

Predicting oscillations in complex networks with delayed feedback

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(Received 5 March 2026; accepted 23 July 2026; published 17 August 2026)

Oscillatory dynamics are common features of complex networks, often playing essential roles in regulating function. Across scales from gene regulatory networks to ecosystems, delayed feedback mechanisms are key drivers of system-scale oscillations. The analysis and prediction of such dynamics are highly challenging, however, due to the combination of high-dimensionality, nonlinearity and delay. Here, we investigate how structural complexity and delayed feedback are associated with oscillatory dynamics in complex systems, and provide an analytical approach combining theoretical dimension reduction and data-driven prediction. We reveal that oscillations emerge from the interplay of structural complexity and delay, with reduced models uncovering their critical thresholds and showing that greater connectivity lowers the delay required for their onset. Our theory is empirically tested in an experiment on a programmable electronic circuit, where oscillations are observed once structural complexity and feedback delay exceeded the critical thresholds predicted by our theory. Finally, we deploy a reservoir computing pipeline to accurately predict the onset of oscillations directly from time-series data. Our findings deepen understanding of oscillatory regulation and provide useful insights into predicting dynamics in complex networks.

DOI: [10.1103/physreve.114.024214](https://doi.org/10.1103/physreve.114.024214)

I. INTRODUCTION

Oscillatory behaviors are widely observed across complex networked systems and often emerge near transitions characterized by reduced stability [1–6]. Such oscillations arise in diverse contexts, including neural activity, power-grid dynamics, social and technological networks, and ecological systems, where predator-prey interactions and population cycles provide a paradigmatic example [7–10]. External variability—whether periodic forcing, stochastic perturbations, or slow environmental modulation—can trigger or reshape oscillatory dynamics, sometimes leading to abrupt or irreversible transitions [11–15]. Oscillations may also be triggered by intrinsic stochasticity, particularly when close to a transition point [16–20]. In this way, oscillatory dynamics may serve as early warning signals of declining resilience and impending functional loss [21]. Understanding and predicting the emergence of oscillatory behavior in complex networks is, thus, essential for anticipating transitions and preventing large-scale functional degradation.

A central mechanism shaping dynamics in many networked systems is memory, whereby past states influence present evolution. In ecosystems, this phenomenon is often termed *ecological memory*, reflecting delayed demographic,

environmental, or biogeochemical feedbacks [22–24]. For example, soil organic matter decomposition depends not only on current conditions but also on past microbial and environmental states [25], while tree growth reflects cumulative climatic influences, leading to the so-called prior growth effect [26]. Analogous memory effects arise in neural adaptation, delayed control in engineering systems, and epidemic propagation in social networks [27,28]. Mathematically, such memory can be incorporated through time delays or hereditary terms [29–32], which capture delayed responses and can induce oscillations [33–37]. Yang *et al.* showed that short delays may enhance stability whereas long delays destabilize communities [35], and Jiang *et al.* demonstrated Hopf bifurcation in delayed two-species systems beyond critical thresholds [36]. Although low-dimensional systems have provided valuable insight into memory-driven oscillations, the interplay between memory effects and structural complexity in high-dimensional nonlinear networks remains largely unexplored [38].

Complex networks composed of many interacting units exhibit structural richness that plays a decisive role in shaping their collective dynamics [39–41]. Increased structural complexity may undermine stability and drive systems toward instability [1–5,42–44] and drive other emergent phenomena including oscillations [45–49]. Ecological networks provide a well-studied realization of these principles, yet similar mechanisms operate in engineered, biological, and socio-technical networks. Modeling and predicting such behaviors remain challenging: large network size renders the system intrinsically high-dimensional, while heterogeneous interactions limit the applicability of traditional tools developed for low-dimensional or weakly nonlinear systems [50,51]. These challenges motivate the development of predictive

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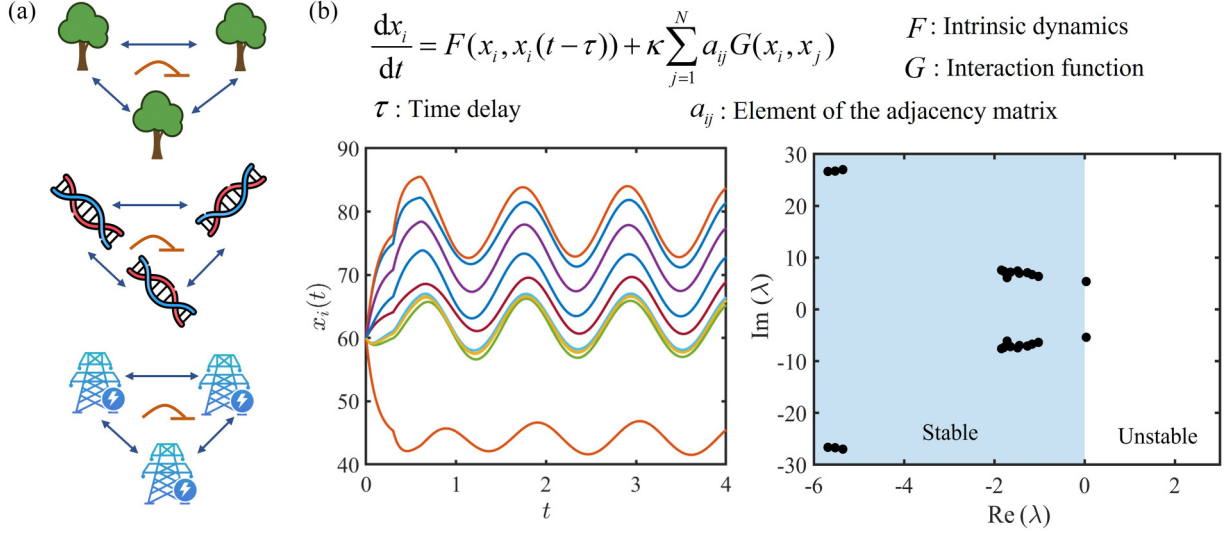


FIG. 1. Complex oscillations in dynamical systems. (a) Schematic illustration of real-world systems exhibiting oscillatory behavior: forest ecosystems (trees), gene regulatory networks, and power grids. Blue arrows denote interspecific interactions, while orange lines represent intraspecific time-delay effects. (b) General formulation of a complex time-delayed system. Left: Temporal evolution of species abundances in system (1), exhibiting sustained oscillatory dynamics. Right: The roots of the characteristic equation for system (1) are marked by black dots. The underlying structure follows an ER network with $N = 10$, connectivity $C = 0.5$, and parameters $r_i = 0.4$, $s_i = 0.08$, $D_i = 1$, $E_i = 1$, and $\tau = 0.3$.

frameworks capable of capturing high-dimensional, nonlinear dynamics in general complex networks [52,53].

