

Characterization of electrostatic discharge currents in electron-charged polymethyl methacrylate as a proxy for natural compact intracloud discharges

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Electrostatic discharges occur in numerous media on a range of length scales, from microscopic discharges inside electronic materials to kilometer-long channels in air during natural lightning events. To study the mechanisms and behavior of electrostatic discharges inside materials, we measured the discharge currents produced during dielectric breakdown of electron-charged polymethyl methacrylate. The resulting current waveforms were analyzed and the relevant parameters for characterizing the waveform morphology were extracted and investigated to build a functional model to describe and characterize the waveforms from these types of discharges. The model involves a distinction between one-dimensional and two-dimensional discharge channel growth, which are governed by separate principles and thus have different contributions to the total measured discharge current. We show that this model, which is derived from the discharge channel growth during the event, allows us to infer characteristics about the dynamics of the breakdown, such as the instantaneous velocity of the channels during breakdown. We applied this model to the context of natural compact intracloud discharges, or narrow bipolar events, by scaling the one-dimensional waveforms in size and time to those relevant to compact intracloud discharges. We find that the discharge currents derived from centimeter-scale and submicrosecond-duration electrostatic discharge events in electron-irradiated polymethyl methacrylate obey the same functional form of those of natural compact intracloud discharges, leading to the conclusion that the polymethyl methacrylate–(PMMA) based discharges may be a useful laboratory tool to reproduce narrow bipolar events discharge currents. We calculated the electric field that would be radiated by these scaled discharge currents using published measurement parameters for compact intracloud discharges and found that the magnitude and pulse shape agree with observations very well. In addition, we find that energy spectral density of the scaled PMMA discharge waveforms have a similar frequency dependence in the 3- to 300-MHz band to those of compact intracloud discharges. These results support the idea that space charge in and around thunderclouds may be the driving mechanism of compact intracloud discharges and that electron-irradiated solid dielectrics may be a useful laboratory-based analog to study natural lightning events.

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I. INTRODUCTION

Many of the dielectric materials commonly used in charged particle radiation environments demonstrate the tendency to retain and store incident particles within the material's bulk [1–3]. These charges in the bulk remain trapped for a period of time that depends on the material's properties, temperature, and thickness, as well as the charged particle's energy and mass [3–10]. In some cases, the irradiated material can retain and continue to collect charge throughout its volume until the internal electric field produced by these trapped charges exceeds the material's breakdown threshold and leads to a spontaneous dielectric breakdown, or electrostatic discharge,

event [10–12]. During these electrostatic discharge (ESD) events, the trapped charge is rapidly released from the dielectric, producing significant currents through and around the material. ESD events of this nature can last from several nanoseconds to a few microseconds, depending on the geometry of the sample and the implanted charge density [10,13–15]. The form of the discharge currents produced during an ESD depends on many factors, including the material geometry, internal charge density, temperature, and discharge initiation orientation relative to the charge implantation depth [16,17].

Most previous work in this field has focused on thin films charged with lower energy (sub-MeV) electrons [5,6,10,12] or on surface discharges [18–25]. For dielectrics with larger dimensions (thickness $\gtrsim 2.5$ cm), such as those studied in this work, with internal electric fields with magnitudes between 0.2 and 0.4 GV/m [26,27] due to implanted trapped charges, the dynamics of the breakdown currents are not well understood. In addition, there is currently no complete physics-based model that can describe the behavior of electrostatic breakdown in these types of bulk-charged dielectric materials.

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The transient discharge currents produced during the breakdown of space charge-containing dielectrics are interesting for a variety of reasons. The neutralization of the implanted charge produces plasma filaments inside of the dielectric [14,28]. These breakdown channels are reminiscent of other ESD phenomena [29,30], such as lightning or gaseous breakdown. However, unlike these breakdowns in gases, the plasma channels in the solid material leave permanent damage patterns, known as electrical trees or Lichtenberg figures, which can be studied postdischarge. When considered alongside the measured discharge current waveforms of the ESD, these electrical trees allow for a deeper understanding of the charge dynamics that occurred during the breakdown event, which can be scaled and applied for understanding other ESD phenomena. In addition to aiding in understanding the physical principles surrounding dielectric breakdown in various media in greater detail, investigating these current waveforms can aid efforts to mitigate unwanted ESD events due to internal deep dielectric charging, such as on satellite and spacecraft components [10,31–35].

One of the primary analogies of interest to other ESD phenomena is natural lightning. There are several distinct categories of natural lightning, including intracloud lightning, cloud-to-ground lightning, intercloud lightning, and so on [36]. Of particular interest to this work is intracloud lightning, which forms entirely inside storm clouds and is not directly visible to an observer on the ground, making it a difficult phenomenon to study [12,37–46]. Occurring between regions of opposite polarity charge inside the cloud, intracloud discharges are analogous to the breakdown observed inside internally charged solid dielectric materials [12,43]. The similarities between intracloud lightning and ESDs in electron-charged polymethyl methacrylate (PMMA) have been discussed in previous work [12,47].

A specific subcategory of intracloud lightning, called compact intracloud discharges (CIDs) or narrow bipolar events (NBEs), is characterized by short, bipolar electric fields in the radiation zone and an accompanying burst of radio frequency (RF) radiation during the event [44,48–55]. These pulses occur on very short timescales compared to other intracloud flashes [50], similarly to the narrow, transient waveforms of the discharges in PMMA presented in this work.

There have been numerous attempts to explain the electromagnetic observations of compact intracloud discharges, including modeling these events as streamer bursts [48,56–58], as bouncing-wave propagation along a discharge channel [50,52,59], as a modified current-carrying transmission line [53], and as relativistic runaway electron avalanche-extensive air showers (RREA-EAS) [60,61]. The models falling under the umbrella of transmission line-type models do not necessarily explain how a highly conductive channel could form quickly in the cloud, and the RREA-EAS models predict unphysical parameters of the discharge-initiating cosmic rays, as well as the resulting peak discharge currents from such an event [60]. These model shortcomings point to areas in which a laboratory-scale analog could be mutually beneficial to understanding both cloud-based discharges and solid dielectric discharges, as the initiating mechanism is an area of uncertainty for both fields, and similar physical principles may be important to both phenomena.

While the initiating mechanism of CIDs remains unclear, the authors of Refs. [56,62] and others demonstrated that modeling CIDs as a streamer burst has the potential to explain the fine structure of CIDs, including variations in the structure of streamer development that contribute to the observed variations in the measured electric fields and the high-frequency (HF) and very high frequency (VHF) radiation produced concurrently with the discharge. This streamer breakdown approach is useful to aid in the investigation of measured current waveforms from electrical breakdown in electron-charged PMMA, and, as a result, the information gleaned from the analysis in this work can aid in developing further explanations for CID phenomena.

The following sections will detail the methods used to prepare the electron-charged PMMA samples of three different geometries. Each sample set provides either a one-dimensional (1D), two-dimensional, or three-dimensional example of breakdown formation inside the electron-charged PMMA. The one-dimensional discharge will be demonstrated as a possible analogy to CIDs, while the two-dimensional discharge will be used to motivate a space charge-driven breakdown model. We will show that the three-dimensional breakdown is a superposition of the one-dimensional and two-dimensional discharges that is useful for understanding how the initial one-dimensional breakdown is influenced by the global system properties. We will discuss the techniques used to measure each of these discharge currents and the analysis performed to understand the correlation between measured electrical parameters. The parameters of interest are extracted from each waveform to build a model of the breakdown channel growth and the expected current produced during a given breakdown for each geometry. Finally, the one-dimensional model is scaled to the time and length scales of a compact intracloud discharge. We find that the one-dimensional PMMA-based model can reproduce measured electric field waveforms for CIDs with reasonable accuracy, demonstrating that electron-charged PMMA can be a useful analog to study natural NBEs inside the laboratory.

