

Emergence of coordinated gait patterns via physically coupled active rotators

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In contrast to the central pattern generator hypothesis, which posits that neural networks generate rhythmic motor patterns without sensory feedback, recent robotics studies have demonstrated that independent oscillating agents with load-dependent feedback can organize coordinated gaits in quadrupedal robots. In this study, we develop minimal mathematical models to describe how such coordination emerges from physical interactions through the trunk and environment. By employing active rotators as limb controllers, we demonstrate their capacity to generate distinct gait patterns, including the trot, pace, and bound. We can also predict gait transitions with walking speed. These models provide insight into why different animals with specific physiques have limited gait patterns and offer suggestions for designing quadrupedal robots.

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I. INTRODUCTION

Quadrupeds exhibit versatile gait patterns, such as the pace, trot, and bound, depending on the species, locomotion speed, and environmental conditions [1,2]. Experiments using decerebrated cats have suggested that an intraspinal neural network called the central pattern generator (CPG) is in part responsible for self-organizing coordinated gait patterns in a decentralized manner [3]. Inspired by this biological finding, various CPG neural network models have been proposed [4–14]. While these models often involve arbitrarily designed neural connections to generate desired gait patterns, many of them do not account for the body's physical dynamics.

Two of the authors and others have constructed a quadrupedal robot consisting of pacemakers with load-dependent local sensory feedback [15–18]. Unlike conventional CPG models, where pacemakers directly interact via neural connections, the pacemakers of this robot do not directly interact with each other. In other words, they are “neurologically decoupled.” However, as each pacemaker is influenced by the posture of its corresponding limb via sensory feedback, the four oscillators eventually interact with

each other and produce versatile gait patterns similar to those observed in animals. Nevertheless, the mechanisms underlying these phenomena remain unclear.

To understand the mechanisms by which the pacemakers organize into specific gait patterns, here we develop minimal mathematical models that describe their physical interactions. By analyzing these models, we demonstrate how a variety of gait patterns can emerge from the effective phase coupling. We also predict gait transitions as walking speed changes. It turns out that nonuniform oscillators with sensory feedback are universally useful for organizing well-coordinated patterns in assemblies of oscillating elements.

II. MODEL DESCRIPTION

When designing a quadrupedal robot, two of the authors and others considered driving the limbs with independent nonuniform pacemakers [15,16]. As a nonuniform pacemaker, they adopted the active rotator model, which was originally introduced by one of the authors and Kuramoto to represent an excitable element, such as a neuron [19]. To represent a pacemaker for each limb, the active rotator is given as

$$\dot{\phi}_i = \omega - \sigma N_i \cos \phi_i, \quad (1)$$

where ϕ_i is the phase of the i th limb ($i = 1, 2, 3$, and 4), measured from the back ($\phi_i = 0$), then lifting up ($\pi/2$), pushing forward (π), touching down ($3\pi/2$), and returning to the back (2π or 0) [Fig. 1(a)]. Here, ω denotes the intrinsic angular velocity, and σ represents the degree of deviation from a uniform oscillation. The pacemaker was designed to be influenced by the normal ground reaction force, N_i , which acts on each limb.

