

Permeability of deforming soft particle packs

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The permeability of the fluid between packs of particles is an important parameter for a range of natural and industrial applications. Here, we start from experiments in which soft spherical particles—“soft balls”—are compressed uniaxially, resulting in particle-particle flattening and an overall decrease in interstitial volume fraction ϕ down to very low values. During deformation, the soft particle packs were scanned in three dimensions using x-ray tomography; we segment these data to isolate the fraction between the particles. We then run lattice Boltzmann fluid flow simulations through the domains to determine the permeability. Additionally, we determine the specific surface area s and ϕ . Together, we use these parameters to test and validate a semiempirical extension to a theoretical percolation model, to propose a framework for predicting permeability in deforming soft particle systems. Once validated, we compare this model to the permeability of the fluid in packs of hard particles and during particle sintering; we conclude that this model is applicable to a wide range of initially particulate systems with all the geometric information encoded in s .

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I. INTRODUCTION

Compression of granular matter typically increases the assembly density up to a jammed state [1–5]. The density at which granular matter jams depends on the friction between the particles [6], the particle size and shape [6–8], and the loading history [2]. Crucially, if the particles are rigid, the packing density cannot exceed a maximum at which the particles have rearranged to an optimal configuration [3,9], often referred to as random close packing (cf. Ref. [4]). By contrast, if the particles break or are soft and able to deform against one another, far higher densities can be achieved [10–12] beyond random close packing.

The dynamics of compressed soft particle systems are relevant to understanding the jamming of cells (blood cells in clots, in particular [13]) and in many industrial processes including viscous sintering under pressure where glass particles deform against one another [14–16], with applications to nuclear waste vitrification [17]. Similarly, in volcanic systems, pyroclasts—the product of explosive fragmentation [18]—can expand because bubbles nucleate and grow internally; this pushes and flattens the pyroclasts against one another [19] in a process very similar to soft ball compression. In all applied cases, the transport of fluid between the compressing or expanding particles is an important process.

The fluid permeability of packs of jammed particles is a measure of the efficiency with which fluid can percolate

between the grains [20–24]. Constraining this permeability is crucial to understanding the role that packed particles play as filters [25,26] or for understanding the separated flow of fluid as the particles jam [27,28]. While the permeability of packs of hard spheres is relatively well understood [29–31], with explicit analytical models for idealized packings [32,33], the permeability of soft particle systems has not received attention.

There is a technical challenge associated with determining the permeability *during* compression of soft particles. Namely, the experimental determination of permeability requires that a pore pressure gradient is established between the particles. However, any pore pressure can also resist the continued compression of the particles by imparting an effective pressure [34]. To get around this, and to decouple the compression of the particles from the determination of permeability, we turn to published three-dimensional (3D) x-ray tomographic datasets of soft particle compression tests [11]. We use the 3D tomographs to initialize a fluid flow simulation, which, once converged to steady-state flow, can be used to obtain the permeability at each time step, without affecting the physics of the experiments themselves. In general, the permeability of packs of particles depends on the specific surface area of the pore space and the porosity of the interconnected pore space [35–37]. And so here, in addition to determining the permeability, we also track the porosity and the specific surface area. Our goal is to constrain a semiempirical but generally applicable model for the permeability of packs of soft particles during particle compression down to very low porosities.

From a more fundamental perspective, transport through a porous medium is ultimately controlled by the connectivity of the void phase. As a packing is compressed, deformation of the constituent particles progressively modifies the geometry and topology of the pore space, causing fluid pathways to

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narrow, disconnect, and eventually lose system-spanning connectivity. The resulting decline in permeability can therefore be viewed as a manifestation of an underlying percolation problem: the evolution and eventual disappearance of a connected pore network capable of supporting long-range fluid transport. Understanding how particle deformability influences this connectivity transition is important not only for predicting permeability, but also for establishing the geometric and statistical mechanisms by which transport ceases in compressed soft granular materials. Here we investigate how compression-induced deformation of soft particles alters both the connectivity of the pore space and the permeability of the resulting medium. By tracking the evolution of the spanning pore network alongside the hydraulic response, we connect a measurable transport property (i.e., permeability) to the underlying percolation transition occurring within the void phase.

A. Theoretical background

The flow of fluid between packs of particles can be modeled using Forchheimer's law [38]

$$\nabla p = -\frac{\mu}{k}q - \frac{\rho}{k_i}q^2, \quad (1)$$

where ∇p is the fluid pressure gradient driving flow, q is the flow rate of the fluid that results from the pressure gradient, μ and ρ are the fluid viscosity and density, respectively, and k and k_i are the Darcian and inertial permeability of the porous medium, respectively. Note that here we neglect tensor notation while acknowledging that ∇p , q , k , and k_i are all tensor quantities. Once k and k_i are known, then the flow of fluid through a porous medium can be modeled via Eq. (1).

The first and second terms on the right-hand side of Eq. (1) are the viscous and inertial contributions to flow, respectively. Darcy's law is a special case of Eq. (1) for where the inertial contribution can be neglected and is

$$\nabla p = -\frac{\mu}{k}q, \quad (2)$$

such that q is a linear function of ∇p . Here, we focus on Eq. (2) and models for k , while acknowledging that in order to use the more general theory [Eq. (1)], a model for k_i would be required, which can be readily found in the form of $k_i = f(k)$ [39,40].

