

Boundary layer beneath a tornado-like vortex

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We examine experimentally the boundary layer beneath a Rankine vortex, which consists of a central vortex tube surrounded by an irrotational free vortex. We do this by setting up a tornado-like vortex in a large water tank, and then we examine the resulting boundary layer using particle image velocimetry. The boundary layer beneath a Rankine vortex has attracted considerable attention in the past, as it is thought to play an important role in the dynamics of a tornado R. Rotunno [*Annu. Rev. Fluid. Mech.* **45**, 59 (2013)]. In particular, Burggraf [*Phys. Fluids* **14**, 1821 (1971)] have performed an approximate asymptotic analysis of the steady, laminar boundary layer beneath a free vortex, predicting a complex, two-layer structure for the boundary layer. Their predictions have been found to be in qualitative agreement with experiments on the flow surrounding a tornado-like vortex, but to date there is no detailed, quantitative comparison. We provide such a comparison here. We find that the analysis of Burggraf *et al.* is in good agreement with our measurements when the boundary layer is steady, but discrepancies start to arise as the Reynolds number increases and the flow becomes unsteady.

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I. INTRODUCTION

The flow associated with a tornado is extremely complicated. There is, of course, the turbulent motion within the tornado itself, which is characterized by spiraled fluid trajectories that reach up from the ground to the cloud above. However, since the tornado may be considered as a vortex tube, which carries axial vorticity, there is also a free vortex surrounding the tornado, and that free vortex establishes a swirling boundary layer near the ground. Crucially, this boundary layer is not merely a passive by-product of the tornado, but rather plays a central role in the maintenance of the tornado itself [1]. That is, there is a spiraled inflow within the boundary layer, directed along the ground towards the base of the tornado. This inflow is driven by a radial pressure gradient established above the boundary layer, within the free vortex. So, the air spirals inwardly along a surface layer, spinning faster as it approaches the tornado. This air then collides at the base of the tornado, erupting upwards into the tornado itself, and carrying mass and angular momentum with it. In short, the boundary layer beneath the free vortex provides the tornado with a continual supply of angular momentum and axial mass flux [1].

The nature of the laminar boundary layer beneath a free vortex has been analyzed by Burggraf *et al.* [2] using asymptotic methods. This approximate analysis suggests that the boundary layer has a surprisingly rich structure, more complicated than that beneath a fluid in rigid-body rotation [3]. In particular, Burggraf *et al.* [2] suggest that the boundary layer beneath a free vortex has a two-layer structure, with an inner layer next to the surface, which carries most of the shear stress needed to

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satisfy the no-slip condition, and an outer layer, which carries most of the radial mass flux. There have also been several experimental investigations of the boundary layer beneath a free vortex, such as the experiments of Phillips [4,5] and those of Ying and Chang [6]. The experiments of Phillips, in particular, lend tentative support to the analysis of Burggraf *et al.* [2], showing qualitative agreement with the asymptotic theory.

However, the asymptotic analysis of Burggraf *et al.* makes a number of simplifications, such as neglecting the azimuthal motion in the inner layer, and it remains important to test quantitatively its various predictions. That is one of the aims of this paper. Another is to examine how the structure of the boundary layer changes as the Reynolds number increases, and we move out of the steady, laminar regime. We do this by setting up a tornado-like vortex in a water tank and examining the resulting velocity field using particle image velocimetry (PIV).

The structure of the paper is as follows. In Sec. II, we examine the asymptotic theory of Burggraf *et al.* [2], with a particular emphasis on those predictions that may be tested in an experiment. Here we expand on the discussion in Ref. [7]. Next, in Sec. III, we describe the experimental setup, and in Sec. IV we detail the experimental results in light of the theoretical predictions. We conclude in Sec. V, noting that the model of Burggraf *et al.* fares remarkably well.

II. A MODEL OF THE BOUNDARY LAYER BENEATH A RANKINE VORTEX

The strong updraft within a tornado or dust devil is a universal and striking feature of these vortices, with the swirling and axial velocities typically of similar magnitudes near the ground [1]. The origin of this updraft may be understood by considering the analysis of Burggraf *et al.* [2], who examine the laminar, axisymmetric boundary layer beneath a Rankine vortex. This, in turn, is a variant of Bödewadt's [3] famous 1940 solution for the boundary layer beneath an external flow consisting of rigid-body rotation. In both cases, the boundary layer is characterized by a radial inflow, driven by a radial pressure gradient. That is, above the boundary layer there is a radial force balance between $\partial p/\partial r$ and $\rho u_\theta^2/r$, resulting in a low pressure on the axis. (We use cylindrical polar coordinates throughout.) This radial pressure gradient is imposed on the fluid within the boundary layer, where the centrifugal force is smaller, and the resulting imbalance between $\partial p/\partial r$ and $\rho u_\theta^2/r$ drives the fluid radially inward.

The geometry considered by Burggraf *et al.* [2] is shown in Fig. 1. A vortex tube of radius r_0 has its centerline normal to, and coaxial with, a disk of radius R , with $R \gg r_0$. The axis of the vortex

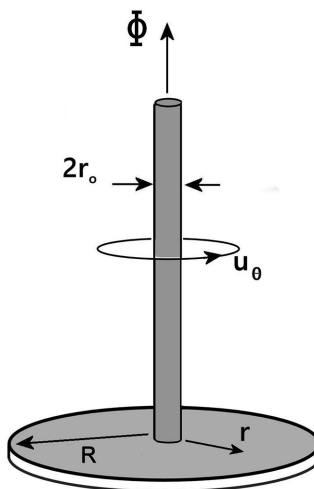


FIG. 1. A thin vortex tube is normal to, and coaxial with, a disk of radius R .

lies on the z -axis, and the disk is located on the plane $z = 0$. We adopt cylindrical polar coordinates centered on the vortex tube and disk, so that outside the boundary layer the azimuthal velocity is a function of r only. Indeed, Stokes' theorem tells us that, above the boundary layer and outside the vortex tube, the azimuthal velocity takes the form $u_\theta = \Phi/2\pi r$, where Φ is the net flux of vorticity along the vortex tube. Within the vortex tube, however, u_θ will increase with radius, increasing linearly near the axis. In short, the overall velocity profile remote from the disk is that of a Rankine vortex. Of course, near the disk a boundary layer forms, across which the azimuthal velocity drops down to zero.

Consider that part of the boundary layer which lies outside the thin vortex tube, $r > r_0$, where the external velocity is $u_{\theta, \text{ex}} = \Phi/2\pi r$. Given Bödewadt's solution for the boundary layer beneath a rigidly rotating external flow, in which z is rescaled using a single viscous length scale, it is natural to look for a similar boundary-layer structure here. While such a structure does indeed exist, Burggraf *et al.* [2] have shown that this is only one part of the story, with the boundary layer for $r > r_0$ comprising two distinct layers. That is to say, the boundary layer has a composite structure, comprising a thin, viscous, Bödewadt-like solution adjacent to the surface, capped by a thicker outer layer. The function of the inner layer is to meet the no-slip condition on the disk, while that of the outer layer is to carry the bulk of the inward radial mass flux.

The main features of the inner viscous layer can be established using a scaling analysis.

Near the boundary, the external radial pressure gradient, $\partial p/\partial r = \rho u_{\theta, \text{ex}}^2/r$, is resisted by the radial viscous force $\rho \nu \nabla^2 u_r$. It follows that there is a force balance of the form

$$\rho \nu |\nabla^2 u_r| \sim \rho \nu u_r / \delta_{\text{in}}^2 \sim \partial p / \partial r \sim \rho u_{\theta}^2 / r, \quad (1)$$

where δ_{in} is the thickness of the inner boundary layer. Now suppose that the radial and azimuthal velocities are of similar magnitudes, as observed in practice. Then

$$\nu / \delta_{\text{in}}^2 \sim u_\theta / r \sim \Phi / 2\pi r^2, \quad (2)$$

or equivalently

$$\delta_{\text{in}}(r) \sim r \sqrt{2\pi \nu / \Phi}. \quad (3)$$

Given this estimate of the inner boundary layer thickness, Burggraf *et al.* introduce the scaled coordinate $\eta = z/\delta_{\text{in}}$, where $\delta_{\text{in}} = r \sqrt{2\pi \nu / \Phi}$. It is useful to outline the inner layer solution of Burggraf *et al.* [2], which we now do, following Davidson [7]. Since the external azimuthal velocity is $u_{\theta, \text{ex}} = \Phi/2\pi r$, we look for a laminar, axisymmetric solution of the form

$$u_r = \frac{\Phi}{2\pi r} f(\eta), \quad u_\theta = \frac{\Phi}{2\pi r} g(\eta), \quad u_z = \frac{\sqrt{\nu \Phi / 2\pi}}{r} h(\eta). \quad (4)$$

