

Beyond holography: The entropic quantum gravity foundations of anisotropic diffusionGinestra Bianconi ^{*}*School of Mathematical Sciences, Queen Mary University of London, London E1 4NS, United Kingdom*

(Received 17 March 2025; accepted 3 September 2025; published 16 October 2025)

Recently, thanks to the development of artificial intelligence (AI), there is increasing scientific attention in establishing the connections between theoretical physics and AI. Traditionally, these connections have been focusing mostly on the relation between string theory and image processing and involve important theoretical paradigms such as holography. Recently, G. Bianconi has formulated the Gravity from Entropy (GfE) approach to quantum gravity in which gravity is derived from the geometric quantum relative entropy (QRE) between two metrics associated with the Lorentzian spacetime. Here, it is demonstrated that the famous Perona-Malik algorithm for image processing is the gradient flow of the GfE action in its simple warm-up scenario. Specifically, this algorithm is the outcome of the maximization of the GfE action between two Euclidean metrics: the one in support of the image and the one induced by the image. As the Perona-Malik algorithm is known to preserve sharp contours, this implies that the GfE action does not, in general, lead to uniform images upon iteration of the gradient flow dynamics as it would be intuitively expected from entropic actions maximizing classical entropies. Rather, the outcome of the maximization of the GfE action is compatible with the preservation of complex structures. These results provide the geometrical and information theory foundations for the Perona-Malik algorithm and might contribute to establish deeper connections between GfE, machine learning, and brain research.

DOI: [10.1103/gkkt-yysp](https://doi.org/10.1103/gkkt-yysp)

Recently, there is an increasing recognition of the common mathematical foundations of theoretical physics and artificial intelligence (AI) algorithms [1–8]. Specifically, there is a growing consensus on the fundamental role of topology and geometry to inform the most recent developments of AI and network theory. This scientific interest is currently leading to the fast developments of very vibrant research fields, such as topological and geometrical machine learning [9,10] and to topological higher-order network dynamics [7,11,12] that might lead to a more comprehensive understanding of brain dynamics [13–17]. At the interface between problems in brain research and in AI, a key research question is whether theoretical physics can inspire geometric diffusion models [18,19] which can be crucial to developing the foundation of the next generation of unsupervised or self-supervised learning algorithms [20]. In addition to geometry and topology, information theory is also recognized as fundamental for both AI [21,22] and brain research [23]. In particular, the relative entropy, also known as Kullback-Leibler entropy, is acquiring a fundamental role in AI and has given rise to very successful theoretical concepts and algorithms as the information bottleneck principle [24] and diffusion models [25,26]. In this work, we define the geometric quantum relative entropy (QRE) in Euclidean spaces and we show that the QRE provides

solid information theory foundations to geometric anisotropic diffusion algorithms. By doing so, we establish the connection between these anisotropic diffusion algorithms and the recently proposed *Gravity from entropy* (GfE) approach to quantum gravity [27].

The relation between artificial intelligence, network theory, and theoretical physics has a long history. Theoretical physics inspiration from computer vision has led to important conceptual frameworks, such as the formulation of the holographic principle [28–30]. The holographic principle was originally motivated [28] to obtain the area law for the entropy of black-holes. In general, this principle states that our three-dimensional universe might be encoded in a two-dimensional surface as an hologram and leading to a very vibrant and active research direction in theoretical physics [29,31].

On the other side, important proposals to reconcile image processing algorithms and, in general, AI algorithm with theoretical physics are at the forefront of physics-inspired AI [18,19]. The root of this field can be traced back to the work [32] by Sochen, Kimmel, and Malladi. This work is the first work that has framed image processing into a differential geometry theory involving the metric induced by the image. Moreover, this work has proposed the adoption of the Polyakov action of string theory in order to formulate modified anisotropic diffusion models for image processing.

