

Patterning surface textured plates with a viscoplastic fluidVanessa R. Kern,¹ Marcel Moura ,² Pål E. S. Olsen ,¹ and Andreas Carlson ^{1,3,*}¹*Mechanics Division, Department of Mathematics, [University of Oslo](#), 0316 Oslo, Norway*²*PoreLab, the Njord Center, Department of Physics, [University of Oslo](#), 0316 Oslo, Norway*³*Department of Medical Biochemistry and Biophysics, [Umeå University](#), 901 87 Umeå, Sweden*

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The deposition of a viscoplastic fluid onto a substrate can be achieved by moving apart two plates initially separated by a fluid-filled gap, where the footprint shape depends on the initiation of a fingering instability. Here, we present an approach for controlled deposition of a viscoplastic fluid by designing the macroscopic structure of the solid substrate. Through experiments in a lifted Hele-Shaw cell, we explore how slot, square, pyramid, and triangular patterns affect the dynamics of liquid deposition. The substrate structures directly control the symmetry and final shape of the deposited viscoplastic footprint, while having little systematic influence on the maximal normal adhesive force or on the force-gap scaling exponents once the total entrapped volume is accounted for through an effective initial gap. We develop a scaling description for the normal force of a Herschel–Bulkley fluid, showing that the measured force decay reflects a superposition of capillary, yield stress, and viscous contributions. For experiments producing stable patterning, the measured effective exponents are consistent with the theoretical range predicted by this model. Finally, we interpret the pattern-selection mechanism through a dynamical permeability analysis where the substrate texture creates an anisotropic permeability field whose angular maxima correlate with the preferred inward propagation directions of the invading air. Our results highlight a direct link between substrate geometry, interfacial dynamics, and the final deposited morphology, showing how millimetric deposition patterns can be programed using submillimetric surface structures.

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The deposition of a complex fluid onto a substrate appears across a wide range of applications and natural processes, including how saliva coats tongues [1], spreading mayonnaise on bread [2], and medical diagnostics [3]. One such type of non-Newtonian fluid is a yield stress or viscoplastic fluid, which can sustain a stress without flowing, i.e., they behave as a solid until a critical stress threshold is exceeded [4–6]. These fluids are found in industrial applications (drilling fluids [7]), in food processing (mayonnaise, ketchup, and chocolate [8]), in cosmetics (toothpaste and hair gel), as well as in geological processes (lava flows [9,10]). Yield stress fluids are topical in the development of 3D printing [11,12], which relies on the detailed control of the deposition of the fluid on a solid substrate. We demonstrate a route to controlling the deposition patterns of yield

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stress fluids deposited during simple plate separation through the design of macroscopic substrate textures.

When a viscous fluid enters a narrow gap filled with a more viscous immiscible fluid, the interface can become unstable and start forming a finger-like pattern. The transition from a circular advancing interface to a finger-like pattern has been well characterized for Newtonian liquids [13–15], which is governed by the viscosity contrast, the surface tension, and the flow rate, and is known as the Saffman–Taylor instability [15–17]. If the liquids are viscoplastic, the interface pattern changes [6,18,19] and the description of the stability threshold becomes more involved [20,21]. A yield stress fluid profoundly alters the fingering dynamics [19,20,22,23] and, as such, affects the deposition of liquid on a surface using a lifted Hele-Shaw cell [18,24,25].

Experiments have shown a criterion for the onset of fingering in colloidal gels under tension [18], distinguishing a stable (elastic deformation) from an unstable (air invasion and fingering) regime. It has been argued that a fingering instability can only be initiated if a sufficiently large stress is locally transferred to fluidize the gel at the finger tips. Once the fluid yields, the patterns that form appear to have strong similarities with the classic viscous-fingering scaling predictions for Newtonian fluids, but the difference stems from parts of the liquid appearing to be in a yielded and unyielded state. In addition to the yield stress properties of the fluid, the interaction between the fluid and the confining surfaces can also affect the flow pattern [26,27]. Model yield stress fluids, such as Carbopol, can also exhibit fluid slip if the surface is smooth [28,29]. One way to ensure a no-slip condition on the surface is to introduce surface texture or to chemically functionalize the material [30]. It has been demonstrated that wall slip can indeed influence finger growth in viscoplastic fluids [31].

In the context of adhesion, Bikerman’s pioneering work [32] established that the normal force required to separate two plates joined by a fluid depends both on surface energy and bulk deformation (and therefore on the adhesive’s rheology). It was later shown that yield stress fluids sustain a significant force even when fingering occurs [33] due to the resistance of the bulk material to cohesive failure and the surprising result that the fingering instability did not seem to have any significant effect on the work of adhesion. These works paint the picture that the interplay between substrate roughness (slip lines and pinning sites) and complex rheology determines not only the final pattern morphology of the process, but also the forces imposed by the fluid on the surrounding solid boundaries, with immediate impact on adhesion applications. In this work, we explore how surface-textured plates affect the flow and final deposited footprint of a yield stress fluid during the separation of two circular plates in a lifted Hele-Shaw cell. By varying the texture geometry of the plates, effective initial plate separation, and pull-off speed, we identify the transition between ramified invasion, controlled patterning, and stable puddle formation, see Fig. 1.

II. EXPERIMENTAL SETUP

To test how the deposition pattern of a yield stress fluid can be controlled by the substrate topography/pattern, we designed an experimental setup around an Anton Paar 702 rheometer, see Fig. 1(a). Lifting the upper plate, the rheometer is equivalent to a lifted circular Hele-Shaw cell. The use of the rheometer allows precise control of the initial distance $h_0 \in [0.1 - 5]$ mm that is separating the two plates in the wall-normal direction, where initially $t = 0$ we always have a fluid fully wetting the rheometer plates with radius $R_0 = 25$ mm. In addition, the rheometer provides control of the lifting speed of the upper plate v_l , and we can simultaneously measure the plate’s wall-normal force $F_N(t)$ with time. The dynamic interface of the yield stress fluid is monitored through side and bottom view imaging, allowing for characterization of the footprint it leaves on the bottom plate, giving a measure of the effective wetted radius $R(t)$ and the radial speed of the liquid front v_r [Fig. 1(b)]. To enable bottom view imaging, the plates were made from transparent Plexiglas. A tilted mirror positioned beneath the lower plate reflects the image toward a camera mounted on the side of the setup.

After the yield stress fluid is sandwiched between the rheometer plates, the upper plate is lifted at a constant speed $v_l \in [10 - 1000]$ $\mu\text{m/s}$. Consequently, the gap between the plates increases linearly

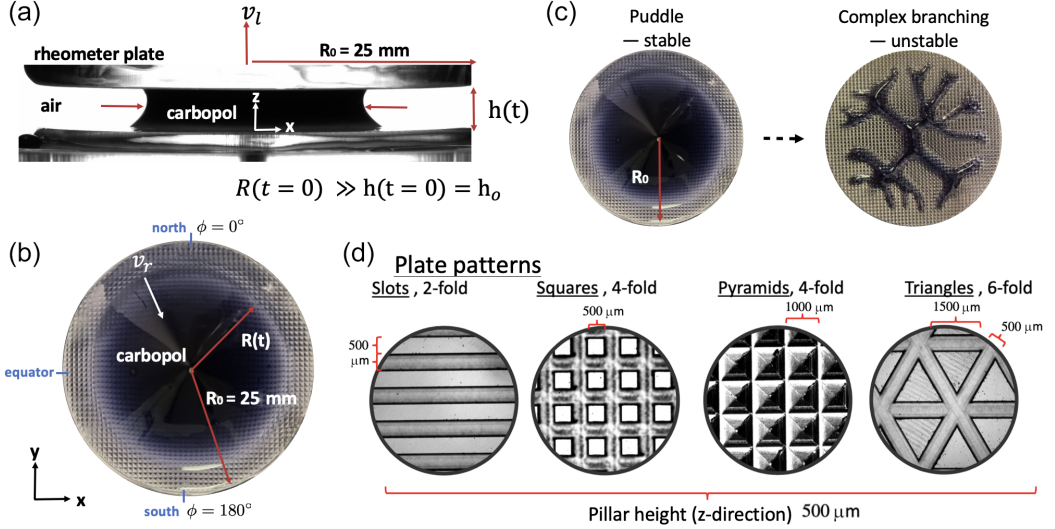


FIG. 1. (a) Side-view image of the experimental setup, where a Carbopol mixture is used as a model yield stress fluid and squeezed between two circular plates in a rheometer. Initially ($t = 0$), the plates are separated by a distance h_0 that is varied in the experiments (around 1 mm). As the plates are lifted by a velocity v_l , air enters the gap, forming in the end a single bump (puddle) or a complex pattern. (b) An image of the bottom plate illustrates the deposited pattern and its radial dynamics $R(t)$, where liquid initially fills the gap and entirely wets the plate with $R_0 = 25 \text{ mm}$. The orientation around the rim of the plate is defined based on the angle ϕ following a left-hand with north $\phi = 0^\circ$ and south $\phi = 180^\circ$ with an “equator” across the center of the plate from $\phi = 90^\circ$ to $\phi = 270^\circ$. (c) Two characteristic features observed in the experiments, where the yield stress fluid either leaves a single bump on the plate or leaves finger-like structures on the substrate. (d) The influence of substrate pattern has been explored through different shapes including parallel slots, squares, pyramids and triangles, where all these shapes have a height of $500 \mu\text{m}$. These micromilled surfaces are also semitransparent, still allowing for bottom and side-view imaging.

with time $h(t)$, and the yield stress fluid is pulled to the center of the plates as air enters the gap. In this process, the wetted area on the plates reduces as air is pulled in, and the intrusion dynamics can transition from the formation of an axially symmetric puddle with a nearly constant radial interfacial velocity v_r in the azimuthal direction to a highly complex (unstable) branching pattern, see Fig. 1(c), as also demonstrated by others [18,31,33]. Our interest lies in understanding how the dynamic retraction pattern and the final deposited footprint are affected by a macroscopic surface pattern of the two plates. Figure 1(d) illustrates the design of the different semitransparent plates with structures of slots, pillars, pyramids, and triangles, which are used to quantify the effect of the patterning on the adhesive (normal) force and the deposited footprint of a yield stress fluid represented here by a Carbopol mixture.

