

Bifurcation-based mechanical sensing of microchannels using microrobotsAndrew Bickerdike *Exeter Small-Scale Robotics Laboratory, Engineering Department, [University of Exeter](#), Exeter EX4 4QF, United Kingdom*Joseph Páez Chávez *Center for Applied Dynamical Systems and Computational Methods (CADSCOM), Faculty of Natural Sciences and Mathematics, [Escuela Superior Politécnica del Litoral](#), P.O. Box 09-01-5863, Guayaquil, Ecuador
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We present a minimally invasive method for probing the mechanical properties of soft-walled microchannels by analyzing the bifurcations in the periodic motion of a magnetically actuated microrobot. Under oscillatory magnetic excitation, the microrobot undergoes transitions, including grazing and fold bifurcations, during wall interactions that are highly sensitive to local stiffness and damping. Experiments and numerical continuation reveal consistent bifurcation shifts across channels of varying compliance, enabling mechanical characterization through frequency-dependent response. A piecewise-smooth dynamical model incorporating fluid damping and compliant boundaries captures the observed behaviors. This bifurcation-based approach leverages dynamic signatures to provide a sensitive method for mechanical sensing in simplified confined environments. It lays the groundwork for future extensions towards applications in tissue diagnostics, vascular health monitoring, and soft material characterization.

DOI: [10.1103/PhysRevE.112.045405](https://doi.org/10.1103/PhysRevE.112.045405)**I. INTRODUCTION**

The exploration and characterization of microscale channels, such as in biological tissues [1,2] or advanced synthetic materials [3,4], pose significant challenges due to their inaccessibility and the delicate nature of their environments. Conventional techniques, such as atomic force microscopy [5,6] and optical coherence tomography [7], often fall short in probing these confined spaces due to their invasive nature, limited resolution, or risk of structural damage. This limitation underscores a critical need for innovative approaches capable of minimally invasive interaction and measurement at the microscale.

Recent advancements in microrobotics offer promising solutions by deploying small-scale autonomous agents that can navigate and sense within these restrictive environments [8,9]. In particular, the use of microrobots to explore and detect mechanical properties of tiny channels presents a transformative approach to accessing and understanding spaces that

were previously beyond the reach of existing technologies. By leveraging principles from nonsmooth dynamical systems, those in which the governing equations exhibit abrupt changes due to impacts, friction, or switching behaviors, these microrobots can interact with the walls of such channels, enabling precise assessments of their structural integrity and functional characteristics [10]. These assessments are crucial for understanding the underlying mechanics and potential abnormalities within the channels.

A key challenge in microscale diagnostics is the ability to accurately characterize the mechanical properties of soft biological tissues [11], particularly in dynamic environments such as blood vessels [1,10]. Vascular elasticity and stiffness play essential roles in various physiological and pathological processes, influencing the progression of diseases such as atherosclerosis and cancer metastasis [12]. Traditional methods for assessing vascular mechanics, including ultrasound elastography [13] and optical coherence tomography [14], provide valuable insights but often lack the spatial resolution and accessibility needed for precise local measurements at the cellular and subcellular scale. Microrobots, on the other hand, offer an unprecedented opportunity to probe these properties directly, providing localized mechanical feedback in ways that were previously unattainable.

In this paper, we present a minimally invasive method for determining the mechanical properties of soft microchannel

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walls by examining the bifurcations in the periodic motion of a magnetically actuated microrobot. By analyzing transitions in the microrobot's dynamic behavior, specifically, stability changes during wall interactions, we establish a direct correlation with wall stiffness. Experimental observations, supported by numerical simulations, demonstrate the sensitivity and reliability of this approach. This methodology differs fundamentally from previous droplet-based sensing techniques used to probe soft biological tissues, such as those by Serwane *et al.* [15] and Pirnat *et al.* [16]. In those works, ferrofluid droplets or optical microcavities embedded in tissue were deformed statically by external magnetic or optical forces to infer mechanical stresses or stiffness. These methods rely on quasistatic equilibrium shapes and require imaging to extract deformation, limiting real-time responsiveness, and introducing complexity in calibration. In contrast, our method employs dynamic, impact-driven bifurcations (e.g., grazing and fold points) arising from the active periodic motion of a magnetically driven microrobot. This dynamic perspective enables high-sensitivity, frequency-resolved stiffness estimation without requiring precise equilibrium imaging, offering an alternative paradigm for mechanical characterization in confined soft environments. To the best of our knowledge, the use of bifurcation signatures as sensing markers in microrobotic systems for soft-channel diagnostics has not been previously reported. This work lays the foundation for using dynamic bifurcation analysis as a tool to detect mechanical variations in simplified, confined microscale environments. While our current validation is performed in symmetric and well-controlled channels, the underlying sensing mechanism can, in principle, be extended to irregular or more anatomically realistic geometries through appropriate calibration and modeling refinements. This offers a path toward future applications such as tissue stiffness mapping and early-stage disease detection [17,18]. Furthermore, our approach bridges nonlinear dynamics, microrobotics, and soft-matter sensing, illustrating how tools from dynamical systems theory can be applied to biomedical microenvironment and thus contributing to cross-disciplinary advances in physics and engineering.

