

Plasma discharge undulator: Concept, theory, and numerical study

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Plasma discharge devices have recently emerged as compact and versatile tools for particle beam manipulation. Building upon the active plasma lens (APL) and its curved extension, the active plasma bending, this work introduces the concept of the plasma discharge undulator (PDU). In a PDU, a high-current discharge within a capillary generates an azimuthal magnetic field providing strong linear focusing [$O(kT/m)$], while a controlled and periodical spatial modulation of the discharge axis acts as a geometric driving term. The resulting beam dynamics can be modeled as a forced harmonic oscillator, yielding a well-defined oscillation at wavelength λ_{PDU} , distinct from the natural betatron wavelength λ_β related to APL focusing. Proper injection conditions result in the suppression of collective betatron oscillations, significantly reducing the intrinsic undulator strength spread typical of conventional plasma undulators while allowing for matched beam transport thanks to strong APL focusing. Analytical models for particle trajectories and radiation emission are developed, and the one-dimensional requirements for free-electron laser (FEL) emission are evaluated, providing scaling relations and feasibility criteria for FEL operation in the proposed scheme. Theoretical estimates are validated through multiparticle simulations of beam dynamics and radiation emission, confirming that the PDU can operate in the short-period regime (λ_{PDU} of millimeters to centimeters) with tunable undulator strength K_{PDU} , supporting narrow-band radiation emission. Numerical studies further demonstrate that the PDU is capable of sustaining seeded longitudinal microbunching analogous to that occurring in conventional magnetic undulators. The PDU thus provides a pathway toward miniaturized, tunable, fully plasma based light sources with enhanced control over focusing and spectral properties.

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I. INTRODUCTION

Conventional plasma undulators (CPUs) [1–9] have attracted considerable interest due to their intrinsically short undulation period, which, for a given beam energy, leads to higher radiation intensity and frequency compared to conventional undulators (CUs). CPUs are also currently under investigation as potential compact free-electron lasing (FEL) systems [10–13]. A common feature of these devices is that the undulator period λ_U is replaced by the betatron wavelength $\lambda_\beta \propto \sqrt{\gamma/n_p}$, with γ being the beam's Lorentz factor and n_p being the plasma density. A feature related to this replacement is that, within a particle beam, each element exhibits a different oscillation amplitude r_0 and, consequently, a different undulator strength parameter $K = r_0 \gamma k_\beta$ [14], with k_β being the betatron wave number. For a Gaussian beam with any transverse size σ_r injected on axis into the electrostatic linear focusing field typical of the blowout regime [15,16], it can be shown that the corresponding undulator strength spread is fundamentally limited by $\sigma_K/K \geq 0.523$ for a flat matched Gaussian beam and $\sigma_K/K \geq 0.363$ for a symmetric matched

round Gaussian beam [17]. One possible way to mitigate this intrinsic spread is to inject the beam centroid off axis, thereby inducing a collective oscillation. However, unless the offset exceeds approximately $3\sigma_r$, some particles will remain in counterphase; moreover, this approach may also excite hosing instabilities [18] or damp out toward the axis, leading to a growth of transverse emittance, especially in the case of long propagation distances and/or multiple plasma stages in the presence of finite beam energy spread. Furthermore, energy spread represents an independent source of K spread by itself, affecting both the betatron wavelength and the relativistic Doppler shift, thereby broadening the emitted spectrum.

A key feature of CPUs leading to these intrinsic limitations is the absence of an external driving term in the oscillatory motion. In this work, a method is proposed to introduce such a forcing term, thereby stabilizing the emission process while also freeing the scheme from the need for a high-energy-density driver such as an intense laser pulse or a particle beam: the *plasma discharge undulator* (PDU). In this concept for a plasma-based undulator, the oscillating and focusing field is magnetic in nature, rather than electrostatic as in CPUs: Building upon the principles of the *active plasma lens* (APL) [19–21] and *active plasma bending* [22,23], the PDU relies on a high-current [$O(1\text{--}10)$ kA] discharge within a capillary previously filled with neutral gas and subsequently ionized by applying a high voltage [$O(10)$ kV]. The associated magnetic field follows a Biot-Savart configuration, with azimuthal field

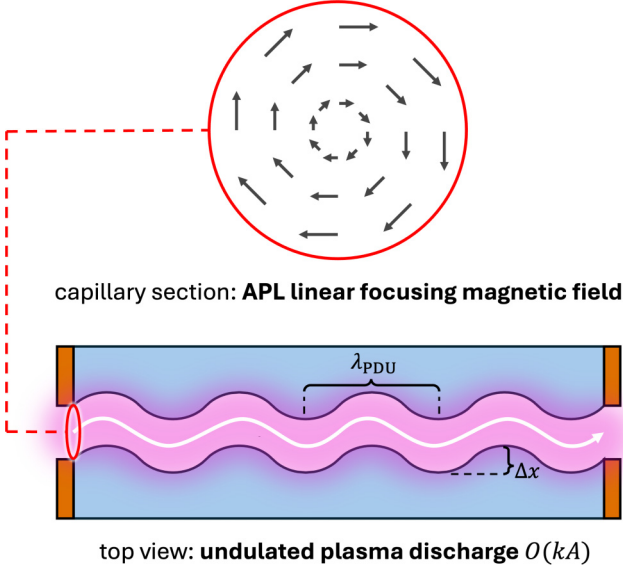


FIG. 1. Conceptual sketch of the plasma discharge undulator (PDU). Top: azimuthal magnetic field distribution induced by the discharge current in a capillary. Bottom: example of a possible geometrical implementation of the periodic discharge modulation.

lines around the capillary axis and a magnitude that increases linearly with radial distance (see Fig. 1, top), providing a linear focusing force on properly injected charged particles. The proposed PDU scheme consists of establishing a spatially sinusoidal (or possibly helical) modulation of the discharge current: The formation of such a modulated current profile would periodically displace the beam off axis with respect to the local zero of the discharge-generated focusing magnetic field, thereby imposing an alternating dipolar kick that results in the overall undulating motion of the beam. Moreover, the intense transverse focusing [$O(\text{kT/m})$] enables effective beam transport under matched APL-like conditions, providing an extra degree of control over the beam dynamics evolution—typically subject to drift or weak focusing in CUs, where matching is usually not achievable. One more advantage of the PDU configuration, now compared to CPUs, is that the undulator wavelength becomes independent of the beam energy and that the K spread is intrinsically lower; both feature reducing the broadening of the emitted spectrum.

