

Nonlinear kinetic simulations of Jeans instability in a magnetized dusty plasma

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The Jeans instability in a magnetized dusty plasma is considered a fundamental process in space, where magnetic fields are common. We investigate the Jeans instability in a magnetized dusty plasma using 1D and 2D particle-in-cell simulations, in which dust grains are treated as particles and the Poisson's equation for the corresponding gravitational and electrostatic potentials is solved in a self-consistent manner. We first confirm that when the magnitude of the dust cyclotron frequency is larger than the Jeans frequency, the Jeans instability is completely stabilized, and the PIC simulation of the stable case shows the existence of cyclotron harmonics in the dispersion relation as predicted by the kinetic linear theory. In unstable cases, we find that the linear growth obtained by the simulations and the fluid and kinetic linear theory are consistent with each other, showing that the linear growth rate decreases as the magnetic field becomes stronger. The magnetized Jeans instability is also qualitatively modified due to the gyromotion of dust grains: (1) a dipole-like structure is observed in phase space and the density peak at the bottom of the gravitational potential is 3 times larger than the nonmagnetized case in the 1D simulation; and (2) dust grains form a filament, which corresponds to the direction in which the gyrating dust grains are accelerated by the gravitational potential, during the linear stage of the instability in the 2D simulation while dust grains condensate at a point in the nonmagnetized case.

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I. INTRODUCTION

A dusty plasma is common in space, astrophysical, and laboratory plasmas. A dusty plasma consists of a fully or partially ionized plasma and charged (and neutral) dusty grains. The typical dust grain size is the order of a μm , and the charge-to-mass ratio is significantly smaller than the ion and electron charge-to-mass ratio [1]. We can find dusty plasma systems, for example, in Saturn's rings, comet tails, the interstellar medium, and nebulae, and it is thought that dust grains play an important role in those environments. Experimental studies have been conducted in the Magnetized Dusty Plasma Experiment (MDPX) device to investigate a dusty plasma with a background magnetic field [2]. Evidently, dusty plasma systems are easily found in space and astrophysical plasma environments, and it behooves us to better understand the physics of dusty plasma.

The Jeans instability is a distinctive phenomenon in dusty plasmas in contrast to electron-ion plasmas. Due to the large mass of dust grains, gravitational forces among dust grains

cannot be ignored in a dusty plasma system and they attract each other by overcoming thermal pressure, which leads to local condensation. This is called the Jeans/self-gravitational instability, which was first recognized by Jeans [3]. The Jeans instability is not usually considered in an electron-ion plasma system because the electrodynamic force is much larger than the self-gravitational force. The Jeans instability is therefore unique to dusty plasmas and determines the spatial distribution of dust grains.

There have been several theoretical and computational studies of the Jeans instability in a dusty plasma system without a background magnetic field. A standard dispersion relation describing the fluid Jeans instability for charged dusty plasma systems was given by [4,5]. Shukla and Stenflo [6] found that the effect of dust charge fluctuations is to enhance the growth rate, and Verheest *et al.* [7] showed that collisions between charged and neutral dust grains stabilize the instability. The theory of kinetic Jeans instabilities has been developed by [8–11] and generalized for a power-law type of mass distribution [12]. Nonlinear kinetic simulations using particle-in-cell (PIC) simulations have been used to study the electrostatic Jeans instability of dusty plasma systems [13,14], and demonstrated Wollman's prediction in which the electrostatic energy stored in the system is maximized when electrostatic and self-gravitational forces on dust grains are of similar magnitude [15]. In summary, the Jeans instability in an electrostatic dusty plasma has been investigated extensively.

Since the presence of an ambient magnetic field is common in space and astrophysical environments, the Jeans instability in magnetized dusty plasmas has also attracted great attention

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over the past several decades. It was discovered that the Jeans instability in the fluid regime is stabilized by a background magnetic field [16–18]. Mamun and Shukla [19] included a neutral fluid effect in the magnetic Jeans instability and concluded that the neutral fluid brings two new types of instabilities. Besides the electrostatic mode, the linear dispersion relation of electromagnetic waves in a magnetized and self-gravitating dusty plasma has been derived [20,21].

However, the studies of the magnetized Jeans instability are limited to a linear theory in the fluid regime, and the magnetized Jeans instability is poorly understood in the nonlinear and kinetic regime. In the fluid approximation, the velocity distribution function of dust grains is integrated to obtain moments and cannot be correctly resolved during the evolution of the Jeans instability. Related to that, the adiabatic equation of state is typically used to close the fluid equations, despite the justification for such a closure not being justified in the fluid framework. Moreover, dust grains experience transport, heating, and acceleration during the Jeans instability, and they can be properly described only in the kinetic regime. Therefore, it is necessary to investigate kinetic properties of the magnetized Jeans instability, such as the linear growth, velocity distribution, and dust motions, to fully understand the Jeans instability. We note that Mamun *et al.* [22] investigated an electrostatic and low-frequency linear wave dispersion relation in the kinetic regime of a magnetized dusty plasma system.

In this paper, we investigate the Jeans instability in a magnetized dusty plasma using linear kinetic theory and nonlinear kinetic simulations. We derive the linear kinetic dispersion relation for the magnetized Jeans instability to discuss the linear growth and perform PIC simulations to investigate the linear and nonlinear evolution of the dust grains during the instability. PIC simulations have been used previously to investigate the kinetic properties of several dusty plasma systems [13,14,23–25].

II. LINEAR DISPERSION RELATION OF THE JEANS INSTABILITY

In this section, we derive the electrostatic linear dispersion relation of the magnetized Jeans instability in the fluid and kinetic regimes, which are later compared with kinetic simulations. We focus on a wave propagation direction perpendicular to the background magnetic field since magnetic fields have no effect on the Jeans instability along the parallel direction. A static magnetic field is assumed, which means we ignore electromagnetic fluctuations in the system.

A. Fluid Regime

It is instructive to review the dispersion relation of the Jeans instability in a fluid system. We consider the following equations of a fluid dusty plasma:

$$\nabla^2 \Psi = 4\pi G m_d (n_d - n_{d0}); \quad (1)$$

$$(\nabla^2 - \lambda_D^{-2}) \Phi = -4\pi q_d (n_d - n_{d0}); \quad (2)$$

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{u}_d) = 0; \quad (3)$$

$$\frac{\partial \mathbf{u}_d}{\partial t} + (\mathbf{u}_d \cdot \nabla) \mathbf{u}_d = -\frac{\nabla P_d}{m_d n_d} + \mathbf{F}_G + \frac{q_d}{m_d} \left(\mathbf{E} + \frac{\mathbf{u}_d}{c} \times \mathbf{B}_0 \right); \quad (4)$$

$$P_d = P_{d0} \left(\frac{n_d}{n_{d0}} \right)^\gamma. \quad (5)$$

The first and second equations are the Poisson equations for the gravitational potential, Ψ , and electrostatic potential, Φ , due to charged dust grains, respectively. The third and fourth equations are the dust continuity and momentum equations, and the last equation is an adiabatic equation of state for the dust component with the specific heat ratio, γ . The gravitational and electrostatic forces are defined as $\mathbf{F}_G = -\nabla \Psi$ and $\mathbf{E} = -\nabla \Phi$. The dust number density, fluid velocity, and pressure are denoted by n_d , $\mathbf{u}_d$, and P_d , respectively. The subscript 0 indicates a background component, which is assumed to be homogeneous in this study. We can see that the effect of the plasma components (ion and electron) through the form of a Debye length, $\lambda_D^{-2} = 4\pi e^2 (n_{i0}/T_i + n_{e0}/T_e)$, and $(\nabla^2 - \lambda_D^{-2})$ indicates a Debye shielding on dust grains. Here, n_{i0} and n_{e0} are the initial uniform ion and electron number density satisfying the charge neutrality condition, and T_i and T_e are the ion and electron temperatures, which are assumed constant in this study. This plasma description is justified since we consider very low-frequency wave modes associated with dust grains, and plasma inertial effects can be neglected due to $m_i, m_e \ll m_d$. However, at a microscopic scale, ion and electron $\mathbf{E} \times \mathbf{B}$ drifts may cause a plasma instability and contribute to an increase in the plasma temperature (e.g., [26]). Although this is an interesting topic, we cannot capture this effect in our simulation model. To derive Eq. (2), we used the weak potential expansion technique [27]. The other parameters have their standard meanings: G is the gravitational constant, m_d the dust mass, and q_d the dust charge. Here, we assume that the dust mass and charge are constant.