While theoretical models yield mechanistic insight, they often rely on assumptions—such as homogeneous interactions or simplified functional forms—that may not hold in real-world networks [52]. This raises the broader question of how to predict complex network dynamics without imposing restrictive structural hypotheses. As a complementary strategy, data-driven machine learning approaches offer flexible, model-free frameworks capable of learning high-dimensional nonlinear dynamics directly from observations. In particular, reservoir computing has demonstrated strong performance in forecasting complex dynamical systems [54–59]. Xiao *et al.* introduced parameter-aware reservoir computing to predict dynamics under parameter drift [54], and Roy *et al.* showed that echo state networks trained on a single attractor can reconstruct coexisting attractors at new parameter values [55].

Here, we systematically investigate how structural complexity and memory jointly generate oscillatory dynamics in the considered class of complex networks, using ecological mutualistic networks as a representative example. Memory is incorporated as delayed intraspecific competition, and structural complexity is encoded in a random interaction network. The considered class of complex networks is characterized by positive interactions and local delayed feedback. We employ a dimension reduction approach that maps the high-dimensional network system onto an effective one-dimensional system [52]. Our analysis reveals that the interplay between connectivity and delay induces oscillations via Hopf bifurcation, with stronger connectivity lowering the critical delay threshold. We apply our theoretical framework to an experimental electronic circuit that exhibits oscillatory transitions near the predicted thresholds. Finally, to address parameter uncertainty and structural heterogeneity, we apply reservoir computing to forecast oscillations driven by complexity and memory. The

data-driven approach accurately predicts critical transitions, complements theoretical analysis, and may be extended to other nonlinear networked systems similar to those considered in this study.

II. MODELING AND PREDICTION OF OSCILLATIONS

In this section, we present the modeling framework and prediction approach for oscillatory dynamics. Figure 1(a) shows schematic examples of oscillatory behavior in three representative real-world systems: forest ecosystems, gene regulatory networks, and power grids. To characterize memory effects on complex network dynamics, we construct a delayed network model in which memory length is represented by a time delay. Figure 1(b) indicates a general formulation of such systems, where the first term describes intrinsic dynamics and the second term represents a general interaction function G . κ is the interaction strength.

In the development of our theory, we will focus primarily on the application to ecological networks, in which dynamical variables $x_i(t)$ denote the abundance of species i at time t . Mathematically, our model is formulated as follows:

$$\frac{dx_i(t)}{dt} = x_i(t)[r_i - s_i x_i(t - \tau)] + \frac{\sum_{j=1}^N a_{ij} x_i(t) x_j(t)}{D_i + E_i x_i(t)}, \quad (1)$$

where the time delay $\tau > 0$ represents the length of ecological memory. The parameter r_i is the intrinsic growth rate of species i , and s_i controls the strength of intraspecific competition for resources, which depends on the delayed density $x_i(t - \tau)$. The second term reflects saturated mutualistic interactions. Specifically, a_{ij} denotes the adjacency matrix element of the network, where $a_{ij} = 1$ if species i and j are connected and $a_{ij} = 0$ otherwise, with $a_{ii} = 0$. The parameters $D_i > 0$ and $E_i > 0$ determine the saturation level of these mutualistic interactions [52]. The initial condition is $x_i(t) = \varphi_i(t) \geq 0$,

