

Reversal of tracer advection and Hall drift in an interacting chiral fluid

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Chiral fluids are defined by broken mirror or time-reversal symmetry, giving rise to tensorial transport coefficients with antisymmetric components. A key example is the odd mobility tensor, which governs the response of a chiral tracer to an applied force and induces a characteristic transverse drift. While this response is well understood in the infinite dilution limit, the impact of interparticle interactions on the tracer dynamics remains largely unexplored. Here, we conduct an analytical and computational study of a chiral fluid with interparticle interactions and show that, under an external driving force, a chiral tracer can undergo a complete reversal of both its transverse Hall drift and its advection along the force. This reversal emerges from the interplay between odd mobility and interaction-mediated forces, resulting in a phenomenon reminiscent of absolute negative mobility.

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Chiral fluids have garnered significant attention for their unconventional transport properties, with no counterparts in conventional fluids. These systems exhibit striking effects, such as edge flows and optimal transport occurring at boundaries [1–4] and interactions—even purely repulsive ones—enhancing rather than suppressing motion [5–8]. The origin of these anomalous behaviors lies in the broken mirror or time-reversal symmetry inherent to chiral fluids. These unusual transport properties manifest themselves across a diverse range of systems, from condensed matter settings like magnetic skyrmion structures [9–12] and externally driven colloidal or macroscopic assemblies [13–17] to biological systems featuring self-propelled chiral agents such as *Escherichia coli* [18–20], *Thiovulum majus* [21], *Paenibacillus vortex* [22], or swimming starfish embryos [23]. Further examples include vortex fluids [24,25] and even granular systems such as strongly rotating or magnetized plasmas [26,27].

A unifying framework has recently emerged to describe the transport properties of these diverse chiral systems using so-called *odd* transport tensors. At the continuum level, chiral fluids are effectively characterized by concepts such as odd viscosity [28–33], while on the discrete agent-based scale, their dynamics are described in terms of odd mobility [34,35] and odd diffusion [5,36,37]. The defining feature of

odd transport coefficients is their tensorial structure, which includes antisymmetric off-diagonal elements that directly arise from the fundamental symmetry principles of two-dimensional rotation [38] and encode the chiral nature of the fluid. Specifically, the odd mobility tensor is given by [35]

$$\boldsymbol{\mu} = \mu_0(\mathbf{1} + \kappa\boldsymbol{\epsilon}), \quad (1)$$

where $\mathbf{1}$ denotes the identity tensor and $\boldsymbol{\epsilon}$ represents the totally antisymmetric Levi-Civita symbol in two dimensions. The mobility tensor relates the response velocity $\mathbf{v}$ of a tracer to an applied force $\mathbf{f}$ through $\mathbf{v} = \boldsymbol{\mu} \mathbf{f}$. In Eq. (1) μ_0 is the bare mobility with dimensions $[\mu_0] = \text{s/kg}$ and κ is the dimensionless characteristic *odd-mobility* coefficient. While in skyrmionic systems [40,41] or self-spinning biological organisms [42], which are inherently chiral, odd mobility is an intrinsic property, it can also be an emergent phenomenon such as for particles immersed in an odd-viscous fluid [35,43] or particles subjected to an external field, as for paramagnetic colloids in an externally rotating magnetic field [44] or Brownian particles under a Lorentz force [45]. In our approach, we treat odd mobility as an effective transport coefficient that captures the essential chiral dynamics across these diverse systems, making our results broadly applicable to any system exhibiting odd mobility, regardless of its microscopic origin.

In the absence of interactions, the response of a chiral tracer to an applied force follows directly from the odd mobility tensor. For a drift force $\mathbf{f} = f\hat{\mathbf{e}}_x$, the tracer moves with velocity $\mathbf{v} = \mu_0 f(1, -\kappa)^T$, exhibiting a characteristic component transverse to the driving force. This motion, commonly associated with effects such as Hall viscosity [46–48] or lift forces in chiral flows [49], is an intrinsic feature of odd mobility. In the biological context, a transverse drift even is known as *bacterial* or *sperm rheotaxis* [50–52], in the context of skyrmion motion as the *skyrmion Hall effect* [40,41]. While

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this behavior is well understood in the infinite dilution limit, the impact of interparticle interactions on the tracer's drift in a chiral fluid remains an open problem. In this work, we systematically investigate this problem using analytical theory and simulations. By incorporating interactions within a chiral fluid, we reveal that a driven tracer can undergo a complete reversal of both its intrinsic Hall drift and its advection along the direction of the force—a striking effect reminiscent of absolute negative mobility [53,54]. In particular, we show that these effects emerge due to the interplay of interparticle interactions and their odd mobility, which govern how the driving force is mediated through the fluid.