II. METHODS

Electrical tree formations and resulting current waveforms were studied in one-dimensional, two-dimensional, and three-dimensional configurations inside electron-irradiated PMMA. To achieve each of these configurations, different sample geometries and charging conditions were used.

During electron irradiation of dielectrics in atmospheric conditions, positive ions are produced in the surrounding air, which cling to the outer surface of the material as its internal negative charge increases. These positive ions have the effect of screening the electric field produced by the internal negatively charged particles, creating a net neutral object with no externally detectable electric field. Thus, measuring the internal charge density by measuring the external electrostatic field is not possible. Due to the large size of the samples, pulsed-electroacoustic measurements [34,63] are also impractical, prompting us to use a discharge current-based investigative approach.

While there is no detectable external electric field due to the layer of screening positive ions, there is a high internal

electric field. At the discharge point, this internal field is estimated to be on the order of hundreds of megavolts per meter. When breakdown is initiated, the mechanical insult created at the discharge point results in a field amplification effect, increasing the field at the initiation point to above the breakdown threshold of PMMA.

A. One-dimensional sample configuration

We irradiated 2.5-cm cubes of commercially procured PMMA on a single face using the University of Maryland linear electron accelerator. The incident beam energy on the sample was nominally 5 MeV. The incident charge density on the sample was calculated to be 72 mC/m^2 . The peak implanted charge density was approximately 75% of the way through the sample or about 1.9 cm. Each sample was exposed to the beam for 1.7 min.

Electrical breakdown was induced in the center of the beam-facing side of the cube. We measured the transient discharge current using a custom fixture that maintains direct, consistent electrical contact with each sample and creates a low inductance path for the discharge current produced by the sample during the ESD. The discharge current travels around a CLCE-1032 RF current probe, making a complete primary loop through the probe by way of the low inductance closed path created by the fixture. This produces a secondary current in the current probe, and the voltage across the probe terminals is proportional to the primary current in a ratio of 1 V/A in the operating band of 1 MHz to 1 GHz. A Tektronix MSO68B 6-series oscilloscope measures the potential across the current probe, which is an inferred measurement of the discharge current in the primary loop. The signal is padded with 26 dB of attenuation prior to entering the scope, which was set to $50\text{-}\Omega$ input impedance. The output waveforms were corrected for the low-frequency distortion effect. For more details of this correction procedure, see Appendix A.

Due to the approximately one-to-one aspect ratio of the depth of the formed electrical tree channel to its horizontal width, resulting in a narrow, vertically directed discharge channel with the initiation point at the top, these measurements were studied as the one-dimensional breakdown configuration. Figure 1 shows a schematic of the 1D setup overlaid on an image of a discharged 2.5-cm cube used in this experiment.

B. Two-dimensional sample configuration

We irradiated $30 \times 10 \times 2.5 \text{ cm}$ blocks of PMMA on a single $30 \times 10 \text{ cm}$ face and initiated a discharge in the sample in the same plane as the peak implanted charge density in the center of one of the $10 \times 2.5 \text{ cm}$ faces [14,64], resulting in a planar electrical tree that is functionally two dimensional. The incident beam energy was 4.5 MeV using an electron Dynamitron accelerator. The incident charge density of the samples used in this study was 10 mC/m^2 . The discharge currents were measured with the same method as the one-dimensional configuration above, though the signal was padded with a higher attenuation of 40 dB to accommodate a much larger discharge current produced by these samples.

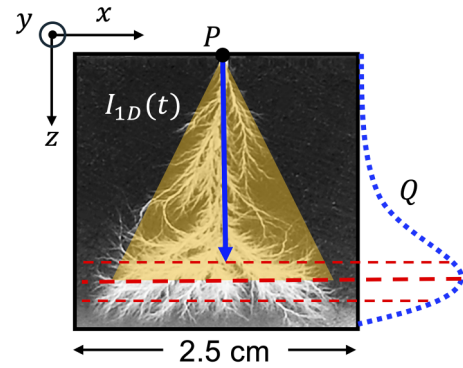


FIG. 1. Side view of sample. Diagram of 2.5-cm cube that has been irradiated on one face in the $+z$ direction overlaid on a real image of a postdischarge 2.5-cm cubic sample. The implanted charge (blue dotted curve) is predicted to peak at roughly 1.9 cm in the $+z$ direction. The shape of the implanted charge density is inferred from simulations presented in Ref. [65]. The charged sample is discharged from point P . The resulting electrical tree forms primarily in the $+z$ direction, with the yellow triangle outlining the discharge envelope. The extent in the $+z$ direction is greater than or equal to the width of the base of the tree, allowing us to treat this electrical tree growth as a one-dimensional process. The current produced during the electrical tree growth is defined as $I_{1D}(t)$.

C. Three-dimensional sample configuration

The three-dimensional configuration essentially consists of a one-dimensional component immediately followed by a two-dimensional component. To achieve a three-dimensional geometry, we irradiated 5-cm cubes of PMMA on a single face and discharged the sample perpendicular to the plane of peak implanted charge density. This results in an electrical tree that is comprised of a narrow channel spanning the distance from the discharge initiation point to the plane of peak implanted charge density before radially branching out in this plane. Figure 2 shows a schematic of the charge distribution and electrical tree growth in the 5-cm cubic samples.

Each sample was irradiated with a 4.5-MeV electron Dynamitron accelerator using a range of beam currents (3, 6, and 9 mA) to achieve different incident charge densities of 0.51, 0.95, and $1.5 \text{ }\mu\text{C/cm}^2$ [13]. Each sample is exposed to the beam for approximately 3 s. After charging the samples, we initiated an ESD event by mechanical insult and measured the current produced during the dielectric breakdown events, which last on the order of 100 ns.

As with the one-dimensional and two-dimensional configurations, we measured the transient discharge current using the aforementioned low-inductance discharge fixture and a CLCE-1032 RF current probe.

In this paper, the planar geometry samples are only used to motivate the functional form of the breakdown currents in the proposed three-dimensional model, and the primary datasets presented here are those of the 2.5- and 5-cm cube configurations.

III. MODEL RESULTS

A. Functional form of the current waveform

The current waveforms produced by samples in the 5-cm cubic discharge orientation exhibit two regimes, which