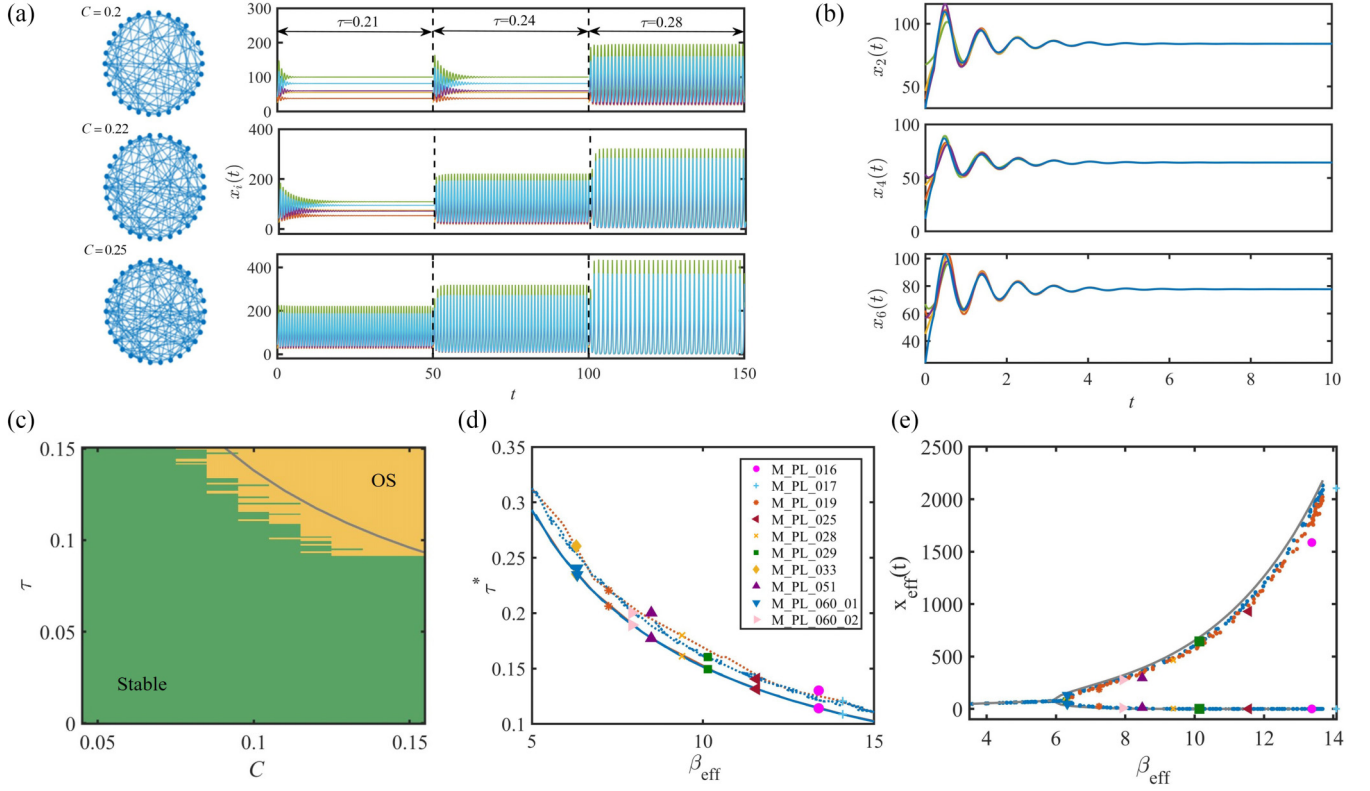


FIG. 2. Critical delay for oscillations induced by network complexity and time delay. (a) The time series of system (1) under various connectivity and delays. For $C = 0.2$, the species abundance remains stable at $\tau = 0.21$ and $\tau = 0.24$, but exhibits oscillatory behavior at $\tau = 0.28$. As connectivity increases to $C = 0.22$, the system also transitions from a stable to an oscillatory regime, with oscillations already appearing at $\tau = 0.24$. When $C = 0.25$, oscillations emerge as early as $\tau = 0.21$, and their amplitude further increases with larger delays. (b) The time series $x_2(t)$, $x_4(t)$, and $x_6(t)$ of the system are observed to converge to the same equilibrium, $\tau = 0.21$. (c) The colored regions correspond to the dynamical states of the original high-dimensional system (1) on ER random networks, whereas the solid lines depict the theoretical estimates (4) derived from the reduced 1D system (2). As τ and C increase, the system states shift from stability to oscillatory behavior, as shown by the green (stable) and yellow (oscillatory, OS) regions. (d) The reduced 1D model (2) predicts the critical delay of the original high-dimensional system (1). The blue and orange lines correspond to the homogeneous and heterogeneous parameter settings, respectively. Increasing β_{eff} decreases the critical delay of the system. Both theoretical (solid lines) and numerical (dotted lines) results are shown for ER networks with varying connectivity C and network size N , while different markers represent results from ten empirical networks. (e) The reduced 1D model accurately reproduces the abundance dynamics of the original high-dimensional model. Solid lines represent the reduced 1D model, while symbols represent the original high-dimensional system. The network structures are the same as those used in panel (d). Heterogeneous parameters: $r_i \in [0.3, 0.4]$, $s_i \in [0.0667, 0.1]$, and $D_i = E_i = 1$. The homogeneous case is considered in panels (d)–(e), with $r_i = 0.4$, $s_i = 0.08$, $D_i = 1$, and $E_i = 1$ for all nodes. $N = 30$ for panels (a)–(b), and $N = 100$ for panels (c)–(e).

$t \in [-\tau, 0]$, where $\varphi_i \in C([-\tau, 0], [0, +\infty))$, and $\varphi_i(0) > 0$. The model captures potential delayed effects, enabling a systematic analysis of how ecological memory influences the network dynamics of the system [25,29,30].

To explore how ecological memory and network topology jointly shape oscillatory dynamics, we couple species through Erdős-Rényi (ER) random networks [60]. The time series of system (1) are shown in Fig. 1(b). For delay-free systems, stability can be evaluated by the largest real part of the eigenvalues of the Jacobian matrix $\mathcal{J}$, denoted $\text{Re}(\lambda_1)$. For delayed systems, the dynamical stability is determined by the roots of the corresponding characteristic equation [35]. The system is stable when all roots have negative real parts. The distribution of the characteristic roots is shown in Fig. 1(b). A pair of complex conjugate roots crosses the imaginary axis, resulting in oscillatory behavior. Then, we further investigate the oscillatory behavior of the system. Figure 2(a) presents the dynamic evolution of system (1) over time. The

results reveal that increasing connectivity and delay jointly destabilize the system, inducing oscillatory behavior and underscoring their combined influence on high-dimensional, nonlinear dynamics. Moreover, for a fixed network structure, a critical delay exists that triggers oscillations, raising the question of how this threshold can be predicted. In addition, the effect of initial conditions is discussed in Fig. 2(b). The time series under different initial values show that all trajectories eventually converge to the same steady state, indicating that the initial conditions do not affect the system dynamics. Therefore, identical initial conditions are used in the following analysis [5,35,52].

A. Derivation of theoretical critical delay

In the following, we derive the critical delay that induces oscillations. Since the complexity of high-dimensional and nonlinear dynamics in the model, we reduce the system

to a lower-dimensional form using the Gao-Barzel-Barabási (GBB) framework [52]. This dimension reduction provides an approximate description of the dynamical features of the original system, enabling a more tractable theoretical analysis and facilitating the identification of the critical delay associated with the onset of oscillations [38,52]. The corresponding reduction operator is defined as follows: $\mathcal{L}(x) = \sum_{i,j} a_{ij}x_j / \sum_{i,j} a_{ij}$. The operator $\mathcal{L}$ takes the vector x as input and outputs a weighted average of its nearest-neighbor components, with weights proportional to the degree of each component. By using the operator, we obtain the following effective one-dimensional (1D) system

$$\frac{dx_{\text{eff}}(t)}{dt} = x_{\text{eff}}(t) \left[r_{\text{eff}} - s_{\text{eff}}x_{\text{eff}}(t - \tau) + \frac{\beta_{\text{eff}}x_{\text{eff}}(t)}{D_{\text{eff}} + E_{\text{eff}}x_{\text{eff}}(t)} \right], \quad (2)$$

where $x_{\text{eff}}(t)$ is the effective abundance, and the role of the network is compressed into a single effective parameter $\beta_{\text{eff}} = \sum_{i,j,k} a_{ij}a_{kj} / \sum_{i,j} a_{ij}$. We use $r_{\text{eff}} = \mathcal{L}(r)$, $s_{\text{eff}} = \mathcal{L}(s)$, $D_{\text{eff}} = \mathcal{L}(D)$, $E_{\text{eff}} = \mathcal{L}(E)$, and $\mathcal{L}(a \circ b) = \mathcal{L}(a)\mathcal{L}(b)$, where $\circ$ is the Hadamard product defined by $a \circ b = (a_1b_1, \dots, a_Nb_N)^T$. This reduction collapses N^2 microscopic parameters a_{ij} into a single macroscopic parameter β_{eff} , while preserving the network's structural features through degree-weighted average. We further compare the dimensionality reduction method with other approaches, and the results are presented in the Methods section.

The model has two equilibria, $x_{\text{eff}} = 0$ and $x_{\text{eff}}^* = (a_1 + \sqrt{a_1^2 + 4r_{\text{eff}}s_{\text{eff}}D_{\text{eff}}E_{\text{eff}}})/2s_{\text{eff}}E_{\text{eff}}$, where $a_1 = r_{\text{eff}}E_{\text{eff}} + \beta_{\text{eff}} - s_{\text{eff}}D_{\text{eff}}$. Next, we conduct a theoretical analysis and focus on the positive equilibrium x_{eff}^* . The oscillatory behavior of delayed systems can usually be analyzed through their characteristic equations. Specifically, the characteristic equation of Eq. (2) at x_{eff}^* is given by

$$\lambda + B_1 e^{-\lambda\tau} - B_2 = 0, \quad (3)$$

where $B_1 = s_{\text{eff}}x_{\text{eff}}^*$, and $B_2 = (\beta_{\text{eff}}D_{\text{eff}}x_{\text{eff}}^*)/(D_{\text{eff}} + E_{\text{eff}}x_{\text{eff}}^*)^2$. According to the equation, we can examine the Hopf bifurcation in system (2) to derive the critical delay theoretically. Let $\lambda = i\omega$, the critical delay can then be obtained by analysis as follows (see Methods section):

$$\tau^* = \arccos \left(\frac{\beta_{\text{eff}}D_{\text{eff}}x_{\text{eff}}^*}{s_{\text{eff}}x_{\text{eff}}^*(D_{\text{eff}} + E_{\text{eff}}x_{\text{eff}}^*)^2} \right) / \omega. \quad (4)$$

The transversality condition is satisfied for the parameter sets used in the numerical simulations.