Model. We model the chiral fluid as a system of $N + 1$ interacting Brownian particles in two spatial dimensions. The chirality is encoded in the odd mobility μ_i for particle $i \in \{0, \dots, N\}$. A complete description of the dynamics requires specifying the diffusion of the chiral particles as well. We consider two scenarios: (i) a *strong fluctuation-dissipation relation (FDR)* where $\mathbf{D}_i = k_B T \mu_i$, which ensures equilibrium steady states [37], and (ii) a *weak FDR* where only $D_{0,i} = k_B T \mu_{0,i}$, permitting nonequilibrium steady states [7]. $k_B T$ here is the thermal energy. The effects we report arise from odd mobility and occur under both FDR conditions. For analytical simplicity and to focus on interaction effects in a well-defined equilibrium background, we assume a strong FDR throughout the main analysis. The weak FDR case, demonstrating that odd mobility alone drives our findings, is analyzed in the Supplemental Material (SM) [55]. Each particle is subjected to a constant external drift $\mathbf{f}_i$, such that the overdamped Langevin dynamics become

$$\dot{\mathbf{r}}_i(t) = \mu_i \mathbf{f}_i - \mu_i \sum_{j \neq i} \nabla_i U(\mathbf{r}_i, \mathbf{r}_j) + \boldsymbol{\eta}_i(t), \quad (2)$$

where U is the two-body interaction potential, giving rise to the interaction force $-\sum_{j \neq i} \nabla_i U(\mathbf{r}_i, \mathbf{r}_j)$ on particle i . Note that the response to both the external drift and the interaction force has transverse components encoded in μ_i . $\boldsymbol{\eta}_i(t)$ is the driving Gaussian but nonwhite noise with mean $\langle \boldsymbol{\eta}_i(t) \rangle = \mathbf{0}$ and correlation $\langle \boldsymbol{\eta}_i(t) \otimes \boldsymbol{\eta}_j^T(t') \rangle = \mathbf{D}_i \delta_{ij} \delta_+(t - t') + \mathbf{D}_i^T \delta_{-}(t - t') \delta_{ij}$ [45], where $\mathbf{D}_i = D_{0,i}(\mathbf{1} + \kappa_i \boldsymbol{\epsilon})$ is the odd diffusion of particle i that obeys strong FDR $\mathbf{D}_i = k_B T \mu_i$. Here $\delta_{\pm}(\cdot)$ represent variants of the Dirac delta distribution defined on the half axis of the real numbers, $\mathbb{R}_{\pm}$, respectively [73]. By tagging one particle ($i = 0$), the tracer particle, we coarse grain Eq. (2) over the interactions with the host particles in the limit of low density $\phi \ll 1$ (see SM [55] for details). We adapt a geometric method in the coarsening [76,77]. This allows us to analytically treat the oddness-affected probability fluxes, originating in the modified repulsive (so-called *oblique* [78]) boundary conditions introduced by the sterically interacting particles. The drift-diffusion equation for the one-body probability density function (PDF) $p_t(\mathbf{x}, t)$ to find the tracer in the interval $[\mathbf{x}, \mathbf{x} + d\mathbf{x}]$ at time t becomes

$$\frac{\partial p_t(\mathbf{x}, t)}{\partial t} = \nabla \cdot [D \nabla - \mathbf{v}] p_t(\mathbf{x}, t). \quad (3)$$

