

Common explanation for coherent patterns in transitional and turbulent wall-bounded flows

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A common explanation for selecting the shape and size of coherent patterns based on the filtering nature of the linearized Navier-Stokes equations is investigated. The filtering properties are examined by generalized optimal perturbations. Computing generalized optimal perturbations with different structure norms in a homogeneous shear flow demonstrates that different generalized optimal perturbations take different structural forms, implying the existence of multiple filtering properties of the linearized Navier-Stokes operators dependent on the filter output. Subsequently, linear simulations with random initial conditions are conducted to further clarify the capability of generalized optimal perturbations in representing the multiple filtering properties. With nonlinearity not taken into consideration, generalized optimal perturbations well predict the temporal evolutions of structural characteristics of multiple flow patterns governed by the linearized Navier-Stokes operators, thereby offering a more refined description of the multiple filtering properties than the optimal perturbation framework. The capability of generalized optimal perturbations in describing the filtering nature of the linearized Navier-Stokes operators is further examined in real turbulence with nonlinearity taken into account. In addition to the spanwise streak spacing, comparing with the direct numerical simulation results reveals that the spanwise length scale of spanwise velocity patterns can also be well-captured by the corresponding generalized optimal perturbations. This supports the effectiveness and intuitiveness of generalized optimal perturbations in representing the selectivity of the linearized Navier-Stokes operators, which is essential in dictating some specific flow patterns in fully developed turbulence.

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I. INTRODUCTION

A. Coherent patterns

Coherent patterns in wall-bounded turbulent and transitional flows have been the subject of many studies. The variety of coherent patterns is large. There are Tollmien-Schlichting waves, lambda-shaped structures, low-speed streaks, kinks, hairpin vortices, spikes, soliton-like coherent structures and coherent structures/solitons, bulges, large-scale motions, very-large-scale motions, and more [1,2]. Many of these structures are unique to the specific circumstances they are observed in. There are, however, structures common to many flows. Probably the most commonly observed and best understood coherent structures are streamwise elongated streaks in the near-wall region of fully developed turbulent flows. Streamwise streaks are narrow regions of high and low speed fluids moving in the mean flow direction. Immediately near the wall, the characteristic spanwise spacing of streaks is

about one hundred wall units [3–6]. The growth of near-wall streaks is linear and transient in viscous wall-bounded flows [7], associated with the nonnormality of the linearized Navier-Stokes operators [8,9]. For a large variety of turbulent flows, there is more recent and still growing evidence supporting the common existence of large-scale streaky motions in the region further away from the wall, whose streamwise and spanwise length scales are much greater than those of near-wall streaks [10–18].

Various coherent patterns can interact with each other, which plays an essential role in transition to and maintaining turbulence. In transitional flows, the mean velocity profile of streamwise velocity in the presence of near-wall streaks can be inflectional with time-dependent fluctuations appearing in a sinuous or varicose pattern in terms of the underlying streaks [19]. The streaky base flow is therefore linearly unstable to infinitesimal disturbances. As such, the small-amplitude perturbations, especially the spanwise velocity perturbations, can strongly interact with the least-stable mode of the near-wall streaks leading to the final streak breakdown and transition to turbulence [20,21].

An important question is why the coherent patterns have their particular shape and size, and what is the “mechanism”, that is a simple intuitive way of explaining them. It has been found that the lift-up mechanism is instrumental in the generation of near-wall streaks [22–33]. This mechanism

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consists of the following. When moving away from the wall, the wall-normal motions “lift up” the slow-moving near-wall fluids, thus creating the regions of slow motion away from the wall. Then the reason for these regions to have streamwise elongated shape with a specific spanwise spacing can simply be that the wall-normal motions have the same shape and spacing. Such wall-normal and the associated spanwise motions would then necessarily have the form of streamwise vortices with similar structural characteristics. This is definitely the case in the traveling wave solutions (see for example [34]), which are periodic in the spanwise direction, so that all structures necessarily have the same spanwise spacing. Traveling wave solutions are considered as a good model for the “self-sustaining process” which exists in both near-wall region [20,35–37] and the region further away from the wall [38–43]. The self-sustaining process is often described with the following three sub-processes: (i) the streamwise elongated streaks are generated by quasistreamwise vortices via the lift-up effect by which the energy is transferred from mean shear to streamwise velocity fluctuations [22–33]; (ii) the amplified streaks oscillate as a result of the secondary instability and/or transient growth [20,44–46]; (iii) the streaks break down finally, and quasistreamwise vortices are regenerated by a nonlinear process [20,35]. Even more in-depth understanding of the mechanism of the self-sustaining process was obtained within an asymptotic approach for large Reynolds number [47], which revealed that the quasistreamwise vortices are generated by a steady streaming effect caused by the oscillations in the critical layer in which the wave speed and streak velocity coincide. The oscillations are caused by the effectively inviscid waves propagating on the streaky flow and made possible by the presence of inflection points in that flow.

At relatively small Reynolds numbers, near-wall turbulent flows can be quite similar to traveling wave solutions qualitatively and even quantitatively. However, certain differences remain, and become more pronounced as the Reynolds number increases. For example, as it was pointed out in Ref. [27], the spacing of streaks and the spacing of quasistreamwise vortices is the same in a traveling wave solution, whereas in a turbulent flow the spanwise spacing of streaks and quasistreamwise vortices at the same distance from the wall differs by almost a factor of two. This difference can be explained by the now commonly accepted idea that turbulent flows have a “skeleton” of a multitude of so-called recurrent solutions, and many of them are traveling waves [48]. The turbulent flow spends time in the vicinity of these recurrent solutions, switching randomly between them. The presence of a multitude of traveling wave solutions allows an explanation of the observed difference between the spacing of vortices and streaks at the same distance from the wall [41,49]: namely, that streaks at a fixed distance from the wall are created by longitudinal vortices located at a different distance from the wall, such that their spanwise spacing is the same. This is possible given the spanwise spacing of streaks and vortices depending on the distance from the wall.

Traveling wave solutions throw more light on the interaction between different structures in turbulent flows than on the question of the reason for the structures to have specific shape and size. Indeed, as far as the traveling wave mechanism is concerned, streaks have a particular shape and size because

the quasistreamwise vortices have a specific shape and size, which in turn are dictated by the shape and dimensions of the critical layer, which in their turn are dictated by those of the streaks, completing the loop. Despite being quite detailed, this is a chicken-and-egg explanation, leaving the answer to the question of why the structures have the specific shape and dimensions somewhat incomplete.

An explanation of the mechanism of selecting the shape and dimensions of streaks was given in Ref. [27], building on the earlier works on optimal perturbations [25,50]. In essence, it amounts to representing the flow velocity as a sum of the mean flow and fluctuations, moving all the terms nonlinear in fluctuations to one side of the Navier-Stokes equations, and postulating, on the basis of observations, that the specific scales and patterns of this side have far lesser influence on the shape and dimensions of the solution than the structure-forming properties of the linearized Navier-Stokes equations, constituting the other side of the equations [i.e., Eqs. (3.1)–(3.3) in Ref. [27]]. This was confirmed by a numerical experiment, consisting in replacing the actual wall-normal motions with a potential random field of wall-normal motions, thus guaranteed to have no vortices, and observing that such a replacement led to streaks of the same shape and dimensions as in the turbulent flow. The linearized part of the Navier-Stokes equations acts similarly to a narrow-band filter, with almost a near-white noise input and the output consisting of streaks of only such shape and dimensions that can pass through. This breaks the chicken-and-egg loop and forms streaks, which then dictate the shape and dimensions of other structures. This applies equally to the developed turbulence and to recurrent solutions like traveling waves, which explains the similarity of organized patterns in such flows.

To predict the shape and dimensions of the organized structures created by this mechanism, the filtering properties of the linearized Navier-Stokes operators should be investigated. In Ref. [27] generalized optimal perturbations, a particular method of studying such properties, was proposed and successfully used to predict quantitatively the dependence of streak spacing on certain distance from the wall. Rather nontrivial applications of this approach are Refs. [51,52]. A deeper introduction to optimal and generalized optimal perturbations is given next.

B. Optimal perturbations

Representing the flow velocity as the sum of a certain given or specifically selected flow field $\mathbf{U}$ and the fluctuation velocity $\mathbf{u}$ around it, the Navier-Stokes equations can be written as [27]

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{U} + \nabla p - \frac{1}{\text{Re}} \nabla^2 \mathbf{u} = \mathbf{F}, \quad (1a)$$

$$\mathbf{F} = -\frac{\partial \mathbf{U}}{\partial t} - \mathbf{U} \cdot \nabla \mathbf{U} - \mathbf{u} \cdot \nabla \mathbf{u} - \nabla P + \frac{1}{\text{Re}} \nabla^2 \mathbf{U}, \quad (1b)$$

$$\nabla \cdot \mathbf{u} = 0. \quad (1c)$$

For given $\mathbf{F}$ and unknown $\mathbf{u}$, Eq. (1a) are the Navier-Stokes equations linearized around $\mathbf{U}$ and subject to the forcing $\mathbf{F}$. Together, Eqs. (1a) and (1b) are just standard fully nonlinear Navier-Stokes equations. Note that an intuitive desire to minimize the nonlinear effects represented by $\mathbf{u}$ by minimizing

$\|\mathbf{u}\|$ over $\mathbf{U}$ leads to the assumption that $\mathbf{U}$ being the averaged velocity [51]. Depending on the averaging method, $\mathbf{U}$ can be time-dependent, as for example was the case, in quite different but relevant contexts, in Refs. [51,53]. It is then assumed that as far as the organized structure of interest is concerned, its properties are largely defined by the filtering properties of the linearized operators rather than by the exact properties of $\mathbf{F}$. This can be exploited by replacing Eq. (1b) with a simpler formula. There are many options, of which three are the most common: to take $\mathbf{F}$ to be a broadband random signal, to take $\mathbf{F} = e^{i\omega t} \mathbf{f}(\mathbf{x})$, or to take

$$\mathbf{F} = \delta(t - t_0) \mathbf{u}_0(t_0, \mathbf{x}). \quad (2)$$

In the context of developed turbulence, these problems have to be considered over an infinite time interval, $-\infty < t < \infty$. In the case of Eq. (2), this can also be interpreted as an initial value problem with zero forcing:

$$\mathbf{F} = 0, \quad \mathbf{u}|_{t=t_0} = \mathbf{u}_0(\mathbf{x}). \quad (3)$$

The three choices differ in terms of their convenience for various purposes. The first choice is closest to the actual turbulent flow case, because the right hand side of Eq. (1b) is also broadband and behaves chaotically. However, it is also the most complicated choice since, if considered rigorously, it turns Eq. (1a) into a stochastic differential equation. Interested readers are referred to a recent review [54]. The second choice leads to the most widely known method, now called resolvent analysis. It benefits from effective elimination of the time variable made possible by the linearity of Eq. (1a), with the analysis performed separately for each ω . Good starting points for investigating the vast area of resolvent analysis are the original paper [55] and an introductory text [56]. The third choice, in particular in the form of Eqs. (1a) and (3), which leads to optimal perturbations, has the advantage of relative easiness of understanding the behavior of the solutions on the basis of hydrodynamic intuition honed on considering linear physical mechanisms of fluid flows with given initial conditions but without the body force term $\mathbf{F}$.

