

Six universality classes govern the critical and multicritical behavior of an active Ising modelMatthew Wong^{✉*} and Chiu Fan Lee^{✉†}*Department of Bioengineering, Imperial College London, South Kensington Campus, London SW7 2AZ, United Kingdom*

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The critical behavior of the equilibrium Ising model, famously governed by the Wilson-Fisher universality class (UC), is a cornerstone of statistical physics. A natural question is whether its properties are modified when the interacting spins are allowed to move. This can be explored within the framework of active motility by, e.g., prescribing that the spins self-propel along the direction of their orientation. In such a system, the spin number density necessarily constitutes a soft hydrodynamic mode and can therefore modify the scaling behavior. In this letter, we show that this is indeed the case in a critical active Ising model where density impedes the system's collective motion. Using a perturbative dynamic renormalization group analysis at a one-loop level, we identify six UCs, including three previously unidentified ones, one of which supersedes the Wilson-Fisher UC as the generic UC governing the critical behavior.

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The equilibrium Ising model, with its critical behavior governed by the Wilson-Fisher universality class (UC) [1], provides a canonical paradigm of universality in statistical physics. A natural question is how this paradigm is modified when detailed balance is broken, for example by endowing the spins with active motility—a defining feature of collective motion in nonequilibrium and biological systems [2,3]. Activity can be incorporated into the Ising framework by prescribing that each spin self-propels along the direction of its orientation [Figs. 1(a) and 1(b)]. This construction introduces a direct coupling between orientational degrees of freedom and *real* space, which underlies much of the novel emergent physics observed in active Ising models (AIMs) [4–7].

Beyond their theoretical interest, AIMs admit realizations in both natural and synthetic systems. In migratory bird flocks, alignment interactions can effectively become axial due to the inclination-based nature of the avian magnetic compass, while avoidance of collisions at high densities provides a biological motivation for contact inhibition. Complementarily, AIMs can be engineered in polymeric gels composed of motile cells: when subjected to an external stretch, the gel imposes an intrinsic easy axis, leading to anisotropic collective motion. These systems thus provide potential experimental platforms in which the interplay between activity, density, and collective order can be probed.

A crucial distinction from the equilibrium Ising model arises when the spins are allowed to move; the locally varying spin number density becomes a conserved soft hydrodynamic (gapless) mode and must be included in the long-wavelength

description. The impact of such a density mode is already evident in the Vicsek model [8], a paradigmatic microscopic model of active matter, where its coupling to orientational order generically renders the order-disorder transition discontinuous [9–11]. Importantly, however, this outcome is not universal: subsequent work has shown that continuous critical behavior can be restored in active systems where collective motion is suppressed at high densities [12–16], as occurs, for example, in systems of motile agents exhibiting contact inhibition of locomotion [17].

Such a continuous transition is naturally described within the Toner-Tu (TT) framework [18–21], which provides a universal hydrodynamic theory of polar active fluids. As we shall see, AIMs can be viewed as a realization of the TT model supplemented by an easy-axis anisotropy [22]. In this formulation, existing analyses of the TT order-disorder transition apply directly to AIMs, yielding the generic phase diagram shown in Fig. 1(c).

Despite this progress, a key theoretical question remains unresolved. While the AIM order-disorder transition is governed by the Wilson-Fisher UC when density fluctuations are neglected [23], it is unknown whether this UC remains stable once the conserved density field is incorporated. More broadly, it is unclear how coupling a hydrodynamic mode modifies the universality structure of the transition. Generally, identifying and classifying UCs in nonequilibrium systems remains a central challenge in statistical physics (see, e.g., Ref. [24]). In this Letter, we address this problem by performing a dynamical renormalization group (RG) analysis combined with a one-loop ϵ expansion. We show that the conserved density field destabilizes the Wilson-Fisher fixed point and leads to multiple RG fixed points, each corresponding to distinct UCs. Crucially, one of the three previously unidentified UCs supersedes the Wilson-Fisher UC as the generic UC governing the critical behavior of AIMs.

