

Correspondence principle, dissipation, and Ginibre ensemble

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The correspondence between quantum and classical behavior has been essential since the advent of quantum mechanics. This principle serves as a cornerstone for understanding quantum chaos, which has garnered increased attention due to its strong impact in various theoretical and experimental fields. When dissipation is considered, quantum chaos takes concepts from isolated quantum chaos to link classical chaotic motion with spectral correlations of Ginibre ensembles. This correspondence was first identified in periodically kicked systems with damping, but it has been shown to break down in dissipative atom-photon systems [*Phys. Rev. Lett.* **133**, 240404 (2024)]. In this contribution, we revisit the original kicked model and perform a systematic exploration across a broad parameter space, reaching a genuine semiclassical limit. Our results demonstrate that the correspondence principle, as defined through this spectral connection, fails even in this prototypical system. These findings provide conclusive evidence that Ginibre spectral correlations are neither a robust nor a universal diagnostic of dissipative quantum chaos.

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Quantum theory has raised a series of intriguing questions since its early development. One key idea is how quantum mechanics aligns with the predictions of classical mechanics, which led to the formulation of the correspondence principle [1,2]. The systematic observation of chaotic motion in nonlinear systems [3], along with its implications for engineering and complex systems [4–7], has sparked a growing interest in understanding how this phenomenon translates into quantum mechanics, giving rise to the fruitful field of quantum chaos [8–11]. The impact of quantum chaos on both theoretical and experimental topics of current interest, such as thermalization [12,13], quantum metrology [14], quantum information [15–18], and high-energy physics [19–22], among others [23], has motivated the study of dissipative quantum systems [24–26], where the chaotic motion in the classical limit is characterized by the emergence of chaotic attractors [27–31].

Traditionally, the correspondence principle links classical chaotic motion in isolated systems with universal spectral correlations, as suggested by the *quantum chaos conjecture* [32,33]. In contrast, classical regular motion corresponds to uncorrelated energy spectra [34]. A key advancement in formulating a correspondence principle in dissipative systems involved extending the correspondence principle established in isolated systems to the periodically kicked top with damp-

ing [35]. These studies demonstrated that the presence of a chaotic attractor in the classical limit corresponds to cubic level repulsion in the Liouvillian spectrum of a Floquet operator with complex phases, similar to what is seen in Ginibre ensembles of non-Hermitian Gaussian random matrices [36]. In contrast, simple attractors correspond to linear level repulsion in the Liouvillian spectrum. These two relationships were recognized as the *dissipative quantum chaos conjecture* [35,37,38].

The description established by the dissipative conjecture has inspired the application of the spectral analysis to define quantum chaos in dissipative many-body quantum systems [38–43], which lack a classical limit, and categorize these systems based on their symmetries [44–47]. Subsequent studies have been conducted to pinpoint specific behaviors of the Ginibre spectral correlations in quantum evolution [48–50], while others have attempted to detect these correlations experimentally [51]. However, dissipative atom-field systems with a well-defined classical limit have detected that the description of the dissipative conjecture fails [52], raising questions about the reliability of its predictions. Alternative frameworks for understanding dissipative chaos have also been proposed, using properties of steady states [53–61] or focusing on dynamical signatures [59,62,63].

The previous discussion provides a perspective on the perplexing nature of what dissipative quantum chaos entails. In this work, we investigate the periodically kicked top with dissipation [35] that inspired the formulation of the dissipative quantum chaos conjecture. A variety of research has been conducted in this system, including studies of quantum dynamics [64,65], classical motion in presence of dissipation and pumping [66], semiclassical analysis [67–70], and alternative methods for modeling dissipation, such as stochastic methods [71–74] and non-Hermitian Hamiltonian approaches

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[75–81]. The characteristics of non-Hermitian Hamiltonians have led to useful correspondence principles between quantum and classical dynamics [82–85]. The latest studies of the kicked top have shown a connection between dissipative classical chaos and measures of quantum complexity [86]. Moreover, the kicked top in absence of dissipation is a paradigm of classical and quantum chaos [87–94] and has been the subject of fruitful research, including mixed-type phase space dynamics [95,96] under the Berry-Robnik picture [97–99], scarring [100–102], entanglement [103–113], localization [114], fractality [115–117], as well as experimental realizations [104,105,118–120] and metrological aspects [121].