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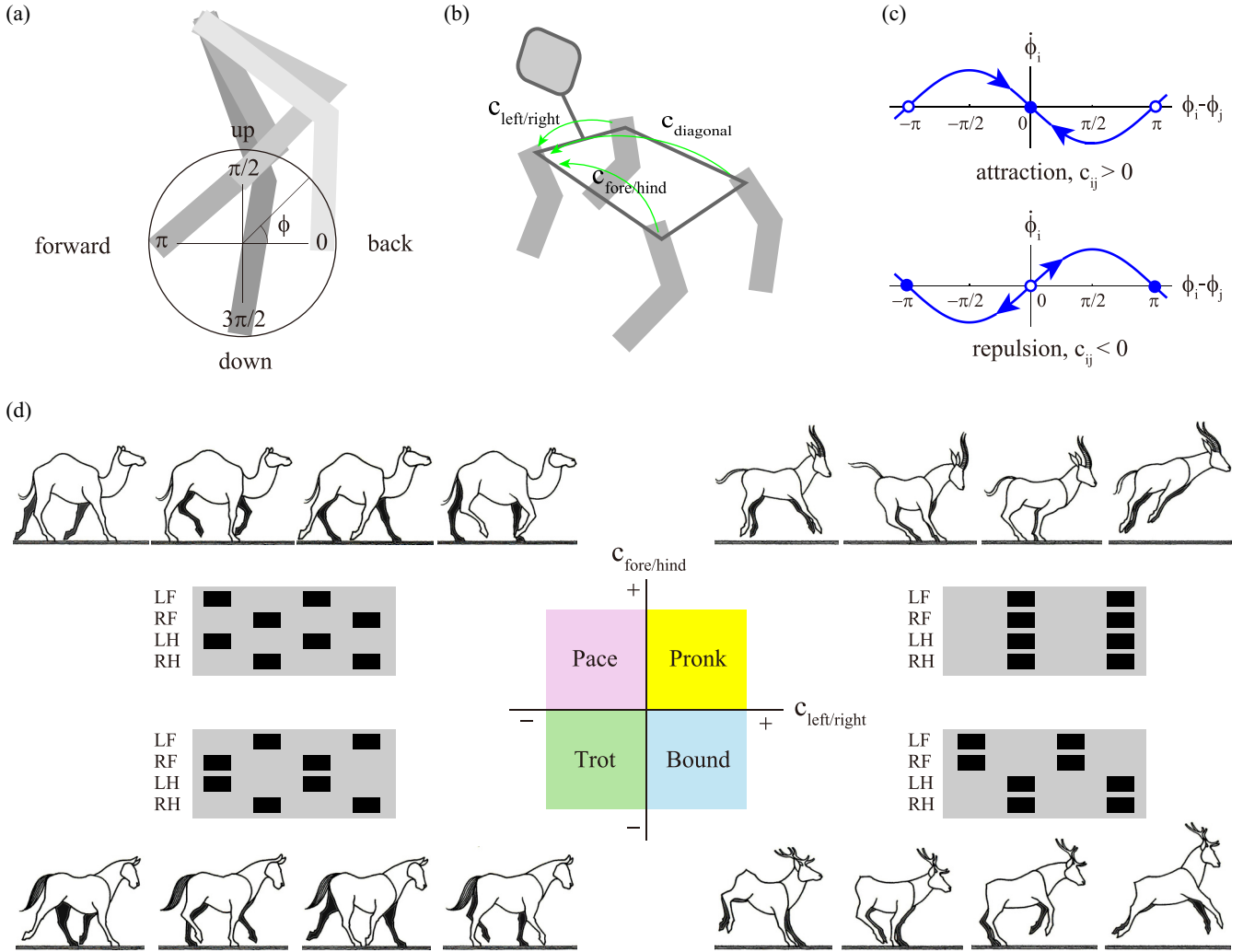


FIG. 1. (a) Schematic representation of limb phase ϕ_i during a locomotor cycle, indicating key positions: backward (0), lift-up ($\pi/2$), forward (π), touch-down ($3\pi/2$), and return (2π or 0). (b) Effective coupling scheme between limbs. (c) Phase interaction represented by the Kuramoto coupling $-c_{ij} \sin(\phi_i - \phi_j)$; the i th phase tends to be attracted to the j th phase in phase if $c_{ij} > 0$, and repelled out of phase if $c_{ij} < 0$. (d) A phase diagram of the gait patterns produced by the phase-coupled Kuramoto oscillators in a plane of the effective coupling parameters $c_{\text{left/right}}$ and $c_{\text{fore/hind}}$, with another parameter fixed at $c_{\text{diagonal}} = 0$. LF, RF, LH, and RH in the footprint diagram stand for left fore, right fore, left hind, and right hind limbs, respectively. The schematic figures depicting the pace, trot, bound, and pronk gaits of a camel, horse, deer, and gazelle are courtesy of Kai Shinomoto.