An optimal perturbation (but not yet a generalized optimal perturbation) is defined as the solution of the initial value problem for the linearized Navier-Stokes equations Eqs. (1a) and (3) that gives the maximum transient growth of its energy,

$$G_{\max} = \max_{t, \mathbf{u}_0} \frac{\|\mathbf{u}(t)\|^2}{\|\mathbf{u}_0\|^2}. \quad (4)$$

Here, the simplest L_2 norm is used, which is the perturbation energy density averaged over the entire flow domain $\|\mathbf{u}\|^2 = 1/V \int_V (u^2 + v^2 + w^2) dx dy dz$, where $\mathbf{u} = (u, v, w)$ denotes perturbation velocity components in the streamwise (x), wall-normal (y), and spanwise (z) directions, respectively, and V is the volume of the computational domain.

Optimal perturbations have been computed for almost all canonical shear flows in the past three decades [8,26,50,57]. Their straightforward interpretation is related to the question of the stability of the base flow profile $\mathbf{U}$ with respect to small perturbations [8]. In the case when $\mathbf{U}$ is linearly stable, all sufficiently small perturbations are guaranteed to decay. However, in many flows at high Reynolds numbers, transition to turbulence is observed even when perturbations are initially quite small and $\mathbf{U}$ is linearly stable. It turns out

that optimal perturbations can transiently grow very significantly, thus explaining this long-standing contradiction. With respect to the flow patterns in developed turbulence, optimal perturbations also offered hope. It is well known that when the growth of small perturbations saturates and nonlinearity becomes essential, the patterns of the resulting flow resemble, sometimes quite strongly, the pattern of the fastest-growing small perturbation. The most famous example is probably one of the classical problems of fluid mechanics: the Rayleigh-Bénard convection [58,59]. A less known but quite remarkable example is the flow in a mixing layer [60]. Similarly, one can expect that in the case when there is a significant transient growth of small perturbations, the structures corresponding to the maximal transient growth will also be observed in developed turbulence. However, the patterns of a developed near-wall flow are not at all similar to Tollmien-Schlichting waves. Calculation of the optimal perturbations for a turbulent channel flow gave mixed results [25]: streaks are predicted, but with the characteristic size very different from that of the streaks observed in near-wall turbulent flows. To obtain an agreement, instead of maximizing over time in Eq. (4) the authors prescribed t to be equal to the eddy turnover time at a certain distance from the wall. Since the eddy turnover time is determined by nonlinear effects, this implies that the analysis of linearized Navier-Stokes equations alone does not allow to predict the characteristic dimensions of near-wall streaks. With the use of an eddy viscosity, optimal energy growth exhibits two peaks in spanwise wavelengths, the first one scaled in outer units corresponding to large-scale streaks, and the second one scaled in inner units corresponding to the streaks in the near-wall region retrieving the spanwise spacing of around 100 inner units [28–30]. However, the use of an eddy viscosity in the prediction of organized patterns would shield the native filtering properties of the linearized Navier-Stokes equations and is also argued in Ref. [51]. Generalized optimal perturbations, however, were shown to do exactly that: predict the dimensions of organized patterns purely based on the analysis of linearized equations only.

C. Generalized optimal perturbations

The hope that the pattern in developed turbulent flows will be similar to the fastest growing, transiently or indefinitely, small perturbation implies a certain, “stability-like”, physical interpretation. One can imagine that the process of the growth of such a perturbation actually occurs in the turbulent flow frequently. This requires that from time to time the velocity field becomes close to the sum of the mean profile and a small perturbation. Then $\mathbf{F}$ in Eq. (1a) becomes small and Eqs. (1a) and (3) applies over a certain period of time. However, in a developed turbulent flow, such occurrences are not very likely. In contrast, the filtering interpretation, already briefly discussed in Sec. I A, does not imply such occurrences. It only requires the linear operator to be “filtering”, that is allowing to pass through the input of a certain structure with noticeably bigger gain (or noticeably smaller attenuation) than the gain or attenuation of all other structures. Then, as long as $\mathbf{F}$ is sufficiently “broadband”, the linearized analysis can correctly predict the flow pattern regardless of the nature of the actual process in the flow. Moreover, while the stability-like

interpretation requires the growth of the perturbation to be large, the filtering explanation implies only the ratio of the gains among different perturbations to be large, while the optimal perturbations themselves might even monotonically decay.

These differences become particularly significant when one considers how to beneficially affect the flow properties. Turbulent drag reduction is one such example. If one assumes, as it was and maybe still is commonly assumed, that the turbulent flow structures are instrumental in increasing turbulent friction, and if the mechanism of the optimal perturbation leading to known structures is the actual mechanism in the turbulent flow, then one can envisage a method of flow control that destroys such mechanism, thus reducing the turbulent drag. This is sometimes referred to as the “throwing a spanner in the works” method. If the “stability-like” interpretation is correct, then throwing a spanner in the works should give good results, but if the filtering idea is correct, then targeting the mechanism of the optimal perturbation might be irrelevant, as that mechanism might not be present in the actual flow. A well-known example of the success of “spanner in the works” idea is the opposition control [61,62], which targets the lift-up mechanism instrumental in many optimal perturbations and described in Sec. IA. A study of another method of drag reduction, that of in-plane oscillations of the wall [63] throws an interesting light on this question. In Ref. [51] it has been found that in the flow with drag reduction by oscillating wall, the maximum gain of the (generalized) optimal perturbation is noticeably greater than the gain in the flow with nonmoving wall. In other words, with respect to structure generation, wall oscillations promote rather than suppress structure generation but reduce the drag nevertheless. In fact, results of Refs. [51,64,65] suggest that the reason for drag reduction by wall oscillations is the reduction of the amplification of energy of all perturbations rather than the reduction of the gain of the optimal mode. This also applies to opposition control, which is not particularly selective with respect to the form of perturbations.

Recognizing that optimal perturbations should be used as a tool for explaining the filtering properties of the operators rather than a tool for revealing a mechanism of turbulence leads to a refinement of the notion of optimal perturbations. The entire flow field might contain a variety of organized structures and not all of them might even be related. It is logical then to define the gain of the filter as the ratio of the norm most appropriate for the structure of interest to the energy norm of the input, and replace Eq. (4) with [27]

$$G_{u_i, \max} = \max_{t, \mathbf{u}_0} \frac{\|\mathbf{u}(t)\|_s^2}{\|\mathbf{u}_0\|^2}, \quad (5a)$$

$$G_{\omega_i, \max} = \max_{t, \mathbf{u}_0} \frac{\|\boldsymbol{\omega}(t)\|_s^2}{\|\mathbf{u}_0\|^2}, \quad (5b)$$

where $\|\mathbf{u}\|_s^2$ and $\|\boldsymbol{\omega}\|_s^2$ are the appropriate structure norms. In the present study, the structure norms are defined as the average of the perturbation energy of a specific velocity component $\|\mathbf{u}\|_s^2 = 1/S \int_{y=y_0} u_i^2 dx dz$ and vorticity component $\|\boldsymbol{\omega}\|_s^2 = 1/S \int_{y=y_0} \omega_i^2 dx dz$ in a particular horizontal plane at a certain distance from the wall, respectively. Here, $i=1,2,3$ with $(u_1, u_2, u_3) = (u, v, w)$ and $(\omega_1, \omega_2, \omega_3) = (\omega_x, \omega_y, \omega_z)$

interchangeably denoting the perturbation velocity and vorticity components. In addition, y_0 is the wall-normal coordinate of the particular plane where the visualization will be made, and S is the area of the cross-section of the computational domain at $y = y_0$. The initial condition norm here is the same as in Eq. (4) and the structure norm is consistent with that used in Refs. [27,51]. Given the difference between the definitions of the structure norm and the initial condition norm, the gain $\|\mathbf{u}(t)\|_s^2 / \|\mathbf{u}_0\|^2$ can be dimensional, which underlines that its value is meaningful only when the gains for different $\mathbf{u}_0$ or G are compared.

If the initial condition norm and the structure norm coincide, then the generalized optimal perturbation problem reduces to the original optimal perturbation problem. On this occasion, only longitudinal streaks, as the output of the original optimal perturbation problem whose magnitude is maximized over all possible inputs, are expected in fully developed turbulence. In addition, the energy norm in the original optimal perturbation problem in Ref. [25] represents a kind of averaging across the full turbulent channel. In a turbulent channel flow, however, the streaks are essentially a near-wall feature whose strength reduces and spanwise dimension grows with increasing distance from the wall. In this sense, the energy norm of the solution with strong streaks in the near-wall region and weak streaky structures in the other part of the domain could be very small, which may lead to the omit of the solutions corresponding to the strong near-wall streaks in real turbulence. This logic favors the generalized optimal perturbations with the structure norm defined for a particular pattern of interest in a specific visualization plane at a certain distance from the wall.

D. Scope of the present study

Given the large variety of coherent patterns and their importance in both transition to and maintaining turbulence, the objective of the present study is to explore a common explanation for selecting the shape and dimensions of the coherent patterns, which applies to streaks, streamwise vortices, spanwise velocity structures, and others. Particular attention is paid to the interpretation based on the filtering properties of the linearized Navier-Stokes operators. For this purpose, generalized optimal perturbations are used as the tool to effectively describe the multiple filtering properties. We will see that the temporal evolutions of pattern dimensions governed by linearized Navier-Stokes equations with random broadband initial conditions in the homogeneous shear flow are well captured by the corresponding generalized optimal perturbations, supporting the capability of generalized optimal perturbations in depicting the filtering properties of the linearized Navier-Stokes operators. In a fully developed turbulent channel flow with nonlinearity taken into account, the generalized optimal perturbations are able to qualitatively capture the shape and dimensions of some essential coherent patterns, whereas there are also other flow patterns for which generalized optimal perturbations cannot predict their structural characteristics. On the basis of these comparisons, discussions on the interaction and competition between linear selectivity and nonlinear forcing in dictating the structural characteristics of flow patterns are provided.

