

Flow generation via catastrophic loss of equilibrium in weakly rotating self-gravitating fluids: A minimal idealized model

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This paper explores the catastrophic energy transformations, in particular the ones leading to the generation of a flow in a weakly rotating self-gravitating fluid/gas found, for instance, in the vicinity of a massive compact object. Because of the similarity in the governing equations, the system dynamics is worked out exactly in parallel to the methods developed for investigating catastrophic relaxation in stellar plasmas [S. Ohsaki *et al.*, *Astrophys. J.* **559**, L61 (2001); **570**, 395 (2002); E. Saralidze *et al.*, *Astrophys. Space Sci.* **370**, 64 (2025)]. In the latter, a more “complex” equilibrium state, on slow changes in the environment, can lose its equilibrium (catastrophe), and transform to a less complex state with a very different energy mix from the original. It is shown that a similar transformation in the weakly rotating self-gravitating fluid/gas will convert much of its gravitation energy into kinetic energy in the flow. Since flows are a perennial ingredient of high-energy astrophysical systems, the energy transformation processes revealed in the present study, can advance our understanding of a variety of them. Some particularly relevant examples are macroscale flows / structures in galaxies, accretion discs, and the dynamics and stability of a rotating star / its atmosphere.

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I. INTRODUCTION

In this paper, we develop a minimal idealized model for catastrophic (when an equilibrium is suddenly lost) flow generation in a weakly rotating self-gravitating neutral fluid/gas surrounding astrophysical objects like extremely massive compact object [active galactic nucleus (AGN), quasars]. Such flows could become the source of relativistic jets that emerge from these formations; these jets could extend a million parsecs in length.

The accretion disks providing the “material” (matter, energy, and angular momentum) in outflows/jets/winds [1,2] play a crucial role in the star formation process—naturally, the resulting disk-jet structure, appropriately, dictates the evolution of the star [3–6].

In order to explore the problem of flow generation, we will employ, here, a framework similar to the one developed to study catastrophic (as well as quiescent) phenomena in plasmas constituting the stellar atmospheres [7–10]. The systems have tremendous similarities despite the fact that the stellar plasmas are controlled, mostly, by electromagnetism while the dynamics of the weakly rotating self-gravitating neutral matter in accretion disks is purely gravitational. This happens because linear but relativistic GR (general relativity) is not Newtonian in the weak-field limit of GR, the rotation of a self-gravitating fluid can twist the background space time surrounding it (known as frame dragging identified with a magnetic-type gravitational field [11,12], gravitomagnetic field); this dynamics, governed by the equations derived by taking the weak-field, slow-velocity limit of Einstein’s equation [13,14], has a structure that has almost one-to-one

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correspondence with Maxwell equations. Because of this mathematical similarity in the governing equations, the results are expected to have much in common. In fact, it was shown in [15] that the systems like the accretion disk-jet outflows, even when they are vastly different in characteristic parameters (like the Lorentz factor, Reynolds number, Lundquist number, ionization rate, etc.) have a common macroscopic geometry in addition to a shared mathematical structure.

The general shared mathematical structure consists of what has been described as Beltrami-Bernoulli (BB) equilibria [16–19]; the details of the system simply change the parameters without affecting the structure of the system. Since many authors have analyzed BB equilibria in various settings, we will refer to them at the appropriate stage of our calculation.

II. DESCRIPTION OF THE THEORETICAL MODEL, MODEL EQUATIONS

We will now go directly to the set of equations governing the dynamics of the weakly rotating self-gravitating matter. Our equations and initial (equilibrium) methodology is guided by the original paper of [16] (recently [13] used it for the system under consideration) while the latter part of the calculation (dealing with catastrophic loss of equilibrium) is heavily influenced by [7,8].

When the space-time metric is almost Minkowskian, terms of $O(c^{-4})$ and higher can be neglected, and the linearized Einstein equations appear almost identical to Maxwell equations:

$$\nabla \cdot \mathbf{E}_g = -4\pi G_s \rho, \quad (1)$$

$$\nabla \cdot \mathbf{B}_g = 0, \quad (2)$$

$$\nabla \times \mathbf{E}_g = -\frac{1}{c} \frac{\partial}{\partial t} \mathbf{B}_g, \quad (3)$$

$$\nabla \times \mathbf{B}_g = \left(-\frac{4\pi}{c} G_s \rho \mathbf{v} + \frac{1}{c} \frac{\partial}{\partial t} \mathbf{E}_g \right). \quad (4)$$

The total gravitational force, “Lorentz force,” acting on a fluid particle is

$$\mathbf{f}_g = \rho \left(\mathbf{E}_g + \frac{1}{c} \mathbf{v} \times \mathbf{B}_g \right).$$

Also, the equation of motion governing the self-gravitating charge less fluid [14,20],

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla p - \rho \left(\mathbf{E}_g + \frac{1}{c} \mathbf{v} \times \mathbf{B}_g \right), \quad (5)$$

is simply a replica of the equivalent “Lorentz force” equation of a negatively charged fluid [14]. The gravitating fluid obeys the continuity equation (holds everywhere):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \quad (6)$$