Achieving the proposed current modulation is currently under investigation and could be realized through capillaries with sinusoidal or stepwise-offset geometries (see Fig. 1, bottom). A favorable aspect of this approach is the extensive body of work developed in recent years on plasma discharge capillaries [24–27], accompanied by a growing interest in nonlinear channel configurations [22,23]—of which the PDU represents a specific realization—and by the parallel ongoing development of numerical tools capable of reliably simulating these systems [28], including thermal transport and boundary effects. There are also sound physical reasons to expect that the discharge current should follow, at least partially, a spatial profile imposed by nonlinear capillary walls. The first argument is of thermal nature: Regardless of the current direction, the plasma temperature in a discharge capillary is minimized at the inner walls, which act as an

effective heat sink [29,30]. This gives rise to a transverse temperature profile that encodes information about the wall geometry into the plasma volume. Since the electrical resistivity decreases with increasing temperature, the region of maximum temperature—and therefore minimum resistance—is expected to be influenced by the wall shape, favoring a current distribution correlated with the capillary geometry. A second argument concerns the nature of the current carriers in the plasma, namely, electrons and ions, and their large mass disparity (exceeding a factor $m_i/m_e \approx 1836$). Typical discharge capillary parameters, with current densities $J = O(10^9\text{--}10^{10}) \text{ A/m}^2$ and plasma densities $n_p = O(10^{17}) \text{ cm}^{-3}$, correspond to electron drift velocities $v_d = O(10^5) \text{ m/s}$. Over a characteristic discharge duration $\tau = O(1) \mu\text{s}$, electrons can therefore travel distances of $O(1\text{--}10) \text{ cm}$, comparable with typical capillary lengths. Conversely, ions are several thousand times slower and can be regarded as effectively stationary within the capillary, remaining distributed according to the initial ionization profile, which for small capillaries typically fills the entire volume bounded by the walls. Any local charge separations relax on timescales of order $1/\omega_p \ll \tau$, effectively enforcing quasineutral plasma flow influenced by capillary geometry. This latter reasoning does not hold for possible negative charge accumulation at the capillary walls [31], which, if present, would further assist the spatial modulation of the current direction.

II. BEAM DYNAMICS

The single-particle trajectory model adopted for the PDU is a simple forced harmonic oscillator:

$$\frac{d^2x}{dz^2} = -k_\beta^2[x - x_{\text{eq}}(z)], \quad (1)$$

where k_β is the betatron wave number determined by the magnetic focusing strength [19,23],

$$k_\beta = \sqrt{\frac{e\mu_0 J}{2m_e c \gamma}}, \quad (2)$$

with e being the elementary charge, μ_0 being the vacuum magnetic permeability, J being the current density, and c being the speed of light, and where $x_{\text{eq}}(z)$ describes the spatial dependence of the magnetic field kernel imposed by the sinusoidal modulation of the discharge current,

$$x_{\text{eq}}(z) = \frac{\Delta x}{2} \cos(k_{\text{PDU}} z), \quad (3)$$

where $\Delta x/2$ is the oscillation amplitude, $k_{\text{PDU}} = 2\pi/\lambda_{\text{PDU}}$ is the undulation wave number, and λ_{PDU} is the PDU modulation wavelength (see Fig. 1, bottom). The solution for the transverse trajectory $x(z)$, assuming vanishing initial transverse velocity, is

$$x(z) = x_0 \cos(k_\beta z) + \frac{\Delta x}{2} \frac{k_\beta^2}{k_\beta^2 - k_{\text{PDU}}^2} [\cos(k_{\text{PDU}} z) - \cos(k_\beta z)], \quad (4)$$

which represents a superposition of oscillations at the betatron wavelength λ_β and at the undulation wavelength λ_{PDU} . By choosing the initial amplitude x_0 such that the λ_β oscillation

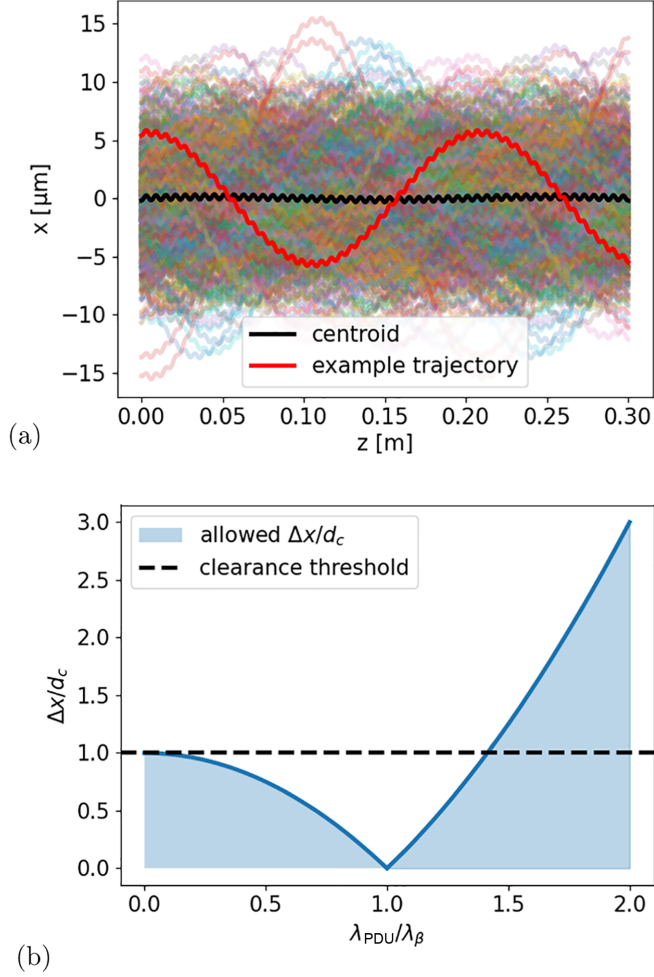


FIG. 2. (a) Matched beam propagation in the PDU field for $I = 4$ kA, $r_c = 0.5$ mm, $\Delta x = 0.5$ mm, $\lambda_{\text{PDU}} = 6$ mm, $\gamma = 2000$, and $Q = 100$ pC. Proper centroid injection offset $x_{\text{inj}} = x_0$ provides collective oscillation only at $\lambda = \lambda_{\text{PDU}}$ (solid black line). Individual particle trajectories are subject to mixed betatron-undulator motion (see the solid red line as an example), arising from their initial displacement from the beam centroid due to finite transverse emittance. (b) Geometric constraints on the capillary offset Δx normalized over the capillary diameter d_c as a function of $\lambda_{\text{PDU}}/\lambda_\beta$, showing that for $\lambda_{\text{PDU}} < \lambda_\beta$ the system necessarily requires clearance (i.e., a line of sight through the device). Note that the curves shown refer to only the beam centroid: Accounting for the finite transverse beam size imposes additional constraints, yielding a lower limit on the admissible $\Delta x/d_c$ ratio.

term is canceled, one obtains

$$x_0 = \frac{\Delta x}{2} \frac{k_\beta^2}{k_\beta^2 - k_{\text{PDU}}^2} := x_0. \quad (5)$$

Injecting the beam centroid at $x = x_0$ therefore eliminates any collective centroid oscillation at $\lambda = \lambda_\beta$, leaving only the motion at $\lambda = \lambda_{\text{PDU}}$ [see Fig. 2(a), solid black line]. Even under these conditions, individual particles still possess a finite transverse distribution, resulting in random injection amplitudes and, in general, giving an oscillatory motion which superposes λ_β and λ_{PDU} [see the highlighted example in Fig. 2(a), solid red line].

