

Comment on “Instabilities of rotating inclined buoyancy layers”

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This comment addresses the article “Instabilities of rotating inclined buoyancy layers” by Xiao *et al.* [*Phys. Rev. E* **108**, 025102 (2023)], which presents linear stability results of Prandtl’s slope flow subjected to surface cooling and the additional effect of rotation. Our main point of contention concerns the steady base flow used in their work, which the authors claim fulfills all physically plausible boundary conditions, including vanishing temperature difference and upslope velocity at infinite height. While the analytical base flow solution provided in their study satisfies the flow equations and boundary conditions of the standard Prandtl model in a mathematical sense, it suffers from a major physical inadequacy that greatly restricts its applicability to real flows, previously highlighted in literature. This restriction is due to the imposition of an additional (that is, not included in the original Prandtl model) upslope wind forcing term representing an upslope perturbation pressure gradient force. To obtain a steady state, this term must be artificially linked to the slope angle, surface buoyancy, and other governing parameters. Moreover, this term must be imposed externally and becomes infinite for vanishing slope angle, both conditions that are physically unjustifiable. As a second point, we contest the authors’ classification of the traveling waves in their study as Tollmien-Schlichting modes, which are characterized by the destabilizing effect of viscosity. Instead, we assert that the nature of the traveling waves—also present in nonrotating Prandtl slope flows as shown in previous literature—are inviscid and should be attributed to an inertial-type Kelvin-Helmholtz instability, amplified by the infinite number of inflection points in the analytical base flow profile.

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I. INTRODUCTION

Xiao *et al.* [1] examine down-slope flows over a cooled inclined plate in a stably stratified rotating fluid, modeling katabatic winds at high latitudes. Their linear stability analyses identifies five unstable modes for fixed temperature and isoflux boundary conditions: stationary longitudinal roll (LR), oblique roll with low (OR-1) and high (OR-2) streamwise wave numbers, and Tollmien-Schlichting (TS) waves with low (TS-1) and high (TS-2) streamwise wave numbers. The Coriolis effect induces three-dimensional critical modes and increases the instability of TS wave modes with greater tilt angles and stronger rotation. OR-1 and OR-2 modes are most unstable at low and high tilt angles, respectively, while TS-1 dominates when the plate is nearly vertical. Spanwise phase

velocities of TS waves vary with tilt angles, except for TS-1 under fixed temperature conditions.

In this Comment, we raise two major points regarding the results presented in Ref. [1]:

(1) The steady base flow in the study is unphysical since it requires an artificial perturbation pressure gradient¹ throughout the depth of the atmosphere (and the cross-slope wind far above the slope) that depends on the surface temperature deficit as well as the slope angle, without any physical justification. Even more problematically, this stipulated perturbation pressure gradient would blow up in the limit of vanishing slope angles.

We dispute the identification of the instability modes as Tollmien-Schlichting (TS) waves. These traveling waves originate from a Kelvin-Helmholtz (KH) instability.

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¹By pressure (or temperature) perturbation, we exclusively refer to finite, one-dimensional deviations from the background pressure (temperature) field, consistent with the convention adopted in Ref. [1].

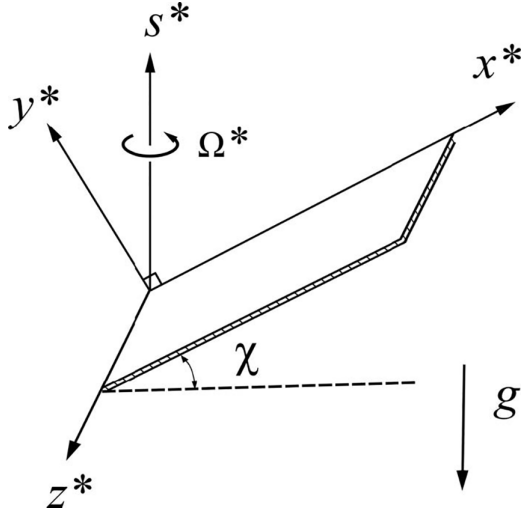


FIG. 1. Coordinate system reproduced from Fig. 1 in Ref. [1].