The cross-fertilization of ideas between theoretical physics and, specifically, quantum information and network science is also very fertile [33–39]. In this context, quantum entropy has found applications in the characterization of complex network structure, thanks to the use of the Von Neumann entropy associated with the graph Laplacian. This so-called Von Neumann entropy of networks has been originally proposed in Ref. [33]

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by Passerini and Severini and, since then, has found wide applications in the theory of simple and multilayer networks [34–36,38] and provides the underlying theoretical framework for the recently proposed renormalization group approach in network theory [37]. However, the quantum relative entropy that is so central in quantum information theory [40] has not yet been recognized to have wide applications in either AI or network science.

Recently, in Ref. [27], the author has combined quantum mechanics with gravity in a comprehensive statistical mechanics and quantum information theory framework known as *Gravity from Entropy* (GfE). Specifically, she has formulated a geometrical definition of quantum relative entropy called GQRE in Lorentzian spacetimes. In this framework, the metrics associated with spacetime are treated as quantum operators and the GfE action is determined by a Lagrangian given by the GQRE between the metric of the manifold and the metric induced by the matter field and curvature. Thus, the action of GfE is a quantum information theory action that fully captures the interplay between geometry and the matter fields defined on the manifold. Interestingly, the GQRE can also be calculated for the Schwarzschild black hole, giving rise to an area law for large Schwarzschild radius [41] without obeying the holographic principle.

Here, we demonstrate that the action of GfE is key for geometric anisotropic diffusion models. In particular, we reveal that in one of its simplest incarnations, the GfE action, associated to a GQRE Lagrangian defined on Euclidean spaces, provides the quantum information theory foundations for the famous Perona-Malik algorithm [42] for image processing. The Perona-Malik algorithm is a fundamental reference model in the field of image processing [43–45], which denoises an image by implementing a diffusion process. This diffusion is, however, anisotropic and takes place in the presence of a metric that is chosen to have an *ad hoc* functional form dependent on the contrast of the image. Previous attempts to relate the Perona-Malik algorithm to string theories and, in particular, the Polyakov action, [32] have not been able to justify the empirical choice of the functional form of the metric assumed in the Perona-Malik algorithm. Here, we provide solid quantum information theory foundations for the Perona-Malik algorithm by defining the GQRE in an Euclidean setting and showing that the Perona-Malik algorithm is given by the gradient flow of the GfE action proposed in Ref. [27]. The definition of the GQRE is, here, shown to be rooted in the theory of von Neumann algebras [46,47] and to constitute a significant advance with respect to the Araki entropy [48,49]. The GQRE provides an information theory quantity that is built from two metrics associated with the image: the true metric of the 2D flat Euclidean space that offers the support to the image and the metric induced by the image defined on it. Anisotropic diffusion emerges from the tension between these two metrics to reduce their differences, quantified through the GQRE Lagrangian of GfE. As the Perona-Malik is known to preserve sharp contours of the image, the GfE foundation of the Perona-Malik action also points out an important property of the GfE action. Indeed, the maximization of the GfE action is compatible with the preservation of both the structure and complexity of the image. This is a notable feature of the GfE action as it reveals that, contrary

to the expectations in scenarios when the classical entropy is maximized, the maximization of the GfE action, based on the GQRE Lagrangian, is compatible with nonhomogeneous and complex outcomes.

The Perona-Malik algorithm. We consider a 2D flat Euclidean manifold Ω of coordinates $\mathbf{r} = (x_1, x_2) \in \Omega$ with metric $g_{\mu\nu} = \eta_{\mu\nu}$ with $\eta_{\mu\nu} = 1$ if $\mu = \nu$ and $\eta_{\mu\nu} = 0$ otherwise. The infinitesimal distance ds between points in 2D is defined by this metric and obeys $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Note that the metric $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$ will be central to transform vectors in one-forms and vice versa by lowering or raising the indices, i.e.,