II. EXPERIMENTAL SETUP AND MATHEMATICAL MODEL

Figure 1 illustrates the experimental setup used to capture the dynamic response of a magnetically actuated microrobot within a soft-walled microfluidic channel, along with a schematic of its equivalent dynamical model. The microrobot consists of a 1-mm diameter spherical N52 neodymium-iron-boron permanent magnet suspended in water inside an agar-based soft microchannel. Actuation was achieved using an electromagnetic system composed of Helmholtz-Maxwell coil pairs. The Helmholtz coils generated a uniform magnetic field that aligned the particle's magnetic moment $\mathbf{m}_i$ with the field, producing a torque $\mathbf{T} = \mathbf{m}_i \times \mathbf{B}$, where $\mathbf{B}$ is the applied magnetic field. To induce dynamic motion, a sinusoidal current was applied to the Maxwell coils, generating a time-varying magnetic field gradient $\nabla B \sin(\Omega t)$. This resulted in a magnetic force $F = \mathbf{m}_i \cdot \nabla B \sin(\Omega t)$, assuming the particle acts as a point dipole. The channel samples were fixed within a three-dimensional (3D)-printed enclosure, ensuring that the

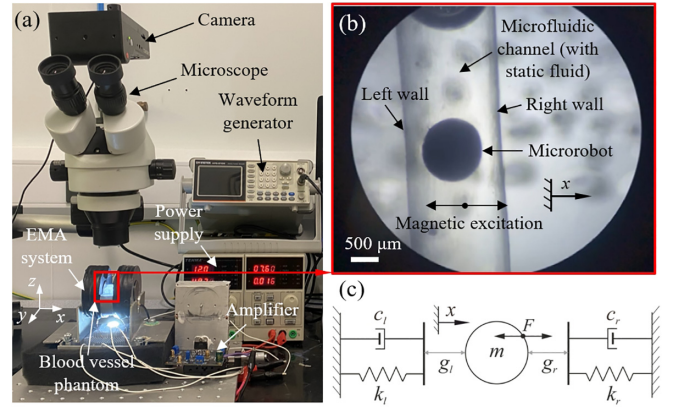


FIG. 1. (a) Experimental setup for capturing the dynamic response of a magnetically actuated microrobot within a soft-walled microfluidic channel. The system integrates an electromagnetic actuation (EMA) unit, comprising Helmholtz and Maxwell coils, into a microscope stage. Coil current is driven by a wave-form generator and amplifier, with power supplied by a dedicated unit. The microrobot is suspended in a static fluid medium inside an agar-based blood vessel phantom and is actuated by a sinusoidal magnetic force. Its motion is visualized through the microscope and recorded with a high-speed camera. (b) Microscopic view of the microrobot interacting with the compliant left and right walls of the microchannel under magnetic excitation. (c) Schematic of the equivalent dynamical model, where the microrobot is treated as a mass m subject to magnetic forcing F . The compliant walls are modeled as spring-damper systems with stiffness k_ℓ, k_r and damping coefficients c_ℓ, c_r . Gaps g_ℓ and g_r denote the initial distances between the microrobot and the left and right walls, respectively.

channel's central axis was precisely aligned with the center of the electromagnetic system and oriented perpendicular to the applied fields. Owing to the cylindrical geometry of the channel and the spherical shape of the microrobot, the particle naturally settled at the bottom center of the channel when at rest, prior to excitation. As shown in Fig. 1(c), the channel walls were modeled as linear spring-damper systems with fixed stiffness and damping coefficients, while fluid damping due to water viscosity was modeled as a constant viscous term. Frequency sweeps from 20–95 Hz in 1 Hz increments were conducted, allowing 0.5 s of stabilization time at each frequency before recording. Both increasing and decreasing sweeps were performed to identify bifurcation phenomena in the microrobot's periodic motion. A Chronos 2.1 high-speed camera recorded the particle's trajectory, and post-processing was used to extract displacement, velocity, and acceleration data.

Three channel types were tested, each with different agar concentrations and stiffness: Material I (0.8% wt), Material II (1.2% wt), and Material III (1.6% wt). Channels were fabricated by casting agar into 3D printed moulds [19]. To identify the mechanical properties of the channel walls, we employed a genetic algorithm [20] to fit our dynamic model to experimental displacement time series. The model, implemented in MATLAB, simulated the microrobot motion using candidate stiffness (k_ℓ, k_r) and damping (c_ℓ, c_r) parameters. These were optimized by minimizing the mean squared error between simulated and experimental trajectories. Long-duration simulations ensured steady-state behavior, and particular attention

TABLE I. Model parameters and vessel wall properties identified from experiments and applied in simulations.

Parameters	Values	Units
Microrobot mass (m)	3.9×10^{-6}	kg
Wall clearance ($g_r = g_\ell$)	0.25×10^{-3}	m
Fluid damping coefficient (c)	1.0×10^{-3}	Ns/m
Material I wall stiffness ($k_r^1 = k_\ell^1$)	4.5	N/m
Material I wall damping ($c_r^1 = c_\ell^1$)	4.0×10^{-4}	Ns/m
Material II wall stiffness ($k_r^2 = k_\ell^2$)	18.44	N/m
Material II wall damping ($c_r^2 = c_\ell^2$)	1.44×10^{-3}	Ns/m
Material III wall stiffness ($k_r^3 = k_\ell^3$)	32.2	N/m
Material III wall damping ($c_r^3 = c_\ell^3$)	2.32×10^{-3}	Ns/m
Forcing magnitude ($\mathbf{m}_i \cdot \nabla B$)	2.0×10^{-4}	N
Forcing frequency (Ω)	20–95	Hz

was paid to reproducing wall impact dynamics. The resulting optimal parameters are listed in Table I and were subsequently used in bifurcation analysis.

Next, we consider the dynamics of the magnetically actuated microrobot subjected to the harmonic force F , constrained to translate along the x axis within the soft-walled microchannel. The particle experiences three distinct motion states, depending on its position relative to the channel walls.