In the preceding equations, $\mathbf{v}$ is the fluid velocity, ρ is the mass density of neutral matter, p is local fluid pressure, $\mathbf{E}_g = -\nabla \phi_g - \frac{1}{c} \frac{\partial \mathbf{A}_g}{\partial t}$ is the local gravitational field (so called “gravitoelectric” field) with the corresponding local gravitational potential ϕ_g , and $\mathbf{B}_g = \nabla \times \mathbf{A}_g$ is the “gravitomagnetic” field with the corresponding $\mathbf{A}_g$ “vector” potential. Hence, we are taking into account the self-gravity of the fluid/gas, with G_s

being the gravitational constant characterizing the fluid/gas. Equations (1)–(6) (reflecting the attractive nature of gravity) will serve as the foundations for this work; following the methodology of [7,8,10,18], we will now formulate the gravitational equivalent of catastrophic flow generation. We will also compare and contrast the behavior of flows generated by weakly rotating self-gravitating and electromagnetic systems. Notice, that in the present paper we study the equilibrium configurations of a weakly rotating self-gravitating neutral fluid in the weak field limit; in the PPN coordinate frame, in the weak field slow motion limit, Eqs. (1)–(6) with the corresponding scalar and vector potentials of “gravitoelectromagnetic field” (GEM) look like Maxwell equations ([14,20–25] and references therein). We understand that for the stationary background space time, the time-dependent term in (3) is absent. In our equilibrium analysis, this term does not appear anyway (it is written just to display the similarity to Maxwell equations). However, in the time dependent analysis we plan to carry out in our next paper, this term, though small, is likely to play a significant role. Even though the effects of GEM are small, their study is important for more complex gravitational phenomena on different scales (in [26] this issue is well discussed). In fact, in our next paper, we will emphasize the physics that originates in GEM. In the present equilibrium analysis, it does not play a role, and the “gravitoelectric limit” is sufficient.

III. QUASIEQUILIBRIUM ANALYSIS: EXPLOSIVE EVENT LEADING TO CATASTROPHIC FORMATION OF FLOW

In analogy with [7,8], we expect two distinct scenarios for explosive/eruptive events for the weakly rotating self-gravitating system: (i) when a slowly evolving structure finds itself in a state of no equilibrium, and (ii) when the process of creating a long-lived structure is prematurely aborted; the flow shrinks/distorts the structure, which suddenly shines and/or releases energy or ejects particles. The latter mechanism requires a detailed time-dependent treatment and is not the subject matter of this study. The former semiequilibrium, collisionless gravitomagnetofluid treatment will be pursued here. For simplicity, our analysis is limited to constant density structures. Though somewhat drastic, this step will help us capture the underlying physics in a much simpler analytically tractable calculation. Including density variation, though, is straightforward [19,27,28].

A. Beltrami-Bernoulli equilibrium states

We begin by introducing some definitions: $\mathbf{P}_g = \mathbf{v} - \frac{\mathbf{A}_g}{c}$ is the generalized momentum, $\Phi = \frac{p}{\rho} + \frac{v^2}{2} - \phi_g$ is the generalized pressure, and

$$\mathbf{\Omega} = -\frac{\mathbf{B}_g}{c} + \nabla \times \mathbf{v} \quad (7)$$

is the generalized vorticity (analog of the generalized vorticity of negatively charged fluid).

Then, using Eqs. (1)–(4), we can derive a stationary double Beltrami (DB) equilibrium—a Beltrami equilibrium results from the alignment of an effective velocity along its (generalized) vorticity: $\mathbf{\Omega} = -|b|\rho \mathbf{v}$, the constant of propor-

tionality b is a parameter of the system. The equilibrium does, simultaneously, satisfy the Bernoulli condition—the vanishing of gradient forces ($\nabla\Phi = 0$) and the time-independent continuity equation $\nabla(\rho\mathbf{v}) = 0$. The principal result of the procedure, however, is the emergence of the promised double Beltrami (DB) equation

$$\nabla \times \nabla \times \mathbf{v} + |b|\nabla \times (\rho\mathbf{v}) + \frac{4\pi G_s \rho}{c} \mathbf{v} = 0; \quad (8)$$

the preceding equation, in this context, was first derived by [13] to describe the relaxation of self-gravitating weakly rotating astrophysical flows. Notice, that all the terms in (8) are with positive sign. That is because of the minus sign of current in “Ampère’s law” [see the r.h.s. of the Eq. (4)]. DB equation and its solutions were studied in detail in the context of Hall MHD (see, e.g., [10,18,19,28]) both for astrophysical and laboratory plasmas [29].

By assuming $\rho = \rho_0 \approx \text{const}$, and introducing normalized variables,

$$\mathbf{r}' \rightarrow \lambda_j^{-1} \mathbf{r}, \quad \mathbf{v}' \rightarrow \frac{\mathbf{v}}{c}, \quad \mathbf{B}'_g \rightarrow \frac{\lambda_j}{c^2} \mathbf{B}_g, \quad (9)$$

$$p' \rightarrow \frac{p}{\rho_0 c^2}, \quad |b|\rho_0 \lambda_j \equiv b' > 0$$

in terms of the characteristic parameters of the system: $\omega_j = \sqrt{4\pi G_s \rho_0}$ (Jean’s frequency), $\lambda_j = c_{s0}/\omega_j$, (Jean’s length) (where c_{s0} is the speed of sound), and

$$\kappa = \frac{\omega_j^2 \lambda_j^2}{c^2} = \frac{c_{s0}^2}{c^2} \ll 1.$$

Equation (8) is written in a dimensionless form ($b > 0$ and $\kappa > 0$ by definition) as follows (primes are omitted):

$$\nabla \times \nabla \times \mathbf{v} + b\nabla \times \mathbf{v} + \kappa \mathbf{v} = 0. \quad (10)$$

Comparing this equation with the DB equation used for the two-fluid plasma [7] we find a sign difference in the last terms; also, the sign of the second term is now fixed. For the neutral fluid both terms are with positive signs. We will see the consequences of this effect below.

