

Cross-scale echoes of ultrafast oscillations in the human brain

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Ultrafast oscillations (UFOs) are brief bursts of high-frequency (2–8 kHz) activity recorded using depth electrodes in epileptic brain tissue. While reliably detected at small scales using microelectrodes, UFOs have thus far likely not been observed at larger scales using macroelectrodes. Here we show that faint echoes of these bursting patterns can be detected in macroelectrode signals using a model of cross-scale propagation. To model this cross-scale effect, we extend the classical damped wave equation to include an explicit scale coordinate governed by the dilation operator. We then use Green's functions to obtain an expression for the spectral echo expected at the macroelectrode scale. This echo takes the form of a carrier envelope modulated by the temporal structure of the original microscale bursting event. We test the predicted signature echo pattern in human intracranial recordings from eleven epilepsy patients implanted with both micro- and macroelectrodes. Statistically significant echoes are confirmed in two out of eleven patients, appearing in 2% of all tested micro-macro pairs.

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I. INTRODUCTION

Ultrafast oscillations (UFOs) are brief bursts of activity in the 2–8 kHz range that originate in epileptic mesiotemporal structures [1] and occur at substantially higher frequencies than other reported bursting patterns such as ripples (80–250 Hz) and fast ripples (250–500 Hz) [2–4]. UFOs are reliably detected using microelectrodes but are absent in macroelectrode recordings, leading to their common interpretation as a fundamentally microscale phenomenon. However, we propose that the original microscopic burst does not vanish at larger scales, but rather transforms into slower, spatially broader echoes as it propagates outward from smaller to larger structures in the brain. Here we derive the power spectrum that a UFO echo is expected to produce at the macroscale and use this as a template to search for echoes in empirical macroelectrode recordings.

It is well established that microscale neural events can have macroscale correlates. For instance, discharges confined to single microelectrodes (microseizures) can precede

clinical seizures visible in macroelectrode recordings [5]. More broadly, high-frequency bursts generated by local neuronal assemblies have been proposed as electrophysiological substrates of engram activity [6]. Recent work has shown that such bursts co-occur across widespread cortical areas during memory processing, suggesting that microscale activity routinely organizes into large-scale patterns [7]. What has been lacking, however, is a model describing precisely how a signal is expected to change as it propagates across scales. Our framework addresses this gap by deriving the predicted transformation from first principles.

In building the theoretical framework underlying cross-scale echoes, we begin with the damped wave equation commonly used to describe neural field propagation [8,9] along a spatial coordinate and adapt it to instead describe how activity propagates across scales. To this end, we note that, unlike spatial position, the coordinates associated with scale do not shift—they stretch. Formally, position transforms additively under translation, whereas scale transforms multiplicatively under dilation. The natural coordinate for scale is therefore logarithmic rather than linear (Table I).

In contrast to models that describe neural dynamics at a fixed spatial scale [10,11], we model cross-scale propagation by replacing the spatial translation generator in the wave equation with the dilation operator. We then use a Green's function approach to derive the spectral signature of the UFO echo expected in macroelectrode recordings.

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TABLE I. Summary transformation properties of space and scale.

	Space	Scale
Transformation	Additive	Multiplicative
Generator	Translation	Dilation
Geometry	Euclidean	Logarithmic

II. METHODS

Theoretical framework. We describe a neural variable $z(t, x)$ evolving in time and propagating along a spatial coordinate x using the standard damped wave equation:

$$\frac{\partial^2 z}{\partial t^2} + 2\gamma \frac{\partial z}{\partial t} + \omega^2 z - v_x^2 T^2 z = 0, \quad (1)$$

where ω is the natural angular frequency, γ is the damping rate, v_x is the propagation speed, and $T = \partial/\partial x$ the generator of infinitesimal translations in space.

In contrast with the additive transformations of the spatial coordinate x , a scale coordinate s transforms multiplicatively, with a generator of infinitesimal dilations given by

$$D = s \frac{\partial}{\partial s}. \quad (2)$$

Our key modeling assumption is that cross-scale propagation obeys the same structure as spatial propagation, if we replace the spatial translation generator T in Eq. (1) with the dilation generator D in Eq. (2),

$$\frac{\partial^2 z}{\partial t^2} + 2\gamma \frac{\partial z}{\partial t} + \omega^2 z - v_s^2 D^2 z = 0, \quad (3)$$

where v_s is the cross-scale propagation speed.