$$g_{\mu\nu}V^\nu = V_\mu, \quad g^{\mu\nu}V_\nu = V^\mu. \quad (1)$$

On top of this manifold, we define a function $\phi(\mathbf{r}) \in \mathbb{R}$ indicating the intensity of the color of the single color, black-white image in our simple setting. Given an initial noisy image determined by the function $\psi(\mathbf{r})$ the Perona-Malik algorithm [42,44,45] proposes to reconstruct the true image by performing a Laplace-Beltrami diffusion with metric. Specifically, the reconstructed image $\phi(\mathbf{r})$ is found by integrating the system

$$\begin{aligned} \frac{d\phi(\mathbf{r}, t)}{dt} &= \nabla_\mu \rho(|\nabla\phi|^2) \nabla^\mu \phi(\mathbf{r}, t) \\ \phi(\mathbf{r}, 0) &= \psi(\mathbf{r}), \end{aligned} \quad (2)$$

where the metric $\rho(|\nabla\phi|^2)$ is taken to be

$$\rho(|\nabla\phi|^2) = \frac{1}{1 + \alpha|\nabla\phi|^2}, \quad (3)$$

and where $\alpha \in \mathbb{R}^+$ is a parameter of the model. The power and beauty of this model is that the metric evolves together with the diffusion model and the reconstruction of the image. It is clearly highly desirable to derive the specific functional form of the metric $\rho(|\nabla\phi|^2)$ in a principled way. However, in the original work of Perona and Malik [42], there is no fundamental information theory principle driving this choice, which remains an *ad hoc* choice in order to achieve anisotropic diffusion and a good performance of the algorithm. Additionally, the string theory approach to anisotropic diffusions proposed in Ref. [32] does not provide this explanation.

As we will see in this article, this particular choice of the metric $\rho(|\nabla\phi|^2)$ is exactly what is predicted by the GfE action.

Induced metric. The set of points $(\mathbf{r}, \phi(\mathbf{r}))$ with $\mathbf{r} \in \Omega$ defines a 2D surface $\mathcal{K}$ immersed in 3D (see Fig. 1). At any given point of $\mathcal{K}$, the tangent vectors are given by $\mathbf{e}_1 = (1, 0, \nabla_{x^1}\phi)$ and $\mathbf{e}_2 = (0, 1, \nabla_{x^2}\phi)$. Let us assume that the 3D embedding space of $\mathcal{K}$ has a flat Euclidean metric with diagonal elements $(1, 1, \alpha)$, where α is a positive real constant. The infinitesimal distance $d\hat{s}$ between points $\mathbf{X}$ and $\mathbf{X} + \delta\mathbf{X}$ with $\delta\mathbf{X} = \mathbf{e}_1 dx^1 + \mathbf{e}_2 dx^2$ in $\mathcal{K}$ obeys $d\hat{s}^2 = G_{\mu\nu}dx^\mu dx^\nu$, where $G_{\mu\nu}$ are the elements of the real and symmetric rank 2 tensor which defines the induced metric on Ω . Specifically, we show that the induced metric $\mathbf{G}$ on the 2D manifold Ω , which provides the support of the image, is given by

$$G_{\mu\nu} = g_{\mu\nu} + \alpha \nabla_\mu \phi \nabla_\nu \phi. \quad (4)$$

Thus, the considered manifold Ω is associated with two metrics: the metric $g_{\mu\nu}$ and the metric $G_{\mu\nu}$. Note that in the