Equation (10) can be formally written as ($\text{curl} = \nabla \times$)

$$(\text{curl} - \lambda_+)(\text{curl} + \lambda_-)\mathbf{v} = 0, \quad (11)$$

where λ_+ and λ_- are the solutions of quadratic equation

$$\lambda^2 + b\lambda + \kappa = 0, \quad (12)$$

yielding ($b > 0$)

$$\lambda_{\pm} = \frac{1}{2}(-b \pm \sqrt{b^2 - 4\kappa}). \quad (13)$$

The general solution of the DB equation is a linear combination of single Beltrami fields ($\mathbf{G}_{\pm}$), which satisfy the following condition: $\nabla \times \mathbf{G}_{\pm} = \lambda_{\pm} \mathbf{G}_{\pm}$ [18,30]. The velocity $\mathbf{v}$, then, may be expressed as a linear superposition

$$\mathbf{v} = C_+ \mathbf{G}_+ + C_- \mathbf{G}_-. \quad (14)$$

Using (14) and the expression for vorticity $\boldsymbol{\Omega} = -b\mathbf{v}$, the generalized vorticity

$$\mathbf{B}_g = b\mathbf{v} + \nabla \times \mathbf{v} \quad (15)$$

becomes

$$\mathbf{B}_g = (b + \lambda_+)C_+ \mathbf{G}_+ + (b + \lambda_-)C_- \mathbf{G}_-. \quad (16)$$

The nonzero coefficient κ contains the Jean’s length λ_j that is not zero due to the inclusion of fluid self-gravity. The relation between κ and b will determine the characteristic scales (could be highly separated) of the system.

If $\frac{\kappa}{b^2} \ll 1$, the roots $\lambda_{\pm}$ will be vastly separated (system exhibits the macro- and microscale structures):

$$\lambda_+ = \frac{1}{2}(-b + \sqrt{b^2 - 4\kappa}) \approx -\frac{\kappa}{b} < 0, \quad (17)$$

$$\lambda_- = \frac{1}{2}(-b - \sqrt{b^2 - 4\kappa}) \approx -b < 0, \quad (18)$$

implying

$$|\lambda_-| \gg |\lambda_+|, \quad (19)$$

admitting macro- (inverse of $|\lambda_+|$) and micro- (inverse of $|\lambda_-|$) structures.

B. Conservation laws

We consider adiabatic processes when the parameters change quite slowly, assuming that a given structure at every stage can be described by the DB equilibrium solutions [7,8]. For a system governed by DB equation, there are three conserved quantities: the total energy and two helicities. The general DB field is characterized by four parameters—with its eigenvalues ($\lambda_{\pm}$) and amplitudes ($C_{\pm}$) [31]. The helicity is generally defined as follows (with $\boldsymbol{\Omega}$ being the vorticity):

$$h = \frac{1}{2} \int (\text{curl}^{-1} \boldsymbol{\Omega}) \cdot \boldsymbol{\Omega} d\mathbf{r}. \quad (20)$$

For the total energy of our weakly-rotating, self-gravitating fluid, we have the following [13,14,22,24]:

$$E = \frac{1}{2} \int \left(v^2 - \frac{B_g^2}{\kappa} \right). \quad (21)$$

Notice, that unlike the conventional plasma magnetic field energy, the gravitomagnetic field energy is negative reflecting the attractive character of gravity [14,22]. The conventional fluid helicity

$$h_{\min} = \frac{1}{2} \int \mathbf{v} \cdot (\nabla \times \mathbf{v}) d\mathbf{r} \quad (22)$$

is also changed to

$$h_{\text{gen}} = -\frac{b}{2} \int \left(v^2 - \frac{B_g^2}{\kappa} \right) d\mathbf{r} = -bE \quad (23)$$

for the self-gravitating fluid.

For simplicity, our domain is a Cartesian cube of length L (similar to [7,8]). We model the Beltrami field $\mathbf{G}_{\pm}$ by the simple two-dimensional ABC field ([32]),

$$\mathbf{G}_{\pm} = \mathbf{g}_{x\pm} \begin{pmatrix} 0 \\ \sin(\lambda_{\pm} x) \\ \cos(\lambda_{\pm} x) \end{pmatrix} + \mathbf{g}_{y\pm} \begin{pmatrix} \cos(\lambda_{\pm} y) \\ 0 \\ \sin(\lambda_{\pm} y) \end{pmatrix}, \quad (24)$$

with the normalization $|\mathbf{g}_{x\pm}|^2 + |\mathbf{g}_{y\pm}|^2 = 1$. Periodic boundary condition $L = n_+ \frac{2\pi}{\lambda_+} = n_- \frac{2\pi}{\lambda_-}$ defines the intergers with $n_{\pm}$; in addition, $\int (\mathbf{G}_+ \cdot \mathbf{G}_-) d\mathbf{r} = 0$, $\int \mathbf{G}_{\pm}^2 d\mathbf{r} = L^2$.

In terms of the parameters of the DB solutions, we may write the minimal helicity (fluid without self-gravity) and the generalized (fluid with self-gravity) helicity:

$$h_{\min} = \frac{L^2}{2} (\lambda_+ C_+^2 + \lambda_- C_-^2), \quad (25)$$

$$h_{\text{gen}} = h_{\min} - L^2 ((b + \lambda_+)^2 C_+^2 + (b + \lambda_-)^2 C_-^2) + \frac{L^2}{2} \left((b + \lambda_+)^2 \frac{C_+^2}{\lambda_+} + (b + \lambda_-)^2 \frac{C_-^2}{\lambda_-} \right), \quad (26)$$

and, then, the amplitudes C_+^2 and C_-^2 are now expressible in terms of helicities

$$C_{\pm}^2 = \frac{2h_{\min}}{L^2 \lambda_{\pm}} - \frac{\lambda_{\mp}}{\lambda_{\pm}} C_{\mp}^2. \quad (27)$$