We linearize the governing equations by decomposing the variables into a background and perturbative part as follows:

$$\Psi = \delta \Psi; \quad (6)$$

$$\Phi = \delta \Phi; \quad (7)$$

$$n_d = n_{d0} + \delta n_d; \quad (8)$$

$$\mathbf{u}_d = \delta \mathbf{u}_d, \quad (9)$$

and perform the Fourier transformation, i.e., taking $\nabla \rightarrow i\mathbf{k} = i(k, 0, 0)$ and $\partial_t \rightarrow -i\omega$. We set the background magnetic field along the z -direction, $\mathbf{B}_0 = (0, 0, B_0)$. These operations yield the following linearized and Fourier transformed equations:

$$-k^2 \delta \hat{\Psi} = 4\pi G m_d \delta \hat{n}_d; \quad (10)$$

$$(-k^2 - \lambda_D^{-2}) \delta \hat{\Phi} = -4\pi q_d \delta \hat{n}_d; \quad (11)$$

$$-i\omega \delta \hat{n}_d + ik n_{d0} \delta \hat{u}_{dx} = 0; \quad (12)$$

$$-i\omega \delta \hat{u}_{dx} = -\frac{ik \delta \hat{P}_d}{m_d n_{d0}} - ik \delta \hat{\Psi} + \frac{q_d}{m_d} \left(-ik \delta \hat{\Phi} + \frac{\delta \hat{u}_{dy}}{c} B_0 \right); \quad (13)$$

$$-i\omega \delta \hat{u}_{dy} = -\frac{q_d}{m_d} \frac{\delta \hat{u}_{dx}}{c} B_0. \quad (14)$$

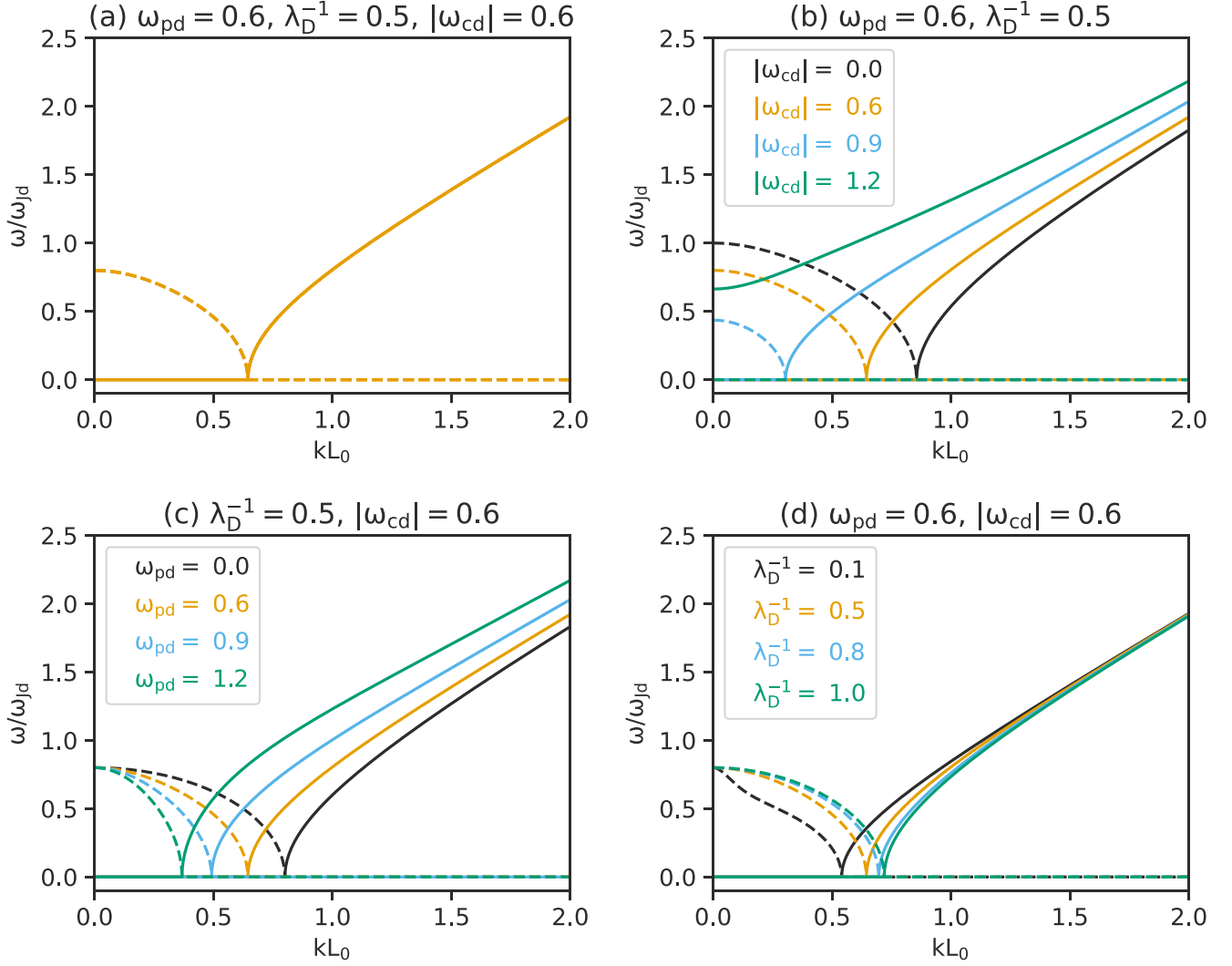


FIG. 1. Fluid dispersion relation, Eq. (15). The solid and dashed lines correspond to the real frequency and growth rate, respectively. Panel (a) shows a reference case, and panels (b)–(d) show the dependence in the dust cyclotron frequency, dust plasma frequency, and Debye length, respectively. Note that the orange line for each panel is identical, and unstable modes are a zero-frequency mode.

Note that we have used $1/n_d \sim 1/n_{d0} - \delta n_d/n_{d0}^2$ and $\delta P_d = (\gamma P_{d0}/n_{d0})\delta n_d$. Here, the hat symbol denotes the Fourier coefficient of the respective variable.

The dispersion relation for linear wave modes can then be found by combining the linearized equations, yielding

$$\omega^2 = C_d^2 k^2 - \omega_{Jd}^2 + \omega_{pd}^2 \frac{k^2}{k^2 + \lambda_D^{-2}} + \omega_{cd}^2. \quad (15)$$

Here, $C_d = \sqrt{\gamma P_{d0}/(m_d n_{d0})}$ is the dust sound speed, $\omega_{Jd} = \sqrt{4\pi G m_d n_{d0}}$ the dust Jeans frequency, $\omega_{pd} = \sqrt{4\pi q_d^2 n_{d0}/m_d}$ the dust plasma frequency, and $\omega_{cd} = q_d B_0/(m_d c)$ the dust cyclotron frequency. It is straightforward to see that gravitation destabilizes the system while dust thermal, electrostatic, and magnetic effects stabilize the system. A similar dispersion relation was derived in [17].

Panel (a) of Fig. 1 shows an example of Eq. (15) with parameters of $\omega_{pd} = 0.6\omega_{Jd}$, $\lambda_D^{-1} = 0.5L_0^{-1}$, and $|\omega_{cd}| = 0.6\omega_{Jd}$. Here, $L_0 = C_d\omega_{Jd}^{-1}$. The solid and dashed lines are the real frequency (ω_r) and growth rate (ω_i), respectively. We

can find an unstable mode ($\omega_i > 0$), which is nonpropagating ($\omega_r = 0$), at wavelengths smaller than $\sim 0.5L_0^{-1}$. The maximum growth rate occurs at the long-wavelength limit $k \rightarrow 0$ and is estimated as $\omega_i^{max} = \sqrt{\omega_{Jd}^2 - \omega_{cd}^2} = 0.8\omega_{Jd}$. For large wave number modes, the linear mode is stabilized and asymptotically approximated by $\omega \sim C_d k$, which can be regarded as a dust sound wave.

Panels (b)–(d) in Fig. 1 show the effect of different choices of parameters, (b) ω_{cd} , (c) ω_{pd} , and (d) λ_D^{-1} , on the fluid dispersion relation. When the magnetic field is absent ($\omega_{cd} = 0$, black line), the growth rate is overall increased and the maximum growth rate corresponds to ω_{Jd} . On the other hand, when the dust cyclotron frequency is larger than the Jeans frequency ($|\omega_{cd}| > \omega_{Jd}$), the unstable mode is stabilized and a cutoff frequency, $\omega_{cut} = \sqrt{-\omega_{Jd}^2 + \omega_{cd}^2} \sim 0.66\omega_{Jd}$, appears at $k = 0$. No linear wave modes are allowed to propagate below the cutoff frequency. Although increasing the dust plasma frequency reduces the unstable region [see panel (c)], the maximum growth rate is unchanged due to the effect of the plasm Debye shielding in the term, $\omega_{pd}^2 k^2/(k^2 + \lambda_D^{-2})$.