II. GENERALIZED OPTIMAL PERTURBATIONS IN HOMOGENEOUS SHEAR FLOW

A. Exact analytic solutions

The filtering properties of the linearized Navier-Stokes operators are first explored by computing the generalized optimal perturbations based on the exact analytic solutions in a homogeneous shear flow. The linearized version of the governing equations Eqs. (1a) and (1c) with $\mathbf{F} = 0$ admits the following Kelvin-mode solution [66–68]:

$$\mathbf{u} = \hat{\mathbf{u}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (6a)$$

where $\mathbf{x} = (x, y, z)$, $\mathbf{k} = (\kappa_1, \kappa_2, \kappa_3)$ the dimensionless wave-number vector. The streamwise (κ_1) and spanwise (κ_3) wave numbers are constants, while the vertical (κ_2) wave number is a linear function of time $\kappa_2(t) = \kappa_{2,0} - \kappa_1 t$ with $\kappa_{2,0}$ also constant. In the homogeneous shear flow, the Kelvin-mode

solution Eq. 6(a) can be rearranged as

$$\mathbf{u} = \hat{\mathbf{u}}(t)e^{i[\kappa_1(x-Ut)+\kappa_{2,0}y+\kappa_3z]}, \quad (6b)$$

where $U = y$ is the streamwise velocity of the base flow. This form of solution implies that $\hat{\mathbf{u}}(t)$ is a Fourier mode of $\mathbf{u}$ in the co-moving frame with the local base flow. Also note that the linear time dependence in $\kappa_2(t)$ is a result of the downstream advection of the perturbation velocity due to the basic flow with shear [69]. Here, the velocity and length scales are selected in such a way that the dimensionless base flow becomes $\mathbf{U} = (y, 0, 0)$ and the analysis is limited to the perturbation with a fixed vertical wave number at zero time, such that its initial dimensionless value $\kappa_{2,0} = 1$. The Reynolds number is therefore defined based on the base flow shear S^* and the vertical length scale of the perturbation at zero time $\kappa_{2,0}^*$ as $\text{Re} = S^*/[\nu(\kappa_{2,0}^*)^2]$. The Fourier mode $\hat{\mathbf{u}}(t)$ in Eq. (6) has the following analytic solutions [70]

$$\hat{u}(t) = \hat{u}_0 e^{\phi(t)} - \hat{v}_0 \kappa_1 \left(\frac{\kappa_0^2 \kappa_2(t)}{\kappa_h^2 \kappa^2(t)} \right) e^{\phi(t)} - \hat{v}_0 \left(\frac{\kappa_0^2 \kappa_3 [\theta(t) - \theta(0)]}{\kappa_1 \kappa_h^3} \right) e^{\phi(t)}, \quad (7a)$$

$$\hat{v}(t) = \frac{\hat{v}_0 \kappa_0^2}{\kappa^2(t)} e^{\phi(t)}, \quad (7b)$$

$$\hat{w}(t) = \hat{w}_0 e^{\phi(t)} - \hat{v}_0 \kappa_3 \left(\frac{\kappa_0^2 \kappa_2(t)}{\kappa_h^2 \kappa^2(t)} \right) e^{\phi(t)} + \hat{v}_0 \left(\frac{\kappa_0^2 \kappa_3 [\theta(t) - \theta(0)]}{\kappa_h^3} \right) e^{\phi(t)}, \quad (7c)$$

where $\hat{\mathbf{u}}_0 = (\hat{u}_0, \hat{v}_0, \hat{w}_0)$ is the initial condition, $\kappa(t) = [\kappa_1^2 + \kappa_2^2(t) + \kappa_3^2]^{1/2}$, $\kappa_0 = \kappa(0)$, $\kappa_h = (\kappa_1^2 + \kappa_3^2)^{1/2}$ and

$$\theta(t) = \arctan \left(\frac{\kappa_h}{\kappa_2(t)} \right), \quad (8a)$$

$$\phi(t) = -\frac{1}{\text{Re}} \int_0^t \kappa^2(\tau) d\tau. \quad (8b)$$

Here, $\theta(t) \in [0, \pi)$ is the angle between the horizontal (κ_h) and vertical (κ_2) components of wave-number vector, and $\phi(t)$ incorporates the viscous effect with the temporal evolution of the solution [69]. The physical implications of Eq. (7) have been summarized in previous studies; see Refs. [69,70] for further details.

B. Generalized optimal perturbations

In this section, the generalized optimal perturbations for individual velocity and vorticity components are computed, which shares some similar ideas of the componentwise optimal response in Ref. [71]. In homogeneous shear flow, the optimization problem in Eq. (5) is easily solved using the analytical solution Eq. (7).

Figure 1 shows the contours of the maximum energy growth for each velocity and vorticity component at $\text{Re} = 100$ on the κ_1 - κ_3 plane. It can be seen well from Figs. 1(a), 1(e), and 1(f) that $G_{u,\max}$, $G_{\omega_y,\max}$ and $G_{\omega_z,\max}$ reach their peaks with $\kappa_1 \ll \kappa_3$, suggesting the generalized optimal perturbations for $G_{u,\max}$, $G_{\omega_y,\max}$ and $G_{\omega_z,\max}$ share similar streamwise elongated structural characteristics. However, Fig. 1(b) shows that $G_{v,\max}$ exhibits its peak with $\kappa_1 \gg \kappa_3$, implying that the patterns corresponding to $G_{v,\max}$, whose streamwise

dimensions are much smaller than the spanwise ones, are very different from those corresponding to $G_{u,\max}$, $G_{\omega_y,\max}$ and $G_{\omega_z,\max}$. In addition, $G_{w,\max}$ and $G_{\omega_x,\max}$ show their peaks between the peak locations of $G_{u,\max}$ and $G_{v,\max}$ with $\kappa_1 \approx \kappa_3$, which reflects the corresponding generalized optimal perturbations taking the form of oblique structures [see Figs. 1(c) and 1(d)]. As such, generalized optimal perturbations for different velocity or vorticity components exhibit different structural characteristics. Also note that the gains of different generalized optimal perturbations are different (i.e., $G_{u,\max}$ much greater than $G_{v,\max}$ and $G_{w,\max}$; $G_{\omega_z,\max}$ much greater than $G_{\omega_x,\max}$ and $G_{\omega_y,\max}$). Given different generalized optimal perturbations displaying different behaviors in structural characteristics and energy growth, the linearized Navier-Stokes operators can be considered as a filter with multiple collected filtering properties. In addition, the filtering properties appear to be strongly dependent on the choice of the filter outputs corresponding to the structure norms.

C. Linear simulations

Now, the capability of generalized optimal perturbations in predicting the evolution of flow patterns in homogeneous shear flow is further examined. To this end, we introduce a perturbation with random initial conditions taking the form of

$$\mathbf{u} = \text{Re} \left[\sum_{m=-32}^{m=32} \sum_{n=-32}^{n=32} \hat{\mathbf{u}}^{(m,n)} e^{i[m\kappa_{1,0}x + n\kappa_{3,0}z + (1-m\kappa_{1,0}t)y]} \right] \quad (9)$$

into the homogeneous shear flow. Here, $\text{Re}[\cdot]$ represents the real part and $\hat{\mathbf{u}}^{(m,n)} = (\hat{u}^{(m,n)}, \hat{v}^{(m,n)}, \hat{w}^{(m,n)})$ denotes the

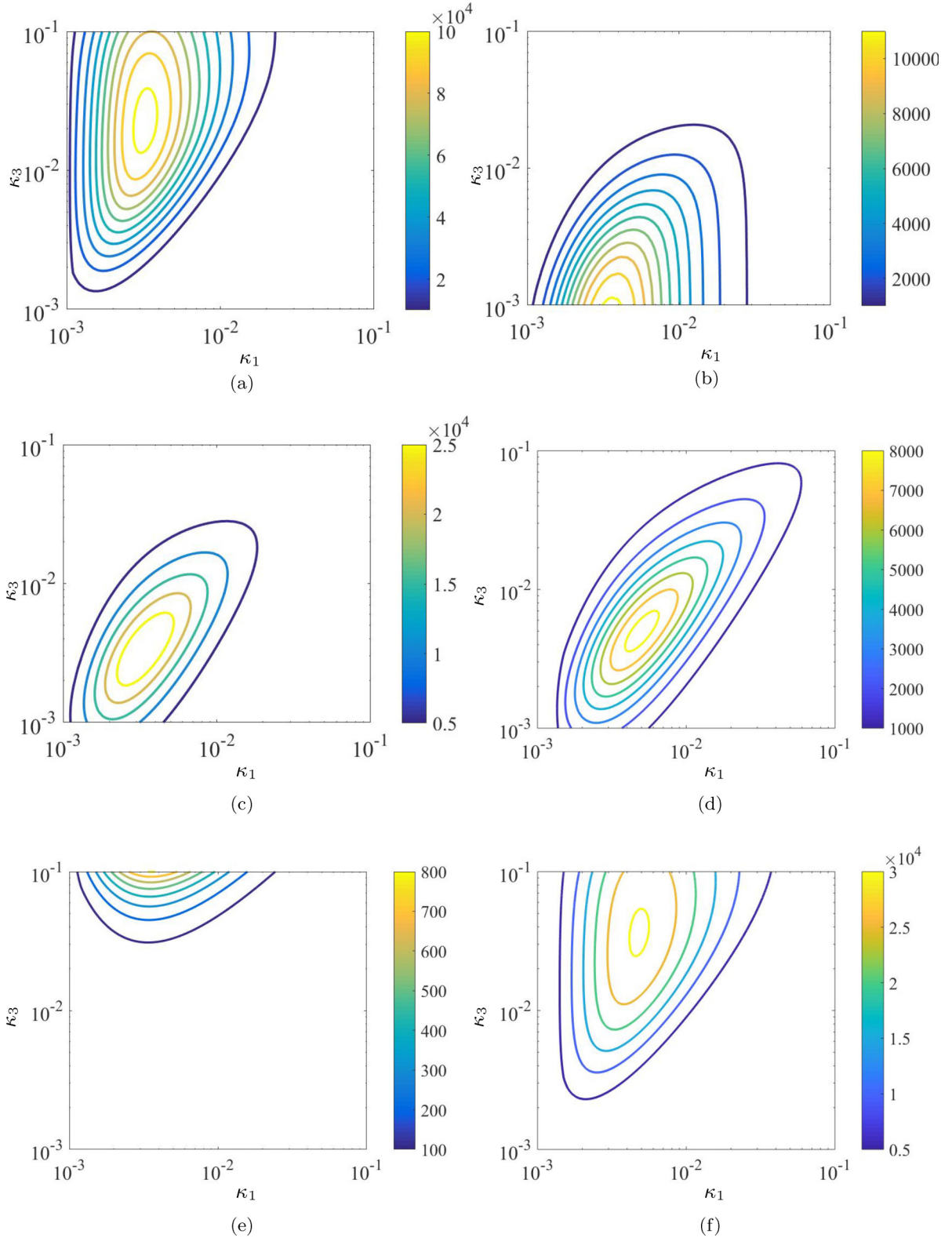


FIG. 1. Contours of (a) $G_{u,\max}$, (b) $G_{v,\max}$, (c) $G_{w,\max}$, (d) $G_{\omega_x,\max}$, (e) $G_{\omega_y,\max}$, and (f) $G_{\omega_z,\max}$ in the κ_1 - κ_3 plane for $\text{Re} = 100$. Here, panels (a)–(c) are reproduced based on the data reported in Fig. 1 from Ref. [69] for completeness and direct comparison with panels (d)–(f).