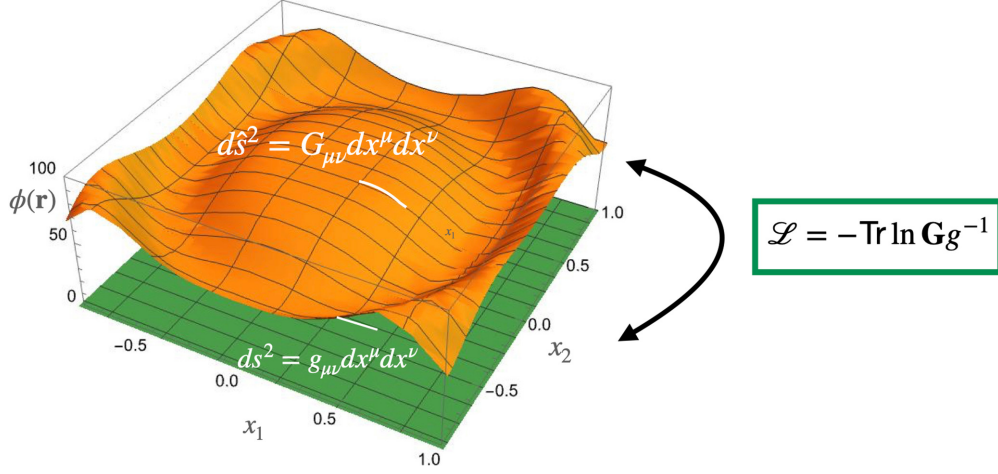


FIG. 1. Schematic representation of the metrics associated with a black-and-white image and the GQRE Lagrangian of the GfE action, which provides the foundations of the Perona-Malik algorithm. The flat 2D Euclidean manifold Ω (in green), which offers the support of the image, has points $\mathbf{r}$ of coordinates $\mathbf{r} = (x_1, x_2)$ and a Euclidean metric $g_{\mu\nu} = \eta_{\mu\nu}$. The infinitesimal distance ds between points in Ω is defined by this metric and obeys $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. The image is associated with the surface $\mathcal{K}$ (in orange) embedded in a flat 3D Euclidean metric, thus every point of $\mathcal{K}$ has coordinates $(\mathbf{r}, \phi(\mathbf{r}))$. The infinitesimal distance $d\hat{s}$ between two points in this surface obeys $d\hat{s}^2 = G_{\mu\nu} dx^\mu dx^\nu$, where $G_{\mu\nu}$, given by Eq. (4), is the symmetric rank 2 tensor that defines the metric induced by the surface $\mathcal{K}$ on the 2D flat support of the image Ω . The GfE action, associated to the Lagrangian $\mathcal{L}$, is given by the GQRE between the metrics $\mathbf{G}$ and $\mathbf{g}$. In this work, we show that the Perona-Malik algorithm is the gradient flow that maximizes the GfE action. Therefore, the Perona-Malik algorithm emerges from the principle of maximization of the GfE action between the metric induced by the image $\mathbf{G}_{\mu\nu}$ and the metric $g_{\mu\nu}$ of the 2D image support.

following we will use $\hat{\mathbf{G}}_{\mu\nu}$ when referring to either one of these two metrics.

According to the GfE approach here, we will treat both $\mathbf{g}$ and $\mathbf{G}$ as quantum operators and we will consider the action $\mathcal{S}$ given by the GQRE between these two metrics. We will show that the Perona-Malik algorithm can be obtained as the gradient flow maximizing the GfE action.

Eigenvalues of the metrics associated with the manifold. The GQRE will be defined in terms of the eigenvalues of the true metric $\mathbf{g}$ and the induced metric $\mathbf{G}$. In order to define the eigenvalues and eigenvectors of these two metrics in a rotationally invariant way, we define λ as an eigenvalue of $\hat{\mathbf{G}}_{\mu\nu}$ if it solves the eigenvalue problem

$$\hat{\mathbf{G}}_{\mu\nu} [V^{(\lambda)}]^\nu = \lambda V_\mu^{(\lambda)}. \quad (5)$$