The Debye length has a similar effect as the dust plasma frequency, reducing the unstable region as the Debye length decreases. We also see that the profile of the growth rate is modified at $k < \lambda_D^{-1}$ [see the $\lambda_D^{-1} = 0.1L_0^{-1}$ case in panel (d)].

B. Kinetic regime

A few remarks are in order regarding the derivation of a dispersion relation in a kinetic dusty plasma system. In deriving the linear kinetic dispersion relation, we use the Vlasov equation for the dust grains, assuming there are no collisions between species. We assume a static magnetic field as we did in the fluid regime. Since the following derivation largely follows the derivation of Bernstein modes as presented in Gurnett and Bhattacharjee [28], readers are referred to the textbook for more details.

We adopt the following equations to describe a kinetic dusty plasma:

$$\nabla^2 \Psi = 4\pi G m_d (n_d - n_{d0}); \quad (16)$$

$$(\nabla^2 - \lambda_D^{-2}) \Phi = -4\pi q_d (n_d - n_{d0}); \quad (17)$$

$$\frac{\partial f_d}{\partial t} + \mathbf{v} \cdot \nabla f_d + \left[\mathbf{F}_G + \frac{q_d}{m_d} \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}_0 \right) \right] \cdot \nabla_{\mathbf{v}} f_d = 0. \quad (18)$$

The third equation is the Vlasov equation for the dust velocity distribution function, $f_d(t, \mathbf{x}, \mathbf{v})$. The dust number density is calculated by $n_d = \int_{-\infty}^{\infty} f_d d\mathbf{v}$. We have made the same assumption on the plasma, and the other parameters and variables are the same as used in the fluid regime.

By means of the standard normal mode analysis, we derive the dispersion relation by linearizing and taking the Fourier transformation in the configuration space over the kinetic equations, as was done with the derivation of the fluid dispersion relation, except now we decompose the dust velocity distribution function into the background and small perturbation as $f_d = f_{d0} + \delta f_d$. We obtain the following linearized and Fourier transformed equations:

$$-k^2 \delta \hat{\Psi} = 4\pi G m_d \delta \hat{n}_d; \quad (19)$$

$$(-k^2 - \lambda_D^{-2}) \delta \hat{\Phi} = -4\pi q_d \delta \hat{n}_d; \quad (20)$$

$$(-i\omega + i\mathbf{k} \cdot \mathbf{v}) \delta f_d - \omega_{cd} \frac{\partial \delta f_d}{\partial \phi} - \left(\delta \hat{\Psi} + \frac{q_d}{m_d} \delta \hat{\Phi} \right) i\mathbf{k} \cdot \nabla_{\mathbf{v}} f_{d0} = 0. \quad (21)$$

Here, we have introduced cylindrical coordinates for the velocity space using $v_x = v_{\perp} \cos \phi$, $v_y = v_{\perp} \sin \phi$, and $v_z = v_{\parallel}$ where ϕ is the azimuth. Assuming the wave number vector lies in the $x-z$ plane, which yields $\mathbf{k} \cdot \mathbf{v} = k_{\parallel} v_{\parallel} + k_{\perp} v_{\perp} \cos \phi$, we can derive a general form of the dispersion relation as

$$1 + \frac{\omega_{jd}^2}{k^2} \left(1 - \frac{\omega_{pd}^2}{\omega_{jd}^2} \frac{k^2}{k^2 + \lambda_D^{-2}} \right) \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \frac{J_n^2(k_{\perp} v_{\perp} / \omega_{cd})}{k_{\parallel} v_{\parallel} - \omega + n\omega_{cd}} \left[k_{\parallel} \frac{\partial F_{d0}}{\partial v_{\parallel}} + \frac{n\omega_{cd}}{v_{\perp}} \frac{\partial F_{d0}}{\partial v_{\perp}} \right] 2\pi v_{\perp} dv_{\perp} dv_{\parallel} = 0, \quad (22)$$

where $F_{d0} = f_{d0}/n_{d0}$ is the normalized background distribution function, and J_n the n -th order Bessel function.

Assuming a Maxwellian distribution velocity function as the simplest case, we can further proceed with the calculation of the integration, which eventually results in the dust Bernstein modes

$$1 - \frac{2}{\beta_{cd}^2 \omega_{cd}^2} \left(\omega_{pd}^2 \frac{k^2}{k^2 + \lambda_D^{-2}} - \omega_{jd}^2 \right) e^{-\beta_{cd}^2} \sum_{n=1}^{\infty} \frac{I_n(\beta_{cd}^2)}{(\omega/n\omega_{cd})^2 - 1} = 0, \quad (23)$$

where $\beta_{cd} = kv_{thd}/\omega_{cd}$ corresponds to a scale factor between the wavelength and the cyclotron radius, and I_n is the n -th order modified Bessel function of the first kind. Since our interest is only in the perpendicular mode, we have set $k_{\parallel} = 0$. A similar dispersion relation has been derived in [22,29,30]. We can easily recover the same form as the electron Bernstein mode by taking the limit of $\lambda_D \rightarrow 0$ and removing the gravitational term, $\omega_{jd} = 0$ [28].

Figure 2 shows the dependency of the dispersion relation, Eq. (23), for several parameters. The solid and dashed lines correspond to the real frequency and growth rate, respectively, and the dotted lines are the cyclotron harmonics ($= n|\omega_{cd}|$) for each case. We calculate the dispersion relation up to the fifth harmonic. Panel (a) uses the same parameters as panel (a) of Fig. 1, and we find an unstable and nonpropagating mode in the first harmonic mode at $0 < k < 0.7$, showing a similar growth rate profile as the fluid case. In the higher wave number region, the linear mode becomes

stable and asymptotically approaches the first cyclotron harmonic. There exist higher harmonic modes, which have no unstable region. When the dust cyclotron frequency is larger than the Jeans frequency [see panel (b)], the unstable mode vanishes and a cutoff frequency appears in the first harmonic mode. By taking the limit of $k \rightarrow 0$, we find that the cutoff frequency is the same as that found in the corresponding fluid case. The higher harmonic modes are slightly modified. Panel (c) shows a case in which the dust plasma frequency is larger than the Jeans frequency. In this case, the unstable region is reduced while maintaining the same maximum growth rate as found in panel (a). We also notice that the real frequency crosses over their own harmonic and then approaches the harmonic asymptotically from above. The turning point is around $k = \sqrt{\lambda_D^{-2} \omega_{pd}^2 / (\omega_{pd}^2 - \omega_{jd}^2)}$. Decreasing λ_D^{-1} to $0.1L_0^{-1}$ in panel (d) modifies the growth rate profile as it does with the fluid case, and merely affects the harmonic modes. Overall, the Jeans unstable modes in the linear kinetic