Our analysis of the kicked top with dissipation examines a broad parameter space and reveals that the expected correspondence between quantum and classical behavior is not entirely valid. This provides conclusive evidence of the limitations of Ginibre spectral correlations as an overall test of dissipative quantum chaos.

Quantum kicked top. A periodically kicked top with dissipation [35,64] is represented with the Markovian Lindblad master equation [24–26], setting $\hbar = 1$,

$$\frac{d\hat{\rho}}{dt} = \hat{\mathcal{L}}(t)\hat{\rho} = -i[\hat{H}(t), \hat{\rho}] + \hat{\Lambda}\hat{\rho} = (\hat{\Lambda} - i\hat{\mathcal{L}}(t))\hat{\rho}, \quad (1)$$

where $\hat{\mathcal{L}}(t)$ is the Liouvillian operator, $\hat{\mathcal{L}}(t)\hat{\rho} = [\hat{H}(t), \hat{\rho}] = \hat{L}_0\hat{\rho} + \hat{L}_1\hat{\rho} \sum_n \delta(t - n)$, and the dissipation process is chosen as occurs in the theory of super-radiance [122]

$$\hat{\Lambda}\hat{\rho} = \frac{\Gamma}{2j}(2\hat{J}_-\hat{\rho}\hat{J}_+ - \{\hat{J}_+\hat{J}_-, \hat{\rho}\}), \quad (2)$$

where Γ is the dissipation strength. The terms $\hat{L}_{0,1}\hat{\rho} = [\hat{H}_{0,1}, \hat{\rho}]$ come from the unitary dynamics represented by the time-dependent Hamiltonian

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_1 \sum_n \delta(t - n), \quad (3)$$

where $\hat{H}_0 = p\hat{J}_z + (k_0/2j)\hat{J}_z^2$ is a precessional motion around the z axis controlled by the parameters p and k_0 . The term $\hat{H}_1 = (k_1/2j)\hat{J}_y^2$ is a rotational motion around the y axis that appears periodically with unit period and is modulated by the kick strength k_1 . The Dirac delta function in Eq. (3) represents the periodic kick. The operators $\hat{J}_{x,y,z}$ and $\hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$ are usual angular momentum operators.

The operator $\hat{\mathbf{J}}^2$ with eigenvalues $j(j+1)$ is a conserved quantity due to the commutation relations with the Hamiltonian in Eq. (3), the Floquet operator $\hat{F} = e^{-i\hat{H}_1}e^{-i\hat{H}_0}$, and the Liouvillian in Eq. (1), $[\hat{H}(t), \hat{\mathbf{J}}^2] = [\hat{F}, \hat{\mathbf{J}}^2] = [\hat{\mathcal{L}}(t), \hat{\mathbf{J}}^2] = 0$. The angular momentum basis $|j, m_z\rangle$ provides a Hilbert space with finite dimension $2j+1$ and a Liouville space with dimension $(2j+1)^2$. The parity symmetry of the Hamiltonian system appears as a weak symmetry [123,124] in the dissipative system, $[\hat{D}, \hat{\mathcal{P}}] = [\hat{F}, \hat{\Pi}] = 0$, where $\hat{\Pi} = e^{i\pi(\hat{J}_z + j\hat{\mathbb{I}})}$ is the parity operator, $\hat{\mathcal{P}}\hat{\rho} = \hat{\Pi}\hat{\rho}\hat{\Pi}^\dagger$, and $\hat{D}$ is the dissipative Floquet operator that provides a kick-to-kick stroboscopic description of the density operator

$$\hat{\rho}_{n+1} = \hat{D}\hat{\rho}_n = e^{-i\hat{L}_1}e^{(\hat{\Lambda} - i\hat{L}_0)}\hat{\rho}_n. \quad (4)$$