Continuity then requires that f and h are related by

$$\eta f'(\eta) = h'(\eta), \quad (5)$$

while the radial and azimuthal components of the Navier-Stokes equation yield,

$$h f'(\eta) - f \frac{d}{d\eta}(\eta f) + (1 - g^2) = f''(\eta) + \frac{2\pi \nu}{\Phi} \left[\frac{1}{\eta} \frac{d}{d\eta}(\eta^3 f'(\eta)) \right], \quad (6)$$

$$(h - \eta f) g'(\eta) = g''(\eta) + \frac{2\pi \nu}{\Phi} \left[\frac{1}{\eta} \frac{d}{d\eta}(\eta^3 g'(\eta)) \right]. \quad (7)$$

For large Reynolds numbers, $\Phi \gg \nu$, the contribution to the viscous Laplacian from the radial velocity gradients may be dropped, giving us

$$h f'(\eta) - f \frac{d}{d\eta}(\eta f) + (1 - g^2) = f''(\eta), \quad (8)$$

$$(h - \eta f) g'(\eta) = g''(\eta). \quad (9)$$

Next, it is convenient to introduce the Stokes stream-function, defined in the usual way by

$$(u_r, 0, u_z) = \nabla \times [(\Psi(r, z)/r)\hat{\mathbf{e}}_\theta]. \quad (10)$$

Following Burggraf *et al.*, we express Ψ in terms of an auxiliary function, $\psi_0(\eta)$:

$$\Psi(r, z) = -r\sqrt{\nu\Phi/2\pi} \psi_0(\eta). \quad (11)$$

It is readily confirmed that (10) ensures

$$f = \psi'_0(\eta), \quad h = \eta\psi'_0(\eta) - \psi_0, \quad (12)$$

and so (8) and (9) can be rewritten in terms of $\psi_0(\eta)$ and g , according to

$$\psi_0'''(\eta) + \psi_0\psi_0''(\eta) + \psi_0'^2 + (g^2 - 1) = 0, \quad (13)$$

$$g''(\eta) + \psi_0g'(\eta) = 0. \quad (14)$$

Finally, Burggraf *et al.* suggest that, within the inner layer, u_θ is considerably smaller than u_r , and so g may be omitted from (13). Given that $\psi'_0(0) = \psi_0(0) = 0$, this then integrates to give

$$2\psi'_0(\eta) + \psi_0^2 = \eta^2 - 2\alpha\eta, \quad (15)$$

where α is a constant of integration. However, $f = \psi'_0(\eta)$ must remain finite and of order unity for large η , and so (15) requires $\psi_0^2(\eta \gg 1) \approx \eta^2 - 2\alpha\eta$. It follows that $\psi_0(\eta \gg 1) \approx \alpha - \eta$, and hence $\psi'_0(\eta \rightarrow \infty) = -1$. This constraint is sufficient to determine α , and Burggraf *et al.* show that $\alpha = 1.087$, a result we have confirmed. For large η , we then have the power-law expansion

$$\psi_0(\eta \gg 1) = -\eta + \alpha - \frac{2 - \alpha^2}{2\eta} - \frac{\alpha(2 - \alpha^2)}{2\eta^2} + O(\eta^{-3}), \quad (16)$$

from which (12) gives us

$$f(\eta \gg 1) = \psi'_0(\eta \gg 1) = -1 + \frac{2 - \alpha^2}{2\eta^2} + \dots, \quad (17)$$

$$h(\eta \gg 1) = [\eta\psi'_0(\eta) - \psi_0]_{\eta \rightarrow \infty} = -\alpha + \frac{2 - \alpha^2}{\eta} + \dots \quad (18)$$

For small η , on the other hand, (15) gives $\psi'_0(\eta \rightarrow 0) = -\alpha\eta$, from which $h(\eta \rightarrow 0) = -\frac{1}{2}\alpha\eta^2$. Crucially, the axial velocity within the inner layer is negative for both small and large η . This is the precise opposite of Bödewadt's solution, which has $u_z > 0$, and so it detains mass for all r . Of course, there is an outer region on the disk (where $r \sim R$) within which this solution does not apply, since the asymptotic structure of the boundary layer takes time to build up.

Now the inward radial mass flux within the inner boundary layer is evidently

$$\dot{m}_{\text{in}}(r)/\rho = 2\pi r \int_0^{\delta_{\text{in}}} |u_r| dz = \Phi \delta_{\text{in}} \int_0^1 |f| d\eta \sim r\sqrt{2\pi\nu\Phi}. \quad (19)$$

This has a value of $\dot{m}_{\text{in}}/\rho \sim R\sqrt{2\pi\nu\Phi}$ towards the outer edge of the disk, but approaches zero as we get close to the vortex tube. We conclude that the inner boundary layer continually detains mass, despite the fact that u_z is everywhere negative. Burggraf *et al.* show that this mass flux gradually transfers to an upper capping layer, whose height scales as $\delta_{\text{outer}} \sim R\sqrt{2\pi\nu/\Phi}$, or $\delta_{\text{outer}} \sim \delta_{\text{in}}(r = R)$. The reason why the lower layer loses mass to the upper one, despite the fact that $u_z < 0$, is because the interface between the two layers is steeper than the streamlines in the lower boundary layer [see (23) below]. Indeed, the numerical simulations of Burggraf *et al.* show this interface to lie at the critical height $\eta_{\text{crit}} \approx 2$, or equivalently, $z_{\text{crit}} \approx 2\delta_{\text{in}}$.

In summary, then, as we approach the central vortex, the outer boundary layer becomes much thicker than the inner one, and so it sustains less shear stress. It also carries nearly all of the radial mass flux as it approaches the central vortex tube, with $\dot{m}_{\text{outer}}/\rho \sim R\sqrt{2\pi\nu\Phi}$. This mass is then

redirected upward into the vortex core, causing an eruption of boundary layer fluid into the vortex. (Such an eruption does *not* occur in Bödewadt's solution because this boundary layer detrains mass.) The characteristic azimuthal and axial velocities at the base of the vortex are then

$$u_\theta \sim \frac{\Phi}{2\pi r_0}, \quad u_z \sim \frac{\dot{m}_{\text{outer}}/\rho}{\pi r_0^2} \sim \frac{\Phi R \sqrt{2\pi\nu/\Phi}}{\pi r_0^2} \sim \frac{\Phi \delta_{\text{outer}}}{\pi r_0^2}. \quad (20)$$

Now, it is observed that $u_z \sim u_\theta$ at the base of the vortex tube [5], and so we conclude from (20) that the radius of the vortex is determined by the relationship $r_0 \sim \delta_{\text{outer}}$.

There are many aspects of this analysis that are testable in a laboratory experiment. For example, the prediction that $u_z < 0$, in contrast to Bödewadt's solution, clearly demands attention. Also, the height of the inner layer is predicted to be $z_{\text{crit}} \approx 2\delta_{\text{in}}$, which satisfies

$$z_{\text{crit}}/r \approx \sqrt{8\pi\nu/\Phi} = \text{const}, \quad (21)$$

while the vortex core is predicted to scale as $r_0 \sim \delta_{\text{outer}}$. Finally, (17) and (18), tentatively applied at the top of the inner layer, yield $f(\eta = \eta_{\text{crit}}) \approx -0.90$ and $h(\eta = \eta_{\text{crit}}) \approx -0.68$. This, in turn, gives

$$ru_r(z = z_{\text{crit}}) \approx -0.9 \frac{\Phi}{2\pi} = \text{const}, \quad ru_z(z = z_{\text{crit}}) \approx -0.68 \sqrt{\frac{\nu\Phi}{2\pi}} = \text{const}, \quad (22)$$

and

$$(u_z/u_r)_{z=z_{\text{crit}}} \approx 0.38 z_{\text{crit}}/r, \quad (23)$$

which confirms that the streamlines in the lower layer are shallower than the interface. Of course, all of these predictions apply only to intermediate radii, being invalid close to the edge of the disk and also close to the central vortex.

