2 \rangle^2$), closely aligning with previous findings (see [34] as a reference) at similar values of the Taylor Reynolds number. The first index is a typical hallmark of the emergence of anisotropic effects at small scales, even in globally isotropic turbulence, driven by the presence of persistent vortex stretching. In contrast, the non-Gaussian value of the flatness (larger than 3) indicates the presence of intermittency, i.e., rare but intense fluctuations of the velocity gradients [35].

In the right panel of Fig. 2 the second-order structure function, $S_2(r) = \langle [u(x_i + r_i) - u(x_i)]^2 \rangle$, is reported and compared with two theoretical scaling laws [35]: the r^2 scaling in the dissipation range (small values of r , fine-scale pseudolaminar regime) and the Kolmogorov scaling, $r^{2/3}$, in the inertial range (larger values of r corresponding to larger-scale structures). The same agreement with the existing literature is seen for the results of the simulation at higher Taylor Reynolds number case (i.e., $Re_\lambda = 230$). For the sake of conciseness, results have only been shown for the case $Re_\lambda = 150$.

B. Bubble-induced turbulence modulation in homogeneous isotropic turbulence

In the following, we examine the modulation of HIT due to the presence of a dispersed gas phase. To this aim, we performed simulations across a range of bubble volume fractions (ϕ_B), viscosity ratios, and density ratios. In particular, the focus is on the impact of gas-liquid interfaces on higher-order turbulence statistics and their connection to the modified turbulent energy spectra.

The simulations were performed on a 512^3 domain, consistent with the single-phase reference case. The baseline physical parameters consist of a viscosity ratio $\nu_G/\nu_L = 1$ and a density ratio of $\rho_L/\rho_G = 100$. The void fraction was varied from 0% (single-phase flow) to 24%, as shown in Fig. 3, reporting instantaneous snapshots of the dispersed bubbly phase and of the velocity field for the volume fractions investigated. We wish to point out that the maximum gas volume fraction considered in this study ($\phi_B = 24\%$) lies at the upper edge of the dispersed bubble regime. Higher void fractions would lead to percolation or foamlike behavior, departing from the physics of bubble-laden HIT. The range inspected in this work covers most experimental and DNS conditions reported in the literature. As for the single-phase flow, we investigate two different Taylor-scale Reynolds

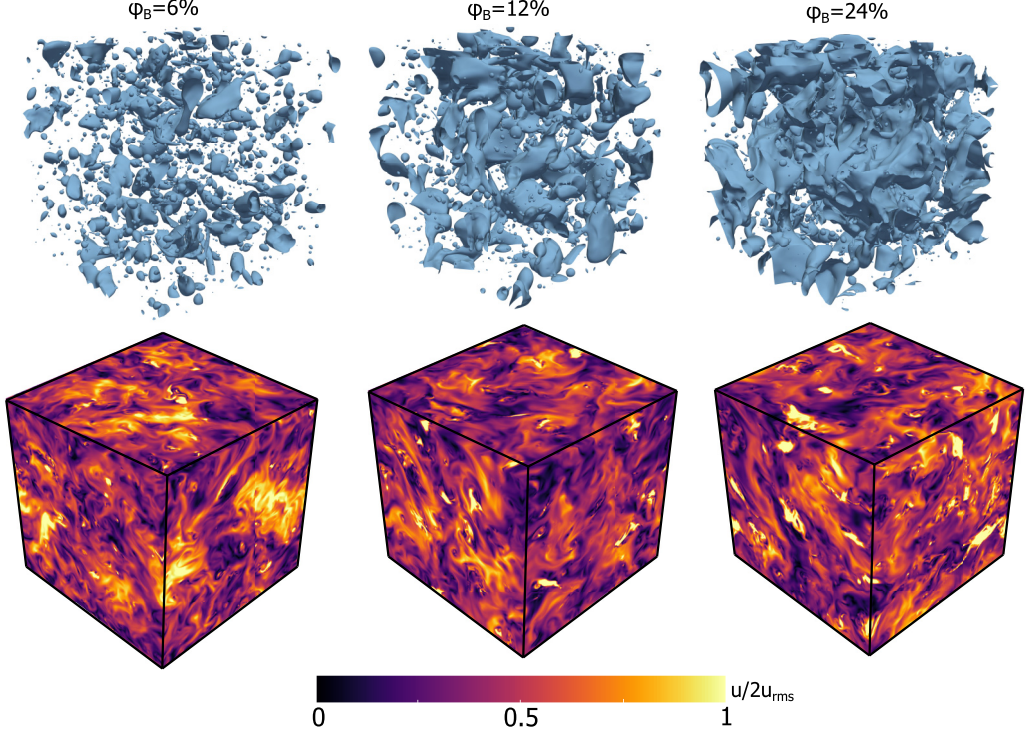


FIG. 3. Instantaneous snapshots of the dispersed bubble phase and velocity field for three different bubble volume fractions, ϕ_B . The snapshots are taken after HIT has reached an averaged steady state. The density and viscosity ratios are set to 100 and 1, respectively.

numbers, $Re_\lambda = 150$ and 230 . The surface tension has been set to $\sigma = 5 \times 10^{-3}$ (expressed in lattice units) delivering a turbulent Weber number $We = \rho_L u_{rms}^2 \lambda / \sigma \sim 3$. The main numerical and physical parameters used in the simulations are summarized in Table I.

For the present set of physical parameters, the Hinze length scale has been inferred *a posteriori* by employing the formula proposed by Deane and Stokes [36], $l_H = 2^{-8/5} \varepsilon^{-2/5} (\frac{\gamma We_c}{\rho})^{3/5}$ assuming a critical Weber number of $We_c \sim 4$. This choice is in agreement with the findings of Martínez-Bazán

TABLE I. Numerical and physical parameters used in the simulations (lattice units). The liquid density is fixed at $\rho_L = 10.0$ in all cases; density- and viscosity-ratio variations are obtained by modifying ρ_G and ν_G , respectively. The turbulent Weber number is approximately $We \approx 3$ for all configurations.

