

Leveling of MHD turbulence imbalance in shear flows

Mariami Kavtaradze ^{1,2}, George Mamatsashvili ^{3,1,*}, George Chagelishvili ^{1,4} and Elene Uchava ¹

¹*Abastumani Astrophysical Observatory, Abastumani 0301, Georgia*

²*Department of Physics, Faculty of Exact and Natural Sciences, Tbilisi State University, Tbilisi 0179, Georgia*

³*Helmholtz-Zentrum Dresden-Rossendorf, Bautzner Landstr. 400, D-01328 Dresden, Germany*

⁴*M. Nodia Institute of Geophysics, Tbilisi 0193, Georgia*



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We investigate magnetohydrodynamic (MHD) turbulence in plane shear flows with a streamwise background magnetic field in the super-Alfvénic regime, i.e., when the flow velocity is larger than the local Alfvén speed. In this regime, turbulence is energetically supplied by large-scale shear of the flow via transient or nonmodal amplification of Alfvén waves, which are the building blocks of MHD turbulence, and streamwise-uniform, mostly nonmagnetic modes. We show that shear reduces turbulence imbalance, driving the system toward a balanced state—the energies of counter-propagating Alfvén waves become essentially equal, even for an initially perfectly imbalanced Alfvénic turbulence. This balancing occurs due to the shear-induced linear nonmodal dynamics of Alfvén waves—their nonmodal growth, resulting in over-reflection, and coupling between counter-propagating wave branches. Because of its linear origin, this process of balancing of MHD turbulence by shear is fundamentally different from the nonlinear processes operating in standard MHD turbulence in the absence of a mean shear flow. It can have direct implications for understanding the balanced/imbalanced nature of MHD turbulence in shear regions of the solar wind.

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I. INTRODUCTION

Turbulence in magnetized plasmas, often described within magnetohydrodynamics (MHD), is widespread in natural and laboratory settings. Its first theoretical model was proposed by Iroshnikov [1] and Kraichnan [2] as an extension of Kolmogorov's [3] theory of isotropic hydrodynamic turbulence, incorporating a magnetic field while still assuming isotropy. Later, Goldreich and Sridhar [4] dropped this limitation and addressed anisotropy of MHD turbulence, which is naturally caused by a sufficiently strong mean background magnetic field. As is well known in incompressible MHD, such a mean field supports Alfvén waves with two distinct polarizations: pseudo-Alfvén and shear-Alfvén waves. Each polarization consists of counterpropagating wave components. A key factor in MHD turbulence is the difference between the amplitudes of these counterpropagating Alfvén waves, which determines the balanced or imbalanced nature of the turbulence and has been the subject of extensive study over the past decades (e.g., [5–11]). The growing interest in this area has been largely motivated by observations of MHD turbulence in the solar wind (e.g., [12–15]). Studying the degree of imbalance in MHD turbulence is essential to understand energy and momentum transfers across scales (e.g., [8,10,16,17]), as well as heating and particle acceleration [18]. These processes are

crucial for interpreting astrophysical and heliospheric (e.g., solar wind) plasma observations.

In these settings, magnetized plasma, as a rule, exhibits large-scale inhomogeneous kinematics, or shear flows that act as a significant source of dynamical activity. For example, shear flows in the solar wind have long been recognized as a source of turbulence therein [19–26]. Of course, in astrophysical plasma flows, complex thermodynamic effects are at work alongside kinematic ones. Nevertheless, shear is often the main source of energy for turbulence in the flow. So, omitting thermodynamic factors for the moment, we aim to grasp shear-driven processes that constitute the core of turbulence dynamics.

Shear flow—one of the central drivers of plasma dynamics—is a likely factor determining the balanced/imbalanced nature of MHD turbulence as well. Indeed, observations of the solar wind indicate that, despite being mostly imbalanced, regions of balanced turbulence also exist that coincide with strong velocity shear regions [20,21,27–32]. This has motivated us to investigate in more detail the physics underlying the effect of shear on the degree of imbalance of MHD turbulence.

Previous studies of forced, strong, imbalanced MHD turbulence [9,33] showed that numerical simulations are sensitive to the forcing method, complicating the description of the inertial range and energy spectra in Fourier space. However, in shear flow turbulence, the inertial range, which is a range of action of nonlinear processes only, is strictly speaking absent, because both shear-induced linear and nonlinear processes operate in parallel over a broad range of length scales [34–36]. This naturally calls into question the applicability of the inertial range concept in shear flows.

*Contact author: g.mamatsashvili@hzdr.de

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Shear-induced linear dynamical processes originate from the non-self-adjointness, or non-normality, of the mathematical operators governing linear evolution of perturbations in smooth shear flows without inflection points. The corresponding eigenfunctions of non-normal operators are not mutually orthogonal due to shear and can therefore strongly interfere [37,38]. Because of this interference, a particular solution composed of a superposition of eigenfunctions can still exhibit substantial growth over a finite time interval even when all eigenfunctions are modally stable, i.e., contain no exponentially growing components and decay in time (see, e.g., [39–41] for hydrodynamic and [42] for MHD shear flows as well as references therein). This phenomenon, caused by flow shear, is referred to as transient, or nonmodal growth of perturbations. It occurs over a broad range of length scales, depends on the orientation of the wavevector of perturbation harmonics relative to the shearwise direction, and hence is anisotropic in Fourier space [36,43–45]. It is regarded as an important mechanism for triggering transition to turbulence in shear flows that are otherwise modally stable [39–41,46]. However, due to its transient character, the nonmodal growth itself has a drawback: it requires nonlinear positive feedback to continuously operate and energetically supply turbulence [46–50].

The key role of nonlinearity in the sustenance of subcritical turbulence and, in particular, its interplay with linear nonmodal dynamics was subsequently studied in detail. Based on low-dimensional models of wall-bounded hydrodynamic shear flows, a nonlinear cycle underlying the self-sustaining process (SSP) was identified and described in [49,51]. This cycle consists of three basic interacting elements: (1) streamwise rolls which, by redistributing momentum, lead to the nonmodal growth of (2) streaks, i.e., spanwise modulations of the mean streamwise velocity of the flow, via the lift-up mechanism; these streaks, in turn, become unstable via (3) an inflectional, or secondary, instability. The nonlinear self-interaction of this secondary instability provides positive feedback that regenerates the rolls, thereby closing the self-sustaining cycle. This SSP, which is widely accepted in the fluid dynamics community, is therefore based on the dynamical interplay of coherent roll-streak structures in physical space, which are the characteristic features of wall-bounded shear flows, such as plane Couette flow, channel flow, and boundary layer flow. In this SSP, the main emphasis is placed on the role of nonlinearity, while boundaries/walls are essential because they support coherent roll-streak structures and determine the near-wall regeneration cycle [50].

Another type of SSP operating in unbounded or effectively boundary-free modally stable hydrodynamic shear flows (as is usually the case in many natural flows) was proposed in [47,48] using again a simplified reduced-order model that does not rely on rigid boundaries. It is based on the subtle interplay of linear nonmodal growth of perturbations and nonlinear processes. Specifically, nonlinearity, conserving the total energy, redistributes (mixes) energy among modes and continuously replenishes “misfit” perturbations oriented relative to the base flow in such a way that the latter can undergo nonmodal growth. Thus, in this SSP of unbounded modally stable shear flows, the linear nonmodal dynamics plays a central role because it enables

perturbations to extract energy from the base flow and inject it into turbulence.