(i) Contact with the right wall:

$$m\ddot{x} + (c + c_r)\dot{x} + k_r(x - g_r) = \mathbf{m}_i \cdot \nabla B \sin(\Omega t), \quad \text{for } x > g_r, \quad (1)$$

(ii) No contact with the walls:

$$m\ddot{x} + c\dot{x} = \mathbf{m}_i \cdot \nabla B \sin(\Omega t), \quad \text{for } -g_\ell \leq x \leq g_r, \quad (2)$$

(iii) Contact with the left wall:

$$m\ddot{x} + (c + c_\ell)\dot{x} + k_\ell(x + g_\ell) = \mathbf{m}_i \cdot \nabla B \sin(\Omega t), \quad \text{for } x < -g_\ell. \quad (3)$$

$$z' = \begin{pmatrix} a \sin(\omega\tau) - 2\xi\tilde{v} - H_{k_\ell}(\tilde{x} + e_\ell + 2\xi_\ell\tilde{v}) - H_{k_r}(\beta_k(\tilde{x} - e_r) + 2\beta_c\xi_\ell\tilde{v}) \\ \tilde{v} \end{pmatrix} = f_{\text{ROB}}(z, \lambda, H_{k_r}, H_{k_\ell}). \quad (5)$$

In the equation above, the prime symbol (') represents the derivative with respect to the nondimensional time τ and the flags H_{k_r} , H_{k_ℓ} stand for discrete variables governing the operation mode of the system, according to

$$H_{k_r} = \begin{cases} 1, & \tilde{x} - e_r \geq 0 \text{ and } \beta_k(\tilde{x} - e_r) + 2\beta_c\xi_\ell\tilde{v} \geq 0, \text{ (contact with } k_r), \\ 0, & \text{otherwise, (no contact),} \end{cases} \quad (6)$$

$$H_{k_\ell} = \begin{cases} 1, & \tilde{x} + e_\ell \leq 0 \text{ and } \tilde{x} + e_\ell + 2\xi_\ell\tilde{v} \leq 0, \text{ (contact with } k_\ell), \\ 0, & \text{otherwise, (no contact),} \end{cases} \quad (7)$$

For the path-following analysis using the continuation platform COCO, the discrete variables H_{k_r} , H_{k_ℓ} will be employed to distinguish the sequence of operation modes along a periodic trajectory of the microrobot-vessel model (5). For this purpose, each feasible pair (H_{k_r} , H_{k_ℓ}) will be associated to a physical state of the microrobot-vessel: *No contact* ($H_{k_r} = 0$, $H_{k_\ell} = 0$), *Contact with k_r* ($H_{k_r} = 1$, $H_{k_\ell} = 0$) and *Contact with k_ℓ* ($H_{k_r} = 0$, $H_{k_\ell} = 1$).

To analyze the periodic responses of this piecewise-smooth system under harmonic excitation, we employed the numerical continuation software COCO [21]. This tool allows tracking of periodic solutions and detection of bifurcations as control or system parameters vary. Our primary aim is to investigate how variations in stiffness, damping, and excitation parameters influence the microrobot's dynamics and lead to changes in the qualitative behavior of its motion. For numerical analysis, we define the following dimensionless variables and parameters:

$$\begin{aligned} \Omega_0 &= \sqrt{\frac{k_\ell}{m}}, \quad \tau = \Omega_0 t, \quad \omega = \frac{\Omega}{\Omega_0}, \\ \xi &= \frac{c}{2m\Omega_0}, \quad \xi_r = \frac{c_r}{2m\Omega_0}, \quad \xi_\ell = \frac{c_\ell}{2m\Omega_0}, \\ \beta_c &= \frac{\xi_r}{\xi_\ell}, \quad \beta_k = \frac{k_r}{k_\ell}, \quad a = \frac{\mathbf{m}_i \cdot \nabla B}{\ell_0 k_\ell}, \\ \tilde{x} &= \frac{x}{\ell_0}, \quad e_r = \frac{g_r}{\ell_0}, \quad e_\ell = \frac{g_\ell}{\ell_0}, \end{aligned} \quad (4)$$

where ℓ_0 is an arbitrary reference length. In our investigation, the vectors $z = (\tilde{x}, \tilde{v})^T \in \mathbb{R}^2$ and $\lambda = (\omega, a, \beta_k, \beta_c, \xi, \xi_\ell, e_r, e_\ell) \in \mathbb{R}^8$ will denote the state variables and parameters of the system, respectively, with $\tilde{v}$ being the nondimensional velocity of the oscillating mass. The original velocity v (with dimensions) can be calculated via $v = \ell_0 \Omega_0 \tilde{v}$. Although the nondimensional form of the equations of motion of the microrobot-vessel will be employed in our numerical implementation, the numerical results will be presented using the original physical dimensions, in order to achieve a better understanding of the practical implications.

With the rescaling introduced above, we can now formulate the mathematical model of the microrobot-vessel (1)–(3) in the framework of piecewise-smooth dynamical systems as

III. RESULTS AND ANALYSIS

Next, let us consider an initial periodic solution computed for the excitation frequency $\Omega/2\pi = 82$ [Hz]. At this point, a microrobot-vessel presents a stable oscillatory motion whose amplitude is not high enough to touch either of the secondary springs k_r or k_ℓ , see Fig. 2(d) (solution at the test point P1). When the excitation frequency decreases, the oscillation

