

# Nonthermal electron acceleration and transition probability density in turbulent wakefields driven by an intense laser pulse

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(Received 16 February 2026; revised 15 August 2026; accepted 10 September 2026; published 23 September 2026)

Nonthermal power-law spectra with index  $-2$  are ubiquitously observed in turbulent wakefield acceleration, yet their statistical origin remains unresolved. Using one-dimensional particle-in-cell simulations with particle tracking, we analyze the stochastic energy evolution of electrons accelerated by turbulent wakefields and construct the transition probability density (TPD) of energy jumps. We show that the particle population separates into three dynamical regimes. Low-energy particles exhibit a superposition of Brownian diffusion and Lévy-type jump process, moderate-energy particles are dominated by Lorentzian-type TPDs indicating a nonlocal Lévy-type jump process in momentum space, and high-energy particles undergo ballistic acceleration. The Lorentzian-type TPD in the nonthermal regime naturally leads to a fractional Fokker-Planck description with fractional index  $\alpha = 1$ , whose analytical solution yields a robust  $f(\gamma) \propto \gamma^{-2}$  spectrum. Comparison with the  $\kappa$  distribution confirms consistency with strongly nonthermal plasmas while revealing fractional transport as the dynamical origin of the universal index  $-2$  heavy tail.

DOI: [10.1103/nc7w-yr34](https://doi.org/10.1103/nc7w-yr34)

## I. INTRODUCTION

Cosmic rays have been investigated for more than a century since their discovery by Hess in 1912 [1], yet their origins remain one of the central unresolved problems in astrophysics [2]. Their energy spectra follow a nonthermal power-law form rather than a Maxwell-Boltzmann distribution, a feature that continues to motivate extensive theoretical and observational efforts [3,4]. For galactic cosmic rays, first-order Fermi acceleration or diffusive shock acceleration (DSA) provides the standard mechanism and naturally yields a spectral index of  $-2$  in strong shocks [5,6]. In contrast, the acceleration sites responsible for extragalactic ultra-high-energy cosmic rays remain uncertain [7,8].

Wakefield acceleration has emerged as a promising alternative to DSA in high-energy astrophysical environments [9]. Originally formulated for laboratory plasmas [10], wakefield acceleration has since evolved into a major research area. In laser wakefield acceleration, the ponderomotive force of an intense laser pulse excites a plasma wake in which trapped electrons are rapidly accelerated to relativistic energies [11].

In astrophysical systems, numerical studies have shown that large-amplitude precursor electromagnetic waves are generated ahead of relativistic collisionless shocks via the synchrotron maser instability [12]. These waves excite wakefields in the upstream plasma, enabling efficient nonthermal particle acceleration [13,14]. Two-dimensional particle-in-cell (PIC) simulations further demonstrate that turbulent wakefield acceleration (TWFA) robustly generates power-law spectra with an index near  $-2$ , largely independent of plasma parameters or wave conditions [15–22]. Unlike DSA, wakefield acceleration proceeds through longitudinal fields and is therefore free from synchrotron losses [9].

Direct observational validation of these analytical and numerical models remains impossible, as the relevant astrophysical environments cannot be accessed *in situ*. Nevertheless, the similarity principles of laboratory astrophysics [23] enable scaled analogs to be reproduced experimentally. High-intensity laser experiments provide such an opportunity by modeling the upstream plasma with a laser-produced plasma and replacing precursor electromagnetic waves with an intense laser pulse [24–26].

In this paper, we employ the fractional Fokker-Planck equation (FFPE) together with an one-dimensional PIC simulation to investigate the nonthermal distributions reported in previous studies [15,19–22,24,25]. We show that particles accelerated in turbulent wakefields exhibit intermittent nonlocal energy jumps characteristic of a Lévy-type stochastic transport process [23]. Lévy flights are stochastic processes characterized by step-length power-law distributions that permit rare but arbitrarily large jumps, resulting in superdiffusive transport with divergent variance [27].

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To verify this behavior, we track individual particles and record their temporal energy histories to construct the transition probability density (TPD) describing the energy jumps in successive stochastic events. Histograms of these jumps reveal that the TPD can be modeled as a superposition of Gaussian and Lorentzian components. According to the TPD characteristics, the particle population can be classified into three stochastic transport regimes. (1) High-energy particles undergo monotonic acceleration in the wakefields and correspond to ballistic motion [11]. (2) Moderate-energy particles exhibit intermittent acceleration through repeated trapping and scattering by turbulent wakefields. Their dynamics resemble Lévy-type transport and produce the characteristic index-2 power-law spectrum [15,19–22,24,25]. (3) Low-energy particles display Brownian-like stochastic motion arising from the coexistence of Gaussian diffusion and nonlocal transport [23].

Moreover, understanding how stochastic transport processes generate superthermal particle populations and non-thermal power-law distributions has long been a central topic in plasma kinetic theory. A variety of stochastic acceleration mechanisms driven by wave-particle interactions and plasma turbulence have been proposed to explain the formation of superthermal particles and the emergence of non-thermal power-law distributions beyond thermal equilibrium. In particular, Yoon *et al.* demonstrated the self-consistent generation of superthermal electron tails through beam-plasma interactions and plasma turbulence [28]. Such nonequilibrium plasmas are often characterized by the  $\kappa$  distribution, which provides a phenomenological description of suprathermal power-law tails governed by the parameter  $\kappa$  [29].

Motivated by these studies, the present work focuses on the stochastic transport process underlying nonthermal particle acceleration in turbulent wakefields. Rather than focusing solely on the resulting particle energy distribution, we directly construct the TPD of energy jumps from tracked particle trajectories and investigate the underlying stochastic transport using the FFPE [23]. We further compare the resulting particle distributions with the  $\kappa$  distribution to examine their consistency with strongly nonthermal plasmas. The present approach therefore complements previous studies by directly linking particle trajectories, stochastic transport, and the resulting nonthermal energy distributions.

## II. METHOD

We performed an one-dimensional and three-component velocity particle-in-cell (1D-3V PIC) simulation using the open-source code EPOCH [30]. A  $y$ -polarized laser pulse propagates along the  $x$  direction. The simulation parameters are chosen based on the J-KAREN-P high-intensity laser system at the Kansai Institute for Photon Science [31]. The laser wavelength is  $\lambda_L = 810$  nm, corresponding to the critical density  $n_c = 1.7 \times 10^{21} \text{ cm}^{-3}$ , with the normalized laser amplitude  $a_0 = 40$  and the pulse duration  $\tau_L = 30$  fs.

The simulation box is  $L_x = 2073.6 \text{ } \mu\text{m}$  with the grid number  $N_x = 81920$ , grid size  $\Delta x = 25.3$  nm, and particle number per cell  $\text{PPC} = 512$ . A super-Gaussian electron-proton plasma profile is initialized at  $x = 1000 \text{ } \mu\text{m}$  with an index of 10, radius  $800 \text{ } \mu\text{m}$ , and initial density  $n_0 = 0.2n_c$ . Particle tracking is used to record the temporal evolution

of particle energies and construct the TPD associated with stochastic acceleration in TWFA. The total simulation time is  $t_{\text{end}} = 6.00$  ps.

The present work focuses on the statistical properties of longitudinal stochastic acceleration in turbulent wakefields. Therefore, a 1D geometry is adopted to enable complete particle tracking and TPD analysis for the entire particle population. In multidimensional moving-window simulations, energetic particles may leave the simulation domain, making complete particle statistics difficult [15,20–22].