It is worth noting that, for the special case $\lambda_{\text{PDU}} = \lambda_\beta$, the optimal offset x_0 diverges (theoretical resonance), while it becomes negative (positive) for $\lambda_{\text{PDU}} < (>) \lambda_\beta$. A regime of primary interest is $\lambda_{\text{PDU}} < \lambda_\beta$, where particles undulate in counterphase with respect to the geometric forcing: This is the favorable regime in terms of emitted photon energy, and the counterphase condition allows for a natural line of sight for the emitted radiation to exit the device. While advantageous for dynamics and radiation emission, the off-axis injection condition $x_{\text{inj}} = x_0$ requires further contextualization with respect to the beam quality degradation mechanism discussed in Ref. [23] for curved plasma discharge channels. Equations (20) and (21) of Ref. [23] describe, respectively, the saturation rms beam size and the saturation length of an initially matched beam injected with a finite offset into a focusing channel with nonzero bending radius and energy spread: The saturation transverse size grows hyperbolically with both the injection offset and the energy spread (the latter entering only for a finite bending radius) due to collective betatron oscillations, while the saturation length is inversely proportional to the energy spread. In the PDU, the beam traverses alternating curved sections with opposite bending directions. In the regime $\lambda_{\text{PDU}} < \lambda_\beta$, the bending direction alternates on a scale shorter than the betatron wavelength, making the effective bending radius infinite (i.e., the alternating curvature averages to zero over some betatron wavelengths). In this case the degradation equations of Ref. [23] are modified as follows:

$$\sigma_{\text{sat}} = \sqrt{\sigma_M^2 + \frac{\Delta x_{\text{inj}}^2}{2}}, \quad (6)$$

$$L_{\text{sat}} = \frac{2\pi\gamma k_\beta \tilde{I}_0 r_c^2}{3I\sigma_{\Delta\gamma/\gamma}}, \quad (7)$$

where Δx_{inj} is the injection offset with respect to the equilibrium condition, $\tilde{I}_0 = 2\pi m_e c / \mu_0 e \approx 8.5$ kA is half the Alfvén current, r_c is the circular capillary section's radius, $\sigma_{\Delta\gamma/\gamma}$ is the rms relative energy spread, and σ_M is the APL transverse matching condition [19,23] [see Fig. 2(a), where a matched beam transverse envelope evolution is presented]:

$$\sigma_M = \left(\frac{2m_e c \gamma \epsilon_{\text{rms}}^2}{e \mu_0 J} \right)^{1/4}, \quad (8)$$

with ϵ_{rms} being the geometric root-mean-square emittance. Since x_0 is precisely the amplitude that suppresses collective betatron oscillations of the centroid, the injection offset reads $\Delta x_{\text{inj}} = x_{\text{inj}} - x_0$, meaning that injection at $x_{\text{inj}} = x_0$ is the condition which simultaneously minimizes beam size growth and emittance degradation. The PDU injection condition therefore plays a dual role: It defines the single-particle pure undulation regime and represents the optimal operating point for beam quality preservation in the presence of finite energy spread.

Moreover, the x_0 expression defines important design guidelines for the device. Imposing the condition that the beam centroid must not touch the capillary wall yields

$$\Delta x < \frac{2r_c}{\frac{k_\beta^2}{k_{\text{PDU}}^2 - k_\beta^2} + 1}, \quad \lambda_{\text{PDU}} < \lambda_\beta, \quad (9)$$

and

$$\Delta x < \frac{2r_c}{\frac{k_\beta^2}{k_\beta^2 - k_{\text{PDU}}^2} - 1}, \quad \lambda_{\text{PDU}} > \lambda_\beta. \quad (10)$$

In the former case, the total offset between adjacent sections always satisfies $\Delta x < 2r_c$ (forced clearance), while in the latter, the forced clearance condition $\Delta x < 2r_c$ applies for $\lambda_\beta < \lambda_{\text{PDU}} < \sqrt{2}\lambda_\beta$, and a no-clearance configuration ($\Delta x > 2r_c$) becomes possible for $\lambda_{\text{PDU}} > \sqrt{2}\lambda_\beta$ [see Fig. 2(b)]. In either case, practical radiation applications will require maintaining sufficient clearance through the device to allow the emitted photon beam to propagate without interception by the capillary walls. The matched beam size σ_M provided by Eq. (8) allows us to further refine the centroid clearance conditions of Eqs. (9) and (10): Since individual particles are displaced from the centroid by the finite beam emittance, the capillary wall must accommodate not only the centroid excursion but also the beam envelope. This translates into replacing $r_c \rightarrow (r_c - S)$ on the right-hand side of both equations, where S is a safety margin based on the matched beam size, e.g., $S = 3\sigma_M$, resulting in more stringent oscillation amplitude constraints.

III. RADIATION

The radiative properties of the PDU are directly determined by the features of particle trajectories. The primary parameter defining the emission features in an undulating system is the undulator strength parameter K . In the following sections, simple limiting expressions for the PDU undulator strength are first introduced, corresponding to the pure CU and CPU cases. These limits are then unified in a general expression for the effective undulator strength in the PDU configuration.

A. PDU undulator strength

The simplest expression for the PDU undulator strength is obtained by substituting x_0 from Eq. (5) and k_{PDU} into the standard formula [14,32], yielding

$$K_{\text{PDU}} = x_0 \gamma k_{\text{PDU}} = \frac{\gamma \Delta x}{\frac{\lambda_{\text{PDU}}}{\pi} - \frac{4\tilde{I}_0 \gamma}{J \lambda_{\text{PDU}}}}, \quad (11)$$

which should always be taken in absolute value. Equation (11) shows that K_{PDU} always grows linearly with the capillary amplitude offset Δx . In the regime $\lambda_{\text{PDU}} < \lambda_\beta$, the second term in the denominator is dominant, leading to a linear dependence $K_{\text{PDU}} \propto \lambda_{\text{PDU}} J \Delta x$ and removing any dependence on γ . Conversely, for $\lambda_{\text{PDU}} > \lambda_\beta$, one obtains $K_{\text{PDU}} \propto \gamma \Delta x / \lambda_{\text{PDU}}$.