In particular, for undirected networks with node degrees k_i that are uncorrelated, we have $\beta_{\text{eff}} = \langle k^2 \rangle / \langle k \rangle$. For random ER networks, $\langle k^2 \rangle = \langle k \rangle (\langle k \rangle + 1)$ yielding $\beta_{\text{eff}} = \langle k \rangle + 1$. Using $\langle k \rangle = (N-1)C \approx NC$ in the large- N limit, we obtain $\beta_{\text{eff}} \approx NC + 1$. We use Fig. 2(c) to show the states of the original system (1) along with the analytically derived expressions (4) across different values of τ and C . The system is numerically integrated with a time step $h = 0.01$ up to $t = 500$. To identify sustained oscillations, the entire time series is first examined for persistent periodic behavior. The last 10 time steps are then analyzed: a trajectory is classified as oscillatory if the difference between the maximum and minimum values of the

state variable exceeds 10^{-3} . By comparison, it can be seen that the theoretical results are in good agreement with the numerical simulations, indicating that the theoretical analysis can effectively predict the oscillatory behavior of the high-dimensional system.

B. Oscillations in synthetic and real networks

Further, we verify the validity of the theoretical critical time delay. The critical delay as a function of β_{eff} is shown in Fig. 2(d), where the solid line represents the theoretical prediction for ER random networks derived from Eq. (2), while the dotted line shows the corresponding numerical results from Eq. (1). The two critical delays are basically consistent for both homogeneous-parameter and heterogeneous-parameter cases. This indicates that the one-dimensional simplified system (2) can be used to predict the oscillations of the original high-dimensional system (1). When the reduced system undergoes oscillations, the high-dimensional system correspondingly exhibits oscillatory behavior. To assess the accuracy of the reduction more directly, Fig. 2(e) compares the state evolution between system (1) and system (2). The effective variable $x_{\text{eff}}(t)$ of the high-dimensional system (1) is constructed as a weighted average of the node states $x_i(t)$. Under the same parameter settings, both systems exhibit nearly identical state evolution, with minor differences in the location of the critical points. Moreover, under homogeneous parameter settings, we examine the critical delay in the system (1) with empirical mutualistic networks [5], and compare it with that of the reduced system (2). The ten networks shown here are randomly selected illustrative examples from the full set of analyzed mutualistic pollinator-plant networks available from the Web of Life database [5,52,61]. The results remain consistent, illustrating the robustness of the reduced model in capturing the onset of oscillations.

III. EXPERIMENTAL TEST AND DATA-DRIVEN PREDICTION OF OSCILLATIONS

A. Experimental realization using programmable micro-controller

To demonstrate our predictions, we employ an experimental system designed around circuit simulation [62]. The structural composition of the network oscillation control circuit system is formed by successively connecting in series a microcontroller unit (MCU) chip, an H-bridge drive circuit, a subtractor circuit, and an integrator circuit, as depicted in Fig. 3(a). Correspondingly, a printed circuit board (PCB) has been fabricated, as shown in Fig. 3(b). The information regarding network nodes and their topological structures is encoded and stored within the registers of the MCU. Simultaneously, by leveraging its internal register set, a delay controller is constructed. This enables precise regulation of time delays and dynamic storage of node state information. The MCU generates level-conversion instructions in real time according to the right-hand-side function of Eq. (1). These instructions are used to perform closed-loop regulation of the physical quantities of the peripheral circuits (details can be found in the Methods section). This allows the circuit dynamics