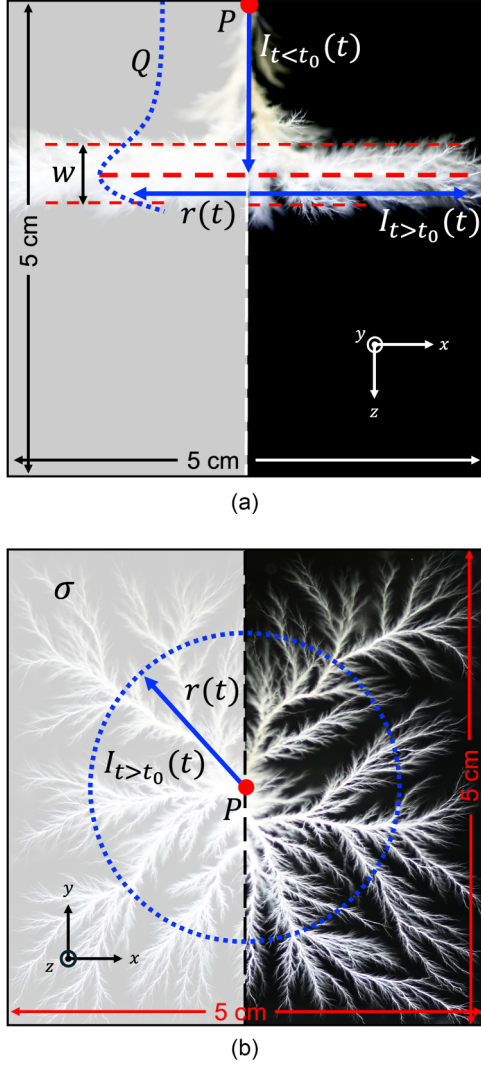


FIG. 2. (a) Side view of sample. Diagram of 5-cm cube that has been irradiated on one face in the $+z$ direction overlaid on a real image of a postdischarge 5-cm cubic sample. The implanted charge (blue dotted curve) is predicted to peak at roughly 2 cm in the $+z$ direction. The shape of the implanted charge density is inferred from simulations presented in Ref. [65]. The charged sample is discharged from point P . The resulting electrical tree forms primarily in the $+z$ direction initially ($t < t_0$), then expands circularly in the plane of peak implanted charge. The thickness w of the tree is small compared to the full extent of the formed tree (bounded by the red dashed lines), allowing us to treat this electrical tree growth for $t > t_0$ as a two-dimensional process. The two-dimensional circular growth in the plane of peak charge is described by $r(t)$, and the currents produced during the electrical tree growth are defined as $I_{t < t_0}(t)$ and $I_{t > t_0}(t)$. (b) Top view of sample. Schematic of the two-dimensional circular expansion of the electrical tree in a uniform plane of charge with charge density σ overlaid on a real image of a postdischarge 5-cm cubic sample. The left-hand sides of the images are opaque for diagram clarity.

correlate to the two phases of electrical tree formation in these samples: the initial growth from the point of initiation to the plane of peak charge implantation and the following radial expansion of the discharge channels in that plane. We

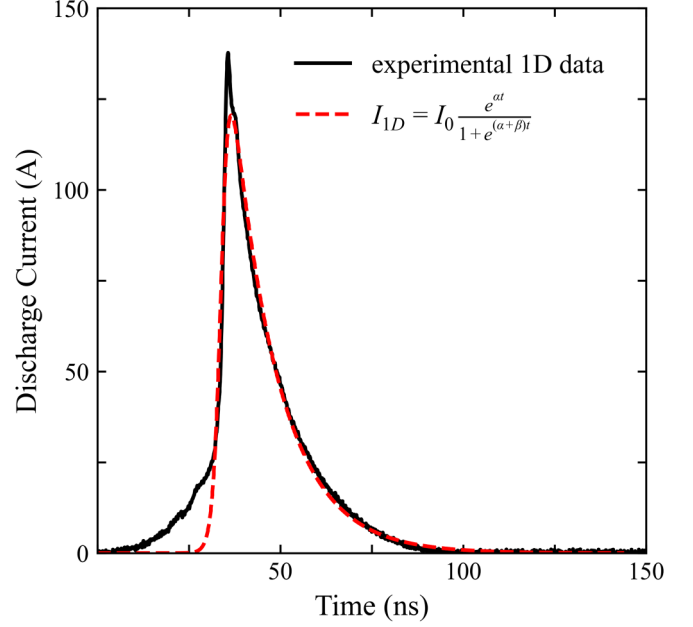


FIG. 3. Example one-dimensional discharge current waveform (black solid line) overlaid with its fitted model (red dashed line). The optimal fitted parameters for α , β , and I_0 are 0.84 ns^{-1} , 0.080 ns^{-1} , and 160 A , respectively. Note that the waveform is strictly positive since it is electron flow current not conventional current.

model these regimes as a 1D breakdown followed by a 2D, or planar, breakdown. The functional form of these two regimes are distinct, as the charge density distributions are different.

1. One-dimensional channel growth

The initial segment of the current waveforms is the 1D channel growth, which describes the current up to a time t_0 that is correlated to the peak current. We have empirically found that this one-dimensional current obeys a double exponential function of the form

$$I_{1D}(t) = I_0 \frac{e^{\alpha t}}{1 + e^{(\alpha+\beta)t}}, \quad (1)$$

where α is the reciprocal of the rise time constant and β is the reciprocal of the decay time constant. I_0 is proportional to the peak current at time $t = t_0$. This fit was applied to each of the 1D configuration measured waveforms, as well as the assumed 1D component of the 3D configuration measured waveforms.

This is the standard double exponential equation used by Refs. [51,62] and others to estimate the current profile in a compact intracloud discharge, which we will revisit when discussing the analogy to NBEs.

The expression given in Eq. (1) represents the experimental 1D waveforms produced by the 2.5-cm cubic samples with very good agreement ($R^2 > 0.95$). An example of the experimental one-dimensional current waveform and its fit to Eq. (1) is given in Fig. 3.

If we assume that the current produced during the one-dimensional discharge is a result of the motion of the discharge channel, where the discharge current is proportional to the speed of the streamer head at a given time, then it

is possible to estimate the head velocity during the channel growth via

$$I_{1D}(t) = \lambda \frac{dr(t)}{dt}, \quad (2)$$

where λ is the presumed linear charge density subtended by the channel during its 1D growth. The validity of this assumption is discussed under Sec. III C. Comparing Eqs. (1) and (2), we find

$$\frac{dr(t)}{dt} = \frac{I_0}{\lambda} \frac{e^{\alpha t}}{1 + e^{(\alpha+\beta)t}}. \quad (3)$$

We estimate λ as the total charge recovered (area under the current curve) for the 2.5-cm samples divided by the length of the discharge channel. Using this relationship, the average peak 1D velocities in the 2.5-cm cubic samples is $(1.12 \pm 0.03) \times 10^6$ m/s, which is in the expected range from prior work [14].

2. Two-dimensional channel growth

In addition to the current waveforms, we utilized the results from a high-speed imaging experiment [14,64,66] which captured the formation of the breakdown channels in real time inside the planar $30 \times 10 \times 2.5$ cm samples to determine the functional form to describe the breakdown channel growth along a uniform plane of charge. The cumulative length of the discharge channel as a function of time during breakdown was fit to the empirically derived function

$$r(t) = r_0 \frac{(t/\tau_{\text{growth}})^n}{1 + (t/\tau_{\text{growth}})^n}, \quad (4)$$

where r_0 is the total one-dimensional extent of the fully formed channel absent limiting boundary conditions, τ_{growth} is the characteristic time constant of this growth, and n is a constant that describes the slope of the radial expansion of the channels over time, referred to as the channel growth order parameter. The measured values of this function depend on the charge density in the plane and the enforced boundary conditions due to the geometry of the sample. For the samples used in the planar discharge geometry experiments [14], which had a maximum length of 30.5 cm, the fitted parameters of the growth curve are 551 ns, 36.2 cm, and 1.10 for τ_{growth} , r_0 , and n , respectively, which closely match the expected quantities extracted from the current waveforms from samples of this size and charge density. The average channel formation rate for the planar geometry samples and the resulting fit are shown in Fig. 4. It is important to note that as the channel length is cumulative, the final length of the channel is larger than the length of the sample, since the channel does not grow in a straight line but has some tortuosity. In addition, the last 50 ns of the data are influenced by the boundary conditions, as the channels are forced to stop ahead of the end of the sample, which causes the deviation of the data from the fitted curve starting at 400 ns.