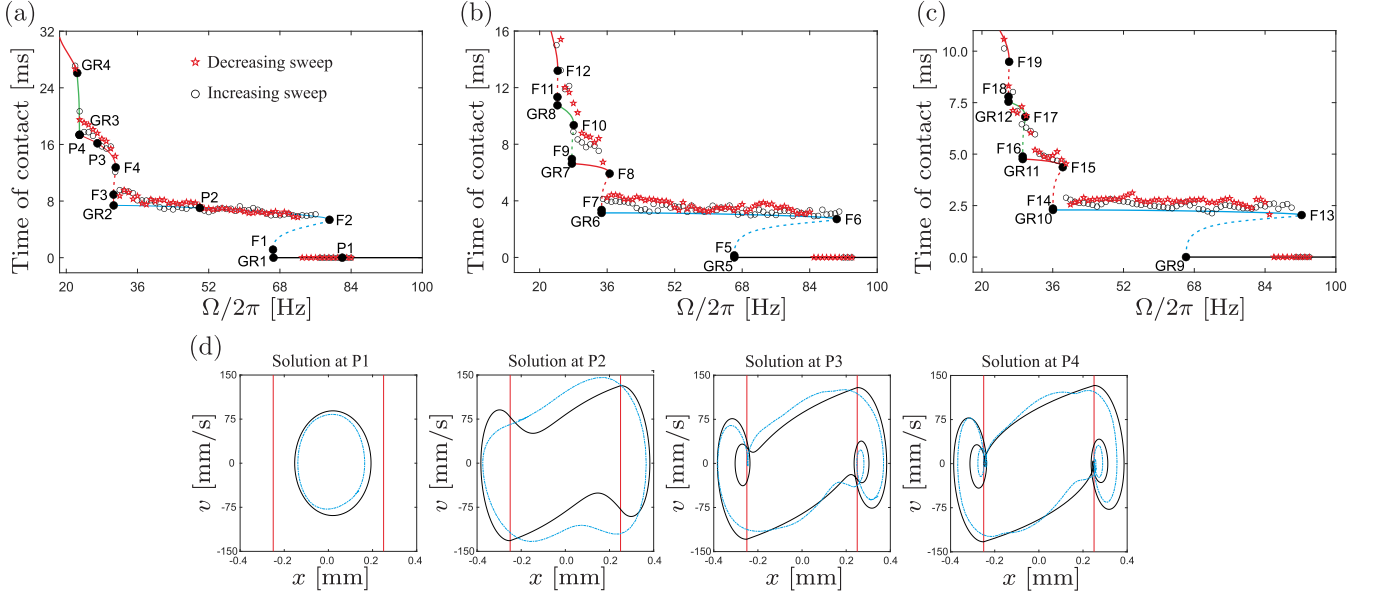


FIG. 2. Numerical continuation of the periodic response with respect to excitation frequency, using parameters from Table I: Material I (a), Material II (b), and Material III (c). The vertical axis represents contact duration with the left and right walls per period. Solid and dashed lines indicate stable and unstable periodic solutions, respectively. Labels F_i and GR_i correspond to fold and grazing bifurcations (listed for Material I only): F1 (66.4476 Hz), F2 (79.1289 Hz), F3 (30.6223 Hz), F4 (31.1041 Hz); GR1 (66.5550 Hz), GR2 (30.6453 Hz), GR3 (22.9628 Hz), GR4 (22.4637 Hz). P1–P4 denote test points at 82, 50, 27, and 23 Hz. Stars and circles indicate decreasing and increasing experimental sweep directions. (d) Periodic solutions at these test points. Red vertical lines denote impact boundaries at $x = g_r$ and $x = -g_l$; black and blue curves show numerical and experimental results, respectively. (A video clip showing experimental observations corresponding to points P1–P4 is provided in the Supplemental Material [22].)

amplitude gradually grows, until the periodic solution hits tangentially the impact boundaries $x = g_r$ and $x = -g_l$ simultaneously due to symmetry ($g_r = g_l$). This event is detected as a grazing bifurcation at $\Omega/2\pi \approx 66.5550$ [Hz], labeled GR1, in Fig. 2(a). Right after, a fold bifurcation of limit cycles (F1) is found for $\Omega/2\pi \approx 66.4476$ [Hz]. At this point, the periodic orbit loses stability, giving rise to a branch of unstable solutions [dashed blue curve in Fig. 2(a)] that finishes at another fold bifurcation (F2, $\Omega/2\pi \approx 79.1289$ [Hz]), where the periodic solution recovers stability. The solid blue branch shown in Fig. 2(a) denotes the resulting family of stable periodic orbits, showing two impacts per orbital period, one with the left secondary spring k_l and one with the right secondary spring k_r . The solution type described above changes qualitatively when the frequency decreases below the critical value $\Omega/2\pi \approx 30.6453$ [Hz], where a grazing bifurcation GR2 is detected. Due to the system symmetry mentioned earlier, a family of periodic solutions with four impacts per orbital period is born at GR2 [solid red branch in Fig. 2(a)], with two impact events at each secondary spring. After this, a similar pattern as before of fold bifurcations is found along the solid red branch until another grazing bifurcation is detected at GR3 ($\Omega/2\pi \approx 22.9628$ [Hz]). Here, a branch of solutions with six impacts per orbital period is created (solid green curve), which finishes at another grazing bifurcation labeled GR4 ($\Omega/2\pi \approx 22.4637$ [Hz]). Here, the solution type is the same as the one found along the solid red branch described above. The bifurcation diagram presented here also includes small stars and circles standing for decreasing and increasing frequency sweeps taken experimentally, which fit remarkably well with the theoretical model. The periodic re-

sponses calculated in Fig. 2(d) display a comparison between the numerical simulations (black lines) and experimental observations (blue lines), showing a good qualitative agreement.