To mirror the T^2 operator in Eq. (1), we apply the dilation operator in Eq. (2) twice:

$$D^2 = s^2 \frac{\partial^2}{\partial s^2} + s \frac{\partial}{\partial s}. \quad (4)$$

By substituting Eq. (4) into Eq. (3), we obtain the following expression:

$$\frac{\partial^2 z}{\partial t^2} + 2\gamma \frac{\partial z}{\partial t} + \omega^2 z - v_s^2 \left(s^2 \frac{\partial^2 z}{\partial s^2} + s \frac{\partial z}{\partial s} \right) = 0. \quad (5)$$

This describes how activity changes as it propagates across scales.

Switching to log-scale coordinates $u = \log s$ allows us to write the dilation operator in terms of an additive transformation $s \frac{\partial}{\partial s} = \frac{\partial}{\partial u}$ and $D^2 = \frac{\partial^2}{\partial u^2}$, such that Eq. (5) reduces to

$$\frac{\partial^2 z}{\partial t^2} + 2\gamma \frac{\partial z}{\partial t} + \omega^2 z - v_s^2 \frac{\partial^2 z}{\partial u^2} = 0. \quad (6)$$

This cross-scale wave equation therefore adopts an equivalent form to the expression for spatial wave motion in Eq. (1).

Next, we model a UFO by taking the homogeneous expression in Eq. (6) and considering its response to a rapid sequence of $N + 1$ impulses separated by the inverse burst frequency $1/\Omega$. Given that this burst has a total duration τ , originates at a microscale s_μ (corresponding to $u_\mu = \log s_\mu$) and occurs at times $t_n = n/\Omega$, we can consider Eq. (6) as being driven by an

input $\delta(s - s_\mu) \sum_{n=0}^N \delta(t - t_n)$, where $N = \lfloor \tau\Omega \rfloor$. The driven form of Eq. (6) therefore becomes

$$\begin{aligned} \frac{\partial^2 G}{\partial t^2} + 2\gamma \frac{\partial G}{\partial t} + \omega^2 G - v_s^2 \frac{\partial^2 G}{\partial u^2} \\ = s_\mu^{-1} \delta(u - u_\mu) \sum_{n=0}^N \delta(t - t_n), \end{aligned} \quad (7)$$

where the factor s_μ^{-1} on the right-hand side arises from expressing the spatial delta function $\delta(s - s_\mu)$ in logarithmic coordinates.

We note that Eq. (7) takes the form of the 1D driven Klein-Gordon equation, with a known causal Green's function [12] given by a decaying wavefront spreading in the (t, u) plane:

$$\begin{aligned} G = \frac{1}{2v_s} e^{-\gamma t} J_0 \left(\sqrt{(\omega^2 - \gamma^2)(t^2 - v_s^{-2}|u - u_\mu|^2)} \right) \\ \times H(t - v_s^{-1}|u - u_\mu|), \end{aligned} \quad (8)$$

where J_0 is the zeroth-order Bessel function and H is the causal Heaviside function. H enforces causality by restricting the solution to $t \geq v_s^{-1}|u - u_\mu|$, which means the Green's function is zero outside the causal cone, i.e., the region where the argument of the square root in Eq. (8) is negative.

We then consider a macroscopic observation scale u_M , at which point the total UFO echo is simply given by the sum of the Green's responses in Eq. (8) to each delta impulse:

$$z_{\text{echo}} = \sum_{n=0}^N G(t - t_n, u_M; u_\mu). \quad (9)$$

We now move to the frequency domain, given that the model predicts that a UFO should reappear at the macroscale with a specific spectral signature shaped by the Green's function. Applying the Fourier transform $\tilde{G}(\sigma, u) = \int_{-\infty}^{\infty} G(t, u) e^{i\sigma t} dt$ to Eq. (7) gives

$$\alpha(\sigma) \tilde{G} - v_s^2 \frac{\partial^2 \tilde{G}}{\partial u^2} = s_\mu^{-1} \delta(u - u_\mu), \quad (10)$$

where σ is the Fourier frequency and $\alpha(\sigma) = \omega^2 - \sigma^2 - 2i\gamma\sigma$. Equation (10) yields the frequency-domain form of a 1D Klein-Gordon operator in log-scale space.

