

Order parameter for morphodynamic transitions in braided rivers: A stochastic compositional framework

Samuele De Bartolo ^{*}

Department of Engineering for Innovation, [University of Salento](#), via per Monteroni S.P.6, 73100 Lecce, Italy

Carlo De Michele 

Department of Civil and Environmental Engineering, [Polytechnic University of Milan](#), Piazza L. Da Vinci 32, 20133 Milan, Italy



(Received 10 June 2026; accepted 19 August 2026; published 8 September 2026)

We develop a stochastic simplex framework for width partitioning in braided river cross sections, modeling normalized channel widths as random compositional vectors. Two complementary models are introduced: the symmetric Dirichlet distribution, which models width allocation as exchangeable stochastic fluctuations around a uniform baseline with no preferred channel, and the isotropic logistic-normal distribution, which accommodates systematic width concentration through a single scale parameter. Both models carry independent physical content: the Dirichlet characterizes morphodynamic states in which all channels are statistically equivalent, while the logistic-normal identifies and quantifies organized concentration regimes. We derive exact closed-form expressions for the expected concentration index and width concentration fraction under both models, establish a parameter-free structural breakdown criterion, and analyze the temporal evolution of model parameters across 11 annual surveys of the Brahmaputra-Jamuna River (Bangladesh) spanning 1976–2007. The analysis reveals a decadal-scale morphodynamic transition: for 1976–1985, width is distributed among channels in a statistically uniform fashion consistent with the Dirichlet model. From 1995 onwards, a progressive increase in the logistic-normal scale parameter σ^* , optimized independently for each survey year and fitted by a logistic regression ($R^2 = 0.997$), indicates systematic growth of channel width concentration. A parameter-free structural breakdown criterion certifies the failure of the Dirichlet model between 1995 and 1999, where the logistic-normal model recovers goodness-of-fit values above 0.95. Both σ^* and the Dirichlet parameter c^* retain memory of the concentration phase above pretransition levels throughout the observational window.

DOI: [10.1103/PhysRevE.114.034305](https://doi.org/10.1103/PhysRevE.114.034305)

I. INTRODUCTION

Identifying order parameters that distinguish disordered, purely stochastic configurations from structurally organized ones is a recurring theme across statistical physics, from spin systems to network topology. Here we show that an analogous distinction applies to the spatial organization of braided river networks: a single scale parameter, extracted from a stochastic compositional model of channel width partitioning, behaves as an order parameter for a decadal-scale morphodynamic transition, with a rigorous, parameter-free criterion certifying when the disordered (stochastic) description structurally breaks down.

Braided rivers are among the most morphodynamically active fluvial systems on Earth, characterized by multiple active channels that continuously shift, merge, and bifurcate around midchannel bars and islands [1–5]. Their planimetric

complexity is routinely summarized by the braiding index B , defined as the number of active wet channels intersected by a cross section, which quantifies the degree of channel multiplicity at the reach scale [6,7]. Nevertheless, B provides no information on the internal partitioning of width among channels, a quantity that directly controls the lateral distribution of discharge, sediment flux, and morphodynamic activity across the braided reach [8].

Despite its physical significance, the statistical organization of width partitioning across braided cross sections has received limited attention. Over the past two decades, significant effort has been devoted to uncovering scaling laws and self-similar properties of braided channel networks, revealing power-law behaviors in channel counts, fractal structures, and analogies with self-organized criticality [9–15]. More recently, multiscaling approaches have characterized the spatial variability and intermittency of the braiding index [16–18], and geometric network descriptors have been proposed to capture two-dimensional channel organization [19]. Entropy-based indices such as the entropic Braiding Index [20] incorporate width information into a scalar metric, but treat width fractions as proxies for discharge rather than as primary geometric observables, and compress cross-sectional structure into a single effective channel count. However, none of these approaches characterizes the joint distribution of

^{*}Contact author: samuele.debartolo@unisalento.it

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](#) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