Table II compares the key bifurcation frequencies observed in both simulations and experiments across microchannel materials with increasing stiffness and damping, as illustrated in the bifurcation diagrams of Fig. 2. A clear trend emerges: as the wall material becomes stiffer and more heavily damped (from Material I to III), the onset frequencies of major bifurcations consistently shift to higher values. This pattern holds for most cases and is consistent between simulation and experimental data, except for slight discrepancies in the grazing bifurcations GR5 and GR9 in simulation, and fold bifurcations F6 and F13 in experiment. These variations may arise from fabrication tolerances in the agar channels, computational deviations, or experimental measurement errors. Importantly, the consistent shift in bifurcation frequencies across materials demonstrates that such transitions can serve as sensitive indicators of microchannel mechanical properties. As a result, analyzing the frequencies of these grazing and fold bifurcations offers a practical method for estimating the local stiffness and damping characteristics of confined soft-walled environments.

In summary, Fig. 2 presents a detailed bifurcation picture showing qualitative changes in the periodic response of the microrobot-vessel system (5), when the frequency of periodic external excitation varies. The next step in our numerical investigation is to understand how some of the encountered bifurcations are affected when a second system parameter is allowed to change and what transitions can be observed in the system dynamics upon such parameter variations. This

TABLE II. Comparison of selected bifurcation frequencies [Hz] across Materials I–III.

Bifurcation	Material I	Material II	Material III
Simulation			
Grazing (No impact $\rightarrow$ two impacts)	66.5550 (GR1)	66.1281 (GR5)	66.1281 (GR9)
Fold (Two impacts $\rightarrow$ no impact)	79.1289 (F2)	90.4108 (F6)	92.2628 (F13)
Grazing (Two impacts $\rightarrow$ four impacts)	30.6453 (GR2)	34.6810 (GR6)	36.0504 (GR10)
Fold (Four impacts $\rightarrow$ two impacts)	31.1041 (F4)	36.5796 (F8)	38.2683 (F15)
Grazing (Four impacts $\rightarrow$ six impacts)	22.9628 (GR3)	27.6437 (GR7)	29.2132 (GR11)
Grazing (Six impacts $\rightarrow$ eight impacts)	22.4637 (GR4)	24.2201 (GR8)	26.0002 (GR12)
Experiment			
Grazing (No impact $\rightarrow$ two impacts)	72 (GR1)	84 (GR5)	86 (GR9)
Fold (Two impacts $\rightarrow$ no impact)	77 (F2)	91 (F6)	90 (F13)
Grazing (Two impacts $\rightarrow$ four impacts)	32 (GR2)	33 (GR6)	40 (GR10)
Fold (Four impacts $\rightarrow$ two impacts)	33 (F4)	35 (F8)	39 (F15)
Grazing (Four impacts $\rightarrow$ six impacts)	24 (GR3)	29 (GR7)	33 (GR11)
Grazing (Six impacts $\rightarrow$ eight impacts)	22 (GR4)	27 (GR8)	28 (GR12)

study is presented in Fig. 3. Here, we have carried out a two-parameter continuation with respect to excitation frequency and damping ratio β_c of the bifurcation points F4 (red curve), GR2 (black curve) and F2 (blue curve) identified in Fig. 2(a). Figure 3(a) displays the so obtained curves in the two-parameter space, while Fig. 3(b) presents system solutions computed for selected test points (P1–P6) as indicated in Fig. 3(a). These simulations serve the purpose of illustrating the sort of transitions in the system response observed when the calculated critical curves are crossed. For instance, an operation point located to the left of the red curve produces period-1 solutions with four impacts with the secondary springs per period. If the parameter values are chosen to cross the red curve, then the solution loses one impact with the left spring k_l . Here, it is important to note that the solutions are not symmetric anymore as, for instance, observed in Fig. 2(b), because now the damping ratio β_c is allowed to be different than 1, i.e., the left and right impacting surfaces have different physical properties, hence inducing asymmetric responses. Another case can be observed when crossing the black curve corresponding to the two-parameter continuation of the grazing bifurcation GR2 encountered earlier. To the right of this curve, we have the test point P4, where a period-1 solution with two impacts per period is found. If this black curve is crossed precisely from the right (test point P3) we obtain a periodic response with three impacts per period, of the same type detected at P2. A third transition is observed when crossing the blue curve in Fig. 3, illustrated by the test points P5 and P6. From P5 we obtain a period-1 orbit with two impacts per period, and after a small change of parameters from P5 to P6, this solution loses stability, and the system exhibits oscillations without wall contact. Here, it is important to note that such transition is direction-dependent due to the presence of hysteresis phenomena produced by the interaction of fold bifurcations, as can be observed in Fig. 2 (fold points F1 and F2). Among these three bifurcation curves, GR2 (black curve) exhibits a pronounced horizontal spread across the parameter plane, indicating that the grazing bifurcation is highly sensitive to changes in the damping asymmetry between the left and right walls. In contrast, the fold bifurcations F4 (red curve) and F2 (blue curve) appear as relatively steep curves,

suggesting a weaker dependence on the damping ratio β_c and thus reduced sensitivity for detecting asymmetry. This implies that GR2 is particularly well-suited for estimating or distinguishing differences in wall damping, as small variations in β_c lead to significant shifts in the excitation frequency at which the grazing transition occurs.