Case	Re_λ	ϕ_B	ρ_L	ρ_L / ρ_G	ν_L	ν_G / ν_L	σ
Single-phase (baseline)	150	0	10.0	1	1.0×10^{-3}	1	5.0×10^{-3}
Bubble-laden (baseline)	150	0.06–0.24	10.0	100	1.0×10^{-3}	1	5.0×10^{-3}
Single-phase (baseline)	230	0	10.0	1	5.0×10^{-4}	1	5.0×10^{-3}
Bubble-laden (baseline)	230	0.06–0.24	10.0	100	5.0×10^{-4}	1	5.0×10^{-3}
Density-ratio sweep	150	0.24	10.0	10	1.0×10^{-3}	1	5.0×10^{-3}
[Fig. 11(a)]	150	0.24	10.0	100	1.0×10^{-3}	1	5.0×10^{-3}
Viscosity-ratio sweep	150	0.24	10.0	500	1.0×10^{-3}	1	5.0×10^{-3}
[(Fig. 11(b)]	150	0.24	10.0	100	1.0×10^{-3}	0.1, 1, 10	5.0×10^{-3}

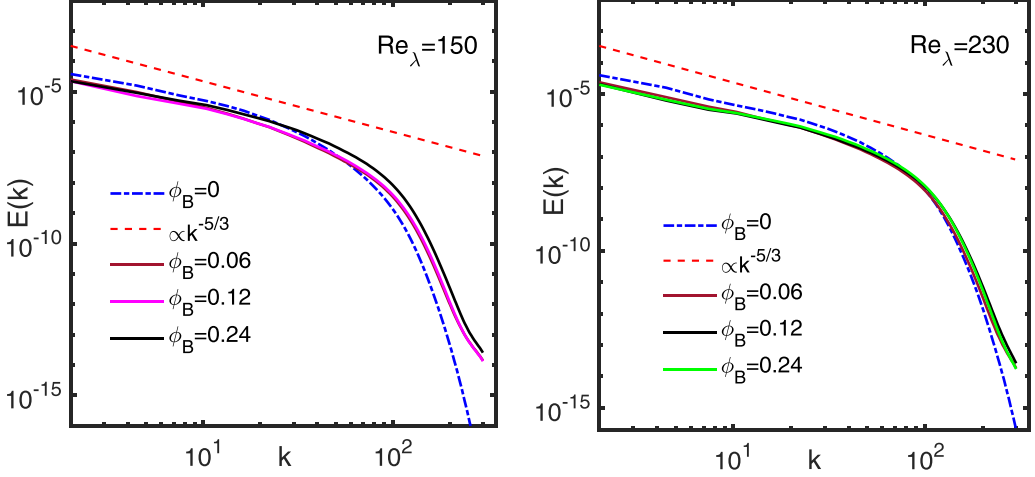


FIG. 4. Energy spectra at $\text{Re}_\lambda = 150$ and 230 for different bubble volume fractions, $\phi_B = 0.06, 0.12$, and 0.24 .

et al. [37], who reported similar critical conditions for bubble fragmentation in fully developed turbulence. The resulting estimate is $l_H \approx 24$, expressed in lattice units.

1. Effect of bubble volume fractions

The analysis of energy spectra from HIT simulations, with and without bubbles, reveals that the presence of bubbles leads, in the first place, to a redistribution of energy across scales. As shown in Fig. 4, the dispersed phase causes a certain decrease in the energy content at low wave numbers (k), which is almost independent of the gas volume fraction (i.e., ϕ_B), and an increase in the energy content at the highest k , corresponding to small-scale structures, which is more pronounced when increasing ϕ_B .

As observed in earlier works for emulsions with unitary density ratio [10,38], such redistribution is to be attributed to the role of the surface-tension force, which removes energy at the largest scale by deforming and breaking the larger bubbles and introduces additional energy at small scales during bubble merging and interface relaxation. As demonstrated below, the energy increase at high k points to the presence of positive time-averaged work of the surface-tension force at high wave numbers, which is effectively injecting energy into small-scale turbulence. In [10,39,40], it is shown that surface-tension effects enhance energy content at small scales by modifying the energy transfer pathways of the two-phase, emulsion system.

Note that, as pointed out above, the spectral modifications weakly depend on the bubble volume fraction for the parameters of our simulations, except for the smallest active scales. As demonstrated by the spectra in Fig. 4, simulations with increasing bubble volume fractions ($\phi_B = 0.06, 0.12$, and 0.24) at Taylor Reynolds numbers of $\text{Re}_\lambda = 150$ and 230 exhibit similar trends of the energy cascade. This consistency suggests that, at least for the range of volume fractions under scrutiny, the presence of bubbles, rather than their concentration, plays the dominant role in altering the energy transfer dynamics and that the mass fraction of the dispersed phase, rather than the volume fraction, plays an important role for the turbulence modulation. Indeed, in the case of emulsions (i.e., unitary density ratio) in [10] the increase of the turbulence modulation with the volume fraction of the dispersed phase is more pronounced, especially at the larger scales.

Further insights on the energy redistribution across scales can be obtained by examining the statistical distribution of bubble sizes in our simulations. We define the bubble size as the diameter of a sphere of the same volume of the bubble under consideration. The probability density functions