III. THE EXPERIMENTAL SETUP

The experiment described below is similar to that of Phillips [4], who demonstrated that it was possible to simulate a tornado-like flow in water in the laboratory. His apparatus comprised a circular cylinder that rotated steadily above a fixed flat plate, and, using flow visualization, he showed that an intense vortex could be created in the center of the cylinder. He also determined some of the characteristics of the boundary layer that flowed towards the vortex over the flat plate. Later, in conjunction with Phillips and Khoo [5], he extended this study, using laser-Doppler anemometry, and showed that the distribution of velocity in this boundary layer was qualitatively consistent with the structure proposed by Burggraf *et al.* [2].

The present equipment, shown in Fig. 2, is in many respects similar to that of Phillips. The flow is generated within a Perspex cylinder that is 600 mm in diameter and 550 mm tall. It has a solid lid, and a flow is created by extracting water through a 35 mm hole at the center of the lid using a variable speed pump. The cylinder is submerged in a 1 m cubic water tank, into which this water is returned. The angular momentum required to form the vortex is generated by a concentric array of short static inlet turning vanes situated at the base of the cylinder. These feed fluid from the tank directly into the boundary layer that forms on the flat plate. Thus, unlike Phillips, we have no rotating mechanical parts, which avoids any vibration that might arise from the use of a mechanical system to rotate the cylinder.

Flow visualization confirms that a concentrated tornado-like vortex, similar to that of Phillips, is formed. This extends along the axis of the cylinder from the base to the exit in the lid. The incidence of the turning blades is variable but is set at 60° for all of the results presented here. A porous foam annulus positioned upstream of the blades ensures a uniform distribution of flow from the tank. Although swirl is only imparted to the flow immediately above the base, after a suitable settling time a potential vortex forms, $u_{\theta,\text{ex}} = \Phi/2\pi r$, induced in the bulk of the flow by the central vortex column. To ensure the flow has reached a statistically steady state in each set of measurements, the flow was left running for 20 minutes before any data were taken. Our experiment

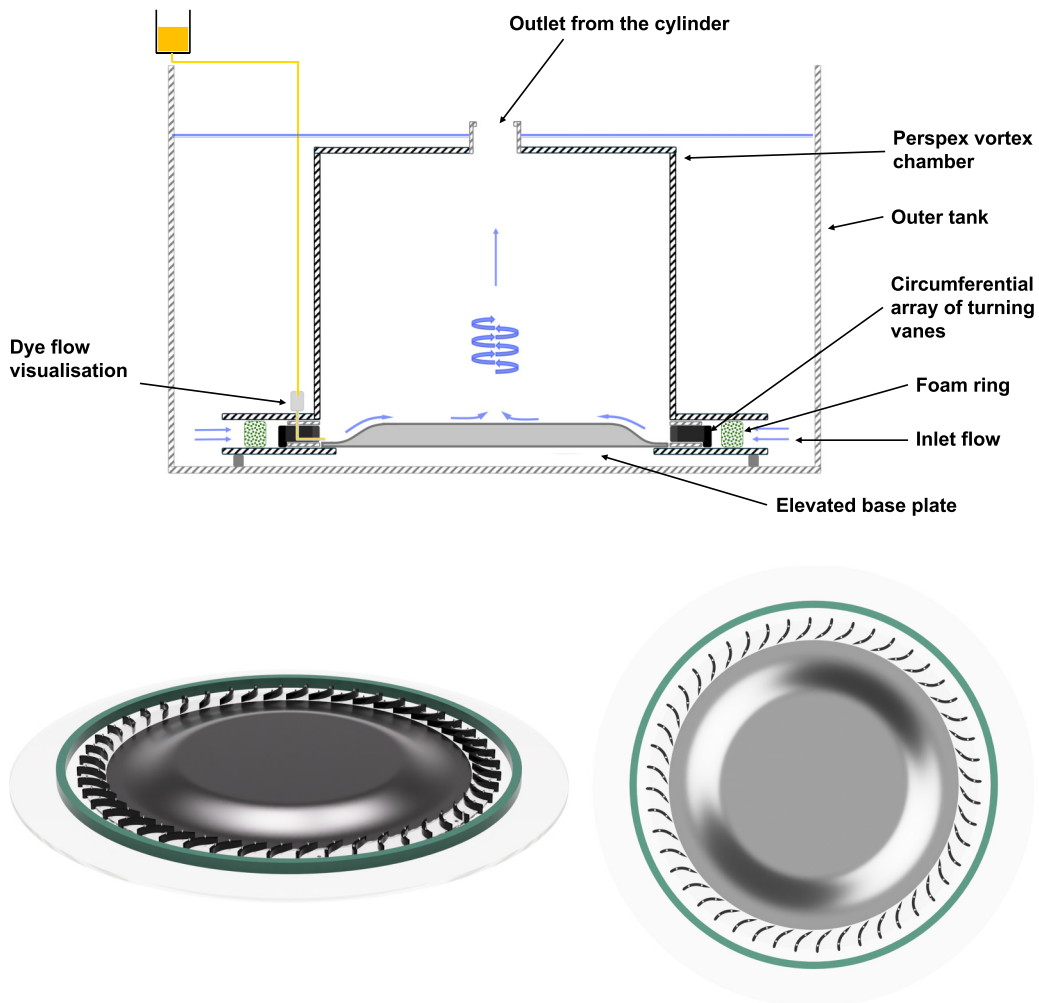


FIG. 2. Schematic of the vortex simulation apparatus. The top panel is a side view of the apparatus and the bottom panels show the base plate and turning vanes.

is designed to confront the predictions of Burggraf *et al.* [2] for the boundary layer beneath a potential vortex. To this end, we measured the radial, azimuthal, and axial velocity components in the tank, (u_r, u_θ, u_z) , concentrating on the boundary layer. This is done using particle image velocimetry (PIV). Measurements reveal that the radial and azimuthal velocities are of similar order, and so care has had to be exercised to ensure that each of these components is measured without mutual interference, as discussed below.

A consequence of installing the turning vanes at the base of the cylinder is that, for a flat base plate, it would be necessary to incline the PIV camera downwards to observe the flow below the top of the blades. This oblique view gives rise to some difficulty in distinguishing between the radial and azimuthal velocity components. Moreover, a consequence of looking downwards towards the plate is that the camera receives light scattered from it, which precludes extracting PIV data for the region close to the surface. These issues are overcome by elevating the central section of the base plate to a height above the turning vanes, as shown in Fig. 2. The ramp is carefully contoured and does not show any signs of materially affecting the growth of the boundary flow over the central

TABLE I. Maximum measured velocity component (m/s), vorticity flux in the central vortex, $\bar{\Phi}$ (m²/s), and the Reynolds numbers $\text{Re} = 2\bar{u}_r R/\nu$ and $\text{Re}_\Phi = \bar{\Phi}/\nu$, for the three cases observed.

	$u_{\theta,\text{max}}$	$u_{r,\text{max}}$	$\bar{\Phi}$	$\frac{u_{r,\text{max}}}{u_{\theta,\text{max}}}$	Re	Re_Φ
Re_1	0.0122	0.0075	0.00505	0.615	3.85×10^3	4.44×10^3
Re_2	0.0768	0.0276	0.0265	0.360	1.02×10^4	2.32×10^4
Re_3	0.1010	0.0321	0.0373	0.318	1.71×10^4	3.27×10^4

region of interest here. The diameter of the flat portion of the base is 360 mm and it is raised 2 mm above the turning vanes, thus giving an unimpeded horizontal view of the boundary layer. This improves the quality of the PIV measurements, and avoids much of the light scattering from the surface of the plate.

We measure all three velocity components in the near-wall region using a pulsed light sheet generated by an ILA.5150 GmbH LPS v3 module and high-brightness LEDs, positioned outside the tank. The LEDs are triggered by a signal from the Analogue Discovery 2 wave generator. The light sheet has a thickness of 0.8 mm and is pulsed at 74 Hz, matching the maximum frame rate of a JAI GO-5100M-USB camera which for most of the measurements is position 1 m from the containing tank. Each velocity profile measurement consists of the average of three runs at a fixed Reynolds number, with 800 image pairs acquired per run, processed using PIVlab software. 60- μm -diam polyamide particles are used for the particle correlation. The accepted methods to ensure accurate PIV measurements are adopted, and no interpolation or postprocessing is applied to the raw data.