Using (16) and (27) in (21), we obtain

$$E = \frac{L^2}{2} \left(1 - \frac{(b + \lambda_+)^2}{\kappa} \right) C_+^2 - \frac{L^2}{2} \left(1 - \frac{(b + \lambda_-)^2}{\kappa} \right) \frac{\lambda_+}{\lambda_-} C_+^2 + \left(1 - \frac{(b + \lambda_-)^2}{\kappa} \right) \frac{h_{\min}}{\lambda_-}, \quad (28)$$

from which, using (27) and the following relations

$$\lambda_+ + \lambda_- = -b, \quad \lambda_+ \lambda_- = \kappa,$$

we derive

$$C_+^2 = \frac{2\lambda_-}{L^2} Q^{-1} \left(E - \frac{h_{\min}}{\lambda_-} \left(1 - \frac{\lambda_+^2}{\kappa} \right) \right), \quad (29)$$

$$C_-^2 = -\frac{2\lambda_+}{L^2} Q^{-1} \left(E - \frac{h_{\min}}{\lambda_+} \left(1 - \frac{\lambda_-^2}{\kappa} \right) \right), \quad (30)$$

where

$$Q = \lambda_- \left(1 - \frac{\lambda_-^2}{\kappa} \right) - \lambda_+ \left(1 - \frac{\lambda_+^2}{\kappa} \right) = (\lambda_- - \lambda_+) \left(2 - \frac{b^2}{\kappa} \right). \quad (31)$$

Notice, that when $\frac{\kappa}{b^2} \ll 1$, $\lambda_- < 0$ and (19) is satisfied, $Q > 0$; also Q^{-1} diverges at the coalescence of the roots (when $\lambda_+ = \lambda_-$). For an acceptable equilibrium the amplitudes $C_{\pm}$ and the microlength (corresponding to one of the $\lambda_{\pm}$) must remain real as the other goes over real values. Therefore, $C_{\pm}^2$ must remain positive. When we deal with the equilibria with vastly separated scales, slow changes in the parameters can cause the loss of equilibrium only by making one of the Beltrami components vanish (either of $C_{\pm}^2$ becomes zero starting from positive values) for real $\lambda_{\pm}$. The other possible route for losing equilibrium when two solutions become the same ($\lambda_{\pm}$ coalesce) is not likely to happen via adiabatic changes.

IV. ANALYSIS OF CATASTROPHIC LOSS OF EQUILIBRIUM

For $\kappa \ll b^2$ the roots are real and vastly separated; then the only catastrophe allowed in this setup is when the amplitude corresponding to any state/structure approaches zero.

In what follows, λ ($\equiv \lambda_+$) denotes the macro-, and μ ($\equiv \lambda_-$) the microscale; these are related to the basic parameters of the equilibrium via

$$\lambda + \mu = -b; \quad \lambda \mu = \kappa. \quad (32)$$

The catastrophic loss of equilibrium (the higher order relaxed state is reduced to lower order one) will occur at either $C_{\mu}^2 = 0$ —the disappearance of the microscale constituent of the DB field, or when $C_{\lambda}^2 = 0$, implying the disappearance of the macroscale constituent of DB field—the DB state, then, must relax to a single Beltrami state. When either of them goes to zero, the energy mix of the State must change. The change could lead to state richer in gravitational (fluid) energy—the flow generation must necessarily mean that the final state must have larger fluid kinetic energy than the original one. The catastrophe condition is met at the critical value of control parameter when [see (30) for the former scenario]

$$E - \frac{h_{\min}}{\lambda_{\text{crit}}} \left(1 - \frac{\mu^2}{\kappa} \right) = 0, \quad (33)$$

which, using (32), reduces to the following equation in terms of the energy and helicities characterizing the DB state:

$$\frac{h_{\min}}{\kappa} \lambda_{\text{crit}}^2 + \left(E + 2b \frac{h_{\min}}{\kappa} \right) \lambda_{\text{crit}} + h_{\min} \left(\frac{b^2}{\kappa} - 1 \right) = 0. \quad (34)$$

Solving the above equation yields an expression for the critical value of control parameter ($\lambda = \lambda_{\text{crit}}$):

$$\lambda_{\text{crit}} = \frac{-F + \sqrt{D}}{2K}, \quad (35)$$

where

$$D = F^2 - 4KM, \quad K = \frac{h_{\min}}{\kappa}, \quad F = \left(E + 2b \frac{h_{\min}}{\kappa} \right), \quad M = h_{\min} \left(\frac{b^2}{\kappa} - 1 \right),$$

which must be satisfied together with Eqs. (26) and (28).

The unfolding of the process of the loss of equilibrium for the self-gravitating weakly rotating fluid follows the same path as described in [8]. Starting from a positive $C_{\mu(\lambda)}^2$, the ambience must change to push it past the critical point $C_{\mu(\lambda)}^2 = 0$. Negative $C_{\mu(\lambda)}^2$ implies an imaginary $C_{\mu(\lambda)}$ and the loss of original equilibrium.

Notice, that due to the attractive nature of gravity, the current system allows a negative (total) energy state (different from the electromagnetic plasma state) for weak background flows with strong self-gravity; we also emphasize that due to (17), (18), and (25), both λ and μ are negative as well $h_{\min} < 0$ with $C_{\mu(\lambda)}^2 > 0$.