The limb controller of real animals is known to have a two-layer structure, consisting of a rhythm-generating layer and a pattern-generating layer beneath it. The former produces the oscillating phase, while the latter controls how the muscles in each limb respond to it. Our model primarily focuses on the rhythm-generating layer. The pattern-generating layer has been simplified to create a one-to-one mapping between the phase and limb position.

Though the governing equations of the controllers of the four limbs [Eq. (1)] appear to be mutually independent, they exhibit distinct phase relationships that comprise coordinated gait patterns. This occurs because the phase of each limb is indirectly influenced by the phases of the other limbs via the normal ground force, N_i , in Eq. (1). As walking speed increased with ω , the robot transitioned between different gait patterns.

It is noteworthy that Tero *et al.* attempted to describe the physical interactions between the limbs using modal analysis with a linear approximation, a variable transformation, and analysis in each eigenspace [20]. By incorporating many differential equations, they found that their model could demonstrate coordinated gaits, as well as transitions between gait patterns. Because they considered all possible physical interactions, however, the model became rather complicated to understand the mechanisms by which the gait transitions occur.

To capture the mechanisms by which pacemakers organize into specific gait patterns, we introduce the simplest interaction, in which the normal force of the i th limb is influenced by the j th limb by $-2c_{ij} \sin \phi_j$. Specifically, stretching the j th limb with an angle $\phi_j = 3\pi/2$ (or $\sin \phi_j = -1$) adds the normal force to the i th limb through the trunk if the

coupling parameter is positive $c_{ij} > 0$, and conversely reduces the normal force otherwise [Fig. 1(b)]. Though this seems like an oversimplification, we have confirmed that the resulting locomotion pattern is robust against modifications, such as adding a phase shift to the interaction.

Accordingly, we propose here the model of “interacting active rotators”:

$$\dot{\phi}_i = \omega - \sigma \left(N_0 - \sum_{j(\neq i)} 2c_{ij} \sin \phi_j \right) \cos \phi_i. \quad (2)$$

Although the four limbs are driven independently, they interact indirectly with the term N_i through the trunk and the external world. In our mathematical model, this interaction is represented by the cross term: $\sin \phi_j \cos \phi_i$. Consider the case where the original angular velocity is high, such that $\omega \gg \sigma N_i$. Then the interaction leads to the in-phase Kuramoto coupling between the limbs,

$$\begin{aligned} 2 \sin \phi_j \cos \phi_i &= \sin(\phi_i + \phi_j) - \sin(\phi_i - \phi_j) \\ &\approx -\sin(\phi_i - \phi_j), \end{aligned} \quad (3)$$

because $\sin(\phi_i + \phi_j) \approx \sin 2\omega t$ on average does not contribute to the interphase interaction. Accordingly, we obtain “phase-coupled active rotators”:

$$\dot{\phi}_i = \omega - \sigma N_0 \cos \phi_i - \sigma \sum_j c_{ij} \sin(\phi_i - \phi_j). \quad (4)$$

If we further ignore the nonuniform oscillation term $N_0 \cos \phi_j$, we obtain the “phase-coupled Kuramoto oscillators”:

$$\dot{\phi}_i = \omega - \sigma \sum_j c_{ij} \sin(\phi_i - \phi_j). \quad (5)$$

This equation clearly represents the situation where the i th phase is attracted in phase to the j th phase if $c_{ij} > 0$, and repelled out of phase if $c_{ij} < 0$ [Fig. 1(c)].

III. ANALYSIS OF GAIT PATTERNS

To simplify the analysis, we reduce the number of degrees of freedom of the set of coupling parameters, $\{c_{ij}\}$, from 12 to 3 by grouping them into $c_{\text{left/right}}$, $c_{\text{fore/hind}}$, and c_{diagonal} , as shown in Fig. 1(b). Figure 1(d) depicts the phase diagram of the gait patterns manifested by the phase-coupled Kuramoto oscillators in a plane of $c_{\text{left/right}}$ and $c_{\text{fore/hind}}$, with c_{diagonal} being fixed at zero. In this model, the gait patterns are simply determined by the attraction ($c_{ij} > 0$) and repulsion ($c_{ij} < 0$) of the phases, which are caused by sensory feedback to the four pacemakers.