This SSP, based on the interplay between linear nonmodal growth and nonlinear processes, was further studied beyond low-dimensional models using direct numerical simulations of fully developed turbulence in modally stable hydrodynamic shear flows, which account for spectral anisotropy in Fourier space [34,36,46,52,53]. In particular, using the model of homogeneous shear turbulence, it was shown in [36] that: (i) a generic nonlinear process in shear flows is an anisotropic redistribution of modes over wavevector orientation in Fourier space—the nonlinear transverse cascade—which is caused by shear and is thus topologically distinct from conventional (inverse/direct) nonlinear cascades and (ii) the transverse cascade provides a positive feedback by continually replenishing nonmodally growing modes, closing the sustenance loop. In other words, turbulence is self-sustained through a cooperative and refined interplay between linear nonmodal growth and nonlinear transverse cascade. This process was also shown to be responsible for the sustenance of turbulence in MHD shear flows [35,44,54,55].

In this paper, we investigate the dynamics of MHD turbulence in plane shear flows subject to a background magnetic field, focusing on its balanced/imbalance nature. We consider a super-Alfvénic regime, that is, when the flow velocity exceeds the Alfvén speed associated with the field, so that shear-induced nonmodal processes play a dominant role in turbulence sustenance and dynamics. Our main result is that sufficiently strong shear can reduce the degree of turbulence imbalance, equalizing the amplitudes of counter-propagating Alfvén waves even when these amplitudes differ substantially initially. This process of turbulence balancing is governed by the linear nonmodal dynamics of Alfvén waves, which is based on two basic shear-induced processes: (i) nonmodal growth of Alfvén waves, resulting in wave over-reflection, and (ii) linear coupling between different Alfvén wave branches, whereby an initially excited wave branch generates a counter-propagating one [56–61].

The essence of the over-reflection phenomenon of Alfvén waves in unbounded shear flows is as follows [61]. Consider an incident Alfvén wave propagating at a certain angle to the flow, when its wave numbers k_y in the streamwise (y) and k_x in the shearwise (x) directions have different signs, $k_x/k_y < 0$ (assuming the flow with velocity U_{0y} along the y -axis and shear $S = -dU_{0y}/dx > 0$ along the x -axis). The shear of the flow leads to the nonmodal growth of the wave amplitude and makes the shearwise wave number change with time $k_x(t)$, so that the latter decreases in absolute value, becomes zero, and changes sign. At the time when $k_x(t)$ approaches and crosses zero, another branch of the Alfvén wave emerges due to the mode coupling process, which propagates opposite to the incident wave. This counterpropagating branch is referred to as the reflected wave, while the incident wave becomes a transmitted wave with $k_x(t)$ and k_y now having the same signs $k_x(t)/k_y > 0$. As a result of nonmodal growth, both of these wave branches—the transmitted and reflected waves—attain amplitudes larger than that of the incident wave by extracting energy from the base flow; hence, this process is referred to as wave over-reflection (see sub-Sec. II). Thus, in this case of an unbounded shear flow, transmission and reflection of Alfvén

waves occur in time: initially there is only an incident wave, which later, when the shearwise wave number changes sign, is converted into transmitted and reflected waves.

In this paper, we analyze the balance/imbalance nature of MHD turbulence in shear flows based on the shear-induced linear processes—nonmodal growth, resulting in the wave over-reflection, and mode coupling—of Alfvén waves discussed above rather than by their nonlinear interactions only as is the case in classical, i.e., shearless and externally forced, MHD turbulence.

The paper is organized as follows: The physical model of the magnetized shear flow, the main equations, and the linear dynamics of pseudo- and shear-Alfvén waves are described in Sec. II. Numerical setup, general nonlinear evolution, and a demonstration of the balancing of MHD turbulence by shear and energy spectra are presented in Sec. III. Conclusions are given in Sec. IV.

II. PHYSICAL MODEL

We consider an unbounded isentropic plane flow along the y -axis with a constant shear $S > 0$ of velocity along the x -axis, $\mathbf{U}_0 = -Sx\mathbf{e}_y$, in the Cartesian coordinate system (x, y, z) , where $\mathbf{e}_y$ is the unit vector along the y -axis. A uniform background magnetic field $\mathbf{B}_0 = B_0\mathbf{e}_y$, $B_0 > 0$ is applied parallel to the flow. Owing to its kinematic and thermodynamic simplicity, this basic model of a magnetized flow with constant shear provides a useful framework for isolating and understanding the effects of large-scale velocity shear on MHD turbulence, with relevance, for example, to shear regions in the solar wind [58,60,62]. The perturbations of velocity $\mathbf{u}$ and magnetic field $\mathbf{b}$ about this base flow are governed by the equations of incompressible nonideal MHD,

$$\frac{D\mathbf{u}}{Dt} + (\mathbf{u} \cdot \nabla)\mathbf{u} - S u_x \mathbf{e}_y = -\frac{1}{\rho} \nabla P + (\mathbf{U}_A \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{u} + \nu \Delta \mathbf{u}, \quad (1)$$

$$\frac{D\mathbf{b}}{Dt} = -S b_x \mathbf{e}_y + (\mathbf{U}_A \cdot \nabla)\mathbf{u} + \nabla \times (\mathbf{u} \times \mathbf{b}) + \eta \Delta \mathbf{b}, \quad (2)$$

$$\nabla \cdot \mathbf{u} = \nabla \cdot \mathbf{b} = 0, \quad (3)$$

where ρ is the constant density, P is the total pressure, equal to the sum of the thermal and magnetic pressures, and $\mathbf{U}_A = \mathbf{B}_0/(4\pi\rho)^{1/2}$ is the Alfvén speed corresponding to the background field. ν is the kinematic viscosity and η is the Ohmic resistivity. Here, $D/Dt \equiv \partial/\partial t - Sx\partial/\partial y$ is the advective derivative along the base flow. The magnetic field perturbation is measured in velocity units $\mathbf{b}/(4\pi\rho)^{1/2} \rightarrow \mathbf{b}$. Following a standard approach in MHD turbulence studies [4,6,9], we rewrite Eqs. (1)–(3) in terms of Elsässer variables, $\mathbf{Z}^\pm = \mathbf{u} \pm \mathbf{b}$:

$$\begin{aligned} \left(\frac{D}{Dt} \mp \mathbf{U}_A \cdot \nabla \right) \mathbf{Z}^\pm + (\mathbf{Z}^\mp \cdot \nabla) \mathbf{Z}^\pm \\ = -\frac{1}{\rho} \nabla P + S \mathbf{Z}_x^\mp \mathbf{e}_y \\ + \frac{\nu + \eta}{2} \Delta \mathbf{Z}^\pm + \frac{\nu - \eta}{2} \Delta \mathbf{Z}^\mp, \end{aligned} \quad (4)$$

$$\nabla \cdot \mathbf{Z}^\pm = 0. \quad (5)$$

The main difference in Eq. (4) compared to the standard equations for $\mathbf{Z}^\pm$ used in MHD turbulence theory in the shearless case ($S = 0$) is the term $S \mathbf{Z}_x^\mp \mathbf{e}_y$ arising from shear S on the right-hand side. This term plays a key role in the present case with shear—it is responsible for the linear coupling of counterpropagating Alfvén waves, characterized by $\mathbf{Z}^\pm$, as well as for their energy exchange with the base flow, resulting in the nonmodal growth, which we describe in more detail in sub-Sec. II below. By contrast, in the classical case without a base shear flow, these waves are coupled through the nonlinear term $(\mathbf{Z}^\mp \cdot \nabla) \mathbf{Z}^\pm$ on the left-hand side of Eq. (4) and through the part of the diffusion term on the right-hand side that is proportional to $\nu - \eta$. From these two coupling terms, the nonlinear one is usually considered dynamically far more important than the dissipation one, since it determines nonlinear cascades and energy spectra of MHD turbulence [6,63]. Below, we take for simplicity $\nu = \eta$, when this diffusion coupling term vanishes. Our goal is to analyze the shear-induced linear nonmodal dynamics of Alfvén waves and its effect on the imbalance/balance properties of MHD turbulence in shear flows.