Three nominal Reynolds numbers are selected for the tests to enable the data to be compared with the theoretical model of Burggraf *et al.*, and to explore the effect of going from laminar to transitional or weakly turbulent flow. A convenient definition of the Reynolds number is $\text{Re} = 2\bar{u}_r R/\nu$, where $\bar{u}_r$ is the mean radial velocity of the flow entering the chamber through the vanes, derived from measurements of the water flow rate, R is the radius of the elevated section of the base plate, equal to 0.18 m, and ν is the kinematic viscosity of water. An alternative, and more physical, definition of the Reynolds number is $\text{Re}_\Phi = \bar{\Phi}/\nu$. The three Reynolds numbers chosen for the tests are shown in Table I, and it can be seen that $\text{Re} = 2\bar{u}_r R/\nu$ and $\text{Re}_\Phi = \bar{\Phi}/\nu$ typically have similar values. Examples of dye flow visualization in the boundary layer and the vortex eruption region are presented in Fig. 3 for the two lower Reynolds numbers. For Re_1 , the flow over the plate is smooth and stable, from which it is concluded that the boundary layer flow is laminar. The central vortex is, however, unsteady. For Re_2 , however, instabilities develop in the boundary layer, and the vortex eruption region is increasingly unsteady. From this it is concluded that at Re_2 , and at the higher value of Re_3 , the boundary layer flows are transitional or weakly turbulent.

The radial and vertical velocities for the boundary layer flow over the plate are measured for all three Reynolds numbers using the PIV light sheet that passes through the centerline of the cylinder. Data are presented up to 1.6 cm from the surface for radial stations between 2 and 10 cm from the center line (which is the maximum radius for which data are recorded) at 1 cm increments, that is, at $r/r_{\text{max}} = \bar{r} = 0.2, 0.3, \dots, 1.0$, with $r_{\text{max}} = 10$ cm. For each of the three Reynolds numbers, the azimuthal velocity, u_θ , is measured for five radial stations by moving the position of the light sheet from the centerline plane towards the camera as indicated in Fig. 4 (that is, at $\bar{r} = 0.2, 0.4, \dots, 1.0$). These measurements are extended into the flow above the boundary layer. The camera is positioned 3 m from the tank to minimize perspective distortions. Data are only extracted for the narrow segment of the sheet (shown highlighted), aligned along the line from the camera to the cylinder center where the flow is tangential to the sheet. This minimizes errors due to cross-contamination with the radial velocity component. The planes are separated by increments of 2 cm and go up to 10 cm from the center so as to include some of the azimuthal flow induced by the vortex.

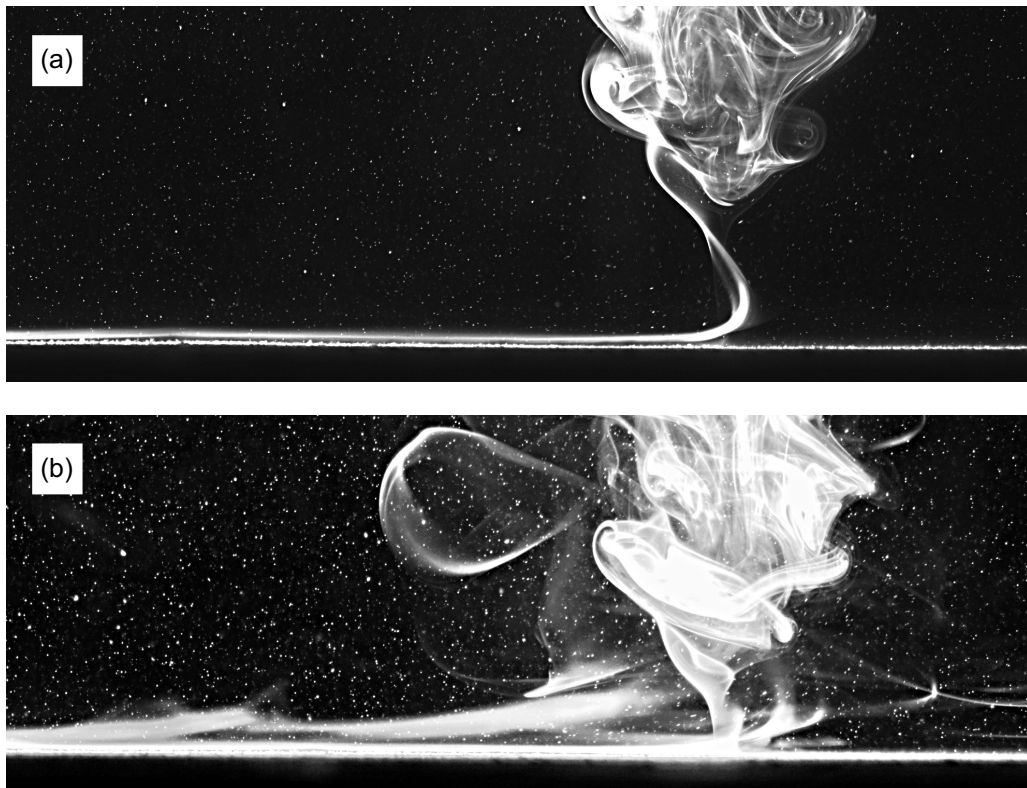


FIG. 3. Dye images of the flow taken at (a) $Re_1 = 3.85 \times 10^3$ and (b) $Re_2 = 1.02 \times 1$.

IV. THE EXPERIMENTAL RESULTS

A. The azimuthal velocity distribution and the vorticity flux in the central vortex

Normalized plots of the measured azimuthal velocity above the cylinder base are plotted in Fig. 5. They show the data for five radial positions up to 10 cm from the vortex center for the three different

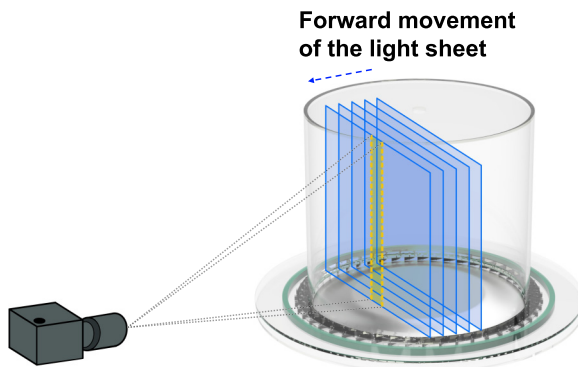


FIG. 4. Azimuthal velocity measurements using PIV. The blue translucent panes represent successive position of the vertical light sheet, which was translated forward in 2 cm increments from the vortex center. At each position, the lateral velocity component was recorded only within the central highlighted region of the pane, to resolve the azimuthal velocity at that radius.