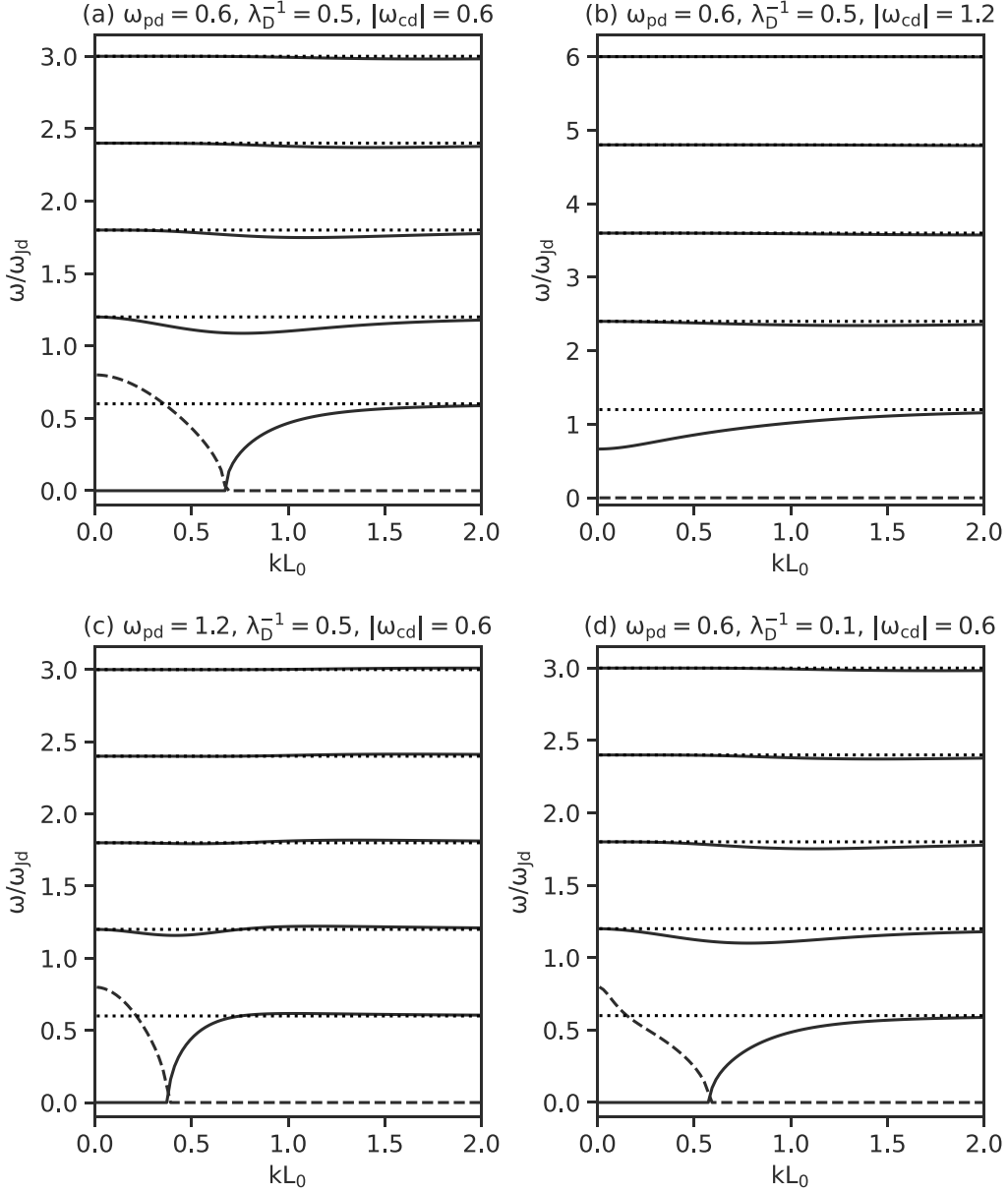


FIG. 2. Kinetic dispersion relation, Eq. (23). The solid, dashed, and dotted lines correspond to the real frequency, growth rate, and cyclotron harmonics ($= n|\omega_{cd}|$), respectively. Panels (a)–(d) adopt the same parameter used in Fig. 1. Note that unstable modes correspond to the smallest harmonic mode ($n = 1$).

dispersion relation behave quite similarly to the fluid dispersion relation.

III. 1D AND 2D KINETIC PIC SIMULATIONS

To investigate the linear and nonlinear evolution of the Jeans instability, we use a PIC method to solve the kinetic dusty plasma equations (16)–(18). We use pseudoparticles to represent the dust distribution function, $f_d(t, \mathbf{x}, \mathbf{v})$, in the 1D/2D configuration space $\mathbf{x} = (X, Y)$ and the three-dimensional velocity space $\mathbf{v} = (V_x, V_y, V_z)$. We define uniform Cartesian grids in the configuration space for the gravitational, electrostatic potentials, and the dust number density. Periodic boundary conditions are imposed, and we

use a spectral method to solve the gravitational and electrostatic Poisson equations. We advance the dust grains by the quadruple Boris method [31]. The second-order shape function is used to calculate the dust number density on grids and to interpolate gravitational and electrostatic forces on the particle position. Our simulation method is almost identical to the standard electrostatic PIC simulation [32]. We randomly and uniformly distribute dust grains in the simulation domain. The initial velocity distribution is a Maxwellian distribution with thermal speed, v_{thd} . We assume that the dust charge number and mass are the same for all dust grains and constant throughout the whole simulation time. The time step is determined by $\Delta t = \frac{C}{|v_{\max}|} \Delta x$, where C is a CFL number (fixed at $C = 0.2$), $|v_{\max}|$ is the maximum speed among all the

dust grains, and Δx is the grid space. Note that we consider negatively charged dust grains, and the dust mass and charge are fixed to a constant.

It is convenient to normalize the governing equations for a numerical calculation. We normalize the time to $\omega_{Jd} = \sqrt{4\pi G n_{d0} m_d}$, length to $L_0 = v_0 \omega_{Jd}^{-1}$, dust number density to n_{d0} , gravitational potential to $4\pi G m_d n_{d0} L_0^2$, electrostatic potential to $q_d L_0^2 / G m_d$, and magnetic field to B_0 . Here, v_0 can be defined by any characteristic speed, and we use the thermal speed of dust grains for the unit speed in the following simulation. Using these units, we obtain the following normalized governing equations:

$$\tilde{\nabla}^2 \tilde{\Psi} = (\tilde{n}_d - 1); \quad (24)$$

$$(\tilde{\nabla}^2 - \tilde{\lambda}_D^{-2}) \tilde{\Phi} = -(\tilde{n}_d - 1); \quad (25)$$

$$\frac{\partial f_d}{\partial t} + \tilde{\mathbf{v}} \cdot \tilde{\nabla} f_d + [-\tilde{\nabla} \tilde{\Psi} - \tilde{\omega}_{pd}^2 \tilde{\nabla} \tilde{\Phi} + \tilde{\omega}_{cd} \tilde{\mathbf{v}} \times \tilde{\mathbf{B}}] \cdot \tilde{\nabla} f_d = 0, \quad (26)$$

where a variable with a tilde denotes a normalized value. For clarity, $\tilde{\omega}_{pd} = \omega_{pd} / \omega_{Jd}$ and $\tilde{\omega}_{cd} = \omega_{cd} / \omega_{Jd}$. Therefore, the system of the above equations is determined by the three parameters: $\tilde{\lambda}_D$, $\tilde{\omega}_{pd}$, and $\tilde{\omega}_{cd}$. Hereafter, we drop the tilde symbol unless necessary.

First, we investigate a 1D stable linear wave case by using a strong dust cyclotron frequency, and compare it to the kinetic dispersion relation. The main purpose for investigating the stable case is to confirm that the PIC simulation yields the same dispersion relation as the theoretical kinetic version. This would be difficult for unstable cases since the spectrum of unstable modes can be significantly strong and mask stable modes. Dust grains are distributed uniformly and randomly, and therefore, initial perturbations are produced due to the finite number of particles. As we have seen in Fig. 2, we set $\omega_{cd} = -1.2$ for a stable case. Other physical parameters are $\omega_{pd} = 0.6$, $\lambda_D^{-1} = 0.5$. We use the number of grid points $N_x = 512$, grid length $\Delta x = 0.1$, and 5000 particles per cell.

Simulated linear stable modes are consistent with the kinetic theoretical dispersion relation for a variety range of frequency and wave number. The color map in Fig. 3 shows the power spectral density of the dispersion relation of the PIC simulation. The dashed and dotted lines correspond to the kinetic theoretical dispersion relation, Eq. (23), and cyclotron harmonics, respectively. We can clearly see that the simulated wave modes agree quite well with the theory. The lowest cyclotron harmonic mode shows a stronger spectrum than the other higher harmonics. Note that the wave mode around $\omega = 0$ can be understood as an entropy mode.

Next, we investigate a 1D unstable case of the kinetic dusty plasma system. We use the same parameters as the stable case, except for keeping $|\omega_{cd}| < \omega_{Jd}$ to make the system unstable to the Jeans instability. Dust grains are initially distributed uniformly and randomly in the simulation domain.

The linear growth rates obtained by the PIC simulation are consistent with the kinetic theoretical growth rate. Figure 4 shows the time evolution of the Fourier coefficient of the density fluctuations at the smallest wave mode ($k = 2\pi/L_x$ where $L_x = N_x \Delta x = 51.2$ or $m = 1$) for $|\omega_{cd}| = 0.0$ (gray), 0.1 (black), 0.3 (orange), 0.5 (blue), 0.7 (green), and 0.9 (red).

