

Erratum: L-based numerical linked cluster expansion for square lattice models [Phys. Rev. E **108**, 034126 (2023)]

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(Received 9 July 2025; published 11 August 2025)

DOI: [10.1103/q7fj-f3mm](https://doi.org/10.1103/q7fj-f3mm)

The critical values g_c cited for the ordered-disordered phase transitions in the transverse-field Ising models in the square and triangular lattices were incorrect by a factor of 2. For the square lattice, we reported $g_c \approx 0.76$ in the caption of Fig. 2 and the discussion in the first paragraph in Sec. II. It should be replaced by $g_c \approx 1.52$, which is 1/2 the value reported in Refs. [1,2] because we use spin- $\frac{1}{2}$ operators (as opposed to Pauli matrices) in our work. Similarly, for the triangular lattice, we reported $g_c \approx 0.42$ in the last paragraph in Sec. V B. This value should be replaced by $g_c \approx 0.82$, which is 1/2 the value reported in Ref. [3].

These values are relevant to the discussions of the results reported in Figs. 9 and 10. The penultimate paragraph of Sec. V B should read as follows.

The convergence above and below T_c observed in the classical Ising model also occurs after introducing quantum fluctuations through a transverse field of strength $g = 0.1$ [see Fig. 9(a)]. Increasing the strength of the transverse field beyond $g = 0.1$ [see results for $g = 0.2$ in Fig. 9(b) and $g = 0.5$ in Fig. 9(c)], we find that higher orders of the L expansion are needed for the bare sums to converge in the scale of the plots. For $g = 0.5$, we find that the convergence is lost at the lowest temperatures for the orders of the L expansion that we are able to calculate. Still, one can see that the results for the energy converge well below $T = 1$, which is where one expects high-temperature expansions to diverge. For $g \gtrsim g_c$, since the system is disordered, the NLCE results converge at all temperatures [see Fig. 9(d) for $g = 1.5$]. For models with moderate to strong fields, which are in many instances of relevance to understanding experimental results, NLCEs can provide very accurate results independently of whether or not there is frustration in paramagnetic phases.

The part of the last paragraph of Sec. V B starting from “In Fig. 10” to the end of the paragraph should read as follows.

In Fig. 10, we show results for the entropy of the transverse field Ising model in the triangular lattice as one increases the strength of the field g and the model departs from the classical $g = 0$ limit. At the weakest ($g = 0.1$) field shown, the entropy follows the classical result at high and intermediate temperatures and then departs at low temperatures, showing indications that it vanishes at the lowest temperatures. As expected, increasing the strength of the field results in a departure from the classical result occurring at higher temperatures. For $g \gtrsim g_c$, we again find that the NLCE results converge at all temperatures. Like in Fig. 9, the results in Fig. 10 show that NLCEs are an excellent choice when it comes to obtaining accurate results for thermodynamic properties of quantum lattice models in the presence of magnetic fields at low temperatures.

DATA AVAILABILITY

No data were created or analyzed in this study.

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