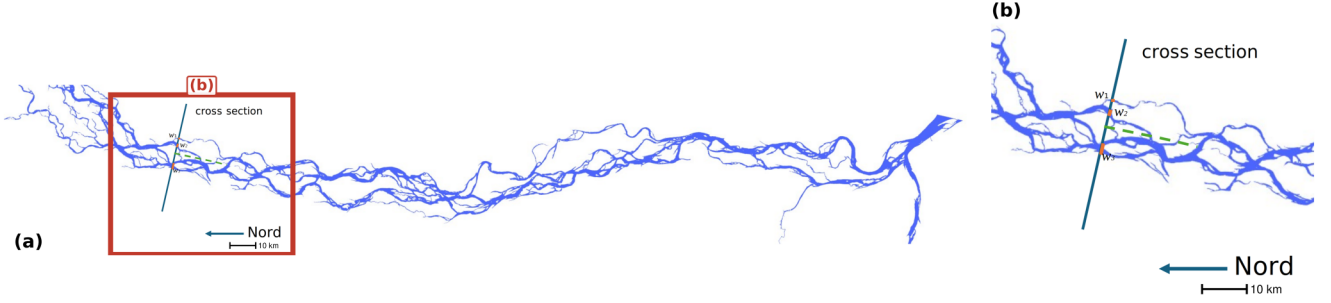


FIG. 1. (a) Planimetric extent of the Brahmaputra braided channel network extracted from 1985 Landsat MSS satellite imagery ($80 \times 80 \text{ m}^2$ spatial resolution), as documented in Rizzello *et al.* [18]. The red rectangle indicates the region enlarged in panel (b). (b) Detail of a representative cross section (blue oblique line, drawn perpendicular to the curvilinear abscissa s , shown as a green dashed segment). The three active wetted widths w_1 , w_2 , w_3 (red segments) intercept the cross section, giving a braiding index $B = 3$ for this transect. The normalized widths $p_i = w_i/W$, where $W = \sum_i w_i$ is the total wetted width, constitute the compositional vector analyzed in this study.

individual channel widths $w_1, \dots, w_B$ conditional on B , nor provides a stochastic model for width partitioning capable of distinguishing random fluctuations from systematic morphodynamic reorganization or detecting regime shifts in the internal structure of the braided network.

Here we address this gap by introducing a stochastic simplex framework for width partitioning. The normalized channel widths $p_i = w_i/W$ satisfy $\sum_i p_i = 1$ and lie on the $(B-1)$ -dimensional simplex Δ^{B-1} [21], making them a natural object for compositional data analysis. We propose two complementary probability models on this simplex, the symmetric Dirichlet distribution and the isotropic logistic-normal distribution, and show that the progressive increase of the logistic-normal scale parameter σ^* provides a quantitative signature of decadal-scale morphodynamic reorganization. Analysis of 11 annual surveys of the Brahmaputra-Jamuna River (Bangladesh) spanning 1976–2007 [18] reveals a slow morphodynamic transition characterized by a progressive growth of channel width concentration from 1995 onwards, captured by a single order parameter σ^* whose temporal evolution follows a logistic regression with $R^2 = 0.997$.

The paper is organized as follows. Section II introduces the stochastic models, the structural indicators, the two central results of the framework, the structural breakdown criterion for the Dirichlet model and the structural compatibility of the logistic-normal model, and the procedures used to fit and compare the models against the empirical data. The Appendix develops the supporting mathematical derivations, including exact moment formulas and the variance of the concentration index. Section III describes the dataset. Section IV presents the empirical results. Section V discusses the physical interpretation and broader implications. Section VI gives the conclusions.

II. METHODS

This section introduces the two stochastic models for width partitioning together with the procedures used to fit and compare them against the empirical data. Supporting mathematical derivations, including exact moment formulas and the variance of the concentration index, are given in the Appendix.

A. Stochastic models for width partitioning

Consider a cross section with B active wet channels (braiding index) of widths $w_1, \dots, w_B$ (Fig. 1). The normalized widths $p_i = w_i/W$, where $W = \sum_i w_i$ is the total wetted width, satisfy $\sum_i p_i = 1$ and constitute a compositional vector $\mathbf{p} = (p_1, \dots, p_B)$ on the $(B-1)$ -dimensional probability simplex [21,22]

$$\Delta^{B-1} = \{\mathbf{p} \in \mathbb{R}^B : p_i \geq 0, \sum_i p_i = 1\}. \quad (1)$$

Two scalar indicators summarize the internal structure of the partition: the *concentration index* $H = \sum_i p_i^2$ [23], equal to $1/B$ for a perfectly uniform partition and to 1 when a single channel captures the entire wetted width, and the *width concentration fraction* $D = \max_i p_i$. Their formal properties and mutual bounds are established in Sec. A 4.