II. FORMULATION AND STEADY BASE FLOW

To explain our first point we consider the same model for stably stratified slope flows including the Coriolis force as adopted in Ref. [1], using the same governing equations, boundary conditions, coordinate system, and notation. The slope is assumed to be uniformly cooled and of infinite extent. The slope-aligned, right-handed, Cartesian dimensional coordinates (x^*, y^*, z^*) represent the upslope, slope-normal, and cross-slope directions, respectively. The corresponding dimensional velocity components are denoted by U^*, V^*, W^* . The true vertical dimensional coordinate s^* is related to y^* and x^* by

$$s^* = \cos(\chi)y^* + \sin(\chi)x^*, \quad (1)$$

where χ is the slope angle. To maintain consistency with the notation in Ref. [1], all dimensional quantities are denoted with a superscript “*.” The flow configuration is depicted in Fig. 1.

According to Ref. [1], there is no major qualitative difference in linear stability results for fixed temperature deficit and constant heat flux surface conditions. We hence limit our discussion to the constant surface temperature deficit case, but our points remain valid for the fixed surface flux case. The ambient (i.e., far above the slope) temperature $T_\infty^*(s^*)$ varies with s^* as

$$T_\infty^*(s^*) = T_\infty^*(0) + N_\infty^* s^*, \quad (2)$$

where $N_\infty^* (\equiv \partial T_\infty^* / \partial s^*)$ is the constant ambient vertical temperature gradient.² The slope (at $y^* = 0$) is a no-slip surface that is colder than the ambient temperature T_∞^* at the same elevation by $\Delta T^* > 0$.

² N_∞^* differs from the buoyancy frequency $N \equiv \sqrt{\frac{g}{\theta_{\infty}^*} \frac{\partial \theta_{\infty}^*}{\partial s^*}}$ commonly used in boundary-layer and mountain meteorology, where θ_{∞}^* is the ambient or potential temperature, $\theta_{\infty cr}^*$ is a constant reference value of θ_{∞}^* , and $g \equiv |g|$, where g is the gravity vector.

The governing equations with the Oberbeck-Boussinesq approximation are the following:

$$\nabla \cdot \mathbf{V}^* = 0, \quad (3)$$

$$\frac{\partial \mathbf{V}^*}{\partial t^*} + (\mathbf{V}^* \cdot \nabla) \mathbf{V}^* = -2\boldsymbol{\Omega}^* \times \mathbf{V}^* - \nabla(P^*/\rho_r) + \nu \nabla^2 \mathbf{V}^* - g\gamma(T^* - T_\infty^*), \quad (4)$$

$$\frac{\partial T^*}{\partial t^*} + \mathbf{V}^* \cdot \nabla T^* = \kappa \nabla^2 T^*. \quad (5)$$

Here $\mathbf{V}^* = (U^*, V^*, W^*)$ is the velocity, T^* is the temperature,³ $\mathbf{g} = -[g \sin(\chi), g \cos(\chi), 0]$ is the gravity vector, and $\boldsymbol{\Omega}^*$ is Earth’s angular velocity.⁴ The reference density ρ_r , viscosity ν , diffusivity κ , and coefficient of thermal expansion γ are considered constant.

Xiao *et al.* [1] included the additional wind forcing term in Eq. (4) in the form of a perturbation pressure gradient $\nabla(P^*/\rho_r)$ but did not discuss the special nature of that term or solve for P^* . (P^* is the deviation of full pressure from the background profile.⁵ In our discussion of perturbation quantities in Secs. II–IV, the perturbation pressure is the deviation from the background pressure, while the perturbation temperature is the deviation from the ambient temperature. Since the gradient of the background pressure is consistent with a state of rest but the gradient of the ambient pressure supports the ambient Coriolis force, the background and ambient pressures are not the same.) We first summarize the steady solution in Xiao *et al.* [1], and then we discuss how their implicit inclusion of this perturbation pressure gradient leads to unphysical ramifications for the steady flow.

Xiao *et al.* [1] present the (steady) base solution and the (unsteady) differential equations for the stability analysis in nondimensional form. For the steady base state, X, Y, Z denote the dimensionless version of the spatial coordinates (x^*, y^*, z^*) , and U_0, V_0, W_0 denote the dimensionless wind components. θ_0 represents the dimensionless temperature perturbation (deviation) from the ambient temperature. These quantities are defined as⁶

$$(X, Y, Z) = \frac{(x^*, y^*, z^*)}{d}, \quad d = \left[\frac{4\nu\kappa}{g\gamma \sin^2(\chi) N_\infty^*} \right]^{1/4},$$

$$(U_0, V_0, W_0) = \frac{(U^*, V^*, W^*)}{v/d}, \quad \theta_0 = \frac{T^* - T_\infty^*}{\Delta T^*}. \quad (6)$$

³When comparing results from this study with those of other slope-wind studies, it should be kept in mind that most of the latter studies work with potential temperature instead of temperature.