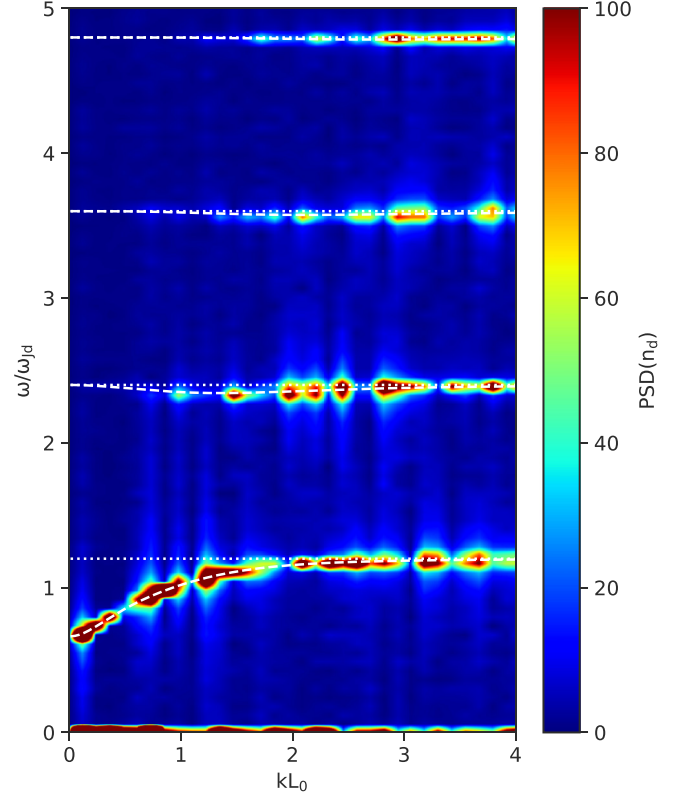


FIG. 3. 1D PIC simulation: Power spectrum of density fluctuations for the stable case ($\omega_{pd} = 0.6$, $\lambda_D^{-1} = 0.5$, and $|\omega_{cd}| = 1.2$). This case corresponds to panel (a) in Fig. 2, and the white dashed and dotted lines are identical to the real frequency and cyclotron harmonics of the appropriate theoretical kinetic dispersion relation.

The linear stage is saturated at around $\omega_{Jd} T = 10$, except for the case of $|\omega_{cd}| = 0.9$. We have shifted these lines along the y axis for ease of viewing. The dashed line corresponds

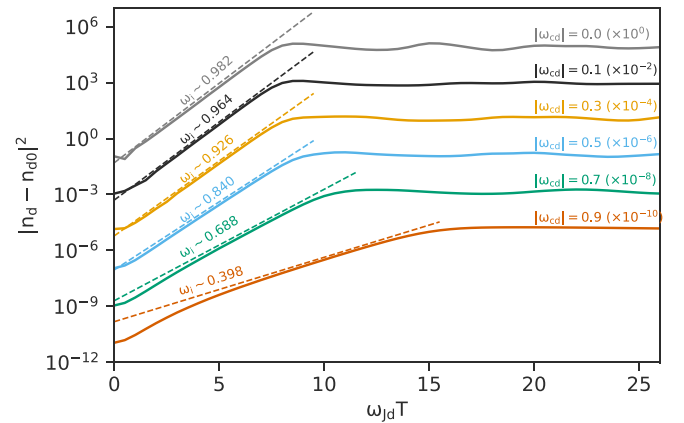


FIG. 4. 1D PIC simulation: Time evolution of the Fourier coefficient of the density fluctuations at the lowest wave mode ($k = 2\pi/L_x$ or $m = 1$) for different dust cyclotron frequencies, $|\omega_{cd}| = 0.0$ (gray), 0.1 (black), 0.3 (orange), 0.5 (blue), 0.7 (green), and 0.9 (red). Here, $L_x = N_x \Delta x = 51.2 L_0$. These profiles are shifted evenly along the y axis for clarity of visualization. The dashed line corresponds to the theoretical kinetic growth rate at the same lowest mode.

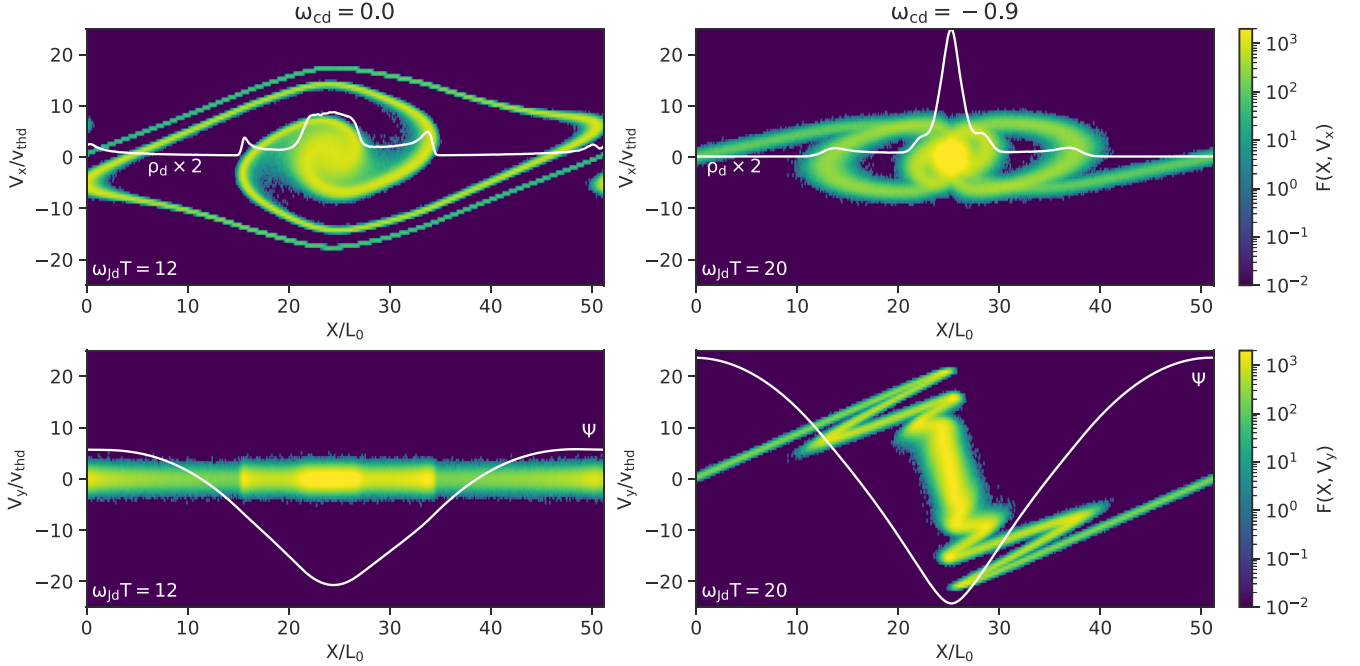


FIG. 5. 1D PIC simulation: $V_x - X$ (top) and $V_y - X$ (bottom) phase-space density of dust grains for $\omega_{cd} = 0.0$ at $\omega_{jd}T = 16$ (left panel) and $|\omega_{cd}| = 0.9\omega_{jd}$ at $\omega_{jd}T = 24$ (right panel). The time is chosen roughly $4\omega_{jd}^{-1}$ after the saturation of the linear stage for each case. Solid white lines represent the dust number density (ρ_d) in the top and gravitational potential (Ψ) in the bottom panel, respectively.

to the theoretical growth rate obtained from the kinetic dispersion relation at the same wave number. We can see that the simulated linear growth rates match the theoretical expectation especially well for weaker magnetic field cases. For the strongest magnetic field case ($|\omega_{cd}| = 0.9$), the simulated growth rate is consistent with the theoretical one in the latter part of the linear stage. The nonlinear stage stays almost constant with slight oscillations for all cases.

The structure of the unmagnetized Jeans instability agrees well with the results of past studies. Figure 5 shows the phase space distribution of V_x (top) and V_y (bottom) for unmagnetized (left) and magnetized (right) cases. The plotted time is

chosen roughly $4\omega_{jd}^{-1}$ after the saturation of the linear stage for each case. We also plot the dust number density profile (top) and gravitational potential (bottom) as white lines. The V_x phase-space plot of the unmagnetized case shows that dust grains oscillate along the gravitational potential and fall to the bottom of the potential. In the same V_y phase space, since there is no magnetic field, dust grains keep their initial thermal spread. Small jumps of the dust density in the top-left panel at $X \sim 25$ and 35 correspond to a location at which the V_x of dust

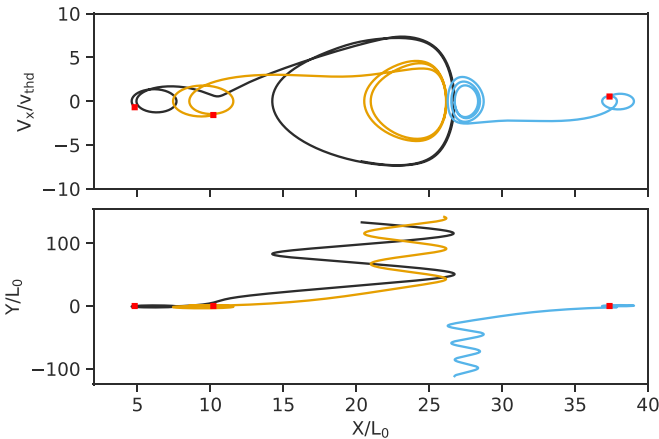


FIG. 6. 1D PIC simulation: Dust grain trajectories in the $V_x - X$ phase space (top) and $X - Y$ plane (bottom) for the case of $|\omega_{cd}| = 0.9\omega_{jd}$. A red square mark indicates the initial position.