Classical limit of the kicked top. The classical limit of the periodically kicked top with dissipation is obtained by a mean-field approximation for the expectation values of the angular momentum operators $\langle \hat{J}_{x,y,z} \rangle = \text{Tr}(\hat{\rho}\hat{J}_{x,y,z})$,

$$\dot{J}_x = - \left[p + k_0 J_z - k_1 J_z \sum_n \delta(t - n) \right] J_y + \Gamma J_x J_z, \quad (5)$$

$$\dot{J}_y = (p + k_0 J_z) J_x + \Gamma J_y J_z, \quad (6)$$

$$\dot{J}_z = -k_1 J_x J_y \sum_n \delta(t - n) - \Gamma(J_x^2 + J_y^2), \quad (7)$$

where we used the evolution law $\partial_t \hat{J}_{x,y,z} = \hat{\mathcal{L}}^\dagger(t)\hat{J}_{x,y,z}$ and decoupled expectation values $\langle \hat{J}_x \hat{J}_y \rangle \approx \langle \hat{J}_x \rangle \langle \hat{J}_y \rangle$. The classical variables $J_{x,y,z} = \lim_{j \rightarrow \infty} \langle \hat{J}_{x,y,z} \rangle / j$ represent an angular momentum vector in the Bloch sphere, $\mathbf{J} = (J_x, J_y, J_z) = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta)$, with conserved unit magnitude $|\mathbf{J}| = 1$.

The Hamiltonian dynamics can be recovered by switching off the dissipation strength Γ in Eqs. (5)–(7), or by a mean-field approximation for the expectation value of the quantum Hamiltonian in Eq. (3) with Bloch coherent states of minimum uncertainty, $|\alpha\rangle = e^{(\alpha\hat{J}_+ - \alpha^*\hat{J}_-)}|j, -j\rangle$, obtaining the classical Hamiltonian

$$H(t) = \lim_{j \rightarrow \infty} \frac{\langle \alpha | \hat{H}(t) | \alpha \rangle}{j} = H_0 + H_1 \sum_n \delta(t - n), \quad (8)$$

where $H_0 = pJ_z + (k_0/2)J_z^2$ and $H_1 = (k_1/2)J_y^2$. The classical variables $J_{x,y,z} = \lim_{j \rightarrow \infty} \langle \alpha | \hat{J}_{x,y,z} | \alpha \rangle / j$ provide the Hamilton equations of motion $J_{x,y,z} = \{J_{x,y,z}, H(t)\}$.

The nonautonomous dynamical system in Eqs. (5)–(7) represents two processes of the classical motion. In the first process, dissipation takes place continuously before the kick and we can set $k_1 = 0$ in Eqs. (5)–(7). The evolution of this reduced dynamical system provides the solution $J_{x,y,z}$. The second process is discrete and is defined by the action of the kick ($k_1 \neq 0$) as a rotation around the y axis of the previous solution, $\tilde{J}_{x,y,z} = R(k_1 J_y) J_{x,y,z}$. The last two processes allow us to define a kick-to-kick stroboscopic description for the dissipative classical motion in the same spirit as occurs with the Hamiltonian motion

$$\tilde{J}_x^{n+1} = \cos(k_1 J_y^n) J_x^n + \sin(k_1 J_y^n) J_z^n, \quad (9)$$

$$\tilde{J}_y^{n+1} = J_y^n, \quad (10)$$

$$\tilde{J}_z^{n+1} = -\sin(k_1 J_y^n) J_x^n + \cos(k_1 J_y^n) J_z^n. \quad (11)$$

In the Supplemental Material [125], we present an alternative classical description that considers dissipation as a decoupled process from the unitary evolution [64,69,70].