## III. RESULT

The line profiles of the laser pulse ( $a_y$ ) at time steps  $t$  (ps) = 0.67, 1.33, 2.00, 2.67, 3.34, 4.00, 4.67, 5.34, and 6.00 are shown in Fig. 1(a). Initially, the maximum amplitude is close to  $a_y \simeq a_0 = 40$ , which is highly relativistic. When a laser pulse propagates to the flat region of the density profile ( $n_0 = 0.2n_c$ ), the pulse rapidly damps down to  $a_y < 1$ . The corresponding turbulent wakefields  $a_x$  are excited behind the laser pulse, as shown in Fig. 1(b). These turbulent wakefields randomly accelerate the electrons, resulting in the TWFA [15,19–22,24,25].

The electron energy distribution functions  $f(\gamma)$  at different times are shown in Fig. 2. For  $t \geq 0.667$  ps, the distributions develop similar nonthermal power-law tails, approximately following  $f(\gamma) \propto \gamma^{-2}$  over a broad energy range. At the end of the simulation,  $t_{\text{end}} = 6.00$  ps, the distribution exhibits a power-law scaling with an index of approximately  $-2$  in the range  $200 < \gamma < 2000$ , together with a pronounced high-energy bump at  $\gamma > 2000$ . Below  $\gamma \approx 200$ , the distribution gradually deviates from the power-law scaling.

To analyze the composition of the distribution function, we track particles with different final particle energies [ $\gamma(t = t_{\text{end}})$ ] to investigate the dynamical behavior of the particles in the TWFA excited by the intense laser pulse, as shown in Fig. 3. In order to extract the features from the distribution function, we randomly select some samples to display the dynamical behavior of the particles in low-energy ( $\gamma < 30$ ), moderate-energy ( $200 < \gamma < 600$ ), and high-energy ( $\gamma > 2500$ ) regimes, as shown in Figs. 3(a)–3(c), respectively. The particles in the high-energy regime (red) exhibit a ballistic motion, i.e., once the particle is trapped in an accelerating phase of the TWFA, the energy rapidly increases in the earlier time. These particles contribute to the monoenergetic electron bunch of the distribution function. The particles in the moderate energy regime (blue) show intermittent accelerations, i.e., the particles have a small probability of having a large energy gain, e.g., Lévy-type jump process. These particles contribute to the nonthermal distribution with a power-law index of  $-2$ . The particles in the low-energy regime (green) show a Brownian-like motion, and the electrons stochastically gain energy.

To quantitatively characterize the classification of dynamical behavior in the three regimes, we calculate the TPD that describes the energy jumps in successive stochastic events. Histograms of these jumps reveal that the TPD can be modeled as a superposition of Gaussian and Lorentzian components. We discuss the details in the next section.

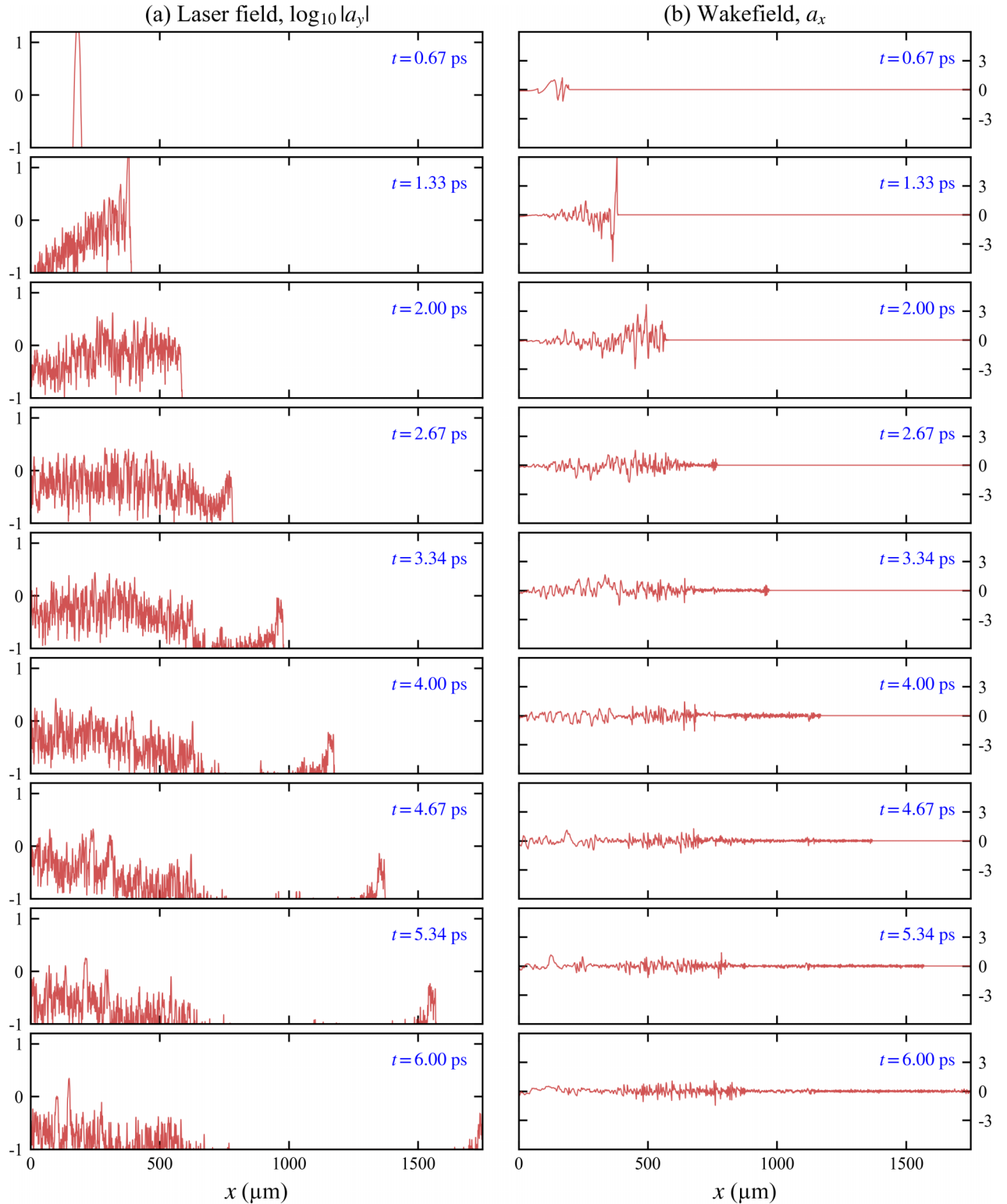


FIG. 1. Snapshots of the laser field  $a_y$  and the resulting wakefields  $a_x$  at different times. (a) Line profiles of the transverse laser field in terms of  $\log_{10}(|a_y|)$  propagating through the plasma density profile  $n_e/n_c = 0.2$ . (b) Corresponding longitudinal wakefields  $a_x$  excited behind the laser front, showing the turbulent structures responsible for stochastic electron acceleration in TWFA.

#### IV. DISCUSSION

To explain the distribution function shown in Fig. 2, we consider the electron transport as a combination of Brownian motion and Lévy flights [23]. Lévy flight and Brownian

motion are both memoryless stochastic processes (Markov property) meaning that the future state of the process depends only on the current state and not on the history of states that preceded it [32]. However, Lévy flight and Brownian motion have distinct characteristics: (1) Brownian motion is

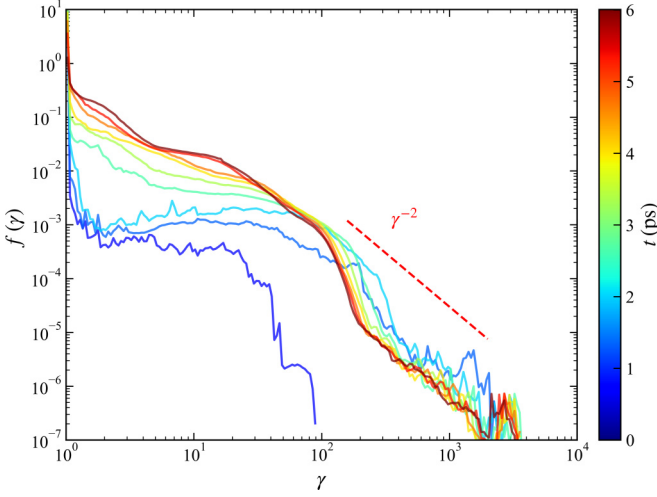


FIG. 2. Time evolution of the electron energy distribution  $f(\gamma)$  at selected time steps showing the emergence of a nonthermal power-law tail. The dashed line indicates a reference slope of  $f(\gamma) \propto \gamma^{-2}$  characteristic of TWFA.

characterized by continuous paths, which means the process varies smoothly over time without any sudden jumps; Lévy flight process includes jumps or discontinuities that occur at random times and have random sizes. (2) The increments of Brownian motion are normally distributed with a mean of zero and some variance; the distribution of increments in a Lévy-flight process is “heavy-tailed,” which means the probability of extreme events (large jumps) is higher compared to

a normal distribution. Hence, the combination of Brownian motion and Lévy flight creates a more comprehensive model that includes both continuous fluctuations and sudden jumps.