The Carbopol samples were created by combining Carbopol powder with 1M NaOH in deionized water (dyed with Nigrosin), followed by mechanical mixing of the solution for about 10 days. The Carbopol concentration was varied by changing the Carbopol-to-deionized-water ratio, while maintaining an approximately constant NaOH-to-Carbopol powder mass ratio of 8. The characterization of Carbopol solutions was conducted on an Anton-Paar Rheometer 702 using striated-parallel-plate geometry to minimize slip. A preshear step was incorporated to disrupt and then reform the fluid’s microstructure; see [34] for further details about the preparation procedure. The stress versus shear-rate data were fitted to a Herschel-Bulkley model $\tau = \tau_y + K\dot{\gamma}^n$, giving the fitting coefficients $\tau_y = 28 \text{ Pa}$, $K = 15.2 \text{ Pa s}^n$, and $n = 0.31$. One caveat with using Carbopol is that it is well known to produce slip on smooth surfaces. All the results reported in this article on smooth surfaces are

with a surface treatment following a protocol similar to [30,31,34]. Note that the structured surfaces have millimeter-sized features that would naturally reduce any slip effects.

We characterize the flow by estimating the Reynolds Re (ratio of inertia and viscous effect) and Capillary Ca (ratio of viscous and surface tension effect) numbers. The characteristic initial shear rate is estimated as $\dot{\gamma} = v_{r,0}/h_{0,\text{eff}}$, where $v_{r,0} = v_l R_0/(2h_{0,\text{eff}})$ is the initial radial velocity and $h_{0,\text{eff}}$ is the initial mean fluid thickness. The quantity $h_{0,\text{eff}}$ is introduced to accommodate the fluid volume that is also found in the grooves of the plate surface patterns (additional details in the coming section). The Reynolds number is defined as $Re = \rho v_{r,0} h_{0,\text{eff}}/\mu$, and the Capillary number as $Ca = \mu v_{r,0}/\gamma$, where $\rho \approx 1000 \text{ kg/m}^3$ is the fluid density and $\gamma \approx 0.05 \text{ N/m}$ is the surface tension [34]. The viscosity is a spatiotemporal parameter in this system as it is linked to the local shear and may vary in the range of $\mu \in [10 - 10^3] \text{ Pa} \cdot \text{s}$. For the experimental conditions reported here, we find that for most experiments $Re \in [10^{-8}, 10^{-3}]$, indicating negligible inertial effects, while $Ca \in [0.1, 100]$, suggesting that viscous forces can be comparable to and dominate over capillary forces in the flow.

III. DYNAMICAL PERMEABILITY ANALYSIS

Next, we show how the lifted Hele-Shaw problem can be reformulated as a flow through a permeability field $k(x, y, t)$ varying both in space and time. This perspective provides a natural framework for understanding how the plate texture influences the morphology and symmetry of the deposited fluid patterns. In a classical Hele-Shaw cell composed of two smooth parallel plates separated by a small gap g , with $g \ll L$ where L is a characteristic lateral dimension of the flow, the depth-averaged velocity obeys Darcy's law with an effective permeability given by $k = g^2/12$, which follows from the parabolic Poiseuille velocity profile established across the gap between the plates. In the present experiments, however, the plate separation is not spatially uniform due to macroscopic surface textures patterned onto the substrates. The local gap thickness therefore becomes a function of both space and time,

$$g(x, y, t) = h(t) + \delta(x, y) = h_0 + v_l t + \delta(x, y), \quad (1)$$

where $\delta(x, y)$ represents the geometrical perturbation induced by the plate texture. Depending on the geometry, $\delta(x, y)$ describes the local protrusions associated with pyramids, slots, squares, triangular elements, or other patterned structures. Notice that $h(t)$ corresponds to the gap at the points where $\delta(x, y) = 0$. For instance, in the case of the pyramids, $h(t)$ corresponds to the gap along the vertical line linking the tips of the pyramids in the top and bottom plates.

The corresponding local permeability field becomes $k(x, y, t) = g(x, y, t)^2/12$. Importantly, the spatial symmetry of $\delta(x, y)$ directly determines the symmetry of the permeability field. As we will later demonstrate, this anisotropic permeability landscape strongly influences the preferred propagation directions of the invading air fingers and therefore the final morphology of the deposited fluid patterns.

We define the effective initial gap of the cell as the spatially averaged gap at $t = 0$,

$$h_{0,\text{eff}} = \langle g(x, y, 0) \rangle = h_0 + \Delta, \quad (2)$$

where $\Delta = \langle \delta(x, y) \rangle$. The quantity Δ is a purely geometrical correction factor and depends only on the substrate texture. For the geometries considered here, we obtain $\Delta_{\text{slots}} = 0.50 \text{ mm}$, $\Delta_{\text{squares}} = 0.75 \text{ mm}$, $\Delta_{\text{pyramids}} = 0.67 \text{ mm}$, and $\Delta_{\text{triangles}} = 0.44 \text{ mm}$. Notice that patterned structures are present on both plates.

Under a Hele-Shaw approximation, the permeability field can therefore be rewritten as

$$k(x, y, t) = \frac{(h_0 + v_l t + \delta(x, y))^2}{12}. \quad (3)$$

Equation (3) highlights two important aspects of the problem. First, the permeability increases globally with time as the plates separate. Second, the spatial heterogeneity induced by the substrate

texture remains imprinted onto the permeability field throughout the experiment, and it is more prevalent in the dynamics in the very early times since, for late times, it becomes less important when compared to the homogeneous lift term $v_l t$.

To quantify the directional anisotropy induced by the plate patterns, it is useful to define the angular permeability field obtained from radial averaging. Using polar coordinates (r, ϕ) centered at the origin of the cell, we define

$$k(\phi, t) = \frac{1}{R_0} \int_0^{R_0} k(r, \phi, t) dr, \quad (4)$$

where R_0 is the radius of the cell. This quantity measures the average permeability along a given angular direction. Following the convention from Fig. 1, we take the value $\phi = 0$ at the “North pole” and let the angle run counterclockwise. We will later also present the angular permeability normalized either by its maximum value $k_{\max}$ or by its angular average $\langle k(\phi, t) \rangle_\phi$. The normalization isolates the anisotropic component of the permeability field independently of its global increase with time during plate separation. As will be shown later, local angular maxima of $k(\phi, t)$ correlate strongly with the preferential propagation directions of the invading air fingers and the minima with the directions for which the air invades the least, thus leaving a more pronounced fluid finger in the plates. The symmetry of the substrate pattern induces corresponding symmetries in the angular permeability field, which in turn are directly reflected in the deposited fluid structures. The permeability anisotropy therefore acts as an effective predictor for both the finger dynamics and the final deposition morphology.

Notice that for the Hele-Shaw assumption to be valid, and consequently the whole permeability analysis based on Eq. (3), we need to obey the condition that the gap is much smaller than the radius of the plate, i.e., $g(x, y, t) \ll R_0$. In the experiments to be shown, this is true only at the beginning of the process. However, as we will see in Sec. V, it is in this initial phase that the heterogeneity in the permeability field is most important and it acts to define the prevailing growth mode for the dynamics.

IV. SCALING LAW FOR THE NORMAL FORCE

In this section, we describe the relevant forces for the dynamics. We will study the case of smooth plates, for which $\delta(x, y) = 0$. We consider a thin liquid film confined between two parallel circular plates of radius R_0 and gap $h(t)$. The upper plate is lifted at constant velocity $v_l = \dot{h}$. Volume conservation gives $\pi R^2 h = \pi R_0^2 h_0$, so that

$$R(h) = R_0 \left(\frac{h_0}{h} \right)^{1/2}, \quad (5)$$

where $R(h)$ is the projected radius of the fluid, which shrinks as the top plate of the cell is lifted. For the sake of simplicity to allow the analytical calculations, we assume that the interface remains circular (an assumption that does not hold as the liquid is deposited).