Classical and quantum dissipative chaos. Chaotic motion in dissipative classical systems is described by the appearance of chaotic attractors through different routes, such as period-doubling cascades, intermittency, or quasiperiodicity [4–7,27–31]. The motion inside the chaotic attractors generates an exponential divergence of the initial conditions, which can be identified with Lyapunov exponents $h = \lim_{t \rightarrow \infty} \lim_{\delta_0 \rightarrow 0} t^{-1} \ln(\delta(t)/\delta_0)$ [126–128], where $\delta(t) =$

$|\mathbf{x}_1(t) - \mathbf{x}_2(t)|$ is the separation in phase space of two trajectories at time t and $\delta_0 = \delta(0)$.

The dissipative quantum chaos conjecture relates the presence of chaotic attractors in the classical dynamics with a level spacing distribution described by the Ginibre unitary ensemble (GinUE) distribution [35,36,38]

$$P_{\text{GinUE}}(s) = \prod_{k=1}^{\infty} \frac{\Gamma(1+k, s^2)}{k!} \sum_{k'=1}^{\infty} \frac{2s^{2k'+1}e^{-s^2}}{\Gamma(1+k', s^2)}, \quad (12)$$

where $\Gamma(k, z) = \int_z^{\infty} dt t^{k-1} e^{-t}$ is the incomplete Gamma function. The spacing $s_k = |\varphi_k^{\text{NN}} - \varphi_k|$ is defined as the minimum Euclidean distance between a reference eigenvalue φ_k and its nearest neighbor φ_k^{NN} in the complex plane. On the contrary, when simple attractors appear in the classical dynamics, the spacing distribution agrees with a 2D Poisson function [35,38]

$$P_{2\text{DP}}(s) = \frac{\pi}{2} s e^{-\pi s^2/4}. \quad (13)$$

The GinUE distribution is normalized with a first moment given by $\bar{s} = \int_0^{\infty} ds s P_{\text{GinUE}}(s) \approx 1.1429$. For numerical comparisons, we scale the GinUE distribution, $\tilde{P}_{\text{GinUE}}(s) = \bar{s} P_{\text{GinUE}}(\bar{s}s)$, to ensure that its first moment is also normalized, $\int_0^{\infty} ds s \tilde{P}_{\text{GinUE}}(s) = 1$. The limit $s \rightarrow 0$ provides the power of level repulsion, $P_{\beta}(s) \propto s^{\beta}$, which is determined by the exponent β . In this way, linear (cubic) level repulsion, $\beta = 1$ ($\beta = 3$), identifies a regular (chaotic) dissipative quantum system [8,35,37].

Quantum analysis of the kicked top. We begin with the spectral analysis of the kicked top with dissipation by considering the complex eigenphases φ_k of the dissipative Floquet operator defined in Eq. (4). The eigenphases are given by the eigenvalue equation $\hat{D}|\lambda_k\rangle = \lambda_k|\lambda_k\rangle = e^{i\varphi_k}|\lambda_k\rangle$. As occurs in isolated systems, the complex eigenphases need to be unfolded to remove system specific behaviors [38,44,129]. To avoid this procedure, we use the complex spectral ratio introduced in Ref. [40]:

$$Z_k = r_k e^{i\theta_k} = \frac{\varphi_k^{\text{NN}} - \varphi_k}{\varphi_k^{\text{NNN}} - \varphi_k}, \quad (14)$$

where NN stands for the nearest neighbor and NNN for the next-to-nearest neighbor of the reference eigenphase φ_k . The average of the quantities $r_k = |Z_k|$ and $\theta_k = \text{Arg}(Z_k)$ provide limit values for numerical comparisons. The averages $\langle r \rangle_{2\text{DP}} = 2/3$ and $-\langle \cos \theta \rangle_{2\text{DP}} = 0$ are obtained using the 2D Poisson distribution. For the GinUE distribution, we get $\langle r \rangle_{\text{GinUE}} \approx 0.74$ and $-\langle \cos \theta \rangle_{\text{GinUE}} \approx 0.24$.