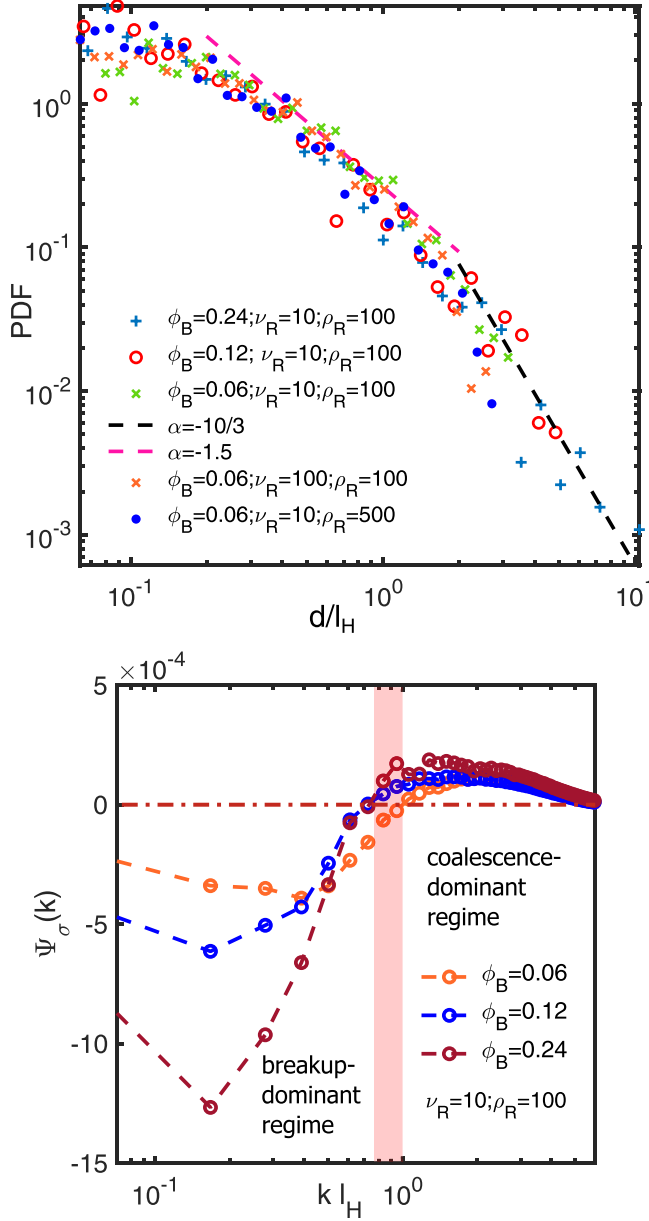


FIG. 5. (Top panel) Probability density functions of equivalent bubble diameters d , normalized by the Hinze scale l_H , for various flow parameters. (Bottom panel) Net surface-tension contribution $\Psi(k)$ as a function of wave number k , normalized by l_H . The shadowed area marks the transition region $k \sim 1/l_H$ separating the breakup- and coalescence-dominated regimes. ν_R and ρ_R denote the viscosity and density ratios.

(PDFs) of the equivalent bubble diameters, normalized by the Hinze scale d/l_H , are reported in Fig. 5(a) for various values of the bubble volume fraction ϕ_B , density ratio $R = \rho_l/\rho_g$, and viscosity ratio. Despite variations in these control parameters, all cases exhibit a two-regime scaling: a $-3/2$ power law for $d/l_H \lesssim 1$, and a steeper $-10/3$ decay for $d/l_H \gtrsim 1$. The latter behavior reflects a universal fragmentation mechanism driven by turbulent stresses, which dominate over surface

tension beyond the Hinze scale. The transition near $d \sim l_H$ signals the scale at which turbulence becomes strong enough to overcome capillary resistance and break up bubbles, in agreement with previous studies [10,36,41]. Notably, the persistence of this scaling across different ϕ_B and R values highlights the self-similar character of bubble size distributions in statistically steady bubble-laden turbulence. The presence of two distinct regimes for bubble dynamics—coalescence-dominated for $d < l_H$ and breakup-dominated for $d > l_H$ —is further corroborated by the spectral distribution of the interfacial energy transfer shown in Fig. 5(b). The figure shows the scale-dependent surface-tension work $\Psi_\sigma(k) = \langle \mathbf{u} \cdot \mathbf{F}^s \rangle$ as a function of the wave number k for the baseline case at different bubble volume fractions. In line with the analysis reported in [42,43], we observe that $\Psi_\sigma(k)$ changes sign near $k \sim 1/l_H$: surface tension drains energy at larger scales ($k < 1/l_H$), associated with bubble deformation and breakup while it injects energy into the carrier flow at small scales ($k > 1/l_H$), indicative of coalescence and interface relaxation. As shown in [42,43], this crossing point defines the Hinze scale dynamically and matches the scale identified in the PDFs of the droplets size. Indeed, the sign inversion in $\Psi_\sigma(k)$ close to the Hinze scale implies a physical mechanism whereby surface tension competes with inertial forces: breaking large bubbles while smoothing small interfacial disturbances. This dual role reflects how surface tension extracts energy from large-scale bubble deformation and releases it at smaller scales through rapid interface relaxation events, such as coalescence-driven recoil, which contribute to localized bursts of kinetic energy and enhanced intermittency in the gas phase. The alignment between the energy-sign inversion in $\Psi_\sigma(k)$ and the slope transition in $P(d)$ supports the interpretation of the Hinze scale as the boundary between nonlocal coalescence-driven dynamics and local fragmentation cascades with the energy increase at high k observed in Fig. 4.

2. Budget of turbulent kinetic energy

Useful information on the energy transfer mechanisms occurring in bubble-laden HIT, beyond spectral analysis, can be obtained by examining the global energy budget in statistically steady conditions for different bubble volume fractions ϕ_B and for the baseline values of viscosity and density ratios, as reported in Fig. 6. The global energy budget equation reads as follows:

$$\frac{dE}{dt} = F - \varepsilon + T_p + \Psi_\phi, \quad (28)$$

where E is the turbulent kinetic energy, $F = \langle \rho \mathbf{u} \cdot \mathbf{f} \rangle / \langle \rho \rangle$ is the power related to the external forcing used to sustain the turbulence in the flow, $\varepsilon = \langle 2\rho\nu S_{ij}S_{ij} \rangle / \langle \rho \rangle$ is the rate of turbulent kinetic energy dissipation, $T_p = -\langle \mathbf{u} \cdot \nabla p \rangle / \langle \rho \rangle$ is the work of pressure forces per unit time, and $\Psi_\phi = \langle \mathbf{u} \cdot \mathbf{F}_\sigma \rangle / \langle \rho \rangle$ is the mean surface-tension power, already defined above. All terms in the present budget are computed using Favre averages, i.e., density-weighted quantities normalized by the domain-average density [39,40].

As evident in the figure, at the global level, the energy balance largely follows the classical single-phase turbulence picture, with the mean energy input by external forcing F balanced by the mean viscous dissipation ε . The surface-tension power Ψ_σ is found to be very small compared to both forcing and dissipation (more than one order of magnitude smaller).

As shown in [41,44], the global average of Ψ_σ in bubble- or droplet-laden turbulence is zero at steady state, when the surface area is statistically constant. However, in our configuration, Ψ_σ takes a slightly positive value, which can be attributed to the fact that we are using a diffuse-interface approach for which the net dissipation is not zero [45].