Entropy-based measures have recently been proposed to characterize the structural diversity of braided rivers, including the entropic Braiding Index (eBI) introduced by Tejedor *et al.* [20]. Three fundamental differences distinguish the present approach. First, Tejedor *et al.* [20] use discharge fractions q_i/Q as probability weights, substituting channel width as a proxy for discharge where measurements are unavailable; here width fractions $p_i = w_i/W$ are the primary geometrical observable, not a surrogate. Second, the eBI exponentiates entropy to an effective channel count ($\text{eBI} = 2^{H_w}$), whereas H is retained in its direct form as a structural indicator. Third, the eBI is a braiding intensity index, while the present framework characterizes the internal width allocation across the simplex without collapsing it to a scalar count.

Two stochastic models are proposed. The first assumes that all channels are statistically equivalent and any width concentration is purely stochastic. Let $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_B)$ with $\alpha_i \in \mathbb{R}^+$ for all $i = 1, \dots, B$; a random vector $\mathbf{p} = (p_1, \dots, p_B)$ follows the Dirichlet($\boldsymbol{\alpha}$) distribution if its density on the open simplex Δ^{B-1} is

$$f(\mathbf{p}; \boldsymbol{\alpha}) = \frac{\Gamma(\sum_{i=1}^B \alpha_i)}{\prod_{i=1}^B \Gamma(\alpha_i)} \prod_{i=1}^B p_i^{\alpha_i-1}, \quad (2)$$

where $\Gamma(\cdot)$ denotes the Euler Gamma function. This is the natural distribution for compositional vectors on the simplex, being the only one closed under aggregation of components [21]. The fundamental identity underlying all moment

calculations is the *Dirichlet integral*: for any exponents $\alpha_1, \dots, \alpha_B \in \mathbb{R}^+$,

$$\int_{\Delta^{B-1}} \prod_{i=1}^B p_i^{\alpha_i-1} d\mathbf{p} = \frac{\prod_{i=1}^B \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^B \alpha_i)}. \quad (3)$$

In the symmetric case $\alpha_i = \alpha$ for all i , we write $\mathbf{p} \sim \text{Dir}_B(\alpha)$, where $\text{Dir}_B(\alpha) \equiv \text{Dirichlet}(\alpha \mathbf{1}_B)$ with $\mathbf{1}_B = (1, \dots, 1)^\top \in \mathbb{R}^B$ and $\sum_i \alpha_i = B\alpha$, and we set $\alpha = cB$ with $c > 0$ [21]. Here $c \in \mathbb{R}^+$ is the free parameter of the theoretical model that controls the degree of homogeneity of the width partition: larger values of c concentrate the distribution near the uniform partition $p_i = 1/B$, implying greater statistical equivalence among channels; smaller values allow wider fluctuations in individual channel widths. The notation c^* indicates the estimated value of c for a given survey year, obtained by maximizing $R_{D,\text{md}}^2$ over the grid of values described in Sec. II C. The higher moments and the variance of H are derived in the Appendix; the key result needed here is the power-law scaling

$$\langle H \rangle \sim B^{-\beta}, \quad \beta \rightarrow 1, \quad (4)$$

where β is the exponent predicted by the model; the corresponding exponent β_{exp} , extracted from the empirical $\langle H \rangle_{\text{exp}}$ vs B scaling by log-log linear regression (Sec. III), is used throughout for comparison with the data.