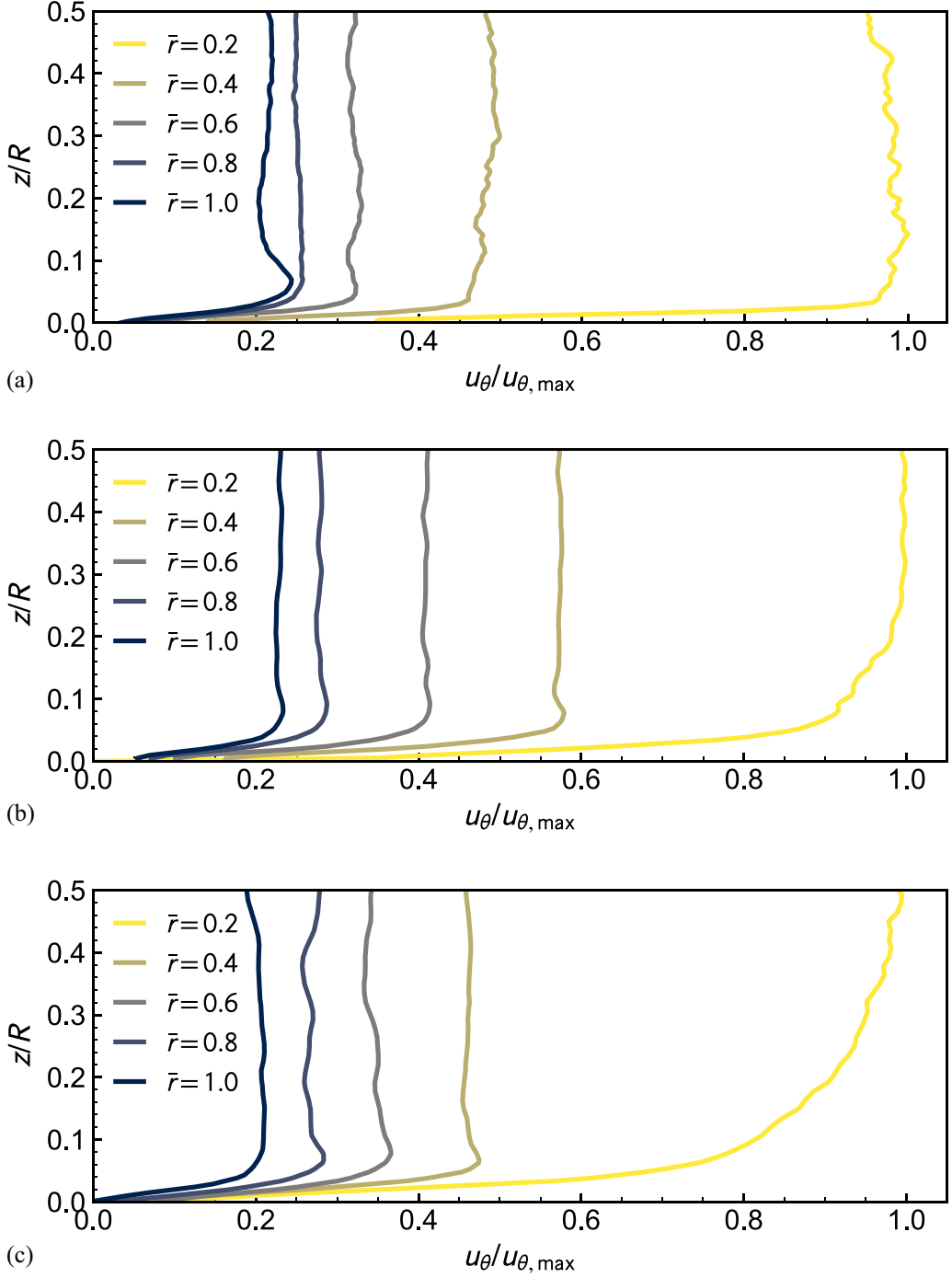


FIG. 5. Azimuthal velocity as a function z/R for (a) Re_1 , (b) Re_2 , and (c) Re_3 . The profiles are given at five dimensionless radial positions, $\bar{r} = r/r_{\max}$, in the range 0.2–1.0.

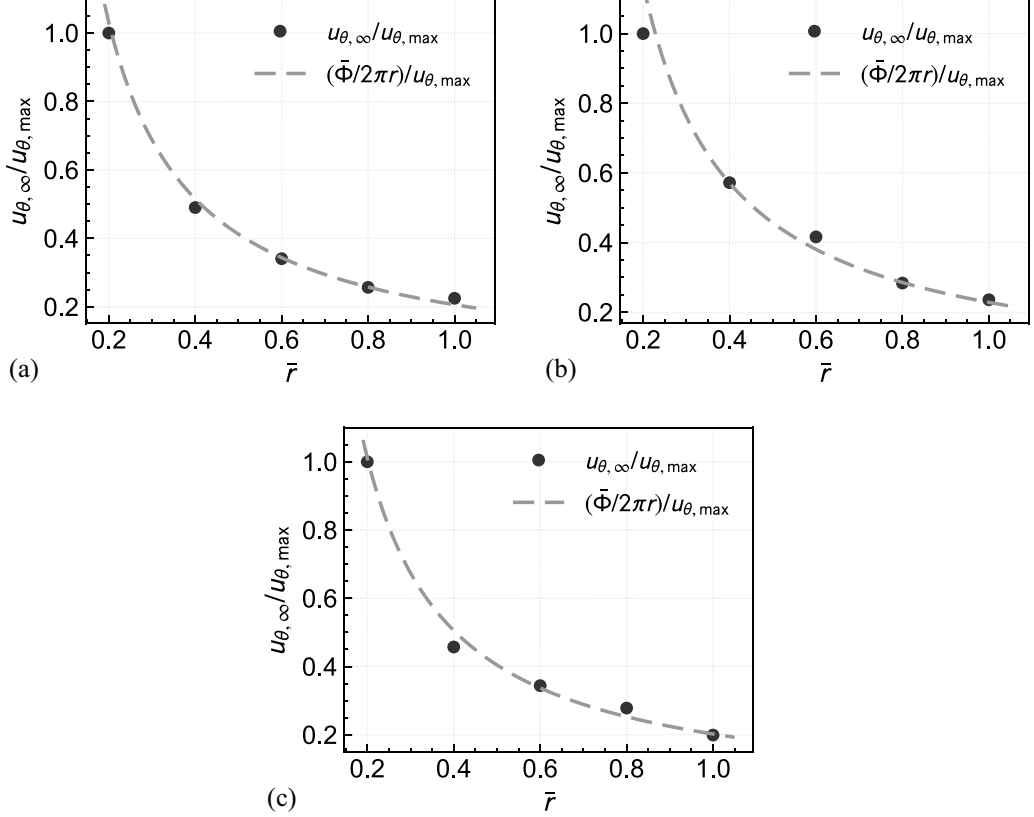


FIG. 6. Azimuthal velocity distributions above the boundary layer for (a) Re_1 , (b) Re_2 , and (c) Re_3 . The dots show the measured velocity ratio $u_{\theta, \infty}/u_{\theta, \max}$. The dashed line is the theoretical distribution.

Reynold numbers. These profiles illustrate how the azimuthal velocity varies within the boundary layer adjacent to the surface (at $z = 0$), as well as above this. They are plotted as $u_{\theta}/u_{\theta, \max}$ versus z/R , where R is the radius of the flat upper surface of the baseplate, and $u_{\theta, \max}$ is the maximum measured azimuthal velocity for a given Re , observed above the boundary layer at $r = 2$ cm. The values of $u_{\theta, \max}$ for each Reynolds number are given in Table I.

A boundary layer between 1 and 2 cm in height is clearly evident. Above this boundary layer, the tornado-like vortex induces an azimuthal flow that is very close to a potential vortex. That is to say, we have $u_{\theta, \infty}(r) = \bar{\Phi}/2\pi r$, where $\bar{\Phi}$ is the measured circulation averaged over measurements made at several radii. (The subscript ∞ indicates "outside the boundary layer.") All of the azimuthal velocity measurements made outside the boundary layer are used to determine $\bar{\Phi}$, with

$$\bar{\Phi} = \frac{1}{n} \sum_{i=1}^n (2\pi r_i) u_{\theta, \infty}(r_i), \quad (24)$$

where n denotes the number of radial positions. The measured values of $\bar{\Phi}$ are listed in Table I for each Reynolds number. These are used in Sec. IV B to normalize the measured velocity field, thus enabling a comparison with the analytical similarity solution of Burggraf *et al.* [2]. Figure 6 shows the averaged measured azimuthal velocities for the three Reynolds numbers compared with that for a corresponding potential vortex of strength $\bar{\Phi}$. The results confirm that the tornado-like vortex induces a free vortex in the region above the boundary layer.

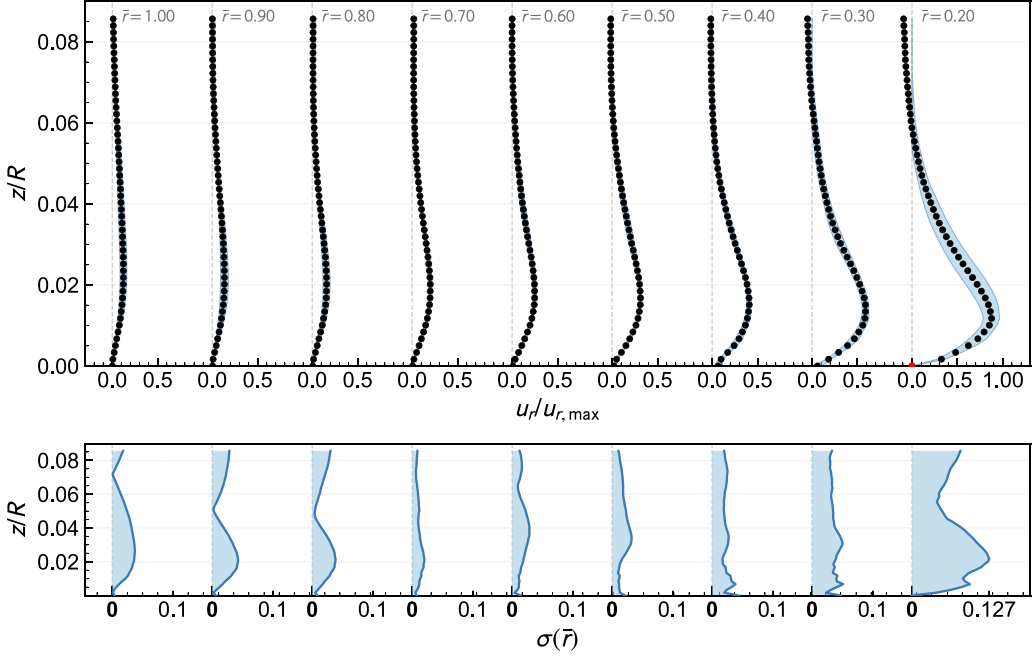


FIG. 7. Averaged radial velocity profiles for the flow at Re_1 . Each profile represents the mean of three independent PIV runs at fixed $\bar{r}$ positions between 0.2 and 1.0. The red marker at $\bar{r} = 0.2$ indicates where the nonslip condition was artificially enforced for the data at the innermost radius. The standard deviation is shown by blue and is repeated below with an exaggerated scale.