Active Ising model. Motivated by the interpretation of the equilibrium Ising model as an $O(n)$ model with an easy axis (i.e., spins preferentially align along a distinguished direction

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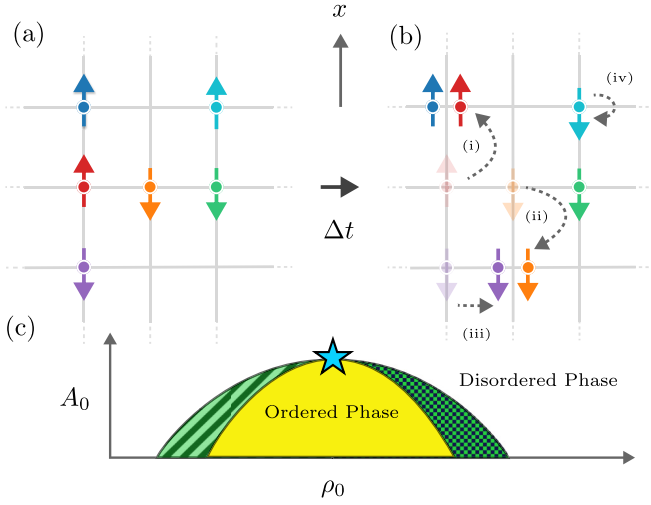


FIG. 1. (a), (b) In the active Ising model (AIM), spins preferentially align along the vertical x axis, and their orientations determine the direction of self-propelled motion, as illustrated in (i) and (ii). Fluctuations can locally reorient spins, giving rise to transverse motion (iii) and spin flips (iv). We further assume that alignment becomes disordered at sufficiently high local spin density, leading to the loss of collective motion. This effect can be modeled, for example, by allowing the alignment parameter to depend on a reference density ρ_{ref} (not necessarily identical to the mean density ρ_0) as $\alpha(\rho) = A_0 + A_1(\rho - \rho_{\text{ref}}) + A_2(\rho - \rho_{\text{ref}})^2$ [Eq. (4)], which qualitatively reproduces the phase diagram shown in (c). The green striped region denotes the banding regime, corresponding to a coexistence between a high-density ordered phase and a low-density disordered phase, while the green meshed region denotes the reverse-banding regime, in which a high-density disordered phase coexists with a low-density ordered phase. A critical point (blue star) occurs where these two coexistence regimes meet [12].

[22]), we formulate our active Ising model (AIM) by introducing an easy-axis anisotropy into the standard Toner-Tu (TT) model in d spatial dimensions.

Without loss of generality, the easy axis can be chosen to lie along the x direction. We denote the mass density field by ρ and the momentum density field by $\mathbf{g} = g_x \hat{\mathbf{x}} + \mathbf{g}_\perp$, which aligns with the local coarse-grained spin orientation. The equations of motion (EOM) of the TT model, with the momentum field $\mathbf{g}$ partitioned to explicitly reveal the easy-axis anisotropy, then read

$$\partial_t \rho + \nabla \cdot \mathbf{g} = K \nabla^2 \rho, \quad (1)$$

$$\begin{aligned} \partial_t g_x + \frac{\lambda}{2} \partial_x g_x^2 + \lambda_1 (\mathbf{g}_\perp \cdot \nabla_\perp) g_x + \lambda_2 g_x (\nabla_\perp \cdot \mathbf{g}_\perp) + \frac{\lambda_3}{2} \partial_x (|\mathbf{g}_\perp|^2) \\ = [-\alpha(\rho) - \beta g_x^2] g_x + (\mu_1 \partial_x^2 + \mu'_1 \nabla_\perp^2) g_x \\ + [\partial_x \mu_2 (\partial_x g_x) + \partial_x \mu'_2 (\nabla_\perp \cdot \mathbf{g}_\perp)] \\ - \partial_x P^x(\rho) + \text{h.o.t.} + f_x, \end{aligned} \quad (2)$$

$$\partial_t \mathbf{g}_\perp = -\Gamma \mathbf{g}_\perp - \nabla_\perp P^\perp(\rho) + \text{h.o.t.} + \mathbf{f}^\perp, \quad (3)$$

where α and P^θ ($\theta = x$ or $\perp$) are scalar functions of ρ ,

$$\alpha = \sum_{n \geq 0} \alpha_n (\rho - \rho_0)^n, \quad P^\theta = \sum_{n \geq 1} \kappa_n^\theta (\rho - \rho_0)^n, \quad (4)$$

with ρ_0 being the mean density, and the noise term $\mathbf{f}$ is Gaussian with vanishing mean and statistics:

$$\langle f_x(\mathbf{r}, t) f_x(\mathbf{r}', t') \rangle = 2D \delta(t - t') \delta^d(\mathbf{r} - \mathbf{r}'), \quad (5)$$

$$\langle f_i^\perp(\mathbf{r}, t) f_j^\perp(\mathbf{r}', t') \rangle = 2D_\perp \delta_{ij}^\perp \delta(t - t') \delta^d(\mathbf{r} - \mathbf{r}'). \quad (6)$$