More generally, the angle-bracket notation denotes two distinct but related quantities, distinguished by context. Under the model distribution, $\langle H \rangle \equiv \mathbb{E}[H]$ is the theoretical ensemble mean of $H = \sum_i p_i^2$ over the distribution of $\mathbf{p}$ (derived in full in the Appendix). When applied to empirical data, the same notation denotes the sample mean

$$\langle H \rangle_{\text{exp}} = \frac{1}{N_B} \sum_{n=1}^{N_B} H_n, \quad (5)$$

where the index $n = 1, \dots, N_B$ runs over distinct cross sections, H_n is the value of the concentration index H measured at the n th such cross section, and N_B , not to be confused with the braiding index B itself, counts how many cross sections in a given annual survey share that same braiding index B . Analogously, $\langle D \rangle \equiv \mathbb{E}[D]$ is the theoretical ensemble mean of $D = \max_i p_i$ over the distribution of $\mathbf{p}$, and the corresponding sample mean is

$$\langle D \rangle_{\text{exp}} = \frac{1}{N_B} \sum_{n=1}^{N_B} D_n, \quad (6)$$

where D_n is the value of the width concentration fraction D measured at the n th cross section, with n and N_B as defined above. The subscript exp is omitted where the distinction is clear from context. This scaling yields the bound

$$\frac{1}{B} \leq \langle D \rangle \leq \sqrt{\langle H \rangle} \lesssim B^{-1/2}, \quad (7)$$

understood under the model distribution (i.e., with $\langle D \rangle \equiv \mathbb{E}[D]$ and $\langle H \rangle \equiv \mathbb{E}[H]$, as defined above). In direct analogy with β , we define the scaling exponent γ of the width concentration fraction through $\langle D \rangle \sim B^{-\gamma}$, where, as above, the same angle-bracket notation distinguishes the theoretical scaling of $\langle D \rangle \equiv \mathbb{E}[D]$ predicted by a given model from the empirical

scaling of $\langle D \rangle_{\text{exp}}$ measured in the data. Accordingly, γ_{mod} denotes the exponent predicted by a candidate model (here the symmetric Dirichlet model) and γ_{exp} denotes the exponent extracted from the empirical $\langle D \rangle_{\text{exp}}$ vs B scaling by log-log linear regression (Sec. III). This bound yields a parameter-free structural breakdown criterion: if the empirical exponent $\gamma_{\text{exp}} < 1/2$, no symmetric Dirichlet model can reproduce the data regardless of parameter choice. To see why, consider the more general parametrization $\alpha(B) = cB^\eta$ with $c > 0$ and an arbitrary exponent $\eta > 0$, which reduces to the case $\alpha = cB$ used above when $\eta = 1$ and allows us to rule out any power-law dependence of α on B , not only the linear one. Under this parametrization, the expected concentration index satisfies $\mathbb{E}[H] = (cB^\eta + 1)/(cB^{\eta+1} + 1) \sim B^{-1}$ for any $\eta > 0$, so that $\mathbb{E}[D] \leq \sqrt{\mathbb{E}[H]} \sim B^{-1/2}$ from the bound above. Hence any symmetric Dirichlet model predicts $\mathbb{E}[D] \geq C_1 B^{-1}$ and $\mathbb{E}[D] \leq C_2 B^{-1/2}$ for positive constants C_1, C_2 , so the effective decay exponent satisfies $\gamma_{\text{mod}} \in (1/2, 1)$ for all $c, \eta > 0$. If $\gamma_{\text{exp}} < 1/2$, the empirical curve decays *slower* than $B^{-1/2}$, exceeding the upper bound for all sufficiently large B , and the model is therefore structurally inconsistent with the data regardless of parameter choice.

When the Dirichlet model fails, a second model accommodates systematic departures from uniform width allocation. We introduce the isotropic logistic-normal distribution $\text{LN}_B(\sigma)$ [22,24], obtained via the additive logistic transformation of latent Gaussian variables $Z_1, \dots, Z_B \in \mathbb{R}$,

$$p_i = \frac{e^{Z_i}}{\sum_{j=1}^B e^{Z_j}}, \quad (Z_1, \dots, Z_{B-1}) \sim \mathcal{N}(\mathbf{0}, \sigma^2 I_{B-1}), \quad Z_B \equiv 0, \quad (8)$$