B. The radial velocity profiles and the similarity structure of the boundary layer

Radial velocity measurements are obtained for all three Reynolds numbers by averaging three independent PIV runs for the nine radial positions of $r/r_{\max} = 0.2 \rightarrow 1.0$ (in increments of 0.1). The results are summarized in Fig. 7 for Re_1 . An acceleration of the flow within the boundary layer towards the central vortex is evident. The no-slip condition is satisfied to within 1% of the local maximum in u_r for all measurements taken adjacent to the surface, with the exception of the data at the innermost location of $\bar{r} = 0.2$. For this case, the nonslip condition is artificially enforced for just the one point (shown in red) nearest the baseplate. Some scatter was observed between runs for the data taken at the innermost radial position, as indicated by the standard deviation between runs (shown by gray). This is almost certainly a consequence of the unsteadiness in the vortex eruption region.

Note that the measured radial velocity profiles are *not* oscillatory. This contrasts with the oscillatory behavior in a Bödewadt boundary layer, where such oscillations are attributed to viscously damped inertial waves (see Fig. 3.3 and the associated discussion in [8]). This nonoscillatory behavior is to be expected, as inertial waves cannot be sustained by a free vortex.

According to Burggraf *et al.* [2], the radial velocity in the *inner* part of the boundary layer should adopt the self-similar form

$$u_r = [\Phi/(2\pi r)]f(\eta), \quad (25)$$

where $\eta = z/\delta_{\text{in}}$, and δ_{in} is given by Eq. (3). In theory, this should hold whenever the radial location is well removed from R , and from the central vortex. Using the vorticity fluxes listed in Table I, the radial velocity measurements can be rewritten in this normalized form. Figure 8(a) shows the same data as Fig. 7, but with the progressive shift in origin removed, while Fig. 8(b) shows the radial

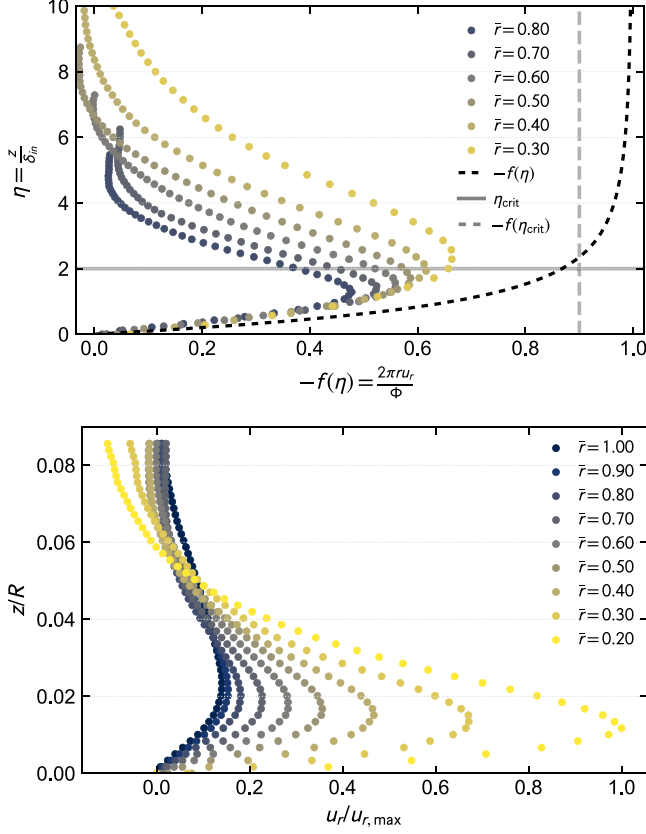


FIG. 8. (a) Radial velocity profiles for Re_1 as a function of z/R for different $\bar{r}$. (b) Self-similar radial velocity profiles, $-f(\eta) = 2\pi r u_r / \Phi$, plotted against η .

velocity for Re_1 rescaled using Eq. (25), rewritten as

$$|f(\eta)| = -f(\eta) = \frac{2\pi r |u_r|}{\Phi}. \quad (26)$$

Figure 8(b) shows that, for small η , all of the normalized velocity profiles collapse onto a single universal curve before reaching the peak value of $f(\eta)$, which we take as a proxy for the interface between the inner and outer parts of the boundary layer. This confirms the existence of a self-similar region close to the surface in the laminar regime, as predicted by Burggraf *et al.* [2]. Data for $\bar{r} = 0.2$ are omitted from this plot as the boundary layer at this location is perturbed by the unsteady vortex eruption region. Also, the data for $\bar{r} = 0.9$ and 1.0 are omitted because the boundary layer may not be fully developed at those locations, adjusting to the flow over the ramp. Figure 9 shows the same normalized velocity profiles, but for Re_2 and Re_3 .

In Figs. 8(b) and 9, the inner-layer radial velocities are compared to Burggraf's theoretical prediction of $f(\eta)$, which is obtained from the integration of Eq. (15) and shown as a dashed line. This exposes one weakness of comparing an asymptotic theory, in which $\text{Re} \rightarrow \infty$, with data taken from an experiment at modest values of Re . Namely, in the asymptotic theory, the inner-layer radial velocity increases in magnitude with η , tending to the far-field velocity, i.e., $f(\eta \rightarrow \infty) = -1$. In reality, the inner-layer velocity cannot reach this limit, but rather transitions to the outer boundary layer, just as shown in Fig. 8(b). In the numerical experiments of Burggraf *et al.* [2], performed at modest Re , they found that the departure from the inner-layer solution, which marks the transition

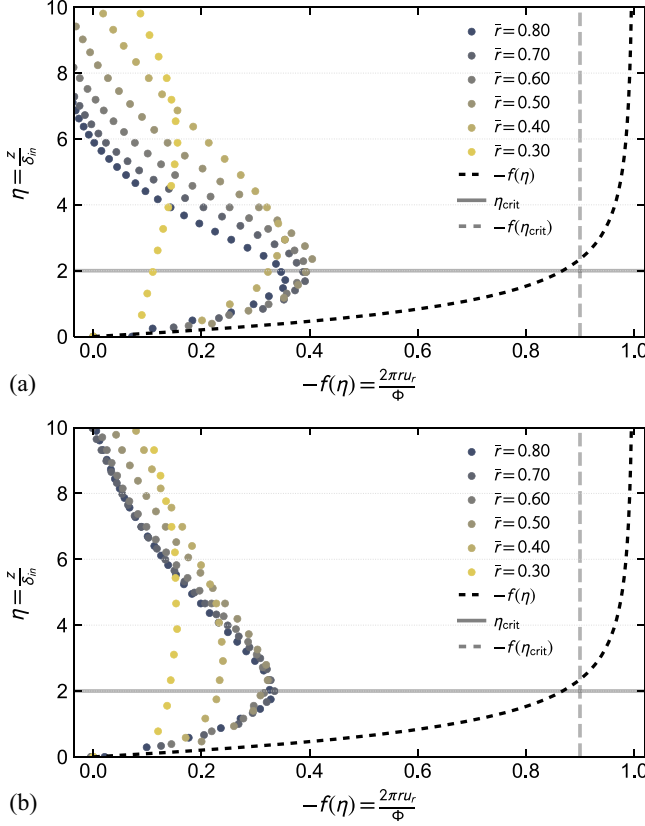


FIG. 9. The velocity profiles showing $-f(\eta) = 2\pi r u_r / \Phi$ vs η for (a) Re_2 and (b) Re_3 .

from the inner to the outer regions, occurs at around $\eta_{crit} \approx 2$. However, they also observed that the value of η_{crit} decreases slightly with increasing radius, consistent with the data shown in Fig. 8(b).