On the other hand, the pressure work, present when the density contrast is not 1, computed here as the difference between F and ε , represents a small fraction (roughly 10%) of the total energy. This is consistent with the fact that, from Gauss's theorem, the term $\int \mathbf{u} \cdot \nabla p dV = -\int p(\nabla \cdot \mathbf{u}) dV$ vanishes in an incompressible fluid, where $\nabla \cdot \mathbf{u} = 0$ (the velocity field is solenoidal within the bulk). Thus we expect this term to be nonvanishing only near gas-liquid interfaces in the presence of strong density contrasts.

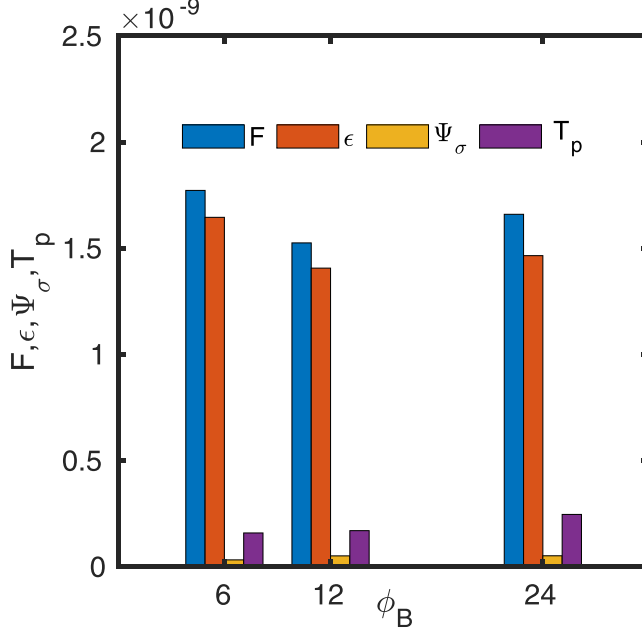


FIG. 6. Global energy budget diagrams for increasing bubble volume fraction and for the baseline values of viscosity and density ratios.

The above observations suggest that in bubble-laden turbulence, energy is redistributed across scales and phases via interfacial stresses, with the dispersed gas phase acting as a site of localized energy injection and the liquid phase serving as the primary reservoir of dissipation. Surface-tension contributions, while playing a role in local intermittency and spectral energy redistribution, do not lead to significant net energy production or loss when integrated over the domain, although they are responsible for phase-specific modifications in the energy transfer process.

3. Third-order structure functions in bubble-laden HIT

To gain a deeper understanding of the mechanisms shaping small-scale turbulence, it is instructive to analyze the third-order turbulent invariant [35]:

$$S_3(r) = \langle (\delta u_r)^3 \rangle, \quad (29)$$

where $\delta u_r = u(x+r) - u(x)$ is the longitudinal velocity increment at distance r . In classical Kolmogorov theory, $S_3(r)$ follows a linear scaling in the inertial range (i.e., $S_3(r) = -\frac{4}{5}\epsilon r$, ϵ being the energy dissipation rate), which implies that the energy transfer rate is scale invariant within the inertial range or, in other words, that the turbulence exhibits a form of self-similarity across scales [12,35]. Deviations from the linearity may stand as a clear signature of a nonclassical cascade, providing important insights into energy transfer mechanisms in bubble-laden flows.

As shown in Fig. 7, single-phase turbulence closely follows the Kolmogorov prediction for the larger separations r (i.e., the small-wave-number regime). However, in the presence of a dispersed gas phase, the shape and magnitude of $S_3(r)$ appear significantly altered, now with significant differences for the different values of ϕ_B .

At $\phi_B = 6\%$, $S_3(r)$ remains largely consistent with a forward energy cascade, although its magnitude is noticeably reduced compared to the single-phase flow, which points to the presence of extra dissipative mechanisms in the intermediate range of scales for bubble-laden flows.

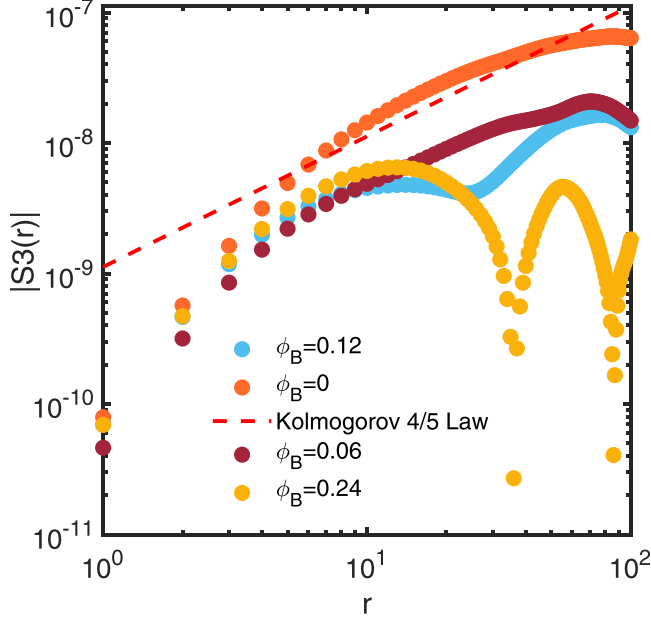


FIG. 7. Third-order structure functions, $S_3(r)$, for the cases under investigation. The data pertaining to the single-phase flow are shown as a reference, along with Kolmogorov’s theoretical 4/5 scaling law.

As the bubble volume fraction increases ($\phi_B = 12\%$ and 24%), $S_3(r)$ further deviates from the Kolmogorov scaling, dropping below the single-phase curve, particularly at intermediate scales. This is an indication of the presence of a reduced net energy transfer across scale due to vortex stretching and nonlinear interactions as bubble concentration increases, which can be attributed to the disruption of the large-scale coherence by the presence of the bubbles. As noted in previous studies, interfacial stresses provide the alternative path for energy transfer to the smallest scales.

Note that in Fig. 7 we report the modulus of the third-order structure function and, consequently, $S_3(r)$ at $\phi_B = 0.24$ exhibits an oscillatory behavior with the presence of cusps at $r \sim 40$ and ~ 90 (grid units). The third-order structure function has been chosen in place of S_2 since the second-order function, being an even-order moment, is primarily sensitive to the energy distribution across scales, but not to the direction of the energy transfer. As such, it masks non-Kolmogorov dynamics such as non-local energy injection, backscatter, or anisotropy introduced by bubbles [35]. This can be readily observed in Fig. 8 reporting the $S_2(r)$ for the baseline case as a function of the bubble volume fraction. The third-order structure function directly reflects the energy flux through scales and is therefore more sensitive to deviations from classical turbulence assumptions (e.g., scale locality, isotropy, and homogeneity). The observed departure in $S_3(r)$ therefore reveals the presence of altered cascade dynamics in bubble-laden flows that remain hidden in the second-order statistics.