where $\mathcal{N}(\mathbf{0}, \sigma^2 I_{B-1})$ denotes the $(B-1)$ -dimensional multivariate Gaussian distribution with zero mean vector and isotropic covariance $\sigma^2 I_{B-1}$, I_{B-1} being the identity matrix, so that $Z_1, \dots, Z_{B-1}$ are independent and identically distributed as $\mathcal{N}(0, \sigma^2)$. Here $\sigma > 0$ is the sole free parameter of the model: it is the standard deviation of the latent log ratios Z_i , and therefore directly controls the dispersion of channel widths around the uniform partition. As $\sigma \rightarrow 0$ the partition approaches uniformity; as $\sigma \rightarrow \infty$ it concentrates on a single channel. Unlike the Dirichlet model, $\text{LN}_B(\sigma)$ is not subject to the breakdown criterion. To see why, write $p_i = e^{Z_i}/S$ with $S = \sum_j e^{Z_j}$ and $\bar{Z} = B^{-1} \sum_j Z_j$; a second-order expansion for small σ gives $p_i \approx B^{-1}(1 + Z_i - \bar{Z})$ and $H \approx B^{-1} + B^{-2} \sum_i (Z_i - \bar{Z})^2 + O(\sigma^3)$; taking expectations and using $\mathbb{E}[\sum_i (Z_i - \bar{Z})^2] = (B-1)\sigma^2$ yields

$$\mathbb{E}[H] = \frac{1}{B} + \frac{B-1}{B^2} \sigma^2 + O(\sigma^4) \sim \frac{\sigma^2}{B}, \quad B \rightarrow \infty, \quad (9)$$

with finite- B correction $O(B^{-2})$, producing a more linear log-log scaling over finite B ranges than the Dirichlet model. Correspondingly,

$$\mathbb{E}[D] \leq \sqrt{\mathbb{E}[H]} \sim \frac{\sigma}{\sqrt{B}}, \quad (10)$$

which places no lower bound of order $B^{-1/2}$ on $\mathbb{E}[D]$, unlike the symmetric Dirichlet model. More precisely, $\text{LN}_B(\sigma)$ is structurally compatible with any scaling exponents $\beta \in (0, 1)$

TABLE I. Comparison of competing parametric models for the temporal sequence $\{\sigma^*(t_i)\}_{i=1}^{11}$ of independently optimized scale parameters ($n = 11$ survey years), assessed by the Bayesian Information Criterion (BIC). k : number of free parameters; SSE: residual sum of squares; R^2 : coefficient of determination; ΔBIC : difference relative to the logistic model (positive values favor the logistic). The logistic model is strongly preferred over the constant and linear alternatives; the cubic polynomial achieves a marginally lower BIC but lacks physical interpretability.

Model	k	SSE	BIC	R^2	ΔBIC
Logistic (A, k, t_0, B_0)	4	0.0271	-56.48	0.902	—
Linear ($a + bt$)	2	0.0520	-54.11	0.812	+2.4
Constant (mean)	1	0.2764	-38.13	0.000	+18.4
Cubic polynomial	4	0.0224	-58.55	0.919	-2.1

and $\gamma \in (0, 1/2]$ for appropriate σ : from (9), $\mathbb{E}[H]$ is a smooth increasing function of σ for each B , covering $(1/B, 1)$ as σ ranges over $(0, \infty)$, so the effective exponent $\beta_{\text{eff}}(\sigma, B)$ covers $(0, 1)$; and from (10), $\gamma \leq 1/2$ is achievable for fixed σ and large B , spanning the full range of exponents observed in the channel concentration state. This is the structural counterpart of the breakdown criterion established above: while the symmetric Dirichlet model cannot reproduce $\gamma_{\text{exp}} < 1/2$, the logistic-normal model does so naturally, escaping the structural breakdown that constrains the symmetric Dirichlet model.