Here $D = \text{diag}(\mathbf{D}_t[\mathbf{1} - 4\phi \mathbf{\Gamma} \mathbf{D}_h])$ is the effective diffusion coefficient, where $\mathbf{\Gamma} = (\mathbf{D}_t + \mathbf{D}_h)/\det(\mathbf{D}_t + \mathbf{D}_h)$. The tracer

drift $\mathbf{v}$ is given by

$$\mathbf{v} = \mu_{\text{dir}} \mathbf{f}_t + \mu_{\text{indir}} \mathbf{f}_h, \quad (4)$$

where the mobilities

$$\mu_{\text{dir}} = \mu_t - 4\phi \mathbf{D}_t \mathbf{\Gamma} \mu_t, \quad (5)$$

$$\mu_{\text{indir}} = 4\phi \mathbf{D}_t \mathbf{\Gamma} \mu_h \quad (6)$$

are referred to as the *direct* and *indirect* mobility of the tracer particle, respectively. Here subscripts t and h refer to tracer and host particles, respectively.

Both μ_{dir} and μ_{indir} are influenced by interactions between the tracer and host particles. In an ordinary, nonchiral fluid, these mobilities correspond to distinct physical scenarios; when only the tracer particle is driven ($\mathbf{f}_h = \mathbf{0}$), μ_{dir} describes how the tracer advection is hindered by continuous collisions with the host particles at rest. Conversely, when the tracer remains at rest ($\mathbf{f}_t = \mathbf{0}$), while the host particles are driven, μ_{indir} quantifies how the tracer is advected by the host particle stream, an effect mediated by interparticle interactions. In both cases, the tracer drifts in the direction of the applied force [79] (see also the SM [55]), a trend seen at all densities of the host particles. This intuitive behavior is reflected in the positivity of both transport coefficients—direct and indirect mobility. However, this is fundamentally different in a chiral fluid. We demonstrate that in a chiral fluid with interacting particles these mobilities can turn negative leading to reversed tracer drift.

Results. We model the chiral fluid by fixing $\mu_t = \mu_h$ and $\mathbf{D}_t = \mathbf{D}_h$ and in particular $\kappa_t = \kappa_h \equiv \kappa \neq 0$. Despite this simplification, the model remains broadly applicable to describe systems with distinct particle responses, such as driven binary colloidal systems [80–82] and particularly tracer particles in chiral fluids [3,6,34,35]. We first consider the case where the tracer particle and host particles are subjected to dragging forces parallel to each other along a direction, which we chose as the x direction, i.e., $\mathbf{f}_t = f_t \hat{\mathbf{e}}_x$ and $\mathbf{f}_h = f_h \hat{\mathbf{e}}_x$. As model parameters, we vary the magnitudes of the dragging forces. The results are shown for a chosen f_t and the ratio $\tilde{f} = f_h/f_t$. The drift of the tracer is decomposed into its components along and perpendicular to the tracer force, which are denoted by $v_{\parallel}$ and $v_{\perp}$, respectively.

Consider the scenario in which $f_t, f_h > 0$, i.e., the chiral fluid and the tracer are dragged in the same direction. In contrast to the normal fluid, in Fig. 1 the chiral tracer exhibits highly counterintuitive dynamics: for sufficiently large densities, the tracer's drift velocity along the applied force reverses direction despite $\tilde{f} > 0$. The tracer drift becomes negative when the oddness exceeds a threshold $\kappa > \kappa_{\text{rev}}^{\parallel}(\tilde{f}, \phi)$ that depends both on $\tilde{f}$ and ϕ , which need to obey $\tilde{f} > 1 + 1/(6\phi)$ (for analytical details, see SM [55]). The latter gives rise to the state diagram for negative tracer drift shown in Fig. 1(b) and we test the predictions against Brownian dynamics simulation results (see SM [55] for details on simulations) for $\tilde{f} = 3.77$ in Fig. 1(a). We observe three distinct regimes. At lower densities ($\phi = 0.04$), the tracer exhibits ordinary drift, consistent with the applied force. At higher densities ($\phi = 0.08$), in contrast, the chiral tracer exhibits a negative drift ($v_{\parallel} < 0$). At the threshold density $\phi = 0.06$, the theory predicts an asymptotic negative drift as $\kappa \rightarrow \infty$. While the theory and simulations are