The lubrication approximation ($h \ll R$) assumes a flow mainly in the radial direction. The depth-averaged radial velocity scales as $v_r(r) \sim v_l r / (2h)$, leading to a characteristic shear rate $\dot{\gamma} \sim v_r / h \sim v_l r / (2h^2)$. In cylindrical coordinates, the radial Stokes equation reduces to $\partial p / \partial r = \partial \tau_{rz} / \partial z$, where τ_{rz} is the radial shear stress acting on planes normal to the gap direction z . Variations across the gap occur over a scale h , so $\partial \tau_{rz} / \partial z \sim \tau / h$, giving $dp / dr \sim \tau / h$. As the plate is lifted, a meniscus forms at the edge $r = R$, leading to a capillary pressure jump. The boundary condition becomes $p(R) \sim -2\gamma / h$. For a Herschel–Bulkley fluid, $\tau = \tau_y + K \dot{\gamma}^n$. By combining the different parts above, we get

$$\frac{dp}{dr} \sim \frac{\tau_y}{h} + \frac{K v_l^n r^n}{2^n h^{2n+1}}. \quad (6)$$

The pressure inside the fluid can be estimated by integrating from r to R , also including the effect from capillarity at the edge (R)

$$p(r) \sim -\frac{2\gamma}{h} - \frac{\tau_y(R-r)}{h} - \frac{K v_l^n}{2^n h^{2n+1}} \frac{R^{n+1} - r^{n+1}}{n+1}. \quad (7)$$

The normal force is given by $F_N = -2\pi \int_0^R p(r) r dr$, which can be decomposed into different parts $F_N = F_\gamma + F_y + F_{HB}$, where the capillary contribution scales as

$$F_\gamma \sim \pi R^2 \frac{2\gamma}{h}, \quad (8)$$

the yield stress contribution scales as,

$$F_y \sim \frac{\tau_y}{h} \int_0^R (R-r) r dr \sim \frac{\tau_y R^3}{h}, \quad (9)$$

and the viscous Herschel–Bulkley contribution scales as

$$F_{HB} \sim \frac{K v_l^n}{2^n h^{2n+1}} \int_0^R (R^{n+1} - r^{n+1}) r dr \sim \frac{K v_l^n R^{n+3}}{2^n h^{2n+1}}. \quad (10)$$

Thus, using the relation for $R(h)$ from Eq. (5), the total normal force becomes

$$F_N(h) \sim 2\pi \gamma R_0^2 h_0 h^{-2} + \tau_y R_0^3 h_0^{3/2} h^{-5/2} + \frac{K v_l^n}{2^n} R_0^{n+3} h_0^{(n+3)/2} h^{-5(n+1)/2}. \quad (11)$$

In the experiments, we expect the capillary term to be more relevant at early times in the dynamics, when the gap h is small. Later on, the capillary term contribution to Eq. (11) will be much smaller (by at least an order of magnitude) than the remaining yield stress and viscous Herschel–Bulkley terms. The relative importance of the those two terms can be assessed by taking the ratio of the two contributions. For this purpose, it is convenient to express the forces in terms of the instantaneous radius $R = R(h)$,

$$\frac{F_y}{F_{HB}} \sim \frac{\tau_y R^3/h}{K v_l^n R^{n+3}/(2^n h^{2n+1})} \sim \frac{2^n \tau_y h^{2n}}{K v_l^n R^n}. \quad (12)$$

By using the characteristic shear rate at the rim $\dot{\gamma} \sim v_l R/(2h^2)$, this can be rewritten as

$$\frac{F_y}{F_{HB}} \sim \frac{\tau_y}{K \dot{\gamma}^n} = Bi, \quad (13)$$

naturally introducing the Bingham number (Bi), representing the relative magnitude of the yield and viscous contributions to the force. By using $\dot{\gamma} \sim v_l R/(2h^2)$ and Eq. (5), one obtains $Bi \propto h^{5n/2}$, so that the Bingham number increases as the plates are separated. This scaling prediction allows us to distinguish two flow regimes: A viscous regime ($Bi \ll 1$) where the Herschel–Bulkley contribution dominates $F \approx F_{HB} \propto h^{-5(n+1)/2}$, and a yield stress regime ($Bi \gg 1$) where the yield stress contribution dominates, $F \approx F_y \propto h^{-5/2}$. As the gap increases during lifting, the system naturally evolves from a viscous-dominated regime at small h to a yield stress-dominated regime at larger h . Equation (11) naturally recovers the limits reported in the literature for the two flow regimes when disregarding the small capillary contribution [18,33]. In the Newtonian case ($n = 1$, $\tau_y = 0$), the viscous contribution dominates and we obtain $F \propto h^{-5}$. Conversely, in the purely yield stress-dominated limit ($K \dot{\gamma}^n \ll \tau_y$), the force reduces to $F \propto h^{-5/2}$.

Since the yield stress contribution decays more slowly with the gap size than the viscous Herschel–Bulkley term, it becomes increasingly important as the plates are separated. This predicts the existence of a crossover gap h_c at which the two contributions are comparable, defined by $\tau_y \sim K \dot{\gamma}^n$, i.e., $Bi \sim 1$. Using $\dot{\gamma} \sim v_l R/(2h^2)$ together with Eq. (5), this yields $h_c \sim (K v_l^n R_0^n h_0^{n/2}/(2^n \tau_y))^{2/(5n)}$.

Experimentally, we also observe that beyond a second critical gap, additional effects associated with extensional flow begin to play a role [18]. These correspond to inherently three-dimensional flow contributions that are not captured within the lubrication-based framework developed here. As a result, the force–gap dynamics exhibits a complex evolution, transitioning from a viscous-dominated regime at small gaps, to a yield stress-dominated regime at intermediate gaps, and eventually to a regime where extensional effects become significant. In this latter regime, the force is expected to follow a different scaling, which we denote by an exponent β . Similar transitions and scaling behaviors have been reported in previous studies [18,33].

In practice, extracting these different scaling regimes from experimental force–displacement curves is challenging. A clear identification of multiple power-law exponents would require data spanning several decades in gap, which is typically not accessible experimentally. Moreover, the limited range of accessible gaps in the rheometer also makes it difficult to estimate the crossover scale h_c with great confidence. Nevertheless, the existence of such a transition scale plays an important physical role. In particular, it is associated with the emergence of patterned structures observed on the plates after separation, where the fluid retains its shape. This indicates that, beyond the crossover, the material response is sufficiently dominated by yield stress effects (and possibly extensional contributions) to preserve the imprinted structures.

If we only keep the dominant yield stress and Herschel–Bulkley viscous terms in the force prediction, we can recast it in dimensionless form by the following scaling: $\hat{h} = h/h_0$ and $F_y^* = \tau_y R_0^3 h_0^{-1}$, where the $\hat{}$ markers represent nondimensional quantities,

$$\hat{F} = \frac{F}{F_y^*} = \hat{h}^{-5/2} [1 + \Lambda \hat{h}^{-5n/2}], \quad (14)$$

with $\Lambda = \frac{K v_l^n R_0^n}{2^n \tau_y h_0^{2n}}$. In this form, the crossover between viscous and yield-dominated regimes corresponds to the condition $\Lambda \hat{h}_c^{-5n/2} \sim 1$, and with $\hat{h}_c = h_c/h_0$. This is consistent with the fact that the transition scale h_c introduced above can be interpreted as the value of the gap at which the two terms in the dimensionless force become comparable. Equivalently, up to numerical prefactors,

$$\hat{F} \sim \hat{h}^{-5/2} \left(1 + \frac{1}{Bi(\hat{h})} \right), \quad (15)$$

highlighting that the force results from the competition between yield and viscous contributions. In this representation, the crossover occurs when $Bi(\hat{h}_c) \sim 1$, consistent with the condition $\tau_y \sim K \dot{\gamma}^n$ derived above.

Due to the limited range of gaps available for the experimental data, in the following section, we adopt an approach similar to that used in previous studies, in which two effective regimes in the force versus gap curve are identified and characterized by exponents α and β . We anticipate that the determination of the exponent α is more subtle in the present system than in the case of purely Newtonian or purely yield stress fluids, as both viscous and yield contributions coexist and compete over a significant range of gaps. The experimental analysis is thus inherently limited by this superposition of mechanisms here. The above analysis only considers smooth plates. In the case studied here, where the gap is modulated by the plates' imprinted roughness, the effective quantities of the form h_{eff} should be employed instead.

V. RESULTS

A. Effect of surface texture on the fluid invasion dynamics

To illustrate how the surface texture affects the intrusion dynamics and the patterning between the two plates, we show in Fig. 2 the results for experiments with a smooth plate and a patterned plate at the same lifting velocity $v_l = 64 \mu\text{m/s}$ and mean initial film thickness $h_{0,\text{eff}} = 1.25 \text{ mm}$. When comparing the footprint shapes at different time points, it is clear that the surface texture affects the dynamics. On the smooth plate in Fig. 2(a), the footprint remains nearly axisymmetric as the yield