We stress that shearless MHD turbulence considered in most studies is fueled by some external forcing (e.g., [4,9,10,64]), whereas here, we study the turbulence with an intrinsic energy source—the base MHD shear flow $\mathbf{U}_0$, as is usually the case in many natural settings. Although this constant shear flow subject to a uniform streamwise field is modally stable, i.e., it has no exponentially growing modes according to the modal analysis [65–67], perturbations (Alfvén waves) can still undergo nonmodal growth due to the term $S \mathbf{Z}_x^\mp \mathbf{e}_y$ proportional to shear, which, as noted above, provides their main supply of energy, extracting it from the base flow [35,59,61].

One of the central characteristics of MHD turbulence is the total cross-helicity

$$H_c = \int (\mathbf{u} \cdot \mathbf{b}) dV = \frac{1}{4} \int [(Z^+)^2 - (Z^-)^2] dV,$$

where the integral is taken over the entire flow domain, which measures the correlation of the velocity and magnetic field perturbations, often referred to as Alfvénicity in solar wind studies [24]. Equivalently, it characterizes the degree of turbulence imbalance, that is, the difference between the Elsässer energies $(1/4) \int (Z^+)^2 dV$ and $(1/4) \int (Z^-)^2 dV$ carried by Alfvén waves propagating in the opposite directions. In the strongly imbalanced case, one of the Elsässer energies is much larger than the other. In the ideal case without shear, H_c is conserved if perturbations obey periodic boundary conditions or vanish at infinity [68], implying the conservation of the imbalance/balance of turbulence. However, in shear flows $S \neq 0$, as follows from Eqs. (1)–(3), H_c is no longer conserved and varies in time at a rate proportional to shear

$$\frac{dH_c}{dt} = S \int (u_x b_y - u_y b_x) dV = \frac{S}{2} \int (Z_x^- Z_y^+ - Z_x^+ Z_y^-) dV,$$

which can have consequences for the turbulence imbalance, as will be shown below.

Following [33], we also introduce the normalized cross-helicity $\sigma_c = H_c/E$, where $E = (1/2) \int (u^2 + b^2) dV = (1/4) \int [(Z^+)^2 + (Z^-)^2] dV$ is the total energy of perturbations

in the domain equal to the sum of the kinetic $(1/2) \int u^2 dV$ and magnetic $(1/2) \int b^2 dV$ energies. It is often used as a convenient diagnostic measure of imbalance, or Alfvénicity, in the solar wind MHD turbulence [24,27,69]. Defining a volume-average over the domain $\langle \cdot \rangle = (1/V) \int \cdot dV$, where $V = L_x L_y L_z$ is the domain volume, the normalized cross-helicity can be written as

$$\sigma_c = \frac{2\langle \mathbf{u} \cdot \mathbf{b} \rangle}{\langle u^2 + b^2 \rangle} = \frac{\langle (Z^+)^2 \rangle - \langle (Z^-)^2 \rangle}{\langle (Z^+)^2 \rangle + \langle (Z^-)^2 \rangle}. \quad (6)$$

When the velocity and magnetic field perturbations exhibit a strong correlation (alignment) of a preferred sign $\mathbf{b} \approx \pm \mathbf{u}$, i.e., high Alfvénicity, the turbulence is strongly imbalanced and σ_c is close to unity in absolute value $\sigma_c \approx \pm 1$. By contrast, in the balanced case, counterpropagating Alfvén waves carry equal amounts of energy; the velocity and field perturbations are not correlated and hence $\sigma_c \approx 0$.

Linear dynamics and over-reflection of Alfvén waves

To classify perturbation modes present in the flow, we consider first the linear regime and decompose the quantities into spatial Fourier harmonics,

$$f(\mathbf{r}, t) = \int \tilde{f}(\mathbf{k}, t) \exp(i\mathbf{k} \cdot \mathbf{r}) d^3\mathbf{k}, \quad (7)$$

where $f \equiv (\mathbf{u}, P, \mathbf{b})$ and their Fourier transforms are $\tilde{f} \equiv (\tilde{\mathbf{u}}, \tilde{P}, \tilde{\mathbf{b}})$. Assuming perturbations to be small, linearizing Eqs. (1)–(3) and using decomposition (7), we get

$$\frac{d\tilde{\mathbf{u}}}{dt} - S\tilde{u}_x \mathbf{e}_y = -\frac{i\mathbf{k}(t)}{\rho} \tilde{P} + i(\mathbf{U}_A \cdot \mathbf{k}(t))\tilde{\mathbf{b}}, \quad (8)$$

$$\frac{d\tilde{\mathbf{b}}}{dt} = -S\tilde{b}_x \mathbf{e}_y + i(\mathbf{U}_A \cdot \mathbf{k}(t))\tilde{\mathbf{u}}, \quad (9)$$

$$i\mathbf{k}(t) \cdot \tilde{\mathbf{u}} = i\mathbf{k}(t) \cdot \tilde{\mathbf{b}} = 0, \quad (10)$$

where we have taken into account that the wave number k_x of Fourier modes in the shearwise x -direction changes linearly with time due to shear $k_x(t) = k_x(0) + Sk_y t$, which physically means shearing of the spatial harmonics by the base flow [43,59,61], while the wave numbers k_y in the streamwise y -direction, and k_z in the vertical z -direction are constant in time. As a result, the wave vector $\mathbf{k}(t) = (k_x(t), k_y, k_z)$ also varies with time and changes direction. For simplicity, viscous and resistive dissipation terms are omitted in Eqs. (8) and (9), respectively, as they are not important for mode classification and dynamics discussed below.

It follows from Eqs. (8)–(10) that two main types of linear modes exist in the flow: (i) Alfvén waves, which oscillate with frequencies $\omega = \pm U_A k_y$, each for $\mathbf{Z}^\pm$, and vary in the streamwise y -direction, parallel to the background field $\mathbf{B}_0$, with the nonzero wave number $k_y \neq 0$, and (ii) nonoscillatory, uniform in the streamwise direction ($k_y = 0$) modes, which have independent magnetic and velocity perturbations. The Alfvén wave mode can be further divided into pseudo-Alfvén and shear-Alfvén waves according to their polarizations [4,6,61], which in Fourier space take the form:

(1) *Pseudo-Alfvén wave* (PAW) with velocity $\tilde{\mathbf{u}}_p = (\tilde{u}_x, -k_x(t)\tilde{u}_x/k_y, 0)$ and magnetic field $\tilde{\mathbf{b}}_p = (\tilde{b}_x, -k_x(t)\tilde{b}_x/k_y, 0)$ perturbations lying in the (x, y) -plane

and satisfying $k_x(t)\tilde{u}_{p,x} + k_y\tilde{u}_{p,y} = k_x(t)\tilde{b}_{p,x} + k_y\tilde{b}_{p,y} = 0$. The corresponding Elsässer variables are $\tilde{\mathbf{Z}}_p^\pm = (\tilde{Z}_x^\pm, -k_x(t)\tilde{Z}_x^\pm/k_y, 0)$ and obey $k_x(t)\tilde{Z}_{p,x}^\pm + k_y\tilde{Z}_{p,y}^\pm = 0$.