Such Lévy-type nonlocal jump processes can be modeled by the FFPE [23], i.e.,

$$\frac{\partial f}{\partial t} = D \frac{\partial^\alpha f}{\partial |\gamma|^\alpha}, \quad (1)$$

where  $D$  is the diffusion coefficient and

$$\frac{\partial^\alpha}{\partial |\gamma|^\alpha} = \frac{1}{2\pi} \int_{-\infty}^{\infty} |k|^\alpha e^{-ik\gamma} dk \quad (2)$$

is the fractional derivative operator with a fractional index  $\alpha$ . The absolute value in the fractional operator denotes the symmetric Riesz fractional derivative corresponding to a bidirectional Lévy-type jump process in the energy increment  $\Delta\gamma$ .

Since the present work mainly focuses on relativistic electrons with  $\gamma \gg 1$ , the momentum and Lorentz factor approximately satisfy  $p \simeq \gamma m_e c$ . Therefore, the stochastic transport process can be equivalently formulated in momentum space or  $\gamma$  space. In the present work, the Lorentz factor  $\gamma$  is adopted as the stochastic transport variable because the particle tracking analysis directly measures the temporal evolution of  $\gamma$  in the PIC simulations. A TPD,  $\Psi(\gamma, \Delta\gamma)$ , describing the energy jump  $\Delta\gamma$  in each stochastic event, is introduced to characterize the time evolution of the distribution function, i.e.,

$$f(\gamma, t + \Delta t) = \int_{-\infty}^{\infty} \Psi(\gamma, \Delta\gamma) f(\gamma - \Delta\gamma, t) d(\Delta\gamma), \quad (3)$$

where the TPD is normalized to unity. It is known that Eq. (1) can be solved analytically only for the cases with  $\alpha = 1$  or  $\alpha = 2$ . The common diffusion equation corresponds to the case with  $\alpha = 2$ , where the corresponding TPD  $\Psi(\gamma, \Delta\gamma)$  is Gaussian. For  $\alpha < 2$ , long-distance jumps or Lévy flight are expected. The Lorentzian-type TPD corresponding to  $\alpha = 1$  is associated with a nonlocal Lévy-type jump process in energy space [23].

In our previous work [33], the TPD  $\Psi$  was constructed using a fixed temporal sampling interval  $\Delta t$  and was assumed to be independent of the evolution time  $t$ . Consequently, its possible dependence on  $\Delta t$  was not examined. However, the representative particle trajectories shown in Fig. 3 indicate that the measured energy increments vary with the temporal sampling interval. This observation motivates us to revisit the TPD by explicitly examining its dependence on  $\Delta t$ . For this purpose, we define the energy increment over a sampling interval  $\Delta t$  as

$$\Delta\gamma = \gamma(t + \Delta t) - \gamma(t), \quad (4)$$

where  $\Delta t$  denotes the time interval over which the energy change is evaluated. We then construct histograms of  $\Delta\gamma$  for different values of  $\Delta t$  to characterize the corresponding TPDs. As will be shown below, the dependence of the increment statistics on  $\Delta t$  is not universal but varies among the three energy regimes.

In the low-energy regime [Fig. 3(a)], most particles exhibit randomly continuous energy fluctuations over comparable amplitudes and timescales. Hence, increasing  $\Delta t$  primarily

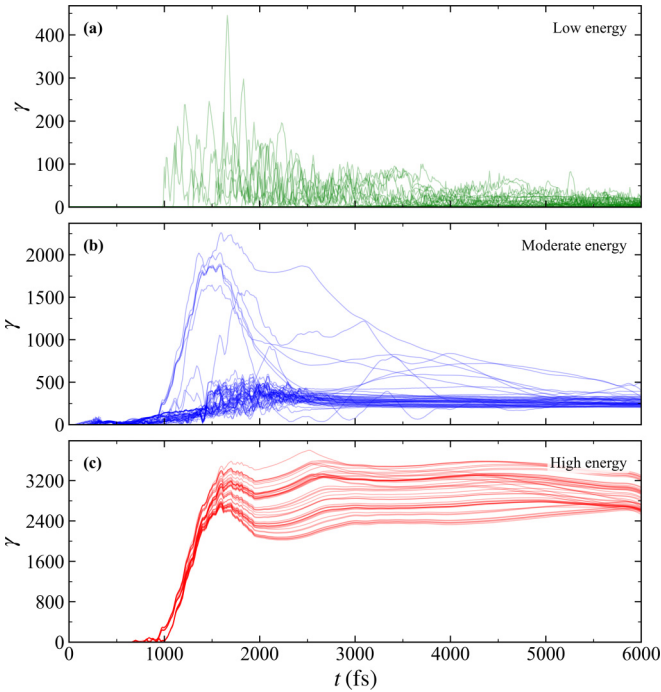


FIG. 3. Time evolution of the Lorentz factor  $\gamma$  for representative particles in (a) low-energy, (b) moderate-energy, and (c) high-energy regimes. Each regime contains 50 particles to present the behavior by randomly sampling.

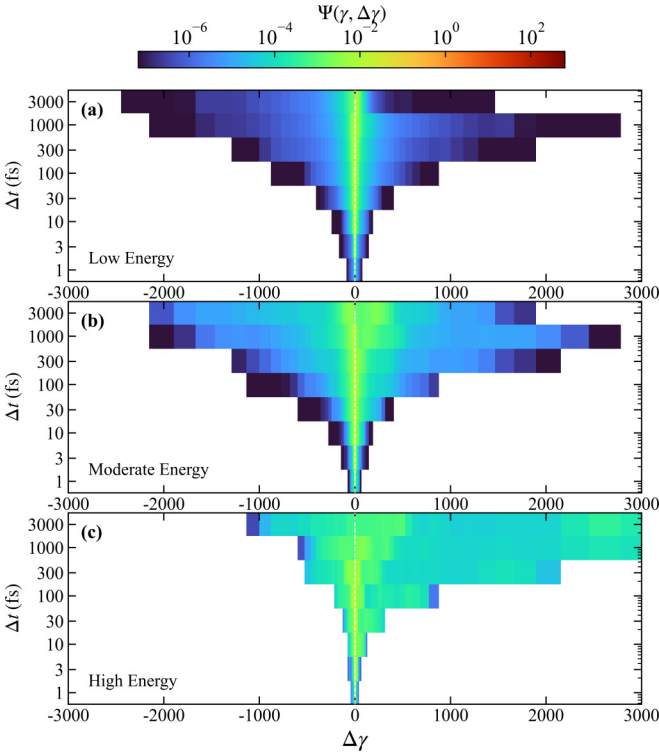


FIG. 4. The TPDs  $\Psi(\gamma, \Delta\gamma)$  with different  $\Delta t$  in (a) low-energy, (b) moderate-energy, and (c) high-energy regimes.

extends the interval over which these fluctuations are sampled, without substantially changing the characteristic of larger acceleration or deceleration events.