Thus, this eigenvalue problem is the usual eigenvalue problem for the matrix $\hat{\mathbf{G}}_{\mu\nu}$, as the above equation reduces to

$$\hat{\mathbf{G}}_{\mu\nu} g^{\nu\rho} V_\rho = \lambda V_\mu. \quad (6)$$

It follows that all the eigenvalues λ' of $g_{\mu\nu}$ are equal to one $\lambda'_n = 1$, for $n \in \{1, 2\}$ independently of the choice of the metric $g_{\mu\nu}$. Instead, the eigenvalues λ and the associated (non-normalized) eigenvectors V_μ of the induced metrics $\mathbf{G}_{\mu\nu}$ are given by

$$\begin{aligned} \lambda_1 &= (1 + \alpha |\nabla \phi|^2), & V_\mu &= \nabla_\mu \phi, \\ \lambda_2 &= 1, & \left(\eta_{\mu\nu} - \frac{\nabla_\mu \phi \nabla_\nu \phi}{|\nabla \phi|^2} \right) V^\nu &. \end{aligned} \quad (7)$$

An action that is only dependent on these eigenvalues will be clearly rotational invariant.

The GfE action. The GfE action [27] is associated with a Lagrangian given by the GQRE between two Lorentzian metrics: the metric of the manifold and the metric induced by the

matter fields and curvature. In the context of the Perona-Malik algorithm, we consider the warm-up scenario of GfE where the two metrics associated with the GQRE are Euclidean of the metric induced by the matter-field and curvature is played by $\mathbf{G}_{\mu\nu}$, which is defined in Eq. (4) and schematically described in Fig. 1. The action $\mathcal{S}$ of GfE is given by

$$\mathcal{S} = \frac{1}{2} \int_\Omega \sqrt{|\mathbf{g}|} \mathcal{L} d\mathbf{r}, \quad (8)$$

where the Lagrangian $\mathcal{L}$ is given by the GQRE between $\mathbf{g}$ and $\mathbf{G}$. Anticipating the results of the next paragraph, the GQRE Lagrangian $\mathcal{L}$ in Euclidean space that encodes for the Perona-Malik algorithm is given by

$$\mathcal{L} = -\text{Tr} \ln \mathbf{G} \mathbf{g}^{-1}. \quad (9)$$

This definition of the GQRE implies that the GQRE that encodes for the Perona-Malik algorithm can also be expressed in terms of the eigenvalues λ_n of $\mathbf{G}$ and the eigenvalues λ'_n of $\mathbf{g}$ in a familiar form for the relative entropy. Indeed, we have

$$\mathcal{L} = - \sum_{n=1}^2 \ln \lambda_n = \sum_{n=1}^2 \lambda'_n (\ln \lambda'_n - \ln \lambda_n), \quad (10)$$

where we have used $\lambda'_n = 1$ for $n \in \{1, 2\}$. Using the explicit expression of λ_n given by Eq. (7), we obtain

$$\mathcal{L} = -\ln(1 + \alpha |\nabla \phi|^2), \quad (11)$$

which constitutes the Euclidean version of the warm-up scenario of GfE proposed in Ref. [27].

First principles derivation of the GQRE in Euclidean space Eq. (9) and its connection with Araki entropy. In this paragraph, we formulate the GQRE in Euclidean space. We discuss the quantum information theory foundations of the GQRE Lagrangian $\mathcal{L}$ [Eq.(9)] and we relate it to the Araki entropy [48,49].

Consistently with the GfE approach [27], here we consider a Dirac-Kähler [50,51] interpretation of the gradient in which $\nabla_\mu \phi$ describes the component of a one-form. Therefore, in order to provide the theoretical foundations for our definition of the GQRE, we will consider Hilbert spaces formed by the direct sum of a zero-form and a one-form. The generic vector $|\Psi\rangle$ of the considered Hilbert space $\mathcal{H}$ is given by

$$|\Psi\rangle = \phi \oplus \omega_\mu dx^\mu. \quad (12)$$

Thus, both the image (encoded by $\phi \oplus 0_\mu dx^\mu$) and the gradient of the image encoded by $0 \oplus \nabla_\mu \phi dx^\mu$ can be interpreted as elements of this Hilbert space. The scalar product between $|\Phi\rangle$ and another generic vector $|\Phi\rangle$, given by $|\Phi\rangle = \hat{\phi} \oplus \hat{\omega}_\mu dx^\mu$, is defined as