modes with random initial conditions. The initial conditions of $\hat{u}$ and $\hat{v}$ are given randomly with uniform distribution, whereas the initial condition of $\hat{w}$ is solved based on those of $\hat{u}$ and $\hat{v}$ according to the continuity equation to ensure

that the incompressibility condition is satisfied. Here, the evolution of the perturbation is analyzed through linear simulations governed by the linearized Navier-Stokes equations. The size of the horizontal computational domain is prescribed

as $L_x = 2000\pi$ and $L_z = 2000\pi$ with the number of grid points $N_x = 401$ and $N_z = 401$. Given the mean shear homogeneously distributed across the vertical direction with no wall boundaries, a small vertical domain size should be enough in the present study which is set as $L_y = 2\pi$ based on the vertical wave number at $t = 0$. The number of grid points in vertical direction is $N_y = 65$. The spatial resolutions are verified via doubling the number of grid points with the simulation results matching well with the present ones. The wave numbers $\kappa_{1,0}$ and $\kappa_{3,0}$ are set to be equal to 0.001 in the simulations.

The instantaneous velocity and vorticity fields at $t = 0, 200, 400, 600$ are presented in Figs. 2 and 3, respectively. Initially, as illustrated in Figs. 2(a)–2(c), and Figs. 3(a)–3(c), the initial velocity and vorticity fields exhibit a random distribution. Subsequently, distinct patterns of different velocity and vorticity components begin to emerge and the entire velocity and vorticity fields contain a variety of organized structures. In particular, Figs. 2 and 3 demonstrate that the patterns of streamwise velocity, vertical vorticity and spanwise vorticity, taking the form of streamwise elongated structures, remain consistent in form across different time instances, despite an increasing elongation in the streamwise direction with time progression. Furthermore, a noticeable similarity is observed between the patterns of spanwise velocity and streamwise vorticity, where their streamwise dimensions are consistently comparable to their spanwise dimensions and increase over time. However, changes are evident in the patterns of vertical velocity with time. Specifically, at $t = 200$, the spanwise dimension of the vertical velocity patterns is much greater than the streamwise dimension, coinciding with a strong amplification [see Fig. 2(e)]. In contrast, at $t = 400$ and $t = 600$, the patterns become much weaker and exhibit greater elongation in the streamwise direction [see Figs. 2(h) and 2(k)]. These linear evolutions of pattern characteristics, governed by the linearized Navier-Stokes equations, clearly reveal the different filtering properties that allow the certain structures to pass through to form the output of certain patterns at a specific time instant.

To quantitatively describe the temporal evolutions of horizontal dimensions of the flow patterns, the streamwise and spanwise wave numbers of the dominant modes for an individual velocity or vorticity component, defined as the modes having the maximum energy amplification at a certain time instant, are determined by computing the spectrum from the linear simulations. Given the random initial conditions, the linear simulations are repeated ten times, and the horizontal wave numbers of the dominant modes at each time instant are identified by averaging among the ten samples obtained from the series of linear simulations. The error bars of the averaged wave numbers correspond to the plus/minus standard deviation of the averaged values at a certain time instant. Figure 4 presents the time dependence of the averaged wave numbers of the dominant modes. Consistent with the observations on flow patterns, the averaged streamwise wave numbers of the dominant modes for each individual velocity and vorticity component indeed decrease over time (red solid lines in Fig. 4). However, oscillations are observed in the averaged spanwise wave numbers, particularly for the vertical velocity component [blue solid line in Fig. 4(b)]. Notably, the averaged spanwise wave numbers for vertical velocity

are close to zero and smaller than the corresponding streamwise wave numbers at $t = 200$ and 500, highlighting that the dominant modes of vertical velocity patterns exhibit a near spanwise-independence at these time instants. Given the process governed by the linearized Navier-Stokes equations, the evolutions of the flow patterns can be effectively interpreted through the analytical solutions [i.e., Eq. (7)]. For the patterns of vertical velocity, in particular, the mode with $\kappa_1 = 0.005$ is tilted being exactly orthogonal to the x axis at $t = 200$, which allows the vertical velocity energy of the spanwise-elongated mode ($\kappa_3 \ll \kappa_1$) to undergo significant transient energy amplification due to the Orr mechanism [69,72]. Similarly, the mode with $\kappa_1 = 0.002$ is tilted orthogonally to the x axis at $t = 500$, activating the Orr mechanism. In contrast, at other time instants, there are no modes in Eq. (9) orthogonal to the x axis. As such, the Orr mechanism is not active to facilitate any transient amplification, leading to the vertical velocity patterns being fairly weak [Figs. 2(h) and 2(k)], whose streamwise wavelengths are significantly larger than the spanwise ones. Indeed, the linear evolutions of multiple flow patterns are dictated by the narrow-band filters of the linearized Navier-Stokes equations, which can exhibit different filtering properties with certain dominant modes passing through at different time instants.

After uncovering the filtering nature of the linearized Navier-Stokes operators, it is then essential to evaluate whether the generalized optimal perturbations are effective tools to describe these important properties. As such, generalized optimal perturbations are employed to predict the temporal evolutions of the pattern dimensions, and the predictions will be compared with those of the dominant modes in the random linear simulations. Figure 4 illustrates the time dependence of the optimal horizontal wave numbers predicted by the generalized optimal perturbations. Notably, the predicted optimal wave numbers generally fall within the error bar ranges of the averaged wave numbers of the dominant modes. Indeed, the predicted optimal streamwise wave numbers match well with the averaged ones of the dominant modes (see red solid and dashed lines in Fig. 4). In addition, the near spanwise independence of vertical velocity patterns at $t = 200$ and 500 can also be well predicted. However, there are also some time instances when the predicted optimal spanwise wave numbers lie outside the error bar ranges. Several factors could account for this discrepancy. First, the averaged wave numbers of the dominant modes are derived from a limited sample size (ten samples), which might introduce variability. Increasing the sample size for averaging could potentially lead to a better match. The horizontal wave-number range of the perturbation could be another reason. Specifically, the small-amplitude perturbation in Eq. (9) restricts the horizontal wave-number range to a maximum of 0.032 and a minimum of -0.032 . Consequently, the averaged wave numbers of the dominant modes cannot exceed this range, whereas the generalized optimal perturbations can predict optimal horizontal wave numbers beyond this limit. This constraint probably gives another explanation to the occasional deviation of the predicted optimal wave numbers from the error bar ranges. It is worthwhile to note that the shape and distribution of the flow patterns in vertical direction also evolve with time in the linear simulation. Given the vertical wave number

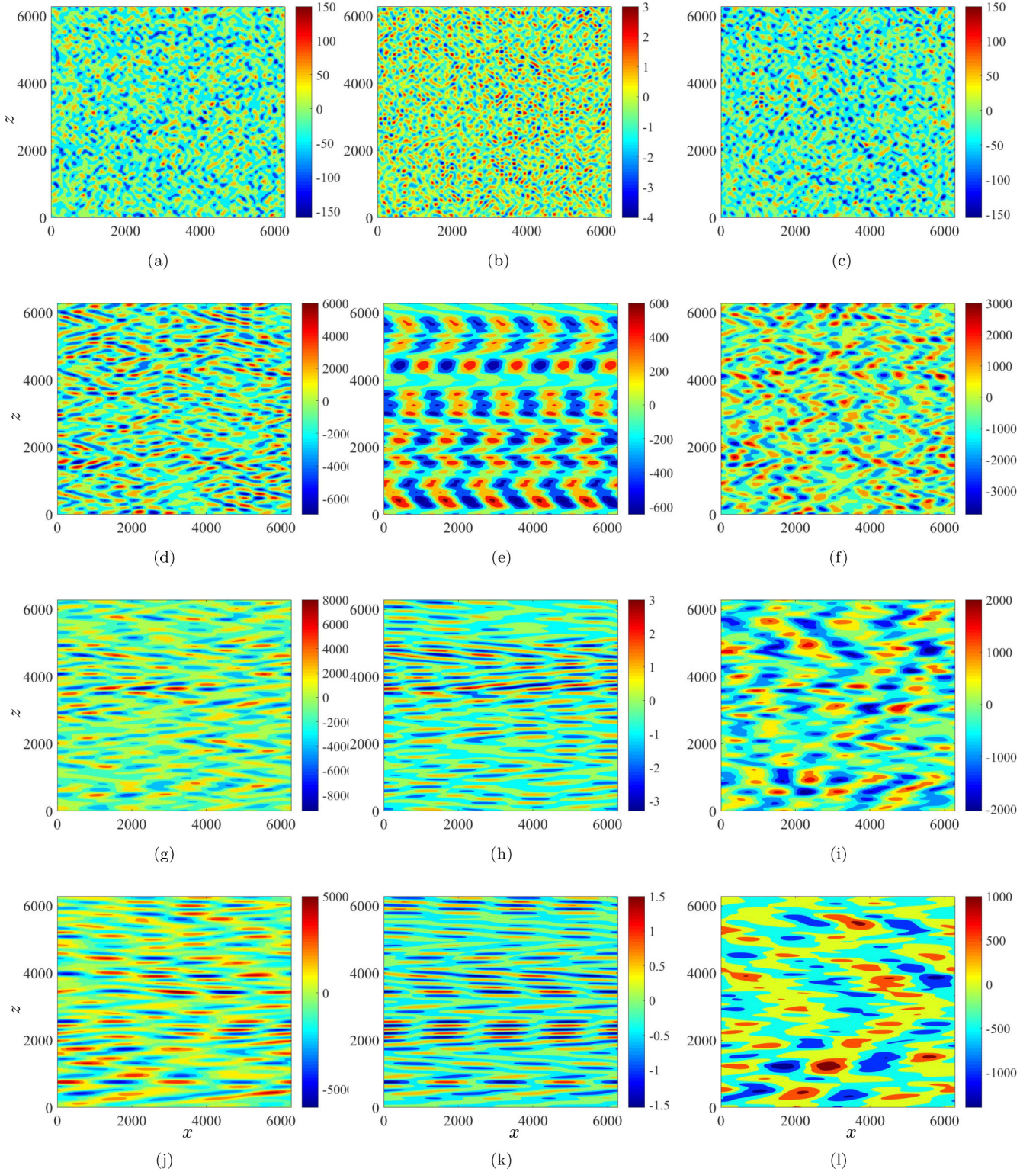


FIG. 2. Contours of (a), (d), (g), (j) streamwise velocity, (b), (e), (h), (k) vertical velocity, (c), (f), (i), (l) spanwise velocity at the x - z plane at (a)–(c) $t = 0$, (d)–(f) $t = 200$, (g)–(i) $t = 400$, and (j)–(l) $t = 600$ for $\text{Re} = 100$.