In the moderate-energy regime [Fig. 3(b)], in addition to the majority of particles exhibiting relatively small energy variations, a small fraction undergoes substantially larger acceleration or deceleration events. A sufficiently long  $\Delta t$  is therefore required to capture these relatively intermittent, large energy increments that contribute to the tails of the TPD. The situation is qualitatively different in the high-energy regime [Fig. 3(c)], where the selected particles undergo a pronounced acceleration stage, particularly around  $t \simeq 1000$ – $1500$  fs. Increasing  $\Delta t$  in this case allows progressively larger energy changes associated with this acceleration stage to be sampled.

To quantify this dependence, Fig. 4 shows the TPDs calculated using  $\Delta t = 1, 3, 10, 30, 100, 300, 1000$ , and  $3000$  fs for the (a) low-, (b) moderate-, and (c) high-energy regimes. In the low-energy regime, the characteristic width of the TPD changes only weakly once  $\Delta t \gtrsim 10$  fs. Here the energy-regime classification is based on the final particle energy rather than on the instantaneous value of  $\gamma$ . Particles classified in the low-energy regime may therefore experience transient large energy excursions during the TWFA process before eventually returning to low energies. This explains why events with  $|\Delta\gamma| > 2000$  can still appear for  $\Delta t \gtrsim 1000$  fs.

In the moderate-energy regime, the TPD broadens systematically with increasing  $\Delta t$  and approaches a nearly saturated width at  $\Delta t \sim 1000$  fs. This behavior is consistent with the characteristic acceleration and deceleration intervals visible in Fig. 3(b), which are typically several hundred femtoseconds.

Once  $\Delta t$  becomes comparable to or longer than these characteristic timescales, increasing the sampling interval produces only a limited further increase in the accessible energy increments.

For the high-energy regime, the TPD develops a pronounced asymmetry between positive and negative  $\Delta\gamma$ . This behavior reflects the predominantly directed acceleration of these particles during the early stage of the interaction, as seen in Fig. 3(c). The high-energy particles are trapped and subsequently accelerated by the laser-driven wakefield, resulting in a substantial population of large positive energy increments. At sufficiently long  $\Delta t$ , the TPD therefore reflects not only short-time stochastic fluctuations but also the finite-time energy gain associated with the directed acceleration process.

Based on these observations, we choose  $\Delta t = 3, 30$ , and  $300$  fs for the quantitative analysis below. The resulting TPDs are subsequently characterized using several candidate distribution functions. The first is a Gaussian distribution,

$$F_G(\Delta\gamma) = a_G \exp \left[ -\frac{(\Delta\gamma - x_G)^2}{2\sigma_G^2} \right], \quad (5)$$

which represents thermal-like fluctuations. The second is a Lorentzian distribution,

$$F_L(\Delta\gamma) = a_L \frac{\sigma_L}{(\Delta\gamma - x_L)^2 + \sigma_L^2}, \quad (6)$$

characterized by broader wings and slower decay in the tails. The third is the Gaussian-Lorentzian (GL) distribution, defined as a linear combination of the two,

$$F_{GL}(\Delta\gamma) = F_G(\Delta\gamma) + F_L(\Delta\gamma), \quad (7)$$

which captures both a narrow core and extended tails. Finally, we employ the well-known  $\kappa$  distribution [29], which is widely used in space and astrophysical plasmas to describe nonthermal particle populations with enhanced high-energy tails. Unlike the Maxwellian distribution, which assumes thermal equilibrium and exponential decay, the  $\kappa$  distribution exhibits a power-law tail governed by the spectral index  $\kappa$ . Smaller values of  $\kappa$  correspond to stronger deviations from thermal equilibrium and more pronounced suprathermal populations, whereas the limit  $\kappa \rightarrow \infty$  recovers the Gaussian form.  $\kappa$  distribution is expressed as

$$F_\kappa(\Delta\gamma) = a_\kappa \left[ 1 + \frac{(\Delta\gamma - x_\kappa)^2}{\kappa \sigma_\kappa^2} \right]^{-\kappa}, \quad (8)$$

where  $a_{G,L,\kappa}$ ,  $x_{G,L,\kappa}$ , and  $\sigma_{G,L,\kappa}$  in the above distributions denote the amplitude, central position, and width parameters, respectively.

The fitting results for  $\Delta t = 3, 30$ , and  $300$  fs are shown in Fig. 5. The corresponding goodness-of-fit metrics are summarized in Table I, while the detailed fitting parameters and their uncertainties are listed in Tables II–IV. The fitting quality is evaluated using the adjusted coefficient of determination  $R_{\text{adj}}^2$ , defined as

$$R_{\text{adj}}^2 = 1 - (1 - R^2) \frac{N - 1}{N - p - 1}, \quad (9)$$

where  $N$  is the number of fitted histogram bins and  $p$  is the number of fitting parameters. Unlike the conventional