Away from the point source at $u = u_\mu$, the right-hand side of Eq. (10) is zero and the resultant expression has exponential solutions of the form $e^{-q(\sigma)|u - u_\mu|}$ with a complex scale-propagation wavenumber $q(\sigma) = v_s^{-1} \sqrt{\alpha(\sigma)}$. The causal branch is selected by imposing $\text{Im}\{q\} > 0$, ensuring that the signal attenuates with increasing scale separation. Imposing the standard jump condition across the delta source at $u = u_\mu$ determines the normalization, yielding the frequency-domain Green's function:

$$\tilde{G} = \frac{1}{2s_\mu v_s^2 q(\sigma)} e^{-q(\sigma)|u - u_\mu|}. \quad (11)$$

Taking the Fourier transform of the macroscale time domain UFO signal in Eq. (9) gives $\tilde{G}(\sigma) \sum_{n=0}^N e^{-i\sigma n/\Omega}$. This summation has a closed form expression given by $e^{-i\sigma N/(2\Omega)} \sin[(N + 1)\sigma/(2\Omega)] / \sin[\sigma/(2\Omega)]$, with an absolute

value yielding the Dirichlet kernel:

$$D_N(\sigma) = \left| \frac{\sin[(N+1)\sigma/(2\Omega)]}{\sin[\sigma/(2\Omega)]} \right|. \quad (12)$$

This represents the comblike spectral modulation created purely by the periodic timing of the impulses.

Combining Eqs. (11) and (12), we obtain the macroscopic power spectrum $P(\sigma) = |\tilde{G}(\sigma)|^2 D_N(\sigma)^2$:

$$P(\sigma) = \frac{e^{-2\text{Re}\{q(\sigma)\}|u_M - u_\mu|}}{4s_\mu^2 v_s^4 |q(\sigma)|^2} \left| \frac{\sin[(N+1)\sigma/(2\Omega)]}{\sin[\sigma/(2\Omega)]} \right|^2. \quad (13)$$

The first factor $e^{-2\text{Re}\{q(\sigma)\}|u_M - u_\mu|}$ describes exponential attenuation with scale separation, the second factor $\frac{1}{4s_\mu^2 v_s^4 |q(\sigma)|^2}$ shapes the Green's function spectral envelope through the complex wavenumber, and the third factor $\left| \frac{\sin[(N+1)\sigma/(2\Omega)]}{\sin[\sigma/(2\Omega)]} \right|^2$ encodes the comblike spectral structure of the original burst.

SEEG recordings. Intracranial SEEG recordings were obtained from eleven epilepsy patients across three clinical centers: Mayo Clinic (Rochester, MN; $n = 5$), Brno Epilepsy Center (Brno, Czech Republic; $n = 3$), and Wrocław Medical University Hospital (Wrocław, Poland; $n = 3$). All patients were implanted with hybrid depth electrodes carrying both clinical macrocontacts and research microcontacts. Electrode designs and acquisition systems differed across centers and are described separately below.

Mayo Clinic (Patient IDs MAYO00038, MAYO00397, MAYO00448, MAYO00452, and MAYO00453) custom hybrid depth electrodes (AD-Tech Medical Instrument Corporation, Racine, WI) were used, based on standard four- and eight-contact clinical depth electrode designs. Each electrode consists of a 1.3 mm diameter polyurethane shaft with platinum/iridium (Pt/Ir) macrocontacts (2.3 mm long, 5 or 10 mm center-to-center spacing, surface area 9.4 mm²). The eight-contact design carries 8–24 microcontacts positioned radially along the shaft between 1–4 macrocontacts; the four-contact design carries 16 microcontacts. Each electrode additionally has a bundle of 9 Pt/Ir microwires (40 μ m diameter, surface area 10^{−3} mm², 0.5–1 mm spacing) protruding 1–3 mm from the shaft tip.

Patients received one hybrid electrode in each temporal lobe; in one patient, subdural strips and additional intracerebral electrodes were implanted to monitor the parietal and occipital cortex. Electrode positions were verified using CT with electrodes *in situ*. Signals were acquired on a Neuralynx system (Neuralynx Inc., Bozeman, MT) using a wideband DC-coupled analog front end (passband DC to ~ 9 kHz) and digitized at 32 kHz. Macrocontact signals were low-pass filtered (Bartlett-Hanning, 1 kHz) and downsampled to 5 kHz. A stainless steel subgaleal suture placed at the scalp vertex between the 10–20 Cz and Fz positions served as the common reference.

Brno Epilepsy Center (Patient IDs FNUS00061, FNUS00072, and FNUS00079). In addition to standard multicontact platinum depth electrodes (0.8 mm diameter, 2 mm macrocontact length, 1.5 mm intercontact distance; Microdeep, DIXI Medical, Besançon, France, or Depth Electrodes Range, ALCIS, Besançon, France), one custom hybrid depth electrode (2069-ECP-8C6-35T06-2G-26,

ALCIS, Besançon, France) was implanted in each patient. This hybrid electrode has eight macrocontacts (surface area 5 mm²) and six microcontacts (surface area 1075 μ m²) positioned radially along the shaft between the first and third macrocontacts.

Each patient received 3–14 depth electrodes, implanted orthogonally in the temporal lobes and, where clinically indicated, in the frontal lobes. Electrode positions were verified using coregistered MRI and CT with electrodes *in situ*. Signals were recorded using a 192-channel research acquisition system (M&I BrainScope, Prague, Czech Republic) at a sampling rate of 25 kHz with a dynamic range of ± 25 mV. The system uses an oversampling analog-to-digital converter with an internal low-pass decimation filter providing antialiasing at the final 25 kHz output rate. The acquisition unit was battery powered to eliminate line noise, and microcontacts were connected to high-impedance amplifiers (>1 G Ω).

Standard epilepsy monitoring unit protocols were followed, with additional measures to minimize electromagnetic interference and 50 Hz power grid contamination; no shielded environment was used. The ground electrode served as the reference for microcontacts; common average reference across all macrocontacts was used for macrocontacts.

Wrocław Medical University Hospital (Patient IDs: WROC00001, WROC00003, and WROC00004). Custom hybrid depth electrodes (AD-Tech Medical Instrument Corporation, Racine, WI) of two designs were used. Both share the same 1.3 mm diameter polyurethane shaft with Pt/Ir macrocontacts (2.3 mm long, 5 or 10 mm center-to-center spacing, surface area 9.4 mm²) described above for Mayo Clinic. The six-contact design carries ten microcontacts positioned radially along the shaft between macrocontacts 1–3. The eight-contact Behnke-Fried design has a bundle of 8 Pt/Ir microwires (40 μ m diameter) protruding 1–3 mm from the electrode tip. Patients were implanted with one or two six-contact hybrid electrodes in the temporal or frontal lobe, and three to four Behnke-Fried electrodes in the temporal cortex unilaterally, with additional standard clinical depth electrodes in other lobes as clinically indicated. Electrode positions were verified using CT with electrodes *in situ*.

Signals were acquired on a Neuralynx system (Neuralynx Inc., Bozeman, MT) using a wideband DC-coupled analog front end (passband DC to ~ 9 kHz) and digitized at 32 kHz. Macrocontact signals were downsampled to 5 kHz. A subgaleal strip of four macrocontacts placed at the scalp vertex between the 10–20 Cz and Fz positions served as the common reference for macrocontacts. Microcontact signals were referenced locally to a quiescent microchannel showing no neuronal spiking or other neural activity.

Software. EEGWave was used for interactive graphical analysis of multichannel data, custom Python scripts for batch processing and event detection, and R for statistical analysis.

UFO detection. Detection was performed in a sliding 30-second window. A spectrogram of the input signal was computed using a short-time Fourier transform with 15 ms analysis windows, 97% overlap, and linear detrending of each windowed segment. Each frequency row of the spectrogram was normalized by dividing by its running median in a one-second sliding window. Analysis was restricted to the 2–8 kHz range, keeping the maximum analyzed frequency below the

practical limit of $f_s/3$ and within the passband of the recording hardware.

A one-dimensional detection signal was created by summing all rows of the normalized spectrogram within the analyzed frequency band. Peaks exceeding five times the 99th percentile of this detection signal were identified as candidate events, and the region around each peak bounded by its inflection points in the detection signal was marked as a putative high-frequency event. For each candidate detection, two quantities were retained for further analysis: (i) the original raw signal within the detection window, and (ii) a spectral profile created by summing all time columns of the normalized spectrogram within the detection boundaries [13].

Classification. Each candidate event was classified as either rhythmic or nonrhythmic based on the ratio of the maximum to the mean of its spectral profile. Events with a ratio exceeding ten were classified as rhythmic, indicating a dominant oscillatory frequency; events below this threshold were classified as nonrhythmic. This ratio served as a tuning parameter and was chosen heuristically: lowering it increased the number of detections but reduced the dominance of the peak frequency. At a threshold of 10, $34.8 \pm 22.1\%$ of primary detections were retained.