$\kappa_2 = 1 - m\kappa_{1,0}t$ in Eq. (9), the vertical dimensions of the flow patterns at a specific time instant are primarily determined by the corresponding streamwise wave numbers of the dominant modes. Considering the predicted optimal streamwise wave numbers showing good agreement with those of the dominant modes, the generalized optimal perturbations are also

capable to predict the linear evolutions of flow patterns in vertical direction. Thus far, the generalized optimal perturbations have shown their capability and predictability in describing the linear evolutions of multiple velocity and vorticity patterns in the linear simulations with random broadband initial conditions.

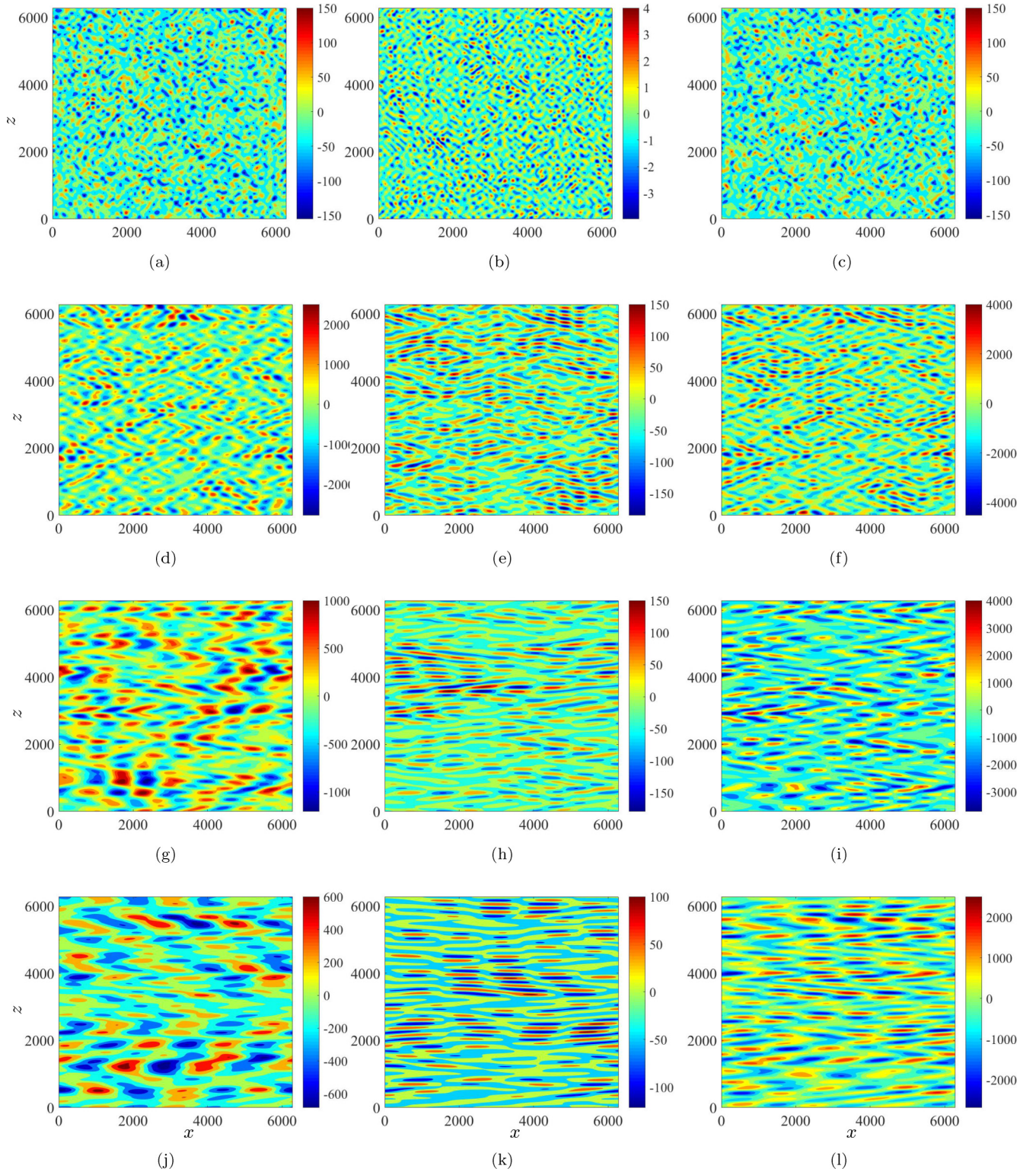


FIG. 3. Contours of (a), (d), (g), (j) streamwise vorticity, (b), (e), (h), (k) vertical vorticity, (c), (f), (i), (l) spanwise vorticity at the x - z plane at (a)–(c) $t = 0$, (d)–(f) $t = 200$, (g)–(i) $t = 400$, and (j)–(l) $t = 600$ for $Re = 100$.

For comparison, Fig. 4 also displays the optimal horizontal wave numbers predicted by the original optimal perturbation framework in Eq. (4). The streamwise and spanwise wave numbers predicted by this tool locate within the error bar ranges for streamwise velocity patterns, suggesting that the original optimal perturbation framework is capable of

predicting the linear temporal evolution of streamwise velocity patterns with random initial conditions [see black dashed and dot dashed lines in Fig. 4(a)]. However, as shown in Figs. 4(b)–4(f), the original optimal perturbation framework fails to accurately capture the linear evolutions of vertical velocity, spanwise velocity, streamwise vorticity, vertical

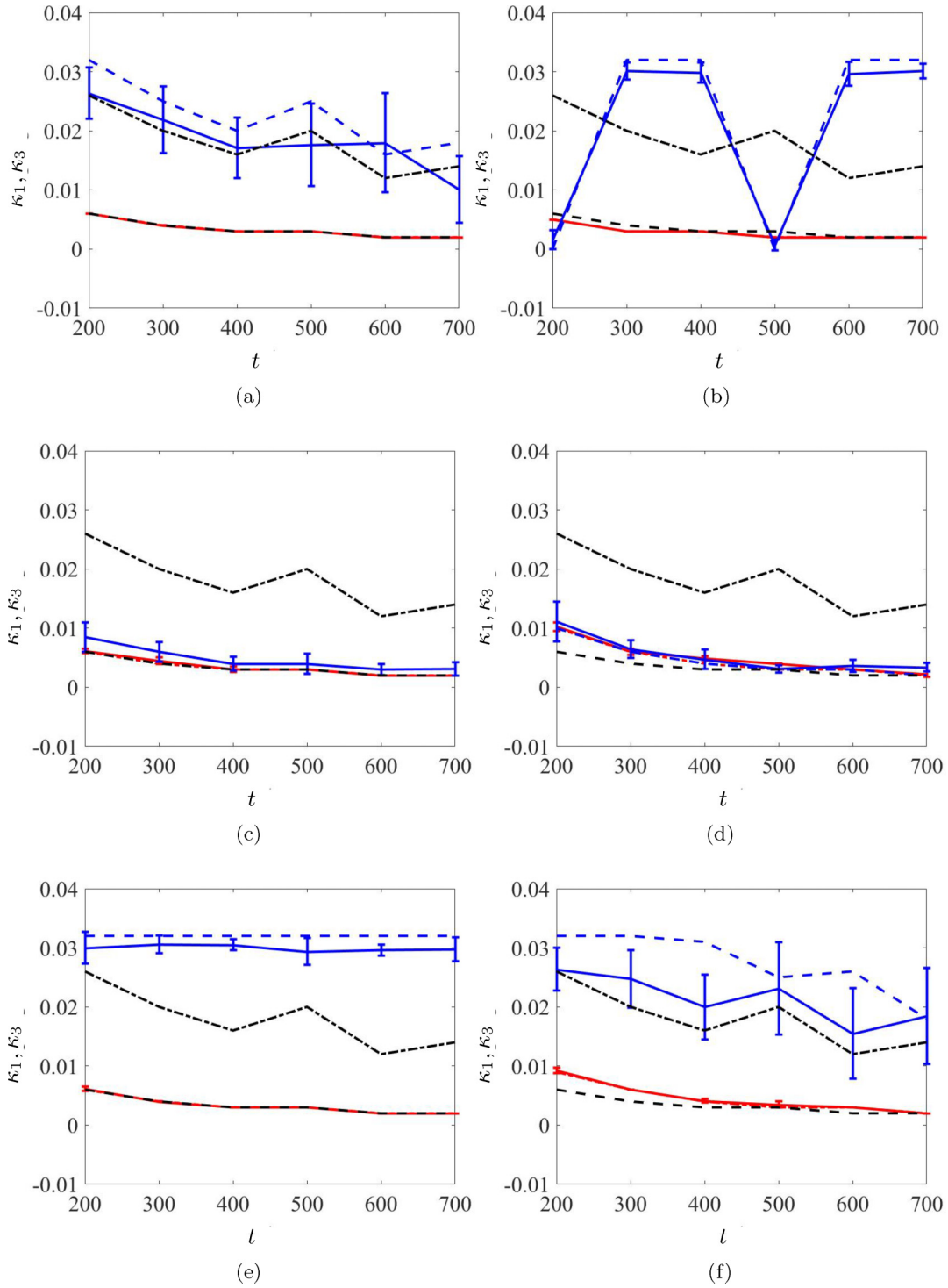


FIG. 4. Comparison between the time dependence of the streamwise (red lines) and spanwise (blue lines) wave numbers predicted by generalized optimal perturbations (dashed lines) and those from the dominant modes with random initial conditions in linear simulations (solid lines) for (a) u , (b) v , (c) w , (d) ω_x , (e) ω_y , and (f) ω_z at $\text{Re} = 100$. The black dashed and dot dashed lines represent the streamwise and spanwise wave numbers predicted by the original optimal perturbation framework.

vorticity and spanwise vorticity patterns. Compared to the original optimal perturbation framework, the improvement in predicting the linear evolutions of flow patterns via generalized optimal perturbations reveals its refinement in explaining the filtering properties of the linearized Navier-Stokes equations. Indeed, with the choice of structure norms appropriate

for patterns of interest, generalized optimal perturbations can effectively represent the strong dependence of the filtering properties on the choice of filters' outputs, leading to the improved predictions.