Further numerical investigations are presented in Fig. 4, where two-parameter continuation analyses are used to explore how the selected bifurcations evolve under variations in system asymmetry and microrobot confinement. Figure 4(a) examines the influence of stiffness asymmetry between the left and right channel walls, quantified by the stiffness ratio β_k . As β_k increases, indicating a stiffer right wall, the bifurcation curves shift to higher frequencies, reflecting the dynamic sensitivity to wall stiffness imbalance. However, as the stiffness imbalance becomes more pronounced, i.e., as β_k increases beyond a certain threshold, all three bifurcation curves (F4, GR2, and F2) become noticeably steeper in the frequency–stiffness-ratio plane. This steepening indicates a reduction in the frequency response to incremental changes in β_k , thereby diminishing the system’s sensitivity to further increases in stiffness asymmetry. In other words, while moderate changes in wall stiffness can be effectively detected through shifts in bifurcation frequency, the system becomes less responsive to larger stiffness disparities, limiting its resolution for extreme asymmetry. Figure 4(b) shows the effect of varying the right wall clearance g_r while keeping the overall channel diameter constant, thus adjusting g_l accordingly. From a practical perspective, such asymmetric shifts may result from physiological influences, such as blood flow or anatomical irregularities, which can displace the microrobot from the central axis of the channel. These variations affect how early or late the microrobot interacts with the walls, thereby shifting the grazing and fold bifurcation loci in a systematic and interpretable manner. As shown in this panel, the F4 (red) and GR2 (black) curves lie very close to one another across the range of g_r , indicating that the fold and grazing bifurcations occur in close succession under these conditions. This proximity suggests that even small variations in wall clearance can trigger rapid transitions between dynamic regimes. Additionally, as g_r increases, effectively shifting the microrobot further from

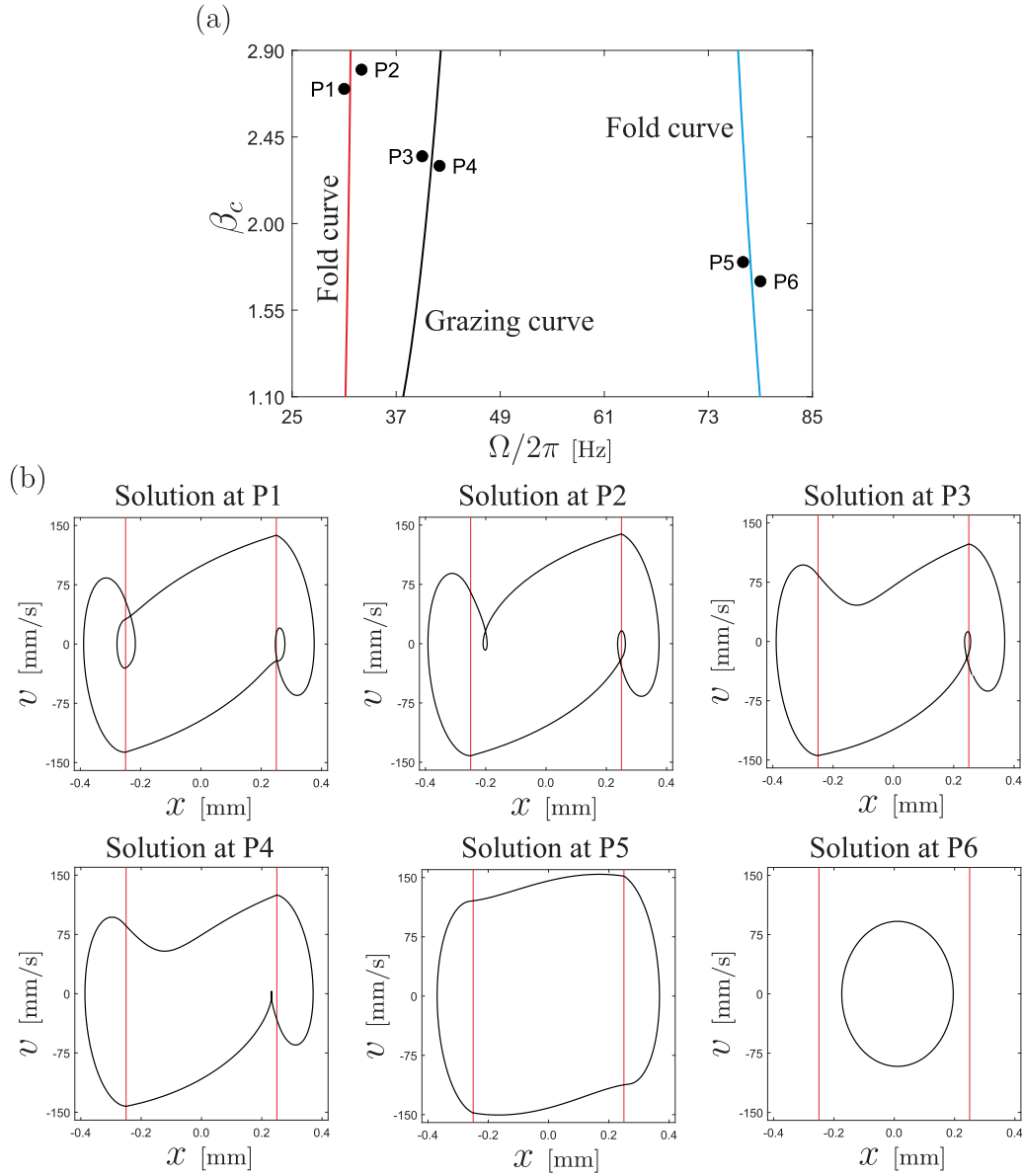


FIG. 3. (a) Two-parameter continuation of bifurcations F4 (red), GR2 (black), and F2 (blue) in the $(\Omega/2\pi, \beta_c)$ plane. (b) Periodic responses at test points: P1 (31 Hz, 2.7), P2 (33 Hz, 2.8), P3 (40 Hz, 2.35), P4 (42 Hz, 2.3), P5 (77 Hz, 1.8), and P6 (79 Hz, 1.7).