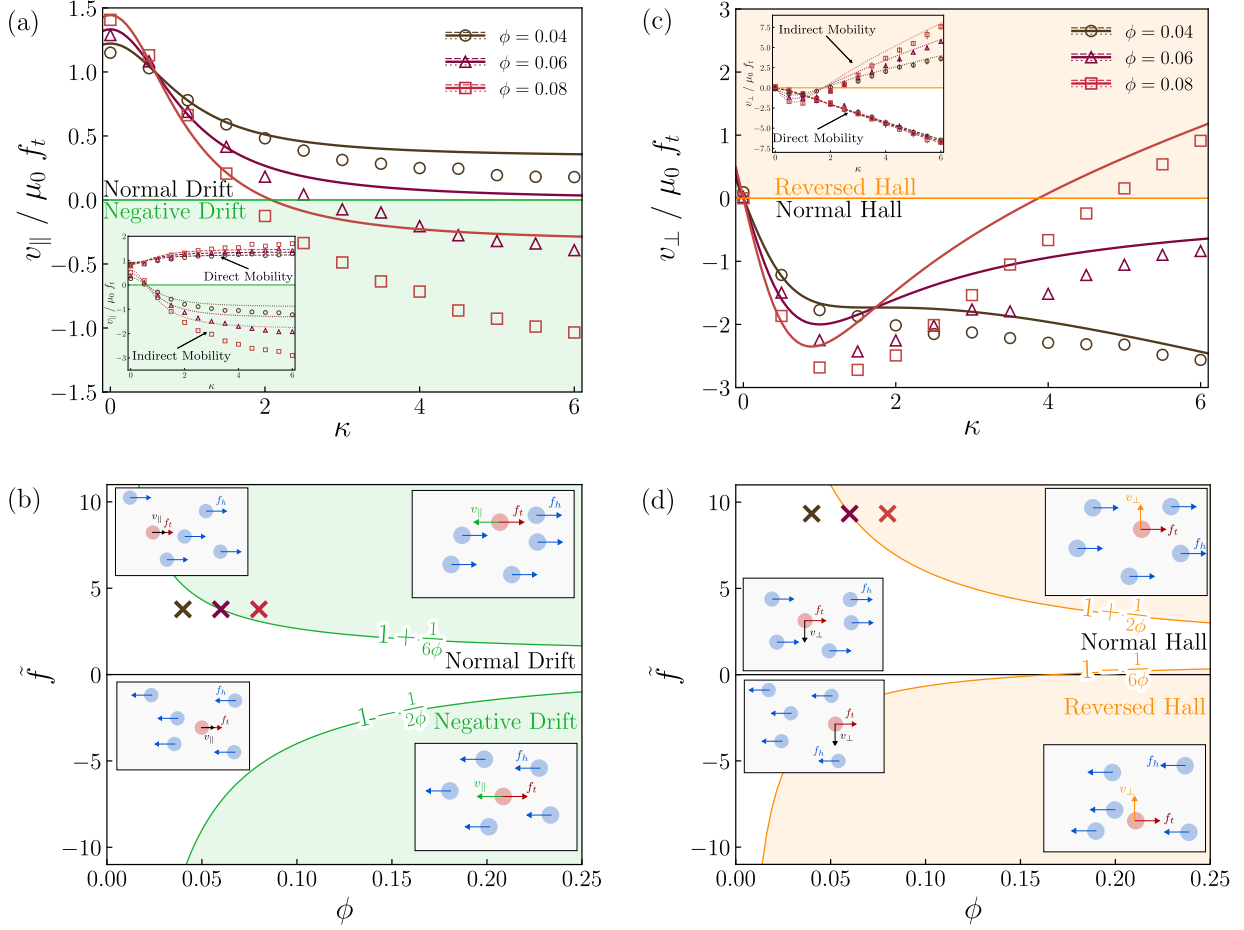


FIG. 1. *Reversal of tracer drift.* Components of the tracer drift $\mathbf{v} = (v_{\parallel}, v_{\perp})$, Eq. (4) (solid lines), of a driven chiral tracer in a driven chiral fluid. Symbols represent data from Brownian dynamics simulations performed in the linear response regime with small absolute force magnitudes; for details see SM [55]. Error bars are smaller than the symbols. Tracer drift $\mathbf{f}_t = f_t \hat{\mathbf{e}}_x$ and host drift $\mathbf{f}_h = f_h \hat{\mathbf{e}}_x$ are taken to be parallel; the ratio $\tilde{f} = f_h/f_t$ of magnitudes serves as a control parameter. (a) Tracer drift along the drift force $v_{\parallel}$ as a function of odd mobility κ for $\tilde{f} = 3.77$. The densities $\phi = 0.04, 0.06, 0.08$ correspond to the three regions of the state diagram (b) of normal drift, asymptotically negative drift, and negative drift (indicated by crosses). (c) Hall drift of the tracer $v_{\perp}$ as a function of odd mobility κ for $\tilde{f} = 9.33$. The densities $\phi = 0.04, 0.06, 0.08$ correspond to the three regions of the state diagram (d) of ordinary Hall drift, asymptotic reversal, and reversal of Hall drift (indicated by crosses). Insets in (a) and (c) show the contribution of μ_{dir} (dashed lines) and μ_{indir} (dotted lines) according to Eqs. (5) and (6) to the negative drift and reversed Hall drift of the tracer. μ_{dir} and μ_{indir} were measured in an auxiliary system with identical parameters but switched off host drift or tracer drift according to Eq. (4). Sketches in (b) and (d) illustrate the tracer's drift in the respective region.

in good agreement for small κ ($\kappa \lesssim 2$), the measured effect is in fact much stronger than the theory predicts; cf. Fig. 1(a). Deviations that occur at larger κ might be due to nonlinear effects outside the linear response regime and are discussed in SM [55]. Whereas both drift reversals become pronounced in these large κ regimes ($\kappa \gtrsim 2$, for the densities shown), the state diagrams [Figs. 1(b) and 1(d)] also indicate that the threshold κ_{rev} depends sensitively on both density ϕ and force ratio $\tilde{f}$, suggesting that the effect may become accessible at lower κ at different parameter regimes. Driven skyrmion systems, that obey the strong FDR condition assumed here [5], experimentally report values $\kappa \lesssim 0.6$ [40,41]. The experiments on bacterial rheotaxis indicate values of $\kappa \lesssim 0.3$ for *Bacillus subtilis* [50] and $\kappa \lesssim 0.6$ for *Escherichia Coli* [52], but strongly depend on details such as the flow profile. Ongoing experiments indicate that these values are substantially larger in (macroscopic) HexbugTM systems [83].

To investigate the origin of the negative drift, we separately analyze the effects of direct and indirect mobility in Eq. (4) by either switching off the host or tracer drift as shown in the inset of Fig. 1(a). The direct mobility, which remains positive for all κ , describes the normal drift response of the tracer in that it describes how the motion of a tracer is impeded due to interactions with host particles. The indirect mobility, in contrast, captures the enhancement of dynamics due to interactions with the chiral fluid. In other words, it is the response of the tracer to the driving of the host particles that ultimately leads to a reversal of the tracer's drift. The theory predicts a second regime with negative drift [$\tilde{f} < 1 - 1/(2\phi)$, i.e., $\tilde{f} < 0$ in the dilute region]; see Fig. 1(b). Here, the tracer and host are dragged in opposite directions and thus a reversal of tracer drift due to interactions might appear less surprising. More intriguingly, however, at sufficiently high oddness [$\kappa > \kappa_{\text{rev}}(\tilde{f}, \phi)$] the tracer overcomes the transferred effect of

interactions and again follows the dragging force, despite the opposing host-particle stream.