With the linear simulations, it has been shown that the multiple filtering properties of the linearized Navier-Stokes

equations can be effectively described by generalized optimal perturbations. Despite the flow patterns in the linear simulations being uncommon in most of the turbulent direct numerical simulation (DNS) studies, it is reminiscent of DNS results for low-Reynolds-number turbulent Couette flow over rough wall. In DNS with modeling virtual rough surfaces under the subcritical conditions, similar flow patterns with the present result in Fig. 1(a) have been observed in Ref. [73]. This similarity implies that the filtering properties of linearized Navier-Stokes operators could play an essential role in the formation of these regular staggered structures in turbulent Couette flow over large roughness at small Reynolds number. In the next section, the filtering properties of the linearized Navier-Stokes equations will be investigated via generalized optimal perturbations in a fully developed turbulent flow, where nonlinearity is taken into consideration.

III. GENERALIZED OPTIMAL PERTURBATIONS IN TURBULENT CHANNEL FLOW

A. Direct numerical simulation

A direct numerical simulation (DNS) is conducted in a turbulent channel flow of a viscous fluid with kinematic viscosity ν between two plates located at $y = \pm h$. Here, all flow variables are nondimensionalized using the channel half-height h and the centerline velocity U_c . The Reynolds number is therefore defined as $Re = U_c h / \nu$, with a value of $Re = 9600$ considered in the present study. When the turbulent flow is fully developed, the friction Reynolds number $Re_\tau = u_\tau h / \nu$ converges to 366.6, where u_τ is the friction velocity. The DNS is performed using Diablo [74], a solver that has been extensively validated in previous works [69,75–77]. In this numerical solver, the streamwise and spanwise directions are discretized using the Fourier-Galerkin method with the 2/3 rule for dealiasing, whereas the wall-normal direction is discretized by a second-order central finite difference method. Time integration is carried out via the fractional-step method. In particular, all the viscous terms are advanced in time using the Crank-Nicolson method, while the remaining terms are integrated with a low-storage third-order Runge-Kutta method. The computational domain is $L_x = 2\pi h$, $L_y = 2h$, and $L_z = \pi h$, with the number of grid points $N_x = 256$, $N_y = 193$, $N_z = 256$ in the streamwise, wall-normal and spanwise directions, respectively. The DNS is performed with imposing a constant mass flux and no-slip boundary conditions. The characteristics of the flow patterns obtained from the DNS will then be compared with those predicted by generalized optimal perturbations. The impact of nonlinearity on the determination of the flow patterns in real turbulence will also be discussed.

B. Generalized optimal perturbations

The generalized optimal perturbations for individual velocity components are computed in the turbulent channel flow at different wall-normal locations (i.e., y_0). Following the previous studies [25,27,78], the mean velocity profile for turbulent channel flow is approximated using the Reynolds-Tiederman profile. Here, $Re_\tau = 366$ is considered such that it matches with the converged DNS data. The optimization problem in Eq. (5) is solved using the standard method [79]. Specifically,

TABLE I. Optimal wavelengths predicted by the generalized optimal perturbations for individual velocity components at certain distance from the wall at $Re_\tau = 366$.

y^+	$\lambda_{u,x}^+$	$\lambda_{u,z}^+$	$\lambda_{v,x}^+$	$\lambda_{v,z}^+$	$\lambda_{w,x}^+$	$\lambda_{w,z}^+$
10	∞	69.81	/	/	83.47	78.54
20	∞	132.28	/	/	168.80	130.75
30	∞	191.69	/	/	228.48	182.42
40	∞	263.02	75.15	∞	305.67	248.57
50	∞	353.43	89.76	∞	411.26	332.64
60	∞	403.92	106.19	∞	471.24	383.38
70	∞	461.62	119.68	∞	551.69	443.52
80	∞	579.99	151.81	∞	706.86	565.49
90	∞	628.32	174.00	∞	807.84	646.27
100	∞	685.44	191.69	∞	904.78	706.86

the Orr-Sommerfeld and Squire system is discretized using a Chebyshev collocation method [80] with 160 collocation points in wall-normal direction. The maximum energy growth and the corresponding generalized optimal perturbations are computed by formulating a variational problem, the solution of which is obtained via the singular value decomposition. The validity of the numerical solver is confirmed by reproducing the previous results of Ref. [27].

The optimal wavelengths predicted by the generalized optimal perturbations for individual velocity components at varying distances from the wall are presented in Table I. Notably, the generalized optimal perturbations for streamwise velocity are streamwise elongated, whose spanwise wavelengths increase with arising distance away from the wall. The generalized optimal perturbations for wall-normal velocity, however, do not experience any transient energy growth for $y^+ \leq 30$, owing to the transient amplification caused by the mean shear insufficient to overcome the viscous effects in the Reynolds numbers considered in this study. However, the generalized optimal perturbations for wall-normal velocity undergo weak amplifications, exhibiting spanwise independence for $y^+ \geq 40$. In addition, $\lambda_{v,x}^+$ also increases with increasing distance from the wall (see Table I). The generalized optimal perturbations for spanwise velocity take the form of oblique structures, with their horizontal dimensions also expanding with increasing distance from the wall. It is worthwhile to note that the structural characteristics of generalized optimal perturbations in the fully developed turbulent channel flow are qualitatively similar to those observed in the homogeneous shear flow. This similarity is not unexpected and provides additional evidence that the optimal wavelengths of the generalized optimal perturbations are primarily governed by the universal properties of the linearized Navier-Stokes operators, rather than by the small details of specific flow configurations. Here, the generalized optimal perturbations are computed using the governing equations Eq. (1) linearized around the Reynolds-Tiederman turbulent mean velocity profile with $\mathbf{F} = 0$. Consequently, this formulation eliminates the need to model the nonlinear Reynolds stress terms, allowing the present study to focus on the essential roles of intrinsic filtering properties of pure linearized Navier-Stokes operators in dictating patterns in real turbulence. A discussion about

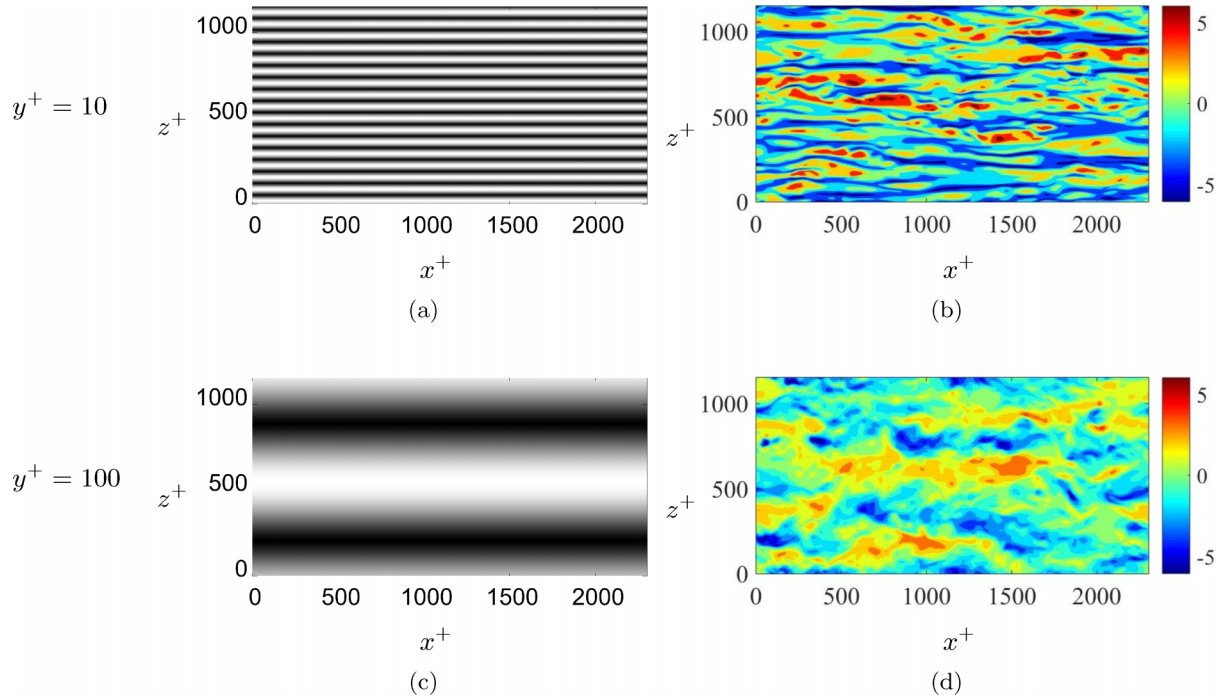


FIG. 5. Comparisons of the predictions from (a), (c) generalized optimal perturbations with (b), (d) instantaneous DNS snapshots at $Re = 9600$. Streamwise velocity u at (a), (b) $y^+ = 10$ and (c), (d) $y^+ = 100$.

the impact of including an eddy viscosity term to model the Reynolds stress on the computations of generalized optimal perturbations is provided in the Appendix.

C. Linear filtering properties in fully developed turbulence

It has been well-established that the generalized optimal perturbations for streamwise velocity can predict the dimensions of streamwise streaks in both near-wall region and outer layer in fully developed turbulent flows [27]. However, it remains unclear whether generalized optimal perturbations, representing the filtering properties of the linearized Navier-Stokes operators, are effectively predictive to capture the dimensions of other flow patterns in fully developed turbulence. To address this question, the instantaneous flow fields for each velocity component at $y^+ = 10$ and $y^+ = 100$ from the DNS are analyzed and compared with the patterns predicted by the corresponding generalized optimal perturbations (see Figs. 5–7). As illustrated in Fig. 5, near-wall streamwise streaks, which extend across the entire streamwise domain, are clearly visible. At $y^+ = 100$, there exists large-scale quasistreamwise streaks characterized by greater spanwise dimensions. Notably, the patterns predicted by the generalized optimal perturbations for streamwise velocity demonstrate qualitative agreement with the instantaneous streamwise velocity patterns derived from the DNS in both the near-wall region and the outer layer. However, the generalized optimal perturbations for wall-normal velocity provide predictions of spanwise-independent patterns, contrasting with the strongly spanwise-dependent wall-normal velocity structures observed in both the near-wall region and the outer layer (see Fig. 6). This strong spanwise dependence observed in the DNS shows that the generalized optimal perturbations are unable to

qualitatively predict the spanwise dimensions of the wall-normal velocity patterns in real turbulence, implying the patterns of wall-normal velocity are caused by mechanisms that are different from the filtering properties of the linearized Navier-Stokes operators. For spanwise velocity, similarities are also found between the patterns predicted by the generalized optimal perturbations and the instantaneous spanwise velocity structures obtained from the DNS, with both exhibiting the form of oblique patterns (see Fig. 7). Note that given the symmetry in spanwise direction, the orientations of the generalized optimal perturbations in Fig. 7 are chosen for better comparison with the counterpart of the DNS snapshots. In this sense, the generalized optimal perturbations for spanwise velocity seem to be capable of predicting the spatial dimensions of spanwise velocity patterns in fully developed turbulent flows.