B. Betatron undulator strength

An interesting feature of the PDU is its inherently two-color nature. Besides the forced λ_{PDU} oscillation, the fundamental λ_β betatron oscillation remains present. Although this component does not reach the submillimeter wavelengths typical of CPUs [8,9,33], the strong magnetic focusing fields involved yield betatron wavelengths down to a few centimeters long. Consequently, the PDU exhibits two reference

fundamental emission wavelengths and an undulator strength parameter limit expression, $(\lambda_{1,\text{PDU}}, K_{\text{PDU}})$ and $(\lambda_{1,\beta}, K_\beta)$, where K_β is associated with the average oscillation amplitude within a Gaussian transverse distribution [17]:

$$K_{\beta,\text{ave}} = \sqrt{\frac{9\pi}{8}} \sigma_r k_\beta \gamma, \quad (12)$$

$$K_{\beta,\text{mode}} = \sigma_r k_\beta \gamma := K_\beta,$$

where both expressions are derived assuming a matched beam condition.

C. Effective undulator strength

The two expressions for the undulator strength introduced above, K_{PDU} and K_β , can be shown to represent limiting cases of a more general formulation. In particular, it can be demonstrated [34] that for a single oscillatory relativistic trajectory the undulator strength parameter K is *unique* and fundamentally defined by the longitudinal component of the particle velocity:

$$K = \sqrt{(1 - \beta_z^2) \gamma^2 - 2}. \quad (13)$$

Under the paraxial approximation, the complete expression of K for a single particle propagating in the PDU field becomes

$$K \approx \gamma \sqrt{k_\beta^2 r_{\text{inj}}^2 + k_{\text{PDU}}^2 x_0^2}, \quad (14)$$

where

$$r_{\text{inj}} = \sqrt{[(x_{\text{inj}} - x_0)^2 + y_{\text{inj}}^2] + \frac{(x_{\text{inj}}'^2 + y_{\text{inj}}'^2)}{k_\beta^2}} \quad (15)$$

and x_{inj} , y_{inj} , x_{inj}' , and y_{inj}' are the particle's transverse injection coordinates and slopes. Interestingly and somewhat intuitively, $K \rightarrow K_{\text{PDU}}$ for vanishing betatron oscillation amplitude when only the λ_{PDU} oscillation survives; conversely, for increasing betatron amplitude $K \rightarrow r_{\text{inj}} \gamma k_\beta := K_{\beta,\text{inj}}$ with a hyperbolic trend, restoring the basic single-particle betatron undulator strength. An example of this behavior is shown in Fig. 3, where the PDU single-particle undulator strength is plotted as a function of transverse injection amplitude. Extending the analysis to a matched Gaussian particle beam, the first step is to normalize the transverse coordinates over the matched beam size σ_M , defining a new variable $\tilde{r}_{\text{inj}}$ which is distributed as a Rayleigh χ_4 function. The resulting K distribution can be analytically evaluated for $\lambda_{\text{PDU}} < \lambda_\beta$, yielding the following average and standard deviation estimators:

$$\mu_K = \frac{3}{2} K_{\text{PDU}} + \frac{\gamma \mathcal{B}}{4 k_\beta \sigma_M},$$

$$\sigma_K = \sqrt{|K_{\text{PDU}}^2 + 4K_\beta^2 - \mu_K^2|}, \quad (16)$$

with

$$\alpha = -\frac{K_{\text{PDU}}}{\sqrt{2} K_\beta},$$

$$\mathcal{Z} = \frac{3K_\beta^2 - K_{\text{PDU}}^2}{\gamma^2},$$

$$\mathcal{B} = \sqrt{2\pi} \mathcal{Z} \exp[\alpha^2] (1 + \text{erf}[\alpha]). \quad (17)$$

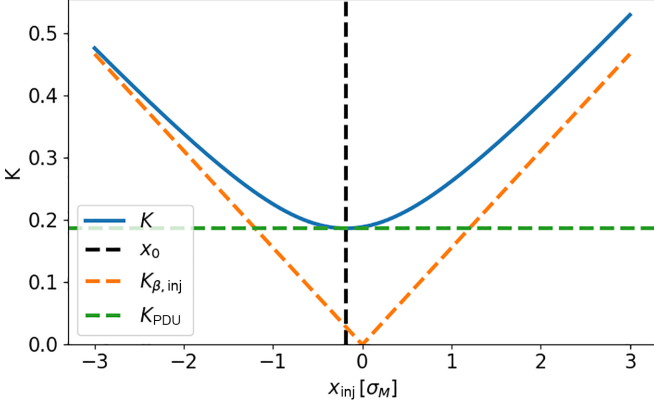


FIG. 3. Comparison of the two asymptotic regimes of the PDU single particle undulator strength K (solid blue line) as a function of the transverse injection position x_{inj} (expressed in units of σ_M) in the plasma channel, where $x_{\text{inj}} = x_0$ (vertical dashed black line) denotes the optimal injection amplitude that suppresses collective betatron oscillations and yields pure undulator motion of the beam centroid. For small amplitudes, K converges to the forced value K_{PDU} (horizontal dashed green line); for increasing amplitudes, it approaches the betatronic limit $K_{\beta,\text{inj}} = r_{\text{inj}} \gamma k_\beta$ (dashed orange line), following a hyperbolic trend.

By evaluating the limits for $\alpha \rightarrow 0$ ($K_{\text{PDU}} \ll K_\beta$) and $\alpha \rightarrow -\infty$ ($K_{\text{PDU}} \gg K_\beta$), one can assess the behavior of the PDU compared with a CPU.