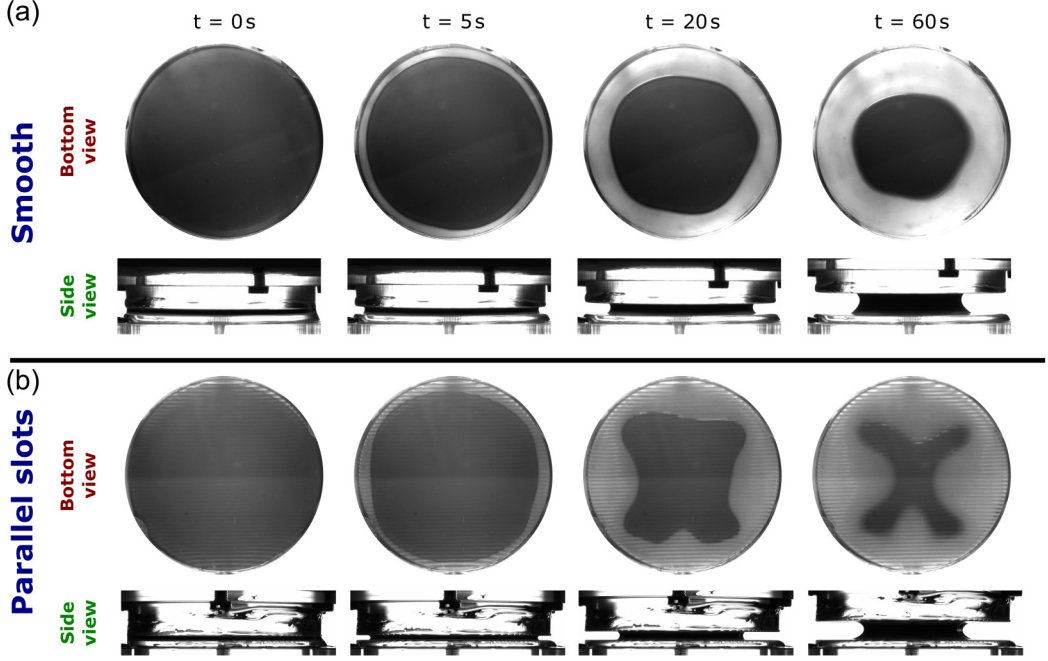


FIG. 2. Bottom and side view images illustrating the interfacial dynamics as two plates in a Hele-Shaw geometry are separated at a velocity of $v_l = 64 \mu\text{m/s}$. The effective mean initial height is $h_{0,\text{eff}} = 1.25 \text{ mm}$, and the fluid is a Carbopol mixture with a yield stress of $\tau = 28 \text{ Pa}$. Panel (a) shows the case with smooth plates (treated to enforce a no-slip boundary condition), while panel (b) corresponds to plates patterned with parallel slots. In the slotted case, the interface shape evolves as the yield stress fluid retracts radially, producing a characteristic cross-shaped deposition pattern with further propagation along the direction that follows the slot lines (seen horizontally in the bottom view). In contrast, the smooth plates generate a more symmetric invasion front, leaving a roughly circular bump. To account for the fluid volume retained within the slots, the initial separation in the slotted case is reduced to $h_0 = 0.75 \text{ mm}$, ensuring the same mean film thickness as in the smooth configuration ($h_{0,\text{eff}} = 1.25 \text{ mm}$), as the depth of the slots is 0.5 mm , see Fig. 1(d). The times given on the top are for both panels.

stress fluid retracts, leaving a fairly symmetric puddle. It has been suggested that, as long as the energy input to the system exceeds a threshold value, the fingering instability becomes independent of the fluid yield stress [18]. In the present smooth-plate experiments, the interfacial perimeter remains close to the axisymmetric base state, with only small deviations from circularity. When the same fluid, mean film thickness, and lifting speed are used with a patterned substrate consisting of macroscopic parallel slots, the dynamics changes markedly, as shown in Fig. 2(b). Instead of forming a symmetric puddle, the deposited footprint becomes cross-shaped. The resulting finger-like structures are much larger than the substrate pattern itself, indicating that the texture affects the global invasion dynamics rather than simply imprinting its local geometry onto the interface. The preferential growth directions coincide with the directions along which the radial motion toward the center of the cell is most strongly promoted by the plate structure. This comparison demonstrates that the surface pattern can strongly influence the final footprint of the deposited yield stress fluid.

B. Phase space for stable and unstable deposition

To determine which experimental parameters control the stability of the flow, we systematically varied both the lifting velocity v_l and the effective mean initial height $h_{0,\text{eff}}$ for several experiments performed with different plate structures. The results are summarized in Fig. 3. When comparing

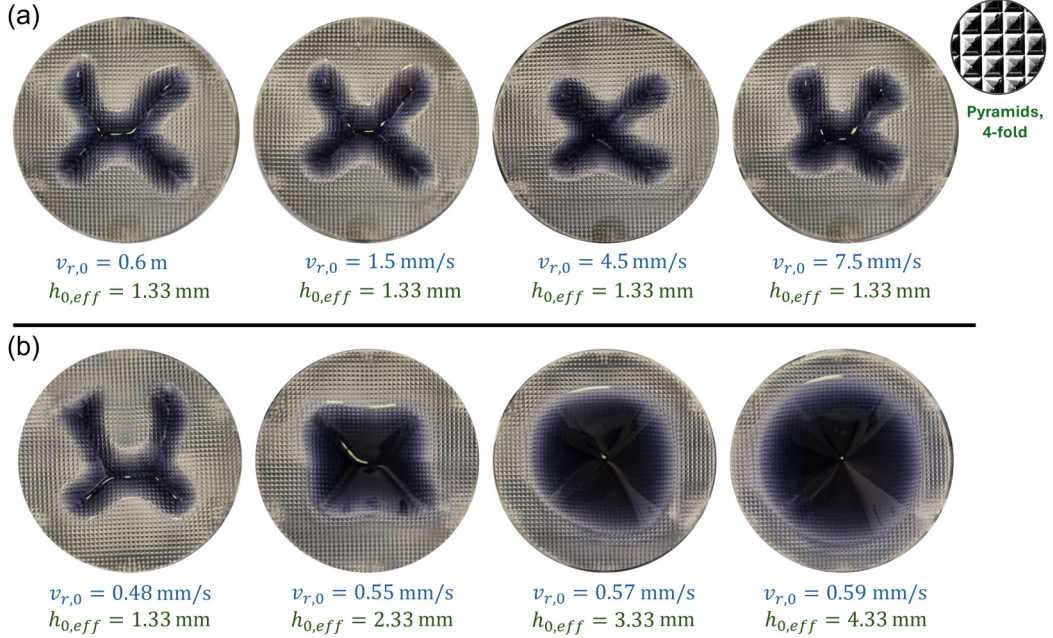


FIG. 3. Sensitivity of the invasion patterns with the initial radial velocity $v_{r,0}$ and effective mean initial height $h_{0,eff}$ with $\tau = 28$ Pa. The plate substrate follows a pyramidal pattern with fourfold symmetry, as shown in the detail. While the patterns seem to be fairly insensitive to $v_{r,0}$ as seen in (a), they are clearly affected by $h_{0,eff}$ as seen in (b), with the substrate influence on the deposition pattern being observed for the smallest values of $h_{0,eff}$ shown here.

experiments with different mean initial heights, it is useful to consider the initial radial velocity rather than the lifting velocity itself, since the same lifting velocity leads to different radial velocities depending on the amount of fluid trapped between the plates. We estimate the initial radial velocity as $v_{r,0} = v_l R_0 / (2h_{0,eff})$.

It appears that the final deposited pattern of yield stress fluid, i.e., initiation of the unstable mode, is fairly insensitive to the radial velocity when varied about one order of magnitude [Fig. 3(a)]. It is partly explained by the modest change to $Bi \sim v_{r,0}^{-0.31}$, whereas a larger change to Bi likely affects the finger length of the deposited pattern. In contrast, the effective mean initial height has a strong effect on the final state [Fig. 3(b)]. Small values of $h_{0,eff}$ lead to strongly ramified (unstable) invasion patterns, whereas increasing $h_{0,eff}$ suppresses the process and eventually produces a stable puddle for very large values of $h_{0,eff}$. In between these two extremes, we find an intermediate range of gaps for which the experiment evolves in a regime that we label “patterning.” In this domain, the dynamics evolve in a stable and repeatable manner, and the liquid is deposited on the plates, reflecting the symmetry of the underlying structure on the plates. In the examples shown on the right side of the figure, we see a fourfold pattern left on the plates as a consequence of the fourfold underlying structure of the pyramidal features on the plates. These observations indicate that the effective initial gap is the primary control parameter for the stability of the deposition process.

Motivated by this observation, we compiled our experimental phase space shown in Fig. 4. The horizontal axis corresponds to the normalized effective initial gap $h_{0,eff}/R_0$, while the vertical axis shows the equivalent initial radial velocity $v_{r,0}$. The color of each marker denotes the observed dynamical outcome, while the marker shape identifies the geometry of the patterned plate. The diagonal lines correspond to constant initial Bingham numbers $Bi_0 = \tau_y / (K \dot{\gamma}_0^n)$, where the initial shear rate is estimated as $\dot{\gamma}_0 \sim v_{r,0}/h_{0,eff}$.