Next, we present the analysis of the complex spectral ratio for the kicked top with dissipation and use the parameters considered in Ref. [35]. We restrict our study to the dissipation region $0 < \Gamma \leq 0.4$. For dissipation strengths $\Gamma > 0.4$, the numerical eigenvalues λ_k collapse to the center of the unit circle and the corresponding eigenphases φ_k are basically random numbers below the machine precision (see the Supplemental Material [125] for complementary information).

In Ref. [35], an ensemble average for the parameter k_0 was considered to minimize quantum fluctuations caused by the small system size examined. In this analysis, we avoid the ensemble average and instead reach the semiclassical limit

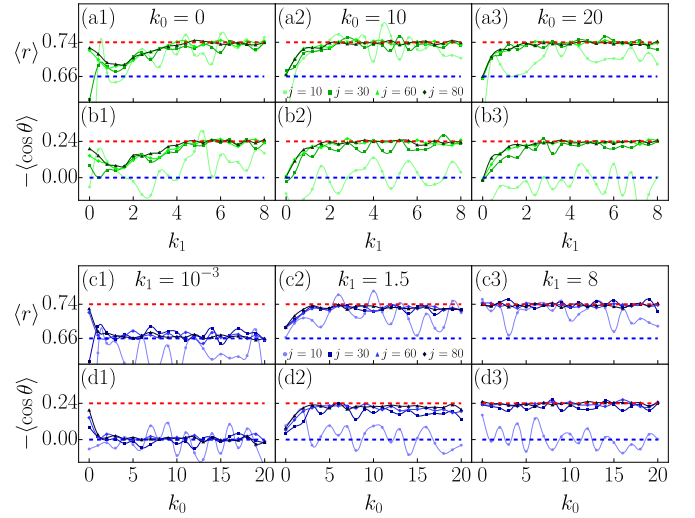


FIG. 1. System size analysis in the kicked top with dissipation. [(a1)–(a3)] Average $\langle r \rangle$ and [(b1)–(b3)] average $-\langle \cos \theta \rangle$ from the complex spectral ratio in Eq. (14) as a function of the kick strength k_1 . Each column identifies a different case: $k_0 = 0$, $k_0 = 10$, and $k_0 = 20$. [(c1)–(c3)] Average $\langle r \rangle$ and [(d1)–(d3)] average $-\langle \cos \theta \rangle$ as a function of the parameter k_0 . Each column identifies a different case: $k_1 = 10^{-3}$, $k_1 = 1.5$, and $k_1 = 8$. In all panels, we show the last averages for different system sizes $j = 10, 30, 60, 80$. The horizontal blue (red) dashed line represents the limit of a quantum system described by 2D Poisson (GinUE) statistics. System parameters: $p = 2$ and $\Gamma = 0.1$.

by increasing the system size j . Additionally, we consider different values of the parameter k_0 to analyze its effect on the system. In Figs. 1(a1)–1(a3), we show the average $\langle r \rangle$ as a function of the kick strength k_1 for three cases: $k_0 = 0$, $k_0 = 10$, and $k_0 = 20$. Figures 1(b1)–1(b3) show similar plots for the average $-\langle \cos \theta \rangle$. In Figs. 1(c1)–1(c3) and Figs. 1(d1)–1(d3), we present the corresponding averages as a function of the parameter k_0 for three cases: $k_1 = 10^{-3}$, $k_1 = 1.5$, and $k_1 = 8$. In all cases, we show the averages for different system sizes. We identify how the numerical results deviate from the 2D Poisson and GinUE statistics for small system sizes. By increasing the system size, the numerical results converge to the expected statistics.