Qualitative analytic results were strengthened by numerical experiments. For this paper, meant to theoretically demon-

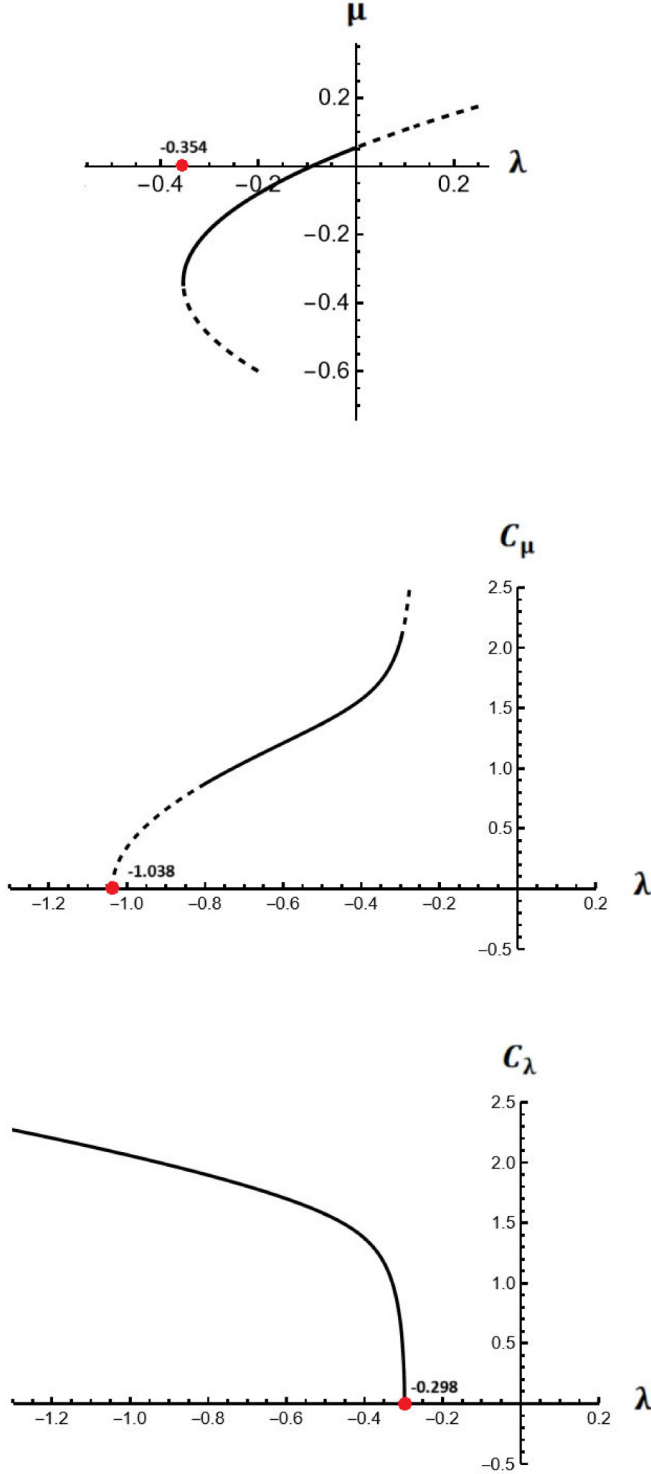


FIG. 1. Plots for microscale μ and $C_{\lambda,\mu}$ vs macroscale λ for catastrophic rearrangement of the original state; $E = -3 > E_{\text{crit}}$, $h_{\text{min}} = -0.5$, $\kappa = 0.04$, $b = 0.5$. The critical point $\lambda_{\text{crit}} = -0.298$ at which the transition happens can be observed on the plot of C_{λ} ; the macroscale constituent is fully vanished.

strate a physical effect, our choice of κ was dictated by considerations of highlighting the importance of self-gravity. This may not correspond to values realistic for neutral weakly rotating astrophysical fluids. We do plan to survey a range

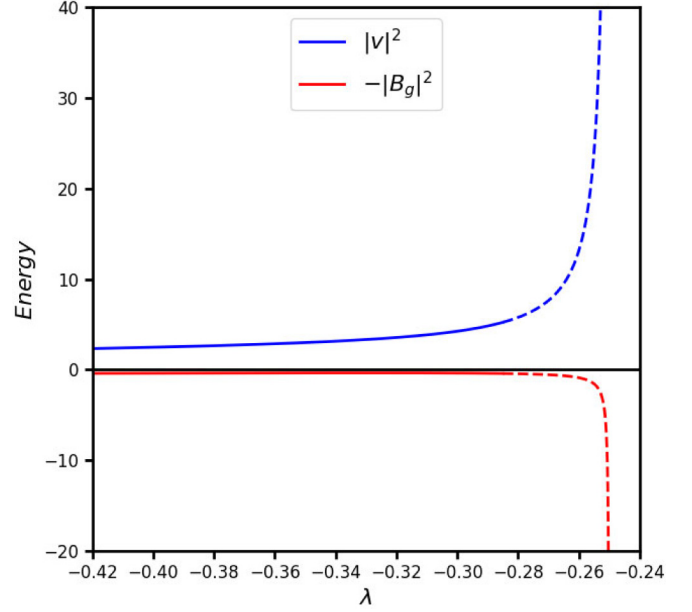


FIG. 2. Plots for flow and gravitomagnetic energies vs λ for catastrophe-prone parameters (see Fig. 1); the microscale/turbulent flow generation is clearly seen.

of κ values in a later publication. The reader may note that $\kappa = 0.33$ for the relativistic gas.

In Figs. 1 and 2, we illustrate the behavior of a special equilibrium with catastrophe-prone parameters: $E = -3$, $h_{\text{min}} = -0.5$, $\kappa = 0.04$, $b = 0.5$, $\lambda_{\text{crit}} = -0.298$. Here $L^2/2$ is normalized out and it was set that $C_{\lambda,\mu} > 0$. In Figs. 3 and 4 we present the results for the parameters: $E = -13$, $h_{\text{min}} = -0.5$, $\kappa = 0.2$, $b = 5$, $\lambda_{\text{crit}} = -3.14$. In both cases, the macroscale constituent completely vanishes while the microscale structures strengthen (see Figs. 1 and 3). One can see that it is the microscale flow energy that slowly increases feeding the local turbulence (see Figs. 2 and 4), which is extremely important for accretion disk local viscosity and disk-jet structure formation [6,33,34].