(2) *Shear-Alfvén wave* (SAW) with velocity $\tilde{\mathbf{u}}_s = (0, -k_z\tilde{u}_z/k_y, \tilde{u}_z)$ and magnetic field $\tilde{\mathbf{b}}_s = (0, -k_z\tilde{b}_z/k_y, \tilde{b}_z)$ perturbations lying in the (y, z) -plane and satisfying $k_y\tilde{u}_{s,y} + k_z\tilde{u}_{s,z} = k_y\tilde{b}_{s,y} + k_z\tilde{b}_{s,z} = 0$. The corresponding Elsässer variables are $\tilde{\mathbf{Z}}_s^\pm = (0, -k_z\tilde{Z}_z^\pm/k_y, \tilde{Z}_z^\pm)$ and obey $k_y\tilde{Z}_{s,y}^\pm + k_z\tilde{Z}_{s,z}^\pm = 0$.

To analyze the dynamics of PAWs and SAWs, their interaction with each other and with the base flow due to shear S , we derive linear equations for $\tilde{\mathbf{Z}}_{p,x}^\pm$ and $\tilde{\mathbf{Z}}_{s,z}^\pm$, which are the independent components of the corresponding Elsässer variables for these waves, using Eqs. (8)–(10) or, equivalently, linearizing Eq. (4) and applying the Fourier transform (7),

$$\left(\frac{d}{dt} \mp ik_y U_A + S \frac{k_x(t)k_y}{k^2(t)} \right) \tilde{\mathbf{Z}}_{p,x}^\pm = -S \frac{k_x(t)k_y}{k^2(t)} \tilde{\mathbf{Z}}_{p,x}^\mp, \quad (11)$$

$$\left(\frac{d}{dt} \mp ik_y U_A \right) \tilde{\mathbf{Z}}_{s,z}^\pm = -S \frac{k_y k_z}{k^2(t)} (\tilde{\mathbf{Z}}_{p,x}^+ + \tilde{\mathbf{Z}}_{p,x}^-), \quad (12)$$

where $k^2(t) = k_x^2(t) + k_y^2 + k_z^2$. In the absence of shear $S = 0$, Eqs. (11) and (12) decouple and describe the individual dynamics of counterpropagating PAWs and SAWs, respectively. These waves evolve independently from each other and, as mentioned above, oscillate with the same Alfvén frequencies,

$$\tilde{\mathbf{Z}}_{p,x}^\pm \propto e^{\pm ik_y U_A t}, \quad \tilde{\mathbf{Z}}_{s,z}^\pm \propto e^{\pm ik_y U_A t}.$$

The situation qualitatively changes in the presence of shear, which dynamically couples the waves both to one another and to the base flow. Namely, it gives rise to two main effects for PAWs in Eq. (11): energy exchange with the base flow for each $\tilde{\mathbf{Z}}_{p,x}^\pm$ component through the third term on the left-hand side $Sk_x(t)k_y\tilde{\mathbf{Z}}_{p,x}^\pm/k^2(t)$, and coupling between these two counterpropagating components through the term $-Sk_x(t)k_y\tilde{\mathbf{Z}}_{p,x}^\mp/k^2(t)$ on the right-hand side. Physically, this means that initially imposing one of these two components generates another—counterpropagating—one in the course of evolution [59,61]. The coefficient $Sk_x(t)k_y/k^2(t)$, which is responsible for these two shear effects, is appreciable at $|k_x(t)| \lesssim |k_y|$, during the shear time $\sim S^{-1}$, and vanishes at large wave number $k(t)$. Therefore, for the shear-induced nonmodal growth and coupling of Alfvén waves to be effective, the wave period should be larger than the shear time, i.e., the wave frequency should be smaller than the shear $k_y U_A \lesssim S$ [35,61]. Note that SAWs do not enter Eq. (11) and hence do not affect the dynamics of PAWs in the linear regime. These two shear-induced effects jointly give rise to the nonmodal growth of both $\tilde{\mathbf{Z}}_{p,x}^\pm$ branches of PAWs and their over-reflection.

To clearly demonstrate this over-reflection phenomenon and its relation to the turbulence balancing, we use the wave action conservation for PAWs [59,61]. Multiplying Eq. (11) by the complex conjugate $(\tilde{\mathbf{Z}}_{p,x}^\pm)^*$, adding to its associated conjugate equation, and then subtracting the equation for $|\tilde{\mathbf{Z}}_{p,x}^-|^2$ from the equation for $|\tilde{\mathbf{Z}}_{p,x}^+|^2$, we get

$$k^2(t)(|\tilde{\mathbf{Z}}_{p,x}^+|^2 - |\tilde{\mathbf{Z}}_{p,x}^-|^2) = \text{const}. \quad (13)$$

Suppose that only $\bar{Z}_{p,x}^+(0) \neq 0$ is present initially in the flow, which we refer to as the incident wave, while $\bar{Z}_{p,x}^-(0) = 0$, i.e., the initial state, is perfectly imbalanced. As discussed above, the nonmodal growth and coupling of waves occur during the time interval in which $|k_x(t)| \lesssim |k_y|$, resulting in the generation and amplification of the other—counterpropagating—component $\bar{Z}_{p,x}^-$ by the initially imposed one $\bar{Z}_{p,x}^+$. We apply Eq. (13) at the initial moment $t = 0$, when $k_x(0)/k_y < 0$ and there is only $\bar{Z}_{p,x}^+ \neq 0$, and at a final moment t_f , after the growth and coupling processes have taken place, when $k_x(t_f)/k_y > 0$ and there are both $\bar{Z}_{p,x}^+$ and newly generated $\bar{Z}_{p,x}^-$. Choosing the initial and final moments such that $k_x(0) = -k_x(t_f)$, we get

$$|\bar{Z}_{p,x}^+(0)|^2 = |\bar{Z}_{p,x}^+(t_f)|^2 - |\bar{Z}_{p,x}^-(t_f)|^2. \quad (14)$$

This relation indicates that the amplitude of $\bar{Z}_{p,x}^+$ is higher after the nonmodal growth than its initial amplitude. The counter-propagating component $\bar{Z}_{p,x}^-$ is referred to as the reflected wave, while the $\bar{Z}_{p,x}^+$ becomes a transmitted wave after the coupling process. It was shown in [35,61] that PAWs undergo a stronger nonmodal growth $|\bar{Z}_{p,x}^\pm(0)| \ll |\bar{Z}_{p,x}^\pm(t_f)|$ at those k_y for which the shear effects are important satisfying $k_y U_A \lesssim S$; therefore, it follows from Eq. (14) that the amplitudes of both transmitted and reflected waves are nearly equal and larger than the amplitude of the incident wave $|\bar{Z}_{p,x}^+(t_f)| \approx |\bar{Z}_{p,x}^-(t_f)| \gg |\bar{Z}_{p,x}^+(0)|$, and therefore this process is referred to as wave over-reflection. Energy for both PAW components is extracted from the base flow via the nonmodal growth process due to shear, making over-reflection possible. The shear-induced over-reflection can thus bring an initially imbalanced state toward balance, a process that, as we demonstrate in the next section, also occurs in shear-driven MHD turbulence composed of Alfvén waves. The degree of Alfvén wave balance increases as the ratio $k_y U_A/S$ decreases [61].

As for SAWs, their dynamics is governed by Eq. (12), from which it follows that these waves are coupled only to PAWs. In other words, SAWs gain energy from PAWs solely through the shear-induced coupling term on the right-hand side of this equation, without directly exchanging energy with the base flow.

Thus, PAWs play an important dynamical role in the presence of shear. The nonmodal growth and over-reflection processes they exhibit, together with their shear-induced coupling to SAWs, necessitate accounting for PAWs in the dynamical balances of shear-driven MHD turbulence. By contrast, in shearless MHD turbulence driven by external forcing, SAWs are generally assumed to play a dominant role, whereas PAWs are usually neglected [4,11,64,70].