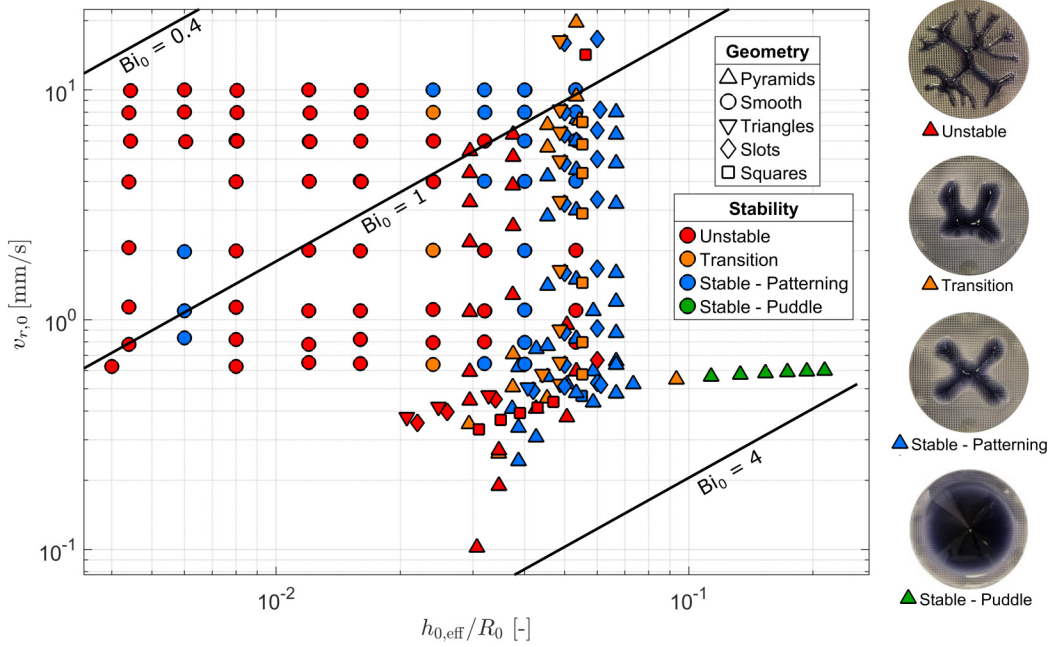


FIG. 4. Experimental phase space for the lifting Hele-Shaw cell. Initial radial velocity $v_{r,0}$ as a function of the normalized effective initial gap $h_{0,\text{eff}}/R_0$. The symbols represent individual experiments, with marker shape indicating the plate geometry and marker color denoting the observed dynamical outcome (unstable, transition, stable patterning, or stable puddle). The diagonal lines correspond to constant initial Bingham numbers $Bi_0 = 0.4$, $Bi_0 = 1$, and $Bi_0 = 4$. The figure reveals that the effective initial gap is the primary parameter controlling the stability of the process. Experiments with small values of $h_{0,\text{eff}}/R_0$ are predominantly unstable and produce highly ramified invasion patterns, whereas stable patterning emerges for $h_{0,\text{eff}}/R_0 > 0.03$. At larger gaps, approximately $h_{0,\text{eff}}/R_0 > 0.1$, the influence of the substrate texture becomes weak, and the deposited footprint approaches a nearly circular puddle. All experiments fall within the range $0.4 < Bi_0 < 4$, indicating that the instability develops under conditions where both the yield stress and Herschel–Bulkley viscous contributions are important.

Several observations can be made in Fig. 4. First, the effective initial gap appears to be the dominant parameter controlling the stability of the process. For small values of $h_{0,\text{eff}}/R_0$, the system is predominantly unstable and develops highly ramified invasion patterns. As the initial gap is increased, the system enters a transitional regime, and stable patterning emerges at approximately $h_{0,\text{eff}}/R_0 \approx 0.03$. In this regime, the final deposited footprint reflects the symmetry imposed by the plate texture, and the resulting structures can be reproduced reliably between experiments. For even larger gaps, approximately $h_{0,\text{eff}}/R_0 > 0.1$, the influence of the patterned substrate becomes weak, and the deposited footprint approaches a nearly circular puddle. It should be noted that in this limit the assumptions underlying the Hele-Shaw and lubrication approximations gradually lose validity, since the gap is no longer small compared to the lateral dimensions of the cell.

A second important observation is that the transition between the different dynamical regimes is primarily organized along the $h_{0,\text{eff}}/R_0$ direction, whereas the dependence on the initial radial velocity is comparatively weak over the experimentally accessible range. This is consistent with the results shown in Fig. 3, where changing $h_{0,\text{eff}}$ produces a clear transition between stable patterning and puddle formation, while comparable variations in $v_{r,0}$ have a much weaker effect on the final deposited pattern. Together, these observations suggest that geometric confinement plays a more

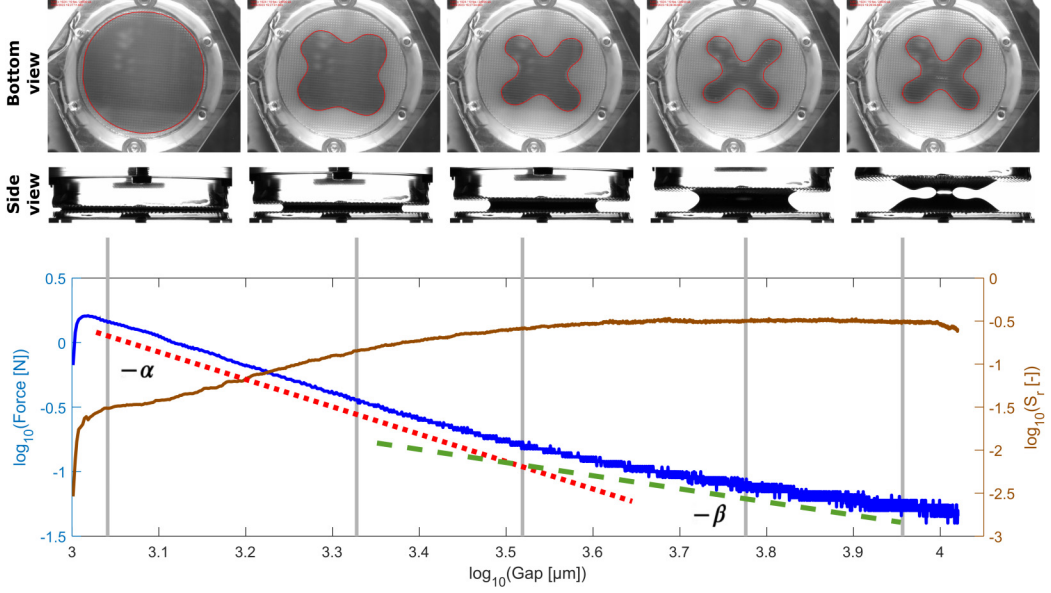


FIG. 5. Top row: bottom- and side-view images of the yield stress fluid interface during the separation of two pyramid-patterned plates. The plate radius is $R_0 = 25$ mm and acts as a scale bar. Bottom plot: pulling force (blue, left axis) and geometric factor S_R (brown, right axis) as functions of the effective cell gap h_{eff} . The pulling speed, initial radial velocity, effective initial gap, and yield stress were $v_l = 51.2 \mu\text{m/s}$, $v_{r,0} = 0.48 \text{ mm/s}$, $h_{0,\text{eff}} = 1.33 \text{ mm}$, and $\tau_y = 28 \text{ Pa}$, respectively. The dotted and dashed lines are guide-to-the-eye power laws indicating the slopes α and β of the force response in the two regimes. The five image panels above the plot correspond to the gap heights indicated by the vertical gray lines. The crossover between the force regimes described by α and β coincides with the stabilization of the geometric factor S_R . Beyond this point, the dynamics is primarily controlled by extensional flow and neck thinning, eventually leading to structural breakup, while the contact line remains pinned. A video of the experiment and measurements shown here can be found in the Supplemental Material [35].

important role than the lifting speed in determining whether the instability develops into a ramified invasion pattern, a reproducible patterned state, or a stable puddle.

Finally, the diagonal solid lines are for a constant Bingham number, showing that all experiments were performed in the range $0.4 < Bi_0 < 4$, corresponding to conditions where neither the yield stress nor the Herschel–Bulkley viscous contribution clearly dominates the force balance. Consequently, the instability develops in a regime where all terms in Eq. (11) are expected to contribute simultaneously, thus making the extraction of a single force scaling exponent α rather challenging in practice. The pattern-selection process cannot be interpreted as arising solely from either a purely viscous or a purely yield stress mechanism.

C. Coupling between force decay and footprint evolution

Our experiments enabled simultaneous visualization of the evolving pattern from both the side and the transparent bottom of the cell, while also recording the corresponding normal force. Figure 5 shows a representative experiment with pyramid-patterned plates, illustrating how the force evolution correlates with the changing footprint of the yield stress fluid. At early times, air intrudes into the gap while the interface remains approximately circular. During this stage, the normal adhesive force rapidly increases and reaches a peak, which defines the maximal normal force $F_{N,\text{max}}$. After this peak, the force decreases as the effective gap increases. This decay contains capillary, yield stress, and Herschel–Bulkley viscous contributions, as described by Eq. (11). Because the

experimentally accessible range of gaps is limited, these contributions cannot be fully separated in individual force curves. We therefore characterize the early decaying regime by an effective power law, $F_N \sim h_{\text{eff}}^{-\alpha}$.

As the plates are separated further, the initially circular footprint evolves into a cross-shaped pattern. The image panels in Fig. 5 correspond to the gap heights indicated by the vertical gray lines in the force curve. The first panel shows the early nearly circular invasion front, highlighted by a red line, while the following panels show the emergence and stabilization of the patterned footprint. Around the central image panel, the contact line at the bottom plate appears to become pinned, and the wetted area changes only weakly thereafter. This pinning coincides with a change in the slope of the force curve, from the initial exponent α to a second effective exponent β , such that $F_N \sim h_{\text{eff}}^{-\beta}$. Beyond this crossover, the dynamics is dominated by the stretching and thinning of the fluid neck connecting the upper and lower plates, as seen in the side-view images. In this later regime, three-dimensional and extensional-flow effects become increasingly important, consistent with previous observations in yield stress fluids [18,33]. A video showing the temporal evolution of the fluid pattern and the associated force measurements can be found in the Supplemental Material [35].