⁴It can be seen from Fig. 1 that only the part of the Earth’s angular velocity about the local vertical axis is considered. Such a treatment of the Earth’s angular velocity is standard in analyses of midlatitude weather phenomena [2].

⁵It can be shown using the equation of motion in its nonperturbation form (full pressure and density) that the background state adopted in Ref. [1] is a state of rest with density equal to the ambient (far above-slope) density and a hydrostatic pressure field that varies only with the true vertical coordinate s^* (so there is no geostrophic wind associated with it).

⁶The velocity scales in Eqs. (6) were not given in Xiao *et al.* [1] but can be inferred from the form of their nondimensional solution.

The steady version of Eqs. (3)–(5) reduce to a set of linear equations with the solutions U_0 , V_0 , W_0 , and θ_0 of the form

$$U_0(Y) = -\frac{G}{2}\mu^2 e^{-\mu Y} \sin(\mu Y), \quad (7)$$

$$V_0(Y) = 0, \quad (8)$$

$$W_0(Y) = \frac{G}{2}\tau \cos(\chi)[e^{-\mu Y} \cos(\mu Y) - 1], \quad (9)$$

$$\theta_0(Y) = -e^{-\mu Y} \cos(\mu Y), \quad (10)$$

where

$$\mu = [1 + (\tau \cos \chi)^2]^{1/4}, \quad G = \frac{g\gamma \sin(\chi) \Delta T^* d^3}{\nu^2}, \quad \tau = \frac{\Omega^* d^2}{\nu}. \quad (11)$$

Here G is the Grashof number and τ is the reciprocal Ekman number (referred to in Ref. [1] as a Coriolis number).

III. DERIVATION OF THE PERTURBATION PRESSURE GRADIENT FOR THE STEADY BASE FLOW

To show that the base flow considered by Ref. [1] [Eqs. (7)–(10)] is unphysical, we derive the solution for the base flow perturbation pressure gradient and the associated cross-slope geostrophic wind, which is also the far-above-slope cross-slope wind (no such derivation was provided in Ref. [1]). Our analysis is similar to Egger’s analysis of slope winds over Antarctica [3], which extends the work of Lykosov and Gutman [4].

We first describe the components of the Coriolis force, $-2\Omega^* \times \mathbf{V}^*$. Note that in Fig. 1, Ω^* points “up” along the local vertical (s^*) axis, which is only appropriate for the northern hemisphere. Whether the theory in Ref. [1] was intended to apply to both hemispheres or just the northern hemisphere was not discussed, and Ω^* , which appears to signify the magnitude of Ω^* , was not clearly defined in Ref. [1]. However, the theory would apply equally well to katabatic flows in both hemispheres if Ω^* were defined as $\Omega^* = 2\pi \sin(\phi)/\text{day}$, where ϕ is the latitude; it is positive in the northern hemisphere ($\phi > 0$) and negative in the southern hemisphere ($\phi < 0$). The cross product $-2\Omega^* \times \mathbf{V}^*$ has an upslope component $-2\Omega^* \cos(\chi)W^*$, a cross-slope component $2\Omega^* \cos(\chi)U^*$, and a slope-normal component $2\Omega^* \sin(\chi)W^*$.

Over a uniformly cooled slope of infinite extent, U^* , V^* , W^* are independent of x^* and z^* . Accordingly, Eq. (3) reduces to $\partial V^*/\partial y^* = 0$, whose y^* integral (due to the surface impermeability condition) is $V^* = 0$ everywhere. Given that the slope is of infinite extent and uniformly cooled, we can assume that the deviation from the ambient temperature, $T^* - T_\infty^*$, is also independent of x^* and z^* . Since T_∞^* itself varies linearly with x^* and y^* [from Eqs. (1) and (2)], we see that T^* is the sum of $T_\infty^*(x^*, y^*)$ and a function that depends only on y^* . Thus, all advection terms drop from Eqs. (4) and (5), except for $U^* \frac{\partial T_\infty^*}{\partial x^*}$ in Eq. (5). This combined with the steady state assumption permits us to write the upslope

component of Eq. (4) as

$$-2\Omega^* \cos(\chi)W^* - \frac{1}{\rho_r} \frac{\partial P^*}{\partial x^*} + g\gamma \sin(\chi)(T^* - T_\infty^*) + \nu \frac{\partial^2 U^*}{\partial y^{*2}} = 0, \quad (12)$$