Rhythmic events were further subdivided into two morphological types. Type I events showed a smooth amplitude envelope (gradual rise and fall) oscillating within a single frequency band, appearing as a circular or ellipsoidal region in the spectrogram. Type II events showed an abrupt amplitude onset followed by a slow decay, often accompanied by a shift in the signal mean, producing a vertical stripe in the spectrogram followed by sustained activity in a narrow frequency band. To distinguish between these types, the raw signal of each detection was analyzed as follows.

The point at which the absolute value of the signal derivative reached half its maximum was designated the true onset. Two flanking sections were defined: section 1 spanning one quarter of the detection length before the true onset, and section 2 spanning one quarter after it. If the true onset fell outside the central half of the detection (before 1/4 or after 3/4 of the total length), the event was automatically classified as type I. Otherwise, an event was classified as type II if either condition 1 $\frac{\text{std}(\text{section 2})}{\text{std}(\text{section 1})} > 10$ or condition 2 $\frac{\text{std}(\text{section 2})}{\text{std}(\text{section 1})} > 4.5$ and $\frac{|\text{mean}(\text{section 1}) - \text{mean}(\text{section 2})|}{\text{std}(\text{section 1})} > 1$ are met. The first condition captures a sudden increase in amplitude variability; the second captures a combined increase in variability and shift in signal mean. The thresholds (10 and 4.5) were chosen heuristically. Using these criteria, approximately 35% of rhythmic events were classified as type II and 65% as type I.

Only type I events—clear rhythmic oscillations with a smooth amplitude envelope with peak frequency above 2 kHz—were used in the subsequent analyses.

Spectral echo detection. For each microelectrode UFO event, we extracted the corresponding macroscale time series from the nearest macroelectrode. Two windows were defined per event: a 200 ms pre-UFO baseline and a 200 ms post-UFO segment. Both windows were demeaned and linearly detrended. The theory predicts that a UFO burst will reappear at the macroscale with a spectrum given by Eq. (13). However, for template matching, we retained only the spectral envelope as this captures the propagation dynamics predicted

by the model, whereas the Dirichlet modulation depends on the specific temporal structure of each individual UFO burst. Taking the squared magnitude of $\tilde{G}$ in Eq. (11) and retaining only the frequency-dependent terms yields:

$$|\tilde{G}(\sigma)|^2 \sim \frac{1}{(\omega^2 - \sigma^2)^2 + (2\gamma\sigma)^2}. \quad (14)$$

This is the power spectrum of a damped oscillator with a resonant peak at the echo frequency $\omega_{\text{echo}} = \sqrt{\omega^2 - \gamma^2}$. For each patient, this template was evaluated using fixed model parameters ($f_0 = 75$ Hz, $\gamma = 40$ s⁻¹), normalized to unit energy, and restricted to the analysis frequency band [60, 100] Hz.

For each window, we estimated the power spectral density using Welch's method (100 ms segments, 50% overlap, minimum 512-point FFT). The template was projected onto each empirical PSD via a non-negative scalar gain: $A = \max(0, \mathbb{T}^\top \mathbb{P})$, where $\mathbb{T}$ is the unit-normalized template and $\mathbb{P}$ the empirical PSD within the analysis band. The echo detection score for each event was defined as $\log_{10} \frac{A_{\text{post}}}{A_{\text{base}}}$, where a positive score indicates greater template-matching power in the post-UFO window relative to baseline.

Statistical testing. Statistical significance was assessed using a time-shifted empirical null procedure. For each micro-macro electrode pair, the echo detection scores across all UFO events were averaged to produce a single empirical statistic. A null distribution was constructed by repeating this procedure 10^4 times at randomly sampled time points from the same macroelectrode channel. For each null iteration, K time points were drawn uniformly from the recording, excluding windows that overlapped with detected UFOs, and processed identically to the empirical UFO events: baseline extraction, PSD estimation, template projection, and scoring. The K null scores were averaged to produce a single null statistic per iteration, yielding a distribution of 10^4 null means.

A one-sided p value for each micro-macro pair was computed as $p = \frac{1+n_{\text{exceed}}}{N_{\text{eff}}+1}$, where n_{exceed} is the number of null means greater than or equal to the empirical mean and N_{eff} is the number of valid null iterations, with the +1 correction ensuring a valid Monte Carlo p value. Multiple comparisons across pairs within each patient were corrected using the Benjamini-Hochberg procedure at a false discovery rate of $q = 0.05$.

III. RESULTS

Macroscopic UFO echoes were verified in two out of eleven patients, accounting for 2% of the tested electrode pairs (Table II).

The remaining nine patients showed no significant pairs after FDR correction, consistent with adequate control of the false positive rate by the null procedure.