We expect the discharge current produced by a circularly symmetric breakdown in a 2D plane of uniform charge density σ to obey the relationship

$$I_{2D}(t) = 2\pi\sigma r(t) \frac{dr(t)}{dt}, \quad (5)$$

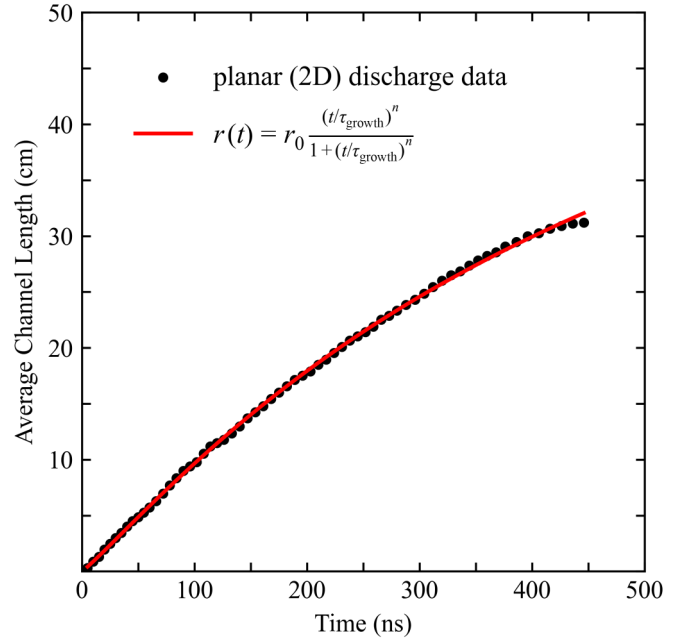


FIG. 4. Average cumulative channel length of a discharge plasma channel in electron-irradiated PMMA as a function of time during a planar geometry discharge. The experimental data (black points) were fit to the functional form (red line) of Eq. (4). The fitted parameters are 551 ns, 36.2 cm, and 1.10 for τ_{growth} , r_0 , and n , respectively. Note that the boundary effects are important in the last 50 ns, where the discharge channels are forced to terminate prematurely ahead of the physical end of the sample. This is the reason for the discrepancy between the expected and measured behavior. For further discussion of this experiment, see Ref. [14].

assuming all current is due to the motion of detrapped charge as the breakdown progresses [15,67]. This form also presumes that the discharge current is directly proportional to the channel growth with no dispersion or losses.

This form of $I_{2D}(t)$ using Eq. (4) for $r(t)$ was fit using a minimization of least-squares method to the two-dimensional part ($t > t_0$) of the measured discharge current waveforms of the 5-cm cubic samples to determine the optimal values of τ_{growth} , r_0 , and n .

3. Three-dimensional channel growth

The total expected current waveform for the 3D configuration is a superposition of the 1D and 2D channel growth components or

$$I_{\text{total}}(t) = \begin{cases} I_0 \frac{e^{\alpha t}}{1 + e^{(\alpha+\beta)t}} & \text{for } t < t_0 \\ I_0 \frac{e^{\alpha t}}{1 + e^{(\alpha+\beta)t}} + 2\pi\sigma n \frac{r^2(t-t_0)}{t-t_0} \left[1 - \frac{r(t-t_0)}{r_0}\right] & \text{for } t > t_0 \end{cases}. \quad (6)$$

The 1D channel growth component of the current waveforms Eq. (1) was fit to each experimentally obtained waveform from the 5-cm cubic samples up to the peak current. After the fitted optimal parameters for I_0 , α , and β were found, these values were input to Eq. (6), which was then fit to the entire current waveform to determine the optimal values of t_0 , r_0 , n , and τ_{growth} . The goodness of fit for every waveform was

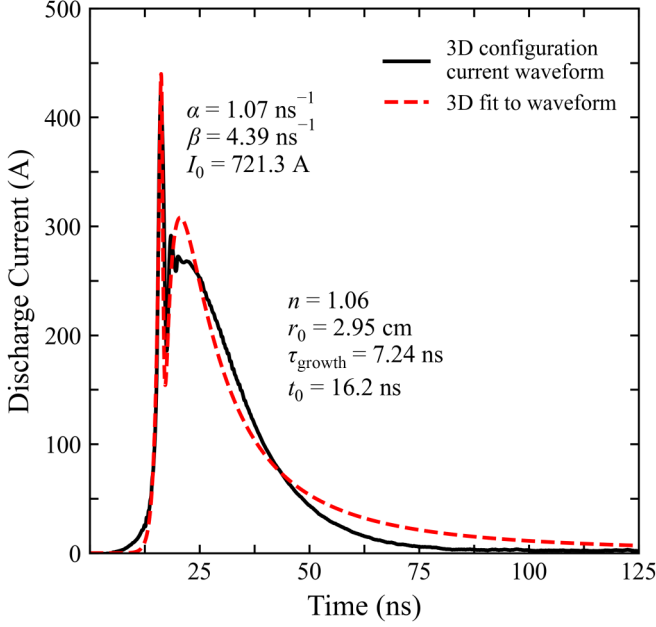


FIG. 5. Example discharge current waveform (black solid line) overlaid with its fitted model (red dashed line). The functional parameters that reproduce this waveform, for both the 1D and 2D components, are displayed. Note that the waveform is strictly positive since it is electron flow current not conventional current.

evaluated using a standard R^2 test. The value for the charge density σ was assumed to be the total charge recovered during discharge (area under the current waveform), which differs for every sample, divided by the area of the 2D plane of charge (25.81 cm^2).

The expected expression for the total current Eq. (6) fits the experimentally obtained waveforms very well, with 91% of the fitted waveforms yielding a R^2 value of greater than 0.95 and 98% yielding a R^2 greater than 0.94. An example waveform compared with its model output is shown in Fig. 5. In addition to good morphology agreement between the experimental waveforms and the model waveforms, the fitted parameters show the expected correlations with each other and the measured parameters from the modeled waveforms. Note that the waveform in Fig. 5 is that of one of the 5-cm cube geometry discharges, which accounts for the differences in time and length scales to those shown in Fig. 4.

B. Correlations between measured parameters

The resulting fitted parameters for both the 1D and 2D channel growth components of the 5-cm cubic waveforms were compared to other measurable quantities and tested for correlation using the Spearman rank correlation coefficient test [68]. The full results of these statistical tests are given in Appendix B in Tables II and III.

The recovered charge (total area under the current curve) is used as a metric of comparison for all samples across the three different beam currents (3, 6, and 9 mA) used in the 3D (5-cm cubic) sample preparation. Despite the fact that the charge leakage rates are different for samples of differing implanted charge densities [13], the total recovered charge still can be

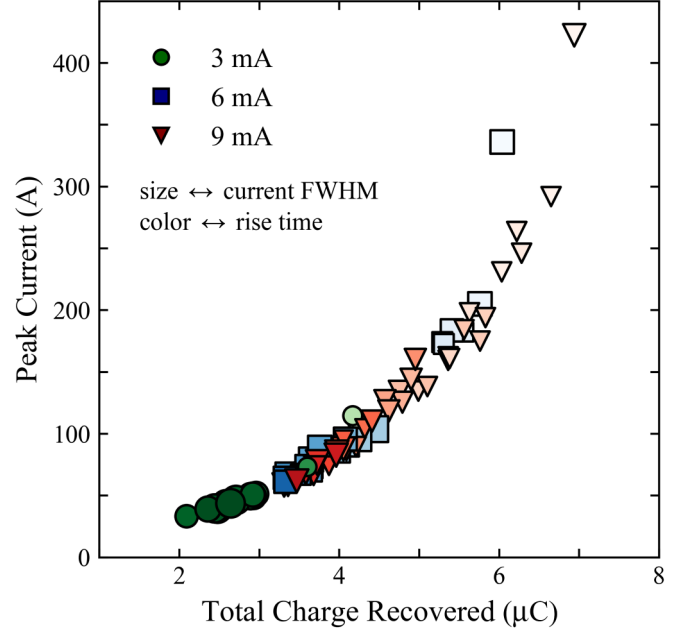


FIG. 6. Experimentally measured quantities extracted from the current waveform data for all 3D (5-cm cubic) samples across all three different incident beam currents. The green circles, blue squares, and red triangles represent data obtained from samples irradiated with 3-, 6-, and 9-mA beam fluence, respectively. The recovered charge is plotted against the peak current. The marker size is proportional to the full width at half maximum of the current waveform, and the darkness of the marker color corresponds to the 10–90% current rise time (a darker color indicates a longer rise time).

used to compare the measured parameters across all three incident beam currents. Figure 6 shows the peak discharge current as a function of the total recovered charge for samples irradiated with all three beam currents discharged at increasing elapsed times postirradiation to allow charge to leak from the sample. The total recovered charge can be seen to change in a consistent manner regardless of the incident beam current used. This indicates that the initial implanted charge density and beam fluence do not impact the measured waveform. The color and marker size correspond to the 10–90% of maximum rise time (darker color indicates a slower rise time) and the full width at half maximum of the current pulse (larger marker indicates a wider pulse), respectively.