All other coefficients in the EOM are independent of the hydrodynamic fields, and “h.o.t.” denotes higher-order terms, such as the pressure anisotropy term, $\mathbf{g}(\mathbf{g} \cdot \nabla)P$, that do not affect the hydrodynamic behavior of the system under RG justifications. Indeed, under similar RG scaling arguments, one can justify the use of the additive Gaussian noise profile seen in Eq. (5), as opposed to one with multiplicative statistics. Further, the superscript/subscript “ $\perp$ ” generically denotes directions orthogonal to $\hat{\mathbf{x}}$, e.g., $\nabla_\perp = \partial_y \hat{\mathbf{y}} + \partial_z \hat{\mathbf{z}}$ for $d = 3$ where d denotes the spatial dimensions. A table detailing physical interpretation of the coupling terms can be found in the Supplemental Material [25].

We now exploit the damping term $(-\Gamma \mathbf{g}_\perp)$ in Eq. (3) to eliminate the transverse momentum fluctuations, thereby enforcing an effective easy axis. This yields

$$\mathbf{g}_\perp \simeq \Gamma^{-1} [-\nabla_\perp P^\perp + \text{h.o.t.} + \mathbf{f}^\perp], \quad (7)$$

which we can then substitute into Eqs. (1) and (2) to eliminate $\mathbf{g}_\perp$, leading to the AIM EOM,

$$\partial_t \rho = -\gamma \partial_x \phi + K_x \partial_x^2 \rho + K_\perp \nabla_\perp^2 \rho, \quad (8)$$

$$\begin{aligned} \partial_t \phi + \frac{\lambda}{2} \partial_x \phi^2 = -\kappa_1 \partial_x \rho - \kappa_2 \partial_x \rho^2 + \mu_x \partial_x^2 \phi + \mu_\perp \nabla_\perp^2 \phi \\ + (-\alpha_0 - \alpha_1 \rho - \alpha_2 \rho^2 - \beta \phi^2) \phi + f_x, \end{aligned} \quad (9)$$

where we have only shown the relevant terms (to be justified *a posteriori*). To ease notation, we have also made the following redefinitions: $\rho - \rho_0 \mapsto \rho$, $\phi = g_x$, $\mu_x = \mu_1 + \mu_2$, $\mu_\perp = \mu'_1 + \mu'_2$, $\kappa_{1,2} = \kappa_{1,2}^x$, and $K_\perp + \kappa_\perp^\perp / \Gamma \mapsto K_\perp$.

The resulting effective theory thus consists of a conserved scalar density field ρ coupled to a nonconserved scalar order-parameter field ϕ . The Z_2 symmetry of the AIM, $\phi \rightarrow -\phi$ under $x \rightarrow -x$, is naturally inherited from the parity invariance of the underlying Toner-Tu model once an easy axis is imposed. In contrast to the minimal hydrodynamic description derived by coarse graining from the microscopic model of Ref. [4], Eqs. (8) and (9) retain all RG-relevant, symmetry-allowed nonlinearities, including anisotropic transport together with nonlinear advective and density couplings, which are essential for the subsequent renormalization-group analysis.

Phase behavior. As in the generic Toner-Tu (TT) model, the phase behavior can be analyzed using a linear stability analysis, yielding the phase diagram shown in Fig. 1(c). The green striped region corresponds to the well-known banding regime, a condensed ordered phase coexisting with a dilute disordered phase, while the green meshed region corresponds to the reverse-banding regime, in which a dilute ordered phase coexists with a condensed disordered phase [12].

The key control parameters are the mean density ρ_0 and the density-dependent alignment strength $\alpha(\rho)$ [Eq. (4)]. Unlike its thermodynamic counterpart in the equilibrium Landau-Ginzburg model, $\alpha(\rho)$ is an emergent kinetic effect that arises in compressible polar active fluids. Importantly, $\alpha(\rho_0) = \alpha_0$

can be interpreted physically as a measure of intrinsic alignment strength, i.e., at the microscopic level, the intrinsic tendency of isolated spins to align with one another. However, $\alpha'(\rho_0) = \alpha_1$ necessarily arises from compressibility. It is best interpreted as the density-induced coupling, i.e., how much local crowding enhances or suppresses alignment. In the hydrodynamic limit, it arises as a collision-induced alignment rate and is responsible for the banding instabilities of Fig. 1(c).