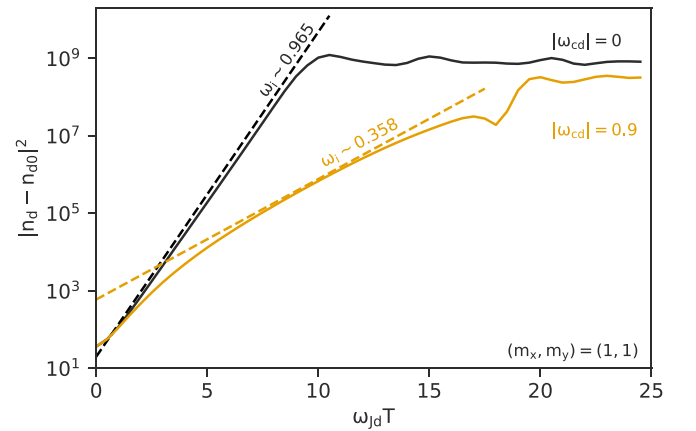


FIG. 7. 2D PIC simulation: Time evolution of the Fourier coefficient of the density fluctuations at the wave mode $((k_x, k_y) = (2\pi/L_x, 2\pi/L_y)$ or $(m_x, m_y) = (1, 1)$) for different dust cyclotron frequencies, $|\omega_{cd}| = 0$ (black), and 0.9 (orange). Here, $L_x = L_y = 51.2L_0$. The dashed line corresponds to the kinetic theoretical growth rate at the same wave mode.

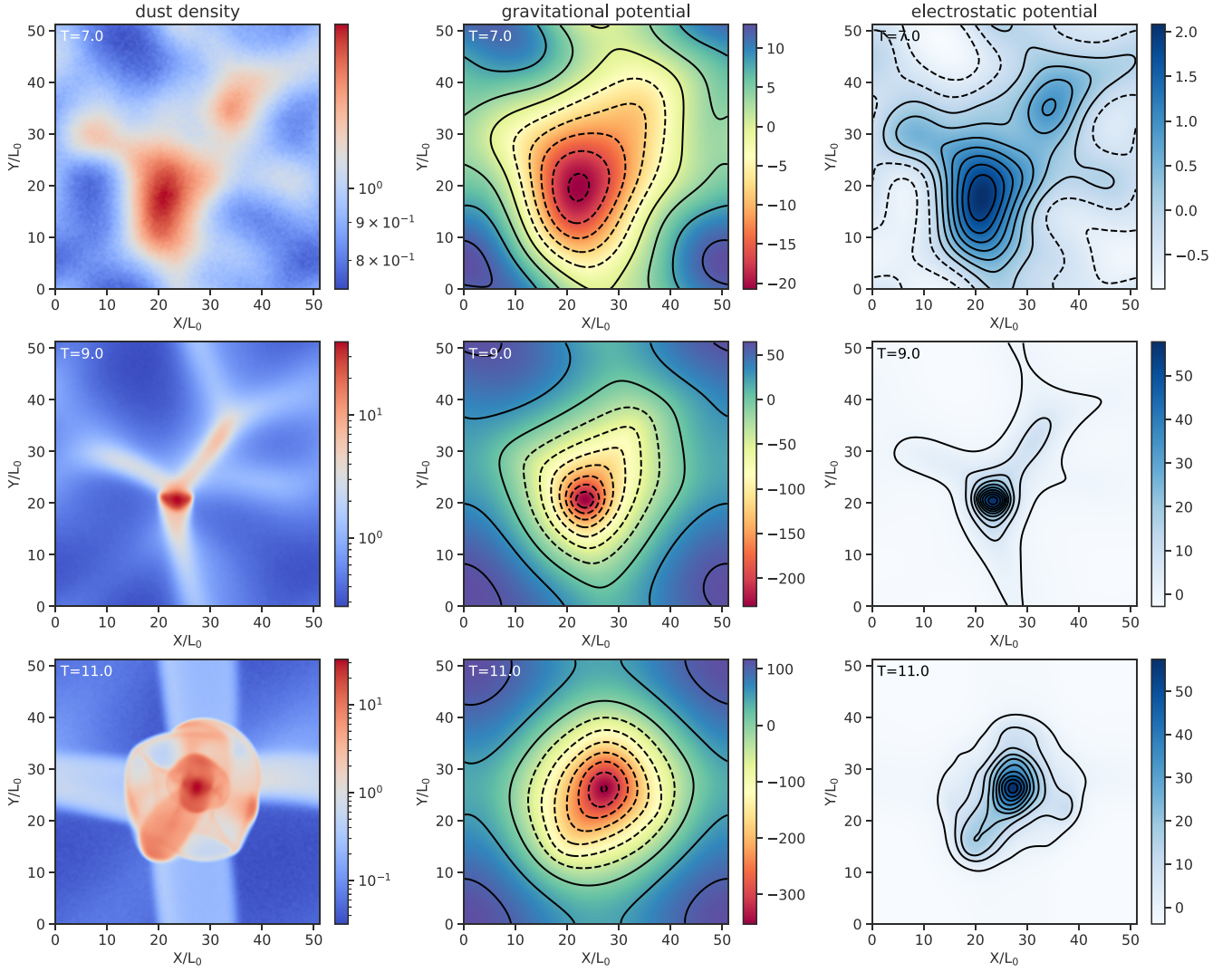


FIG. 8. 2D PIC simulation: Spatial distribution of dust number density (left), gravitational (middle), and electrostatic potential (right) for the unmagnetized case ($|\omega_{cd}| = 0$). Top to bottom, the panels show the time sequence of the Jeans instability. Contour plots in the middle and right panels correspond to the gravitational and electrostatic potentials, respectively.

particles becomes 0. The maximum dust density is $\rho_d \sim 4.5$ at $X \sim 2$, corresponding to the bottom of the gravitational potential. These structures have been found in several studies of Vlasov-Poisson simulations [33,34].

The background magnetic field modifies structures in the nonlinear stage of the Jeans instability. The V_x phase-space plot of the magnetized case (top-right panel in Fig. 5) forms a dipole-field-like structure, and there are no fast dust grains passing by the bottom of the gravitational potential as observed in the unmagnetized case. This can be easily understood by looking into typical trajectories of dust grains as shown in the top panel of Fig. 6. The red dot corresponds to the initial position. Initially, dust grains gyrate around their initial position and are then accelerated toward the center of the simulation domain at $X \sim 26$. Since the magnetic field is strong, dust grains are accelerated along the y -direction due to the magnetic Lorentz force (see the bottom-right panel in Fig. 5). After reaching the center, the dust grains gyrate with a turning point of $V_x = 0$ at the center. The profile of the dust

number density is steeper than for the unmagnetized case, and the maximum dust number density is $\rho_d \sim 12.5$, which is approximately 3 times larger than the unmagnetized case. The steeper dust number density profile reflects the steeper gravitational potential as shown in the bottom-right panel of Fig. 5.

Dust grains drift along the y -direction due to the gravitational force and the magnetic Lorentz force. The bottom panel of Fig. 6 shows dust grain trajectories in the $X - Y$ plane. After the dust grains are accelerated and reach the center, we can see that the grains start drifting along the y -direction associated with the gyration. We integrate the particle velocity V_y over time t to calculate the position Y , such as $Y = \int V_y dt$. The dust drift is caused by $\mathbf{F}_G \times \mathbf{B}_0$, and the drift speed can be written as $\tilde{V}_{Dy} = \frac{1}{\omega_{cd}} \frac{\partial \Psi}{\partial \tilde{x}}$ in normalized form. By linearly approximating the gravitational force from $X = 14$ to 24 as $\frac{\partial \Psi}{\partial \tilde{x}} \sim \frac{-150-10}{24-14} = -16$, we obtain the drift speed, $V_{Dy} = -16/(-0.9) \sim 17.8 v_{thd}$. This corresponds to the simulated drift speed.