The total recovered charge Q , the peak current I_{peak} , the 10–90% of maximum rise time t_{rise} , the full width at half maximum of the current t_{FWHM} , and the recovered charge up to the peak current $Q_{\text{to peak}}$ were each compared to one another to test for statistically significant correlation. All characteristic values were found to be strongly correlated ($|\rho| > 0.80$, where ρ is Spearman's rank correlation coefficient, to one another with high statistical significance ($p < 0.001$), except when compared to the recovered charge up to the peak current.

The recovered charge up to the peak current shows no statistically significant correlation to any other parameter. This implies that the charge subtended up to the peak current has high variability. This could be due to a changing linear charge density in this region, but how it is changing is unknown at present.

The general properties of the current waveforms obey the expected correlative relationships. A larger internal stored charge Q (the recovered charge) leads to a larger peak current I_{peak} , implying that charge is alleviated through the initial 1D breakdown channel at a faster rate for a larger potential gradient between the discharge point and the plane of peak implanted charge. Further, the width of the waveform t_{FWHM} and the rise time t_{rise} both increase as the internal charge decreases. As a result, we can infer that the maximum velocity at which the breakdown channels form decreases with decreasing implanted charge density.

The fitted values of the radial extent of the breakdown channels r_0 are positively correlated with the internal charge density or the total recovered charge during the ESD. This is expected, as there is a critical electric field threshold at the streamer head that must be met for a discharge channel to continue propagation. As the internal stored charge depletes, this condition will be broken at shorter radii of the circular breakdown. Thus, the radial extent r_0 is expected to decrease with decreasing recovered charge, which is what we observe.

In addition, we observe that the channel growth order parameter n decreases with increasing stored charge and r_0 , though the total spread in the values of n is small, ranging only from 1.0 to 1.72.

The fitted values of r_0 , n , and τ_{growth} of Eq. (4) allow for the channel propagation velocity $\frac{dr}{dt}$ to be calculated as a function of time throughout the 2D, or planar, breakdown. We find that the peak propagation velocity is positively correlated with the internal charge density (recovered charge) at the time of breakdown, along with the maximum radial extent r_0 . This is expected, as the speed and extent of breakdown is likely driven by the electric field at the channel head, which increases with the internal charge density. The calculated peak 2D velocity values are consistent with previous studies [14], with a maximum of 3.7×10^6 m/s and a minimum of 9.2×10^5 m/s. The average peak 2D velocity is $(1.8 \pm 0.7) \times 10^6$ m/s.

The relationships between measured and fitted parameters for the 5-cm cubic samples provide insight to how the initial 1D breakdown is influenced by the environment inside the sample during and prior to discharge. For example, the peak 1D velocity and average 1D velocity, calculated using Eq. (2) and the fit results to the 1D component of the 3D samples, have strong positive correlations to the total recovered charge and the peak current, and a strong negative correlation to the 10–90% rise time. This logically follows, as the initial 1D velocity is expected to increase with the internal electric field, which itself increases with internal charge. Additionally, as the peak current is proportional to the peak velocity by Eq. (2), these two parameters are expected to be positively correlated as observed. Further, a shorter rise time indicates a faster 1D breakdown, which is consistent with a higher 1D breakdown velocity over the same traversed distance. Figure 7 demonstrates these relationships.

The fitted parameters to the 1D component of the 5-cm cubic discharges also obey expected relationships, with the 1D growth inverse time constant α and the 1D decay inverse time constant β increasing with internal charge and peak current. This implies that the faster, higher current 1D breakdowns

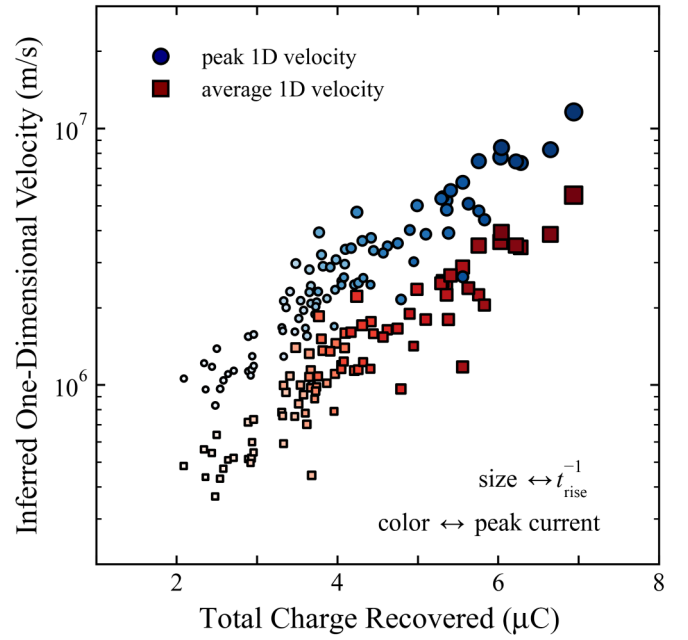


FIG. 7. Inferred velocity calculated using Eq. (2) and the 1D component of the fits to the 5-cm cubic sample waveforms as a function of the total charge recovered from the 5-cm cubic samples. The peak velocity is shown in the blue circles, and the average velocity throughout the 1D discharge is shown in the red squares. The size of the markers is proportional to the inverse of the 10–90% rise time (larger marker indicates faster rise time). The darkness of the marker color is proportional to the measured peak 1D current. Note the log scale on the y axis.

have shorter pulse widths and occur in the presence of a higher electric field, which is expected.

C. Assumptions used in model development

Several assumptions have been made to make this complicated problem of breakdown inside PMMA possible to model with a closed-form solution. For example, the peak currents in the 3D model waveforms are consistently an overestimate of the true, experimentally measured peak current. The second peak associated with the 2D discharge current component in the model is also consistently an overestimate of the true value. This may be attributed to the assumption of the form of the initial growth decay function after the time t_0 , which is estimated as an exponential decay. The overestimate of the peak current due to the superposition of the initial decay and the beginning of the 2D channel growth suggests that the actual decay occurs with a smaller time constant. This is not surprising, as the 1D current essentially “shuts off” once the channel reaches the plane of the peak charge because it immediately transitions to the 2D electrical tree expansion. However, an immediate cutoff of current is nonphysical, as there is some nonzero amount of time required for the disturbed charge to evacuate the material through the fully formed channel.

Another important simplification made in the construction of this model is the considered geometry of the discharge in both the 1D and 2D channel growth regimes. A strictly 1D

representation of the initial discharge channel growth during the breakdown for $t < t_0$ is not a true representation of experimental observations, as there are secondary channels that form off the primary channel in this region. However, the current carried by these channels is expected to be small compared to the current carried by the primary channel, so their contribution to the total initial current is assumed to be negligible.