In patient WROC00003, 119 UFOs were identified, yielding 11 testable micro-macro electrode pairs. One pair—microelectrode CSC73 paired with macroelectrode SMAL1 (see Table III for full list of channels)—showed a significant cross-scale echo after FDR correction ($p = 0.0001$, empirical mean score 0.22, null mean -0.001 ± 0.05). A second pair (CSC69–SMAAL1) was significant at the uncorrected level ($p = 0.01$) but did not survive FDR correction.

TABLE II. Total number of UFOs and micro-macro electrode pairs tested for each of the eleven patients. Patient MAYO00452 did not meet the minimum data requirement for statistical testing, as indicated by the zero usable micro-macro pairs.

Patient ID	UFOs	Pairs
WROC00001	93	10
WROC00003	119	11*
WROC00004	47	5*
FNUS00061	16	1
FNUS00072	19	3
FNUS00079	89	5
MAYO00038	256	32
MAYO000397	306	24
MAYO000448	75	8
MAYO000452	11	0
MAYO000453	20	1

*Asterisks indicate that a single pair survived multiple comparison correction (FDR).

In patient WROC00004, 47 UFOs were identified, yielding five testable micro-macro electrode pairs. One pair—microelectrode CSC65 with macroelectrode AMG1—showed a significant cross-scale echo after FDR correction ($p = 0.0096$, empirical mean score 0.31, null mean 0.002 ± 0.13).

Figure 1 shows a single UFO event in patient WROC00003, for which the corresponding macrochannel

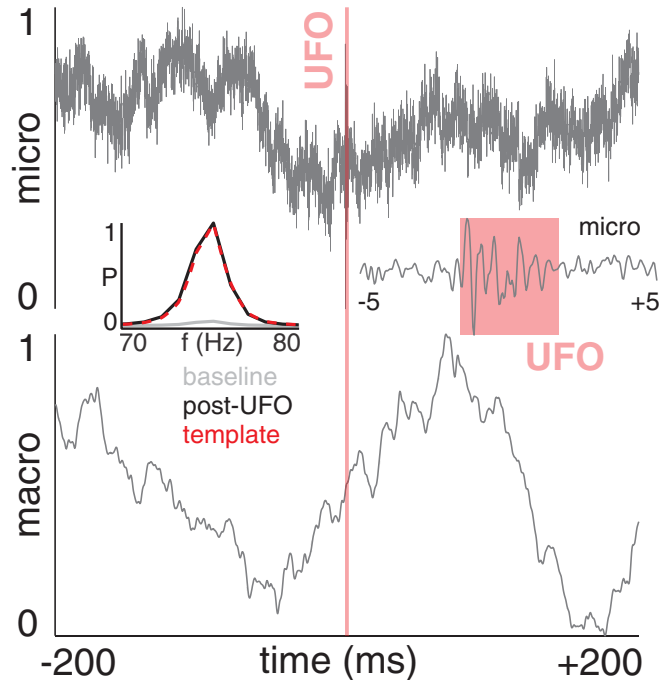


FIG. 1. Micro- (top) and macro- (bottom) traces from patient WROC00003, spanning 200 ms before UFO onset to 200 ms after UFO cessation. The red vertical bar marks the UFO itself. The right inset shows a zoomed-in version of the UFO in the microelectrode recording (± 5 ms around the event). The left inset shows the baseline (gray) and post-UFO (black) power spectra alongside the theoretical Green's function template (red dashed). The echo detection score reflects the projection of each spectrum onto this template; here the post-UFO projection exceeds the baseline, illustrating one of the events contributing to the statistically significant average effect.

TABLE III. Micro-macroelectrode mappings used for spectral echo analysis. Where two macroelectrodes are listed, the microelectrode was analyzed with respect to both adjacent macrocontacts.