B. Permeability $k(\phi)$

Both k and k_i depend on the porosity ϕ of the porous medium and the specific surface area of the pore space s . For example, a widely used constitutive law for k is the Kozeny-Carman law [31,41]

$$k = \frac{\phi^3}{Cs^2}, \quad (3)$$

where C is a dimensionless constant. Across a large range of porous media, $C = 5$ is favored [28,35]. Equation (3) performs especially well at intermediate porosities around $\phi \approx 0.5$ [28]. Because Eq. (3) predicts that $k \rightarrow 0$ only when $\phi \rightarrow 0$, it fails to capture low porosity data where $k \rightarrow 0$ is observed to occur at a finite porosity $\phi > 0$. Specifically, the

porosity at which $k \rightarrow 0$ is called the percolation threshold ϕ_c . To address this, we introduce a percolation model that has been validated down to low porosities across a wide range of media [28,36,37,42]

$$k = \frac{2[1 - (\phi - \phi_c)]}{s^2}(\phi - \phi_c)^b, \quad (4)$$

where b is a dimensionless constant. For granular materials, $b = 4.4$ is theoretically derived [43,44] and matches well to empirically determined values $4 \leq b \leq 4.4$ [36,37].

When considering packs of compressed soft balls, it is also useful to compare with models for idealized packs of hard spheres. These models all use the same reference permeability—termed the Stokes permeability because of its relationship with Stokes drag on single particles—which is

$$k_s = \frac{2R^2}{9(1 - \phi)}, \quad (5)$$

where R is the radius of the particles (assumed here to be spherical). Series expansions exist for spherical particles arranged on simple cubic (sc), body-centred cubic (bcc), and face-centred cubic (fcc) lattices [33]. Sangani and Acrivos [32] found higher-order terms in these expansions and generalized to

$$\frac{k}{k_s} = \left[\sum_{i=0}^{30} c_i \left(\frac{1 - \phi}{1 - \phi_m} \right)^{i/3} \right]^{-1}, \quad (6)$$

where c_i is the i th coefficient in the expansion and ϕ_m is the porosity at maximum packing. Equation (6) is valid down to the maximum packing of particles (i.e., $1 - \phi_m$) in these idealized packings, which are, for sc, $\pi/6 \approx 0.52$, for bcc, $\pi\sqrt{3}/8 \approx 0.68$, and for fcc, $\pi/(3\sqrt{2}) \approx 0.74$. The coefficient values are found tabulated elsewhere [45].

C. Specific surface area $s(\phi)$

The specific surface area is defined as the total surface area of the pore space per unit of volume. Therefore, for solid spheres of radius R packed in a volume at number density ρ_n , the specific surface area is [45–47] $s = \rho_n A$ with $A = 4\pi R^2$. The reduced density η is defined as $\eta = \rho_n V$ with $V = (4/3)\pi R^3$. Noting that for hard spherical particles $\eta = 1 - \phi$, the specific surface area is then

$$s = \frac{3(1 - \phi)}{R}. \quad (7)$$

For fully penetrable spheres, the specific surface area becomes [47]

$$s = -\frac{3\phi \ln(\phi)}{R}. \quad (8)$$

The specific surface area of soft ball assemblies may not follow hard-sphere models [e.g., Eq. (7)] or fully penetrable sphere models [e.g., Eq. (8)]. Instead, it is reasonable to assume that as the soft balls deform against one another, the geometry is akin to a partially penetrable sphere; such a case is referred to as a “concentric-shell model” [48] or a “cherry-pit model” [47]. In this case a control parameter λ is introduced, which can vary between 0 and 1 and where $\lambda = 1$ refers to

hard spheres [Eq. (7)], and $\lambda = 0$ refers to fully penetrable spheres [Eq. (8)]. The model is given by

$$s = \frac{3\phi\eta}{R}H(\eta, \lambda), \quad (9)$$

where $H(\eta, \lambda)$ is as follows:

$$H = 1 + \frac{\eta\lambda^3}{(1 - \eta\lambda^3)^3}(h_0 - h_1\eta\lambda^3 + h_2\eta^2\lambda^6), \quad (10)$$

with $h_0 = 4 - 3\lambda$, $h_1 = 2 + 3\lambda - 3\lambda^2$, and $h_2 = 1$. The reduced density η can be calculated by solving the following equation for a given input ϕ ,

$$\phi = (1 - \eta\lambda^3) \exp[-\eta(1 - \lambda^3)]F(\eta, \lambda), \quad (11)$$

where $F(\eta, \lambda)$ is as follows:

$$F = \exp\left[-\frac{\eta^2\lambda^3}{2(1 - \eta\lambda^3)^3}(f_0 - f_1\eta\lambda^3 + f_2\eta^2\lambda^6)\right], \quad (12)$$

with $f_0 = 8 - 9\lambda + \lambda^3$, $f_1 = 4 + 9\lambda - 18\lambda^2 + 5\lambda^3$, and $f_2 = 2(1 - \lambda^3)$. This system [Eqs. (9)–(12)] can be solved for a given degree of penetrability to provide $s(\phi)$ and will be referred to as the “cherry-pit model” here. Note that for $\lambda = 1$ and $\lambda = 0$ Eqs. (9)–(12) reduce to Eqs. (7) and (8), respectively.

To account for systems of different sphere sizes, we note that R [in Eqs. (5), (7), (8), and (9)] can be replaced by the ratio $\langle R^3 \rangle / \langle R^2 \rangle$ for which $\langle R^n \rangle$ is the n th moment of the distribution of sphere radii [47]. For a discrete distribution of sizes, $\langle R^n \rangle = \sum_i R_i^n v_i / \sum_i v_i$, where R_i represents the i th bin and v_i the associated frequency. Defining v_i as $v_i = N_i / \sum_i N_i$ with N_i the number of type- i particles [47], we then have $\langle R^n \rangle = \sum_i R_i^n N_i / \sum_i N_i$. In the case where there is a bimodal distribution of particle sizes, with two distinct peaks of radius R_1 and R_2 , we can be more explicit about the specific surface area. We find that the characteristic particle size is

$$\frac{\langle R^3 \rangle}{\langle R^2 \rangle} = \frac{R_1^3 \bar{N} + R_2^3}{R_1^2 \bar{N} + R_2^2}, \quad (13)$$

where $\bar{N}$ is the ratio of each sphere number N_1 and N_2 in the pack. As mentioned above, to use Eq. (13), it is appropriate to simply replace R in Eq. (5), (7), (8), or (9) with $\langle R^3 \rangle / \langle R^2 \rangle$ from Eq. (13). For a common case where the mass ratios of each size in the bimodal distribution are known, then $\bar{N} = (R_2^3 / R_1^3)(\mu_1 / \mu_2)$ where $\mu_1 = m_1 / (m_1 + m_2)$ and $\mu_2 = m_2 / (m_1 + m_2)$ with m_1 and m_2 the total masses of particles of types 1 and 2, respectively.