We now focus on $v_{\perp}$, the tracer's drift velocity perpendicular to the applied force. From Eq. (4), we find that $v_{\perp} \propto -\kappa$ in the absence of interactions, which corresponds to the characteristic Hall drift of a chiral tracer. In chiral fluids, interactions with the host particles can lead to a similarly counterintuitive effect as for $v_{\parallel}$ and reverse the direction of $v_{\perp}$ despite $\tilde{f} > 0$ and $\mathbf{f}_t \parallel \mathbf{f}_h$. This occurs if the chirality exceeds a threshold $\kappa > \kappa_{\text{rev}}^{\perp}(\tilde{f}, \phi)$ that depends both on $\tilde{f}$ and ϕ , which have to obey the relation $\tilde{f} > 1 + 1/(2\phi)$ (for analytical details, see SM [55]). The latter again gives rise to a state diagram for reversed Hall drift, shown in Fig. 1(d). In Fig. 1(c) we demonstrate that for the chosen value $\tilde{f} = 9.33$, in a system with sufficiently high density, the chiral tracer reverses its intrinsic Hall drift. Again three distinct regimes are found; at low densities ($\phi = 0.04$), the tracer particle exhibits a Hall-like drift, $v_{\perp} \propto -\kappa$. However, at high densities ($\phi = 0.08$), the Hall drift becomes nonmonotonic as a function of κ , ultimately resulting in a reversal of the Hall drift ($v_{\perp} > 0$). These regimes are separated by an asymptotically reversed Hall drift ($\phi = 0.06$). Analogously to the negative drift, the reversed Hall effect originates in the indirect mobility, as can be seen in the inset in Fig. 1(c). Similar to the negative drift, the reversal of the Hall drift, thus, is induced by the indirect response of the tracer to the host driving in the dilute limit. In fact, as evident from both state diagrams, Figs. 1(b) and 1(d), a reversal of the Hall drift necessarily accompanies a negative drift within the current setup and cannot occur independently.

The simultaneous reversal of both drift components is a direct consequence of the chosen setup, where the dragging forces are strictly parallel. A more general scenario employs nonparallel drift forces, which have been thoroughly investigated, for example, in driven colloidal systems [84,85], in streamline flows [86], or in pedestrian dynamics [87–89]. With nonparallel forces, a broader range of tracer responses exists, allowing for control over its motion in all directions. For instance, Fig. 2 presents the state diagram and simulation results for a scenario in which $\mathbf{f}_t$ is perpendicular to $\mathbf{f}_h$. In this case, the reversed Hall drift does not necessarily imply a negative drift and both effects can be independently realized (see SM for details [55]). The parallel and perpendicular dragging cases illustrate the essential nontrivial behavior of chiral fluids and their tunability. However, our theory provides a general prediction for an arbitrary angle between the tracer and host forces. This fully general response is analyzed in SM [55].

Discussion. We demonstrated that the transport properties of a tracer in a chiral fluid exhibit rich and nontrivial behavior, strongly shaped by interparticle interactions with the host fluid. Driven chiral tracers can undergo a reversal of their intrinsic Hall drift and experience negative drift, emerging from the subtle interplay of chirality, particle density, and drift magnitude.

Our findings have significant theoretical implications for understanding transport in chiral fluids and providing a foundation for future experimental investigations. While our analytical results are derived in the dilute limit, they establish fundamental principles that may guide the study of more complex, higher-density systems [90]. These insights are potentially relevant for tracer dynamics in skyrmionic