Patient	Microelectrodes	Macroelectrodes
WROC00001	CSC(1–8)_AMG	AMG1
WROC00001	CSC1–8	HH1
WROC00001	CSC10	HH1
WROC00001	CSC(1–8)_HH	HH1
WROC00003	CSC64–71	SMAAL1
WROC00003	CSC72–79	SMAL1
WROC00004	CSC64–71	AMG1
WROC00004	CSC72–79	HH1
WROC00004	CSC80–87	HB1
WROC00004	CSC96–105	TP1
FNUS00061/72/79	Bm(1–3)a	B1/B2
FNUS00061/72/79	Bm(4–6)a	B2/B3
MAYO00038	LBundle_01–08	LMacro_01
MAYO00038	RBundle_01–05	RMacro_01
MAYO00038	RBundle_07–08	RMacro_01
MAYO00038	LShaftMicro_01–06	LMacro_01/02
MAYO00038	LShaftMicro_07–12	LMacro_02/03
MAYO00038	LShaftMicro_13–18	LMacro_03/04
MAYO00038	LShaftMicro_19–24	LMacro_04
MAYO00038	RShaftMicro_01–06	RMacro_01/02
MAYO00038	RShaftMicro_07–12	RMacro_02/03
MAYO00038	RShaftMicro_13–18	RMacro_03/04
MAYO00038	RShaftMicro_19–24	RMacro_04
MAYO00397	RBundle_01–07	RMacro_01
MAYO00397	LBundle_01–08	LMacro_01
MAYO00397	RShaft_01–03	RMacro_01/02
MAYO00397	RShaft_05–08	RMacro_02/03
MAYO00397	LShaft_01–03	LMacro_01/02
MAYO00397	LShaft_04–06	LMacro_02/03
MAYO00397	LShaft_07–09	LMacro_03/04
MAYO00448	ADShaft_01–04	PDMacro_01/02
MAYO00448	ADShaft_05–08	PDMacro_02/03
MAYO00448	ADShaft_09–12	PDMacro_03/04
MAYO00448	ADShaft_13–16	PDMacro_04
MAYO00452	RBundle_01–08	RMacro_01
MAYO00452	LBundle_01–08	LMacro_01
MAYO00453	RBundle_01–08	RMacro_01
MAYO00453	LBundle_01–08	LMacro_01
MAYO00453	RShaft_01–03	RMacro_01/02
MAYO00453	RShaft_04–06	RMacro_02/03
MAYO00453	RShaft_07–09	RMacro_03/04
MAYO00453	LShaft_01–03	LMacro_01/02
MAYO00453	LShaft_04–06	LMacro_02/03
MAYO00453	LShaft_07–09	LMacro_03/04

exhibited a post-UFO power increase in the high-gamma range. This boost closely follows the shape predicted by the Green's function envelope, consistent with the presence of a scale-propagated echo.

Raw waveforms, high-frequency components, and spatiotemporal signatures for example UFOs for each patient are shown in Fig. 2.

We note that statistical significance is assessed at the micro-macro pair level by averaging scores across all UFOs, whereas Fig. 1 illustrates a single representative UFO event for visualization purposes.