II. MATERIALS AND METHODS

Here we describe where the raw data originate [11], as well as the methods used to segment the porosity and to simulate fluid flow between the compressed soft balls. Additionally, we describe the algorithms we use to extract the porosity, connected porosities, and the specific surface area.

A. Three-dimensional packs of compressed soft balls

We start from published 3D datasets for soft ball volumes compressed to different degrees from Barés *et al.* [11] made publicly available [49]. Each 3D dataset represents one

snapshot during an experiment of soft balls compressed to a different degree of strain. There are four total independent experiments used, which are each a repeat of the same initial state and test conditions to check for consistency between runs. We use AVIZO to segment the volumes into a binary form where the voxels are assigned to the solid or the pore space, respectively. Given the very high-density contrast between the soft balls and the pore gas, segmentation was straightforward using AVIZO. Reconstruction of 3D data was performed using 8-bit stacks of “.png” files. These were processed using the AVIZO software (version 2024.1). A cubic subvolume of $500 \times 500 \times 500$ voxels was taken from the same coordinates of each experimental step (i.e., each scanned 3D volume). The data were denoised by applying an anisotropic diffusion filter (in three dimensions with a threshold of 90 and six iterations). Interactive thresholding (with an intensity range 80–255) was applied to binarize the denoised data, ensuring the binarization was uniform across the experimental steps. A “remove islands” filter (in three dimensions with maximum size 64 pixels) was applied to the binarized data to remove rare, small spots or holes within the soft balls. The processed data were then output as 8-bit “.tif” stacks for subsequent simulations. In all cases, the z axis in our representations and analysis is aligned with the compression direction from the experiments.

B. Lattice Boltzmann simulations of fluid flow

The binary output of the image processing shows the solid with volume fraction $\phi_s = 1 - \phi$ and fluid with volume fraction ϕ (i.e., the porosity). LBflow is a lattice Boltzmann fluid flow simulator [30,50], which has been calibrated for permeability determination using body-centered cubic sphere arrays [50] and simple cubic sphere arrays [51] and has been used widely for the determination of permeability of binarized digital datasets [20,21,45,52–55], such as those used here. The LBflow algorithm discretizes the pore space into fluid nodes, one per voxel, and “packets” of fluid mass are propagated through the lattice of nodes with numerical time t' . The fluid undergoes collisions using a D_3Q_{15} lattice arrangement (further details are provided in the source publications [30,50]). We apply a numerical pressure gradient of $\nabla p = 0.01 \text{ Pa m}^{-1}$ in a particular direction across the domain, a fluid viscosity of $\mu = 1.8205 \times 10^{-5} \text{ Pa s}$, and a fluid density of $\rho = 1.2047 \text{ kg m}^{-3}$ corresponding to that of air at ambient conditions (values given by the National Institute of Standards and Technology). Steady state is required to output a permeability. It is defined as when the measured relative fluid speed in the domain does not vary by more than 10^{-5} for 50 time steps and this criterion must be met twice consecutively for the result to be recorded.

We perform these simulations in each of the three principal directions across the domains, x , y , and z , to check for homogeneity and isotropy, resulting in permeabilities k_x , k_y , and k_z . We then average these to give $\langle k \rangle$ and use the standard deviation of the average as a measure of the permeability uncertainty. The ultimate output of the LBflow model is the spatial distribution of fluid velocity vectors at steady state in the pore phase. The average fluid velocity in the direction of the pressure gradient is then $\langle u \rangle$ and the so-called Darcy velocity is found as $q = \langle u \rangle \phi$. Darcy’s law is then

used to compute the permeability $k = \mu q / \nabla p$ [see Eq. (2)]. Care is taken to ensure that the input conditions of μ , ρ , and ∇p , all resulting in q , are such that the dimensionless Reynolds and Mach numbers, Re and Ma , are small, which ensures creeping viscous flow. The Reynolds number is $Re = \rho q L / \mu$ and the Mach number is $Ma = q / u_s$, where L is a characteristic length scale and u_s is the speed of sound in the fluid (set by LBflow). We find that for our simulations $1.36 \times 10^{-8} \leq Re \leq 4.82 \times 10^{-3}$ and that $7.99 \times 10^{-14} \leq Ma \leq 2.83 \times 10^{-8}$. Under these conditions of $Re \ll 1$ and $Ma \ll 1$, Darcy's law is valid, and our simulations are robust.

As a final check on our results, we additionally used domain sizes larger or smaller than those reported here; we varied the spatial position of the simulation domain within the wider tomography domain and ensured that these changes did not substantially affect the permeability results. Quantitatively, this entailed ensuring that the error associated with moving the simulation domain or with changing the simulation domain size is ultimately smaller than the uncertainty associated with the averaging to find $\langle k \rangle$.

C. Measuring porosities and specific surface areas

The porosity within a pack of soft balls can comprise connected and isolated porosities, termed ϕ_p (i.e., the percolating porosity) and ϕ' , respectively [56,57]. We can label them as such numerically by looking for connections that span the sample, edge to edge, in a given direction. This results in a measure of ϕ_p that depends on the direction in which system-spanning connections are assessed. Here, we determine ϕ_p in each of the three principal directions of the segmented x-ray scan domain and find an average ϕ_p for the system. By counting the total number of pore voxels, we know the total porosity ϕ and then $\phi' = \phi - \phi_p$. We therefore can determine the fraction of all the pores that are percolating (i.e., the connectivity of the network) as $\Phi = \phi_p / \phi$ so that when $\Phi = 1$, the whole network is connected, and when $\Phi = 0$, the network is fully isolated.