Now, we choose the maximum system size $j = 80$ and present in Figs. 2(a1)–2(a3) density plots of the average $\langle r \rangle$ as a function of the kick and dissipation strengths (k_1, Γ) for three cases: $k_0 = 0$, $k_0 = 10$, and $k_0 = 20$. Figures 2(b1)–2(b3) present similar plots for the average $-\langle \cos \theta \rangle$. Figures 2(c1)–2(c3) and Figs. 2(d1)–2(d3) show the last averages for selected values of Γ . For $k_0 = 0$, we can see an unexpected behavior for weak kick regions and low dissipation, where the system captures GinUE statistics around $k_1 \approx 0$, tends to reach 2D Poisson statistics at $k_1 \approx 1.5$, and recovers GinUE statistics for stronger kick regions. In contrast, when dissipation increases, the system tends to reach GinUE statistics for all kick strengths. The cases with $k_0 = 10$ and $k_0 = 20$ show an overall transition from weak kick regions described by 2D Poisson statistics to strong kick regions that capture GinUE statistics. Surprisingly, the last transition is irrespec-

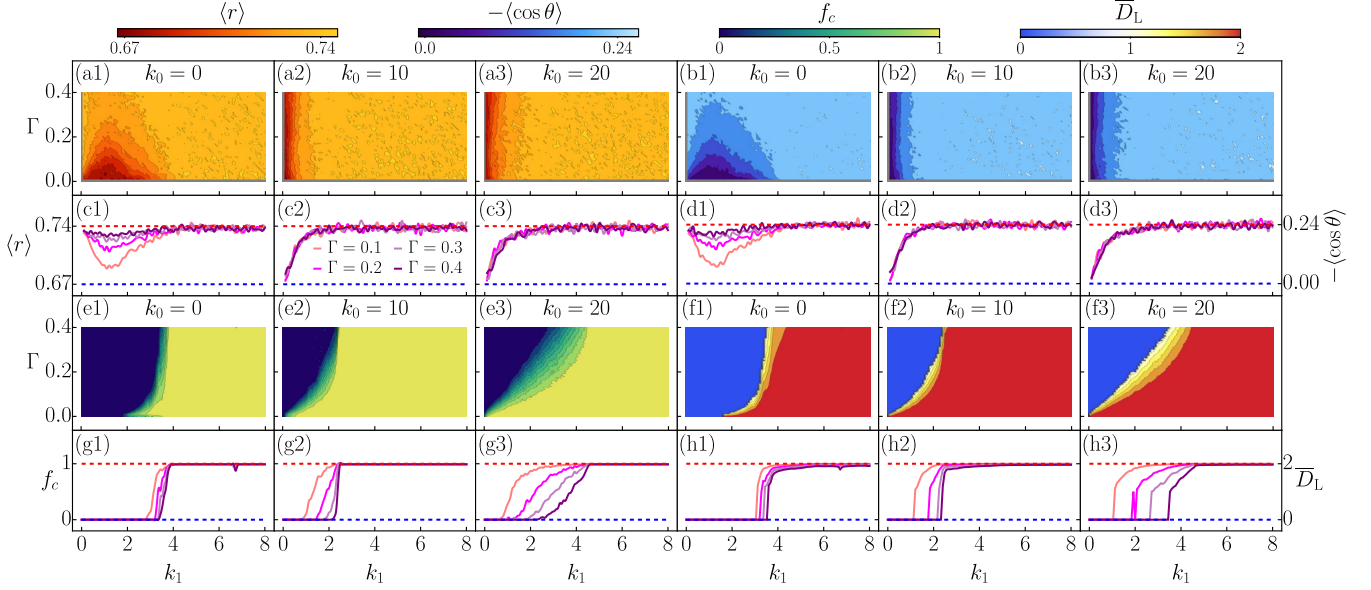


FIG. 2. Quantum and classical analysis of the kicked top with dissipation. [(a1)–(a3)] Average $\langle r \rangle$ and [(b1)–(b3)] average $-\langle \cos \theta \rangle$ from the complex spectral ratio in Eq. (14) as a function of the kick and dissipation strengths (k_1, Γ). Panels (c1)–(c3) and (d1)–(d3) show the last averages for specific dissipation strengths, $\Gamma = 0.1, 0.2, 0.3, 0.4$. Each column identifies a different case: $k_0 = 0, k_0 = 10$, and $k_0 = 20$. In panels (a1)–(a3) and (b1)–(b3), the gray stripes represent the limits ($k_1 \rightarrow 0$ and $\Gamma \rightarrow 0$), where the 2D Poisson and GinUE statistics do not hold. In panels (a1)–(a3) and (b1)–(b3), the horizontal blue (red) dashed line represents the limit of a quantum system described by 2D Poisson (GinUE) statistics. In panels (a1)–(d3), we use a system size $j = 80$ that provides 12 961 eigenvalues with positive parity. [(e1)–(e3)] Fraction of chaotic initial conditions in Eq. (15) and [(f1)–(f3)] average Lyapunov dimension in Eq. (16) as a function of the kick and dissipation strengths (k_1, Γ). Panels (g1)–(g3) and (h1)–(h3) show the last quantities for specific dissipation strengths, $\Gamma = 0.1, 0.2, 0.3, 0.4$. Each column identifies a different case: $k_0 = 0, k_0 = 10$, and $k_0 = 20$. In panels (g1)–(g3), the horizontal blue (red) dashed line represents the limit of a classical system with simple (chaotic) attractors, while in panels (f1)–(f3), corresponds to the Lyapunov dimension of a point (surface). The average in phase space takes 1245 initial conditions evolved until 10^3 periods. We use $p = 2$ in all panels.