For the steady laminar flow of Re_1 , the agreement with the self-similar model of Burggraf *et al.* [2] for the inner layer is encouraging. While a small offset from the theoretical slope is evident in Fig. 8(b), the observed velocity profiles more or less follow the self-similar solution. This is perhaps surprising, as the model of Burggraf *et al.* [2] incorporates the doubtful assumption that, in the inner layer, the azimuthal velocity is much smaller than the radial component.

The corresponding data for the two higher Reynolds numbers are shown in Fig. 9. It is evident that, as the Reynolds number is increased, the extent of the agreement with the laminar theory diminishes. The velocity profiles still collapse reasonably well for the entirety of the inner layer, up to the point where the vortex eruption zone begins to influence its growth, but there is a significant difference between the measured near-wall velocity gradient and the theoretical prediction. The breakdown of self-similarity on approaching the eruption zone occurs at $\tilde{r} = 0.3$ for Re_2 , as opposed to 0.2 for Re_1 . For the highest Reynolds number, Re_3 , these discrepancies are more pronounced, with a larger difference in the near-wall velocity gradient, and the breakdown of self-similarity due to the influence of the central vortex extending out to $\tilde{r} = 0.4$.

In summary, for the steady laminar regime of Re_1 , the analytical solution of Burggraf *et al.* [2] for the inner layer describes the observed flow reasonably well. As the Reynolds number is increased, however, an inner layer self-similarity is still evident, but the observed velocity gradients exceed the model predictions. This is possibly related to the onset of weak turbulence.

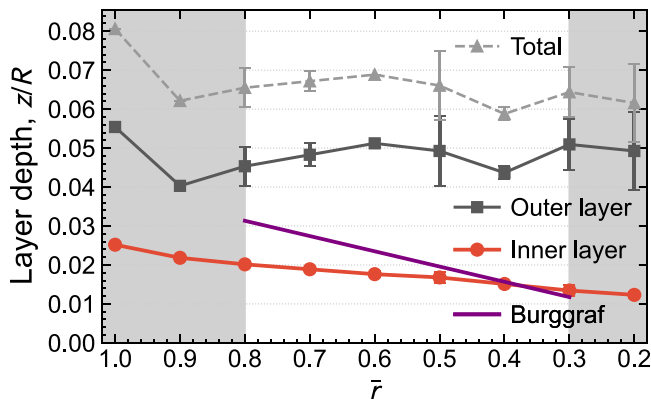


FIG. 10. The thicknesses of the various parts of the boundary layer for Re_1 . The solid straight line is prediction (27), based on the assumption of self-similarity.

Further details as to how the inner and outer boundary layers merge can be obtained by examining the way in which their thicknesses vary with distance from the vortex axis. Here, we find it convenient to define the top of the inner boundary layer to be the vertical position of the peak in the radial velocity. The total boundary layer depth, on the other hand, is defined as the height above the surface where the radial velocity first drops below 1% of the local peak value. This latter criterion is somewhat sensitive to experimental error and so some scatter is observed in the values. The thickness of the outer boundary layer is then defined as the difference between these two values. The results for Re_1 are shown in Fig. 10.

The prediction of Burggraf *et al.* [2] for a self-similar inner layer has the inner boundary layer scaling as $\delta_{in} = r\sqrt{2\pi\nu/\Phi}$, and their numerical experiments give the thickness as

$$z_{crit} \approx 2r\sqrt{\frac{2\pi\nu}{\Phi}}. \quad (27)$$

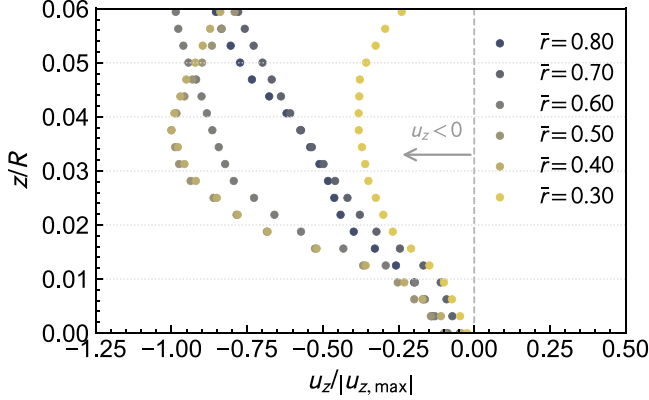
This suggests that the inner layer thins as it approaches the central vortex. This is consistent with Fig. 10, which shows that the measured inner-layer depth decreases with decreasing radius. However, the inner boundary layer is thinner than expected at larger radii, and so the measured rate of decrease of z_{crit}/R is less rapid than predicted. This discrepancy is likely to arise from our somewhat arbitrary definition of the inner layer thickness, as the peak in radial velocity, and from our attempt to compare an asymptotic theory for $Re \rightarrow \infty$ with measurements made in an experiment at modest Re that ensures laminar flow. The inner layer may be influenced in the range $r = 0.6 - 0.8$ by the transition of the flow onto the raised base. However, the effective correlation of the data within the inner layer seen in Fig. 8(b) for this range would suggest otherwise.

C. The axial velocity structure and self-similarity

The measured axial velocity for Re_1 is given in Fig. 11, showing a uniformly negative u_z and a downflow into the boundary layer from above. This is consistent with Burggraf's computations and analytical predictions [see Eq. (18)]. However, it is the opposite of Bödewadt's solution for the boundary layer beneath a fluid in solid-body rotation, which detains mass, i.e., $u_z > 0$.

This downflow is a consistent feature across all three Reynolds numbers, and across each of the 72 runs—a downward axial velocity is observed for $z/R < 0.08$ in all cases. This is clear from Fig. 12, which shows, for all three values of Re , the measured distributions of

$$h(\eta) = \frac{ru_z}{\sqrt{\nu\Phi/2\pi}}. \quad (28)$$


 FIG. 11. Axial flow measurements for Re_1 .

In Fig. 12, the dashed curve represents the Burggraf-Stewartson solution [2], with gray lines marking η_{crit} and $h(\eta_{\text{crit}})$. At Re_1 , the profiles collapse reasonably well onto the analytical curve up to $\eta \approx 2$, suggestive of a laminar self-similar flow within the inner layer. As the Reynolds number increases, however, the collapse becomes poor, reflecting a breakdown of self-similarity, presumably as a result of unsteady transitional flow or weak turbulence.

To understand why $u_z < 0$ in the boundary layer, it is useful to consider the self-similar continuity Eq. (5) for the inner layer, i.e., $\eta f'(\eta) = h'(\eta)$. Now, in the inner layer we have $f'(\eta) < 0$, because u_r is negative and its magnitude increases monotonically with distance from the surface. It follows that $h'(\eta) < 0$, which is consistent with Eq. (18). It follows that the axial velocity decreases with height near the surface, i.e., there is downward axial flow within the inner boundary layer. This downflow is also seen in the outer layer.

This behavior stands in stark contrast to the classical Bödewadt solution, where the self-similar continuity equation now requires $2f(\eta) = -h'(\eta)$, as discussed in [7,8]. In this case, the radial inflow has $f < 0$, and hence continuity requires $h'(\eta) > 0$. In this case, the axial velocity increases with height within the boundary layer, leading to a positive axial velocity and to fluid being detrained from the boundary layer.

As discussed in Sec. II, the self-similar theory predicts that the velocity components at the inner-outer interface satisfy

$$(u_z/u_r)_{z=z_{\text{crit}}} \approx 0.38 z_{\text{crit}}/r, \quad (29)$$

as shown by Eq. (23). This is important as it tells us that the streamlines in the r - z plane are less inclined (more horizontal) than the inner-outer layer interface. It is this that allows the inner layer to detrain mass into the outer layer, despite the fact that the vertical velocity is everywhere negative. This prediction is tested against the measured data for Re_1 in Fig. 13.

Figure 13 shows that, although not a particularly good match, the measured data are at least of the same order of magnitude as the theoretical prediction. For $\bar{r} < 0.7$, it is consistently observed that $(u_z/u_r)_{\eta=2} < z_{\text{crit}}/r$, indicating that the streamlines are less steep than the interface. This confirms that the inner layer detrains mass to the outer layer while still maintaining a downward axial flow within it.