The determination of the specific surface area is achieved by using the marching cubes algorithm [58,59]. This can be used with the total porosity, which gives a specific surface area s of the total pore volume-solid interfaces in any given sample.

III. RESULTS AND ANALYSIS

The 3D volumes analyzed herein are presented in rendered form in Fig. 1. It is clear from the volume renderings that the intraparticle pore volumes remain interconnected down to relatively low ϕ . In Figs. 1(i)–1(l) we color the pore space gray or blue for the components ϕ_p and ϕ' , respectively. It is clear that all porosity is connected [gray and ϕ_p ; see Figs. 1(i)–1(k)] up until a final moment at the end of the compression, when it abruptly isolates [blue and ϕ' ; Fig. 1(l)]. Given the abruptness of this transition, hereafter we treat ϕ as the main variable, neglecting any notion of a sequential or slow partitioning of pore space between connected and isolated domains.

A. Normalized permeability of deforming soft ball systems

To analyze the datasets across a range of scales and to ensure the analysis is independent of scale, we normalize the

data and the models in a few steps. First, we compare the data to the hard-sphere models [Eqs. (6) and (7)] with the expectation that these models will fail to reproduce the data beyond some initial packing result at moderate ϕ . To do this, we use Eq. (5) as the normalization factor, yielding a dimensionless permeability $\bar{k}$ that is $\bar{k} = k / k_s = 9k(1-\phi) / (2R^2)$. The median R is 1.7 mm, but we use Eq. (13) to compute a weighted $\langle R^3 \rangle / \langle R^2 \rangle$ [11]. We find that the permeability of the soft ball system compares well to hard-sphere descriptions [Eq. (6)] initially but diverges as expected toward lower values of ϕ than those descriptions can accommodate (see cubic lattice model in Fig. 2).

Next, we introduce another dimensionless permeability, but with a percolation threshold invoked, which is

$$\bar{k} \equiv \frac{k}{k_r} = \frac{ks^2}{2[1 - (\phi - \phi_c)]}, \quad (14)$$

where we use the measured s . Using Eq. (14) to transform the data, we can then plot Eq. (4) easily, which transforms to $\bar{k} = (\phi - \phi_c)^b$. We can also plot a transformed Eq. (3) (i.e., $\bar{k} = \phi^3 / [2C(1-\phi)]$), and Eq. (6), where for those equations we use $\phi_c = 0$, consistent with their hard-sphere origins. To plot the data, we fit for the best-fit ϕ_c across all four experiments using a least squares regression (using the sum of square residuals of the logarithm of the permeability) using available tools (e.g., Ref. [60]). We find that the best-fit value to all data is $\phi_c = 0.020 \pm 0.001$, which is discussed later. The result is that Eq. (4) provides an excellent description of the data across all ϕ (see percolation model in Fig. 2) when specific surface area remains a measured quantity; Eq. (4) outperforms other models. This validates Eq. (4) as a reasonable model for the permeability of compacting soft ball assemblies.

In the inset to Fig. 2 we confirm the abruptness of the transition from $\phi = \phi_p$ to $\phi = \phi'$. This abrupt transition is the percolation threshold, which occurs at a value of ϕ that we here treat as discrete and singular.

B. Comparison with sphere packs

We compare our results with published data for packs of hard spheres, including across a range of sphere size distributions and associated polydispersities. These include packs of spheres arranged in face-centered, body-centered, and simple cubic lattices [45,50] and random maximally packed sphere packs [21] (Fig. 2). These data occur on the cubic lattice law at high ϕ and track smoothly onto the Kozeny-Carman law at intermediate ϕ , thereby confirming the standard $C = 5$ choice [35]. Note that for the cubic lattice data, they occur at a range of ϕ , whereas for the random maximally packed spheres they occur at a ϕ related to that maximum packing. We then also compare against published data for fully penetrable spheres where the intersphere overlap allows for the packs to access lower ϕ values such that the resultant data track down onto the percolation law [Eq. (14); Fig. 2] [20,45]. These datasets act as calibration “standard” datasets to ensure that the framework we propose here is robust for sphere packs with and without soft deformation.