We further characterize the evolution of the front by measuring the shape factor S_R , introduced in [31], and defined as the relative deviation from a circular pattern,

$$S_R = \sqrt{\left\langle \left(\frac{R(\zeta)}{\bar{R}} - 1 \right)^2 \right\rangle}, \quad (16)$$

where $R(\zeta)$ is the distance to the center of the cell along the perimeter coordinate ζ and $\bar{R}$ is its average. The measurement of S_R is shown by the solid brown line in Fig. 5. We notice that, as expected, this quantity stabilizes at the position marked by the center image panel, corresponding to the point where the contact line appears to pin in the side-view images. This point correlates well with the approximate transition between the α and β scaling regimes for the force. Our results therefore indicate that pinning of the interface marks the onset of the extensional-flow-dominated part of the dynamics, in which the neck connecting the upper and lower fluid regions thins while the footprint at the plate remains nearly fixed.

D. Force-scaling exponents and maximum normal force

Since surface patterning can clearly affect the dynamics of the Carbowol mixture, we investigated how it affects the force-scaling exponents α and β , as well as the maximal normal adhesive force $F_{N,\text{max}}$. In addition to the four patterns shown in Fig. 1(d), we also analyzed three types of smooth geometries: a transparent Plexiglas plate, a metallic plate, and a surface covered with 800-grit waterproof sandpaper. We also included a beveled slot geometry, similar to the parallel slot configuration but with angled slot walls.

The exponents α and β extracted from the force curves are shown in Fig. 6(a) as functions of $h_{0,\text{eff}}/R_0$. Despite the strong influence of plate texture on the final deposited footprint, the measured force exponents do not seem to show any systematic dependence on the plate texture. This indicates that the force response is controlled primarily by the trapped fluid volume, rather than by the detailed surface geometry. From the model in Eq. (11), the force decay results from a superposition of capillary, yield stress, and Herschel–Bulkley viscous contributions. These contributions correspond respectively to the scaling exponents $\alpha = 2$, $\alpha = 5/2$, and $\alpha = 5(n+1)/2 = 3.275$ for $n = 0.31$. The blue shaded region in Fig. 6(a) therefore marks the theoretical range expected from these contributions. Most experiments classified as stable patterning or in a transition regime (see Fig. 4) fall within this theoretical band, suggesting that a single effective exponent measured over a finite fitting range captures the combined action of these mechanisms. The highly unstable experiments, however, show larger values of α on average. This is not unexpected, since the simplified model leading to Eq. (11) assumes an approximately compact fluid domain and does not account for

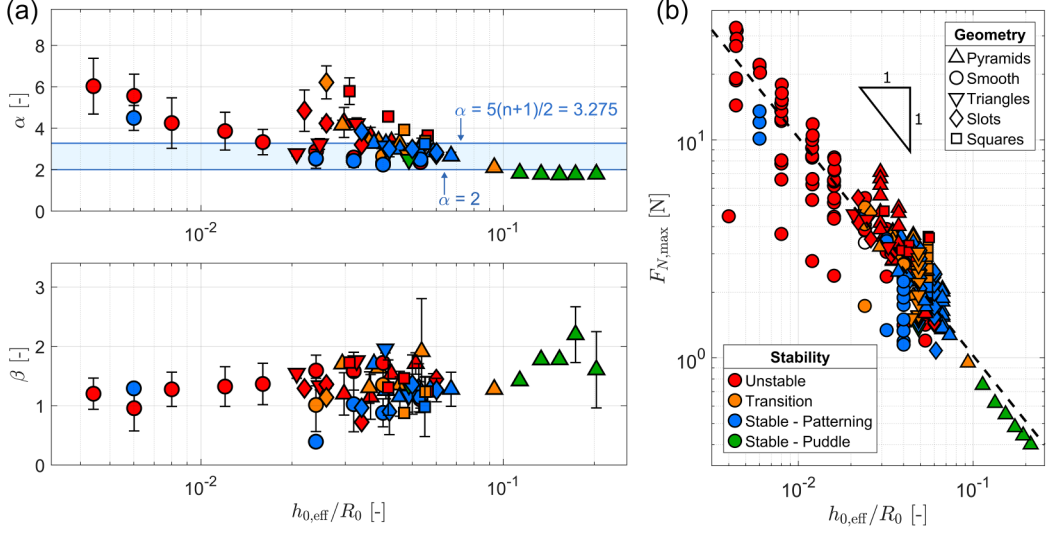


FIG. 6. Force-scaling exponents and maximum force as a function of the effective initial gap. (a) Binned values of the force-scaling exponents α and β as functions of the normalized effective initial gap $h_{0,\text{eff}}/R_0$, where $R_0 = 25$ mm. Marker shape denotes the plate geometry, while marker color denotes the observed stability of the deposited pattern. The blue shaded region indicates the theoretical range $2 \leq \alpha \leq 5(n+1)/2$ predicted by Eq. (11), corresponding to the capillary, yield stress, and Herschel–Bulkley viscous contributions. Most stable-patterning and transition experiments fall within this theoretical band, whereas highly unstable experiments tend to show larger values of α . Error bars combine the spread across experiments within each $h_{0,\text{eff}}$ bin with the fitting-window uncertainty. The β measurements give $\langle \beta \rangle = 1.3 \pm 0.4$ across all experiments. (b) Maximum normal force $F_{N,\text{max}}$ as a function of $h_{0,\text{eff}}/R_0$ for the same set of experiments. The dashed black line shows the fixed-slope scaling $F_{N,\text{max}} \propto (h_{0,\text{eff}}/R_0)^{-1}$.

the highly branched geometry that develops in the unstable regime. The second exponent is less sensitive to the initial gap, with $\langle \beta \rangle = 1.3 \pm 0.4$ across all experiments, consistent with previous measurements of viscoplastic lifting flows [18].

The error bars shown in Fig. 6(a) are mainly from two independent sources of uncertainty. The first arises from the determination of the power-law exponent within each experiment. Exponents α and β are obtained from manually selected fitting windows in log–log space. To assess the robustness of this choice, the boundaries of each fitting window were perturbed by an amount corresponding to approximately 5% of the total logarithmic span of h_{eff} in the corresponding curve for $F_N(h_{\text{eff}})$. For each perturbed window, the exponent was recomputed, generating a distribution of values. The standard deviation of this distribution defines the fitting-window uncertainty for that experiment, denoted σ_{win} . The second source of uncertainty arises from the variability across independent experiments within each $h_{0,\text{eff}}$ bin. Experiments are clustered into bins of similar $h_{0,\text{eff}}$, and the spread of the measured exponents within each bin is quantified by the standard deviation σ_{spread} . The total uncertainty shown in the error bars is computed by combining these two independent contributions in quadrature,

$$\sigma_{\text{total}} = \sqrt{\sigma_{\text{spread}}^2 + \sigma_{\text{win,RMS}}^2}, \quad (17)$$

where $\sigma_{\text{win,RMS}} = \langle \sigma_{\text{win}}^2 \rangle_{\text{bin}}^{1/2}$ denotes the root-mean-square fitting-window uncertainty of the experiments contributing to that bin.

Intuitively, one might expect the surface pattern to affect the force because it changes the local geometry and the wetted surface area. However, Fig. 6(b) shows that the maximal normal force $F_{N,\text{max}}$ is primarily controlled by the effective mean initial height $h_{0,\text{eff}}$. The data follow

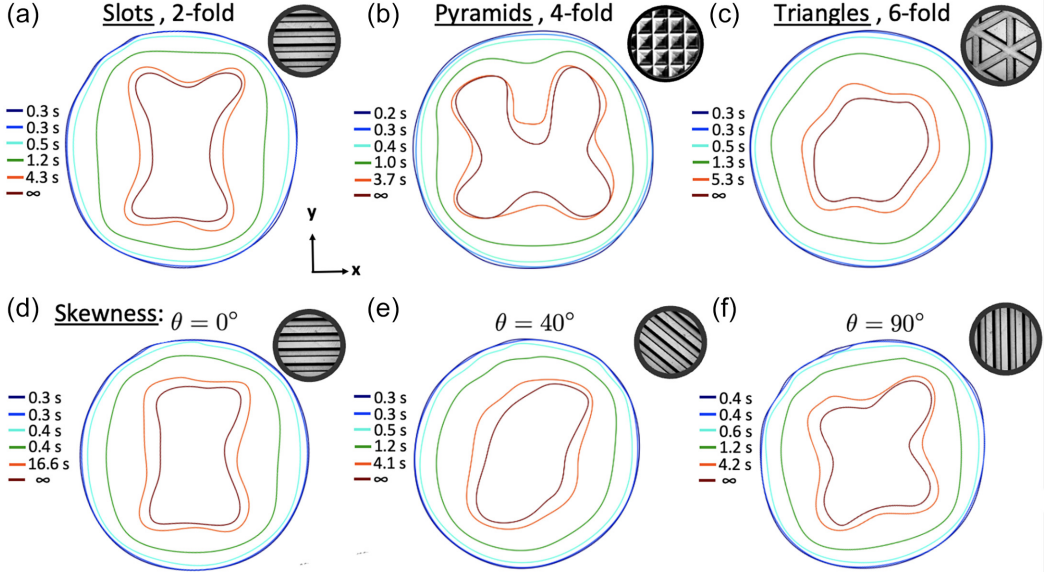


FIG. 7. Illustration of how the dynamic interface of yield stress fluid for (a) twofold (slots), (b) fourfold (squares), (c) sixfold (triangles) pattern with $v_{r,0} \in (0.48 - 0.52)$ mm/s and $h_{0,\text{eff}} \in (1.22 - 1.33)$ mm. In (d)–(f) the angle θ between the slotted patterns of the upper and lower plates is varied to control their alignment, with $v_{r,0} = 0.53$ mm/s and $h_{0,\text{eff}} = 1.5$ mm. At perfect alignment ($\theta = 0$), a symmetric two-fold footprint emerges. As θ increases, asymmetry develops due to differential invasion speeds along azimuthal directions. This demonstrates that pattern skewness significantly influences fluid deposition. The interfacial contours are extracted at different time points (shown in the figure) during the dynamic retraction of the yield stress fluid. See Supplemental Material for videos of the evolving patterns [35].