The observations above show that the dispersed gas phase significantly modifies the turbulent energy transfer across scales, apparently suppressing a classical forward cascade at high volume fractions. In particular, the deviation from the theoretical $-4/5r$ behavior highlights the fundamental impact of gas-phase inclusions on the energy transfer in homogeneous and isotropic turbulence. However, the latter observation seems to contradict, at least partially, the results stemming from the inspection of the energy spectra, which still suggest the presence of a net forward cascade over a broad range of wave numbers. To reconcile this apparent discrepancy, we resort to the analysis of the energy transfer mechanisms within each phase separately.

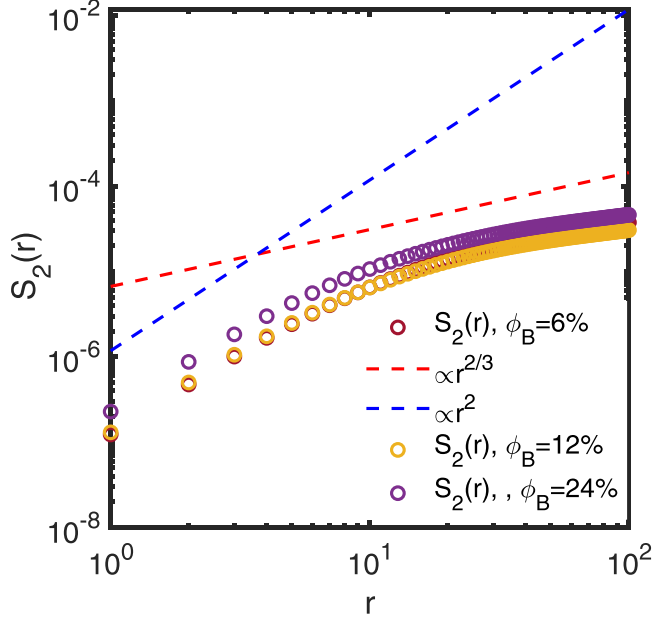


FIG. 8. Second-order structure functions for the baseline case and different bubble volume fraction. As a reference, the $\propto r^2$ and the $\propto r^{2/3}$ scaling laws are reported.

4. Spectral energy distribution in continuous and dispersed phases: Departure from Kolmogorov scaling

To understand the modifications of the energy transfer mechanisms in bubble-laden HIT, we examine the scale-by-scale energy dynamics within each phase.

As shown in Fig. 9, the energy spectrum in the liquid phase still follows the classical scaling at small-to-intermediate wave numbers and transitions to a steeper regime (the dashed reference lines denote k^{-3} scaling) before reaching the dissipation range. In contrast, the gas phase exhibits a nearly flat spectrum at low wave numbers, followed by a k^{-3} scaling and an even faster decay at smaller scales. It should be noted that the spectra in the gas phase agree with those observed in [46] and, interestingly, they closely overlap the analytical expression proposed by Risso [47] of a k^{-3} behavior in dispersed multiphase flows. In [47], the author noted that the spectrum of a signal composed of a sum of localized random bursts can, under certain conditions, exhibit an intermediate subrange that follows a power-law decay of k^{-3} . For this behavior to emerge, the bursts should be characterized by a smooth and regular pattern, their strength and size should be statistically independent, and their size should be uniformly distributed between two finite wavelengths.

The above results suggest that, despite the turbulence within each phase deviating from the Kolmogorov cascade, the global energy spectrum, as observed in the previous subsection, still recovers the universal $k^{-5/3}$ across a wide range of wave numbers.

Remarkably, the emergence of a k^{-3} scaling in both the gas and liquid energy spectra occurs close to the Hinze scale, where the third-order structure function $S_3(r)$ also undergoes a sign inversion. This spatial and spectral coincidence indicates that the Hinze scale marks a transition point at which classical, scale-local energy transfer breaks down, and interfacial mechanisms, such as coalescence-driven recoil and bubble deformation, dominate the dynamics. These processes at the Hinze scale may be associated to the localized random bursts producing the k^{-3} spectra [47]. The sign change in $S_3(r)$ further suggests the presence of nonlocal or even reversed energy flux, highlighting the critical role of surface-tension effects at this scale.

To summarize, the k^{-3} scaling observed in the dispersed gas phase is to be attributed to the dominance of interfacial deformation, coalescence-driven recoil, and localized velocity bursts,