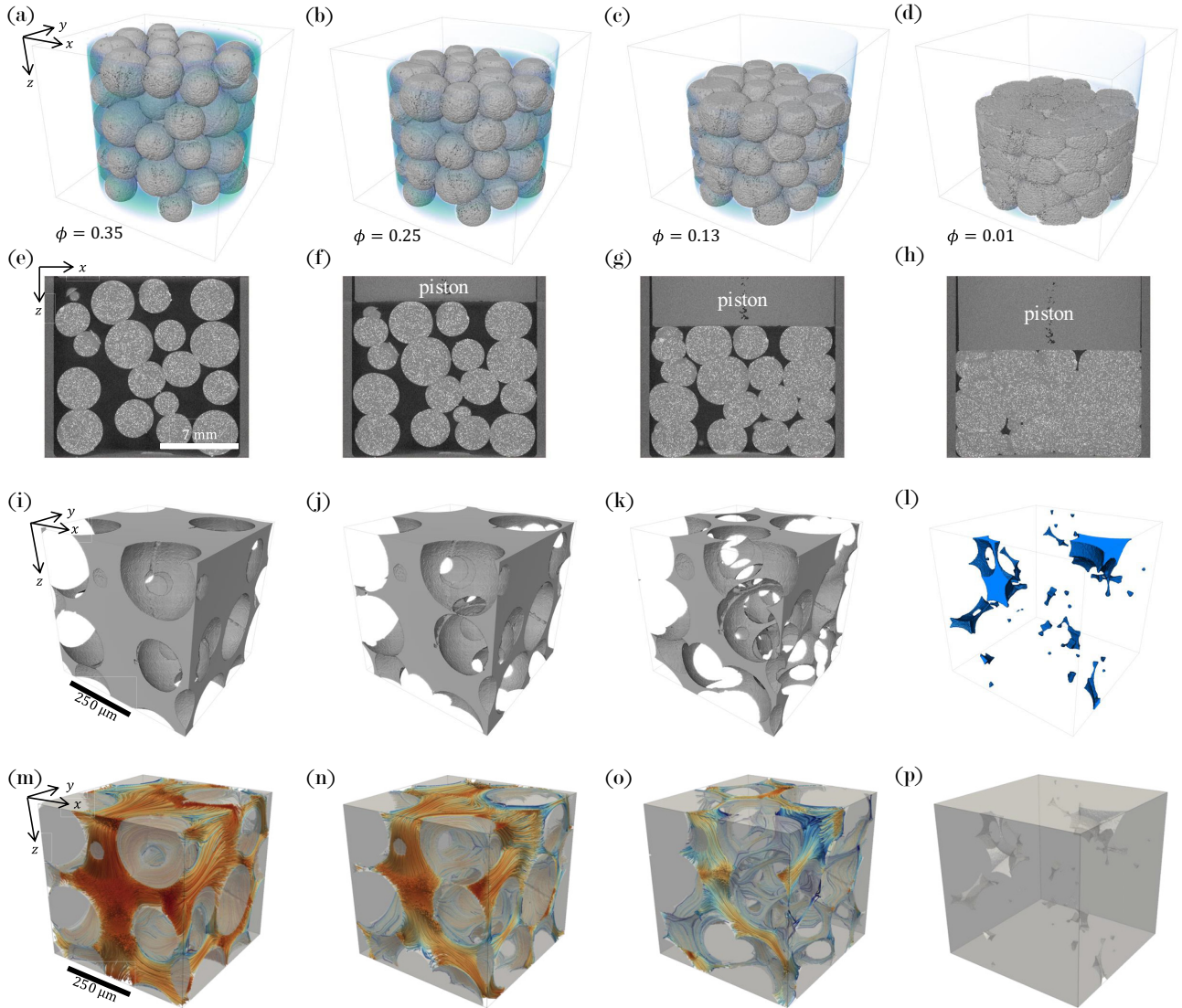


FIG. 1. Visualizations and renderings of experimental results. [(a)–(d)] Renders of soft balls at different degrees of compaction (gas volume fraction ϕ is labeled). Balls appear in gray, the pore space is semitransparent blue, and the piston is not rendered. [(e)–(h)] Two-dimensional slices in the same four snapshots as shown in (a)–(d) where the balls appear in gray and the pore space is black. [(i)–(l)] Rendered segmented pore volumes only (balls appear transparent), colored gray for fully connected pore space and blue for isolated pore space. [(m)–(p)] The result of fluid flow simulations in the phase between the soft balls, where the steady-state fluid flow velocity streamlines are colored according to relative velocity (absolute color scale is arbitrary, but red represents high velocity, and blue represents low velocity). Note that in (p) there is no flow because all the pore space is isolated [compare with panel (l)]. Panels (i)–(p) represent 421.875 mm^3 subvolumes extracted from the total volumes shown in (a)–(h). The cylinder seen in (a)–(d) and in rectangular cross section in (e)–(h) is 15 mm in diameter and 15 mm in height initially (e).

C. Specific surface area and predicting permeability

It is useful to assess the extent to which specific surface area can be predicted across experiments so that all the parameters in Eq. (4) can be determined *a priori*. To do so, we compare our determinations of s to the models for hard spheres [Eq. (7)], fully penetrable spheres [Eq. (8)], and partially penetrable spheres [Eqs. (9)–(12)]. All the “standard” data for hard and fully penetrable spheres [20,21] match the theoretical predictions, as expected (Fig. 3).

As we might expect, neither the hard nor the fully penetrable sphere models do a good job of capturing the new data

presented here for soft ball packs (Fig. 3) and this is clearly attributable to the fact that particles that are compressed and flattened against one another are not microstructurally identical to hard or fully penetrable sphere systems (Fig. 1). However, the data are well matched by the cherry-pit model with a value of λ that linearly decreases as $\phi \rightarrow 0$. We capture that effect with

$$\lambda = \lambda_0 + (1 - \lambda_0)\phi, \quad (15)$$

where λ_0 is an adjustable parameter that physically represents the degree of penetrability achievable when $\phi = 0$. We find