Using the value of λ_{crit} , we may confirm that, at the critical point, the coefficient C_{λ} which measures the strength of the macroscale fields, identically vanishes, and the equilibrium changes from DB to a SB state labeled by $\lambda = \lambda_{\text{crit}}$, i.e., $\mathbf{v} = C_{\mu} \mathbf{G}_{\mu}$ or $\nabla \times \mathbf{v} = \mu \mathbf{v}$. The amplitude of the microscale component (the only one remaining) turns out to be $C_{\mu}^2 = \lambda_{\text{crit}}^{-1} h_{\text{min}}$. The transition leads to a gravitomagnetically more relaxed state with the gravitomagnetic energy reaching its minimum (taking into account that it has a “minus” sign in total energy) with an appropriate gain in the flow kinetic energy (turbulent). At the transition, the kinetic energy is significantly greater than the gravitomagnetic energy with the ratio $-B_g^2/v^2 \sim -(b + \mu)^2$ (with $b < 1$; $|\mu| < 1$; see Figs. 2 and 4). Starting from vastly separated scales (at $\kappa \ll b^2$), the initial gravitomagnetic energy is transferred mainly to flow energy of the microscopic scale.

In both cases, the inverse macroscale $|\lambda|$ decreased as we reached the catastrophic transition to the SB state (starting from the DB state). In the complementary case, when the initial conditions are favorable to a catastrophic change with

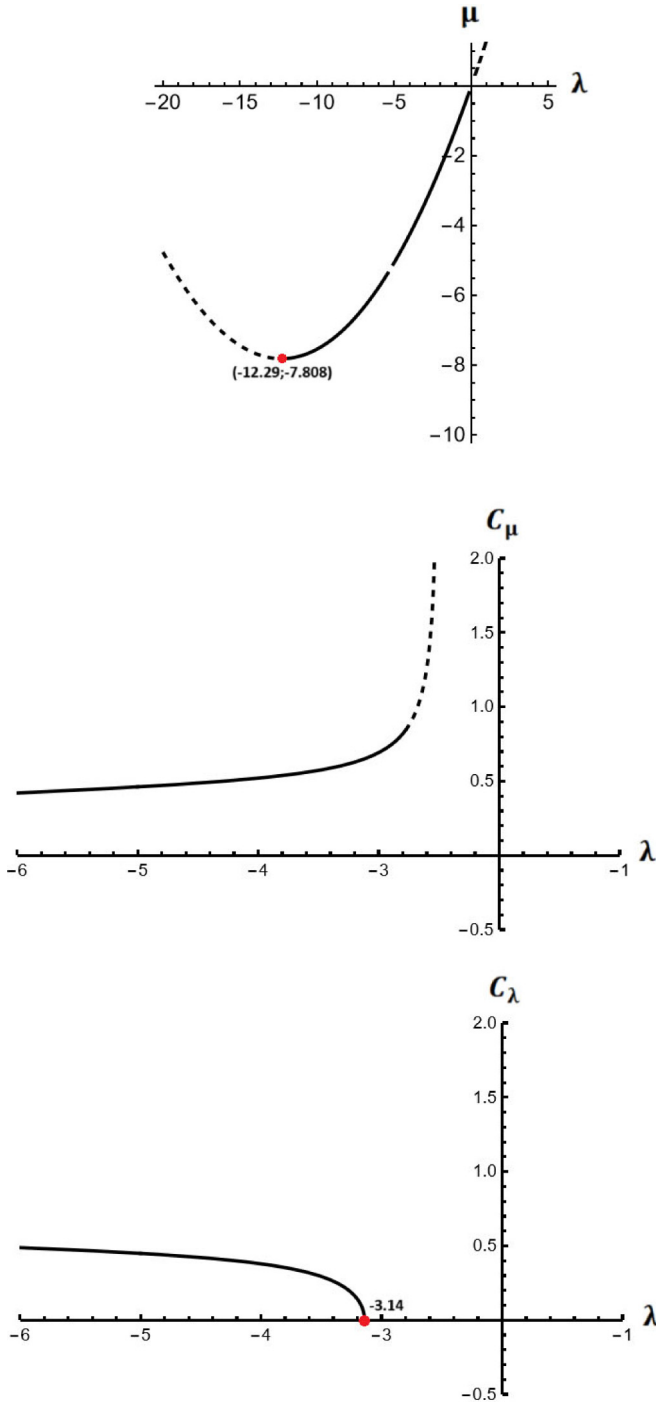


FIG. 3. Plots for microscale μ and $C_{\lambda,\mu}$ vs macroscale λ for catastrophic rearrangement of the original state; $E = -13$, $h_{\min} = -0.5$, $\kappa = 0.2$, $b = 5$. The point $\lambda_{\text{crit}} = -3.14$ at which the transition to turbulent state happens can be observed on the plot for C_{λ} .

increasing $|\lambda|$ (see Figs. 5 and 6 following the variation of this inverse scale from right to left, starting from 0), it is the macroscale energy that increases, and the microscale energy practically vanishes; parameters are the same as for Figs. 3 and 4 but the range of control parameter is different with $\lambda_{\text{crit}} = -1.86$. Specifically, it is interesting that for such scenarios it is the macroscale flow energy that increases, and the