tive of the dissipation strength Γ , due to the inclusion of the parameter k_0 .

Classical analysis of the kicked top. The classical analysis of the kicked top with dissipation, as presented in Ref. [35], does not consider the effects of phase space contraction induced by dissipation or those related to the phase space average of multiple initial conditions. Studying the transition to chaos in classical dynamical systems requires a well-defined invariant measure to quantify the fraction of chaos in phase space [4]. In conservative systems this role is played by the phase space volume, whereas, in nonconservative systems, identifying such a measure is far more difficult. For leaking or absorptive systems, where excluded coordinates break phase space preservation, conditionally invariant measures can restore normalization [130–132]. In contrast, for dissipation induced by channels in the Lindblad master equation, which contract phase space, a practical formulation based on such invariant measures is unclear and still lacking. In view of this difficulty, and since the phase space volume measure can correctly capture both the fully regular and fully chaotic regimes, we define a fraction of chaos in direct analogy with the conservative case

$$f_c = \frac{1}{V_0} \int_{\mathcal{M}} dQ dP \Upsilon(Q, P), \quad (15)$$

where Υ is a characteristic function that takes the value $\Upsilon = 1$ if the maximal Lyapunov exponent of an evolved initial condition is positive, or $\Upsilon = 0$ otherwise [133]. The classical

variables $J_{x,y,z}$ can be transformed to the variables Q and P , which provide a bounded phase space $\mathcal{M} = \{(Q, P) | Q^2 + P^2 \leq 4\}$ with volume $V_0 = \int_{\mathcal{M}} dQ dP = 4\pi$ (see the Supplemental Material [125] for further details). We utilize the phase space $\mathcal{M}$ linked to the initial conditions to average the characteristic function Υ at time $t = 0$ before the phase space contraction, scaling the average by the initial phase space volume V_0 . In this way, $f_c = 1$ defines a classical system completely governed by chaotic attractors, and $f_c = 0$ a system with only simple attractors. Additionally, we use the Lyapunov dimension [134–137]

$$D_L = k + \frac{1}{|h_{k+1}|} \sum_{i=1}^k h_i, \quad (16)$$

where $k < N$ is the largest index of the Lyapunov spectrum of an N -dimensional system, $\{h_1 > \dots > h_N\}$, which fulfills the constriction $\sum_{i=1}^k h_i > 0$. The Lyapunov dimension constitutes an upper limit to the Hausdorff or fractal dimension [4, 138], which provides a measure of a chaotic attractor. In the Supplemental Material [125], we present the correspondence between the Lyapunov dimension and the Hausdorff dimension.