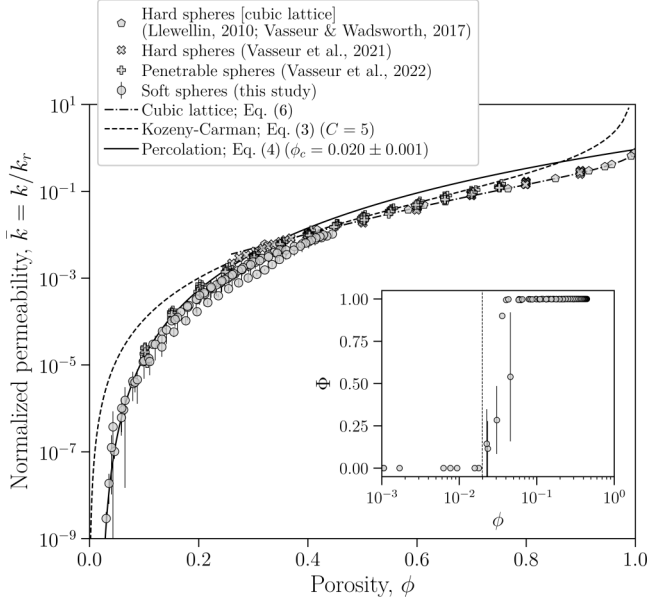


FIG. 2. The permeability of packs of compressed soft balls that flatten against one another. Here the porosity ϕ is a measure of how compressed the pack of balls is (lower porosity indicates larger strain and therefore degree of compression). The permeability is normalized by the reference permeability (see text). Models shown are the cubic lattice model [Eq. (6); here we show just the model for the fcc lattice, but note that the models for the bcc and sc lattices are very similar, albeit with different ϕ values below which they terminate and are invalid]; the classic Kozeny-Carman model with $C = 5$ [Eq. (3)]; and the percolation model [Eq. (4)] with a percolation threshold of $\phi_c = 0.020 \pm 0.001$, which is a result of minimization to the data (see text). Also shown for reference are hard and penetrable simulated sphere packs in random or cubic arrangements [20,21,45,50]. Inset: The connectivity $\Phi = \phi_p/\phi$ of the pore space as a function of the compression metric ϕ (note the near first-order transition as $\phi \rightarrow \phi_c$); the vertical dashed line is at $\phi = \phi_c$ as determined from the permeability datasets.

good agreement with the data with $\lambda_0 = 0.7$ (Fig. 3). This leads to the interpretation that soft ball assemblies are analogous to assuming that compaction is in fact microstructurally similar to sequentially [linearly, according to Eq. (15)] increasing the degree of apparent penetrability between spheres. This makes physical sense since the particle-particle flattening can be thought of as increasing the effective radius of curvature in any particle-particle pair, confirmed as contact areas go up during compression [11].

Using Eq. (15) with Eqs. (9)–(12) to predict s , we can then return to Eq. (14), but this time use the computed value of s in place of the measured value. This results in a new plot of $\bar{k}$ (Fig. 4), and again we find excellent agreement between experiment and our model, confirming that s can be predicted to sufficient accuracy via Eq. (15) to provide good predictive power for the permeability.

D. Percolation near the transition to an impermeable state

The rapid decline in permeability observed during compaction can be interpreted as a percolation transition in the void phase (qualitatively, see Fig. 1). As the deformable

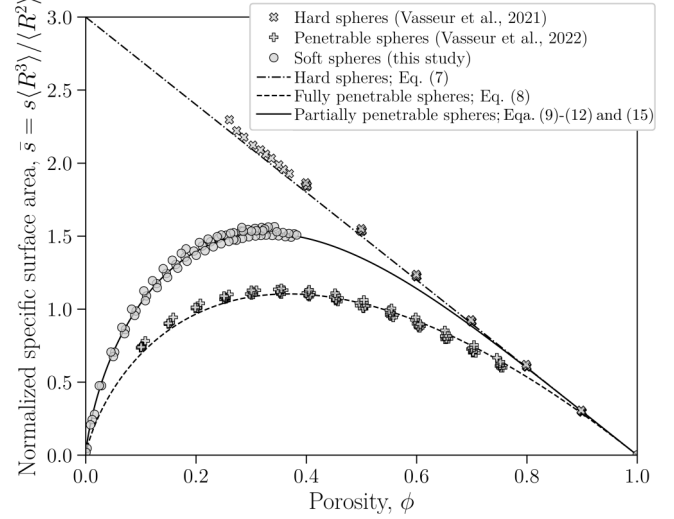


FIG. 3. The evolution of specific surface area s with porosity during compaction of soft balls. The data tend toward the hard-sphere model [Eq. (7); $\lambda = 1$] at high porosity ($\phi \gtrsim 0.4$) while they deviate during compaction toward the fully penetrable sphere model [Eq. (8); $\lambda = 0$] at very low porosity. We find that the path they take from initial hard spheres toward penetrable spheres is via a variant of the penetrable sphere model given by Eqs. (9)–(12) with Eq. (15), where the empirical adjustment is given by $\lambda_0 = 0.7$. Also shown for reference are hard and fully penetrable simulated sphere packs [20,21,45,50].

particles are compressed, the volume fraction and connectivity of the pore space decrease progressively (see inset to Fig. 2). Fluid transport remains possible while a system-spanning pore cluster exists, but once the connected void network loses percolation, long-range flow pathways are destroyed

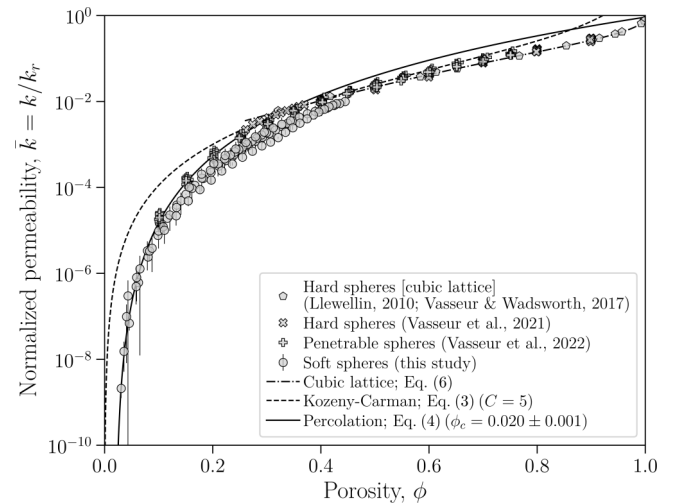


FIG. 4. The normalized permeability as a function of porosity but where the specific surface area is given by Eqs. (9)–(12) with Eq. (15) and is not the measured value, thereby demonstrating that the model for surface area does a good job when embedded in the permeability model. The models are the same as given in Fig. 2. Also shown for reference are hard and penetrable simulated sphere packs in random or cubic arrangements [20,21,45,50].

and permeability approaches zero. Within this framework, the relevant control parameter is not permeability itself but the occupancy and connectivity of the void phase. Increasing compaction drives the system toward a critical state characterized by a percolation threshold ϕ_c , below which no spanning pore cluster exists. Because particle deformation introduces geometric correlations and modifies local connectivity, the numerical value of ϕ_c is not expected to be universal and depends on the details of the microstructure.