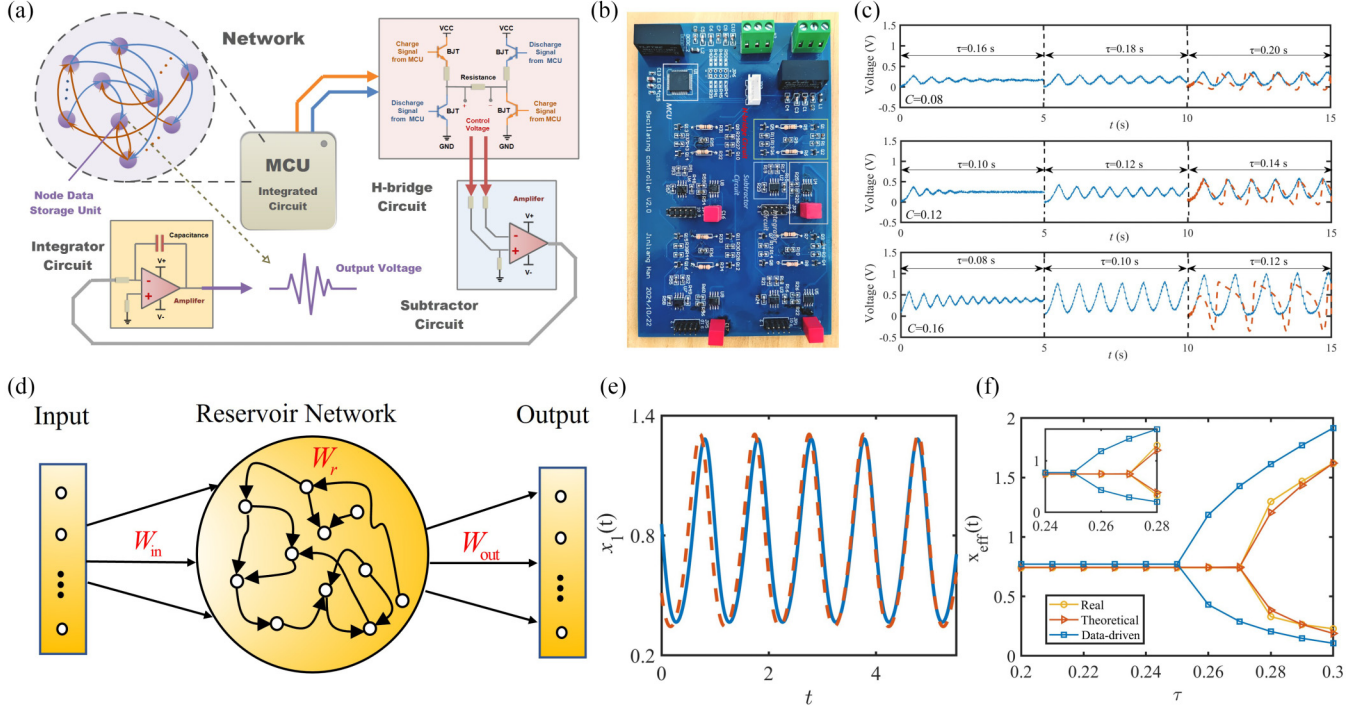


FIG. 3. Prediction and validation of oscillatory behavior. (a) The design block diagram of the oscillation control circuit experimental system consists of the MCU chip, the H-bridge driver circuit, the subtractor circuit, and the integrator circuit connected in series. The network nodes and their topological information are stored in the registers of the MCU. (b) The physical PCB circuit, realized based on the design block diagram, is capable of simultaneously detecting the state waveforms mapped from up to four nodes. (c) Experimental voltage u for various delays and ER random networks with different connectivity: $C = 0.08$, $C = 0.12$, and $C = 0.16$. The dashed line represents the predictions obtained from reservoir computing. All networks have $N = 100$ nodes, and model parameters are set to $r_i = 0.4$, $s_i = 0.08$, and $D_i = E_i = 1$. (d) Schematic diagram of the reservoir computing architecture, which includes three layers: input, reservoir (hidden), and output. The reservoir is trained using time series corresponding to three delays ($\tau = 0.23, 0.24$, and 0.25) and demonstrates generalization beyond the training range by adjusting the input delay parameter. (e) Prediction of oscillatory dynamics in system (1). Time series of $x_1(t)$ from system (1) and its prediction at $\tau = 0.28$ are shown to validate the accuracy and robustness of the training. (f) Effective system abundance $x_{\text{eff}}(t)$ as a function of τ for three cases. Theoretical results are obtained using the GBB method [Eq. (4)], real data are generated from system (1), and data-driven results are produced by the reservoir computing.

to qualitatively reproduce the evolution of node states in the considered system. Consequently, the output of the system can visually exhibit the dynamic characteristics and control mechanisms of complex networks with delay. This demonstrates qualitative consistency between experimental observations and theoretical predictions.

We randomly generate ER networks with three different connection probabilities $C(N = 100)$. By adjusting the system delay within the MCU output, we monitored the dynamic response of the circuit output voltage in real-time. This was done to determine whether oscillations occurred in the network and to analyze the characteristics of its amplitude regulation. Taking the second node as the object of observation, we used an oscilloscope to collect and record signals. The results are presented in Fig. 3(c). When $C = 0.08$, at $\tau = 0.16$, the voltage signal tended towards a static equilibrium state. However, when $\tau = 0.18$, distinct oscillations emerged, indicating that the critical delay τ^* for the onset of oscillations satisfies $0.16 < \tau^* < 0.18$. Substituting the relevant network parameters into Eq. (1), the critical delay of the reduced system was approximately $\tau^* = 0.18$. Analogously, for $C = 0.12$, the critical delay range was $0.1 < \tau^* < 0.12$, corresponding to the theoretically predicted value of

$\tau^* = 0.12$. When $C = 0.16$, the range $0.08 < \tau < 0.1$ was observed. Combining with Eq. (2), the critical delay of the reduced system was $\tau^* = 0.09$. These results demonstrate that the oscillation behavior of high-dimensional networks can be effectively predicted by the critical delay of a one-dimensional simplified model. Moreover, it was further discovered that as the delay increases, the oscillation amplitude gradually enlarges, and this effect is more pronounced at higher connection densities. This finding is highly consistent with the simulation results shown in Fig. 2. Through the construction of a circuit system capable of implementing the delayed network dynamics of the model under study, this experiment successfully captures the onset of oscillations. This is consistent with the oscillatory transition observed in the numerical simulations (see the Supplemental Material video [64]).

B. Reservoir computing pipeline

Although the theoretical analysis provides a clear characterization of the onset of oscillations, its practical applicability is often limited due to system parameter uncertainty and structural heterogeneity. To address this limitation and bridge theory with data-driven prediction, we introduce a reservoir

computing framework as an alternative tool to the analytical approach. The method learns from observed time-series data and predicts oscillatory behavior without requiring explicit knowledge of the model parameters.

As a powerful data-driven approach, reservoir computing effectively captures the nonlinear characteristics of complex systems and reproduces their dynamical properties, providing a practical alternative to traditional methods by avoiding intricate theoretical derivations [55,56,58]. The prediction of coexisting attractors and the identification of basins of attraction have been studied in multistable systems [55]. Rakshit *et al.* investigated the aging transition in networks composed of active and inactive Landau oscillators and employed reservoir computing to predict oscillation collapse and the critical inactive ratio [56]. Furthermore, Liu *et al.* improved the conventional reservoir computing framework by proposing the local information flow echo state network (LIF-ESN), which enhances prediction performance under different parameter regimes and initial conditions [58]. Different from the above works, the present study focuses on delay-induced critical transitions and oscillation emergence in high-dimensional systems. The scheme of reservoir computing is shown in Fig. 3(d), where the dynamics are described with the following map:

$$\gamma(t+1) = (1-\alpha)\gamma(t) + \alpha \tanh[W_r \cdot \gamma(t) + W_{in} \cdot u(t)], \quad (5)$$

where $\gamma(t) = [\gamma_1(t), \gamma_2(t), \dots, \gamma_N(t)]^T$ represents the state of the reservoir at time t . $\alpha \in (0, 1)$ is the leakage rate. $W_r \in \mathbb{R}^{N \times N}$ is a random sparse matrix, generated by an ER graph configuration. $u(t) \in \mathbb{R}^{M+1}$ is an $M+1$ dimensional input vector, where the first M elements are the dynamics of the input signal $\tilde{u}(t)$ at time t and the last element is the corresponding parameter, i.e., $u(t) = [\tilde{u}(t)^T \ p]^T$. We randomly choose the elements of W_{in} from the uniformly distributed interval $[-\sigma, \sigma]$ and form W_{in} in such a way that the information of the input data would be equally distributed to the nodes of the reservoir. Each input dimension is projected onto N/M reservoir nodes, while parameter information is embedded across all nodes. This design allows the reservoir to capture the interplay between system dynamics and their associated parameters.

We train the machine for N_p different parameter values one by one and successively store the values of $\gamma(t)$, and the initial state is set to be $\gamma(0) = 0$. It is conventional to define a new vector $\tilde{\gamma}$ with the same odd row elements as those of γ but with the even row elements as the squared values of the corresponding even row elements of $\gamma(t)$. We store the values of $\tilde{\gamma}(t)$ within the matrix R after ignoring t_b time points as the transient dynamics of the reservoir. Therefore, R is a matrix with dimension $N \times N_p(t_f - t_b)$ for training data of length t_f . After calculating R , we can get the readout weight matrix from the system, $U = W_{out}R$, where U is a matrix having targeted data sets with dimension $M \times N_p(t_f - t_b)$. Here, the target output corresponds to the system state at the next time step. To determine the readout matrix W_{out} , we use Ridge regression to minimize the discrepancy between U and $W_{out}R$:

$$W_{out} = UR^T(RR^T + \beta I)^{-1}, \quad (6)$$

where β is the regularization term that prevents overfitting. Once training is completed, the model can generate predictions solely based on the parameter values.