Criticality. Recent work has verified, analytically and through simulations [12], that at a mean-field level, a continuous order-disorder transition can be recovered by appropriately fine-tuning the alignment strength: the density-dependent alignment function $\alpha(\rho)$. The existence of such a critical point was previously overlooked in Ref. [26], where $\alpha'(\rho) \neq 0$ instead leads to a first-order transition. In the present model, however, the transition becomes continuous upon tuning both parameters $\alpha_0 = \alpha_1 = 0$. Experimentally, these conditions can be achieved by tuning the motility and the mean density of the system, i.e., controlling density fluctuations to travel along the spinodal and binodal lines of the phase diagram, towards the critical point.

Analogous to the equilibrium Ising model, this critical point corresponds to an unstable fixed point of the overall system, where α_0 effectively behaves as the reduced temperature that must be fine-tuned to reach it. The RG flow describes the physics in the absence of fine-tuning, demonstrating α_0 's unstable behavior in the form of generated loop corrections. Its flow equation takes the form

$$\frac{d \ln \alpha_0}{d\ell} = z + \frac{1}{4\alpha_0}(2\mu_\perp \Lambda^2 - \alpha_0)(3g_\beta + g_{\alpha_2}). \quad (10)$$

The y_{α_0} exponent quantifies this instability by characterizing how linear perturbations away from the critical manifold (defined by $\alpha_0 = \alpha_1 = 0$) grow under RG flow. In this sense, y_{α_0} determines how rapidly the system departs from criticality when α_0 is not tuned to zero. For perturbations of the form $\alpha_0 = \alpha_0^* + \delta\alpha_0$, Eq. (10) simplifies to

$$\frac{d \ln \delta\alpha_0}{d\ell} = 2 - \frac{1}{4}(3g_\beta + g_{\alpha_2}), \quad (11)$$

$$y_{\alpha_0} = 2 - \frac{1}{4}(3g_\beta + g_{\alpha_2}). \quad (12)$$

Upper critical dimension and relevant terms. We now use a perturbative dynamical renormalization-group (DRG) approach to analyze the universal behavior of the AIM at criticality. As a first step, we must determine (i) the upper critical dimension d_c and (ii) the nonlinear terms that are relevant in the vicinity of d_c .

To achieve this, we analyze the linearized equations of motion, Eqs. (8) and (9), from which the ρ - ρ and ϕ - ϕ correlation functions can be obtained by spatiotemporal Fourier transformation. This standard calculation is outlined in the Supplemental Material [25] and Ref. [27]; here, we simply quote the results. Using the rescaling convention,

$$(t, \mathbf{r}_\perp, x, \rho, \phi) \rightarrow (t e^{z\ell}, \mathbf{r}_\perp e^\ell, x e^{\zeta\ell}, \rho e^{\chi_\rho\ell}, \phi e^{\chi_\phi\ell}), \quad (13)$$

the associated linear scaling exponents are

$$z^{\text{lin}} = 2, \quad \zeta^{\text{lin}} = 1, \quad \chi_\rho^{\text{lin}} = \chi_\phi^{\text{lin}} = \frac{2-d}{2}. \quad (14)$$

Using these exponents, standard power counting [25] immediately yields (i) an upper critical dimension $d_c = 4$ and (ii) that the only RG-relevant nonlinearities are those appearing in Eq. (9). In particular, the theory now contains four nonlinear terms, two of which— $\kappa_2 \partial_x \rho^2$ and $\alpha_2 \rho^2 \phi$ —arise solely from coupling to the density field. As we show below, these density-induced nonlinearities are essential for capturing the correct critical behavior of the AIM.

DRG analysis. We now carry out a DRG analysis in conjunction with an ϵ expansion, with $\epsilon = d_c - d = 4 - d$. A Wilsonian RG transformation consists of integrating out short-wavelength fluctuations and absorbing their effects into the parameters of the rescaled equations of motion (EOM) [28,29].