For $K_{\text{PDU}} \ll K_\beta$,

$$\begin{aligned} \mu_K &\rightarrow \sqrt{\frac{9\pi}{8}} K_\beta = K_{\beta,\text{ave}}, \\ \sigma_K &\rightarrow \sqrt{4 - \frac{9\pi}{8}} K_\beta := \sigma_{K_\beta}, \\ \frac{\sigma_K}{\mu_K} &\rightarrow \sqrt{\frac{32}{9\pi}} - 1 \approx 0.36. \end{aligned} \quad (18)$$

In contrast, for $K_{\text{PDU}} \gg K_\beta$,

$$\begin{aligned} \mu_K &\rightarrow K_{\text{PDU}}, \\ \sigma_K &\rightarrow 0, \\ \frac{\sigma_K}{\mu_K} &\rightarrow 0. \end{aligned} \quad (19)$$

Equation (19) highlights that the regime $K_{\text{PDU}} \gg K_\beta$ is particularly advantageous in terms of radiation quality, as the K spread asymptotically tends to zero. Assuming all other beam and device parameters are fixed, the condition of Eq. (19) is satisfied for a normalized emittance,

$$\epsilon_n \ll \left(\frac{I}{I_0}\right)^{3/2} \frac{\lambda_{\text{PDU}}^2 \Delta x^2}{16\pi^2 r_c^3 \sqrt{\gamma}} := \epsilon_{\text{PDU}}. \quad (20)$$

For given beam and device parameters, less restrictive emittance conditions can therefore be achieved by increasing the discharge current and/or reducing the capillary radius, with $\epsilon_{\text{PDU}} \propto J^{3/2}$.

D. PDU one-dimensional FEL conditions

Relying on established results for one-dimensional (1D) FEL lasing conditions [34] together with the beam matching constraints within the PDU, it is possible to assess the feasibility of FEL processes in this configuration. Given the nature of the PDU device, the FEL emittance constraint must include the effects of betatron motion, yielding [34]

$$\epsilon_{\text{rms}} < \frac{\beta_{\text{ave}}}{2\sqrt{2}\gamma^2} \rho_{\text{FEL}}, \quad (21)$$

where β_{ave} is the average betatron function and ρ_{FEL} is the Pierce parameter. Assuming a matched beam and recalling that $\sigma_M = \sqrt{\epsilon_{\text{rms}} \beta_{\text{ave}}} = \sqrt{\epsilon_{\text{rms}} / k_\beta}$ and that in the absence of energy spread $\epsilon_n = \gamma \epsilon_{\text{rms}}$, one obtains the following limit on the normalized emittance:

$$\epsilon_n < \frac{\rho_{\text{FEL}}}{2\sqrt{2}\gamma k_\beta}. \quad (22)$$

Equation (22) can then be evaluated using the most appropriate expression for ρ_{FEL} for the given case. For simplicity, the one-dimensional formulation reads [34]

$$\rho_{\text{FEL},1\text{D}} = \frac{1}{4\pi\sqrt{3}} \frac{\lambda_{\text{PDU}}}{L_{g,1\text{D}}}, \quad (23)$$

where the 1D gain length is

$$L_{g,1\text{D}} = \frac{1}{\sqrt{3}} \left[\frac{4\gamma^3 m_e}{\mu_0 \mu_K^2 e^2 k_{\text{PDU}} n_{\text{beam}}} \right]^{1/3}, \quad (24)$$

with n_{beam} denoting the beam number density and μ_K being the mean undulator strength from Eq. (16). By estimating the matched beam density as $n_{\text{beam}} = Q/Ve$, where Q is the total bunch charge and $V = 36\pi R \sigma_M^3$ is the 3σ beam volume, with $R = \sigma_z / \sigma_M$ being the beam aspect ratio, the emittance condition for the FEL instability growth becomes

$$\epsilon_n \lesssim \frac{2}{25} \frac{1}{\gamma k_\beta^{1/3}} \left(\frac{\mu_0 e Q \mu_K^2}{m_e k_{\text{PDU}}^2 R} \right)^{2/9} := \epsilon_{\text{FEL}}. \quad (25)$$

The two emittance constraints given by Eqs. (20) and (25) can be combined to yield the general condition

$$\epsilon_n < \min(\eta \epsilon_{\text{PDU}}, \epsilon_{\text{FEL}}), \quad (26)$$

with $\eta \ll 1$. The distinction between the regimes of validity of the two expressions can be expressed in terms of the bunch charge Q and the beam aspect ratio R :

$$\begin{aligned} \left. \frac{Q}{R} \right|_{\text{lim}} &= \frac{144\sqrt{2}\pi m_e}{\mu_0 e k_{\text{PDU}}^9} [-\eta \gamma \Delta x k_\beta^{4/3} (k_\beta^2 - k_{\text{PDU}}^2)]^{9/2} \\ &\times \left(-\frac{\Delta x \gamma k_\beta^2}{k_\beta^2 - k_{\text{PDU}}^2} \right)^{5/2}. \end{aligned} \quad (27)$$

For $Q/R < Q/R|_{\text{lim}}$, one obtains $\epsilon_{\text{FEL}} < \eta \epsilon_{\text{PDU}}$, and therefore, the correct condition in Eq. (26) is $\epsilon_n < \epsilon_{\text{FEL}}$, with the substitution $\mu_K \rightarrow K_{\text{PDU}}$. Conversely, for $Q/R > Q/R|_{\text{lim}}$, the relation $\epsilon_{\text{FEL}} > \eta \epsilon_{\text{PDU}}$ holds, and the correct condition in Eq. (26) becomes $\epsilon_n < \eta \epsilon_{\text{PDU}}$, ensuring both $\mu_K \rightarrow K_{\text{PDU}}$ and the theoretical onset of the FEL instability.

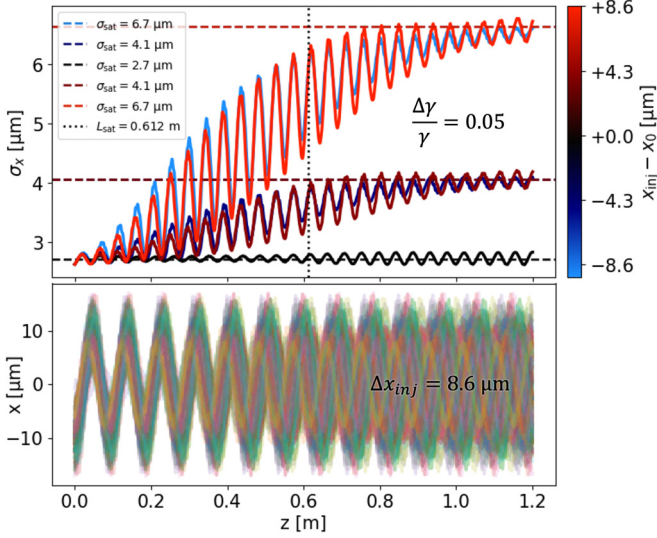


FIG. 4. Transverse beam size evolution in the undulation plane for $\gamma = 2000$, $\Delta\gamma/\gamma = 0.05$, $I = 20$ kA, $r_c = 0.5$ mm, $\lambda_{\text{PDU}} = 6$ mm, $\Delta x = 0.5$ mm, and $\epsilon_n = 1$ mm mrad. Top: beam size σ_x as a function of propagation distance for several injection offsets $\Delta x_{\text{inj}} = x_{\text{inj}} - x_0$ (color scale), with dashed horizontal lines indicating the corresponding theoretical saturation values [Eq. (6)] and the dotted vertical line marking the saturation length [Eq. (7)]. Bottom: single-particle trajectories for $\Delta x_{\text{inj}} = 8.6$ μm , showing the transition from collective centroid betatron oscillations to a constant-envelope evolution recentered at the equilibrium position.