the cross-slope component of Eq. (4) as

$$2\Omega^* \cos(\chi)U^* - \frac{1}{\rho_r} \frac{\partial P^*}{\partial z^*} + \nu \frac{\partial^2 W^*}{\partial y^{*2}} = 0, \quad (13)$$

the slope-normal component of Eq. (4) as

$$2\Omega^* \sin(\chi)W^* - \frac{1}{\rho_r} \frac{\partial P^*}{\partial y^*} + g\gamma \cos(\chi)(T^* - T_\infty^*) = 0, \quad (14)$$

and the thermal energy equation as

$$-N_\infty^* \sin(\chi)U^* + \kappa \frac{\partial^2}{\partial y^{*2}}(T^* - T_\infty^*) = 0. \quad (15)$$

To derive Eq. (15), we have used the fact that T_∞^* varies linearly with y^* [see Eqs. (1) and (2)] to write $\frac{\partial^2 T^*}{\partial y^{*2}}$ as $\frac{\partial^2 (T^* - T_\infty^*)}{\partial y^{*2}}$.

In view of Eq. (14) and the fact that W^* and $(T^* - T_\infty^*)$ are independent of x^* and z^* , we see that $\partial P^*/\partial y^*$ is also independent of x^* and z^* , and therefore $\frac{\partial^2 P^*}{\partial y^* \partial x^*} = \frac{\partial^2 P^*}{\partial y^* \partial z^*} = 0$. Thus, $\partial P^*/\partial x^*$ and $\partial P^*/\partial z^*$ are independent of y^* . Assuming $\lim_{y^* \rightarrow \infty} (T^* - T_\infty^*) = 0$, Eq. (15) shows that $\lim_{y^* \rightarrow \infty} U^* = 0$. Then, assuming that W^* approaches a constant (not necessarily zero) as $y^* \rightarrow \infty$, Eq. (13) shows that $\partial P^*/\partial z^* = 0$ [for all y^* since $\partial P^*/\partial z^*$ is independent of y^*]. As $y^* \rightarrow \infty$, Eq. (12) expresses a (geostrophic) balance between the Coriolis force and the perturbation pressure gradient force in the x^* direction from which we obtain the cross-slope wind far above the slope as

$$\lim_{y^* \rightarrow \infty} W^* = W_g^* \equiv -\frac{1}{2\Omega^* \cos(\chi) \rho_r} \frac{\partial P^*}{\partial x^*}, \quad (16)$$

where W_g^* is the cross-slope geostrophic wind.

To obtain the upslope perturbation pressure gradient, we eliminate U^* between Eqs. (13) and (15) (with $\partial P^*/\partial z^* = 0$):

$$\frac{\kappa}{N_\infty^* \sin(\chi)} \frac{\partial^2}{\partial y^{*2}}(T^* - T_\infty^*) + \frac{\nu}{2\Omega^* \cos(\chi)} \frac{\partial^2 W^*}{\partial y^{*2}} = 0. \quad (17)$$

Integrating Eqs. (17) twice with respect to y^* yields

$$\frac{\kappa}{N_\infty^* \sin(\chi)} (T^* - T_\infty^*) + \frac{\nu}{2\Omega^* \cos(\chi)} W^* = a + by^*, \quad (18)$$

where a and b are integration constants. Since $\lim_{y^* \rightarrow \infty} (T^* - T_\infty^*) = 0$ and W^* approaches a constant as $y^* \rightarrow \infty$, b must be 0. At the slope where $T^* - T_\infty^* = -\Delta T^*$ and $W^* = 0$, Eq. (18) yields

$$a = -\frac{\kappa \Delta T^*}{N_\infty^* \sin(\chi)}. \quad (19)$$

⁷Although not explicitly mentioned in Ref. [1], this condition is consistent with their solution for the cross-slope wind.

Using Eqs. (19), (16), and $\lim_{y^* \rightarrow \infty} (T^* - T_\infty^*) = 0$ to evaluate Eq. (18) at $y^* \rightarrow \infty$ gives

$$\frac{\partial P^*}{\partial x^*} = \frac{4\Omega^{*2} \cos^2(\chi) \rho_r \Delta T^*}{N_\infty^* \sin(\chi) \text{Pr}}, \quad (20)$$

where $\text{Pr} \equiv \nu/\kappa$ is the Prandtl number. From Eq. (16), with $\partial P^*/\partial x^*$ from Eq. (20), we find that the cross-slope wind far above the slope must be

$$\lim_{y^* \rightarrow \infty} W^* = W_g^* = -\frac{2\Omega^* \cos(\chi) \Delta T^*}{N_\infty^* \sin(\chi) \text{Pr}}. \quad (21)$$

Apart from notational differences,⁸ Eq. (21) is the same as Eq. (3.7) in Egger for $\text{Pr} = 1$.