By contrast, percolation theory predicts that transport properties near the threshold follow a power-law scaling of the form $k \propto (\phi - \phi_c)^b$, which is the same form as Eq. (4). The porosity ϕ in this percolation context is a measure of the connected pore occupancy. To test whether the permeability collapse observed here is consistent with a percolation transition, we rescale the data relative to the measured threshold and examine the resulting critical behavior. Figure 5 shows the permeability data collapsed onto a common scaling relation when expressed in terms of distance from the threshold. The theoretical exponent, $b = 4.4$, given in previous sections [43,44], is in close agreement with the data even in close proximity of the transition. This agreement suggests that, although particle deformation strongly influences the location of the critical point, the permeability collapse itself is governed by the same critical transport physics that characterizes percolating systems more generally. The results therefore support a picture in which compaction acts primarily to move the evolving pore network through a deformation-dependent percolation threshold, while the transport behavior near that threshold follows the scaling expected from percolation theory.

IV. DISCUSSION

Here we consider applications and implications of the results presented. In particular, we focus on (1) the application of this model to expanding volcanic pyroclasts; (2) the potential for universality of the proposed percolation framework and models; and (3) the implications of the percolation threshold for compacting soft ball assemblies as compared to other initially particulate systems.

A. Application of the soft ball model to the permeability of packs of expanding volcanic pyroclasts

Volcanic pyroclasts can expand (due to bubble growth) into one another, flattening their surfaces against one another. The details of how these naturally occurring soft ball assemblies are formed is given by Weaver *et al.* [19,61]. Schunke *et al.* [62] determined the permeability of experimentally replicated volcanic soft ball assemblies. In Fig. 5 we show their data transformed in the same way as for the nonvolcanic soft ball assemblies analyzed herein. When plotting the Schunke *et al.* [62] data, we take the same ϕ_c and λ_0 values determined here, and Eq. (15) with Eqs. (9)–(12) to recover a prediction of the specific surface area. When the data are transformed in the same way [Eq. (14)], they match the soft ball systems to a high degree of accuracy (Fig. 5). We therefore propose that our soft ball permeability model is applicable to the permeability of compacting pyroclast systems in all natural volcanic

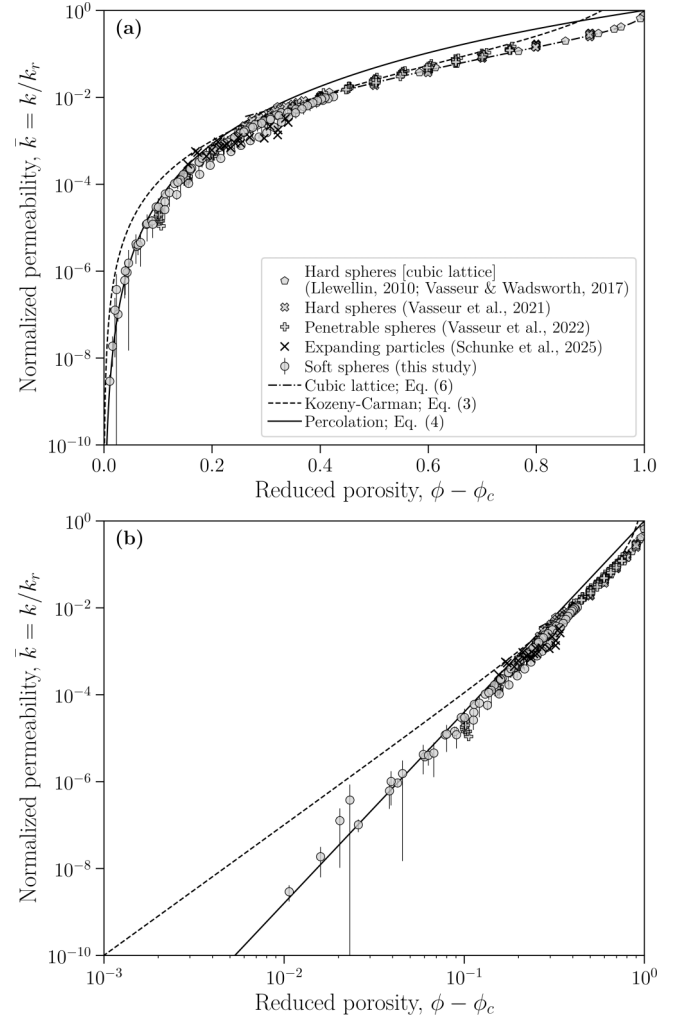


FIG. 5. (a) The normalized permeability as a function of the so-called reduced porosity, $\phi - \phi_c$. The data and models are the same as in Fig. 2 and we additionally show the data from Schunke *et al.* [62] showing that our model approach can be used to predict the permeability of inflating particles as a direct analogy with compressing particles; in both cases the particles are flattening against one another. (b) The same as in (a) but with a logarithmic x axis. The models are the same as given in Figs. 2 and 4. Also shown for reference are hard and penetrable simulated sphere packs in random or cubic arrangements [20,21,45,50].

environments, irrespective of stress configurations. Indeed, the permeability of such compacting pyroclast systems may be central to understanding volcanic eruption dynamics involving rhyolitic and other silicic magmas [63–65].