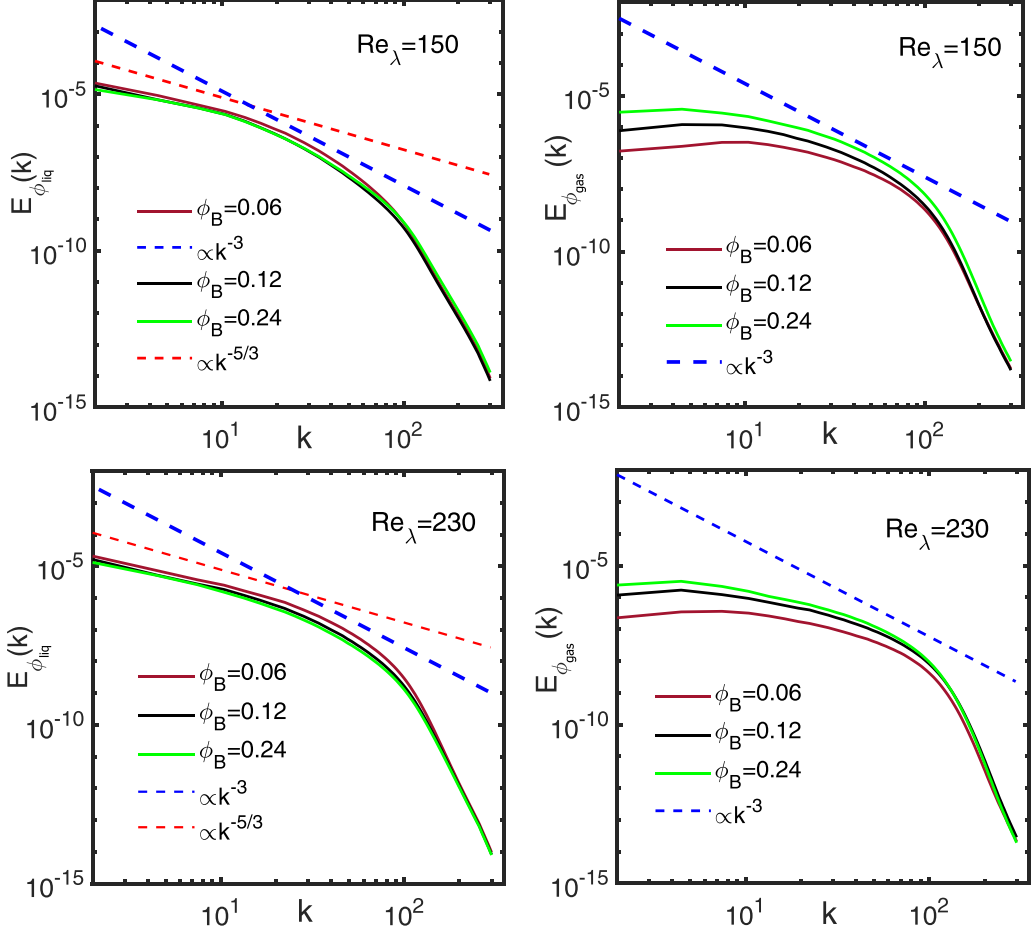


FIG. 9. Phase-specific energy spectra at $Re_\lambda = 150$ and 230 . The left panels show the spectra of the liquid phase, while the right panels correspond to the dispersed gas phase.

which replace the energy injection by the classic inertial cascade inside the confined gas regions. This behavior, as observed above, is consistent with the theoretical model of Risso, showing that a velocity field composed of localized random impulses produces a k^{-3} spectrum. A short k^{-3} scaling is also observed in the liquid phase near the Hinze wave number, reflecting interfacial energy exchange, whereas the classical $k^{-5/3}$ inertial scaling is recovered at larger scales. The robustness of the k^{-3} gas-phase regime is confirmed across all Reynolds numbers, void fractions, and density and viscosity contrasts examined.

Further insights into phase-specific energy transfer mechanisms can be gained by analyzing the phase-specific $S_3(r)$ structure functions. As shown in Fig. 10, in the liquid phase, $S_3(r)$ largely follows the expected Kolmogorov scaling at large separations, confirming the fractal nature of turbulence in the continuous phase. In contrast, the gas phase exhibits a distinctly different behavior. At small separations, $S_3(r)$ follows the expected dissipative-range scaling. However, at intermediate scales, it decreases abruptly, eventually becoming negative as r increases.

This anomalous behavior is to be attributed to the presence of the gas-liquid interface, which introduces an additional mechanism for energy redistribution.

Furthermore, as the bubble volume fraction increases, the magnitude of $S_3(r)$ in the gas phase becomes progressively smaller, indicating a reduced net energy flux at intermediate scales. Such

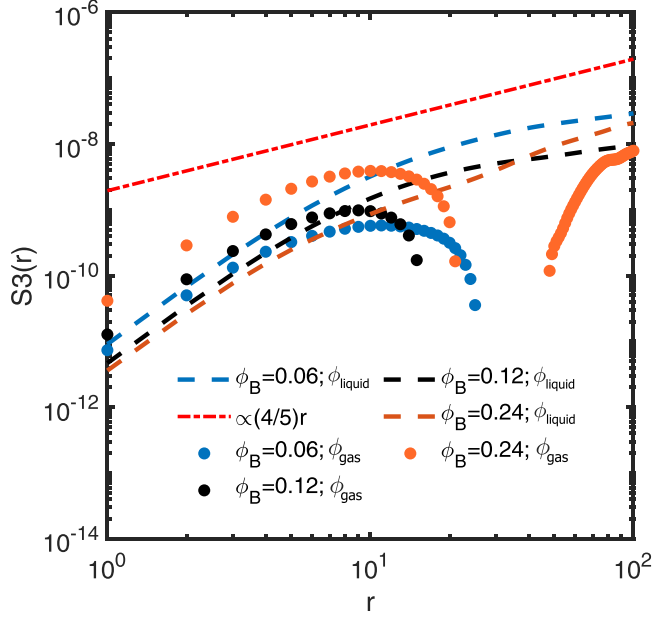


FIG. 10. Phase-specific third-order structure functions, $S_3(r)$, at $\text{Re}_\lambda = 150$. The theoretical Kolmogorov 4/5 scaling law is shown as a dashed red line. Circles represent S_3 in the liquid phase, while pluses correspond to the dispersed gas phase. ϕ_{liquid} and ϕ_{gas} correspond to the liquid and gas phases, respectively.

a suppression of energy transfer aligns with the spectral analysis, where the gas phase does not develop a well-defined inertial cascade, but rather follows a k^{-3} scaling at intermediate wave numbers. This confirms the more intuitive interpretation that turbulence in the dispersed phase remains constrained by interfacial effects, which limits the development of the classical turbulence features.

Taken together, the results from both the energy spectra and the third-order structure function analysis indicate that bubbles significantly modify turbulence by redistributing energy in a nonclassical manner. While the liquid phase still supports a forward cascade, albeit weakened at intermediate scales, the gas phase experiences a fundamentally different energy transfer process, dominated by events at the intermediate scale, that we conjecture is related to droplet breakup and coalescence.

Summarizing, the breakdown of Kolmogorov scaling may be attributed to a combination of factors intrinsic to the gas phase: confinement of turbulent eddies within finite-sized bubbles, low inertia, and surface-tension-dominated dynamics. These effects likely suppress a scale-local energy cascade and instead promote nonlocal, intermittent transfer, associated with interface deformation, coalescence, or recoil. The presence of pronounced oscillations in $S_3(r)$, but not in the second-order structure function $S_2(r)$, further supports this view. While $S_2(r)$ captures the statistical energy content across scales, $S_3(r)$ encodes directional energy flux and reveals the underlying disruption or modification of the cascade mechanism specific to the gas phase. All of the above highlights the critical role of interphase interactions in shaping turbulent energy dynamics in bubble-laden flows.