IV. PHYSICAL IMPLICATIONS OF THE STEADY BASE FLOW

The derivations above have shown that to obtain a steady state in which the temperature perturbation $T^* - T_\infty^*$ vanishes far above the slope, the upslope perturbation pressure gradient $\partial P^*/\partial x^*$ must be of the form given by Eq. (20), and the far-above slope cross-slope wind (i.e., the cross-slope geostrophic wind) must be of the form given by Eq. (21). This implies the following major issues with the model:

(1) There is no physical justification for why the perturbation pressure gradient (and cross-slope wind) far above the slope should depend on ΔT^* , χ , N_∞^* , and Ω^* . Therefore, the model cannot apply to a slope wind embedded in an atmosphere otherwise at rest.

(2) Egger [3] pointed out that this perturbation pressure gradient supports an easterly wind far above the sloping Antarctica ground surface, in contrast to the westerlies that are observed there⁹ (e.g., Ref. [5]).

(3) This pressure gradient and the wind far above the slope blow up as $\chi \rightarrow 0$ for either hemisphere. Egger concluded that because of this perturbation pressure gradient term, the model is unphysical.

Additionally, we argue further why the upslope perturbation pressure gradient force required by the model is unphysical. We may consider the steady-state in Ref. [1] as the terminal state ($t^* \rightarrow \infty$) of an initial value problem, and ask how the upslope pressure gradient force required to obtain the steady-state solution subject to the boundary conditions in Ref. [1] could arise from a flow field initially at rest,

⁸In particular, the coordinates x^* , y^* , z^* and velocity components U^* , V^* , W^* in Xiao *et al.* [1] correspond to the coordinates $-y$, z , $-x$ and velocity components $-v$, w , $-u$ in Egger [3]. Also note that θ_s and $\theta_0 N_0^2$ in Egger correspond to $-\Delta T^*$ and gN_∞^* in Ref. [1] (though the thermal variable in Egger is potential temperature, not temperature).

⁹To deduce the sense of this wind from Eq. (21), note that over Antarctica, the terrain slopes upward toward the south. Accordingly, the coordinate x^* points south, and the cross-slope coordinate z^* points west. Since $\Omega^* < 0$ in the southern hemisphere (the definition $\Omega^* = 2\pi \sin \phi / \text{day}$ allows the theory to apply to either hemisphere) and $\Delta T^* > 0$ for a cooled surface, Eq. (21) yields a positive value for $\lim_{y^* \rightarrow \infty} W^*$ and W_g^* , indicating an easterly geostrophic wind and easterly wind.

i.e., with zero velocity and temperature deficit for $t \leq 0$. It should be clear from Eq. (12) (considered for $t \leq 0$) that a nonzero $\partial P^*/\partial x^*$ is inconsistent with a resting thermally unperturbed state. Accordingly, we consider the scenario where all dependent variables, including the upslope perturbation pressure gradient, are zero until the abrupt decrease of slope temperature at $t^* = 0$. As in Ref. [1], we suppose the velocity components and temperature perturbation are independent of x^* and that $\partial P^*/\partial z^*$ is zero. The relevant equations are the unsteady versions of Eq. (12),

$$\begin{aligned} \frac{\partial U^*}{\partial t^*} &= -2\Omega^* \cos(\chi) W^* - \frac{1}{\rho_r} \frac{\partial P^*}{\partial x^*} \\ &+ g\gamma \sin(\chi) (T^* - T_\infty^*) + \nu \frac{\partial^2 U^*}{\partial y^{*2}}, \end{aligned} \quad (22)$$

Eq. (13) without $\partial P^*/\partial z^*$,

$$\frac{\partial W^*}{\partial t^*} = 2\Omega^* \cos(\chi) U^* + \nu \frac{\partial^2 W^*}{\partial y^{*2}}, \quad (23)$$

and thermal energy equation (15),

$$\frac{\partial T^*}{\partial t^*} = -N_\infty^* \sin(\chi) U^* + \kappa \frac{\partial^2}{\partial y^{*2}} (T^* - T_\infty^*). \quad (24)$$