During the prediction phase, the input $u(t)$ is replaced by the output $v(t)$, creating a closed-loop system that allows the reservoir to evolve autonomously. The system state is then updated iteratively as follows:

$$\begin{aligned} \gamma(t+1) &= (1-\alpha)\gamma(t) + \alpha \tanh[W_r \cdot \gamma(t) + W_{in} \cdot v(t)], \\ v(t) &= [\hat{v}(t)^T \ p_{new}]^T, \\ \hat{v}(t) &= W_{out} \cdot \tilde{\gamma}(t). \end{aligned} \quad (7)$$

By iterating Eq. (7), the reservoir generates predicted time series for new parameter values. The prediction process begins from an initial condition corresponding to a bounded solution, often chosen from the training trajectory. Prior to generating predictions, the reservoir undergoes a warm-up phase to eliminate the influence of its initial state. The scheme involves several hyperparameters-spectral radius, leaking rate, input weight range, and regularization parameter-which are optimized using a simultaneous optimization technique based on the root-mean-square error (RMSE) of the predicted series relative to the target, averaged over multiple reservoir realizations. Once the optimal hyperparameters are obtained, the final reservoir is constructed, ready to predict system dynamics under new parameter values and initial conditions.

C. Proof of concept test

Then, we use this method to predict the oscillation of the system (1) for ER random networks with $N = 100$. We employ the Runge-Kutta method to generate data at $\tau = 0.23, 0.24, 0.25$ by system (1) with a step size of $h = 0.011$ for model training. Under these time-delay parameters, the system exhibits a stable state. The number of training data at each parameter value τ is 10 000. The parameter settings for the model are as follows: the size of the reservoir is 1000, the sparsity of the reservoir weight matrix is 0.1, the leakage rate is 0.3, and the regularization parameter of Ridge regression is 10^{-5} . The reservoir weight matrix is first randomly generated from a uniform distribution in the interval $[-0.5, 0.5]$, then regularized with a spectral radius of 0.8.

We investigate the system behavior at $\tau = 0.26$ and $\tau = 0.28$. The results show that the system remains in a stable state at $\tau = 0.26$, which is accurately captured by the model. When $\tau = 0.28$, the system enters an oscillatory regime, and the predicted results agree well with the actual dynamics. The predicted results of $x_1(t)$ in the system (1) at 0.28 are shown in Fig. 3(e). To further predict the critical delay for the system to transition from a stable state to an oscillatory state, we conduct predictions at $\tau = 0.26, 0.261, 0.262, \dots, 0.278, 0.279$, and 0.28. The results show that the system is in a stable state at $\tau = 0.274$ and an oscillatory state at $\tau = 0.275$, thus the predicted critical delay of oscillation is 0.275. We can see that the model results are consistent with the predicted results, indicating that the prediction method based on reservoir computing is effective. Applying this method to predict the dynamics of the circuit system yields the results shown in Fig. 3(c). We can observe that the reservoir computing

approach is also capable of accurately predicting the dynamics originating from the circuit.

As shown in Fig. 3(f), we further compare the trajectories of $x_{\text{eff}}(t)$ obtained by different methods, using the GBB framework as a theoretical reference. The results indicate that the one-dimensional theoretical model detects the onset of oscillations earlier, providing an effective early warning signal for high-dimensional systems, whereas reservoir computing delivers highly accurate predictions of the actual critical delay. Together, these findings highlight the complementary nature of the two approaches: dimension reduction offers theoretical guidance, while reservoir computing provides data-driven predictions, and their combination enhances the overall reliability of oscillation forecasting.

IV. DISCUSSION

This study investigates the memory effect in intraspecific interactions within the considered class of network systems, with a focus on how structural complexity and time delays influence the emergence and characteristics of oscillatory dynamics [35]. To address the challenges of predicting oscillations in high-dimensional ecological networks, we first applied dimension reduction to transform the system into a one-dimensional representation, enabling analytical tractability. Theoretical analysis of this reduced system allows us to derive an explicit expression for the critical delay, which serves as a threshold for the onset of oscillatory behavior. Using this critical delay, we successfully predicted oscillations in the original high-dimensional networks and demonstrated that, as network connectivity increases, the critical delay decreases, leading to earlier oscillation onset. The GBB method gives the smallest average errors among the methods tested. In addition, we employed reservoir computing as a data-driven framework to predict oscillatory dynamics in complex high-dimensional systems, achieving accurate forecasts without requiring explicit knowledge of all model parameters. Together, these results provide a versatile framework for analyzing and predicting oscillations in the considered class of network systems, highlighting the combined power of theoretical and machine-learning approaches.

While the numerical simulations demonstrate that the reduction can predict the onset of oscillations across a range of networks, the fundamental origin of its effectiveness remains incompletely understood. In particular, the reduction relies primarily on degree-weighted structural information, yet a rigorous theoretical explanation for why degree serves as the dominant determinant of the collective dynamics is still lacking. This issue becomes especially relevant for networks exhibiting strong heterogeneity, degree correlations, or other higher-order structural features. Consequently, developing a theoretical framework to identify the validity limits of the reduction and to quantify the influence of higher-order network structures remains an important direction for future research. Furthermore, although reservoir computing has demonstrated strong predictive capability for systems based on empirical mutualistic networks, its current implementation relies primarily on training data generated from theoretical models. Future studies should incorporate empirical observations to validate the practical performance of the framework [45,63].

Bridging the gap between theoretical models and real-world data will enhance the reliability of oscillation predictions and provide valuable insights for understanding dynamics in ecological and other complex networked systems.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant No. 12271519), the Shenzhen Science and Technology Program (Grant No. JCYJ20240813101300001), the Fundamental Research Funds for the Central Universities (Grants No. 2023ZDYQ11005 and No. 2024KYJD2002).

The authors declare that they have no competing interests.

DATA AVAILABILITY

All real ecological network data supporting the findings of this study are publicly available [61]. The simulation code supporting this work is available for download [64].