We use the same parameters from the quantum analysis of the kicked top with dissipation to conduct its classical analysis. In Figs. 2(e1)–2(e3), we show density plots of the fraction f_c in Eq. (15) as a function of the parameters k_1 and Γ , for the previous cases: $k_0 = 0, k_0 = 10$, and $k_0 = 20$.

For $k_0 = 0$, we identify a wide region of regular motion for weak kick regions, where point attractors or limit cycles appear. In contrast, the motion is dominated by chaotic attractors for stronger kick regions. The cases with $k_0 = 10$ and $k_0 = 20$ show a reduction of the regular region for weak kick regions and low dissipation. In these cases, the classical transition depends on the dissipation strength, as can be seen in Figs. 2(g1)–2(g3) for selected values of Γ , where the highest dissipation produces a delayed transition to the chaotic regime. Complementarily, Figs. 2(f1)–2(f3) show density plots of the average Lyapunov dimension $\bar{D}_L$ in Eq. (16) for the presented cases. Figures 2(h1)–2(h3) show $\bar{D}_L$ for some values of Γ . Regular regions dominated by point attractors show a null Lyapunov dimension, which suddenly changes to a unit value in regions with limit cycles. After this, the Lyapunov dimension increases and reaches values slightly below $\bar{D}_L = 2$, where the regions contain chaotic attractors.

Correspondence principle in the kicked top. There is an alignment between the quantum and classical perspectives in the strong kick regions. However, this correspondence breaks down in the weak kick regions combined with high dissipation. The disagreement is more evident for the case with $k_0 = 0$. The cases with $k_0 \neq 0$ lead to a quantum transition that is independent of the dissipation strength. This leads to partial agreement within the weak kick regions when dissipation is very low ($\Gamma \leq 0.1$) and the dissipative system tends to behave like the isolated one (see the Supplemental Material [125] for a comparison). However, for higher dissipation, the classical transition identified in the cases with $k_0 = 10$ and $k_0 = 20$ clearly depends on the dissipation strength, in contrast to the quantum transition.

As reported in Ref. [35], for the parameters $k_0 = 10$ and $\Gamma = 0.1$ shown in Fig. 2, the individual case with $k_1 = 0$ shows an agreement between classical regular motion and 2D Poisson statistics. In contrast, the case with $k_1 = 8$ demonstrates alignment between classical chaotic motion and

GinUE statistics. However, for higher dissipation ($0.1 < \Gamma \leq 0.4$), the quantum-classical correspondence gradually breaks down in the weak kick region. There is a clear region in Fig. 2(c2) and Fig. 2(d2), $0 \leq k_1 < 2$, where the statistics developed by simple attractors [see Fig. 2(g2) and Fig. 2(h2)] deviate from 2D Poisson statistics and tend to reach GinUE statistics. A similar disagreement is identified when comparing Figs. 2(c3)–2(d3) with Figs. 2(g3)–2(h3) for the case with $k_0 = 20$. The mechanism of why a simple attractor could develop GinUE spectral correlations is still an open question [52].

Discussion. The study presented in this work demonstrated that the correspondence principle fails in the kicked system that inspired the current definition of dissipative quantum chaos. This is supported by achieving a genuine semiclassical limit for the quantum system. As a result, Ginibre spectral correlations cannot be used as a test of dissipative quantum chaos. This observation encourages new approaches to explore this phenomenon. Important remarks arise in this regard. (1) The concept of dissipative quantum chaos should be grounded in a robust framework that incorporates a clear classical interpretation and is not solely based on spectral correlations. (2) Identifying the quantum fingerprints of classical chaotic structures appears to be a promising way to address this problem [139,140].

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Data availability. The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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