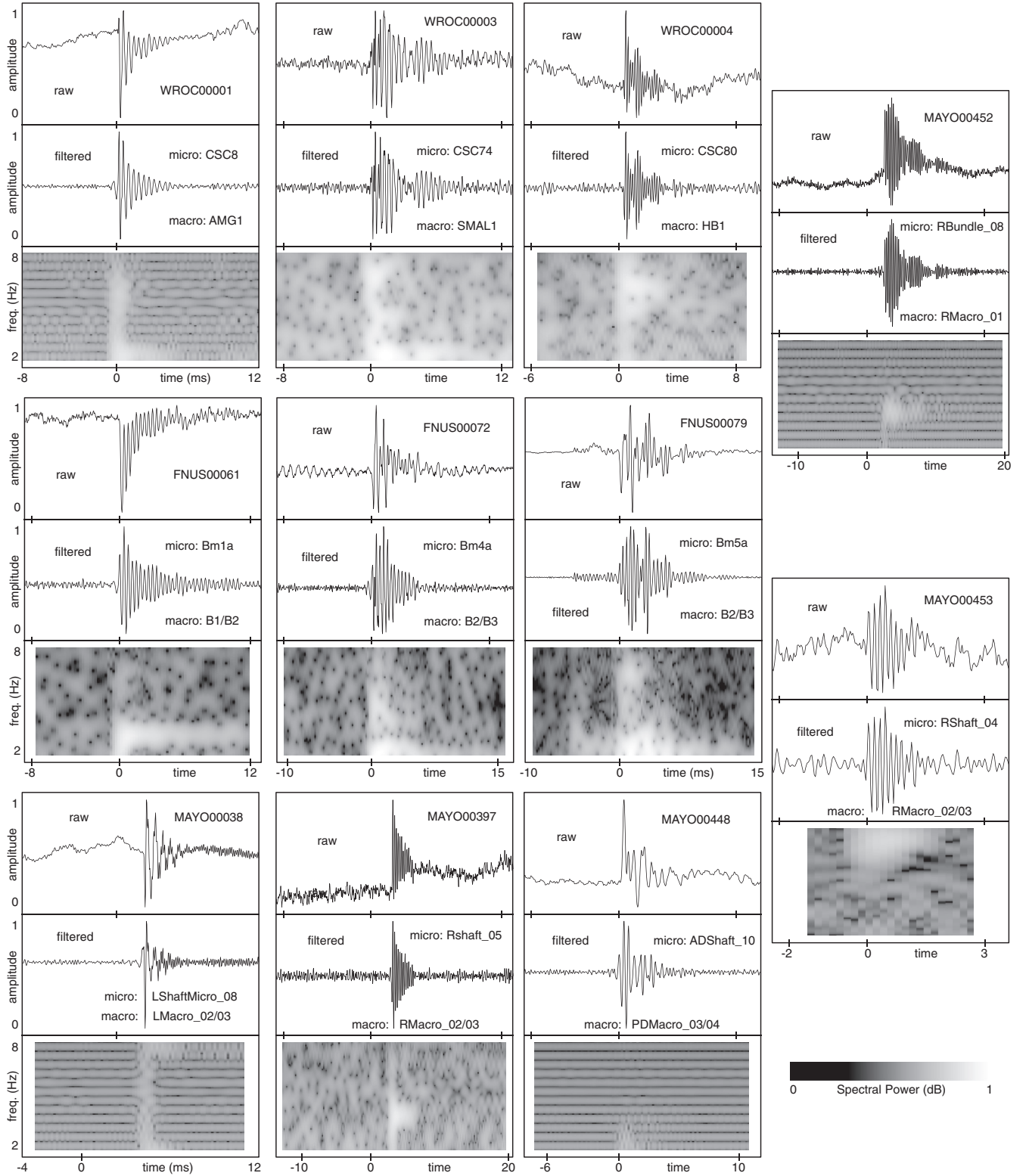


FIG. 2. A representative normalized UFO is shown for each patient. Top panel: raw microelectrode recording centered on the detected UFO. Middle panel: the same signal after 2–8 kHz band-pass filtering. Bottom panel: the corresponding time-frequency spectrogram.

IV. DISCUSSION

We introduce a geometric framework for understanding how ultrafast oscillations (UFOs) transform as they propagate across scales in the brain. The theoretical foundation lies

in replacing the spatial translation operator in the classical damped wave equation with the dilation operator. This yields a cross-scale wave equation whose Green's function describes how microscopic kilohertz bursts give rise to lower-frequency macroscopic echoes. The model predicts a characteristic

spectral signature at the macroscale, with the present findings providing initial empirical support for this prediction.

Statistically significant echoes were rare, being confirmed in 2 out of 100 micro-macro electrode pairs in two out of eleven patients. It is possible that electrode arrays with denser, colocalized micro- and macrocontacts could yield higher detection rates. Furthermore, stimulation experiments that induce controlled microscale high-frequency bursts would provide a complementary test, allowing systematic investigation of the conditions under which predicted macroscopic echoes appear.

The microchannels involved in the two significant micro-macro pairs, CSC73–SMAL1 and CSC65–AMG1 (see Table III), did not show higher average firing rates than the other recorded microchannels. This rules out the possibility that the observed echo was simply a consequence of unusually high local spiking activity [14]. Furthermore, we found no correlation between neuronal spiking recorded on these microchannels and high-frequency oscillations (HFOs) detected on the corresponding macroelectrodes. This further argues against the possibility that the observed echo simply reflects the known relationship between neuronal spiking and HFOs.

There are certain limitations that should be noted with respect to this study. For instance, the number of significant detections was small, and larger datasets are therefore required to establish the generalizability of the predicted spectral signature. Furthermore, recordings are only available at micro- and macroelectrode scales—but not at intermediate scales. This means that we can only sample the transformation

at the available electrode levels, rather than observing the transformation take place across the entire scale hierarchy.

There is a growing body of evidence that microscale bursting can give rise to macroscopically observable brain dynamics. For instance, seizures confined to single microelectrodes have been shown to evolve into macroscopic clinical events [5]. We contribute to this field by introducing a mathematical framework for cross-scale propagation. Specifically, we provide a mechanistic link between microscale UFOs and macroscale spectral responses, offering a testable framework for future investigations of multiscale brain dynamics.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available because of legal restrictions preventing unrestricted public distribution. The data are available from the authors upon reasonable request. All relevant code is made available at [15,16].

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