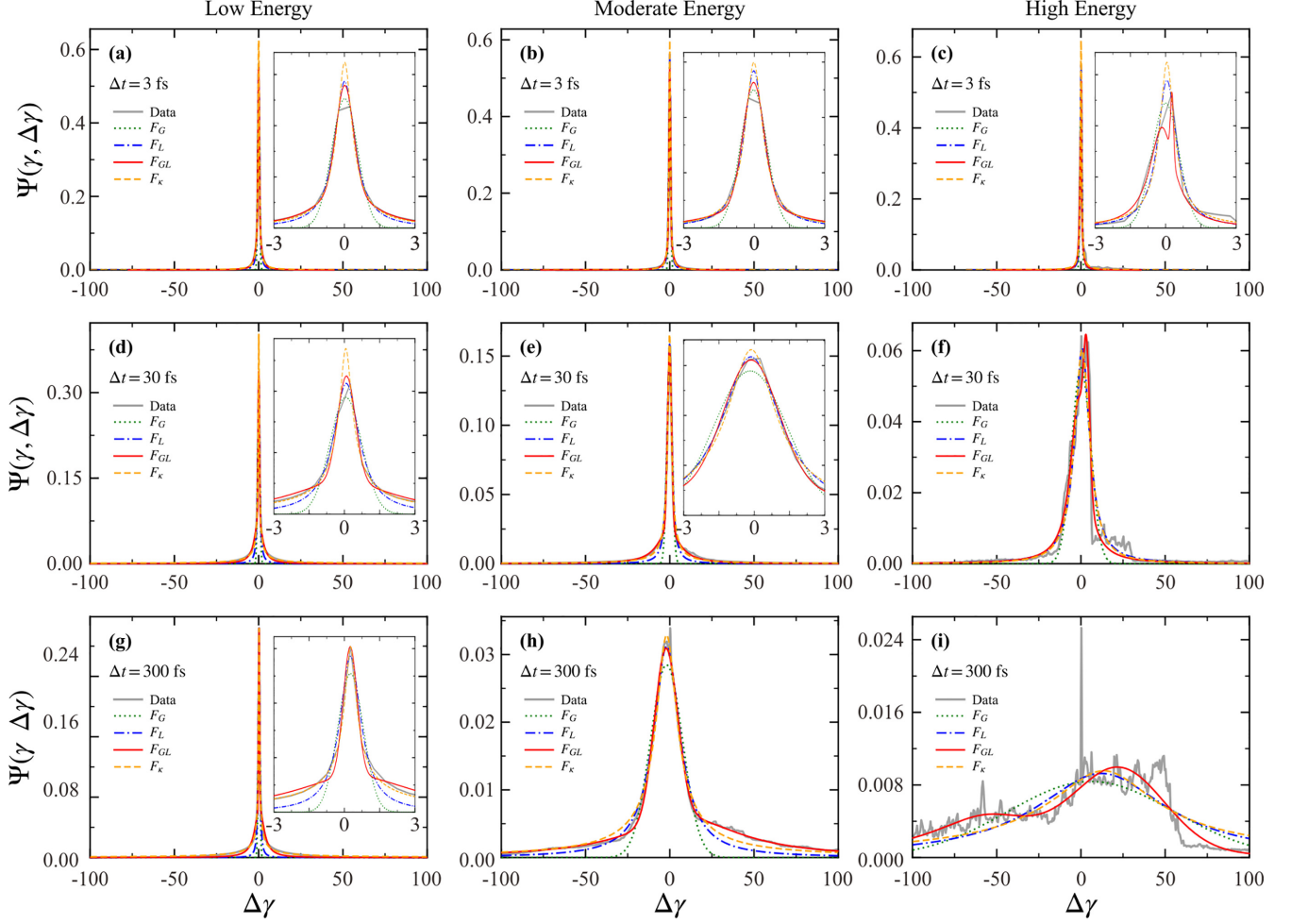


FIG. 5. TPD of the energy jumps  $\Delta\gamma = 3, 30$ , and  $300$  fs in three energy regimes. [(a), (d), and (g)] Low-energy particles, [(b), (e), and (h)] moderate-energy particles, and [(c), (f), and (i)] high-energy particles. The gray curves represent the numerical data obtained from tracked particle trajectories. The red solid line ( $F_{GL}$ ) denotes the Gaussian-Lorentzian superposition fit, while the blue dash-dotted line ( $F_L$ ), green dotted line ( $F_G$ ), and orange dashed line ( $F_\kappa$ ) correspond to Lorentzian, Gaussian, and  $\kappa$  distribution fits, respectively. The subplots are zoomed in with  $-3 < \Delta\gamma < 3$  for showing the details of fitting.

coefficient of determination  $R^2$ ,  $R_{adj}^2$  penalizes the introduction of additional fitting parameters and therefore provides a more appropriate criterion for comparing models of different complexity. In the low-energy regime, the single Gaussian distribution  $F_G(\Delta\gamma)$  provides the poorest description of the TPD, whereas the Lorentzian  $F_L(\Delta\gamma)$ , Gaussian-Lorentzian  $F_{GL}(\Delta\gamma)$ , and  $\kappa$  distribution  $F_\kappa(\Delta\gamma)$  reproduce the non-Gaussian tails substantially better. At  $\Delta t = 3$  fs,  $F_{GL}$  gives the highest  $R_{adj}^2$ , with the  $\kappa$  distribution providing a nearly equivalent fit. As the sampling interval is increased to 30 and 300 fs,  $F_\kappa$  becomes slightly more favorable than  $F_{GL}$ . Therefore, although the relative performance of the two non-Gaussian models depends weakly on  $\Delta t$ , the failure of a single Gaussian distribution to describe the TPD is robust over the sampling intervals considered here.

The evolution of the fitting parameters provides further insight into the origin of this non-Gaussian behavior. For  $F_{GL}$ , the Gaussian width remains nearly unchanged, with  $\sigma_G = 0.36, 0.38$ , and  $0.30$  at  $\Delta t = 3, 30$ , and  $300$  fs, respectively. In contrast, the Lorentzian width increases systematically from

$\sigma_L = 1.39$  to  $2.70$  and  $3.15$  over the same sampling intervals. The different dependences of  $\sigma_G$  and  $\sigma_L$  on  $\Delta t$  suggest that the narrow central component of the low-energy TPD is relatively insensitive to the sampling interval, whereas the broad component associated with large energy increments becomes progressively wider as  $\Delta t$  increases. This interpretation is consistent with Fig. 4(a), where the central structure changes only weakly while increasingly large  $|\Delta\gamma|$  events are sampled at longer  $\Delta t$ .

The  $\kappa$ -distribution fit provides a complementary description of the same tendency. The fitted  $\kappa$  index decreases systematically as  $\Delta t$  increases, from  $0.74$  at the shortest interval to  $0.46$  at the longest interval. Within the present definition of  $F_\kappa$ , decreasing  $\kappa$  corresponds to a more slowly decaying tail. The evolution of  $\kappa$  therefore indicates an increasing relative contribution from large energy-increment events at longer sampling intervals. Importantly, the simultaneous behavior of the two independent fitting models provides a consistent picture: the central part of the distribution remains comparatively stable, as indicated by  $\sigma_G$ , while the tails become broader and

TABLE I. Comparison of the adjusted coefficient of determination ( $R_{\text{adj}}^2$ ) for different models of the TPD at  $\Delta t = 3, 30$ , and 300 fs. The highest  $R_{\text{adj}}^2$  for each energy regime and sampling interval is highlighted in bold.

| Regime   | Model           | 3 fs        | 30 fs       | 300 fs      |
|----------|-----------------|-------------|-------------|-------------|
| Low      | $F_G$           | 0.95        | 0.88        | 0.80        |
|          | $F_L$           | 1.00        | 0.96        | 0.91        |
|          | $F_{\text{GL}}$ | <b>1.00</b> | <b>0.99</b> | <b>0.99</b> |
|          | $F_\kappa$      | 1.00        | 1.00        | 1.00        |
| Moderate | $F_G$           | 0.97        | 0.94        | 0.90        |
|          | $F_L$           | 1.00        | 0.98        | 0.97        |
|          | $F_{\text{GL}}$ | <b>1.00</b> | <b>1.00</b> | <b>1.00</b> |
|          | $F_\kappa$      | 1.00        | 0.99        | 0.99        |
| High     | $F_G$           | 0.94        | 0.90        | 0.55        |
|          | $F_L$           | 0.97        | 0.92        | 0.59        |
|          | $F_{\text{GL}}$ | <b>0.99</b> | <b>0.96</b> | <b>0.76</b> |
|          | $F_\kappa$      | 0.97        | 0.92        | 0.59        |

more prominent, as indicated independently by the increase in  $\sigma_L$  and the decrease in  $\kappa$ .

The moderate-energy regime exhibits a different dependence on the sampling interval. Here  $F_L$ ,  $F_{\text{GL}}$ , and  $F_\kappa$  all provide substantially better descriptions of the TPD than the single Gaussian model. Among them,  $F_{\text{GL}}$  consistently gives the highest or nearly highest  $R_{\text{adj}}^2$  for all three sampling intervals, while the  $\kappa$  distribution also provides excellent agreement. The good performance of both heavy-tailed models demonstrates that the non-Gaussian character of the moderate-energy TPD is not specific to a particular choice of  $\Delta t$ .

In contrast to the low-energy regime, however, both components of  $F_{\text{GL}}$  broaden substantially with increasing  $\Delta t$ . The Gaussian width increases from  $\sigma_G = 0.44$  at  $\Delta t = 3$  fs to 1.18 at 30 fs and 6.88 at 300 fs, whereas the Lorentzian width increases more strongly from  $\sigma_L = 1.74$  to 7.60 and 40.44. Thus, the evolution of the moderate-energy TPD cannot be attributed solely to an extension of its far tails; its central

component also broadens as the sampling interval increases. This behavior is consistent with the trajectories in Fig. 3(b), where the moderate-energy particles undergo appreciable acceleration and deceleration over timescales of several hundred femtoseconds. A longer  $\Delta t$  therefore samples progressively larger energy changes associated with these finite-duration acceleration and deceleration episodes.

The corresponding  $\kappa$  index also decreases with increasing sampling interval. Specifically,  $\kappa$  changes from 0.84 at  $\Delta t = 3$  fs to 0.70 at 30 fs and only slightly further to 0.67 at 300 fs. The relatively small change between the latter two sampling intervals is consistent with the saturation tendency observed in Fig. 4(b). Once  $\Delta t$  becomes comparable to the characteristic timescale of the energy variation, further increasing the sampling interval has a progressively weaker influence on the statistical form of the TPD.