B. Universality of the percolation model

To test the universality of the permeability models presented in Sec. II A, we compile a range of literature data for which the relevant inputs are available to test the model. This compilation includes data for the permeability of numerically generated idealized packs of hard [21] and fully penetrable [20,45] spheres, packed confectionary [52], packs of hard [53] and fully penetrable cuboids [66], sintered glass bead samples [25,28,42,51,67–69], sintered metal bead samples [28],

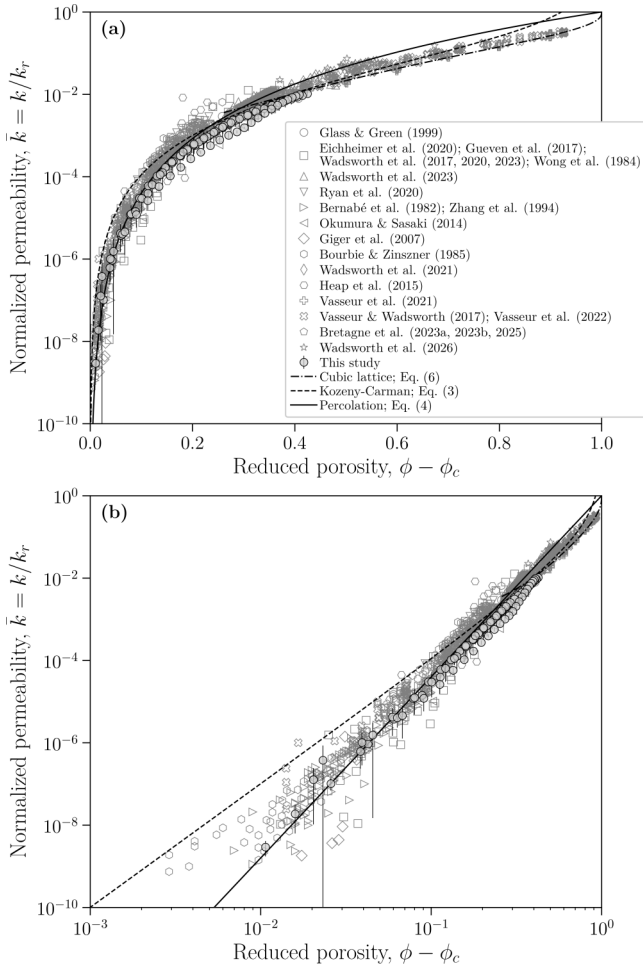


FIG. 6. The same as Fig. 5, but with more datasets compiled in this space, including data that are not directly analogous to soft or expanding balls, but which still conform to the same universality class. This compilation includes data for the permeability of numerically generated idealized packs of hard [21] and penetrable [20,45] spheres, packed confectionary [52], coffee pucks [78], packs of hard [53] and penetrable cuboids [66], sintered glass bead samples [25,28,42,51,67–69], sintered metal bead samples [28], sintered obsidian [70,71], dacitic, calcitic, or quartz gouge particles [72–75], block-and-ash flow deposits (a type of poorly sorted volcanic deposit containing angular fragments) [76], and natural sandstone packs [77].

sintered obsidian [70,71], dacitic, calcitic, or quartz gouge particles [72–75], block-and-ash flow deposits (a type of poorly sorted volcanic deposit containing angular fragments) [76], and natural sandstone packs [77]. These data are shown in Fig. 6.

We find that there is general agreement with the model framework(s) proposed herein. Once the specific surface area has been determined or calculated, it is clear that the data collapse to a universal description. And then, it is also clear that the data transition smoothly from hard-sphere descriptions, such as using Eq. (6), to the percolation model, such as using Eq. (4) (Figs. 5 and 6). The widely used Eq. (3) appears to provide a robust description of the data at the transition

between hard-sphere model(s) and the percolation model, which highlights its broad utility in science and engineering.

C. Anisotropy

An unexpected result is that the anisotropy developed during compression remains relatively weak, even as the system approaches the percolation threshold. Intuitively, one might anticipate that uniaxial compression would generate progressively stronger directional structure within the pore network, leading to preferential closure of pathways parallel to the compression direction and, consequently, increasingly anisotropic transport properties. Instead, we find that the evolution toward the critical state is dominated by an overall reduction in pore connectivity rather than by the development of strong directional bias.

This observation suggests that the approach to the percolation threshold is controlled primarily by the topology of the connected void network rather than by geometric anisotropy. Although compression inevitably deforms individual pores and modifies local connectivity, these changes remain sufficiently distributed throughout the network that no strongly preferred transport direction emerges before loss of percolation. In this sense, the critical point is reached through a nearly isotropic degradation of connectivity despite the anisotropic nature of the applied loading.

The weak anisotropy is also consistent with the successful collapse of the permeability data onto a percolation-style scaling law. Percolation theory predicts that the critical transport behavior is governed by connectivity at large scales, whereas details of local geometry become secondary near the threshold. The results therefore suggest that compression mainly shifts the system toward a deformation-dependent percolation threshold, while leaving the large-scale connectivity statistics sufficiently isotropic that classical critical scaling remains applicable.

V. CONCLUSIONS

We study the permeability of compressed soft ball assemblies. Given that the packs of soft balls we analyze are initially bimodal in size, we adapt existing descriptions of specific surface areas to account for bimodality. We show that initially the permeability matches predictions for packs of nonpenetrable, rigid, spherical (bimodal) balls. With compression, the permeability diverges from those predictions toward predictions for penetrable spheres, albeit with an adapted description of the specific surface area that accounts for the geometrical flattening of the balls one against another. When treated in this manner, we find that soft ball assemblies under compression match a very wide range of hard and penetrable granular media (see Fig. 6) such that we claim a universality class for permeability: granular media. An implication of our approach is that any particulate system can be described in this manner, so long as the specific surface area is well constrained or predicted theoretically. Taken together with models for inertial permeability, our approach can yield universal flow laws for flow between compressing and deformable particles via Eq. (1). We show one applied example of deformable volcanic pyroclasts, while claiming that other applications could be

found in any system in which particle deformation can occur while fluid is flowing between the particles.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [79].

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