The scale parameter σ of the logistic-normal model quantifies the degree of systematic width concentration: it is zero for a perfectly uniform partition and grows as concentration increases. Since both models are fitted independently to the spatial data of each survey year, the sequence of optimized values $\sigma^*(t_i)$ provides a year-by-year structural fingerprint of the braided network that is free of any dynamical assumption. Tracking its temporal evolution across the 11 surveys therefore allows us to identify and quantify morphodynamic transitions directly from planimetric data. For each survey year t_i ($i = 1, \dots, 11$), the optimal parameter $\sigma^*(t_i)$ is estimated independently by maximizing the goodness-of-fit metric $R_{D,\text{md}}^2$ (Sec. III) against the observed $\langle D \rangle$ vs B scaling. The resulting sequence $\{\sigma^*(t_i)\}_{i=1}^{11}$ plays the role of an order parameter for the morphodynamic transition. It is fitted by a logistic regression curve:

$$\sigma^*(t) = \frac{A}{1 + e^{-k(t-t_0)}} + B_0, \quad (11)$$

where A , k , t_0 , and B_0 are fitted parameters and t is the same temporal variable as t_i above, namely, the discrete survey year; we write t_i when referring to one of the 11 specific years at which σ^* was estimated, and t when the variable is treated as continuous for the purpose of fitting and evaluating the curve. The logistic curve is a descriptive interpolation of 11 independent structural estimates, not a dynamical model. A Bayesian Information Criterion comparison confirms that the logistic parametrization is preferred over linear, constant, and polynomial alternatives (Sec. III, Table I).

B. Goodness-of-fit metric

To quantify how well each model reproduces the empirical scaling of the width concentration fraction $\langle D \rangle$ as a function of the braiding index B , we adopt a coefficient of determination computed in log space. The indicator $\langle D \rangle$ is preferred over $\langle H \rangle$ for parameter optimization because it is more sensitive to the tail of the width distribution and provides sharper discrimination between the two models at moderate braiding indices. Working in log space ensures that relative errors are weighted equally across all values of B , avoiding dominance of large- B cross sections. The metric used throughout is

$$R_{D,\text{md}}^2 = 1 - \frac{\sum_B (\ln \langle D \rangle_{\text{exp}} - \ln \langle D \rangle_{\text{mod}})^2}{\sum_B (\ln \langle D \rangle_{\text{exp}} - \overline{\ln \langle D \rangle_{\text{exp}}})^2}, \quad (12)$$

computed in log space, where $\langle D \rangle_{\text{mod}}$ is the value predicted, for each braiding index B , by a given candidate model (Dirichlet or logistic-normal) with a specified parameter value, and the overline denotes the mean taken over all values of B in the dataset, i.e., $\overline{\ln \langle D \rangle_{\text{exp}}} = B_{\text{range}}^{-1} \sum_B \ln \langle D \rangle_{\text{exp}}$, where B_{range} is the number of distinct values of B in the scaling range of the survey year under consideration (i.e., year-specific, as reported in Table II), so that $R_{D,\text{md}}^2$ is computed independently for each annual survey. Negative values of $R_{D,\text{md}}^2$ indicate that the model performs worse than a constant predictor equal to this mean.

C. Parameter optimization procedure

Because $R_{D,\text{md}}^2$ is computed from Monte Carlo model predictions rather than from a closed-form expression, it is not differentiable in c or σ and gradient-based optimization is not directly applicable; a grid search over a finite parameter set is therefore the natural and robust choice, at the cost of a resolution set by the grid spacing. For each year, parameters are optimized by grid search:

$$c \in \{0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.10, 0.12, 0.15, 0.20, 0.25, 0.30\}$$

for the Dirichlet model, and $\sigma \in \{0.2, 0.3, 0.4, \dots, 1.6, 1.8, 2.0\}$ for the logistic-normal model. Both ranges were chosen to bracket the full span of optimal values found across all 11 survey years (Table II), with finer spacing at the lower end of c where the goodness-of-fit surface is most sensitive to small changes in concentration. The optimal parameter maximizes $R_{D,\text{md}}^2$. For each candidate parameter value, model predictions for $\langle H \rangle_{\text{mod}}$ and $\langle D \rangle_{\text{mod}}$ are obtained from 10^4 Monte Carlo realizations per B value, a sample size found to be sufficient for the resulting estimates to be stable to within the third significant digit; a fixed random seed (1234) is used throughout for reproducibility.