We present the full one-loop calculation in the Supplemental Material [25], and here simply quote the resulting RG flow equations at $\mathcal{O}(\epsilon)$, evaluated at criticality (i.e., $\alpha_0 = \alpha_1 = 0$),

$$\frac{d \ln \lambda}{d\ell} = z - \zeta + \chi_\phi - \frac{15}{8}g_\beta - \frac{3}{2}\frac{g_{\kappa_2}g_\beta}{g_\lambda} - \frac{3}{8}g_{\alpha_2}, \quad (15)$$

$$\frac{d \ln \gamma}{d\ell} = z - \zeta + \chi_\phi - \chi_\rho, \quad (16)$$

$$\frac{d \ln \kappa_1}{d\ell} = z - \zeta - \chi_\phi + \chi_\rho, \quad (17)$$

$$\frac{d \ln \kappa_2}{d\ell} = z - \zeta - \chi_\phi + 2\chi_\rho - \frac{1}{8}\frac{g_{\alpha_2}g_\beta}{g_{\kappa_2}} - \frac{3}{8}g_{\alpha_2}, \quad (18)$$

$$\frac{d \ln \mu_x}{d\ell} = z - 2\zeta + \frac{3}{16}g_\lambda + \frac{3}{8}g_{\kappa_2}, \quad (19)$$

$$\frac{d \ln \mu_\perp}{d\ell} = z - 2, \quad (20)$$

$$\frac{d \ln \alpha_2}{d\ell} = z + 2\chi_\rho - \frac{3}{4}g_\beta - \frac{5}{4}g_{\alpha_2}, \quad (21)$$

$$\frac{d \ln \beta}{d\ell} = z + 2\chi_\phi - \frac{9}{4}g_\beta - \frac{3}{4}g_{\alpha_2}, \quad (22)$$

$$\frac{d \ln D}{d\ell} = z - \zeta - 2\chi_\phi - d + 1, \quad (23)$$

$$\frac{d \ln K_\perp}{d\ell} = z - 2, \quad (24)$$

$$\frac{d \ln K_x}{d\ell} = z - 2\zeta, \quad (25)$$

where ℓ corresponds to the level of “coarse graining,” i.e., the larger the ℓ , the more short-wavelength fluctuations have been integrated out. We further define the following effective dimensionless coefficients,

$$g_\beta = \frac{\beta DK_{d-1}}{\mu_x^{1/2} \mu_\perp^{3/2}} \Lambda^{d-4}, \quad g_\lambda = \frac{\lambda^2 DK_{d-1}}{\mu_x^{3/2} \mu_\perp^{3/2}} \Lambda^{d-4}, \quad (26)$$

$$g_{\kappa_2} = \frac{\gamma \kappa_2 \lambda DK_{d-1}}{\kappa_1 \mu_x^{3/2} \mu_\perp^{3/2}} \Lambda^{d-4}, \quad g_{\alpha_2} = \frac{\gamma \alpha_2 DK_{d-1}}{\kappa_1 \mu_x^{1/2} \mu_\perp^{3/2}} \Lambda^{d-4}, \quad (27)$$

where $K_{d-1} = \frac{S_{d-1}}{(2\pi)^d}$, and S_{d-1} is the surface area of a $(d-1)$ -dimensional unit sphere. Additionally, Λ is the arbitrary UV momenta cutoff of our effective theory which corresponds to the inverse of a microscopic length i.e., the distance between the active spin carrying agents.

TABLE I. RG FPs, their stabilities, locations, and critical exponents. Six FPs are obtained in our DRG calculation to order ϵ . The symbols defined next to the Roman numerals are used in Fig. 2 to depict their locations schematically. The integral quantity N_{unst} denotes the unstable direction of each FP within the critical manifold ($\alpha_0 = \alpha_1 = 0$). The FP locations are shown by the g^* 's. The subsequent four columns show the numerical values of the critical exponents to order ϵ : z (dynamic), ζ (anisotropic), χ_ϕ , χ_ρ (roughness exponents for ϕ and ρ , respectively), and y_{α_0} (the exponent associated with the divergence of α_0 under RG transformation). As expected, we recover the Wilson-Fisher FP (II), the Hwa-Kardar FP (IV) first elucidated in the context of self-organized criticality [30,31], and the Gaussian FP (VI). As far as we are aware, the remaining three FPs are derived in this work.