It is also noteworthy that the Lorentzian and  $\kappa$  distributions both provide excellent fits in this regime. The fitted  $\kappa$  values remain of order unity, corresponding to a slowly decaying non-Gaussian tail. Together with the good performance of the Lorentzian model, this behavior supports a Lorentzian-like heavy-tailed transition probability. Within a stochastic-transport interpretation, such a transition probability is consistent with a Lévy-type jump process, in which relatively rare but large energy increments make an appreciable contribution to the particle-energy evolution [23]. This provides a possible statistical connection to the approximately power-law energy spectrum observed in the moderate-energy regime and is qualitatively consistent with previous studies in which  $\kappa$  distributions emerge in nonequilibrium turbulent plasmas characterized by nonlocal stochastic transport [28].

The high-energy regime exhibits statistical properties distinct from those of the low- and moderate-energy populations. As  $\Delta t$  increases, the TPD evolves from a relatively localized distribution into a pronounced asymmetric and multipeak structure. At  $\Delta t = 3$  fs, all four models provide reasonable descriptions of the TPD, with  $F_{\text{GL}}$  yielding the highest  $R_{\text{adj}}^2$ . The agreement progressively decreases with increasing  $\Delta t$ . In

TABLE II. Fitting parameters and uncertainties for different models of the TPD at  $\Delta t = 3$  fs. Here  $a$ ,  $x$ , and  $\sigma$  denote the amplitude, central position, and width parameters, respectively, while  $\kappa$  controls the power-law tail of the  $\kappa$  distribution.

| Regime   | Model           | Fitting parameters and uncertainties |                             |                                  |                          |
|----------|-----------------|--------------------------------------|-----------------------------|----------------------------------|--------------------------|
| Low      | $F_G$           | $a_G = 0.51 \pm 0.01$                | $x_G = 0.01 \pm 0.01$       | $\sigma_G = 0.56 \pm 0.01$       |                          |
|          | $F_L$           | $a_L = 0.30 \pm 0.00$                | $x_L = 0.01 \pm 0.00$       | $\sigma_L = 0.52 \pm 0.00$       |                          |
|          | $F_{\text{GL}}$ | $a_G = 0.40 \pm 0.00$                | $x_G = 0.01 \pm 0.00$       | $\sigma_G = 0.36 \pm 0.00$       |                          |
|          |                 | $a_L = 0.22 \pm 0.00$                | $x_L = -0.01 \pm 0.00$      | $\sigma_L = 1.39 \pm 0.01$       |                          |
|          | $F_\kappa$      | $a_\kappa = 0.65 \pm 0.00$           | $x_\kappa = 0.01 \pm 0.00$  | $\sigma_\kappa = 0.39 \pm 0.00$  | $\kappa = 0.74 \pm 0.01$ |
| Moderate | $F_G$           | $a_G = 0.50 \pm 0.01$                | $x_G = -0.03 \pm 0.01$      | $\sigma_G = 0.56 \pm 0.01$       |                          |
|          | $F_L$           | $a_L = 0.29 \pm 0.00$                | $x_L = -0.02 \pm 0.00$      | $\sigma_L = 0.51 \pm 0.00$       |                          |
|          | $F_{\text{GL}}$ | $a_G = 0.43 \pm 0.00$                | $x_G = -0.03 \pm 0.00$      | $\sigma_G = 0.44 \pm 0.00$       |                          |
|          |                 | $a_L = 0.17 \pm 0.00$                | $x_L = 0.02 \pm 0.01$       | $\sigma_L = 1.74 \pm 0.04$       |                          |
|          | $F_\kappa$      | $a_\kappa = 0.60 \pm 0.00$           | $x_\kappa = -0.02 \pm 0.00$ | $\sigma_\kappa = 0.45 \pm 0.00$  | $\kappa = 0.84 \pm 0.01$ |
| High     | $F_G$           | $a_G = 0.49 \pm 0.01$                | $x_G = -0.00 \pm 0.01$      | $\sigma_G = 0.58 \pm 0.01$       |                          |
|          | $F_L$           | $a_L = 0.28 \pm 0.00$                | $x_L = 0.06 \pm 0.01$       | $\sigma_L = 0.49 \pm 0.01$       |                          |
|          | $F_{\text{GL}}$ | $a_G = 0.25 \pm 842845.33$           | $x_G = 0.27 \pm 1622817.92$ | $\sigma_G = 0.06 \pm 3031243.97$ |                          |
|          |                 | $a_L = 0.26 \pm 0.01$                | $x_L = -0.15 \pm 0.02$      | $\sigma_L = 0.65 \pm 0.01$       |                          |
|          | $F_\kappa$      | $a_\kappa = 0.65 \pm 0.03$           | $x_\kappa = 0.05 \pm 0.01$  | $\sigma_\kappa = 0.37 \pm 0.03$  | $\kappa = 0.75 \pm 0.04$ |

TABLE III. Fitting parameters and uncertainties for different models of the TPD at  $\Delta t = 30$  fs. Here  $a$ ,  $x$ , and  $\sigma$  denote the amplitude, central position, and width parameters, respectively, while  $\kappa$  controls the power-law tail of the  $\kappa$  distribution.

| Regime   | Model      | Fitting parameters and uncertainties |                             |                                 |                          |  |
|----------|------------|--------------------------------------|-----------------------------|---------------------------------|--------------------------|--|
| Low      | $F_G$      | $a_G = 0.28 \pm 0.01$                | $x_G = 0.06 \pm 0.02$       | $\sigma_G = 0.68 \pm 0.02$      |                          |  |
|          | $F_L$      | $a_L = 0.21 \pm 0.00$                | $x_L = 0.08 \pm 0.01$       | $\sigma_L = 0.66 \pm 0.01$      |                          |  |
|          | $F_{GL}$   | $a_G = 0.26 \pm 0.00$                | $x_G = 0.09 \pm 0.00$       | $\sigma_G = 0.38 \pm 0.00$      |                          |  |
|          |            | $a_L = 0.22 \pm 0.00$                | $x_L = -0.07 \pm 0.03$      | $\sigma_L = 2.70 \pm 0.06$      |                          |  |
|          | $F_\kappa$ | $a_\kappa = 0.40 \pm 0.00$           | $x_\kappa = 0.06 \pm 0.00$  | $\sigma_\kappa = 0.34 \pm 0.01$ | $\kappa = 0.53 \pm 0.00$ |  |
| Moderate | $F_G$      | $a_G = 0.14 \pm 0.00$                | $x_G = -0.17 \pm 0.03$      | $\sigma_G = 1.58 \pm 0.03$      |                          |  |
|          | $F_L$      | $a_L = 0.25 \pm 0.00$                | $x_L = -0.14 \pm 0.01$      | $\sigma_L = 1.55 \pm 0.02$      |                          |  |
|          | $F_{GL}$   | $a_G = 0.13 \pm 0.00$                | $x_G = -0.13 \pm 0.01$      | $\sigma_G = 1.18 \pm 0.01$      |                          |  |
|          |            | $a_L = 0.19 \pm 0.00$                | $x_L = -0.63 \pm 0.10$      | $\sigma_L = 7.60 \pm 0.21$      |                          |  |
|          | $F_\kappa$ | $a_\kappa = 0.17 \pm 0.00$           | $x_\kappa = -0.12 \pm 0.01$ | $\sigma_\kappa = 1.26 \pm 0.02$ | $\kappa = 0.70 \pm 0.01$ |  |
| High     | $F_G$      | $a_G = 0.06 \pm 0.00$                | $x_G = 0.13 \pm 0.12$       | $\sigma_G = 5.62 \pm 0.12$      |                          |  |
|          | $F_L$      | $a_L = 0.32 \pm 0.01$                | $x_L = 0.75 \pm 0.10$       | $\sigma_L = 5.34 \pm 0.15$      |                          |  |
|          | $F_{GL}$   | $a_G = 0.04 \pm 0.00$                | $x_G = 3.35 \pm 0.08$       | $\sigma_G = 1.79 \pm 0.10$      |                          |  |
|          |            | $a_L = 0.26 \pm 0.01$                | $x_L = -1.59 \pm 0.22$      | $\sigma_L = 5.70 \pm 0.16$      |                          |  |
|          | $F_\kappa$ | $a_\kappa = 0.06 \pm 0.00$           | $x_\kappa = 0.71 \pm 0.10$  | $\sigma_\kappa = 5.53 \pm 0.24$ | $\kappa = 1.08 \pm 0.08$ |  |