IV. NUMERICAL EVALUATION

In this section, a numerical evaluation of the dynamical and radiative properties of the proposed PDU device is presented. The analysis is carried out using the simulation code RADYNO [8,17,35], which implements fully relativistic particle tracking and radiation postprocessing based on the calculation of Liénard-Wiechert potentials [14]. Both single-particle and beam radiation spectra are investigated, together with the formation of longitudinal microbunching arising from the interaction between the electron beam and an electromagnetic field during propagation through the PDU. Throughout this section, the simulation of relativistic beam dynamics and radiation emission is treated in a decoupled manner: Radiation spectra are computed in postprocessing from the particle trajectories, and the evaluation of microbunching is performed by externally injecting an electromagnetic plane wave, which should not be regarded as self-consistently generated by the beam. This approach is adopted with the specific aim of assessing the theoretical capability of the PDU to sustain longitudinal microbunching dynamics analogous to those occurring in conventional undulators. This distinction is especially relevant when comparing the PDU to conventional plasma undulators (CPUs), where electrostatic focusing is predicted to lead to continuous transverse microbunching [12,36] rather than the longitudinal density modulation characteristic of undulator-driven free-electron laser processes.

Figure 4 presents a numerical study of beam size evolution in the PDU for $\gamma = 2000$, $\Delta\gamma/\gamma = 0.05$, $r_c = 0.5$ mm, $\lambda_{\text{PDU}} = 6$ mm, $\Delta x = 0.5$ mm, and normalized emittance

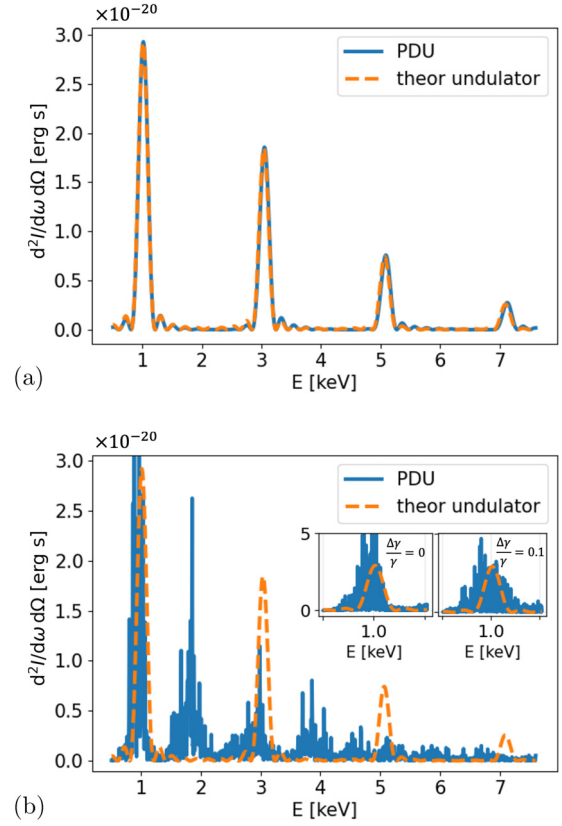


FIG. 5. On-axis radiation spectra for representative PDU parameters. (a) Single-particle spectrum (solid blue line), showing excellent agreement with the theoretical incoherent undulator emission (dashed orange line) at the fundamental wavelength and its harmonics. (b) Spectrum emitted by a matched Gaussian beam (solid blue line), exhibiting a slight broadening and deviation from the ideal undulator case (dashed orange line) due to the superposition of betatron motion on the forced oscillatory motion, which leads to the appearance of both even and odd harmonics on axis, associated with mixed-frequency radiation components. The insets show a close-up of the first harmonic for vanishing energy spread (left) and $\Delta\gamma/\gamma = 0.1$ (right).

$\epsilon_n = 1$ mm mrad. Discharge current is set to $I = 20$ kA to yield beam size saturation within a short propagation length without requiring an excessively large energy spread. The top panel shows the transverse rms beam size evolution in the undulation plane for several injection offsets $\Delta x_{\text{inj}} = x_{\text{inj}} - x_0$ with respect to the pure undulator condition, confirming that injection at $x_{\text{inj}} = x_0$ (i.e., $\Delta x_{\text{inj}} = 0$) corresponds to the beam size preservation condition and providing an accurate numerical validation of the theoretical estimates in Eqs. (6) and (7). The bottom panel shows the trajectory evolution of beam particles for the case $\Delta x_{\text{inj}} = 8.6$ μm , corresponding to the strongest degradation shown in the top panel, clearly illustrating the transition from collective centroid betatron oscillations to a constant-envelope evolution recentered at the equilibrium position after a saturation length $L_{\text{sat}} \approx 0.612$ m.

Figure 5 shows the simulated on-axis radiation spectrum from a single particle [Fig. 5(a)] and from a matched Gaussian beam [Fig. 5(b)] for the parameters $r_c = \Delta x = 0.5$ mm, $\lambda_{\text{PDU}} = 6$ mm, $I = 10$ kA, $K_{\text{PDU}} = 1.12$, $\gamma = 2000$,

$Q = 100$ pC, and normalized emittance $\epsilon_n = 1$ mm mrad. Both are compared to the theoretical incoherent undulator spectrum [14] for equivalent beam energy, undulator strength, and charge. The fundamental wavelength is computed using the standard expression $\lambda_1 = \lambda_{\text{PDU}}(1 + K_{\text{PDU}}^2/2)/2\gamma^2$. The single-particle spectrum is in perfect agreement with theory, whereas the beam spectrum shows a slight deviation due to the mixture of betatron phases superimposed on the forced oscillation, resulting in an effective single-particle oscillation amplitude differing from the nominal x_0 given by Eq. (5). Moreover, because of these phase differences and the relatively large average trajectory slopes, both even and odd harmonics are observed on axis. The latter feature is further justified by the strong superposition of betatron and undulatory motion, which gives rise to radiation at mixed frequencies corresponding to combinations of the fundamental harmonics $\omega_1 = \omega_{1,\beta}$ and $\omega_2 = \omega_{1,\text{PDU}}$ of the form $\omega = n\omega_1 + m\omega_2 + k(\omega_1 - \omega_2) + l(\omega_1 + \omega_2)$, with integer coefficients n, m, k , and l [37]. The insets in the top right corner of Fig. 5(b) show a close-up of the first harmonic evaluated for vanishing energy spread (left) and for $\Delta\gamma/\gamma = 10\%$ (right). In the latter case, the spectrum extends to frequencies slightly above the upper edge of the theoretical undulator spectrum, while the overall behavior remains consistent with the expected trend.