$$\langle\langle\Psi, \Phi\rangle\rangle = \int \sqrt{-|g|} (\bar{\phi}\hat{\phi} + \bar{\omega}_\mu \hat{\omega}^\mu) d\mathbf{r}, \quad (13)$$

where $\hat{\omega}^\mu = g^{\mu\rho} \hat{\omega}_\rho$. Thus, the metric tensor $\tilde{g}^{-1}$ associated with this scalar product is given by

$$\tilde{g}^{-1} = 1 \oplus g^{\mu\nu} dx_\mu \otimes dx_\nu. \quad (14)$$

All vectors $|\Phi\rangle$ in the Hilbert space $\mathcal{H}$ must satisfy

$$\langle\langle\Phi|\Phi\rangle\rangle < \infty. \quad (15)$$

Starting from the induced metric $\mathbf{G}$, we can construct the topological induced metric $\tilde{\mathbf{G}}$, given by

$$\tilde{\mathbf{G}} = 1 \oplus G_{\mu\nu} dx^\mu \otimes dx^\nu. \quad (16)$$

This topological induced metric can be interpreted as a quantum operator $\tilde{\mathbf{G}} : \mathcal{H} \rightarrow \mathcal{H}$, where $\tilde{\mathbf{G}} \cdot |\Phi\rangle \in \mathcal{H}$,

$$\tilde{\mathbf{G}} \cdot |\Phi\rangle = \phi \oplus G_{\mu\nu} \omega^\nu dx^\mu. \quad (17)$$

The metric $\tilde{g}$ of the manifold Ω can be used to define a dual Hilbert space $\mathcal{H}^*$. To this end, we define the dual of $|\Psi\rangle$ as $|\Psi^*\rangle$ and the dual of $|\Phi\rangle$ as $|\Phi^*\rangle$, given by

$$|\Psi^*\rangle = \phi \oplus \omega^\mu dx_\mu, \quad |\Phi^*\rangle = \hat{\phi} \oplus \hat{\omega}^\mu dx_\mu, \quad (18)$$

where $\omega^\mu = g^{\mu\rho} \omega_\rho$, $\hat{\omega}^\mu = g^{\mu\rho} \hat{\omega}_\rho$.

The scalar product $\langle\langle\Psi^*, \Phi^*\rangle\rangle_*$ is mediated by $\tilde{g}$, given by

$$\tilde{g} = 1 \oplus g_{\mu\nu} dx^\mu \otimes dx^\nu \quad (19)$$

and satisfies

$$\langle\langle\Psi, \Phi\rangle\rangle = \langle\langle\Psi^*, \Phi^*\rangle\rangle_*. \quad (20)$$

The dual operator $\tilde{\mathbf{G}}^* : \mathcal{H}^* \rightarrow \mathcal{H}^*$ of $\tilde{\mathbf{G}}$ is given by

$$\tilde{\mathbf{G}}^* = 1 \oplus [G^*]^{\mu\nu} dx_\mu \otimes dx_\nu, \quad (21)$$

which must satisfy

$$\langle\langle\Psi, \tilde{\mathbf{G}} \cdot \Phi\rangle\rangle = \langle\langle\tilde{\mathbf{G}}^* \cdot \Psi^*, \Phi^*\rangle\rangle_* \quad (22)$$

for any arbitrary choices of $|\Psi\rangle$ and $|\Phi\rangle$. Here, the action of $\tilde{\mathbf{G}}^*$ on the generic dual vector $\langle\Phi^*|$ obeys

$$\tilde{\mathbf{G}}^* \cdot |\Phi^*\rangle = \phi \oplus [G^*]_{(1)}^{\mu\nu} \omega_\nu dx_\mu. \quad (23)$$