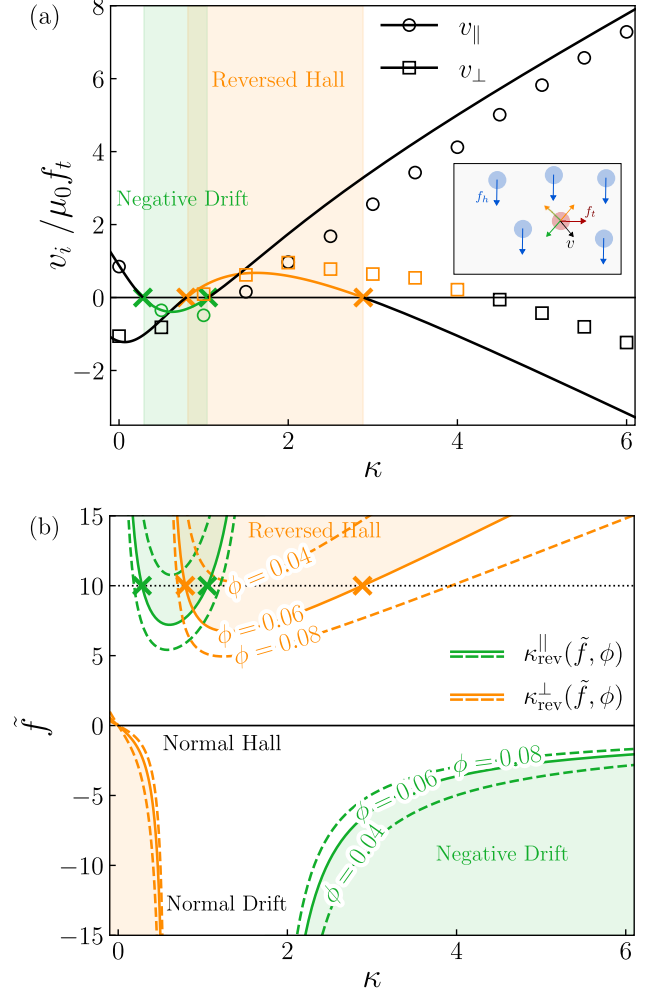


FIG. 2. *Orthogonal drift forces.* (a) Components $v_i/\mu_0 f_t$ of the tracer drift $\mathbf{v} = (v_{\parallel}, v_{\perp})$ from Eq. (4) (solid lines), as a function of odd mobility κ for the case of tracer drift $\mathbf{f}_t = f_t \hat{\mathbf{e}}_x$ perpendicular to host drift $\mathbf{f}_h = -f_h \hat{\mathbf{e}}_y$ in a system with density $\phi = 0.06$. Symbols represent data from Brownian dynamics simulations. Error bars are smaller than the symbols. The ratio of magnitudes is chosen as $\tilde{f} = f_h/f_t = 10$ and is indicated in the theoretical state diagram (b) by the dotted line. The state diagram shows that for different densities $\phi = 0.04, 0.06, 0.08$ the reversed Hall drift (orange) and negative drift (green) can be realized independently or simultaneously. Crosses indicate the theoretically predicted on- and offset of reversed drifts and agree qualitatively well with simulations. Note that the choices $\tilde{f} > 0$ and $\alpha = \arccos(\mathbf{f}_t \cdot \mathbf{f}_h) = -\pi/2$ are equivalent to $\tilde{f} < 0$ and $\alpha = \pi/2$ in (b).

systems [91,92]. For biological suspensions [20,21,50–52], our results apply most directly to low-density regimes or individual tracer studies, though the general principles may inform understanding of denser systems where additional many-body effects become important. Moreover, these results open up new research perspectives on how chirality influences collective behaviors, such as laning transitions in mixtures of oppositely driven species [80–82]. In these systems, spontaneous demixing occurs, leading to the formation of lanes with opposite flow.

The drift asymmetry central to our predictions ($\tilde{f} = f_h/f_t \neq 1$) can be realized experimentally through vari-

ous mechanisms. While we present results in terms of force ratios for analytical clarity, the essential requirement is the existence of different drift velocities between tracer and host particles, governed by $\mu_{0,h}f_h/\mu_{0,t}f_t$. This can be achieved even under identical external forces through differences in bare mobility coefficients, for example, due to variations in particle size, shape, or density leading to different Stokes friction coefficients. Such conditions naturally arise in sedimentation experiments of chiral tracers [93,94] involving particles of different buoyancy [95–97], where gravitational forces are identical but friction differences create distinct sedimentation speeds. Additionally, synthetic bilayer antiferromagnet systems offer promising experimental platforms, where skyrmions in coupled layers can exhibit distinct drift properties [98,99]. Optical or magnetic manipulation techniques can also create controlled drift asymmetries between different particle species in colloidal systems.

Finally, we remark on the intriguing resemblance between absolute negative mobility [53,54] and the negative drift observed in our system. This offers a fascinating example of a chiral fluid under flow acting as an engine, where the tracer, drifting opposite to the applied force, can perform work [100]. The fact that this phenomenon can already occur in a passive system challenges conventional perspectives on nonequilibrium transport and highlights the rich, unexplored dynamics of chiral fluids.

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Data availability. The data that support the findings of this article are openly available [101].

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