5. Effect of density and viscosity ratio

In this subsection we report on the separate effect of density and viscosity ratio on the modulation of the energy spectrum for $\phi_B = 0.24$. The energy spectra, at $\text{Re}_\lambda = 150$, are reported in Fig. 11. Both variations mainly affect the large- and small-wave-number regions of the spectrum, leaving the inertial cascade almost unaltered.

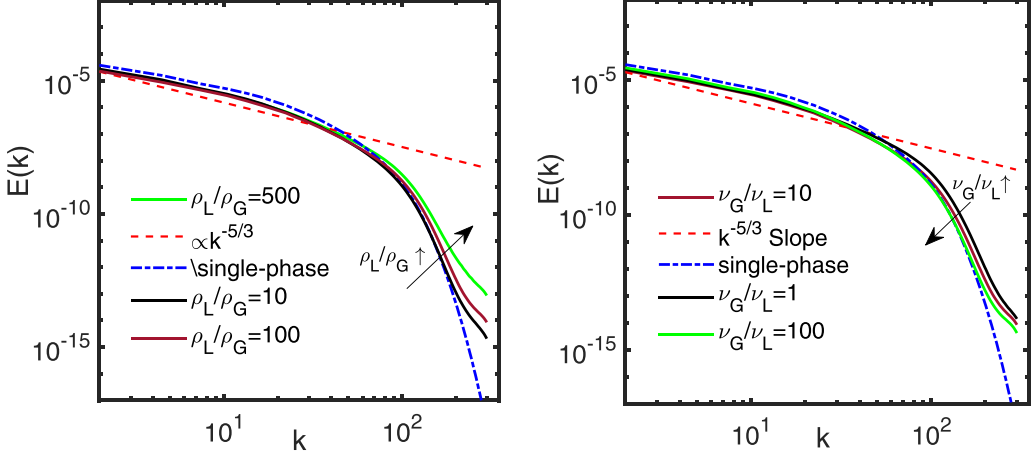


FIG. 11. Energy spectra at $Re_\lambda = 150$ at increasing density and viscosity ratio at 24% gas volume fraction.

In particular, the increase of the density ratio, ρ_L/ρ_G (the viscosity ratio is 1 in these cases), further reduces the slope of the energy spectra in the dissipative range; i.e., smaller and smaller scales are excited in the flow when increasing the density ratio. The opposite effect is instead observed when increasing the viscosity ratio (the density ratio is fixed at $\rho_L/\rho_G = 100$). In any case, the observations made before still hold even in these cases, proving the robustness of the physical mechanisms described above. Indeed, by inspecting the third-order structure function $S_3(r)$ for the case $\rho_L/\rho_G = 500$, $\nu_G/\nu_L = 1$ at $\phi_B = 0.24$ (see Fig. 12), we observe that the Kolmogorov law is

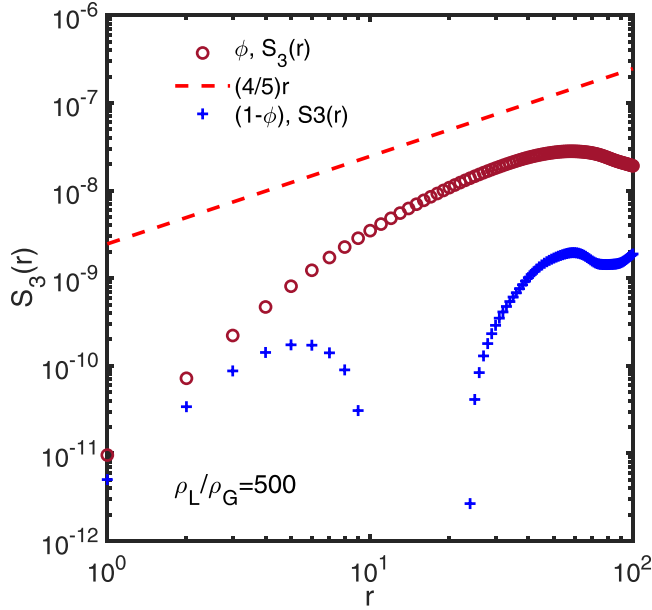


FIG. 12. Phase-specific third-order structure functions, $S_3(r)$, at $Re_\lambda = 150$, $\rho_L/\rho_G = 500$ and $\phi_B = 0.24$. The theoretical Kolmogorov 4/5 scaling law is shown as a dashed red line. Circles represent S_3 in the liquid phase, while pluses correspond to the dispersed gas phase. ϕ and $1 - \phi$ correspond to the liquid and gas phases, respectively.

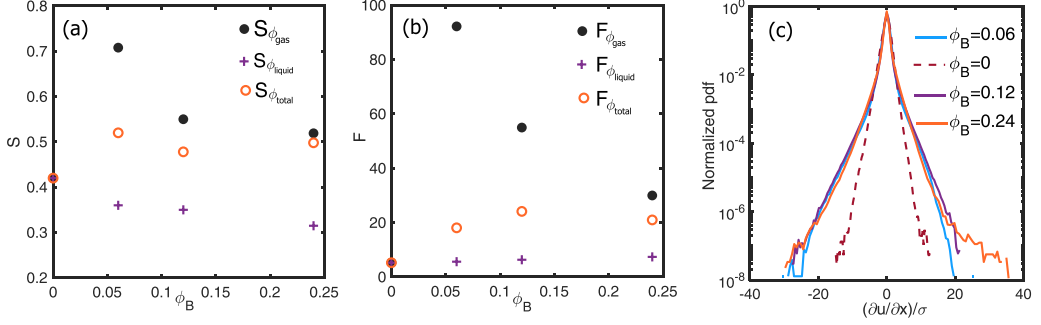


FIG. 13. (a) Skewness S of velocity increments as a function of the bubble volume fraction ϕ_B . The values are reported for the liquid phase, the gas phase, and the homogenized system (subscript *total*). (b) Flatness F of velocity increments as a function of the bubble volume fraction ϕ_B . The values are reported for the liquid phase, the gas phase, and the homogenized system. (c) Normalized PDF (liquid+gas) of the longitudinal velocity gradients at increasing bubble volume fraction.

still clearly visible in the inertial range in the liquid phase while it is completely suppressed within the gas phase. Summarizing, the following conclusions may be drawn:

(i) *Universality holds in the homogenized region.* When considering the flow as a whole—i.e., analyzing the energy spectrum $E(k)$ of the mixture without distinguishing between phases—the behavior closely follows the classical Kolmogorov prediction, at least for the configurations of this study. At large scales, within the inertial range, the presence of dispersed gas does not significantly alter the overall energy cascade in a way that is easily discernible from a homogenized perspective.