particular, the  $R_{\text{adj}}^2$  of  $F_{GL}$  decreases from approximately 0.99 at  $\Delta t = 3$  fs to 0.96 at 30 fs and 0.76 at 300 fs, while the single-component models show an even larger reduction in fitting quality. These results indicate that the high-energy TPD at long sampling intervals cannot be adequately represented by the symmetric statistical distributions considered here.

The asymmetric TPD is directly related to the acceleration dynamics of the high-energy electrons. As shown by the particle trajectories in Fig. 3(c), these electrons undergo strong directed acceleration during the early stage of the interaction and subsequently form a quasimonoenergetic high-energy population. The corresponding finite-time energy increments therefore contain a systematic contribution associated with the directed acceleration, in addition to stochastic energy fluctuations. With increasing  $\Delta t$ , the accumulated contribution of the directed acceleration becomes more significant, resulting in the asymmetric and multippeak structure of the TPD.

Accordingly, the statistical interpretation of the high-energy TPD differs from that of the low- and moderate-energy regimes. Although the  $\kappa$  distribution can reproduce the high-energy TPD over short sampling intervals, its agreement deteriorates as the asymmetric structure develops at longer  $\Delta t$ . The fitted  $\kappa$  parameter in this regime therefore does not provide the same statistical characterization as in the low- and moderate-energy regimes. The high-energy statistics instead reflect the coexistence of stochastic energy fluctuations and directed ballistic acceleration.

The low-energy population is characterized by a relatively stable central component with progressively broadened tails, whereas the moderate-energy population exhibits pronounced non-Gaussian statistics that are well described by Lorentzian- and  $\kappa$ -type distributions. In contrast, the high-energy population is increasingly governed by directed acceleration, producing an asymmetric TPD that deviates from the

TABLE IV. Fitting parameters and uncertainties for different models of the TPD at  $\Delta t = 300$  fs. Here  $a$ ,  $x$ , and  $\sigma$  denote the amplitude, central position, and width parameters, respectively, while  $\kappa$  controls the power-law tail of the  $\kappa$  distribution.

| Regime   | Model      | Fitting parameters and uncertainties |                             |                                  |                          |  |
|----------|------------|--------------------------------------|-----------------------------|----------------------------------|--------------------------|--|
| Low      | $F_G$      | $a_G = 0.25 \pm 0.01$                | $x_G = 0.26 \pm 0.02$       | $\sigma_G = 0.55 \pm 0.02$       |                          |  |
|          | $F_L$      | $a_L = 0.16 \pm 0.00$                | $x_L = 0.26 \pm 0.01$       | $\sigma_L = 0.54 \pm 0.01$       |                          |  |
|          | $F_{GL}$   | $a_G = 0.24 \pm 0.00$                | $x_G = 0.25 \pm 0.00$       | $\sigma_G = 0.30 \pm 0.00$       |                          |  |
|          |            | $a_L = 0.21 \pm 0.00$                | $x_L = 0.45 \pm 0.05$       | $\sigma_L = 3.15 \pm 0.08$       |                          |  |
|          | $F_\kappa$ | $a_\kappa = 0.30 \pm 0.00$           | $x_\kappa = 0.26 \pm 0.00$  | $\sigma_\kappa = 0.31 \pm 0.00$  | $\kappa = 0.46 \pm 0.00$ |  |
| Moderate | $F_G$      | $a_G = 0.03 \pm 0.00$                | $x_G = -1.93 \pm 0.18$      | $\sigma_G = 9.61 \pm 0.18$       |                          |  |
|          | $F_L$      | $a_L = 0.30 \pm 0.00$                | $x_L = -1.86 \pm 0.09$      | $\sigma_L = 9.51 \pm 0.13$       |                          |  |
|          | $F_{GL}$   | $a_G = 0.03 \pm 0.00$                | $x_G = -2.28 \pm 0.04$      | $\sigma_G = 6.88 \pm 0.05$       |                          |  |
|          |            | $a_L = 0.24 \pm 0.00$                | $x_L = 8.34 \pm 0.52$       | $\sigma_L = 40.44 \pm 0.89$      |                          |  |
|          | $F_\kappa$ | $a_\kappa = 0.03 \pm 0.00$           | $x_\kappa = -1.86 \pm 0.06$ | $\sigma_\kappa = 7.49 \pm 0.14$  | $\kappa = 0.67 \pm 0.01$ |  |
| High     | $F_G$      | $a_G = 0.01 \pm 0.00$                | $x_G = 5.91 \pm 1.35$       | $\sigma_G = 49.16 \pm 1.45$      |                          |  |
|          | $F_L$      | $a_L = 0.44 \pm 0.01$                | $x_L = 12.17 \pm 1.21$      | $\sigma_L = 47.40 \pm 1.95$      |                          |  |
|          | $F_{GL}$   | $a_G = 0.01 \pm 0.00$                | $x_G = 23.30 \pm 1.03$      | $\sigma_G = 26.50 \pm 1.25$      |                          |  |
|          |            | $a_L = 0.19 \pm 0.03$                | $x_L = -54.91 \pm 3.36$     | $\sigma_L = 41.61 \pm 6.25$      |                          |  |
|          | $F_\kappa$ | $a_\kappa = 0.01 \pm 0.00$           | $x_\kappa = 13.48 \pm 1.18$ | $\sigma_\kappa = 39.87 \pm 4.02$ | $\kappa = 0.65 \pm 0.11$ |  |

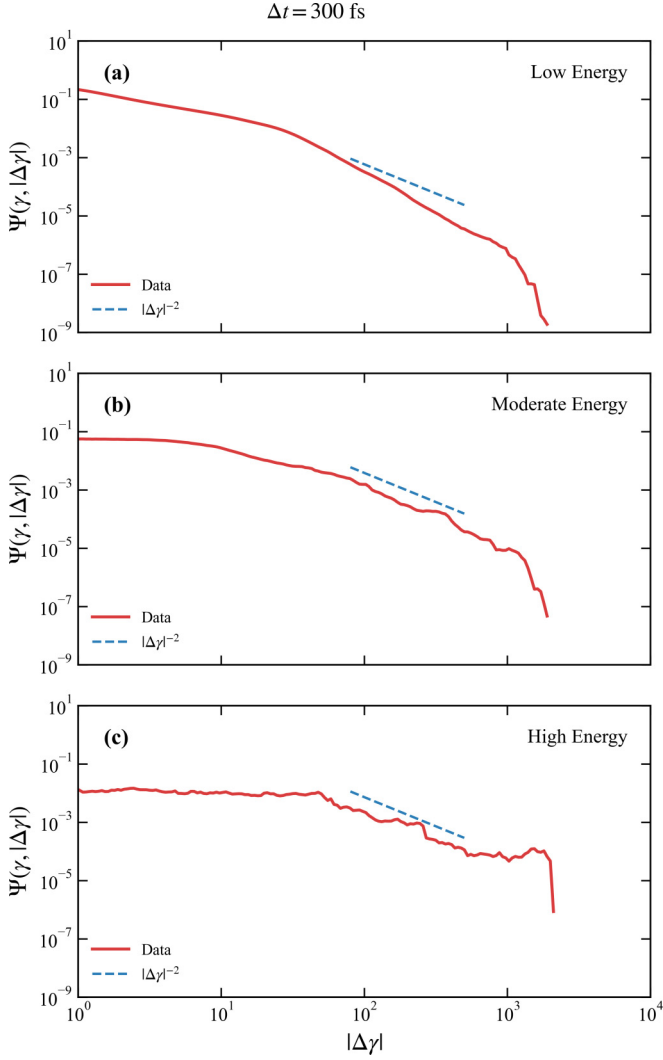


FIG. 6. Heavy-tail behavior of the TPD,  $\Psi(|\Delta\gamma|)$ , at  $\Delta t = 300$  fs for the (a) low-, (b) moderate-, and (c) high-energy regimes. The gray solid curves show the calculated TPDs, while the blue dashed lines indicate the reference scaling  $|\Delta\gamma|^{-2}$ .