III. SIMULATION RESULTS

Having described the linear dynamics of Alfvén waves, we now investigate their nonlinear evolution in shear-driven MHD turbulence by solving the governing equations (1)–(3) in a cubic domain with sizes $(L_x, L_y, L_z) = (2\pi, 2\pi, 2\pi)\ell$ using the pseudospectral code SNOOPY [71], where ℓ is some arbitrary length scale of the order of the flow system size. As is customary in simulations of constant shear flows without rigid boundaries, the boundary conditions are

TABLE I. Simulation parameters: Alfvén Mach number M_A , including the hydrodynamic case $M_A = \infty$, and the volume- and time-averaged (denoted by double brackets) kinetic E_{kin} and magnetic E_{mag} energy densities as well as the Reynolds $u_x u_y$ and Maxwell $-b_x b_y$ stresses in the turbulent state.

M_A	$\langle\langle E_K \rangle\rangle$	$\langle\langle E_M \rangle\rangle$	$\langle\langle u_x u_y \rangle\rangle$	$\langle\langle -b_x b_y \rangle\rangle$
∞	1.53		0.4	
10	0.84	0.24	0.15	0.023
5	0.74	0.21	0.12	0.019
3.3	0.68	0.19	0.11	0.018

periodic in y and z , but shear-periodic in x (e.g., [53,72,73]). In this domain, the wave numbers along the y - and z -directions are discrete, $k_y = 2\pi n_y/L_y$ and $k_z = 2\pi n_z/L_z$, where the integer $n_y, n_z = 0, \pm 1, \pm 2, \pm 3, \dots$, while the wave number in the x -direction, as mentioned above, changes with time as $k_x(t) = 2\pi n_x/L_x + St(2\pi n_y/L_y)$ due to shear for $n_y \neq 0$ to satisfy the shearing-periodic boundary conditions [72], where $n_x = 0, \pm 1, \pm 2, \pm 3, \dots$. k_x coincides with the discrete values $2\pi n_x/L_x$ at specific moments $t_n = nL_y/(SL_x n_y)$, where n is a positive integer.

We nondimensionalize time by S^{-1} , length by ℓ , velocity by $S\ell$ (magnetic field is measured in velocity units too), and energy densities by $\rho_0 S^2 \ell^2$. The main parameters of the problem are the Reynolds number $\text{Re} = S\ell^2/\nu$, magnetic Reynolds number $\text{Rm} = S\ell^2/\eta$ and Alfvén Mach number $M_A = S\ell/U_A = (4\pi\rho)^{1/2} S\ell/B_{0y}$, which characterizes the effect of the shear relative to the Lorentz force due to the background field. We fix $\text{Re} = \text{Rm} = 1000$, so that the magnetic Prandtl number $\text{Pm} = \text{Rm}/\text{Re} = 1$. As noted above, the shear-induced linear processes—nonmodal growth and coupling—of Alfvén waves are efficient when their frequency is lower than the shear $k_y U_A \lesssim S$ or equivalently $k_y \ell \lesssim M_A$. This efficiency increases with decreasing ratio $k_y U_A/S$, i.e., with increasing shear [61]. For this condition to be satisfied at least for the smallest $k_{y,\text{min}} = 2\pi/L_y$ in the domain, it is necessary that $M_A \gtrsim 2\pi\ell/L_y$, or $M_A \gtrsim 1$, which implies the super-Alfvénic regime. Therefore, the following analysis focuses mainly on the case $M_A = 10$, representative of the strongly super-Alfvénic regime $M_A \gg 1$ typically observed in the near-Earth solar wind [29,74]. For comparison, we also did simulations for lower $M_A = 3.3$ and 5 and for the purely hydrodynamic case (i.e., $\mathbf{b} \equiv 0$ and $M_A = \infty$), which are all listed in Table I.

The numerical resolution of the simulations is $(N_x, N_y, N_z) = (256, 256, 256)$. To verify that this resolution is sufficient for the present problem, we compare viscous $\lambda_\nu = (\nu^3/\epsilon)^{1/4}$ and resistive $\lambda_\eta = \text{Pm}^{-3/4} \lambda_\nu$ dissipation scales of MHD turbulence [75], which are equal $\lambda_\nu = \lambda_\eta$ for $\text{Pm} = 1$, with the largest resolved wave number in the domain $k_{\text{max}} = 2\pi N_x/3L_x$ accessible by the pseudospectral code (using the standard 2/3 dealiasing rule). Here, $\epsilon = \nu \langle(\nabla \times \mathbf{u})^2\rangle$ is the volume-averaged dissipation rate of the kinetic energy. For our simulations, we obtain $k_{\text{max}} \lambda_\nu = k_{\text{max}} \lambda_\eta \approx 1$, indicating that both dissipation scales are resolved at the standard spectral resolution threshold [76]. Thus, the adopted resolution is adequate, since the dominant

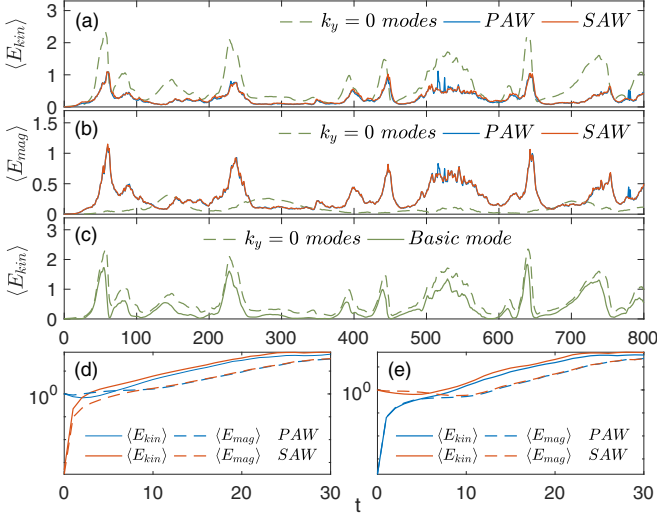


FIG. 1. Evolution of the volume-averaged (a) kinetic $\langle E_{kin} \rangle$ and (b) magnetic $\langle E_{mag} \rangle$ energies of PAW, SAW, and $k_y = 0$ modes for the initially imposed PAWs at $M_A = 10$. (c) Kinetic energies of the basic mode and all of the $k_y = 0$ modes. (d) The same evolution of the wave energies as in (a) and (b), but at early times and normalized by the initial values. (e) The same as in (d) but for initially imposed SAWs.

wave dynamics occurs at wave numbers lower than k_{max} , as shown by the spectral analysis in Sec. III C. As an additional check, we also carried out simulations for $M_A = 10$ by doubling the resolution $(N_x, N_y, N_z) = (512, 512, 512)$. The results show good qualitative and quantitative agreement in terms of the time-averaged energies and stresses with those obtained at $(N_x, N_y, N_z) = (256, 256, 256)$, demonstrating convergence with resolution. Since we are mainly interested in the role of shear in the balancing of MHD turbulence, we consider two types of perfectly imbalanced initial conditions composed of random perturbations of: (i) only PAWs with nonzero $\langle (Z_p^+)^2 \rangle^{1/2} = 0.14$ and zero $Z_p^- = Z_s^\pm = 0$ (i.e., no SAWs), and (ii) only SAWs with nonzero $\langle (Z_s^+)^2 \rangle^{1/2} = 0.16$ and zero $Z_s^- = Z_p^\pm = 0$ (i.e., no PAWs). The corresponding rms amplitudes of initial velocity and magnetic field perturbations are, respectively, $\langle u^2 \rangle^{1/2} = \langle b^2 \rangle^{1/2} = 0.1$ in the first and $\langle u^2 \rangle^{1/2} = \langle b^2 \rangle^{1/2} = 0.11$ in the second type of the initial conditions. The $k_y = 0$ nonoscillatory modes

are initially absent in both cases. Other imbalanced initial conditions with $Z_{p,s}^- \neq 0$ and $Z_{p,s}^+ = 0$ exhibit qualitatively similar evolution (not shown here).

