

Erratum: Difference between memory and prediction in linear recurrent networks **[Phys. Rev. E **96**, 032308 (2017)]**

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Equation (20) of the original paper gave us an upper bound for the predictive capacity (PC) for linear recurrent networks in terms of Wiener filters as

$$\text{PC} \leq \sum_{\tau=0}^{\infty} \mathbf{r}_{\tau}^{\top} R^{-1} \mathbf{r}_{\tau}, \quad \text{where } \mathbf{r}_{\tau} = \begin{pmatrix} R_{ss}(\tau+1) \\ R_{ss}(\tau+2) \\ \vdots \end{pmatrix}, \quad R = \begin{pmatrix} R_{ss}(0) & R_{ss}(-1) & R_{ss}(-2) & \cdots \\ R_{ss}(1) & R_{ss}(0) & R_{ss}(-1) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (1)$$

and $R_{ss}(\tau)$ is the autocorrelation function of signal $s(t)$ with time lag τ . This is originally due to defining PC in the following way:

$$\text{PC} = \sum_{\tau=0}^{\infty} \text{PC}_{\tau}, \quad \text{PC}_{\tau} = \frac{\langle s(t+\tau) \hat{s}(t+\tau) \rangle_t^2}{\langle \hat{s}(t)^2 \rangle_t}, \quad (2)$$

where $s(t)$ is the true signal at time t and $\hat{s}(t)$ is the predicted signal using Wiener filters. In order to derive the first expression seen in Eq. (1), we assumed that $\langle s(t+\tau) \hat{s}(t+\tau) \rangle_t = \langle \hat{s}(t)^2 \rangle_t$; however, this does not hold in general. Instead, we propose a revised upper bound for the predictive capacity. First, we evaluate the numerator. Denoting the Wiener filter as $k_{\tau}(j)$ for the filter at time j with time lag τ :

$$\langle s(t+\tau) \hat{s}(t+\tau) \rangle_t = \left\langle s(t+\tau) \sum_{j=1}^{\infty} s(t-j) k_{\tau}(j) \right\rangle_t \quad (3)$$

$$= \left\langle \sum_{j=1}^{\infty} s(t+\tau) s(t-j) k_{\tau}(j) \right\rangle_t \quad (4)$$

$$= \sum_{j=1}^{\infty} R_{ss}(\tau+j) k_{\tau}(j) \quad (5)$$

$$= \mathbf{r}_{\tau}^{\top} R^{-1} \mathbf{r}_{\tau}. \quad (6)$$

The last step was achieved by referencing Appendix C in the original paper to solve for $\mathbf{k}_{\tau}$, whose entries satisfy $(\mathbf{k}_{\tau})_j = k_{\tau}(j)$. As such, the numerator is $\langle s(t+\tau) \hat{s}(t+\tau) \rangle_t^2 = \mathbf{r}_{\tau}^{\top} R^{-1} \mathbf{r}_{\tau} \mathbf{r}_{\tau}^{\top} R^{-1} \mathbf{r}_{\tau}$. We then evaluate the denominator, where, in the last step, we utilize the symmetry of R since the autocorrelation function is symmetric:

$$\langle \hat{s}(t)^2 \rangle_t = \left\langle \left(\sum_{j=1}^{\infty} s(t-j) k_{\tau=0}(j) \right) \left(\sum_{j'=1}^{\infty} s(t-j') k_{\tau=0}(j') \right) \right\rangle_t \quad (7)$$

$$= \sum_{j,j'} R_{ss}(j-j') k_0(j) k_0(j') \quad (8)$$

$$= \mathbf{k}_0^{\top} R \mathbf{k}_0 \quad (9)$$

$$= \mathbf{r}_0^{\top} (R^{-1})^{\top} \mathbf{r}_0 \quad (10)$$

$$= \mathbf{r}_0^{\top} R^{-1} \mathbf{r}_0. \quad (11)$$

If we denote $S = \mathbf{r}_\tau \mathbf{r}_\tau^\top$ (equivalently, $S = [S_{ij}] = [R_{ss}(\tau + i)R_{ss}(\tau + j)]$), we arrive at the revised expression for predictive capacity in terms of Wiener filters, which is

$$\text{PC} \leq \sum_{\tau=0}^{\infty} \frac{\mathbf{r}_\tau^\top R^{-1} S R^{-1} \mathbf{r}_\tau}{\mathbf{r}_0^\top R^{-1} \mathbf{r}_0}. \quad (12)$$

The conceptual conclusions of the paper remain valid.

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