Figure 6 presents an angle-resolved spectrum simulation for parameters similar to those of Fig. 5, but with $I = 4$ kA, $\sigma_r = 4$ μm , $K_{\text{PDU}} = 0.45$, and device length $L_{\text{PDU}} = 6$ cm (i.e., ten undulation periods). In Fig. 6(a), a numerical integration of the photon yield within a 1% bandwidth is shown as a function of the radiation spot radius at 10 m from the source (color variable), yielding a cumulative incoherent photon number of about 10^7 around 1.4 keV (inset). The corresponding frequency-integrated radiation spot, resolved in angle, is shown in Fig. 6(b), exhibiting the expected divergence consistent with the $K_{\text{PDU}} < 1$ undulator regime ($\theta_{\text{div}} = \pm 1/\gamma$ [14]). The radiation cone divergence of approximately 0.5 mrad, combined with the available clearance of 0.5 mm along the undulation axis (set by the capillary radius $r_c = 0.5$ mm and the offset $\Delta x = 0.5$ mm), implies that the photon beam could propagate within the capillary channel without intercepting the walls over a distance of approximately 1 m, well exceeding the device length considered here. The capillary structure is therefore not expected to shadow the emitted radiation under these parameters. All quoted photon yields refer to a single passage of the electron bunch through the simulated PDU device.

Figure 7 illustrates two representative numerical results obtained for a matched electron beam with Lorentz factor $\gamma = 200$ and an initially uniform longitudinal distribution, showing the onset of seeding-induced longitudinal microbunching for both vanishing energy spread $\Delta\gamma/\gamma = 0$ (top panel) and finite energy spread $\Delta\gamma/\gamma = 0.01$ (bottom panel). Together with the electron beam propagating in the PDU, a plane electromagnetic wave corresponding to 10^8 photons at the fundamental emission wavelength $\lambda_1 = 82$ nm is externally injected. The horizontal axis represents the longitudinal position z of the beam particles with respect to the beam centroid $\bar{z}$, while the vertical axis shows the centroid position along the device. Starting from an initially uniform longitu-

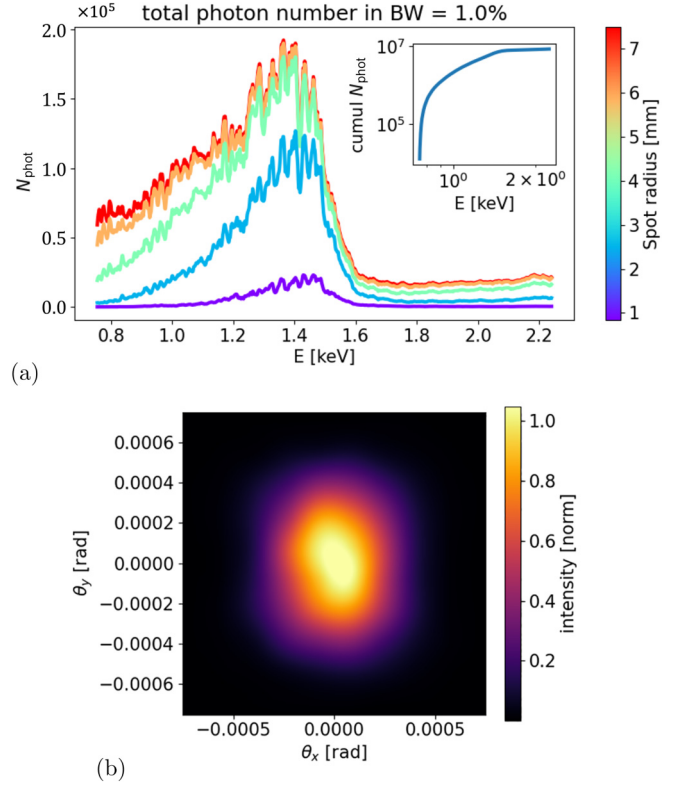


FIG. 6. (a) Photon spectrum integrated within a 1% bandwidth as a function of the radiation spot radius (color scale) for $I = 4$ kA, $\sigma_r = 4$ μm , $K_{\text{PDU}} = 0.45$, $r_c = \Delta x = 0.5$ mm, $\lambda_{\text{PDU}} = 6$ mm, $\gamma = 200$, $Q = 100$ pC, normalized emittance $\epsilon_n = 1$ mm mrad, and total undulator length $L_{\text{PDU}} = 6$ cm (ten undulation periods). The inset shows the cumulative photon number. (b) Frequency-integrated radiation spot at 10 m from the source, showing the expected divergence of $\pm 1/\gamma$ in the $K_{\text{PDU}} < 1$ regime.

dinal distribution, the beam is first propagated through the PDU channel without a seeding field up to $\bar{z} = 0.36$ m (black dashed line), after which the electromagnetic field is switched on and the particle distribution begins to develop oscillating density modulations, which are progressively smoothed by the nonlinearity of the longitudinal betatron motion. The resulting microbunches are evenly spaced along the longitudinal coordinate, with a separation exactly equal to λ_1 . Figure 8 shows, for the same two cases as in Fig. 7, the distribution of the undulator strength parameters of the beam particles (blue binning). For vanishing energy spread (top panel), the numerical distribution is compared with the expected Rayleigh χ_4 distribution (solid orange line), showing excellent agreement. The numerically evaluated mean value and standard deviation of the undulator strength (dashed black lines) are also compared with the corresponding analytical estimates derived in Eq. (16) (dashed green lines), again demonstrating very good quantitative agreement between simulations and theory. For finite energy spread $\Delta\gamma/\gamma = 0.01$ (bottom panel), a deviation of the simulated K distribution from the theoretical prediction is observed, as the latter does not directly account for the effect of energy spread. Nevertheless, for the energy spread considered in this case, the numerical mean value μ_K remains in good agreement with the analytical estimate,