The initial perturbations undergo nonmodal amplification due to shear, and after several shear times, their amplitude becomes sufficiently high to reach a nonlinear regime. As a result, the flow eventually settles into a quasisteady MHD turbulence with characteristic bursts in the kinetic and magnetic energies (Figs. 1 and 2). Because the base shear flow is modally stable, it supports no exponentially growing eigenmodes but only nonmodally growing perturbations and, consequently, the transition to turbulence is subcritical [39]. This means that, in addition to the main parameters of the flow (Re , Rm , M_A), the onset of turbulence also depends on the initial amplitude of the perturbations: turbulence sets in and persists only when this amplitude exceeds a certain threshold. In this case, nonmodal growth of perturbations is considered one of the key mechanisms for triggering transition to turbulence in subcritical regimes in shear flows [46–48]. Here, however, we do not address this transition process itself, that is, how the critical amplitude of the initial perturbations depends on the flow parameters and the domain aspect ratios (L_z/L_x , L_y/L_x), but instead focus on the dynamics and balancing of fully developed MHD turbulence driven by shear. For this purpose, we take initial velocity and magnetic field perturbations with sufficiently high amplitude to ensure that the flow always exhibits transition to sustained turbulence for the given other parameters.

A. Shear-driven MHD turbulence – nonlinear evolution of PAWs, SAWs, and the $k_y = 0$ modes

First, we describe the general properties of the turbulence and then its balanced/imbalanced dynamics. Figure 1 shows the evolution of the volume-averaged kinetic $\langle E_{kin} \rangle = \langle u^2 \rangle / 2$ and magnetic $\langle E_{mag} \rangle = \langle b^2 \rangle / 2$ energy densities of PAWs and SAWs with $k_y \neq 0$ as well as the streamwise-uniform $k_y = 0$ modes, starting from the perfectly imbalanced states described above. Figures 2(a) and 2(b) show the typical structure of this fully developed MHD turbulence through the distribution of the dominant streamwise field component b_y in physical space. The field is filled with chaotic small-scale structures stretched along the y -direction by the shear. Note that the kinetic and magnetic energies of PAWs and SAWs are each

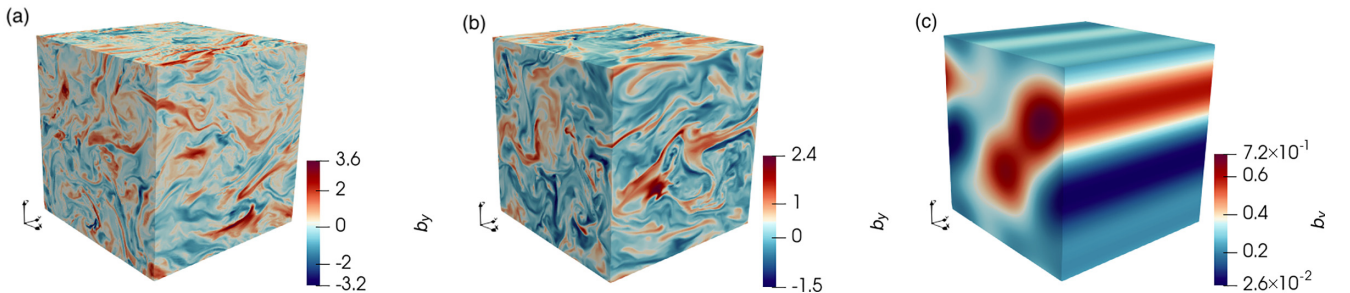


FIG. 2. Structure of the streamwise field component, b_y , in physical space in the fully developed MHD turbulence at (a) $M_A = 10$ and (b) $M_A = 5$, as well as in the laminar state at (c) $M_A = 2.94$. The laminar state is characterized by uniform in the streamwise y -direction large-scale modes. All these snapshots are taken at $t = 670$.

almost equal at all times [Figs. 1(a) and 1(b)], that is, there is equipartition between the PAW and SAW energies in contrast to MHD turbulence without shear, where, as noted above, only SAWs play a role.

The $k_y = 0$ modes, which have been initially absent, emerge as a result of nonlinear interaction of Alfvén waves during the early growth stage and, in the subsequent turbulent state, undergo occasional rapid nonmodal growth events, or bursts [Fig. 1(c)]. These bursts are associated with the bursts of the dominant $\mathbf{k}_b = (0, 0, 2\pi/L_z)$ mode, referred to as the basic mode, since it carries the highest kinetic energy among all the $k_y = 0$ modes. During each burst, the basic mode first rapidly grows and then decays, nonlinearly transferring energy to the Alfvén waves, resulting in similar bursts in the wave energies with a slight time lag [Fig. 1(a)]. In contrast to the waves, the $k_y = 0$ modes remain essentially hydrodynamic, because their magnetic energy is much smaller than the kinetic one and smaller than the magnetic energy of the waves. Therefore, this bursty dynamics of the basic mode, and so all the $k_y = 0$ modes, is governed by the interplay of the mostly hydrodynamic linear nonmodal growth and nonlinear processes [36,77].

Although the evolution of the PAW and SAW energies is quite similar in the turbulent state, their early-time development for the considered both perfectly imbalanced initial states with only PAWs or SAWs differs due to the essentially different linear dynamics of these waves, as discussed in sub-Sec. II. In the first case, initially imposed PAWs undergo nonmodal growth (over-reflection) and also cause SAWs to rapidly grow due to linear coupling [Fig. 1(d)]. By contrast, in the second case, the growth of the imposed SAWs is slower—they initially excite PAWs due to nonlinearity, which in turn nonmodally grow and reinforce the originally imposed SAWs [Fig. 1(e)].

The time and volume averages of the kinetic and magnetic energies of the perturbations as well as the Reynolds $u_x u_y$ and Maxwell $-b_x b_y$ stresses for different M_A in the turbulent state are listed in Table I. Note that the kinetic energy and Reynolds stress are, respectively, about 3.5 and 6 times larger than the magnetic energy and Maxwell stress, implying that the turbulence is driven mainly by shear-induced hydrodynamic nonmodal processes [36,77], but is modified by the magnetic field. Specifically, in the hydrodynamic case ($M_A = \infty$), where the flow is not affected by the field, the kinetic energy and Reynolds stress reach their maximum values and decrease with decreasing M_A , i.e., with increasing field strength. The magnetic energy and Maxwell stress, which are mainly associated with Alfvén waves, exhibit a similar decreasing trend with M_A . This indicates that the background field has a stabilizing effect on the flow: it undergoes a transition from a turbulent to a laminar state at some critical value $M_{A,c}$ as this parameter decreases. For the given initial conditions, such a relaminarization occurs at $M_{A,c} \approx 3$. This laminar state is composed of several large-scale nonoscillatory $k_y = 0$ modes that are uniform in the streamwise y -direction [Fig. 2(c)].