D. Logistic fit to $\sigma^*(t)$

Each value of σ^* is optimized independently from the spatial data of a single survey year, it is not a dynamical time series but a sequence of 11 independent structural estimates. A logistic regression of this discrete sequence against the survey year t_i takes the form of Eq. (11), fitted by nonlinear least squares. The fitted parameters are $A = 0.342$,

TABLE II. Summary of results for all 11 survey years. Res: spatial resolution (m); B range: scaling range used for model fitting; γ_{exp} : empirical width concentration exponent; c^* and σ^* : optimized parameters for the Dirichlet and logistic-normal models, respectively; R_D^2 values refer to Eq. (12). Bold indicates the better model for each year.

Year	Res	B range	γ_{exp}	c^*	σ^*	R_D^2 [Dir(α)]	R_D^2 [LN(σ)]	Best
1976	80	3–14	0.817	0.12	0.7	0.865	0.797	Dir _{B}
1977	80	3–12	0.879	0.25	0.7	0.949	0.844	Dir _{B}
1978	80	3–13	0.776	0.15	0.7	0.912	0.845	Dir _{B}
1980	80	3–10	0.815	0.25	0.7	0.961	0.892	Dir _{B}
1985	80	3–18	0.740	0.10	0.8	0.942	0.887	Dir _{B}
1995	30	3–26	0.603	0.05	0.9	0.789	0.953	LN _{B}
1996	30	3–20	0.535	0.05	1.1	0.626	0.903	LN _{B}
1998	30	3–20	0.521	0.05	1.0	0.628	0.959	LN _{B}
1999	30	3–20	0.431	0.04	1.1	0.353	0.968	LN _{B}
2005	24	3–20	0.650	0.05	1.0	0.943	0.917	Dir _{B}
2007	15	3–26	0.684	0.04	1.0	0.941	0.916	Dir _{B}

$k = 0.327 \text{ yr}^{-1}$, $t_0 = 1987.6$, $B_0 = 0.687$, with $R^2 = 0.997$. Asymptotic values: $\sigma^*(t \rightarrow -\infty) = B_0 = 0.687$ (symmetric state) and $\sigma^*(t \rightarrow +\infty) = A + B_0 = 1.029$ (channel concentration state). The observed values $\sigma^*(2005) = \sigma^*(2007) = 1.0$ remain substantially above the pretransition level $B_0 = 0.687$, despite the Dirichlet model regaining superiority in $R_{D,\text{md}}^2$ for those years, indicating that channel width concentration has not returned to its pre-1985 level within the observational window.

E. Model selection: BIC comparison

To assess whether the logistic parametrization of the $\sigma^*(t_i)$ sequence is justified, we compare it against three alternative models using the Bayesian Information Criterion: a constant model ($\sigma^* = a_0$, mean of the sequence, $k = 1$), a linear trend ($\sigma^* = a_0 + a_1 t$, $k = 2$), and a cubic polynomial ($\sigma^* = a_0 + a_1 t + a_2 t^2 + a_3 t^3$, $k = 4$): although the polynomial is of degree three, it has four free coefficients $a_0, \dots, a_3$, so $k = 4$, matching the number of free parameters of the logistic model. The BIC is defined as

$$\text{BIC} = n \ln \left(\frac{\text{SSE}}{n} \right) + k \ln n, \quad (13)$$

where $n = 11$ is the number of survey years, SSE is the residual sum of squares, and k is the number of free parameters. Lower BIC indicates a better balance between goodness of fit and model complexity. Table I reports the results for all four competing models.

The logistic model is strongly preferred over a constant predictor ($\Delta\text{BIC} = 18.4$), confirming that the temporal structure of σ^* is real. It is positively preferred over a linear trend ($\Delta\text{BIC} = 2.4$). The cubic polynomial achieves a marginally lower BIC ($\Delta\text{BIC} = -2.1$) with the same number of parameters, but its coefficients lack physical interpretation; the logistic model is retained on grounds of parsimony and interpretability.

III. DATA

We analyze 11 annual surveys of a braided sand-bed reach of the Brahmaputra-Jamuna River (Bangladesh), spanning 1976–2007. The channel network configurations were extracted from multisource satellite imagery (Landsat MSS, Landsat TM, IRS LISS, and ASTER) following the methodology documented in Rizzello *et al.* [18]; full details on image preprocessing, channel extraction criteria, and quality control are provided therein. The datasets are publicly available via two Zenodo repositories [25,26]. Each survey provides channel widths for several thousand cross sections. Spatial resolution varies between 15 and 80 m across years (Table II). For each cross section, the braiding index B and the individual channel widths $w_1, \dots, w_B$ are recorded, allowing direct computation of the compositional vector $\mathbf{p}$ and the structural indicators H and D .

