

Reply to “Comment on ‘Instabilities of rotating inclined buoyancy layers’”

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We thank the editor for the opportunity to respond to the Comment [1] on our paper [2]. We also appreciate the careful reading of our original paper by the authors of Ref. [1]. Below, we address the main points raised in the Comment.

The primary point of contention in the Comment concerns the steady base flow obtained in Ref. [2]:

“The steady base flow in the study is unphysical since it requires an artificial perturbation pressure gradient throughout the depth of the atmosphere (and the cross-slope wind far above the slope) that depends on the surface temperature deficit as well as the slope angle, without any physical justification” (Abstract), or “There is no physical justification for why the perturbation pressure gradient (and cross-slope wind) far above the slope should depend on ΔT^* , χ , N_∞^* , and Ω^* ” (Sec. IV). “Even more problematically, this stipulated perturbation pressure gradient would blow up in the limit of vanishing slope angles” (Abstract).

The misunderstanding stems from a misinterpretation of the pressure gradient and can be resolved as follows. The ambient flow is thermally stratified in the s^* direction with a background cross flow W_g^* in a rotation system, as shown in Fig. 1.

The momentum (Navier-Stokes) equation with the Boussinesq approximation in this rotating system is

$$\frac{\partial \mathbf{V}^*}{\partial t^*} + \mathbf{V}^* \cdot \nabla \mathbf{V}^* + 2\boldsymbol{\Omega}^* \times \mathbf{V}^* = -\nabla(P^*/\rho_r) - g\gamma(T^* - T_\infty^*) + \nu \nabla^2 \mathbf{V}^*. \quad (1)$$

In the far field of the rotating buoyancy layer associated with a plate inclined at an angle χ to the horizontal, $\mathbf{V}^* = W_g^*$ = (0, 0, W_g^*) and $T^* = T_\infty^*$. Equation (1) then reduces to

$$2\boldsymbol{\Omega}^* \times W_g^* = -\nabla(P^*/\rho_r) \quad \text{or} \quad \frac{\partial P^*}{\partial x^*} = -2\Omega^* W_g^* \rho_r \cos(\chi). \quad (2)$$

Clearly, as $\chi \rightarrow 0$, $\partial P^*/\partial x^*$ approaches $-2\Omega^* W_g^* \rho_r$ and does not blow up. A steady ambient flow requires a balance between the Coriolis force (due to the background rotation $\boldsymbol{\Omega}^*$ and cross flow W_g^*) and the pressure gradient ∇P^* ; $\partial P^*/\partial x^*$ is simply the x^* -direction projection of the background pressure gradient ∇P^* . Therefore, $\partial P^*/\partial x^*$ is physically justified and must depend on Ω^* and χ .

The rotating buoyancy layer model proposed in Ref. [2] is an extension of Prandtl’s inclined layer [3] that incorporates the additional effects of background rotation and cross flow. It shares several features with the Prandtl layer; for example, both are parallel viscous shear flows whose x^* -direction velocity profiles have the same shape, and the temperature and velocity deviations induced by the inclined plate decay to zero in the far field. A key advantage of this rotating layer model is that it reduces to the Prandtl layer as the background rotation vanishes, which facilitates the study of the rotation effects.

Owing to its parallel-flow nature, the background pressure gradient component $\partial P^*/\partial x^*$ also appears in the rotating buoyancy layer. For a steady boundary layer, this pressure gradient must balance the buoyancy, Coriolis, and viscous diffusion effects, as shown in Eq. (1). These effects involve the surface temperature deficit ΔT^* , the plate or slope angle χ , the ambient temperature gradient N_∞^* , the Prandtl number Pr , and Ω^* . Consequently, W_g^* and hence $\partial P^*/\partial x^*$ are linked to these parameters through Eq. (21) of the Comment [1],

$$\lim_{y^* \rightarrow \infty} W^* = W_g^* = -\frac{2\Omega^* \cos(\chi) \Delta T^*}{N_\infty^* \sin(\chi) Pr} \quad \text{or} \quad \text{ctg}(\chi) \Delta T^* = -\frac{N_\infty^* Pr}{2\Omega^*} W_g^*. \quad (3)$$

In other words, there exist steady parallel basic-flow solutions for the rotating inclined buoyancy layers when Eq. (3) is satisfied. We emphasize that W_g^* (whose dimensionless form $W_0(Y \rightarrow \infty)$ can be obtained analytically from Eq. (6) of Ref. [2]) is the background cross flow and is independent of ΔT^* and χ ; it is the requirement of a steady parallel buoyancy layer that imposes the relation Eq. (3). For instance, a small χ corresponds to a small ΔT^* , while ∇P^* and W_g^* remain unchanged. Polar easterlies are observed in both Antarctica and the Arctic [4,5] and correspond to the background cross flow W_g^* [or $W_0(Y \rightarrow \infty)$] proposed in the model [2].