(1) Pronk (first quadrant, $c_{\text{left/right}} > 0$, $c_{\text{fore/hind}} > 0$): Four limbs are aligned in phase, resulting in the leaping motion as seen in impalas and gazelles.

(2) Pace (second quadrant, $c_{\text{left/right}} < 0$, $c_{\text{fore/hind}} > 0$): The forelimb and hindlimb tend to be in phase while the two limbs on the other side are out of phase, as seen in camels and giraffes.

(3) Trot (third quadrant, $c_{\text{left/right}} < 0$, $c_{\text{fore/hind}} < 0$): Because adjacent limb phases tend to repel each other, the diagonal limbs become phase aligned, while the remaining

two are out of phase, as typically seen in horses, dogs, and cows.

(4) Bound (fourth quadrant, $c_{\text{left/right}} > 0$, $c_{\text{fore/hind}} < 0$): The left and right limbs tend to align in phase while the forelimb and hindlimb alternate out of phase, resulting in the bounding motion as seen in deer and squirrels.

While we developed the mathematical models, Eqs. (2), (4), and (5), to understand the coordinated gaits of quadrupedal robots, it may also be worthwhile to consider real animal locomotion systems because the same argument applies if the pacemaker is influenced by normal ground force, as in Eq. (1). It is noteworthy that different species of animals with various body structures exhibit their own specific gaits, as shown in Fig. 1(d). Deer and squirrels, for instance, tend to perform bounding motions, and their left and right limbs are much closer together than their fore and hind limbs. This seems to be consistent with the bounding regime having coupling parameters of $c_{\text{left/right}} > 0$ and $c_{\text{fore/hind}} < 0$. Conversely, camels and giraffes tend to pace. They have a high center of mass and relatively close fore and hind limbs. This seems to be consistent with the pace regime having coupling parameters of $c_{\text{left/right}} < 0$ and $c_{\text{fore/hind}} > 0$.

IV. PHASE DIAGRAMS OF GAIT PATTERNS

While different species of animals have their own specific gait patterns, some of them can change their gait depending on their walking speed, and the quadruped robot has reproduced such gait transitions [15,16]. While the simplest phase-coupled Kuramoto oscillators, Eq. (5), tend to take a unique gait pattern for a given set of coupling parameters, the original interacting active rotators, Eq. (2), as well as the phase-coupled active rotators, Eq. (4), can exhibit multiple gait patterns for a given set of coupling parameters.

We performed numerical simulations of these models Eqs. (2), (4), and (5), and plotted the phase diagrams on the plane of $c_{\text{left/right}}$ and $c_{\text{fore/hind}}$, within the range of $[-0.5, 0.5]$ [Figs. 2(a)–2(c)]. For each parameter set, we tested 64 initial conditions selected from all combinations of the relative phases of multiples of $\pi/2$ for the four limbs. For each simulation, we identified a gait pattern as pronk, trot, pace, or bound, based on the distance to the four gait patterns (D_{pronk} , D_{trot} , D_{pace} , and D_{bound} , defined in Ref. [21]) computed from the last 10 000 time steps of each simulation.

Experiments with quadrupedal robots have observed gait transitions from trot to bound, as well as hysteresis behavior when walking speed is increased or decreased (see Supplementary Material 7 in Ref. [16]). Our original model of interacting active rotators, Eq. (2), may also exhibit gait transitions at different walking speeds. Figure 3 demonstrates this, showing the case in which the walking speed or the original angular velocity ω was slowly increased from 0.8 to 1.4, and then decreased back to the initial value 0.8. Starting with an initial condition of a trot gait, in which the adjacent limbs are out of phase and the diagonal limbs are in phase ($\phi_1 = \phi_4 = 0$ and $\phi_2 = \phi_3 = \pi$), the system maintained this gait until $\omega \approx 1.1$, at which point it suddenly switched to a bounding gait, in which the left and right limbs tend to align in phase while the forelimbs and hindlimbs alternate out of phase. When ω is decreased, the system continues to