Similarly, for the 2D channel growth, the model treats the area of peak charge implantation as a two-dimensional plane when it is in reality a skewed Gaussian-like profile [65]. However, due to the critical field required for channel propagation during discharge, the electrical tree that is formed in the so-called 2D plane has a small thickness compared to its width and can therefore be approximated as a 2D discharge. Additionally, it is known that not all of the implanted charge is evacuated from the sample during a discharge event, though this remaining charge is negligible compared to the charge recovered during the ESD event and can be ignored. This is also circumvented by assuming the 2D charge density is the recovered charge divided by the area.

Finally, the empirically derived form of the initial 1D channel current Eq. (1) is an exponential growth, though the mechanism underlying this observation is unclear at present. The charge distribution in the z direction of the 5-cm cubic samples is also not well understood. If the charge density in the region of the 1D channel growth is the driving mechanism for the channel growth, then it may make sense for the current to grow exponentially as an exponentially (or quasiexponentially) increasing amount of charge is detrapped. However, if the implanted charge density is the only source for the initial current, then some correlation between the recovered charge up to the maximum current to the total recovered charge would be expected. This is not what we see, leading us to assume that the current is proportional to the local electric field at the channel head, which drives the velocity of growth, and the increasing velocity along with the increasing available charge together create the measured effect. Alternatively, the charge recovered in the initial 1D channel growth could be a result of a different mechanism entirely, such as an electron avalanche, though this seems unlikely due to the required number density of free electrons in the medium to produce this phenomenon [69–72].

The lack of understanding of the mechanism behind the 1D channel growth and discharge current production also means that the velocity estimates of the 1D current growth likely have some inaccuracies, though the range of peak velocities falls within the expected bounds inferred from previous work [14]. For example, the discharge current is most likely not linearly proportional to the streamer velocity, as the linear charge density is not constant with the length of the channel. The true form of the breakdown channel velocity in relation to the measured discharge current is under investigation and requires further study.

However, despite these assumptions to simplify the model, the results are quite good. In general, the model fits the experimental waveforms for both the 2.5- and 5-cm cubic samples well, indicating that this simple model is valid for a simple, generalized understanding of the breakdown in solid, charged dielectrics.

TABLE I. Published electrical parameters from the arithmetic mean of nine measured NBE waveforms [52,73]. The values for h_0 , d , and Δh are used to calculate the electric field E . t_{rise} is the 10–90% rise time for the CID current, I_{peak} is the magnitude of the peak current, and E_{peak} at 100 km is the electric field amplitude calculated for $d = 100$ km using Eq. (9). We calculated the uncertainty on these mean values as the standard deviation divided by $\sqrt{N}$, where N is the number of events used to calculate the average.

Parameter	Published mean value [52]
h_0 (km)	14.5 ± 1.1
d (km)	44.7 ± 6.6
Δh (m)	129.2 ± 4.1
t_{rise} (μs)	2.9 ± 0.5
I_{peak} (kA)	150.2 ± 17.2
E_{peak} at 100 km (V/m)	23.3 ± 2.6

IV. APPLICABILITY TO COMPACT INTRACLOUD DISCHARGES

A. Electric field calculation

The presented model for the dielectric breakdown in electron-irradiated PMMA includes some features which can be extended to compact intracloud discharges. In this comparison, the 1D channel growth current model for a growing streamer discharge in a region between opposite polarity regions of space charge [Eq. (1)] can be extrapolated to a CID discharge oriented vertically occurring between two space-charged clouds. This scenario is well represented by the 1D discharges in the 2.5-cm cubic samples after the timescale, length scale, and electrical scale (current, charge transfer) are adjusted. The time on which the 1D dielectric-based discharge occurs is scaled to the proper CID timescale using the relationship

$$T_s = \frac{t_{\text{rise,CID}}}{t_{\text{rise,1D}}}, \quad (7)$$

where T_s is the time scaling factor, $t_{\text{rise,CID}}$ is the 10–90% rise time of the CID current waveform, and $t_{\text{rise,1D}}$ is the 10–90% rise time of the 1D dielectric breakdown discharge current waveform.

The current scaling factor I_s is given by

$$I_s = \frac{I_{\text{peak,CID}}}{I_{\text{peak,1D}}}, \quad (8)$$

where $I_{\text{peak,CID}}$ and $I_{\text{peak,1D}}$ are the peak currents for the CID discharge and PMMA-based 1D discharge, respectively.

The scaling factors were calculated for each 1D measured PMMA-based discharge current waveform using the values for t_{rise} and I_{peak} given in Table I. The electric field is calculated for the parameters in Table I and compared to measured NBE waveform characteristics.

The electric field radiated by the discharge channel carrying a current $I(t)$ can be calculated using the Hertzian dipole approximation [52], since the frequency content of the radiation is small enough such that $\lambda \gg \Delta h$, where Δh is the length of the dipole. The electric field for a vertically oriented

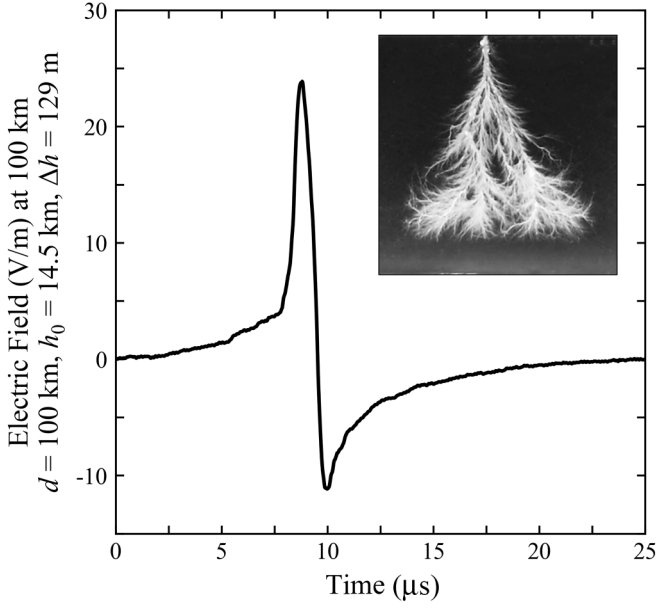


FIG. 8. Calculated electric field waveform using the current waveform for 1D PMMA-based discharge currents and the parameters in Table I. The electric field waveforms are calculated using the Hertzian dipole approximation of Eq. (9). The values used are $h_0 = 14.5$ km, $\Delta h = 129$ m, and $d = 100$ km and assume no dispersion or losses. The resulting peak field, pulse width, maximum to minimum ratio, and rise time agree well with established values [52,57,73]. The inset is a photograph of the discharged 2.5-cm cubic sample that produced this waveform. Note the even growth of the electrical tree to form a smooth, triangular envelope within the sample. The discharge growth (and conventional current) are directed downward in this photo.

Hertzian dipole is

$$E_z(d, t) = \frac{1}{2\pi\epsilon_0} \frac{(2h_0^2 - d^2)\Delta h}{R^5} \int_{t_0}^t I(t' - R/c) dt' + \frac{1}{2\pi\epsilon_0} \frac{(2h_0^2 - d^2)\Delta h}{cR^4} I(t - R/c) - \frac{1}{2\pi\epsilon_0} \frac{d^2\Delta h}{c^2R^3} \frac{dI(t - R/c)}{dt}, \quad (9)$$

where d is the horizontal distance to the observation point from the dipole, h_0 is the height of the dipole, Δh is the length of the dipole (length of the radiator), and $R \equiv \sqrt{d^2 + h_0^2}$ is the distance from the observation point to the dipole. The current $I(t - R/c)$ is the source current.