V. DISCUSSION AND CONCLUSIONS

We have observed that the boundary layer beneath a potential vortex, $u_{\theta,\text{ex}} = \Phi/2\pi r$, is laminar at $\text{Re}_\Phi = \Phi/\nu = 4.4 \times 10^3$, but unsteady for $\text{Re}_\Phi > 2.3 \times 10^4$, being either in transition or mildly turbulent for the highest Reynolds number investigated here, $\text{Re}_\Phi = 3.3 \times 10^4$. For the laminar case,

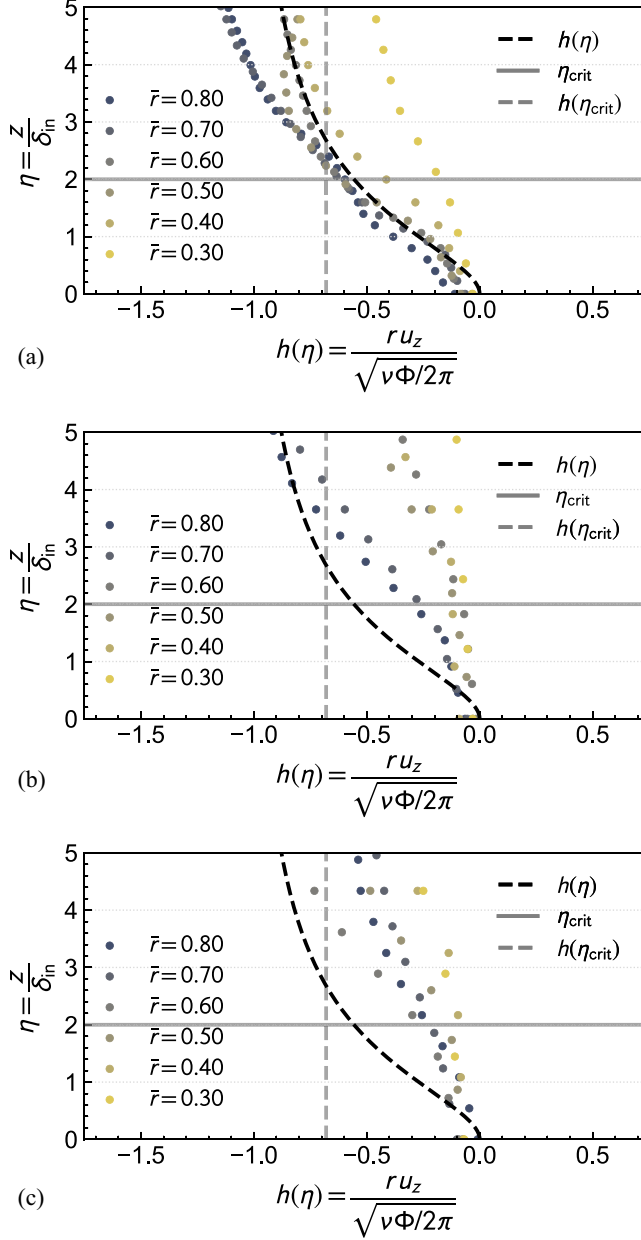


FIG. 12. The self-similar scaled vertical velocity profiles showing $h(\eta) = ru_r/\sqrt{v\Phi/2\pi}$ vs η for (a) Re_1 , (b) Re_2 , and (c) Re_3 .

the analytical model of Burggraf *et al.* [2] is in overall qualitative agreement with our observations. In particular, there is a composite boundary layer structure, with a radial inflow in both layers. The thickness of the near-wall layer scales as $\delta_{\text{in}}(r) \sim r\sqrt{2\pi\nu/\Phi}$, and the outer layer has an approximate thickness of $\delta_{\text{outer}} \sim R\sqrt{2\pi\nu/\Phi}$. Thus, the inner layer thins as it approaches the central vortex, while the outer layer has approximately constant thickness. Moreover, the inner-layer detrains mass into the outer layer, so that close to the central vortex the bulk of the inward mass flux

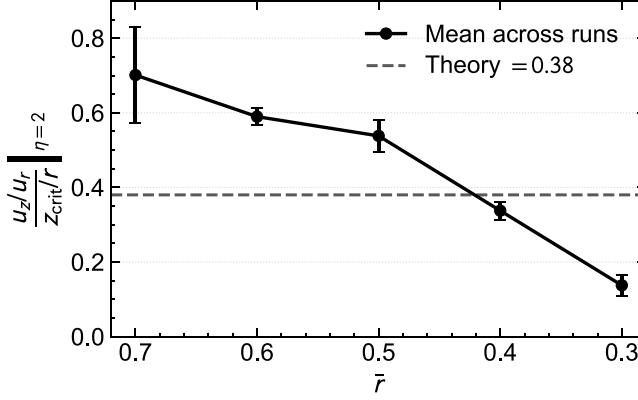


FIG. 13. The ratio of $(u_z/u_r)|_{\eta=2}$ to z_{crit}/r as a function of normalized radius for Re_ϕ .

is carried by the outer layer. However, the thin inner layer absorbs most of the shear stress required to meet the no-slip condition at the surface.

The laminar model of Burggraf *et al.* [2] is asymptotic, and so assumes $Re \rightarrow \infty$. There are therefore practical difficulties in making a quantitative comparison between the model predictions and the laminar measurements made at a modest Reynolds number of $Re_\phi = 4.4 \times 10^3$. Nevertheless, Fig. 8(b) shows a good correlation between theory and measurements for the inner layer, provided that we are not too close to the edge of the disk, or to the central vortex, where the theory necessarily breaks down. This is somewhat surprising as the model of Burggraf *et al.* [2] incorporates the doubtful approximation that, in the inner layer, the azimuthal velocity is much smaller than the radial component of velocity.

At higher Reynolds numbers, the two unsteady boundary layers examined here show that the laminar model of Burggraf *et al.* continues to give a reasonable qualitative description of the overall flow structure, especially its two-layer nature. However, a detailed comparison between theory and measurements progressively worsens as the Reynolds number is increased.

A curious feature of the model of Burggraf *et al.* [2] is that it predicts that $u_z < 0$ at all radii, and so there is a downward entrainment of fluid into the boundary layer. This is precisely the opposite to Bödewadt's solution for the boundary layer beneath a fluid in rigid-body rotation, which has $u_z > 0$, and so detains mass at all radii. The prediction that $u_z < 0$ below a free vortex is indeed observed in all of our measurements. For the inner layer, we have shown that this downward flow is a direct result of the rescaled continuity Eq. (5), i.e., $\eta f'(\eta) = h'(\eta)$. That is, in the inner layer we have $f'(\eta) < 0$, because u_r is negative and its magnitude increases monotonically with distance from the surface. It follows that $h'(\eta) < 0$, which leads directly to negative values of u_z . Moreover, our measurements show that, at the interface between the inner and outer layers, $z = z_{crit}$, we have $(u_z/u_r)_{crit} < z_{crit}/r$, confirming that the streamlines there are less steep than the interface itself. This explains how the inner layer manages to detrain mass into the outer layer while still maintaining a downward axial flow at all points. This is illustrated in Fig. 14.

In conclusion, our measurements lend strong support to the approximate laminar model of Burggraf *et al.* [2]. However, two important issues remain unresolved. First, the prediction that the radius of the central vortex is proportional to the boundary-layer thickness, i.e., r_0 scales as $r_0 \sim \delta_{outer}$, has not been confirmed. (This would require measurements at many more values of the Reynolds number.) Nevertheless, we observe that, purely in terms of orders of magnitude, we do indeed have $r_0 \sim \delta_{outer}$, at least for the Reynolds numbers examined here. Secondly, we have not probed the strongly turbulent regime, and so we cannot say if the observed two-layer structure carries over to a fully turbulent boundary layer. Both of these open questions are of considerable practical importance when it comes to the dynamics of tornadoes.

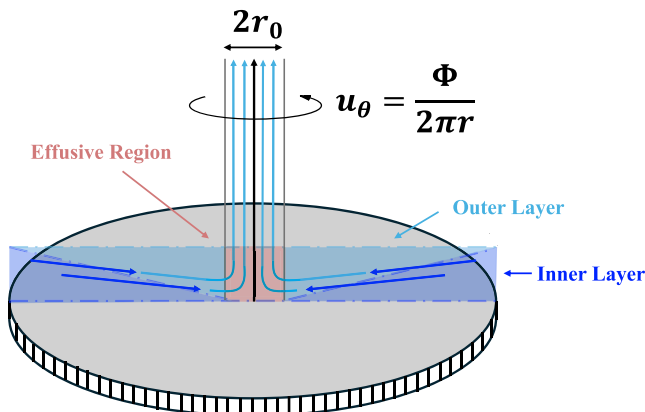


FIG. 14. Schematic of the composite boundary layer.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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