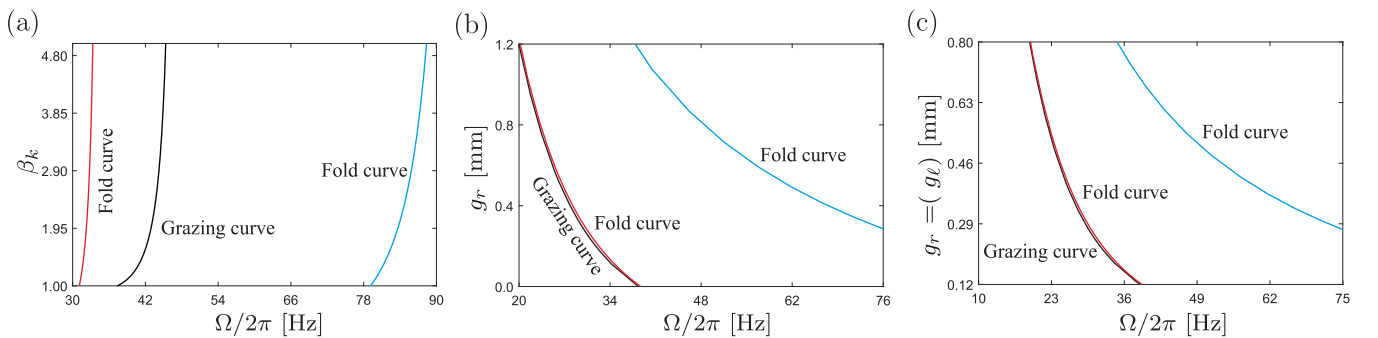


FIG. 4. Two-parameter continuation of bifurcations F4 (red), GR2 (black), and F2 (blue) with respect to (a) stiffness ratio between right and left walls (β_k), (b) variation of right wall clearance g_r with adjusted g_ℓ to maintain fixed channel diameter, and (c) symmetric variation of wall clearances ($g_r = g_\ell$), representing overall microrobot-to-wall proximity.

the right wall, the bifurcation frequencies for F4, GR2, and F2 all decrease. This trend reflects the fact that greater clearances delay wall interactions, thereby requiring lower excitation frequencies to reach the conditions necessary for bifurcation onset. Figure 4(c) considers symmetric variations in both wall clearances ($g_r = g_\ell$), which control the microrobot-to-wall proximity. As the confinement tightens, bifurcation frequencies increase, highlighting the role of spatial constraints in the onset of dynamic transitions. Conversely, as both wall clearances increase, effectively widening the microchannel relative to the microrobot's size, all three bifurcation frequencies (F4, GR2, and F2) decrease. This occurs because reduced confinement lessens the frequency and intensity of microrobot-wall interactions, thereby delaying the onset of key dynamic transitions. These results emphasize that tuning the microrobot diameter relative to the channel size provides an effective means to control the sensitivity and timing of bifurcations in such microrobotic systems.

IV. CONCLUSION

In this work, we conducted a comprehensive experimental and numerical study of a magnetically actuated microrobot vibrating within a soft-walled microfluidic channel. By applying controlled oscillatory magnetic fields, we induced periodic interactions between the microrobot and the compliant channel walls, enabling minimally invasive probing of the vessel's mechanical properties. The system was modeled using a piecewise-smooth dynamical framework incorporating spring-dashpot boundary conditions and fluid damping. Frequency sweep experiments, combined with high-speed imaging, revealed a rich set of dynamic behaviors, including transitions from noncontact to single- and dual-wall impacts. These transitions manifested as bifurcation phenomena, particularly grazing and fold bifurcations, which were systematically characterized using numerical continuation with the software package COCO [21].

To investigate the sensitivity of these bifurcations to system parameters, we performed two-parameter continuation analyses. We demonstrated that increasing wall stiffness and damping causes bifurcation frequencies to shift higher, a trend confirmed across three agar-based microchannel materials. Notably, we found that grazing bifurcations were more sensitive to damping asymmetry than fold bifurcations, suggesting their greater utility in detecting subtle differences in mechanical properties between opposing channel walls. We also showed that the sensitivity of bifurcation frequencies to stiffness asymmetry diminishes when the asymmetry becomes extreme, limiting the resolution for characterizing highly unbalanced systems.

Further analyses revealed that wall clearances play a critical role in shaping bifurcation behavior. Variations in right wall clearance, with a fixed channel diameter, influenced the timing and proximity of microrobot-wall interactions, while symmetric changes in wall gaps (i.e., confinement) strongly affected the onset frequency of bifurcations. In particular, increasing the microrobot-to-wall clearance lowered all key bifurcation frequencies, suggesting that tuning the microrobot diameter relative to the channel size provides a practical means of controlling dynamic sensitivity. Importantly, this also indicates that the bifurcation-based sensing approach remains robust even when perfect microrobot alignment within the channel is not guaranteed. Frequency shifts arising from such asymmetries can still be interpreted within the known bifurcation landscape, enabling reliable inference of local wall stiffness and damping despite potential positional deviations.