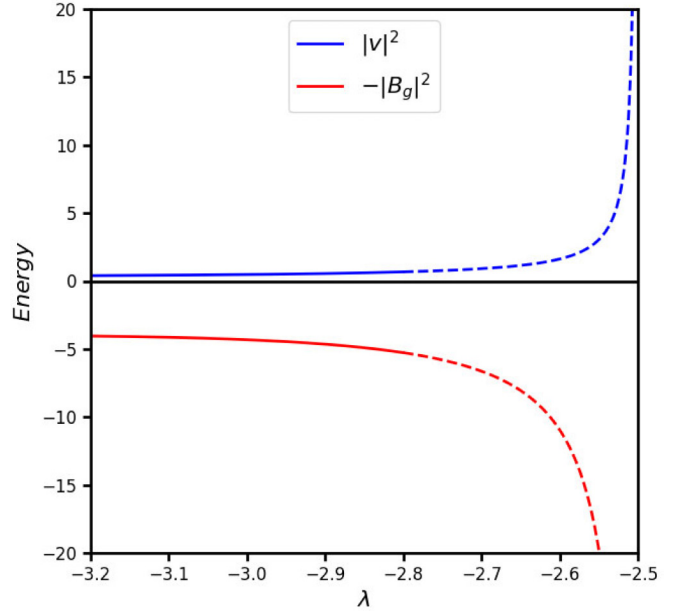


FIG. 4. Plots for flow and gravitomagnetic energies vs λ for the scenario leading to final turbulent state (see parameters in Fig. 3)—flow energy is totally microscale.

macroscale gravitomagnetic energy decreases with increasing $|\lambda|$ ($-|\mathbf{B}_g|^2$, predominantly macroscale, is following the flow energy variation in the opposite direction—a physically reliable transition. Note, that in such scenario the macroscale structure shrinks and is not destroyed.

From these figures it is clear that physical parameters of the system are precisely defined at the critical point. Also, the representation of a changing structure by a DB field persists and the length scales remain well separated since we assume that changes are slow and transport processes can be ignored (see Figs. 1, 3, and 5).

All of the above scenarios dealt with (initially) gravitomagnetically dominated state with a negative total energy. In these, significant self-gravity states defining length scales were vastly separated. In each, the structure at one of the scales vanishes and the DB state relaxes to an SB one; depending on the initial parameter ranges, the final state represents the flows generated at macro/microscales.

There is a very interesting corollary to the second scenario when the resulting state has microscopic scale flows. Such turbulent (we will lump short scale and turbulent flows together) flows have been deeply investigated [10,35–37] as a source of generating macroscale flows and magnetic fields by what has been called the unified dynamo/reverse dynamo mechanism (D-RD). Thus the catastrophic loss of equilibrium may eliminate the macroflow, but (D-RD) could regenerate it; the structure of the new macroflow will, surely, be very different from the original. In fact, in the companion paper, we will investigate this path.

Getting back to the subject matter of this study, one can, alternatively, start from a positive (total) energy initial state in which the flow energy is dominant and self-gravity is insignificant. Then, even though we have a DB state initially, it

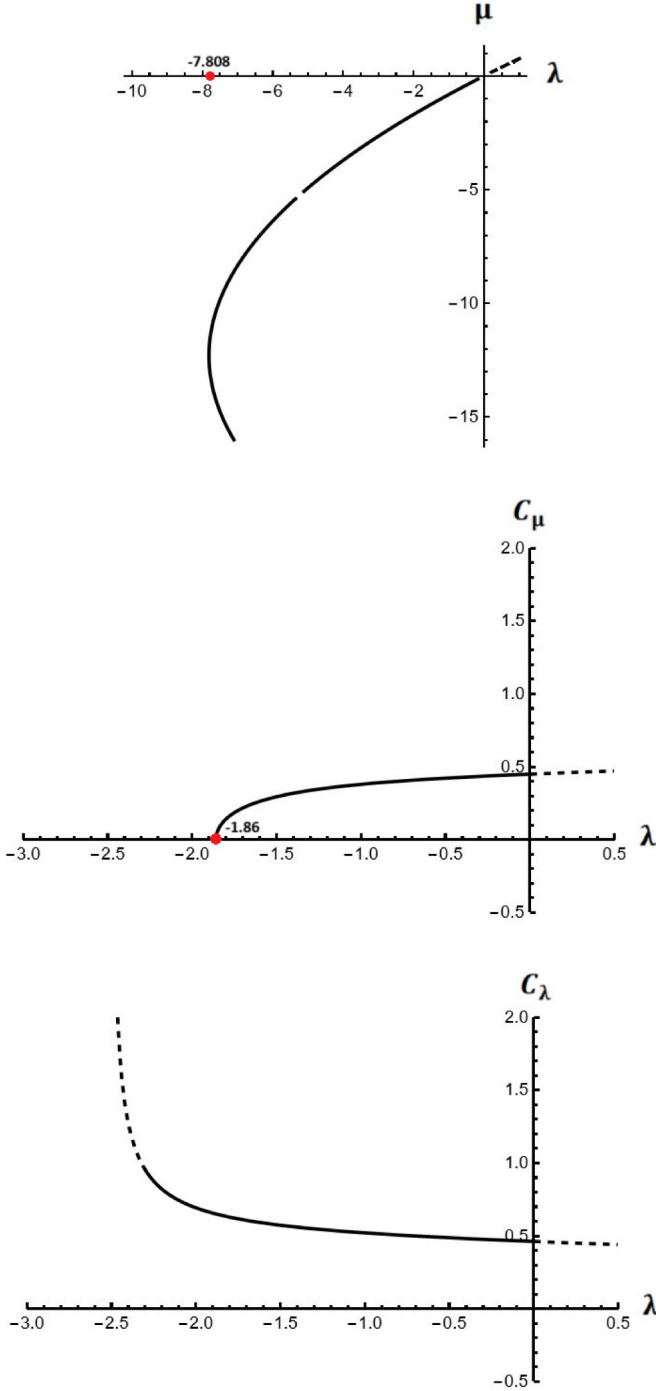


FIG. 5. Plots for microscale μ and $C_{\lambda,\mu}$ vs macroscale λ for catastrophic rearrangement of the original state; $E = -13$, $h_{\min} = -0.5$, $\kappa = 0.2$, $b = 5$. The point $\lambda_{\text{crit}} = -1.86$ at which the transition to macroscale structure dominant state happens can be observed on the plot for C_λ (when we follow the control parameter λ variation from right to left—following the increase of inverse-length scale $|\lambda|$).

is difficult to imagine that the system follows the (i) scenario while relaxation.