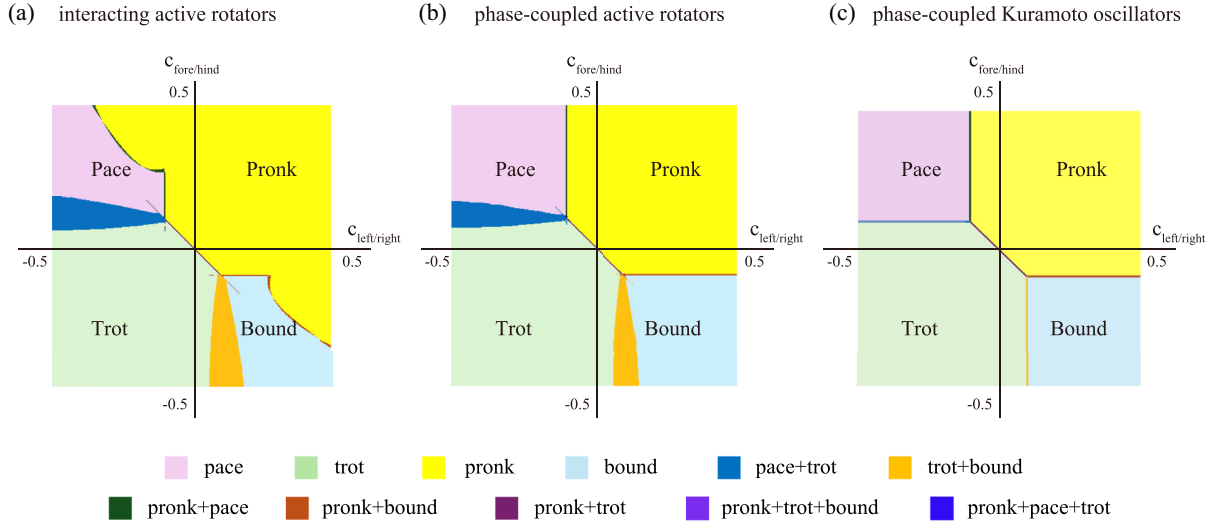


FIG. 2. Phase diagrams of gait patterns on the plane of $c_{\text{left/right}}$ and $c_{\text{fore/hind}}$, with other parameters fixed at $c_{\text{diagonal}} = 0.1$, $\omega = 1.0$, $\sigma = 0.8$, and $N_0 = 1.0$: (a) interacting active rotators, Eq. (2); (b) phase-coupled active rotators, Eq. (4); (c) phase-coupled Kuramoto oscillators, Eq. (5).

bound even at the parameter value at which it was trotting, thus exhibiting hysteresis. We have also demonstrated the time series of $\cos \phi_i$ for the four limbs, $i = 1, \dots, 4$. LF, RF, LH, and RH stand for left fore ($i = 1$), right fore ($i = 2$), left hind ($i = 3$), and right hind ($i = 4$) limbs, respectively.

V. DISCUSSION

In this study, we constructed simple mathematical models to understand the emergence of coordinated gait patterns in quadrupedal robots consisting of pacemakers with load-dependent local sensory feedback. By introducing nonuniform pacemakers influenced by ground reaction forces, the pacemakers effectively couple with each other, thereby exemplifying the embodiment problem [22]. Our models propose

an alternative to the top-down CPG hypothesis, as the gait patterns are primarily determined by bottom-up physical interactions.

Interestingly, in the study of microswimmers, it has been suggested that synchronization of ciliary movement is caused by mechanical interactions through the surrounding fluid. These studies have also shown that the nonuniform oscillation of ciliary movement is important for generating the ordered synchronization patterns [23–30]. The theoretical mechanism discussed in these studies is similar to that of our study, and the simplification from Eq. (4) to Eq. (5) has been extensively discussed for $N = 2$ rotors in the context of cilia synchronization.

In our models, the gait pattern is primarily determined by the physical interaction between the limbs through trunk.