approximately the fixed-slope scaling $F_{N,\text{max}} \propto h_{0,\text{eff}}^{-1}$ and show no clear systematic dependence on plate texture. This suggests that the peak force is set early in the dynamics, when the fluid domain is still relatively compact and before the surface-induced patterning or branching instability has fully developed.

The maximal force also occurs at small gap sizes, where capillary forces are largest and predict a force contribution scaling as $(\gamma/h)R^2$. A competing estimate based on the material yield stress gives a force scale $\tau_y R^2$. Balancing these contributions gives a crossover height $h \sim \gamma/\tau_y \approx 1.7$ mm. Several experiments have initial effective gaps comparable to this threshold, suggesting that both capillary and yield stress effects may contribute to the measured maximal force. A purely yield stress-controlled maximum force would be expected to be independent of $h_{0,\text{eff}}$, in contrast to the trend observed in Fig. 6(b).

E. Permeability anisotropy and pattern selection

The experiments show that, once the total entrapped volume in the gap is accounted for through $h_{0,\text{eff}}$, the substrate pattern has little systematic effect on the force response. In contrast, the substrate patterning clearly affects the final deposition of the yield stress fluid. Figures 7(a)–7(c) illustrate this effect for plates with parallel slots, squares, and triangular patterns, while keeping similar values of the initial radial velocity and effective initial gap. Although the substrate features have a characteristic length scale of only ~ 500 μm , much smaller than the deposited footprint size of ~ 10 mm, their influence propagates upscale and affects the pattern morphology at millimetric scales. As the pattern geometry is changed from slots to squares and triangles, the deposited fluid develops respectively twofold, fourfold, and sixfold symmetries. The time-resolved interface

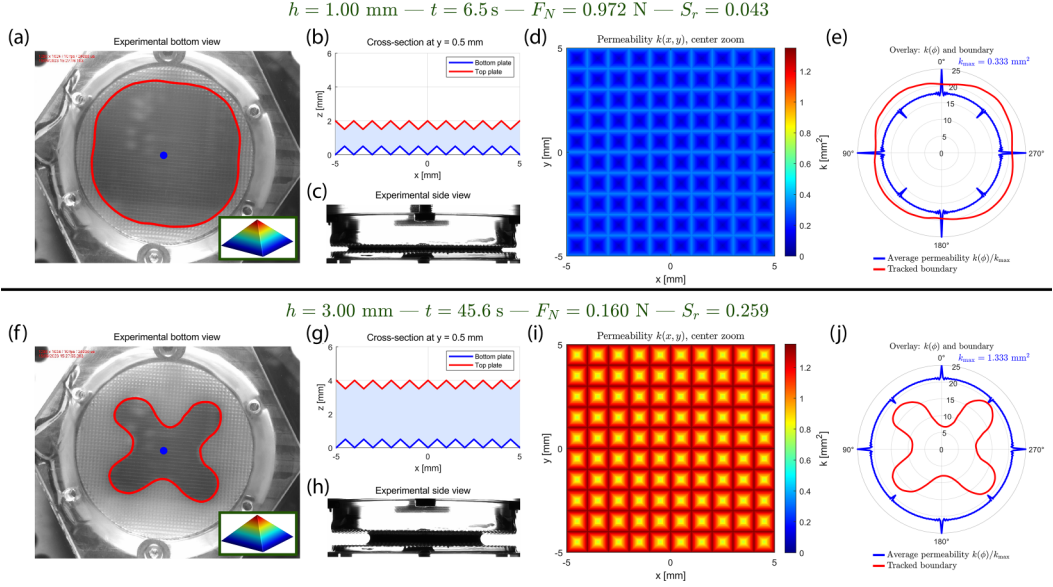


FIG. 8. Evolution of the permeability field in a lifted Hele-Shaw cell with pyramid-textured plates. Panels (a)–(e) correspond to a gap $h = 1 \text{ mm}$, and panels (f)–(j) to $h = 3 \text{ mm}$. The corresponding experimental time t , normal force F_N , and geometrical factor S_r are indicated in the figure. (a) Experimental bottom view, with inset showing the pyramid unit cell. (b) Cross sectional profiles at $y = 0.5 \text{ mm}$. (c) Experimental side view. (d) Permeability field $k(x, y)$, computed as $k(x, y) = g(x, y)^2/12$, where $g(x, y)$ is the local gap between the plates. (e) Radially averaged permeability $k(\phi)/k_{\max}$ (blue) together with the tracked boundary (red). The radial coordinate (0–25) corresponds to the distance from the center in mm, and the normalized permeability is mapped to this scale such that $k_{\max}$ lies at the outer radius. (f)–(j) Same quantities as in (a)–(e) for $h = 3 \text{ mm}$. A clear correlation is observed between permeability anisotropy and interface dynamics as seen in (e) and (j): maxima in $k(\phi)$ occur along $\phi = 0^\circ, 90^\circ, 180^\circ$, and 270° , coinciding with the directions of fastest inward finger propagation, while minima at $\phi = 45^\circ, 135^\circ, 225^\circ$, and 315° correspond to slower interface motion.

contours shown further illustrate how these symmetries emerge progressively during the radial retraction of the fluid. Figures 7(d)–7(f) show that the final footprint is also sensitive to the relative alignment between the top and bottom patterned plates. For slotted plates, changing the skew angle θ between the two plates modifies the symmetry of the deposited pattern, evolving from a nearly twofold footprint for aligned slots to a more complex pattern as the misalignment increases. Videos of the dynamically evolving patterns can be found in the Supplemental Material [35]. These observations suggest that the Carbopol mixture senses an anisotropic resistance to radial motion imposed by the substrate geometry and by the relative alignment of the two confining surfaces. This interpretation will be made more quantitative below through the angular permeability analysis.

In this description, the local gap $g(x, y, t)$ defines a permeability $k(x, y, t) = g(x, y, t)^2/12$, Eq. (3). The plate texture therefore introduces a spatially heterogeneous permeability landscape, while the lifting motion increases the overall permeability with time. The relevant quantity for radial contraction of the fluid domain is the angular permeability $k(\phi, t)$, obtained by averaging $k(x, y, t)$ along radial lines, Eq. (4). Angular maxima in $k(\phi, t)$ correspond to directions where air can invade more easily, and the liquid interface recedes faster. Conversely, angular minima in $k(\phi, t)$ correspond to directions where the air invasion is slower, leaving more pronounced deposited fluid fingers.

Figure 8 illustrates this analysis for pyramid-textured plates. The top row corresponds to a tip-to-tip gap $h = 1 \text{ mm}$, and the bottom row to $h = 3 \text{ mm}$. For each gap, we show the experimental bottom

view, a cross section of the patterned gap, the experimental side view, the computed permeability field $k(x, y)$, and the angular permeability profile compared with the tracked liquid boundary. In the rightmost panels, the normalized permeability $k(\phi)/k_{\max}$ is plotted on the same radial scale as the interface position: the center corresponds to zero permeability, while the outer radius corresponds to $k_{\max}$ (values shown in the figure). This representation allows the angular permeability anisotropy to be compared directly with the instantaneous shape of the retracting liquid domain. The comparison shows a clear correlation between high-permeability directions and the fastest inward propagation of the air. For the pyramid geometry, maxima in $k(\phi)$ occur along $\phi = 0^\circ, 90^\circ, 180^\circ$, and 270° , which coincide with directions where the air penetrates deepest into the initially circular liquid region. The remaining liquid fingers are therefore preferentially deposited in the angular sectors between these high-permeability directions.

The comparison between $h = 1$ mm and $h = 3$ mm also highlights the dynamical nature of the permeability field. As the plates separate, the homogeneous lifting contribution to the gap increases, and the relative amplitude of the angular permeability variations decreases. The permeability anisotropy is therefore strongest at early times, when the gap is smallest. This is precisely the stage at which the interfacial instability is initiated. We therefore interpret the angular variation of $k(\phi, t)$ as setting the preferred directions and characteristic symmetry of the unstable mode, which is then amplified and preserved in the final deposited pattern.