B. Balancing of the turbulence

We now focus on the balance/imbalance properties of the resulting shear-driven MHD turbulence. To this end, we

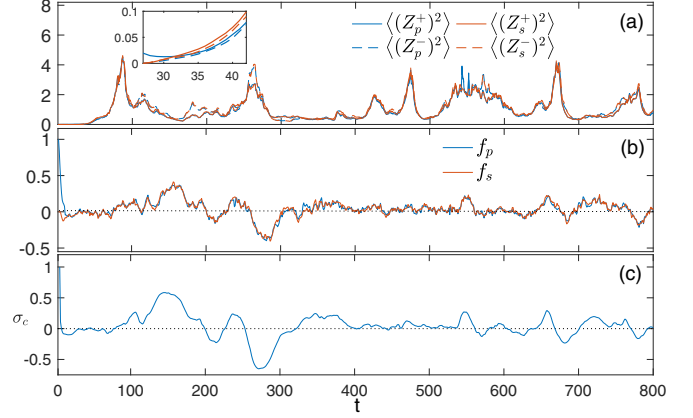


FIG. 3. Evolution of (a) the volume-averaged Elsässer energies $\langle(Z_p^\pm)^2\rangle$ and $\langle(Z_s^\pm)^2\rangle$ (the factor 1/4 is omitted for simplicity) for PAWs (blue) and SAWs (red), respectively, for the initially imposed only PAWs at $M_A = 10$, (b) the corresponding ratios f_p and f_s [given by Eqs. (15) and (16), respectively] and (c) the normalized cross-helicity σ_c [Eq. (6)].

examine the nonlinear evolution of the Elsässer variables, whose linear dynamics under the influence of shear was described in sub-Sec. II.

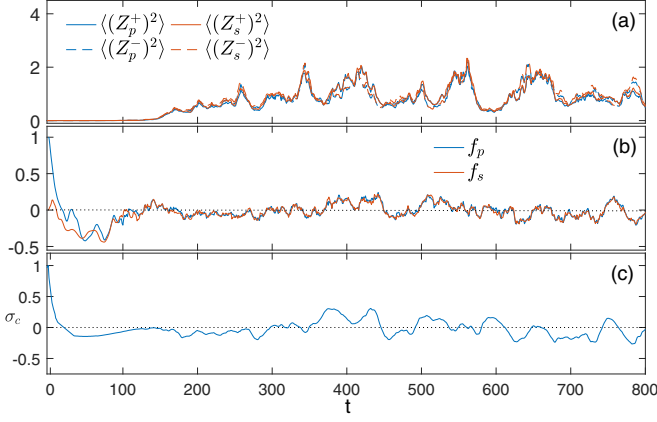
In analogy with the normalized cross-helicity σ_c [Eq. (6)], which characterizes an overall imbalance of MHD turbulence, we also define the ratios f_p and f_s ,

$$f_p = \frac{\langle(Z_p^+)^2\rangle - \langle(Z_p^-)^2\rangle}{\langle(Z_p^+)^2\rangle + \langle(Z_p^-)^2\rangle}, \quad (15)$$

$$f_s = \frac{\langle(Z_s^+)^2\rangle - \langle(Z_s^-)^2\rangle}{\langle(Z_s^+)^2\rangle + \langle(Z_s^-)^2\rangle}, \quad (16)$$

which measure the degree of imbalance separately for PAWs and SAWs, respectively, in the turbulent state. Since the two types of initial conditions described above lead to qualitatively similar saturated turbulent states, as evident in Fig. 1, we focus on the time evolution of the Elsässer energies for the first type of the perfectly imbalanced initial condition, which contains only PAWs with nonzero $Z_p^+ \neq 0$ and no SAWs ($Z_s^\pm = 0$). Therefore, initially $f_p(0) = 1$, whereas $f_s(0)$ is formally undefined because SAWs are absent. However, SAWs are generated by PAWs shortly after the start [Fig. 1(d)], so that f_s becomes well defined already near $t = 0$.

Figure 3 shows the evolution of the volume-averaged Elsässer energies of PAWs and SAWs, the corresponding ratios f_p , f_s , and σ_c . It is clear that the initially imposed only PAWs with $Z_p^+ \neq 0$, its counterpart Z_p^- generated as a result of nonmodal growth (over-reflection), and $Z_s^\pm$ of SAWs excited due to linear coupling with the PAWs all become remarkably close to each other in the turbulent state: $\langle(Z_p^+)^2\rangle \approx \langle(Z_p^-)^2\rangle \approx \langle(Z_s^+)^2\rangle \approx \langle(Z_s^-)^2\rangle$, as seen in Fig. 3(a). Thus, there is equipartition not only between the Elsässer energies of PAWs and SAWs (Fig. 1), but also between Z^+ and Z^- for each of these Alfvén wave branches. Consequently, f_p rapidly drops from its initial value of 1 and fluctuates near zero in the turbulent state, while f_s closely follows that of f_p , remaining approximately equal to it at all times [Fig. 3(b)]. The evolution of the normalized cross-helicity σ_c exhibits a trend similar to that

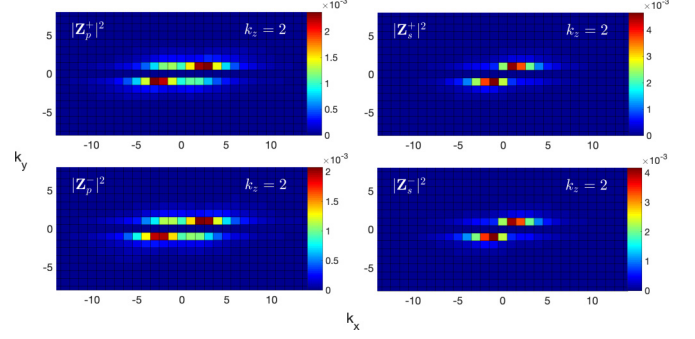
FIG. 4. Same as in Fig. 3 but for $M_A = 3.3$.

of f_p [Fig. 3(c)]. The corresponding time-averaged values of f_p , f_s , and σ_c are close to 0; therefore, the rms amplitudes of their fluctuations provide a more relevant measure of the degree of turbulence balance. We find that the rms amplitudes of these quantities computed over simulation time $\langle f_p^2 \rangle_t^{1/2} = \langle f_s^2 \rangle_t^{1/2} = 0.08$ and $\langle \sigma_c^2 \rangle_t^{1/2} = 0.1$ remain much smaller than 1. This indicates that the turbulence is balanced on average in time, despite being initially maximally imbalanced.

The evolution of the Elsässer energies as well as f_p , f_s , and σ_c toward balanced MHD turbulence also occurs at lower M_A , as shown in Fig. 4 for $M_A = 3.3$, which is qualitatively similar to that for $M_A = 10$ in Fig. 3. The time-averaged Elsässer energies are all close to one another, but are about 1.3 times smaller than those at $M_A = 10$; their peak values are also lower. Being in the balanced state, f_p , f_s , and σ_c fluctuate similarly around 0; however, their rms amplitudes $\langle f_p^2 \rangle_t^{1/2} = \langle f_s^2 \rangle_t^{1/2} = 0.1$ and $\langle \sigma_c^2 \rangle_t^{1/2} = 0.13$ are slightly larger than those at $M_A = 10$, implying that the degree of turbulence balance improves with increasing M_A . This efficient exchange among all four Alfvén wave branches—counterpropagating two PAWs ($Z_p^\pm$) and two SAWs ($Z_s^\pm$)—which results in the emergence of this balanced state from an initially imbalanced one, is therefore due to shear-induced nonmodal dynamics of Alfvén waves, described in sub-Sec. II. Indeed, in the classical case of shearless MHD turbulence, Elsässer fields interact only through nonlinear processes that do not change their respective total energies $(1/4) \int (Z_{p,s}^+)^2 dV$ and $(1/4) \int (Z_{p,s}^-)^2 dV$, but merely redistribute these energies among different modes such that the degree of turbulence imbalance remains conserved in time [4].