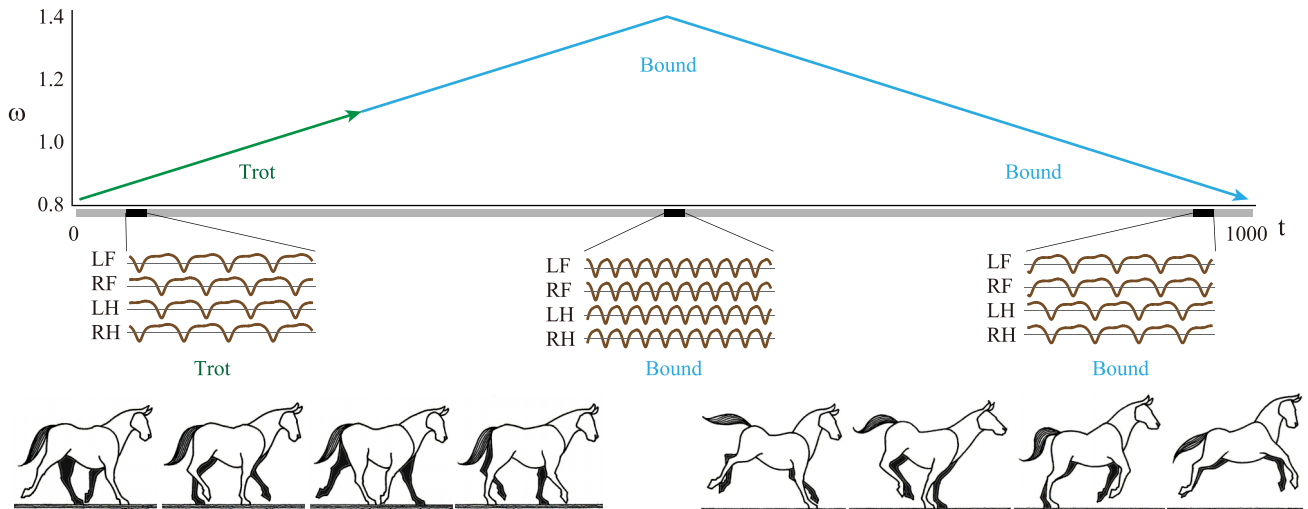


FIG. 3. The gait transition between the trot and the bound in the interacting active rotator model, Eq. (2). The walking speed, or the original angular velocity ω , was slowly increased and then decreased. The system may exhibit either a trot or a bound with the same parameter set. Parameters: $c_{\text{left/right}} = 0.15$, $c_{\text{fore/hind}} = -0.5$, $c_{\text{diagonal}} = 0.1$, $\sigma = 0.8$, and $N_0 = 1.0$. The time series represents $\cos \phi_i$ of four limbs, $i = 1, \dots, 4$. LF, RF, LH, and RH stand for left fore, right fore, left hind, and right hind limbs, respectively.

This suggests that individual animals with particular body structures have their own characteristic gait patterns. To test our physical interaction hypothesis, it would be interesting to observe whether the gait changes when an animal's trunk is perturbed. Owaki *et al.* confirmed that applying a load to their robot induced gait transitions [15]. It would be worthwhile to observe whether gait transitions also occur in animals when a load changes, a limb is manipulated, or the animal is injured. Such observations could provide insight into the mechanisms of gait control. Even for the simplest models proposed here, the full space of the 12 coupling parameters, $\{c_{ij}\}$, has yet to be fully explored. Searching for nontrivial combinations of these parameters may reveal more diverse gait patterns.

Although our discussion has focused on tetrapods, these considerations can also be applied to other animals as well, including myriapods, such as centipedes, and even bipeds, such as humans. In humans, while the hands do not receive normal force from the ground, the hands and legs do interact through the trunk as they swing. Typically, the hands and feet move in opposite phases while running, but there is also

an in-phase gait pattern. This pattern is seen in an ancient Japanese running method and in the martial art of kendo, in which the feet and hands are aligned. Taking advantage of physical phase-locking mechanisms would also be useful when designing robots.

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S.S. developed the theoretical considerations and drafted the manuscript. T.K. performed the numerical analysis. A.I. supervised the study and edited the manuscript.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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