We extend this analysis to different plate geometries and plate alignments in Fig. 9. All permeability fields in this figure, including both the top and bottom rows, are computed for $h = 3$ mm. The top row compares the angular permeability profiles for slots, pyramids, and triangles. The corresponding final deposited patterns are shown at the center of each polar plot. In all cases, the symmetry of the substrate texture is imprinted on the angular permeability field and subsequently on the deposited fluid morphology. Slots generate a twofold permeability anisotropy, pyramids a fourfold anisotropy, and triangles a sixfold anisotropy. The high-permeability directions mark the preferred inward propagation paths of the invading air, while the final liquid fingers remain in the lower-permeability angular sectors between them.

Figures 9(d)–9(f) further show that not only the pattern geometry, but also the relative alignment between the top and bottom plates, affects the permeability anisotropy. For slotted plates with skew angle $\theta = 0^\circ$, the permeability field retains the twofold symmetry of the slots, and the final deposited pattern follows this symmetry. As the skew angle is increased, the angular permeability field is modified because the high- and low-gap regions of the two plates no longer coincide. For $\theta = 90^\circ$, an effective fourfold permeability symmetry emerges. This fourfold symmetry is reflected in the deposited footprint, although the resulting pattern is not identical to that obtained with pyramids. The difference arises because the pyramid geometry contains secondary permeability minima along the diagonal directions, whereas the crossed-slot geometry does not reproduce the same angular structure. Thus, similar global symmetries in $k(\phi)$ can still lead to distinct deposition patterns when the detailed angular distribution of permeability maxima and minima differs.

Together, these results show that the substrate texture has little systematic influence on the global force response once the effective entrapped volume is accounted for through $h_{0,\text{eff}}$, as shown in Fig. 6. However, it strongly affects the final deposition morphology. This contrast can be understood from the anisotropic permeability field: while the total entrapped volume controls the integrated force response, the angular variations in permeability select the preferred directions of air invasion and thereby control the symmetry and shape of the deposited yield stress fluid.

VI. CONCLUSION

Controlled deposition of liquids on substrates is relevant to a wide range of applications. Here, we have shown that the deposition of a yield stress fluid can be guided by macroscopic patterning of the confining plates in a lifting Hele-Shaw geometry. The substrate pattern strongly affects the dynamics

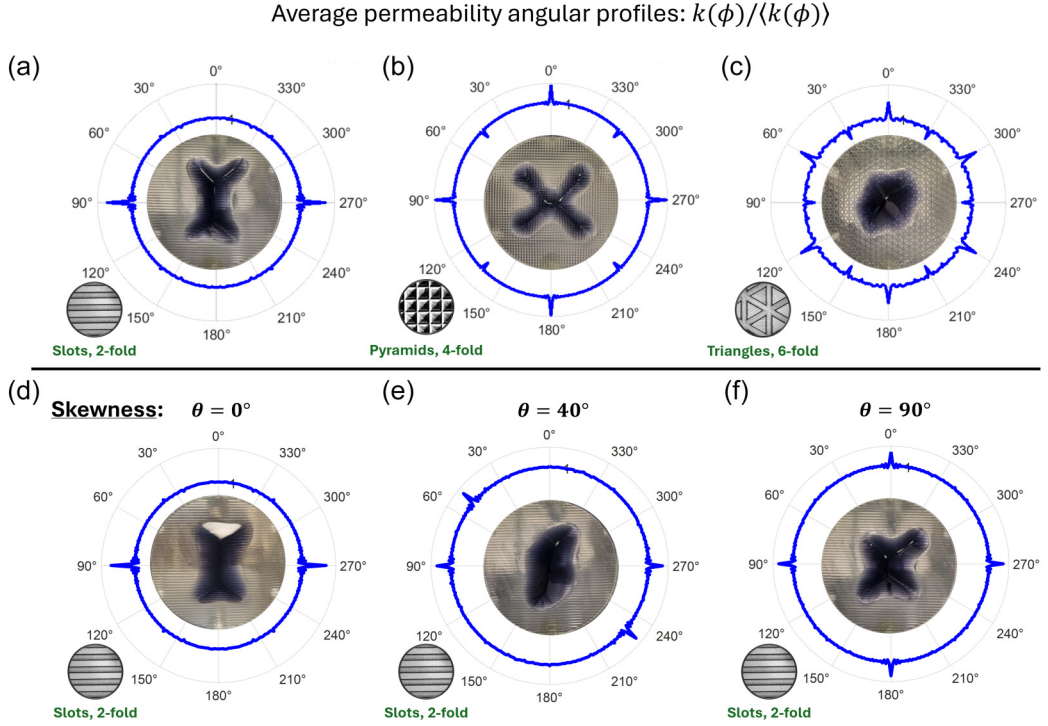


FIG. 9. Radially averaged permeability angular profiles $k(\phi)/\langle k(\phi) \rangle$ for different plate textures and orientations. In all cases, the permeability field is computed for a gap value of $h = 3$ mm, and the images shown correspond to the deposited liquid patterns after complete plate separation. (a) Slot geometry (twofold symmetry). (b) Pyramid geometry (fourfold symmetry). (c) Triangular geometry (sixfold symmetry). In all cases, the angular dependence of the permeability reflects the symmetry of the underlying plate pattern. Maxima in $k(\phi)$ correspond to directions of fastest inward finger propagation, showing a direct link between permeability anisotropy and pattern formation. We also studied the effect of top-bottom plate skewness for a fixed plate geometry (slots). (d) Slot geometry with skew angle $\theta = 0^\circ$. (e) Slot geometry with skew angle $\theta = 40^\circ$. (f) Slot geometry with skew angle $\theta = 90^\circ$. Increasing skewness modifies the angular permeability field and thus the symmetry of the resulting patterns. For $\theta = 90^\circ$, an effective fourfold symmetry emerges. Despite the apparent similarity to the pyramid case, the resulting patterns differ. This is attributed to the absence of secondary permeability minima at $\phi = 45^\circ, 135^\circ, 225^\circ$, and 315° , which are present in the pyramid geometry. These minima reduce radial advancement in those directions, causing the outermost fingers in the pyramid case to remain farther from the center compared to the $\theta = 90^\circ$ skewed slot geometry.

of the intrusion front and the final deposited footprint, producing reproducible twofold, fourfold, and sixfold symmetries depending on the plate geometry. However, once the total entrapped liquid volume is accounted for through the effective mean initial height $h_{0,\text{eff}}$, the same substrate pattern has little systematic influence on the global force response or on the maximal normal force $F_{N,\text{max}}$.

To interpret the force measurements, we developed a scaling relation for the normal force generated during the lifting of a Herschel–Bulkley fluid. The model reduces to the known Newtonian and purely yield stress limits in the appropriate cases, while in the present experiments it predicts a superposition of capillary, yield stress, and Herschel–Bulkley viscous contributions. These contributions correspond to force-decay exponents $\alpha = 2$, $\alpha = 5/2$, and $\alpha = 5(n + 1)/2$, respectively. For the Carbopol mixture used here, with $n = 0.31$, the Herschel–Bulkley contribution gives $\alpha = 3.275$. Thus, experiments for which the force decay can be described by a single effective

exponent are expected to fall within the range $2 \leq \alpha \leq 3.275$. This prediction is consistent with the measurements for experiments classified as stable patterning (or in a transition to stability domain), which mostly fall within the theoretical band shown in Fig. 6(a). In contrast, highly unstable experiments tend to display larger apparent exponents, which is reasonable since the simplified scaling model assumes a compact circular fluid domain and does not include the effects of strongly ramified branching.

We further showed that the transition between the different deposition regimes is primarily governed by the effective initial gap $h_{0,\text{eff}}$, while the dependence on the initial radial velocity $v_{r,0}$ is weaker over the explored range of experimental parameters. Stable controlled patterning occurs over an intermediate range of gaps, where the deposited fluid develops a reproducible symmetry inherited from the underlying substrate. For smaller gaps, the system forms highly ramified unstable invasion patterns, whereas for larger gaps the substrate influence weakens and the final footprint approaches a nearly circular puddle. Further work is needed to precisely determine the boundaries between these regimes in the phase space, in particular the range of gaps over which stable patterning can be achieved.

Finally, we explained the pattern-selection mechanism through a dynamical permeability analysis. The patterned plates create a spatially and temporally varying gap field $g(x, y, t)$, and therefore an anisotropic permeability field $k(x, y, t) = g(x, y, t)^2/12$. By radially averaging this field, we obtained angular permeability profiles $k(\phi, t)$ that reflect the symmetry of the substrate pattern. The directions of maximum angular permeability correlate with the directions of fastest inward air invasion, while the deposited liquid fingers remain preferentially in the lower-permeability angular sectors. This correspondence explains how substrate features with length scales of order $500\text{ }\mu\text{m}$ can influence the final deposition morphology at millimetric scales. The same framework also explains how misalignment between the top and bottom patterned plates modifies the symmetry of the deposited footprint.

Overall, these results show that substrate patterning can be used to guide the deposition morphology of yield stress fluids without strongly altering the integrated normal force response. This opens a route toward controlled deposition by designing the angular permeability landscape of the confining geometry. We hope these results inspire future work, both numerical simulations and experiments, to help identify the yielded and unyielded portions of the flow, potential slip lines, clarify the role of extensional stresses, and establish quantitative criteria for the onset of interfacial instabilities coupled to substrate topography.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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- [35] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/1jz3-5j4d> for videos showing the evolution of the fluid deposition patterns, the force–gap behavior, and the dynamics of the permeability field.