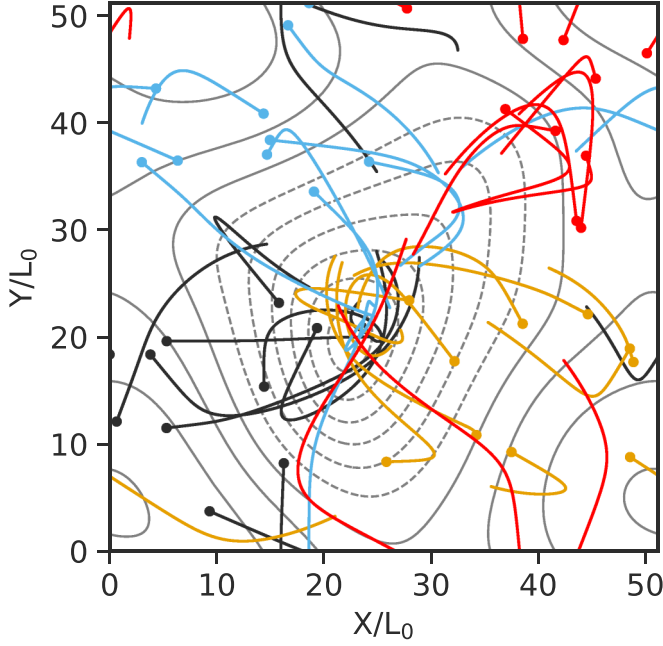


FIG. 9. 2D PIC simulation: Dust grain trajectories in the $X - Y$ plane from $\omega_{Jd}T = 0$ to 10 for the unmagnetized case ($\omega_{cd} = 0$). The trajectories are differentiated by color according to their initial position in a quadrant. A large dot indicates the grain's initial position. The gray lines correspond to the gravitational potential at $\omega_{Jd}T = 10$.

Finally, we perform two-dimensional PIC simulations to investigate the evolution of the magnetized Jeans instability in multidimensions. We set $\Delta x = \Delta y = 0.2$ and $N_x \times N_y = 256 \times 256$. We focus on unmagnetized ($\omega_{cd} = 0$) and magnetized ($\omega_{cd} = -0.9$) cases while fixing the other physical parameters, such as $\omega_{pd} = 0.6$ and $\lambda_D^{-1} = 0.5$. We use 2000 particles per cell. Dust grains are uniformly and randomly distributed in the simulation domain as the initial condition. The background magnetic is directed along the z -direction.

The simulated linear growth rate agrees well with the theoretical growth rate. Figure 7 shows the time evolution of the Fourier coefficient of density fluctuations at $(k_x, k_y) = (2\pi/L_x, 2\pi/L_y)$ or $(m_x, m_y) = (1, 1)$ for the unmagnetized (black) and magnetized (orange) cases. Here, $L_x = N_x \Delta x = 51.2$, and $L_y = N_y \Delta y = 51.2$. The dashed line corresponds to the theoretical growth rate at $k_\perp = \sqrt{k_x^2 + k_y^2}$ using the fluid and kinetic dispersion relations for the unmagnetized and magnetized cases. We can see that the simulated growth rate of the unmagnetized case is identical to the theoretical growth rate. Although the simulated growth rate of the magnetized case is initially larger than the theoretical growth rate, nonetheless, the two are reasonably consistent at later times, which is consistent with the 1D case when $|\omega_{cd}| = 0.9$. Note that the unmagnetized saturation level is larger than the magnetized case, respectively.

Without a background magnetic field, dust grains concentrate at a point isotropically. The left panels of Fig. 8 present the time sequence of the spatial dust-density distribution during the instability. At saturation ($T = 10$), dust grains are concentrated around the point $(X, Y) = (20, 25)L_0$. The shape

of the concentration is isotropic, and the maximum density at the concentration point is about $30n_{d0}$. In the nonlinear state ($T = 12$), it is observed that the concentration structure is broadened. This is because of the inertia of dust grains, allowing them to pass through the concentration point. Figure 9 shows dust grain trajectories with the contour lines of the gravitational potential at $T = 8$. We can see that dust grains primarily move along the gradient of the gravitational potential toward the concentration point.

We observe that the gravitational potential forms a broad shape while the electrostatic potential has a similar shape to the dust density in the unmagnetized case. The middle and right panels of Fig. 8 show the time sequence of the gravitational and electrostatic potential with their contour lines, respectively. Although both of the potentials change their magnitude at the concentration point, their profile keeps a round shape, reflecting the spatial density distribution. Since the distance between two contour lines around the concentration point of the gravitational potential is longer than that of the electrostatic potential, this indicates that the electrostatic potential is steeper than the gravitational potential. This is because dust grains are shielded by the plasma on the scale of λ_D , cutting off longer-range electrostatic forces, and the electrostatic potential resembles the dust density distribution more than the gravitational potential. This can be easily understood by taking the limit of $\lambda_D^{-1} \gg k$, which leads to $\delta\psi = \frac{4\pi q_d \delta n_d}{k^2 + \lambda_D^{-2}} \sim \frac{4\pi q_d \delta n_d}{\lambda_D^{-2}}$.

In the magnetized case ($\omega_{cd} = -0.9$), dust grains condense to form filaments. The left panels of Fig. 10 show the time sequence of the spatial density distribution. From $T = 16$ to 18, we can observe that four large-density filaments form a concave-sided diamond shape, and obviously, there is no point-wise condensation as seen in the unmagnetized case. At saturation ($T = 20$), the filaments move toward the center of the simulation domain and rotate clockwise. Note that the maximum value of the dust density is about $10n_{d0}$.

The density filamentation can be understood as the consequence of the dust cyclotron motion. Figure 11 displays several trajectories of the dust grains in the simulation domain. The contour lines of the gravitational potential at $T = 20$ are also plotted. The dust coloring is the same as Fig. 9. In the initial stage, dust grains gyrate around the background magnetic field at their initial position with a typical radius of $v_{thd}/|\omega_{cd}|$. Most of the dust grains are then accelerated and kicked by the gravitational force, which leads to a larger gyroradius. The accelerated dust grains continue to gyrate with the large gyroradius, and their direction of the gyromotion corresponds to the observed filaments.

The gravitational and electrostatic potential exhibit distinctive profiles in the magnetized case. The middle and right panels of Fig. 10 are the time sequence of the gravitational and electrostatic potential with their contour lines, respectively. As mentioned in the unmagnetized case, since the electrostatic potential is shielded by the plasma, the potential resembles the spatial density distribution. In fact, the maximum value of the electrostatic potential is in filaments, just as with the density distribution. On the other hand, the gravitational potential does not resolve the filamentary structure. This can be because the width of a filament is a few L_0 , and such