FP	N_{unst}	$[g_\beta^*, g_\lambda^*, g_{\kappa_2}^*, g_{\alpha_2}^*]$	z	ζ	χ_ϕ, χ_ρ	y_{α_0}
I ●	0	$\epsilon[\frac{2}{9}, 0, 0, \frac{2}{3}]$	2	1	$-1 + \frac{1}{2}\epsilon$	$2 - \frac{1}{3}\epsilon$
II ◆	1	$\epsilon[\frac{4}{9}, 0, 0, 0]$	2	1	$-1 + \frac{1}{2}\epsilon$	$2 - \frac{1}{3}\epsilon$
III ■	2	$\epsilon[0, \frac{16}{9}, 0, \frac{2}{3}]$	2	$1 + \frac{1}{6}\epsilon$	$-1 + \frac{5}{12}\epsilon$	$2 - \frac{1}{6}\epsilon$
IV ⬠	3	$\epsilon[0, \frac{32}{9}, 0, 0]$	2	$1 + \frac{1}{3}\epsilon$	$-1 + \frac{1}{3}\epsilon$	2
V ⬠	3	$\epsilon[0, 0, 0, \frac{4}{5}]$	2	1	$-1 + \frac{1}{2}\epsilon$	$2 - \frac{1}{5}\epsilon$
VI ⬡	3	$\epsilon[0, 0, 0, 0]$	2	1	$-1 + \frac{1}{2}\epsilon$	2

The universal behavior of the system, as dictated by the RG fixed points, can be elucidated by analyzing the corresponding, much simpler, RG flow equations of the dimensionless coefficients g 's, which effectively are

$$\frac{d \ln g_\beta}{d\ell} = \epsilon - \frac{9}{4}g_\beta - \frac{3}{4}g_{\alpha_2} - \frac{3}{32}g_\lambda - \frac{3}{16}g_{\kappa_2}, \quad (28)$$

$$\frac{d \ln g_\lambda}{d\ell} = \epsilon - \frac{15}{4}g_\beta - 3\frac{g_{\kappa_2}g_\beta}{g_\lambda} - \frac{3}{4}g_{\alpha_2} - \frac{9}{32}g_\lambda - \frac{9}{16}g_{\kappa_2}, \quad (29)$$

$$\frac{d \ln g_{\kappa_2}}{d\ell} = \epsilon - \frac{1}{8}\frac{g_\lambda g_{\alpha_2}}{g_{\kappa_2}} - \frac{3}{4}g_{\alpha_2} - \frac{15}{8}g_\beta - \frac{3}{2}\frac{g_{\kappa_2}g_\beta}{g_\lambda} - \frac{9}{32}g_\lambda - \frac{9}{16}g_{\kappa_2}, \quad (30)$$

$$\frac{d \ln g_{\alpha_2}}{d\ell} = \epsilon - \frac{3}{4}g_\beta - \frac{5}{4}g_{\alpha_2} - \frac{3}{32}g_\lambda - \frac{3}{16}g_{\kappa_2}. \quad (31)$$

RG fixed points. Equipped with the RG flow equations above, we can now analyze the structure of the flows, identify the fixed points (FPs), and determine their stabilities. In total, we find six FPs, together with their associated critical exponents, as summarized in Table I, and visually depicted in Fig. 2. The FPs are ordered by stability, with N_{unst} denoting the number of unstable directions within the critical manifold.

Generically, accessing (multi)critical fixed points requires fine-tuning of control parameters. In the absence of such tuning, the RG flows of the associated couplings run away. The stability of each FP is determined via a linear stability analysis in the space of dimensionless couplings at each FP, where each positive eigenvalue corresponds to an unstable direction; the total number of such directions is denoted by N_{unst} . This analysis is supplemented by direct numerical integration of the flow equations in RG time ℓ , with representative stabilizing and runaway trajectories shown in the Supplemental Material [25].

Crucially, we find that the generic critical behavior of the AIM is controlled by the UC associated with FP I. By additionally fine-tuning α_2 to zero, the flow instead terminates at the Wilson-Fisher UC (FP II). Fine-tuning three parameters leads to four further critical scenarios: (i) a different UC (FP III) for $\kappa_2 = \beta = K_x = 0$; (ii) the UC first identified by Hwa