symmetric stochastic distributions applicable to the lower-energy populations. To examine the asymptotic behavior of the TPD, we fix the sampling interval at  $\Delta t = 300$  fs and plot the distributions as functions of the absolute energy increment  $|\Delta\gamma|$ , as shown in Fig. 6. The reference scaling  $\Psi(\gamma, |\Delta\gamma|) \propto |\Delta\gamma|^{-2}$  is included for comparison.

In the low-energy regime [Fig. 6(a)], the TPD exhibits a non-Gaussian tail, but its decay is steeper than the  $|\Delta\gamma|^{-2}$  scaling. This behavior indicates that large energy increments are suppressed in the low-energy population. In the moderate-energy regime [Fig. 6(b)], the TPD approximately follows the  $|\Delta\gamma|^{-2}$  scaling over the range  $|\Delta\gamma| \sim 10^2$ – $10^3$ . The existence of this power-law-like tail indicates a finite probability of large energy increments and is characteristic of non-Gaussian stochastic transport. As discussed below, this scaling is consistent with the Lorentzian-like statistics obtained from the quantitative fitting of the TPD. In the high-energy regime [Fig. 6(c)], an approximate  $|\Delta\gamma|^{-2}$  dependence is also observed over a limited range of  $|\Delta\gamma|$ .

However, the TPD contains additional structures and deviates from a single power-law scaling. This departure is consistent with the particle trajectories shown in Fig. 3(c), where the high-energy population undergoes monotonic acceleration and subsequently forms a quasimonoenergetic electron bunch. Consequently, the energy increments in this regime contain a substantial contribution from ballistic acceleration in addition to stochastic fluctuations.

The comparison therefore reveals distinct energy-jump statistics among the three energy regimes. The low-energy TPD exhibits a rapidly decaying non-Gaussian tail, the moderate-energy TPD shows an approximately  $|\Delta\gamma|^{-2}$  power-law tail, and the high-energy TPD is characterized by a structured distribution associated with ballistic acceleration. To quantitatively distinguish these statistical properties over the full range of  $\Delta\gamma$ , the TPDs are further analyzed using Gaussian, Lorentzian, Gaussian-Lorentzian, and  $\kappa$  distribution models. It should be pointed out that regarding the TPD, the Brownian and intermittent regimes are not separated by a sharp electron-energy boundary. Instead, they represent distinct physical contributions characterized by the Gaussian and Lorentzian components, respectively. The transition from Brownian-like to intermittent motion occurs continuously, with their relative importance reflected in the corresponding weights of the Gaussian and Lorentzian components. In contrast, the ballistic population constitutes a physically distinct regime. These particles remain trapped in the wakefield for extended periods and undergo coherent resonant acceleration, giving rise to the bump-on-tail feature in the electron energy distribution. Because ballistic motion retains a memory of the preceding dynamics, the single-particle TPDs are not identical. Consequently, the TPDs of the bump-on-tail particles are ensemble-averaged, producing a peak at a finite, nonzero energy increment.

A simple model to address the power-law index of  $-2$  is shown below. The solution of the initial value problem for  $\alpha = 1$  in Eq. (1) is obtained in Lorentzian distribution [23], i.e.,

$$f^L(\gamma, t) = \frac{1}{\pi Dt} \left[ 1 + \left( \frac{\gamma}{Dt} \right)^2 \right]^{-1}. \quad (10)$$

In the limit of  $\gamma \gg Dt$ , the above equation approximately yields

$$f^L(\gamma, t) \propto \gamma^{-2}. \quad (11)$$

This model naturally explains the emergence of the robust  $f(\gamma) \propto \gamma^{-2}$  spectrum observed in Fig. 2 and previous TWFA studies [15,20–22,24,25].

Although the present FFPE interpretation captures the essential statistical behavior observed in the simulations, the present study is based on 1D-3V PIC simulations. Multidimensional effects such as transverse turbulence, filamentation, mode coupling, and cross-field particle transport are suppressed in the 1D geometry. Therefore, the present results should be regarded as an idealized investigation of longitudinal stochastic acceleration and heavy-tail transport in turbulent wakefields. Despite these limitations, the present results suggest that longitudinal stochasticity alone is already

sufficient to generate intermittent acceleration and nonthermal power-law spectra.

## V. CONCLUSION

We investigated nonthermal electron acceleration in TWFA using one-dimensional particle-in-cell simulations with particle tracking. By analyzing the time histories of particle energies, we constructed the TPD of energy jumps and demonstrated that the particle population separates into three dynamical regimes. Low-energy particles exhibit Brownian-like stochastic motion with a Gaussian-Lorentzian superposed TPD. Moderate-energy particles are dominated by Lorentzian-type TPDs, indicating a Lévy-type nonlocal jump process in energy space. High-energy particles undergo ballistic acceleration and no longer follow a well-defined stochastic distribution. The Lorentzian-type TPD naturally leads to a fractional Fokker-Planck description with fractional index  $\alpha = 1$ , whose analytical solution produces a robust  $f(\gamma) \propto \gamma^{-2}$  power-law spectrum [23]. This provides a statistical interpretation of the universal index  $-2$  nonthermal spectrum observed in TWFA simulations and experiments. A comparison with the  $\kappa$  distribution further confirms that the resulting power-law tail is consistent with strongly nonthermal plasmas and previous studies of superthermal particle generation through plasma turbulence and wave-particle interactions [28].

## ACKNOWLEDGMENTS

We gratefully acknowledge the financial support of the National Science and Technology Council (NSTC), Taiwan,

No. NSTC 114-2112-M-006-021-MY3 and No. 111-2112-M-006-011-MY3. This work was also supported by JSPS KAKENHI Grants No. JP19K21865, No. JP20KK0064, No. JP22H01195, No. JP26H02028, and No. JP26K22309 and by the NINS program of Promoting Research by Networking among Institutions (01422301336 and 01412302). We thank the National Center for High-performance Computing (NCHC) for providing computational and storage resources.

## DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

## APPENDIX: DETAILED FITTING PARAMETERS

For completeness, the detailed fitting parameters and their corresponding uncertainties for the TPD analysis are summarized in Tables II–IV, corresponding to  $\Delta t = 3, 30$ , and  $300$  fs, respectively. For each sampling interval, the Gaussian ( $F_G$ ), Lorentzian ( $F_L$ ), Gaussian-Lorentzian ( $F_{GL}$ ), and  $\kappa$ -distribution ( $F_\kappa$ ) models are fitted separately in the low-, moderate-, and high-energy regimes. The parameter uncertainties are estimated from the covariance matrix of the nonlinear least-squares fitting. The associated goodness-of-fit values, quantified by  $R_{\text{adj}}^2$ , are summarized in Table I.

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