Thus, using the symmetry of $G_{\mu\nu}$, we conclude that $\tilde{\mathbf{G}}^*$ is related to $\tilde{\mathbf{G}}$ by

$$\tilde{\mathbf{G}}^* = \tilde{g}^{-1} \tilde{\mathbf{G}} \tilde{g}^{-1}, \quad (24)$$

indicating that

$$[G^*]^{\mu\nu} = g^{\mu\rho} G_{\rho\sigma} g^{\nu\sigma}. \quad (25)$$

The topological metrics $\tilde{\mathbf{G}}$ and $\tilde{\mathbf{G}}^*$ that we consider in this work are, respectively, elements of the algebras $\mathfrak{U}$ and $\mathfrak{U}^*$ which generalize the C^* algebra [47]. The norm associated with the topological metric $\tilde{\mathbf{G}} \in \mathfrak{U}$ is equal to the norm associated with its dual, i.e., $\|\tilde{\mathbf{G}}\| = \|\tilde{\mathbf{G}}^*\|$ with

$$\|\tilde{\mathbf{G}}\| = \|\tilde{\mathbf{G}}^*\| = \int \sqrt{|-g|} \text{Tr}_F(\tilde{\mathbf{G}} \tilde{\mathbf{G}}^*) d\mathbf{r}, \quad (26)$$

where $\text{Tr}_F(\tilde{\mathbf{G}} \tilde{\mathbf{G}}^*)$ is given by

$$\text{Tr}_F(\tilde{\mathbf{G}} \tilde{\mathbf{G}}^*) = 1 + G_{\mu\nu} G^{\nu\mu}. \quad (27)$$

For a topological metric $\tilde{\mathbf{G}}$ interpreted as a quantum operator in $\mathfrak{U}$, we define the square root of the modular operator $\Delta_{\tilde{\mathbf{G}},g}^{1/2} : \mathcal{H} \rightarrow \mathcal{H}$ as

$$\Delta_{\tilde{\mathbf{G}},g}^{1/2} = \sqrt{\tilde{\mathbf{G}} \tilde{\mathbf{G}}^*} = \tilde{\mathbf{G}} \tilde{g}^{-1}, \quad (28)$$

where the last identity is derived under the assumption that $\tilde{\mathbf{G}}$ is positively definite, i.e., it has only positive eigenvalues. By this notation, we indicate the square root modular operator $\Delta_{\tilde{\mathbf{G}},g}^{1/2}$ acting on the topological field $|\Phi\rangle$ as

$$\Delta_{\tilde{\mathbf{G}},g}^{1/2} |\Phi\rangle = \phi \oplus [G_{(1)}]_{\mu\rho} g^{\rho\nu} \omega_\nu dx^\mu. \quad (29)$$

Using the flattened trace (Tr_F) formalism introduced in Ref. [27], we obtain that the Lagrangian $\mathcal{L}$ indicating the GQRE can be expressed as

$$\mathcal{L} = -\text{Tr}_F \ln \Delta_{\tilde{\mathbf{G}},g}^{1/2} = -\text{Tr} \ln \mathbf{G} g^{-1}, \quad (30)$$

where the trace Tr of the last expression indicates the usual trace of a matrix. Therefore, the action $\mathcal{S}$, defined in Eq. (8) and associated with the GQRE Lagrangian given by Eq. (9), extends the definition of Araki [48,49] quantum relative entropy and provides geometrical definition of this entropy by treating metrics as quantum operators.

The Perona-Malik algorithm as the gradient flow of the GQRE. The GfE action, associated with the Lagrangian given by Eq. (11), expressing the GQRE between the metric $g_{\mu\nu} = \eta_{\mu\nu}$ of the 2D support of the image and the induced metric $G_{\mu\nu}$, defined in (4), is given by

$$\mathcal{S} = -\frac{1}{2} \int_\Omega d\mathbf{r} \ln(1 + \alpha |\nabla \phi|^2). \quad (31)$$

Thus, the Perona-Malik algorithm can be easily shown to be the gradient flow maximizing this action, which is given by

$$\frac{d\phi(\mathbf{r}, t)}{dt} = \frac{\delta \mathcal{S}}{\delta \phi(\mathbf{x}, t)}. \quad (32)$$