APPENDIX: METHODS

1. Derivation of the critical delay

Here, we present the detailed derivation of Eq. (4). Assuming $\lambda = i\omega$, we have

$$i\omega + B_1 e^{-\lambda\tau} - B_2 = 0.$$

Then, the real and imaginary parts of the equation are

$$B_1 \cos \omega\tau = B_2, \quad B_1 \sin \omega\tau = \omega. \quad (\text{A1})$$

Hence, we obtain

$$\omega^2 = B_1^2 - B_2^2,$$

which requires $B_1 > B_2$ to ensure a real ω . Then, for $B_2 > 0$,

$$\tau_i^k = \frac{\arccos \frac{B_2}{B_1} + 2k\pi}{\omega} = \frac{\arcsin \frac{\omega}{B_1} + 2k\pi}{\omega},$$

$$k = 0, 1, 2, \dots,$$

and for $B_2 < 0$,

$$\tau_i^k = \frac{\arccos \frac{B_2}{B_1} + 2k\pi}{\omega} = \frac{\pi - \arcsin \frac{\omega}{B_1} + 2k\pi}{\omega},$$

$$k = 0, 1, 2, \dots$$

Due to the periodic nature of inverse trigonometric functions, the critical values of time delay and their corresponding critical frequency values obtained through the method above also exhibit periodicity. Considering the practical significance of time delay in real life, we only take the first positive value of τ in the solution as the critical value of time delay. Then, we can obtain the critical delay as Eq. (4). Let $\lambda(\tau) = \xi(\tau) + i\omega(\tau)$ be the eigenvalue of Eq. (3). Then, to find the transversality condition for the occurrence of Hopf-bifurcation, we need to verify $\frac{d\xi(\tau)}{d\tau}|_{\tau=\tau^*} > 0$. Taking λ into Eq. (3), and equating real and imaginary parts are

$$\xi + B_1 \exp(-\xi\tau) \cos \omega\tau - B_2 = 0,$$

$$\omega - B_1 \exp(-\xi\tau) \sin \omega\tau = 0. \quad (\text{A2})$$

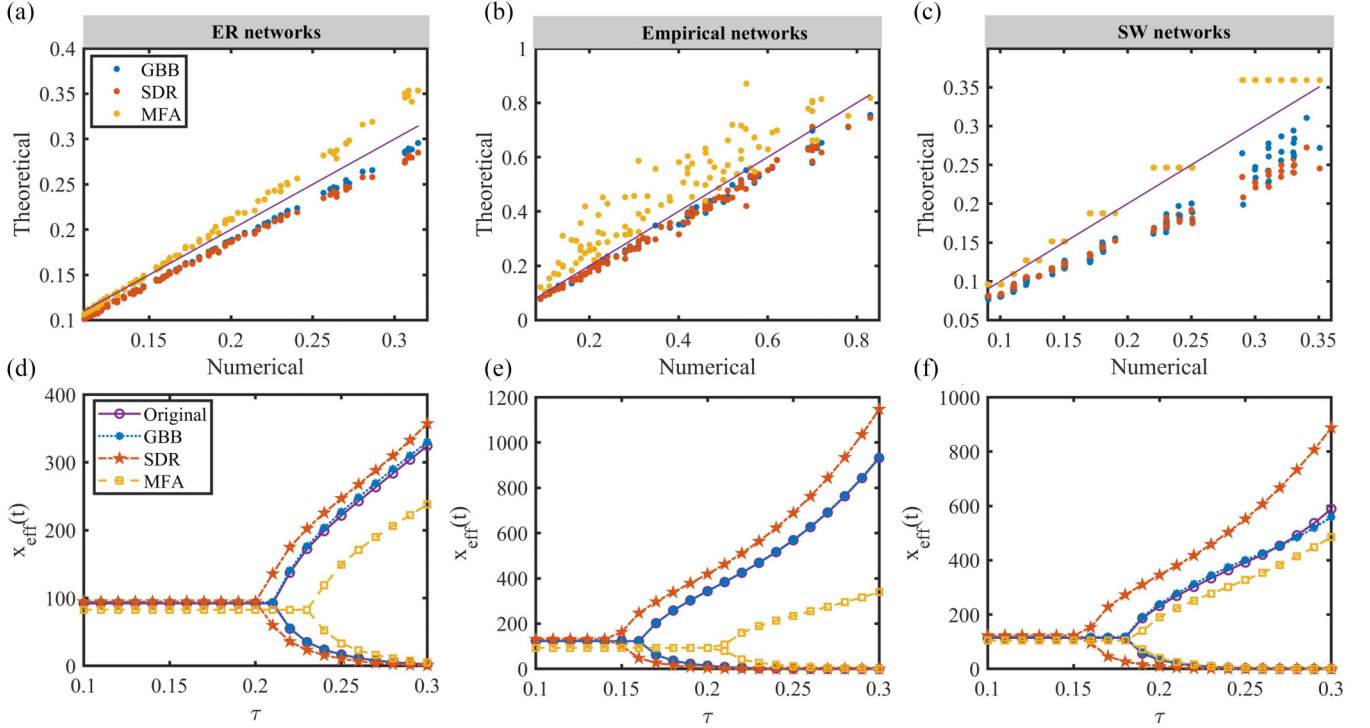


FIG. 4. Comparison of three dimensional reduction methods. (a)–(c) The critical delay τ^* of the original high-dimensional system (1) is compared with that obtained from three reduced representations: the GBB system (2), the SDR system (A6), and the MFA system (A7). Panel (a) shows results for ER random networks with $N = 100$ and varying connectivity C , panel (b) for empirical networks [5,52], and panel (c) for SW networks with $N = 100$, varying nearest neighbors K and $C = 0.15$. Numerical results are obtained from direct simulations, while theoretical results represent the predicted critical delays from Eq. (4) for the reduced systems. The solid line with slope 1 serves as a reference for perfect agreement. (d)–(f) The evolution of the effective system abundance $x_{\text{eff}}(t)$ is shown for the same four systems under randomly selected ER, empirical, and SW networks, with varying delay τ . These panels highlight how the dynamics of each network type are reproduced across the different dimensional reduction methods. Model parameters are fixed at $r_i = r = 0.4$, $s_i = s = 0.08$, $D_i = D = 1$, and $E_i = E = 1$.

Differentiate (A2) and set $\dot{\xi} = 0$, we have

$$\begin{aligned} X(\omega) \frac{d\xi}{d\tau} + Y(\omega) \frac{d\omega}{d\tau} &= Z(\omega), \\ -Y(\omega) \frac{d\xi}{d\tau} + X(\omega) \frac{d\omega}{d\tau} &= W(\omega), \end{aligned} \quad (\text{A3})$$

where $X(\omega) = 1 - B_1 \tau \cos \omega \tau$, $Y(\omega) = B_1 \tau \sin \omega \tau$, $Z(\omega) = B_1 \omega \sin \omega \tau$, and $W(\omega) = B_1 \omega \cos \omega \tau$. Through (A3), we obtain

$$\left. \frac{d\xi(\tau)}{d\tau} \right|_{\tau=\tau^*} = \frac{Z(\omega)X(\omega) - Y(\omega)W(\omega)}{X^2(\omega) + Y^2(\omega)}.$$

When $Z(\omega)X(\omega) - Y(\omega)W(\omega) > 0$, the system will undergo Hopf-bifurcation.