The DNS also provides the premultiplied spanwise wave-number spectra for streamwise and spanwise velocities, as well as the streamwise wave-number spectra for wall-normal and spanwise velocities to facilitate a quantitative comparison between the dimensions of patterns in real turbulence with those predicted based on the generalized optimal perturbations (Fig. 8). Indeed, the spanwise dimensions of streaks are well predicted by the generalized optimal perturbations for streamwise velocity in the range from $y^+ = 10$ to $y^+ = 100$ [see Fig. 8(a)]. Furthermore, the predicted spanwise dimensions of spanwise velocity patterns agree qualitatively well with those from the DNS [see Fig. 8(c)], suggesting the spanwise characteristics of spanwise velocity patterns are also well-captured by the corresponding generalized optimal perturbations. However, the generalized optimal perturbations appear unable to predict the streamwise characteristics of wall-normal and spanwise velocity patterns, since the

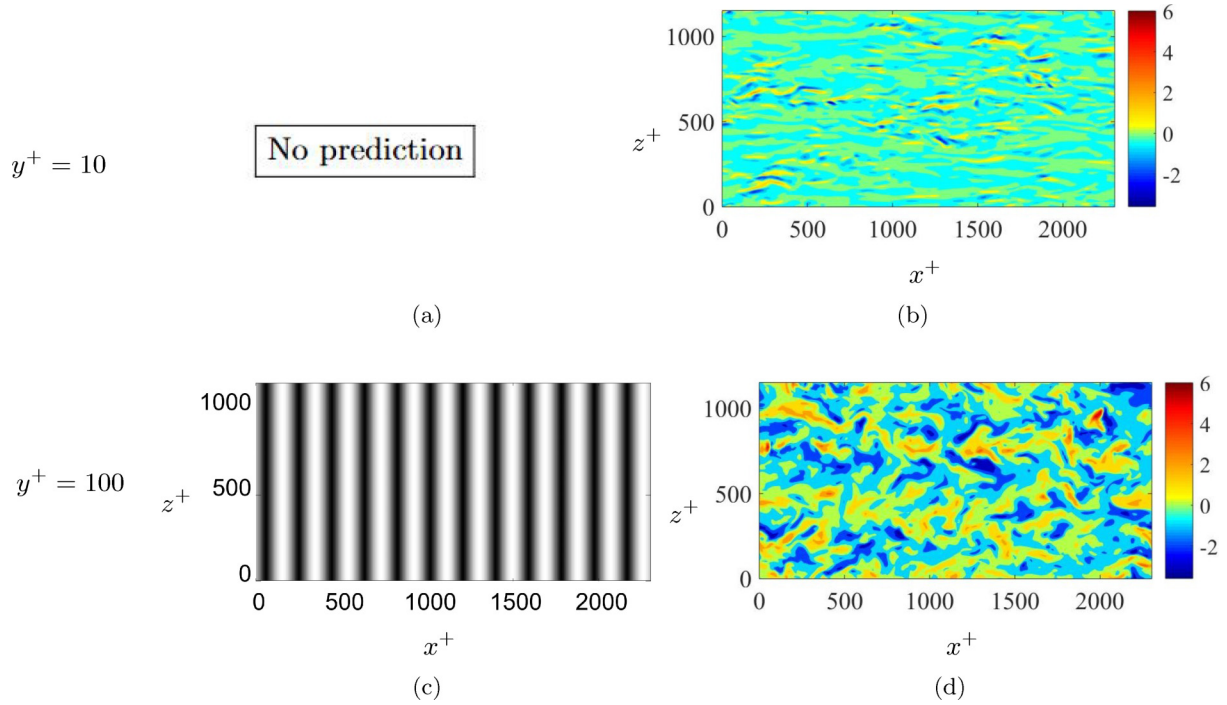


FIG. 6. Comparisons of the predictions from (a), (c) generalized optimal perturbations with (b), (d) instantaneous DNS snapshots at $Re = 9600$. Wall-normal velocity v at (a), (b) $y^+ = 10$ and (c), (d) $y^+ = 100$.

predicted streamwise wavelengths show noticeable qualitative discrepancies with the DNS results. In particular, the streamwise wavelengths of the wall-normal and spanwise velocity structures in real turbulence are almost independent of the distance from the wall, whereas the predicted streamwise

wavelengths exhibit a clear increase with growing y^+ [see Figs. 8(b) and 8(d)].

Given the generalized optimal perturbations purely derived based on the linearized Navier-Stokes equations, it can be inferred that the spanwise dimensions of streamwise and

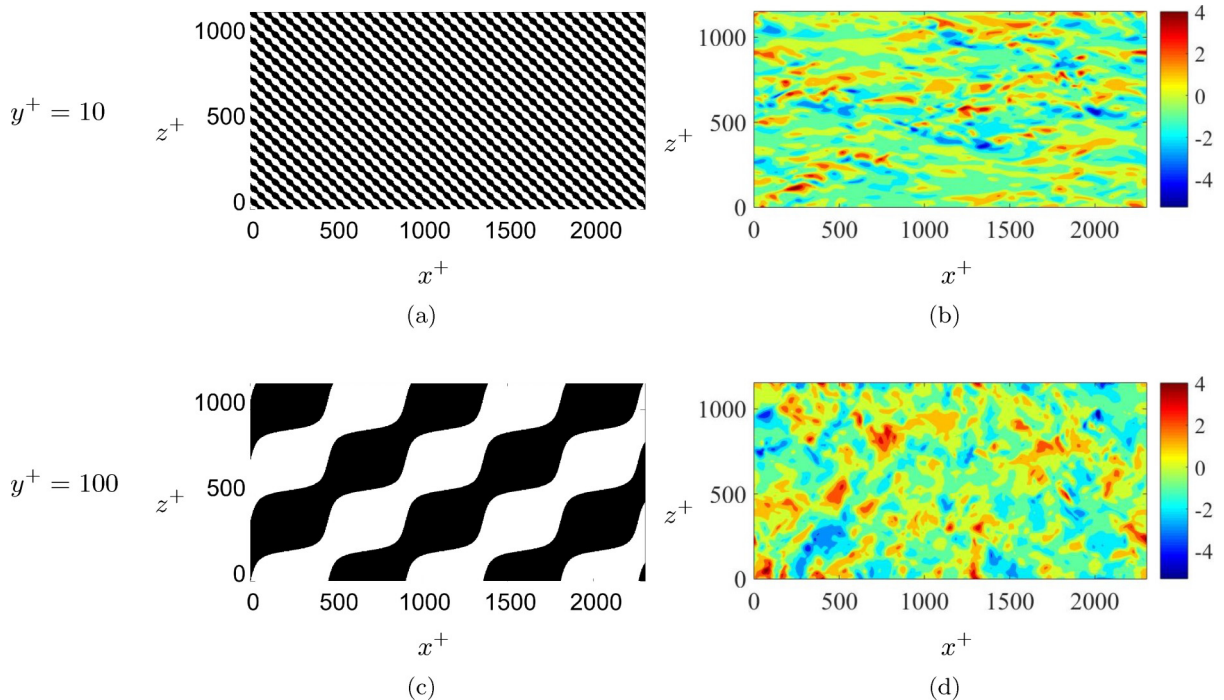


FIG. 7. Comparisons of the predictions from (a), (c) generalized optimal perturbations with (b), (d) instantaneous DNS snapshots at $Re = 9600$. Spanwise velocity w at (a), (b) $y^+ = 10$ and (c), (d) $y^+ = 100$.

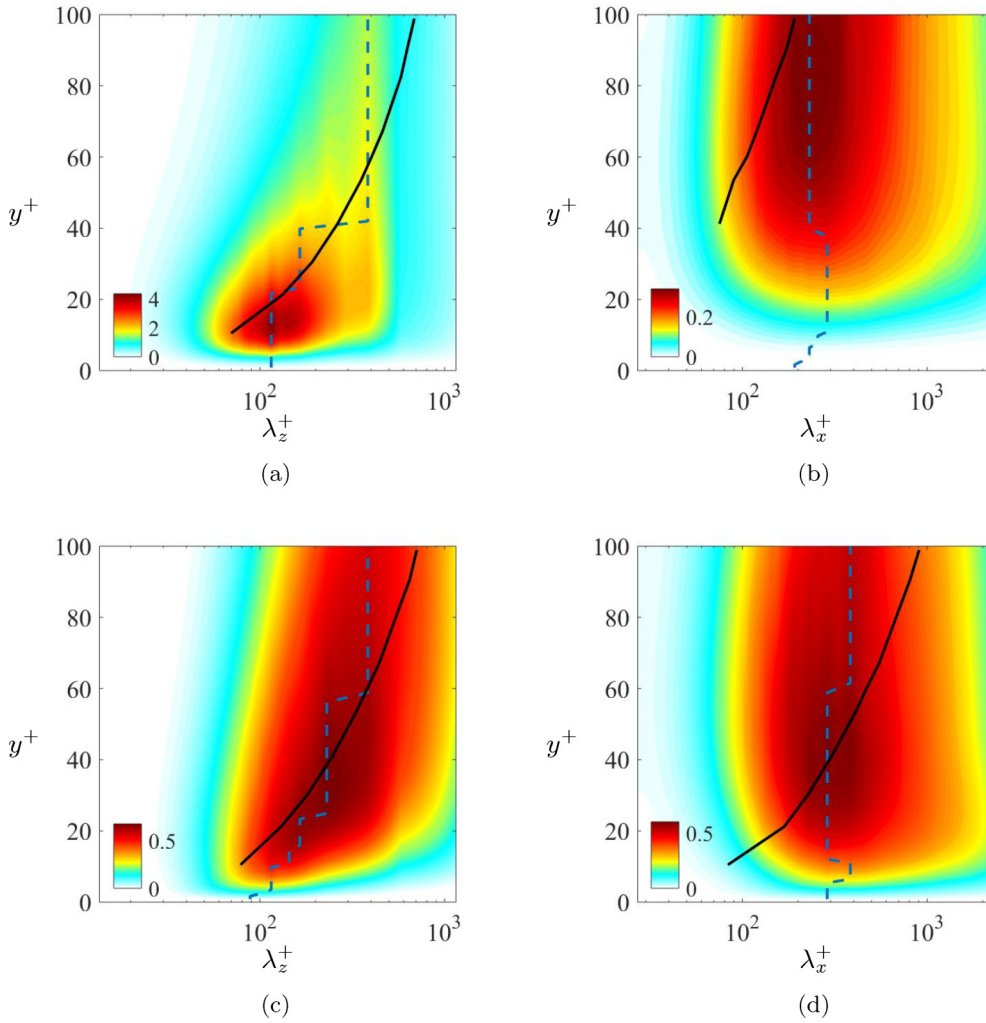


FIG. 8. Premultiplied one-dimensional (a), (c) spanwise and (b), (d) streamwise wave-number spectra of streamwise velocity (a), wall-normal velocity (b), and spanwise velocity (c), (d) at $Re = 9600$. The black solid lines represent the wavelengths predicted by the generalized optimal perturbations whereas the blue dashed lines are the wavelengths obtained from the premultiplied spectra.