The slope-normal equation (14), is unchanged. As previously shown, it follows from Eq. (14) that $\partial P^*/\partial x^*$ is independent of y^* . Moreover, since the wind and temperature perturbation fields are independent of x^* , Eq. (22) shows that $\partial P^*/\partial x^*$ is also independent of x^* . Therefore, $\partial P^*/\partial x^*$ is a function of t^* only. In the equation that results from applying $\partial/\partial t^* - \nu \partial^2/\partial y^{*2}$ to Eq. (22), using Eq. (23) to eliminate W^* and Eq. (24) to eliminate $\partial T^*/\partial t^*$ yields

$$\begin{aligned} &\left(\frac{\partial}{\partial t^*} - \nu \frac{\partial^2}{\partial y^{*2}} \right)^2 U^* \\ &= -[4\Omega^{*2} \cos^2(\chi) + g\gamma N_\infty \sin^2(\chi)] U^* \\ &- \frac{1}{\rho_r} \frac{\partial}{\partial t^*} \frac{\partial P^*}{\partial x^*} + g(\kappa - \nu) \gamma \sin(\chi) \frac{\partial^2}{\partial y^{*2}} (T^* - T_\infty^*). \end{aligned} \quad (25)$$

Imposing the same far-above-slope conditions as in Ref. [1], i.e., $\lim_{y^* \rightarrow \infty} [U, T^* - T_\infty^*] = (0, 0)$ in Eq. (25) yields

$$\frac{\partial}{\partial t^*} \frac{\partial P^*}{\partial x^*} = 0. \quad (26)$$

Integration of Eq. (26) with respect to t^* yields $\partial P^*/\partial x^* = C$, where C is independent of t^* . Since $\partial P^*/\partial x^*$ was initially zero, C must be zero. Accordingly, $\partial P^*/\partial x^* = 0$ for all time; the model is hence incompatible with a nonzero $\partial P^*/\partial x^*$ at $t^* > 0$ if $\partial P^*/\partial x^*$ is initially zero. Accordingly, the nonzero value for that term required by Ref. [1] must be considered to be an external parameter. We can, therefore, conclude that this perturbation pressure gradient is a purely mathematical expediency: there is no physical reason why an externally imposed far-above-slope perturbation pressure gradient given in Eq. (20) should depend on the perturbation temperature at the slope or have a dependence of $\sin(\chi)^{-1}$ on the slope angle.

We have thus shown that provisions for the perturbation pressure gradient as implied in Ref. [1] lead to problematic

conclusions. However, if that term is omitted from Eqs. (3)–(5), then the temperature perturbation cannot vanish at infinite height for any steady state [3,4,6,7]. In addition, Stiperski *et al.* [6] and Shapiro and Fedorovich [7] showed that, when started from rest, a time-dependent Prandtl slope flow model that includes the Coriolis force but not $\partial P^*/\partial x^*$ produces a cross-slope wind and buoyancy that diffuse upward indefinitely, i.e., the flow does not uniformly converge to a steady state over the entire height $y^* \in [0, \infty)$. Thus, for a Prandtl-like flow with Coriolis force, whether the perturbation pressure gradient is included (as in Ref. [1]) or not (as in Ref. [6,7]), the corresponding steady flow is not suitable as base flow for linear stability analysis.

V. IDENTIFICATION OF TRAVELING WAVES

In our final point, we clarify that, contrary to the claim in Ref. [1], the earlier study by Gill and Davey [8] did not identify the traveling waves in their analysis as TS waves,

which are characterized by the destabilizing effect of viscosity without the need for inflection points in the base flow [9,10]. In fact, it has been pointed out in the opening paragraph in Chapter 3.2.1 of Ref. [10] that “According to Rayleigh’s inflection point criterion the Poiseuille profile is stable as $Re \rightarrow \infty$, so one must conclude that viscosity is in this case destabilizing. This unstable eigenmode is called a Tollmien-Schlichting wave.”

According to our prior publications on Prandtl slope flow instabilities [11–14], the underlying mechanism for traveling waves is an inviscid instability and these waves are associated with an inertial type KH instability. This is further supported by the fact that the base flow contains infinitely many inflection points. Our numerical experiments on Prandtl slope flows without Coriolis effect, as presented in Refs. [11,14], have confirmed the inviscid nature of the along-slope traveling instabilities and found that viscosity does not destabilize the flow but instead dampens the growth of the eigenmodes.

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