Overall, this study establishes bifurcation frequency analysis as a robust strategy for mechanical sensing in confined soft-walled environments. The proposed methodology offers high sensitivity to local variations in wall stiffness and damping, and is adaptable through geometric and actuation parameters. We anticipate that this bifurcation-based approach, while currently validated in simplified geometries, may be extended to a wider range of microfluidic and biomedical systems. However, several challenges must be addressed before practical deployment in complex biological settings. These include the rate-dependent viscoelastic behavior of soft tissues, sensitivity to microrobot-to-wall alignment in irregular geometries, and variability in tissue properties. Future work should investigate how these factors affect bifurcation loci and explore calibration strategies for robust operation *in vivo*. With further refinement to accommodate anatomical variability and dynamic tissue properties, this approach could contribute to emerging tools for tissue stiffness mapping, early disease detection, or mechanical characterization in evolving soft environments. These findings also underscore the broader potential of nonlinear dynamics and bifurcation theory to contribute to the development of next-generation diagnostic and sensing tools at the interface of physics, engineering, and biomedicine.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

- [1] D. B. Camasão and D. Mantovani, The mechanical characterization of blood vessels and their substitutes in the continuous quest for physiological-relevant performances. A critical review, *Mater. Today Bio.* **10**, 100106 (2021).
- [2] E. Song, Y. Huang, N. Huang *et al.*, Recent advances in microsystem approaches for mechanical characterization

of soft biological tissues, *Microsyst. Nanoeng.* **8**, 77 (2022).

- [3] S. Biswas, K. Mandal, D. Pramanik, N. Roy, R. Biswas, and A. Kuar, Prediction of quality characteristics of micro-channels by deep learning algorithm during laser transmission ablation of PMMA, *Mater. Today Commun.* **38**, 108142 (2024).

- [4] R. Takahashi, A. Tanaka, T. Saito, S. Ohashi, M. Muto, and M. Yamaguchi, Organ-on-a-chip platforms created through buckled microchannels of porous hydrogel films, *Adv. Mater. Technol.* **10**, 2401468 (2025).
- [5] S. Fatayer, F. Albrecht, I. Tavernelli, M. Persson, N. Moll, and L. Gross, Probing molecular excited states by atomic force microscopy, *Phys. Rev. Lett.* **126**, 176801 (2021).
- [6] F. Savvopoulos, M. C. Keeling, D. Carassiti, N. A. Fogell, M. B. Patel, J. Naser, N. Gavara, R. de Silva, and R. Krams, Assessment of the nano-mechanical properties of healthy and atherosclerotic coronary arteries by atomic force microscopy, *J. R. Soc. Interface* **21**, 20230674 (2024).
- [7] H. Xu, F. Xu, T. Theurer, D. Egloff, Z.-W. Liu, N. Yu, M. B. Plenio, and L. Zhang, Experimental quantification of coherence of a tunable quantum detector, *Phys. Rev. Lett.* **125**, 060404 (2020).
- [8] L. Yang, J. Yu, S. Yang, B. Wang, B. J. Nelson, and L. Zhang, A survey on swarm microrobotics, *IEEE Trans. Robot.* **38**, 1531 (2022).
- [9] V. Iacovacci, E. Diller, D. Ahmed, and A. Menciassi, Medical microrobots, *Annu. Rev. Biomed. Eng.* **26**, 561 (2024).
- [10] G. Wang, A. Bickerdike, Y. Liu, and A. Ferreira, Analytical solution of a microrobot-blood vessel interaction model, *Nonlinear Dyn.* **113**, 2091 (2025).
- [11] L. Rosalia, A. Hallou, L. Cochrane, and T. Savin, A magnetically actuated, optically sensed tensile testing method for mechanical characterization of soft biological tissues, *Sci. Adv.* **9**, eade2522 (2023).
- [12] J. Tian, K. O. Afebu, A. Bickerdike, Y. Liu, S. Prasad, and B. J. Nelson, Fundamentals of bowel cancer for biomedical engineers, *Ann. Biomed. Eng.* **51**, 679 (2023).
- [13] B. Mahmood, C. Ewertsen, J. Carlsen, and M. B. Nielsen, Ultrasound vascular elastography as a tool for assessing atherosclerotic plaques—a systematic literature review, *Ultrasound. Int. Open* **02**, E106 (2016).
- [14] J. Rogowska, N. A. Patel, J. G. Fujimoto, and M. E. Brezinski, Optical coherence tomographic elastography technique for measuring deformation and strain of atherosclerotic tissues, *Heart* **90**, 556 (2004).
- [15] F. Serwane, A. Mongera, P. Rowghanian, D. A. Kealhofer, A. A. Lucio, Z. M. Hockenbery, and O. Campàs, *In vivo* quantification of spatially varying mechanical properties in developing tissues, *Nat. Methods* **14**, 181 (2017).
- [16] M. Pirnat, S. Trigueros, R. Farzan, A. Mukhortava, P. Aymanns, P. Horvath, J. Schäfer, D. Brueckner, and O. Campàs, Quantifying local stiffness and forces in soft biological tissues using droplet optical microcavities, *Proc. Natl. Acad. Sci. USA* **121**, e2314884121 (2024).
- [17] Z. Hu, J. Ding, Z. Ma *et al.*, Quantitative evidence for early metastatic seeding in colorectal cancer, *Nat. Genet.* **51**, 1113 (2019).
- [18] J. M. Alves, S. Prado-López, J. M. Cameselle-Teijeiro, and D. Posada, Rapid evolution and biogeographic spread in a colorectal cancer, *Nat. Commun.* **10**, 5139 (2019).
- [19] V. Saggiomo and A. H. Velders, Simple 3D printed scaffold-removal method for the fabrication of intricate microfluidic devices, *Adv. Sci.* **2**, 1500125 (2015).
- [20] B. Tian, S. Yin, J. Páez Chávez, and Y. Liu, An optimization approach to establish dynamical equivalence for soft and rigid impact models, *Chaos* **34**, 073106 (2024).
- [21] H. Dankowicz and F. Schilder, *Recipes for Continuation* (SIAM, Philadelphia, 2013).
- [22] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/zrcy-c36h> for a video clip showing experimental observations.