2. Dimension reduction methods

To assess the prediction accuracy of dimension reduction techniques, we employ two alternative dimension reduction methods, the spectral dimension reduction (SDR) [65] and the mean-field approximation (MFA) [66], to determine the critical delay. The predictions from all three methods are compared in Figs. 4(a)–4(c), where closer proximity to the diagonal line indicates smaller errors. The empirical networks obtained from the Web of Life database include 174 bipartite

networks. Since these bipartite networks are mapped into $N \times N$ interaction networks for the analysis, only one network was retained for each identical system size N to avoid redundancy. In addition, networks with extremely low connectivity (average degree smaller than 2) were excluded, as such weakly connected systems have limited relevance for collective oscillatory dynamics. After these preprocessing steps, 146 networks are retained for analysis. In Fig. 4(a), the MFA systematically overestimates the critical delay relative to the original system, highlighting its limited accuracy in high-dimensional networks. By contrast, the GBB and SDR methods yield more accurate predictions, with GBB showing smaller deviations, providing a better approximation of the features of the network. Figures 4(b) and 4(c) further confirm these trends across a variety of empirical mutualistic networks and small-world (SW) networks [67]. To more clearly address the quantitative comparison between GBB and SDR, we evaluate their average relative errors across the three networks. The average relative errors of GBB are 6.8051%, 9.1884%, and 16.1629%, while SDR exhibits higher average relative errors of 7.6304%, 10.9217%, and 18.2227%. The results show that GBB consistently yields smaller errors than SDR on average across all networks considered in this study.

To further evaluate their performance, one network is randomly selected from each class. The changes in system state

with delay are shown for an ER random network, an empirical network, and an SW network in Figs. 4(d)–4(f), respectively. The SDR consistently underestimates the critical delay compared with the original system. The MFA overestimates the delay in both ER and empirical networks, although its prediction in the SW network is closer to that of the original system. In contrast, the GBB reduced method provides critical delays that closely match those of the original system across all three network types, thereby reaffirming the robustness and effectiveness of the GBB approach.

The GBB method has been described in the main text, here, we introduce two additional dimension reduction approaches. For spectral dimension reduction, we consider $R(x) = \sum_{i=1}^N a_i x_i$, where a_i is the element of the eigenvector corresponding to the maximum eigenvalue of the adjacency matrix A [65]. This method captures the dominant network mode that governs the system's dynamics. Applying operator R on both sides, we can write Eq. (1) as

$$\begin{aligned} R\left[\frac{dx_i(t)}{dt}\right] &= R\left\{x_i(t)\left[r_i - s_i x_i(t - \tau) + \frac{\sum_{j=1}^N a_{ij} x_j(t)}{D_i + E_i x_i(t)}\right]\right\}, \\ &\approx R(x)\{r_i - R[s_i x_i(t - \tau)]\} + \frac{R\left[\sum_{j=1}^N a_{ij} x_i(t) x_j(t)\right]}{R[D_i + E_i x_i(t)]}. \end{aligned} \quad (\text{A4})$$

After simplification, we have

$$\begin{aligned} \frac{dR(x)}{dt} &= R(x)\{r_i - R[s_i x_i(t - \tau)]\} \\ &\quad + R(k) \frac{R(x)R(x)}{D_i + E_i R(x)}. \end{aligned} \quad (\text{A5})$$

By substituting $R(k) = \alpha_1$, $r_i = r$, $s_i = s$, $D_i = D$ and $E_i = E$ into Eq. (A5), then we can obtain a one-dimensional reduction of the original N -dimensional dynamical system as

$$\frac{dR(x)}{dt} = R(x) \left\{ r - sR[x(t - \tau)] + \frac{\alpha_1 R(x)}{D + ER(x)} \right\}. \quad (\text{A6})$$

For mean-field approximation [68], the system (1) can be written as

$$\frac{dx(t)}{dt} = x(t) \left[r - sx(t - \tau) + \frac{\langle k \rangle x(t)}{D + Ex(t)} \right], \quad (\text{A7})$$

where $\langle k \rangle$ is the average degree of matrix A . It can be seen that Eqs. (A6) and (A7) are similar to Eq. (2), but the specific network parameters are different.

3. Validation methods for electronic circuit experiments

In the circuit, the MCU precisely regulates the resistance voltage at the central node of the H-bridge drive circuit via external I/O pins. This voltage signal is first processed by a subtractor and then fed as an input into the integrator circuit. The dynamic evolution of the capacitor voltage Δu within the integrator serves as an effective indicator of the oscillation behavior of network nodes. By adjusting the delay step m in the MCU's delay register, a systematic investigation of the oscillation characteristics of network nodes under various time delays can be carried out.

The input terminals of the H-bridge circuit are categorized into two sets of control signals for charging and discharging. The operational state of the circuit is determined by the sign function $\text{sgn}_{\Delta u_i}$, which is defined as follows:

$$\text{sgn}_{\Delta u_i} = \text{sgn} \left\{ r_i x_i(n) [1 - s_i x_i(n - m)] + \frac{\sum_{j=1}^N a_{ij} x_i(n) x_j(n)}{D_i + E_i x_i(n)} \right\}. \quad (\text{A8})$$

Here, $x_i(n)$ represents the state variable of node i at the discrete time instant n . Given a time step of h , $x_i(n - m)$ corresponds to the historical state of node i at a delay time of $\tau = hm$. When $\text{sgn}_{\Delta u_i} = +1$, the MCU outputs a positive control signal to charge the external circuit. Conversely, when $\text{sgn}_{\Delta u_i} = -1$, the MCU initiates a discharging operation for the external circuit.

The width of the H-bridge drive pulse, denoted as $T_{\Delta u_i}$, is expressed as

$$T_{\Delta u_i} \propto \left| r_i x_i(n) [1 - s_i x_i(n - m)] + \frac{\sum_{j=1}^N a_{ij} x_i(n) x_j(n)}{D_i + E_i x_i(n)} \right|. \quad (\text{A9})$$

The voltage signal output by the subtractor is given by

$$v_i(n) = V \text{sgn}_{\Delta u_i}, \quad (\text{A10})$$

where V represents the amplitude of the control voltage. Assuming that the input resistance of the integrator is R_1 and the feedback capacitor is C_1 , the voltage increment across the capacitor satisfies the inverse integration relationship:

$$\begin{aligned} \Delta u_i &= -\frac{1}{R_1 C_1} \int_0^{T_{\Delta u_i}} v_i dt = -\frac{1}{R_1 C_1} v_i(n) T_{\Delta u_i} \\ &= -\frac{1}{R_1 C_1} V \text{sgn}_{\Delta u_i} T_{\Delta u_i} \\ &\propto -\left\{ r_i x_i(n) [1 - s_i x_i(n - m)] + \frac{\sum_{j=1}^N a_{ij} x_i(n) x_j(n)}{D_i + E_i x_i(n)} \right\} \\ &= -\Delta x_i. \end{aligned} \quad (\text{A11})$$

Consequently, through the acquisition of the output voltage signal using an oscilloscope and subsequent reverse deduction, the following relationship can be derived:

$$\overline{u_i} = -u_i \propto x_i. \quad (\text{A12})$$

This outcome successfully achieves the electrical mapping of the node state variable x_i . The dynamic response curve displayed on the oscilloscope can provide real-time and intuitive insights into the oscillation state of the network, thereby offering a reliable physical interface for experimental observations and theoretical validations.

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