C. Spectral analysis

We also examine the energy spectrum of Alfvén waves, which is one of the key characteristics of MHD turbulence. Figure 5 shows the (k_x, k_y) -section of the time-averaged 3D spectra of all four Elsässer fields $\bar{Z}_p^\pm$ and $\bar{Z}_s^\pm$ in Fourier space at a given $k_z = 2$ where they reach high values. These spectra appear similar because the turbulence is on average balanced and exhibit pronounced anisotropy due to shear, with more power being concentrated in the $k_x k_y > 0$ region than in the $k_x k_y < 0$ region, as also observed in other studies of

FIG. 5. The (k_x, k_y) -section of the 3D spectra of $|\bar{Z}_p^\pm|^2$ (left) and $|\bar{Z}_s^\pm|^2$ (right) at $k_z = 2$ in the turbulent state at $M_A = 10$.

shear-driven MHD turbulence [35,44,54,55,78]. This shear-induced spectral anisotropy arises because individual Alfvén waves undergo linear nonmodal growth as their time-varying $k_x(t)$ drifts in Fourier space from the region where $k_x k_y < 0$ to $k_x k_y > 0$, as discussed in sub-Sec. II. It is topologically distinct from the well-known spectral anisotropy of forced MHD turbulence without a mean flow shear, which is caused by a background (guide) magnetic field (e.g., [4–6,63,64]): the former occurs between the streamwise and shearwise directions of the base flow, while the latter occurs between the directions parallel and perpendicular to the background field.

Figure 6 shows the 1D spectra of the Elsässer fields in the perpendicular (k_x and k_z) and parallel (k_y) directions to the base flow and background field, which are obtained by integrating their 3D spectra over the other two directions. These 1D spectra also appear to be identical for both PAWs and SAWs, implying that balance is achieved over a broad range of scales. They differ most notably along k_x and k_y , with the latter being somewhat steeper than the former, which is a consequence of the anisotropy of the 3D spectra in the (k_x, k_y) -plane (Fig. 5). On the other hand, the 1D spectra along k_x and k_z are mostly similar, with only a slight difference at low wave numbers. The k_y -spectra follow k_y^{-2} , as is the case in forced MHD turbulence [79]. However, the spectra along k_x and k_z , which follow the same power laws $k_x^{-0.6}$ and $k_z^{-0.6}$ at intermediate wave numbers, differ from the classical ones $k^{-5/3}$ or $k^{-3/2}$ [4,8,16,63], and this difference can be attributed to shear. This again confirms that the spectral anisotropy of MHD turbulence resulting from shear is fundamentally different from that in the classical shearless case.

IV. CONCLUSIONS

In this paper, we studied the dynamics of MHD turbulence in shear flows with a uniform streamwise background magnetic field in the super-Alfvénic regime when the Alfvén Mach number $M_A \gtrsim 1$. We focused on the effect of large-scale shear of the flow velocity on the balanced/imbalanced nature of the turbulence. In this regime, shear plays an important role in the dynamics of Alfvén waves with frequencies smaller than the shear rate and thus affects the overall dynamics of MHD turbulence composed of interacting Alfvén waves. We demonstrated the reduction of turbulence

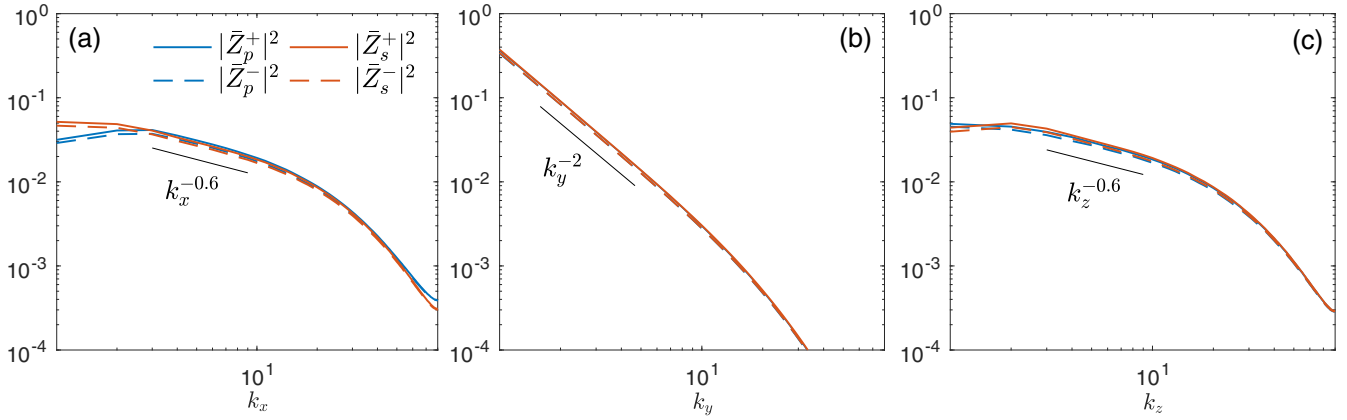


FIG. 6. 1D spectra of $|\bar{Z}_p^\pm|^2$ and $|\bar{Z}_s^\pm|^2$ represented as a function of (a) k_x , (b) k_y , and (c) k_z being integrated in each case over the other two wave numbers.

imbalance in this regime, showing that for sufficiently strong shear—already at $M_A \approx 3$ and higher, i.e., in the strongly super-Alfvénic regime $M_A \gg 1$ —the turbulence becomes essentially balanced. By contrast, in classical forced MHD turbulence in the absence of mean flow shear, the degree of imbalance is conserved. We showed that the dynamics of pseudo-Alfvén and shear-Alfvén waves, which are the building blocks of MHD turbulence, is governed by shear-induced linear processes—nonmodal growth, manifested as the wave over-reflection process, and mode coupling. The nonmodal growth of pseudo-Alfvén waves supplies energy to turbulence, whereas the linear coupling leads to energy exchange and equipartition between pseudo-Alfvén and shear-Alfvén waves as well as between the counter-propagating components $\bar{Z}^\pm$ of each of these wave types. These two main shear-induced processes of linear origin (nonmodal growth and coupling of waves) ultimately lead to balanced MHD turbulence in shear flows.

In conclusion, the balancing of MHD turbulence under sufficiently strong, super-Alfvénic shear of the flow velocity, revealed in this study, is consistent with recent solar wind observations [32] and can provide an interpretation of these observations within the framework of shear-driven MHD turbulence dynamics. Of course, in reality, solar wind dynamics involve other complex factors, such as thermodynamics, compressibility, and geometric effects, which have not been considered here. However, as noted above, shear often remains one of the main drivers of MHD turbulence. The model of an unbounded flow with constant shear adopted in this work is obviously an idealization, but it does allow us

to capture the essence of the dynamics of energy-containing processes driven by shear in more complex flow configurations relevant to the solar wind. Therefore, it is expected that the main results obtained here will also apply to more complex shear profiles. This simplified, yet basic constant shear flow model, including compressibility (which is ignored here), has previously been used in the solar wind context to study the linear nonmodal evolution and shear-induced mutual transformations of Alfvén, fast and slow magnetosonic waves [58,60,62]. Extending the present study to more realistic solar wind flow models will allow us to clarify how geometry, compressibility, thermodynamics and kinetic effects, such as pressure anisotropy, act together with shear to influence the balance/imbalance properties of MHD turbulence in the solar wind.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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