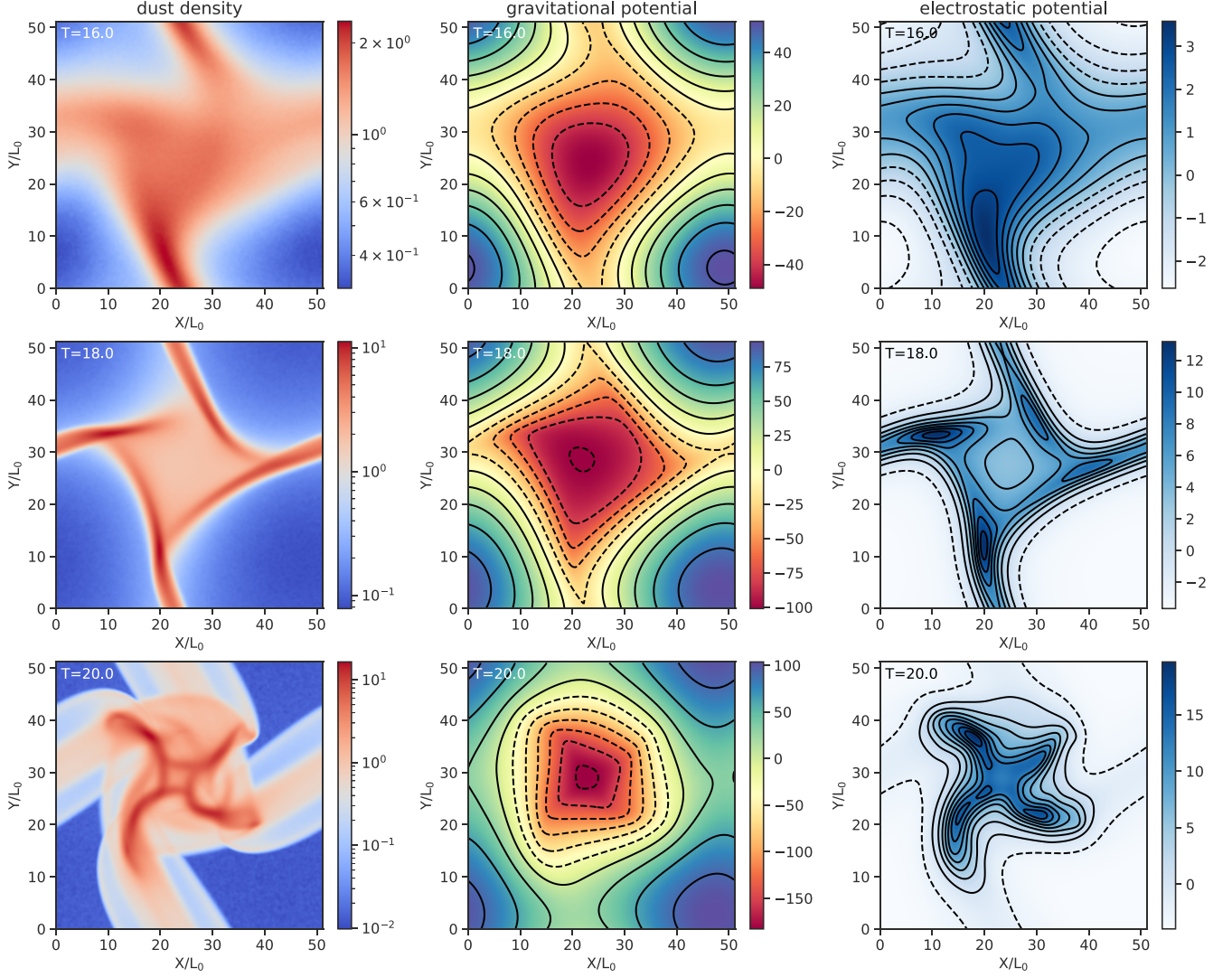


FIG. 10. 2D PIC simulation: Spatial distribution of dust number density (left), gravitational (middle), and electrostatic potential (right) for a strong magnetic field case ($|\omega_{cd}| = 0.9\omega_{jd}$). From top to bottom, the panels show the time sequence of the Jeans instability. The contour plots in the middle and right panels correspond to the gravitational and electrostatic potential, respectively.

a small scale structure is halted by k^2 in the gravitational Poisson equation as $\delta\psi = \frac{4\pi G\delta n_d}{k^2}$ compared to larger scale structures.

IV. SUMMARY

In this study, we have investigated the Jeans instability of a magnetized dusty plasma using PIC simulations and compared with the kinetic linear theory. Our findings are summarized as follows:

(1) When the dust cyclotron frequency is larger than the Jeans frequency, the Jeans instability is stabilized. The kinetic linear theory predicts cyclotron harmonics in the dispersion relation, and the PIC simulation and the kinetic theory are consistent. This validates that the PIC method is useful in studying the magnetized dusty plasma system. The PIC simulation also exhibits an entropy mode at $\omega = 0$.

(2) For unstable cases ($|\omega_{cd}| < \omega_{jd}$), the linear growth rate simulated in 1D and 2D PIC simulations reasonably

agrees well with the kinetic linear growth rate. Although we find that the kinetic growth rate is more consistent with the simulation results of weaker magnetic field cases, the kinetic linear theory is still demonstrated by the PIC simulation. We can therefore conclude that the kinetic linear theory is verified and suitable to predict the growth rate of the magnetized Jeans instability.

(3) We find a dipole-field-like structure in the phase space of the 1D magnetized case, which is quite different from the unmagnetized case, where the V_x component of dust grains is decelerated toward the bottom of the gravitational potential. This is caused by the gyromotion of accelerated dust grains. The accelerated dust grains also drift along the y -direction due to the $\mathbf{F}_G \times \mathbf{B}$ drift mechanism.

(4) In the 2D magnetized simulation, the condensation of dust grains forms a filament, which is quite different from the point-wise condensation seen in the unmagnetized case. Dust grains are accelerated by the gravitational potential steepened by the Jeans instability, so that the gradient is toward the

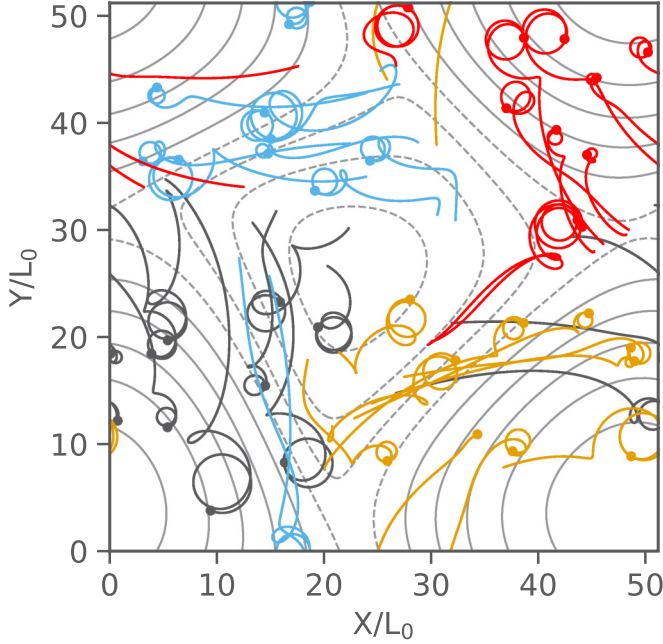


FIG. 11. 2D PIC simulation: Dust grain trajectories in the $X - Y$ plane from $\omega_{Jd}T = 0$ to 20 for a strong magnetic field case. The trajectories are differentiated by color according to their initial position in a quadrant. A large dot indicates the grain's initial position. The gray lines correspond to the gravitational potential at $\omega_{Jd}T = 20$.

center of the simulation domain. The accelerated dust grains gyrate due to the background magnetic field with a large gyro-radius, and this gyromotion corresponds to the filamentation.

(5) The gravitational and electrostatic potential show quite distinctive profiles. This is because an electrostatic potential longer than the Debye length λ_D is shielded by the background plasma, which yields a similar profile to the density distribution. On the other hand, since there is no shielding for the gravitational potential and the filament width is a few L_0 , the gravitational potential is determined by larger-scale structures, and the small filamentary structure is not reflected in the potential.

As a final remark, we comment on the dust parameters and magnetic field. We have seen that the ratio of the dusty cyclotron to the dust Jeans frequency is a critical parameter, therefore, it is useful to estimate parameters that satisfy the condition $\omega_{cd}/\omega_{Jd} \sim 1$. Assuming the dust grains are in a cold interstellar medium [35–38], Fig. 12 shows the magnitude of

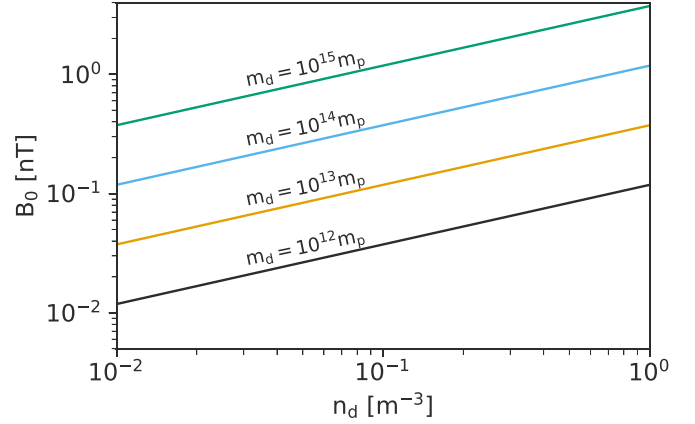


FIG. 12. Strength of a magnetic field satisfying a condition of $\omega_{cd} = \omega_{Jd}$ as a function of the dust density, n_d , for several dust mass values, m_d . Here, we fix the dust charge-to-mass ratio as $10^{-10}(e/m_p)$, and m_p is the proton mass.

a magnetic field satisfying $\omega_{cd} = \omega_{Jd}$, corresponding to solid lines, as a function of the dust density n_d for several dust mass values m_d with a fixed dust charge-mass ratio $10^{-10}(e/m_p)$. Being above (or below) a line indicates that the dust cyclotron frequency is larger (or smaller) than the Jeans frequency. For the case that the magnetic field is 0.1 nT, dust grains with mass $10^{14}m_p$ and $10^{15}m_p$ are not stabilized by the magnetic field, while those with mass $10^{12}m_p$ are stabilized. Dust structures due to the Jeans instability for the dust mass $10^{13}m_p$ and $n_d \sim 10^{-1} \text{ m}^{-3}$ can be critically modified by the magnetic field, as we have discussed in this paper.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author (M.N.), upon reasonable request.

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