spanwise velocity patterns, as well-predicted by the corresponding generalized optimal perturbations, are primarily determined by the selectivity of the linearized Navier-Stokes operators. This finding also indicates that the generation and amplification of streamwise and spanwise velocity structures in fully developed turbulence exhibit a fundamentally linear nature. Indeed, it is well-known that the streamwise velocity structures (i.e., near-wall streamwise streaks and large-scale streaky motions) are generated through the linear lift-up effect. For spanwise velocity patterns in real turbulence, whose spanwise dimensions are comparable to those of the streamwise ones, it could be speculated that the lift-up effect induced by the Orr mechanism plays an essential role in their pattern formations and amplifications. It is also worthwhile to note that the nonlinear optimal perturbations computed in the subspace spanned by the linear optimal perturbation modes show the form of oblique modes [81]. In addition, the localized nonlinear optimal perturbations initially grow via the Orr mechanism and subsequently develop into streamwise elongated streaks via the lift-up effect [82]. Compared to the generalized optimal perturbations for spanwise velocity, the nonlinear optimal perturbations exhibit similar structural

characteristics and growth mechanisms in their early stage of evolutions, implying the importance of the filtering properties of the linearized Navier-Stokes operators in the early stage development of nonlinear optimal perturbations. However, the obvious discrepancies between the DNS observations and the generalized optimal perturbation predictions in pattern dimensions indicate that the nonlinearity has nonnegligible effects on the generation and amplification of wall-normal velocity patterns in fully developed turbulence. Indeed, it has been found that the quasistreamwise vortices are regenerated and amplified through nonlinear mechanisms in the self-sustaining process [20,35]. In addition, nonlinear effects appear nontrivial also in the formation of spanwise velocity patterns with the streamwise dimensions not well predicted by the corresponding generalized optimal perturbations.

Thus far, generalized optimal perturbations have shown their advantages of easiness and intuitiveness on describing the filtering properties of the linearized Navier Stokes operators in the absence of the body forcing as the input. In fully developed turbulence, some important features of flow patterns are well-predicted by the generalized optimal perturbations, suggesting the importance of the selectivity

of the linearized Navier-Stokes operators in patterns formations. However, some linear filtering properties are not strong enough to outweigh the effects of the pattern that might be inherent in the nonlinear forcing. In these cases, the pattern formation, for instance, the generation of wall-normal velocity structures is strongly affected by nonlinear mechanisms. Given the requirement of the linear filtering properties outweighing the nonlinear effects to facilitate qualitatively accurate predictions, generalized optimal perturbation framework also exhibits its potential as a physically interpretable tool which indicates specific flow patterns formed through linear processes.

IV. CONCLUDING REMARKS

In this paper, the utility and effectiveness of generalized optimal perturbations in capturing the filtering properties of linearized Navier-Stokes operators which select the shape and dimensions of coherent patterns without any external forcing are investigated. The generalized optimal perturbations for individual velocity and vorticity components are first computed with the analytic solutions of the Navier-Stokes equations linearized around the basic profile of a homogeneous shear flow to uncover these filtering properties of linearized Navier-Stokes operators. It is found that the generalized optimal perturbations for streamwise velocity, wall-normal vorticity and spanwise vorticity are all streamwise elongated, whereas those for vertical velocity are elongated in spanwise direction. Furthermore, the generalized optimal perturbations for spanwise velocity and streamwise vorticity both take the form of oblique structures. Different generalized optimal perturbations for different velocity and vorticity components exhibiting different forms implies multiple filtering properties of the linearized Navier-Stokes operators, which strongly depend on the choice of the filters' outputs. Subsequently, the capability of generalized optimal perturbations in elucidating the filtering properties is examined by predicting the temporal evolution of a small-amplitude perturbation with random initial conditions in linear simulations of the homogeneous shear flow. With the perturbation linearly evolving over time, distinct patterns are formed. Indeed, the structural characteristics of the flow patterns from the linear simulations match well with those predicted by the generalized optimal perturbations demonstrating that the generalized optimal perturbations are capable to predict the linear temporal evolution of various flow patterns and effective to describe the multiple filtering properties of linearized Navier-Stokes operators. The utility and effectiveness of generalized optimal perturbations which describe the filtering properties of linearized Navier-Stokes operators in predicting the dimensions of flow patterns are further examined in a fully developed turbulent channel flow. Like the spanwise spacing of near-wall and large-scale streaks, the spanwise dimensions of spanwise velocity structures can also be qualitatively predicted using the corresponding generalized optimal perturbations, supporting the effectiveness of generalized optimal perturbations in representing the selectivity of the linearized Navier-Stokes operators, which play important roles in dictating flow patterns in real turbulence. However, it is also observed that the generalized optimal perturbations fail to predict essential spatial

characteristics of some flow patterns, for instance, the wall-normal velocity patterns, as a result of the filtering properties of the linearized Navier-Stokes operators being not strong enough to dominate the nonlinear effects in these aspects.

The explanatory framework based on generalized optimal perturbations for filtering properties of the linearized Navier-Stokes operators is universal and intuitive, which stems from its foundation in the intrinsic properties of the Navier-Stokes equations in the absence of any input body forcing, independent of the specific details of individual flows, and does not require the mean flow profile to exhibit linear instability. Due to its universality and simplicity, the question of which of the numerous organized structures or which of their features can be attributed to the filtering mechanisms of linearized Navier-Stokes operators holds significant interest. Further research could extend these investigations to the soliton-like coherent structures confirmed to be common in a variety of flows [83,84] via generalized optimal perturbations.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: GENERALIZED OPTIMAL PERTURBATIONS WITH EDDY VISCOSITY

In this Appendix, generalized optimal perturbations are computed with the Reynolds stress modeled by eddy viscosity. In particular, the generalized optimal perturbations are calculated based on the linearized equations for small-amplitude "organized" perturbations, in which the random turbulent fluctuations is modeled with eddy viscosity [28,30–32,85]

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{U} = -\frac{1}{\rho} \nabla p + \nabla \cdot [(\nu + \nu_t)(\nabla \mathbf{u} + \nabla \mathbf{u}^H)], \quad (\text{A1a})$$

$$\nabla \cdot \mathbf{u} = 0, \quad (\text{A1b})$$

where ν and ν_t are the kinematic viscosity and eddy viscosity, respectively. The superscript $(\cdot)^H$ indicates complex conjugate transpose. The streamwise turbulent mean velocity $U(y)$ is still approximated by the Reynolds-Tiederman profile and the eddy viscosity ν_t is then adopted from the empirical formulation of Ref. [86],

$$\nu_t(\eta) = \frac{\nu}{2} \left\{ 1 + \frac{\kappa^2 \text{Re}_\tau^2}{9} (1 - \eta^2)^2 (1 + 2\eta^2)^2 \times (1 - \exp[(|\eta| - 1)\text{Re}_\tau/A])^2 \right\} - \frac{\nu}{2}, \quad (\text{A2})$$

with $\eta = (y - h)/h$.

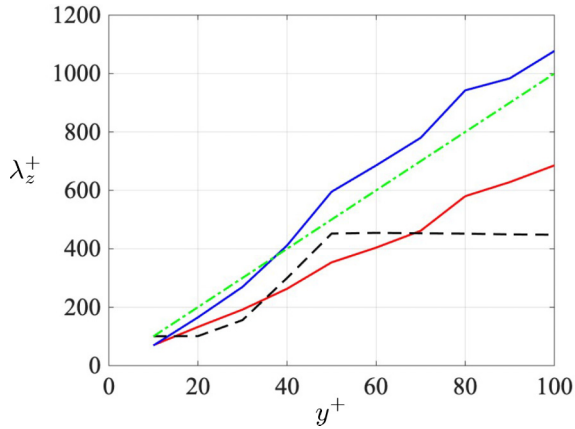


FIG. 9. Comparison among the spanwise dimensions of streamwise streaky patterns obtained from DNS (black dashed), the scaling law $y \approx 0.1\lambda_z$ (green dot dashed), and those predicted by generalized optimal perturbations with eddy viscosity (blue solid) and without eddy viscosity (red solid) at various wall-normal locations and $Re_\tau = 360$.

Figure 9 presents the comparison between the spanwise dimensions of streamwise velocity patterns predicted by generalized optimal perturbations with and without introducing the eddy viscosity in the governing equations. In general, the predictions of spanwise dimensions from the generalized optimal perturbations with and without the inclusion of the eddy viscosity are qualitatively similar, especially in the near-wall region where the difference between these two predictions is small with the effects of eddy viscosity being trivial (i.e., $\nu \gg \nu_t$). However, the discrepancy between these two predictions becomes pronounced in the region further away from the

wall. Indeed, larger predictions for the spanwise dimensions of streamwise streaky patterns are provided by the generalized optimal perturbations with the inclusion of the eddy viscosity, which match better with the scaling law of $y \approx 0.1\lambda_z$ proposed in Ref. [41]. However, the generalized optimal perturbations without introducing the eddy viscosity appear to offer better agreement with the spanwise dimensions of streaky patterns from the present DNS data.

Despite the growing evidence that modeling the nonlinear effects of turbulent fluctuations with eddy viscosity is essential in describing many fundamental features in turbulence [29,87–91], it is also worthwhile to note that there are other successful predictions from generalized optimal perturbations without using eddy viscosity in turbulent flows [27,51]. Furthermore, compared to the scaling law and the DNS data, the overestimation of spanwise dimensions of streaky patterns in outer region by generalized optimal perturbations with eddy viscosity could originate from the theoretical eddy viscosity issues discussed in Ref. [51]. Finally, the present work aims to uncover an universal and intuitive explanation to the structural features of the patterns in transitional and turbulent flows based on the filtering mechanisms of the pure linearized Navier-Stokes operators via generalized optimal perturbations with different structure norms. This is a question that is largely the same regardless of the use of eddy viscosity. Given all above considerations, as such, the generalized optimal perturbations are adopted without modeling the nonlinear effects of turbulent fluctuations via the eddy viscosity model in the current study. However, modifying the eddy viscosity model and exploring the interaction and competition between the linear filtering mechanism and Reynolds stress modeling in fully developed turbulence might represent a valuable direction for future work.

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