Indeed, in this way we get the following dynamical equations:

$$\frac{d\phi(\mathbf{r}, t)}{dt} = \alpha \nabla_\mu \rho (|\nabla \phi|^2) \nabla^\mu \phi(\mathbf{r}, t), \quad (33)$$

where $\rho(|\nabla \phi|^2)$ is given by Eq. (3), i.e.,

$$\rho(|\nabla \phi|^2) = \frac{1}{1 + \alpha |\nabla \phi|^2}, \quad (34)$$

with initial condition $\phi(\mathbf{r}, 0) = \psi(\mathbf{r})$. Therefore, upon a rescaling of the time $t \rightarrow t/\alpha$, we recover the Perona-Malik algorithm defined in Eq. (2). As anticipated above, the GfE action, given by the GQRE between the metric g and the metric induced by the image $\mathbf{G}$ provides the information theory principle which justifies the choice of $\rho(|\nabla\phi|^2)$.

Conclusions. This work shows that the GfE approach that gives rise to modified gravity provides the entropic quantum gravity foundations of the Perona-Malik algorithm for anisotropic diffusion. In particular, we have revealed that the Perona-Malik algorithm is the gradient flow that maximizes the GfE action whose Lagrangian is given by the GQRE between the flat 2D metric $g_{\mu\nu} = \eta_{\mu\nu}$ and the metric $G_{\mu\nu}$ induced by the image. This implies that the Perona-Malik algorithm can be interpreted as the outcome of the tension between the metric of the flat 2D support of the image and the metric induced by the image on the 2D plane trying to maximize their associated GfE action. Interestingly, the maximization of the GfE action is compatible with heterogeneous and complex images with sharp contours. This result is in sharp contrast with the expectations arising from the classical maximum entropy principle and might be reflected also in the properties of the solutions of the GfE modified gravity equations. Therefore, this result might indicate relevant consequences of adopting the GQRE as the Lagrangian for both quantum gravity and image processing algorithms.

From the point of view of artificial intelligence, our findings establish solid quantum information-theoretic grounds for anisotropic diffusion and the Perona-Malik algorithm, justifying the ad hoc choice for the functional expression of the metric adopted by Perona and Malik. From the point of view of entropic quantum gravity, these findings provide a rather immediate application of the GfE approach to machine learning, opening new perspectives on the development of the next generation of AI diffusion algorithms. Moreover, in this application, the 2D metric associated with the image is flat

and Euclidean and remains unchanged during the learning of the image, while, going beyond the warm-up scenario, the GfE approach envisages that this metric is associated with the curvature and can evolve over time. Therefore, future research could explore the full potential of GfE action to develop the next generation of AI algorithms.

Based on these considerations, our expectations are that the GfE approach could provide new perspectives on physics inspired artificial intelligence, unsupervised learning [20], and brain research. On one side, these results might lead to the formulation of a new generation of geometric diffusion algorithms. On the other side, they might inspire research at the interface between topological and geometrical learning and brain research [52–55], which could potentially capture, among the other things, the main mechanisms beyond brain illusions, such as the Kanizsa triangle [56]. Thus, the GfE approach might turn to be a fertile framework for proposing the next generation of diffusion models fully based on the information encoded in the geometrical description of data.

In summary, this work demonstrates the common foundations of GfE and anisotropic diffusion for image processing and indicates that the full GfE action could potentially offer valuable insights for developing future generations of AI algorithms.

Acknowledgments. The author acknowledges interesting scientific discussions with Giovanna Citti and Alessandro Sarti and thanks them for pointing out Ref. [32]. This work was partially supported by a grant from the Simons Foundation. The author would like to thank the Isaac Newton Institute for Mathematical Sciences, Cambridge, for support and hospitality during the programme Hypergraphs: Theory and Applications, where work on this paper was undertaken. This work was supported by EPSRC Grant No. EP/V521929/1.

Data availability. No data were created or analyzed in this study.

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