(ii) *Phase-specific statistics reveal non-Kolmogorov behavior.* By decomposing the flow into liquid and gas phases and examining phase-specific energy spectra or third-order structure functions, deviations from classical turbulence emerge. In particular, the energy transfer within the gas phase diverges substantially from the Kolmogorov scaling, with pronounced departures in small-scale statistics, precisely where multiphase effects become dominant (e.g., bubble interfaces, local slip velocities). In contrast, the liquid phase largely adheres to a near-classical scaling. This distinction implies that when recombining the two phases into a single energy spectrum, the non-Kolmogorov features primarily originating from the gas phase are partially masked. Thus, while turbulence to the inertial-range scales remains close to classical phenomenology, the anomalous k^{-3} scaling emerges as a distinctive feature of the presence of the gas phase at smaller scales.

6. Intermittency footprints in bubble-laden HIT

In the previous sections, we highlighted the presence of localized bursts of velocity within the gas phase, which point to strong intermittent events that alter the small-scale dynamics of bubble-laden turbulence. The statistical analysis of velocity derivatives provides further insights into this behavior. Figure 13(a) reports the skewness S of longitudinal velocity gradients as a function of the bubble volume fraction ϕ_B , while Fig. 13(b) shows the corresponding flatness F .

In the liquid phase, both skewness and flatness remain close to the single-phase reference, confirming that the carrier fluid retains near-classical HIT statistics even in the presence of bubbles. In contrast, the gas phase shows pronounced deviations, especially at low void fractions. At $\phi_B = 6\%$, the skewness reaches a peak of $S \approx 0.7$, while the flatness attains extreme values of $F \approx 92$, pointing to highly intermittent, heavy-tailed fluctuations. The probability density functions of the velocity gradients in Fig. 13(c) further illustrate the enhanced occurrence of rare, intense events in the dispersed phase.

Interestingly, these extreme values decrease with increasing gas volume fraction, suggesting that intermittency is most intense when bubbles are sparse and interfaces are more isolated. At higher

ϕ_B , a more connected gas network appears to reduce the impact of rare events, although sampling limitations due to the reduced gas volume may also contribute to this trend.

When the phases are combined into a homogenized system, the skewness stabilizes around $S \approx 0.5$ and flatness around $F \approx 20$. These aggregate values do not imply that the liquid mediates the gas-phase intermittency, but rather reflect the volume-weighted averaging of the two phases: the nearly Gaussian liquid statistics moderate the extreme gas-phase tails. As a result, the global system displays enhanced intermittency compared to single-phase HIT, but weaker than the raw gas-phase signal.

We wish to clarify that the heavy-tailed PDFs and large flatness values observed in the gas-phase velocity gradients originate from physical interfacial processes, including coalescence-driven recoil, rapid bubble deformation, and the low inertia of the gas. These effects generate localized impulsive events that do not occur in the carrier liquid, which largely follows a classical turbulent cascade and exhibits weaker intermittency. The robustness of these signals is confirmed by mass conservation, proper interface resolution, and the absence of spurious high-frequency features in energy spectra. Moreover, the intermittent bursts correlate with peaks in surface-tension work and bubble-merging events, confirming their physical origin.

Overall, these results complement the spectral and structure-function analysis: the gas phase sustains strong non-Gaussian fluctuations at small scales, while the liquid phase remains close to Kolmogorov statistics. The intermittency diagnostics thus reinforce the view that gas-liquid interfaces act as sites of localized, intermittent energy transfers, but that their net contribution is partly obscured when examining homogenized turbulence statistics.

IV. CONCLUSIONS

In this work, we investigate the dynamics of bubble-laden HIT through high-resolution simulations using a thread-safe, high-performance LB framework. The main focus is to shed light onto the modified energy transfer mechanisms and phase-specific deviations from Kolmogorov scaling in bubble-laden HIT. We find that, at the level of the homogenized system, the energy spectra preserve a Kolmogorov-like behavior across a wide range of wave numbers, confirming the apparent universality of turbulent energy transfer when phase distinctions are neglected. However, a phase-resolved analysis reveals that this global scaling masks important anomalies: the liquid phase largely maintains a classical turbulent cascade, while the gas phase exhibits non-Kolmogorov behavior, including a steeper k^{-3} spectral decay and altered third-order structure functions. These deviations stem from strong interfacial interactions and localized velocity bursts within the gas phase, indicative of pseudoturbulent dynamics. The interface acts as both a source and sink of energy, disrupting the cascade and introducing intermittency, as evidenced by elevated flatness and skewness in the gas-phase velocity gradients. In contrast, the liquid phase shows only modest variations in intermittency, preserving the statistical features typical of HIT.

Our phase-specific energy flux analysis further highlights the fundamentally different roles played by the two phases. While the liquid supports a classical forward cascade, the gas phase lacks a clear inertial range and instead features localized, scale-selective energy injection and dissipation. Interestingly, the combination of these distinct behaviors still results in an aggregate energy spectrum that mimics Kolmogorov scaling. Finally, we assessed the effect of varying density and viscosity ratios, confirming the robustness of our conclusions. Even under extreme contrasts, the qualitative features of energy redistribution and phase-specific cascade dynamics remain consistent.

In summary, our findings demonstrate that phase-specific interactions in bubble-laden turbulence not only disrupt classical scaling laws at small scales but also unveil fundamentally new turbulence dynamics that is obscured in global flow statistics. The distinct cascade dynamics in the gas and liquid phases—governed by localized interfacial bursts and nonclassical energy transfers—challenge the conventional universality of turbulent flows and call for a paradigm shift in the modeling of multiphase turbulence. In future studies, we will extend our investigation to regimes involving

buoyancy, near-contact interactions among fluid interfaces, and higher Reynolds numbers, aiming to bridge the gap between idealized turbulence and real-world multiphase phenomena.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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