An example of the calculated electric field using Eq. (9) using the experimental discharge currents from the scaled PMMA-based 1D discharge and the values in Table I is shown in Fig. 8. The inset is a photograph of the 2.5-cm sample that produced the discharge current used to calculate the electric field. The electric field waveforms are calculated using $h_0 = 14.5$ km, $\Delta h = 129$ m, and $d = 100$ km and assume no dispersion or losses. The resulting average peak electric field between all samples is (27.1 ± 1.0) V/m at 100 km, which agrees with the published average peak field for these values of h_0 , Δh , and d within the uncertainties [52,73]. In addition,

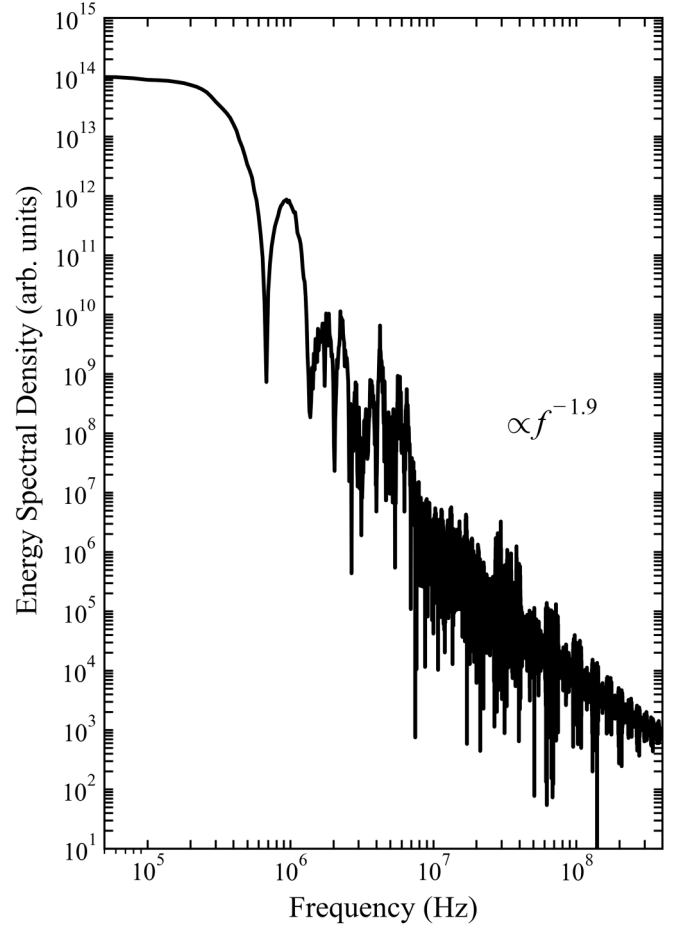


FIG. 9. Calculated energy spectral density in arbitrary units (arb. units) of the electric field waveform calculated using the current waveform for 1D PMMA-based discharge currents and the parameters in Table I. The corresponding electric field waveform is shown in Fig. 8. The spectral density looks as expected for a CID waveform [62], with a slow power-law decay in frequency with a power of -1.9 in the HF to VHF band of 3 to 300 MHz.

the ratio between the initial positive peak and the second negative peak is between 2 and 3, which is in good agreement with established results [57].

B. HF-VHF content of electric field waveforms

A key feature of CIDs is their HF to VHF radiation signatures. The radio spectra between different lightning discharges are distinct and used to categorize the types of discharges. In the case of NBEs, the energy spectral density demonstrates a slow power-law decay in frequency in the HF to VHF band (3–300 MHz) with a power between -2 and -1 [58,62], which is much slower than the decay of other lightning phenomena [74].

We calculated the energy spectral density of the calculated electric field waveforms for the scaled 1D discharges. The result for the waveform of Fig. 8 is shown in Fig. 9. We fit a power-law decay to the 3- to 300-MHz band and found that the frequency content decays as -1.9 in this region, which is consistent with measured NBE signatures, which radiate

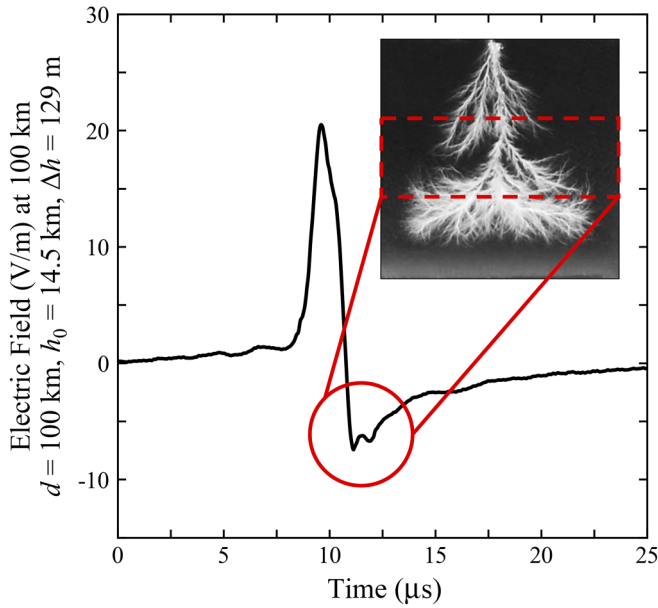


FIG. 10. Calculated electric field waveform using the current waveform for 1D PMMA-based discharge currents and the parameters in Table I. Note the fluctuations in the peaks of the waveform, which correspond to the bilayer structure of the 1D discharge in the sample that produced this waveform. The inset is a photograph of the discharged 2.5-cm cubic sample that produced this waveform. Note the uneven growth of the electrical tree to form bilayer structure. The gap in the discharge structure between the two layers (outlined in the red dashed rectangle) correspond to the disruptions in the waveform (outlined in the solid red circle).

strongly in the HF to VHF band as a defining feature of these events.

C. Fluctuations in electric field waveforms

It is common for measured NBE waveforms to exhibit smaller-scale irregular features [50,52,56,59,62,75], which have been attributed to bouncing wave phenomena [50,52,52] or as fluctuations in channel growth within a streamer burst [56,62]. We have observed similar fluctuations in the dielectric-based discharges that can be traced to the differences in channel formation and morphology inside the discharged samples. For example, the waveforms of Fig. 8 correspond to 1D samples in which the electrical tree left behind postdischarge, which is direct evidence of the streamer paths created during breakdown, shows a smooth, continuous growth as a function of depth (see inset photograph in Fig. 8). As a result, the field waveform is very smooth and continuous.

However, the waveform of Fig. 10 demonstrates some of the fluctuations in the trace that are frequently noticed in real NBE measurements. The PMMA sample that corresponds to this waveform displays an interesting bilayer structure in the electrical tree. We posit that the observed fluctuations in the waveform from this sample correspond to these heterogeneities of the breakdown channel growth. The distance between the two layers and the time between the two fluctuations in the waveform agree with the presumed one-dimensional average velocity of $(1.12 \pm 0.03) \times 10^6$ m/s in

the 2.5-cm cubic samples. The cause of the differences in channel growth between the two examples (Fig. 8 vs Fig. 10) is unclear at present, since both received identical irradiation and charging treatment, but it is likely due to differences in internal charge migration prior to discharge that caused slight discrepancies in the electric field at the streamer head, leading to differences in the growth rates.

The tunability of the charging of PMMA samples provides a convenient method to investigate the variations in the streamer growth as a function of internal charge distribution and the resulting current and electric field waveforms. The direct comparison between charge distribution and charging conditions and the waveforms of NBEs can potentially provide insight to the formation conditions within the thundercloud. Tuning the incident electron beam energy, sample geometry, sample temperature, and internal charge distribution leads to different current waveform morphologies, which can be extrapolated to the natural thundercloud environment.