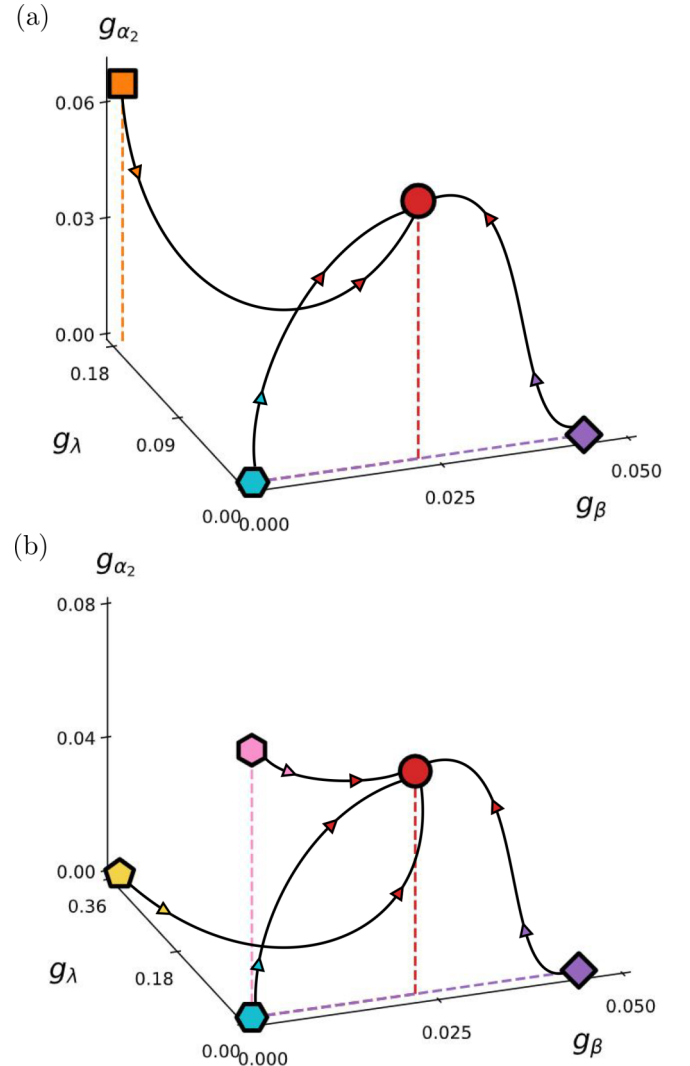


FIG. 2. Schematic RG flow diagrams illustrating the flow structure between fixed points ($\epsilon = 0.1$). For clarity, the six fixed points (FPs) listed in Table I are displayed across two panels. (a) shows the first three FPs together with the Gaussian FP (VI, blue hexagon), while (b) shows the remaining three FPs together with the generic critical FP I (red circle) and the Wilson-Fisher FP II (purple diamond).

and Kardar in the context of self-organized criticality (FP IV) for $\beta = \kappa_2 = \alpha_2 = 0$ [30,31]; (iii) a different UC (FP V) for $\beta = \lambda = \kappa_2 = 0$; and (iv) the Gaussian UC (FP VI) for $\beta = \lambda = \alpha_2 = 0$.

Interestingly, although the generic critical fixed point (FP I) is distinct from the Wilson–Fisher UC, it retains the linear scaling exponents at one-loop order. This follows from the flow equations for $K_\perp$, $\mu_\perp$, γ , and κ_1 , which enforce the constraints $z = 2$ and $\chi_\phi = \chi_\rho$ at this order. This degeneracy is not expected to persist beyond one loop, where higher-order corrections should renormalize all critical exponents. For instance, such deviations have been demonstrated recently in a study of multicritical behavior in the Toner–Tu model [14].

Summary and outlook. We have investigated the critical order-disorder transition of an active Ising model (AIM), formulated as a Toner–Tu active fluid with an easy axis. Using a one-loop dynamical renormalization group (RG) analysis near the upper critical dimension $d_c = 4$, we have characterized its critical behavior.

Our results demonstrate that active motility qualitatively alters the classification of critical phenomena: the coupling to a conserved hydrodynamic mode reorganizes the universality structure, replacing the equilibrium Wilson–Fisher paradigm by a distinct nonequilibrium one. In total, we identify six universality classes (UCs), including three previously unidentified ones, establishing active Ising systems as a minimal setting in which generic nonequilibrium mechanisms can generate previously unidentified UCs. Specifically, the interplay between symmetry, conservation laws, and activity can reorganize the landscape of UCs even when the underlying symmetry remains unchanged.

An important direction for future work is to test these predictions in particle-based simulations and experimental systems with density-dependent motility or contact inhibition. Quantitative refinement of critical exponents using functional RG approaches [32] provides a complementary avenue [33].

Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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