In fact, numerous simulation runs (not displayed here) show that with such insignificant gravity, the macro scale flows may form/accelerate only when the scales are not vastly separated (the condition $\kappa \ll b^2$ is not satisfied; the reader

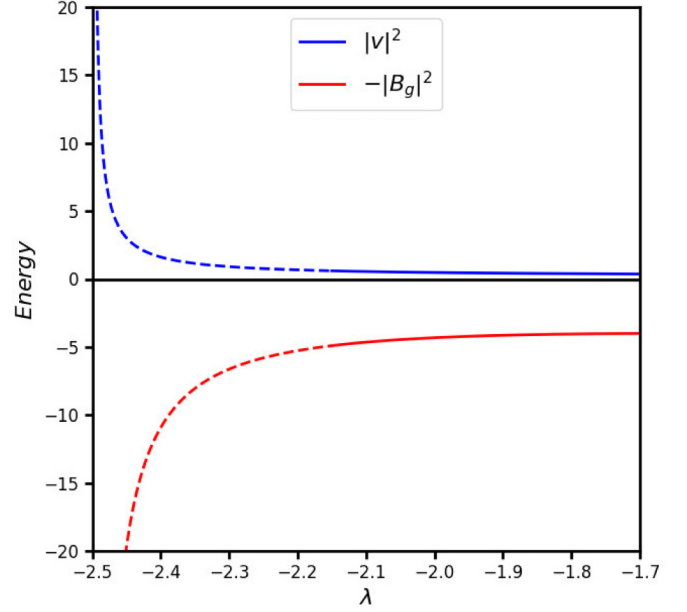


FIG. 6. Plots for flow and gravitomagnetic energies vs λ for the scenario leading to final single Beltrami (macroscale) state (see parameters in Fig. 5)—macroscale flow formation scenario following the increase of inverse-length scale $|\lambda|$.

may consult with the root-coalescence scenario for plasma case explored for astrophysical conditions and well studied in [7–9,38]). It was found that the microscale energy completely vanishes and the macroscale flow formation takes place while relaxation $|\lambda|$ is decreasing, implying the destroy of the structure. Such conditions can be satisfied in the objects with low vortical character ($b \sim \kappa \ll 1$) or with initially hot vortical/turbulent rotating flows ($b \sim \kappa < 1$).

We emphasize here that the transformation characteristics (the rates and direction / mix of energies) are dictated by the initial conditions of the system. A system may be more or less prone to a catastrophe depending on its mode of preparation. It must be noted, however, that the theory presented in this paper can merely work out conditions (and other critical parameters) for the occurrence of the phenomenon, but it has no ability to deal with the phenomenon itself. It is clear that when the system approaches the critical point, the latter will require a fast scale dynamical description that must include, *inter alia*, the transport processes.

V. SUMMARY AND CONCLUSIONS

In the present study, we explored the self-gravity driven flow generation in slowly rotating neutral fluid/gas found in various astrophysical settings. The self-gravitating system, obeying what we call gravitomagnetic field equations (closely resembling the Maxwell-Lorenz system of electrodynamics), follow a trajectory similar to that of its electromagnetic counterpart, and can be analyzed in terms of the Beltrami-Bernoulli equilibrium [7,8], whose principal part is the Double Beltrami equation that the velocity field obeys. Under appropriate conditions (determined by the initial conditions of the system), this DB equilibria can be catastrophically lost; the

resulting single Beltrami equilibrium can have a much larger flow kinetic energy than the original system. Such processes can have significant effects for the formation of macroscale flows/structures in galaxies, accretion discs around the massive compact object, and for the dynamics and stability of a rotating star/its atmosphere, etc. [6,13,34,39].

The process of flow generation manifests in several steps: on a slow change in the ambient parameters, the DB gravitomagnetofluid state, becomes unstable, in particular, one of (macro- or microscale) the two Beltrami components tends to disappear and the resulting readjustment takes place through a catastrophic rearrangement of the energy mix. In the problem of interest, gravitational energy ends up powering strong flows. The critical point at which the transformation takes place depends on the invariants of the collisionless gravitomagnetofluid dynamics. Self-gravity provides the energy for flow generation. In detail:

(1) we have derived the condition of the catastrophic transformation of energies. At the critical point, the double Beltrami state is reduced to a single Beltrami one. The energy for macro- and microscale velocity generation comes from the gravitomagnetic fields.

(2) The most important qualitative result is that well defined initial conditions can lead to a kinetically rich final state—in fact all of the gravitomagnetic field energy is converted to velocity field energy at the catastrophe.

In the end, it must be emphasized that our minimal idealized model analysis is limited to figuring out what DB states are prone to a catastrophic change, even predicting the critical

parameters. But when the critical stage is reached, we have to look for a totally time dependent dynamic theory to work out the fast stages of change. The unified reverse dynamo/dynamo mechanism [10,36,37] could, for instance, be the basis of a theory. Such a study will constitute the scope of future work.

YSOs, HH objects, protoplanetary disks, and rotating planets represent realistic astrophysical conditions for which the theory, developed here, should be applicable (see, e.g., [34] and references therein). Having demonstrated the important role of self-gravity (in an idealized setting) as an important factor in determining the equilibrium state, we plan to take the theory to the aforementioned realistic systems.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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