Note that the pressure gradient term stems from the constitutive relation and is an intrinsic part of the Navier-Stokes

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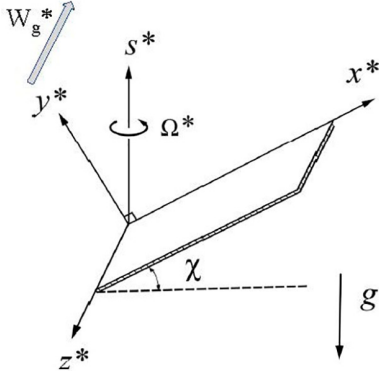


FIG. 1. Schematic geometry of an inclined buoyancy layer rotating about an axis parallel to the direction of gravity.

equations; it cannot be omitted arbitrarily. As stated in the Comment [1], “...if that term (refer to the pressure gradient) is omitted from Eqs. (3)–(5), then the temperature perturbation cannot vanish at infinite height for any steady state [3,4,6,7]. In addition, Stiperski *et al.* [6] and Shapiro and Fedorovich [7] showed that, when started from rest, a time-dependent Prandtl slope flow model that includes the Coriolis force but not $\partial P^*/\partial x^*$ produces a cross-slope wind and buoyancy that diffuse upward indefinitely, i.e., the flow does not uniformly converge to a steady state over the entire height $y^* \in [0, \infty)$,” these previous models in which the pressure term was artificially omitted cannot yield a steady basic flow state.

Another point of contention in the Comment [1] concerns the traveling waves:

“We assert that the nature of the traveling waves—also present in nonrotating Prandtl slope flows as shown in previous literature—are inviscid and should be attributed to an inertial-type Kelvin-Helmholtz instability...” (Abstract [1]); “Our numerical experiments on Prandtl slope flows without Coriolis effect, as presented in Refs. [11,14], have confirmed the inviscid nature of the along-slope traveling instabilities and found that viscosity does not destabilize the flow” (Sec. V [1]); “It can be observed from Fig. 3 that the unstable growth rates increase with increasing Γ_s , hence pointing to general instability at $\Gamma_s \rightarrow \infty$ or $\nu \rightarrow 0$. This implies that the instability mechanism must be caused by an inviscid mechanism due to the oscillatory profile of Prandtl’s laminar solution” ([6] or Ref. [11] of Ref. [1]).

The traveling-wave instability of the buoyancy layer arises from a combination of the viscous shear effect, the buoyancy effect, and the inviscid mechanism associated with inflection points in the velocity profile. Therefore, attributing the instability solely to an inviscid mechanism [1] appears to be an overstatement; the viscous and buoyancy effects should not be neglected. The reasons are as follows. First, these inclined buoyancy layers cannot form without the viscous effects, and viscous diffusion is as important as inertia within the layer.

Second, the no-slip boundary condition at the wall necessitates the inclusion of viscous effects in any stability analysis; indeed, without viscosity there would be no neutral curves and no critical modes. Third, transverse traveling-wave modes were originally obtained for boundary-layer flows by solving the linearized perturbation equations and are named TS waves in honor of Tollmien and Schlichting. In Ref. [2], analogous transverse traveling-wave modes for the buoyancy boundary-layer flows were obtained and are consequently referred to as TS-1 and TS-2 modes. Fourth, the characteristic scales (2.6)–(2.8) in Ref. [6] (Ref. [11] of Ref. [1]) all contain ν . In the strict inviscid limit $\nu \rightarrow 0$ these scales would either vanish or diverge, rendering the nondimensionalization and the entire linear stability analysis ill-posed. Moreover, the observation that the growth rate of an unstable mode increases with decreasing viscosity does not necessarily imply that “the instability mechanism must be caused by an inviscid mechanism” [6]. For example, the limit $\nu \rightarrow 0$ corresponds to very large Reynolds numbers, at which boundary-layer and channel flows become unstable to traveling-wave modes even though their base-flow profiles possess no inflection points; the inviscid mechanism is clearly irrelevant there. Finally, the buoyancy effect is as important as the viscous effect for the instability of the buoyancy layer, and it is intimately linked to the Prandtl number. The ambient thermal stratification leads to a reversal of the x -direction velocity profile in both the Prandtl buoyancy layer [3] and the rotating buoyancy layer [2], and invalidates Squire’s theorem. Tao and Busse (2009) [7] systematically studied the linear instability of the Prandtl buoyancy layer and identified three unstable modes: oblique rolls, longitudinal rolls, and transverse traveling waves. In particular, the (three-dimensional) oblique roll mode can be more unstable than the (two-dimensional) traveling-wave mode for certain ranges of the Prandtl number and inclination angle. The study of the rotating buoyancy layer [2] extends [7] by incorporating the Coriolis effect.

In summary, while we appreciate the commentary in Ref. [1], none of the points raised invalidates the main findings of our original paper. We have provided additional analysis to clarify that the pressure gradient is intrinsically required to balance the Coriolis force induced by the background rotation and cross flow, and that a steady parallel basic flow does exist for the rotating inclined buoyancy layer when the surface temperature deficit and the slope angle satisfy Eq. (3). If Eq. (3) is not satisfied, then a steady basic flow may still be possible, though it would not be a parallel flow—a scenario that merits further investigation. We thank the journal again for the opportunity to respond.

DATA AVAILABILITY

The data that support the findings of this study are available within the article.

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