D. Broader environmental and system considerations

Finally, the investigation into the 3D discharges in the 5-cm cubic samples may give insight to the influence of the background electric field due to charge layers in a thundercloud, as we have shown that certain aspects of the 1D discharge currents in the 5-cm cubic samples, such as the peak current and the rise time, are highly correlated to other aspects of the broader environment, such as the total charge stored and the distribution of charge inside of the sample. The manipulation of the charge distribution inside the PMMA samples and the measurement of the resulting current waveforms will be studied in the future to determine how the internal charge pattern can reproduce the finer structures observed in NBE signatures.

The discharges in the experimental work in this study occur on spatial scales of a few centimeters and in timescales of several nanoseconds. However, using only the model developed for the small-scale dielectric breakdown currents in PMMA along with the reported measured values for CID electrical parameters, we were able to reproduce current and electric field waveforms that represent the actual CID waveforms with good agreement.

The physical properties of the two systems considered here are very different. Besides the obvious difference in the spatial scale (centimeters vs kilometers) and the mass density of the materials (PMMA is approximately 3000 times the density of a thundercloud), the estimated dielectric constant of PMMA is approximately 10 times greater than that of a thundercloud, and the calculated charge density is up to 1500 times greater in the PMMA sample than in a thundercloud [12,76]. Even so, the agreement between the calculated and reported electrical parameters are strikingly similar.

V. CONCLUSIONS

A time-separated, discharge channel growth-based model for the discharge current produced during electrostatic discharge in electron-irradiated PMMA has been presented using empirical observations of PMMA-based ESDs. The model agrees well with experimental discharge current waveforms

obtained for 2.5-cm, room-temperature PMMA cubes demonstrating one-dimensional discharge. The model includes a distinction between 1D, 2D, and 3D electrical tree formation characteristics, which can be used to predict the discharge currents produced from the breakdown of an arbitrary internal charge distribution in PMMA. Further, the model discharge current waveform for an arbitrary internal charge distribution can be used to predict discharge channel growth dynamics, such as the velocity of propagation over time.

We have also investigated the relationship between the measured current waveforms and the system parameters utilizing the 5-cm cubic samples of different internal charge densities. Measured defining characteristics such as the peak current, rise time, and pulse width are strongly correlated to the internal charge density and distribution and thus the internal electric field.

The proposed model describes the empirical results for ESD events in small-scale PMMA samples, and we have shown that it can be applied to large-scale ESD phenomena in vastly different media. This model has a number of potential applications from informing efforts to mitigate unwanted ESD events in charged particle radiation environments to modeling difficult-to-study forms lightning. While this proposed model does not answer every existing question in CID research, it provides a promising avenue for laboratory-based investigations to natural ESD events. The potential for electron-irradiated materials and their accompanying ESDs to be used to investigate analogous phenomena happening on scales 100–10 000 times larger presents a rich and exciting field for further investigation.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: WAVEFORM CORRECTION

It was initially observed that for these discharge current measurements, the raw current waveform obtained does not return to zero at the end of each discharge event. Instead, the waveform crosses the x axis and reaches a peak of opposite polarity before slowly decaying exponentially back to zero. This phenomenon is attributable to the fact that we measured the discharge current using a self-integrating current probe, which cannot measure the dc or low frequency component of a signal. As a result, the low frequency component becomes distorted by the measurement device, resulting in an effect is known as “droop” [77].

The rate at which this effect occurs, called the “droop rate,” on an idealized square pulse is dependent on the time constant

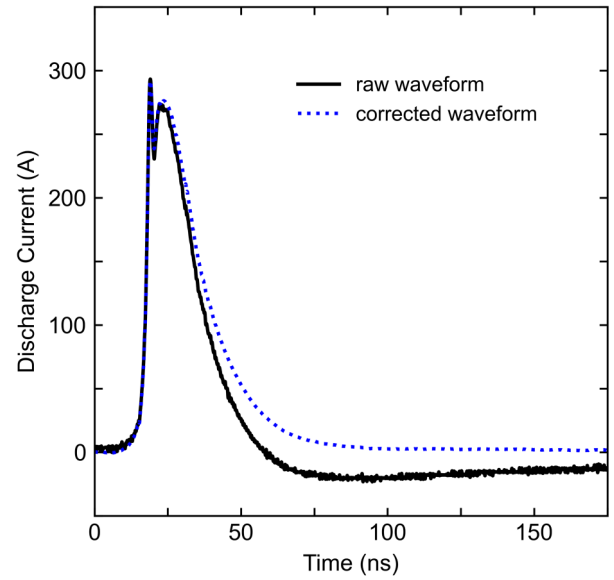


FIG. 11. Example of a raw waveform (black curve) and the droop-corrected waveform (blue dotted curve). Note that the droop-corrected waveform is strictly positive and represents the electron flow current.

of the integrating device. In a measurement that suffers from this effect, the measured voltage is given by

$$V_{\text{out}} = \frac{MI}{RC} - \frac{1}{RC} \int V_{\text{out}} dt, \quad (\text{A1})$$

where V_{out} is the measured voltage, M is the mutual inductance of the device, I is the current, and RC is the integrator time constant. For the current probe used in this work, the ratio M/RC is approximately equal to 1 in the operating band of 1 MHz to 1 GHz, giving the 1 V/A ratio mentioned previously. Using this equation, it is possible to correct for the

TABLE II. Spearman rank correlation coefficients between experimentally measured values extracted from the current waveform data for all 5-cm cubic samples across all three different incident beam currents, along with the correlations calculated for the model output of the same parameters. If there was no statistically significant correlation found, then the cell is filled with NC.

	Experiment			
	I_{peak}	t_{rise}	t_{FWHM}	$Q_{\text{to peak}}$
Q	+0.99	−0.95	−0.96	NC
I_{peak}		−0.96	−0.99	NC
t_{rise}			+0.93	NC
t_{FWHM}				NC
	Model			
	I_{peak}	t_{rise}	t_{FWHM}	$Q_{\text{to peak}}$
Q	+0.99	−0.86	−0.95	NC
I_{peak}		−0.84	−0.98	NC
t_{rise}			+0.83	NC
t_{FWHM}				NC

TABLE III. Spearman rank correlation coefficients for each pair of model parameters for all 5-cm cubic samples across all three different incident beam currents.

	α	β	r_0	τ_{growth}	n
I_0	+0.87	+0.92	−0.58	−0.80	−0.65
α		+0.95	+0.87	−0.65	−0.74
β			+0.79	−0.75	−0.81
r_0				NC	−0.70
τ_{growth}					+0.62

distortion at low frequencies by integrating the output voltage and dividing by the RC time constant of the probe, which is given by

$$RC = \frac{1}{2\pi f_{\text{cutoff}}}, \quad (\text{A2})$$

where f_{cutoff} is the −3 dB point of the current probe [77]. In this work, the RC time constant of the CLCE-1032 RF current probe was measured to be 166.5 ns. This correction was applied to every waveform prior to further analysis. Figure 11 shows a precorrected waveform and a postcorrection waveform.

APPENDIX B: RESULTS OF STATISTICAL TESTS

The Spearman's rank correlation coefficients for every pair of measured parameters from the experimental waveforms and modeled waveforms are shown in Table II. The value of ρ can be positive or negative for positive and negative correlations, respectively. If there was no statistically significant correlation ($|\rho| < 0.25$ or $p > 0.001$), then the cell is marked with NC. Table III shows the correlation coefficients for each pair of model fitting parameters, all of which show the expected behavior.

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