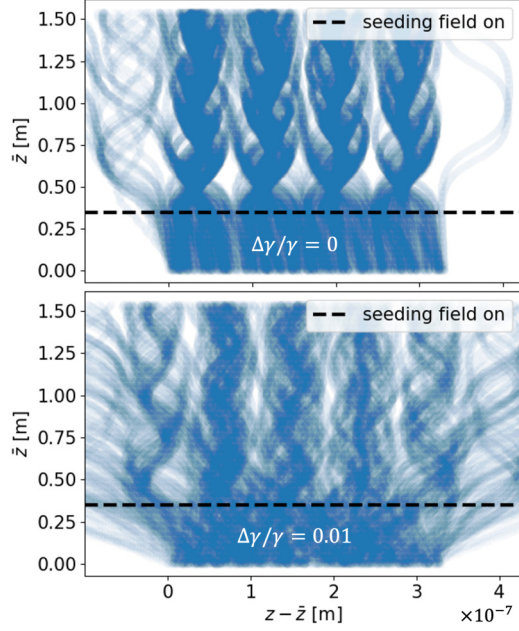


FIG. 7. Longitudinal microbunching of a matched electron beam with $\gamma = 200$ and an initially uniform longitudinal distribution, driven by an externally injected plane electromagnetic wave of 10^8 photons at the fundamental PDU wavelength $\lambda_1 = 82$ nm, for vanishing energy spread $\Delta\gamma/\gamma = 0$ (top) and finite energy spread $\Delta\gamma/\gamma = 0.01$ (bottom). The seeding field is switched on at $\bar{z} = 0.36$ m. The particle longitudinal coordinate z is shown with respect to the beam centroid $\bar{z}$. Density modulations develop with a periodicity equal to λ_1 , progressively smoothed by longitudinal betatron motion nonlinearities.

while the numerical standard deviation σ_K slightly exceeds the theoretical value, although staying within the same order of magnitude.

V. DISCUSSION AND CONCLUSIONS

The plasma discharge undulator was proposed and theoretically characterized as a concept for compact plasma-based radiation sources. By introducing a periodic geometric forcing term through a sinusoidal modulation of a plasma discharge, the PDU enables controlled undulatory motion of relativistic electron beams while freeing the undulation scheme from the need for a high-energy-density external driver such as an intense laser pulse, with related dephasing issues, or a particle beam, as in conventional plasma undulators. Analytical models show that the injection condition $x_{\text{inj}} = x_0$ suppresses collective betatron oscillations, yielding a well-defined oscillation at the imposed wavelength λ_{PDU} and significantly reducing the intrinsic K spread typical of CPUs. This same condition has been shown to simultaneously represent the optimal operating point for beam quality preservation in the presence of finite energy spread. Moreover, the high transverse focusing gradients [$O(\text{kT/m})$] enable matched beam transport within the device. Two simple expressions for the undulator strength associated with forced and betatron motion were presented as limiting cases of a more general formulation, and the analytical evaluation of the resulting

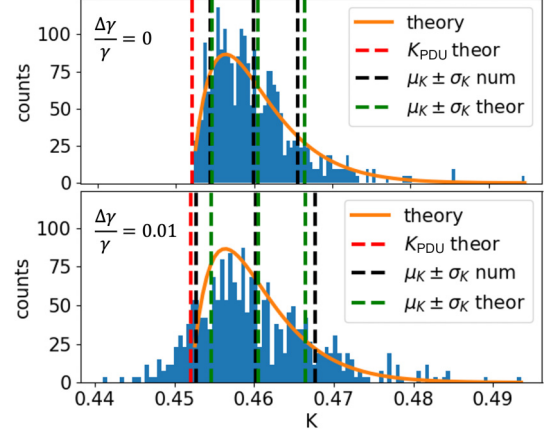


FIG. 8. Distribution of the undulator strength parameter for the same two cases as in Fig. 7 (blue binning), compared with the expected Rayleigh χ_4 distribution (solid orange line). The numerical mean value and standard deviation (dashed black lines) are compared with the analytical predictions of Eq. (16) (dashed green lines). For the vanishing energy spread (top), excellent agreement is found; for the finite energy spread (bottom), μ_K remains well reproduced, while σ_K slightly exceeds the theoretical value.

K distribution highlighted the conditions under which the effective undulator strength converges toward the imposed value K_{PDU} , with a K spread that asymptotically tends to zero. Design laws and scaling relations were derived for particle trajectories and radiation emission, and the one-dimensional requirements for FEL emission were evaluated, providing feasibility criteria for FEL operation in the PDU configuration.

Numerical simulations confirmed the analytical predictions. Single-particle and beam radiation spectra exhibit narrow-band emission in the millimeter-to-centimeter undulator period regime, extending to the soft x-ray range: At the single-particle level and for a proper injection amplitude x_0 , pure undulator motion is achieved with only on-axis emission of odd harmonics. For a beam of finite transverse size, individual particles are displaced from the centroid injection offset, resulting in a superposition of betatron and undulatory motion that may lead to mixed-frequency radiation components and the appearance of both even and odd harmonics on axis. Additional simulations demonstrated that, under the action of an externally injected electromagnetic field, the PDU is theoretically capable of supporting the formation of longitudinal microbunching, analogous to that observed in conventional magnetic undulators. Both the microbunching dynamics and the K -parameter distribution were evaluated for vanishing and finite energy spread, providing an assessment of the robustness of the PDU concept under conditions relevant to laser wakefield acceleration (LWFA)-driven setups.

Regarding radiation wavelength tunability, the PDU follows the same scaling as a conventional undulator: $\lambda_1 = \lambda_{\text{PDU}}(1 + \mu_K^2/2)/2\gamma^2$ scales linearly with the undulator period and inversely with the square of the beam energy, in principle providing continuous tunability through beam energy for a given device geometry. The mean undulator strength μ_K depends nonlinearly on the beam energy and on

the geometric and magnetic properties of the device. A key challenge in practical operation will be identifying configurations that minimize beam-plasma wakefield-type interactions, which tend to increase the energy spread and reduce the mean beam energy; multibunch schemes or appropriate longitudinal charge density modulation could represent viable mitigation strategies.

Concluding with an experimental perspective, the proposed implementation based on geometrically modulated discharge capillaries benefits from the extensive recent development of plasma discharge channels, including nonlinear geometries, providing strong motivation for the experimental feasibility of current modulation through capillary shaping. Overall, the PDU represents a promising step toward compact, fully plasma-based light sources and accelerator beamline components. Future work will focus on the numerical and experimental validation of the discharge modulation mechanism and on the integration of the PDU with conventional RF and possibly plasma-based acceleration stages.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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