For each year and each value of B , we compute the sample mean $\langle H \rangle$ and $\langle D \rangle$ across all cross sections with that channel count. Empirical scaling exponents β_{exp} and γ_{exp} are extracted by log-log linear regression over the range $B \in [B_{\min}, B_{\max}]$, where $B_{\min}$ and $B_{\max}$ are the smallest and largest braiding indices with a sufficient number of cross sections in a given survey year for the sample means to be reliable; both bounds vary by year and are reported as the B range in Table II. Model predictions are obtained by Monte Carlo simulation (10^4 realizations per B value). The goodness-of-fit metric $R_{D,\text{md}}^2$, defined in Sec. II [Eq. (12)], is used to optimize both c^* and σ^* .

IV. RESULTS

The Brahmaputra-Jamuna braided network and the measurement geometry are illustrated in Fig. 1 for the 1985 survey: the full planimetric extent is extracted from Landsat MSS imagery at 80 m resolution, and a representative cross section identifies $B = 3$ active channels with individual widths w_1, w_2, w_3 . This geometry defines the compositional vector $\mathbf{p} = (p_1, p_2, p_3)$ that constitutes the fundamental observable of the framework. A complete summary of quantitative results for all 11 survey years is provided in Table II.

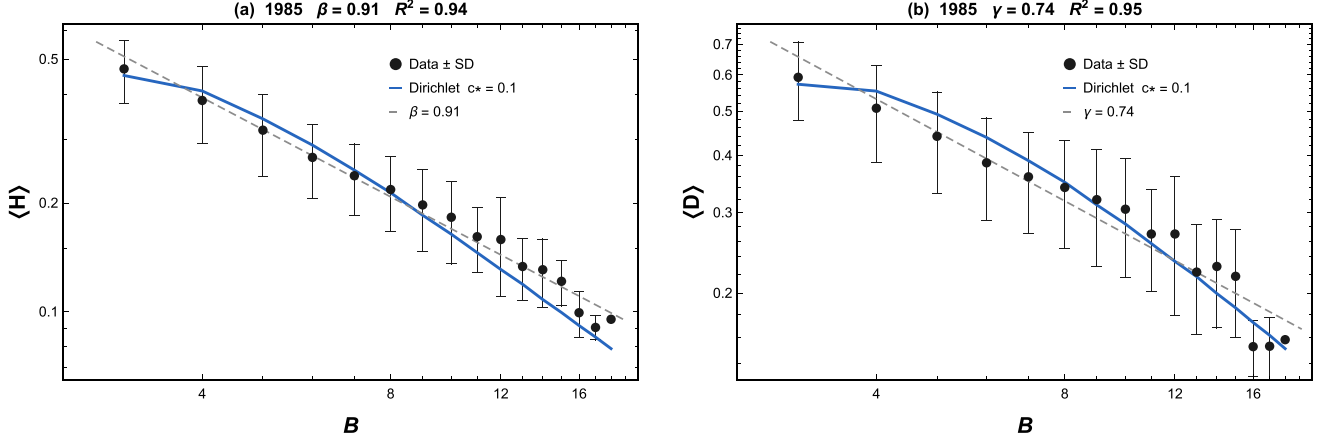


FIG. 2. Scaling of structural indicators for the 1985 dataset (symmetric state). (a) Concentration index $\langle H \rangle$ vs number of channels B ; (b) width concentration fraction $\langle D \rangle$ vs B . Points: Experimental means $\pm$ SD. Solid curve: Dirichlet model with optimized parameter $c^* = 0.10$ ($R_{D,md}^2 = 0.942$). Dashed line: Power-law fit to experimental data. Both axes are logarithmic.

A. Symmetric state: 1976–1985

During the period 1976–1985, the width partition among channels is consistent with a purely stochastic, symmetric allocation: the empirical scaling exponents γ_{exp} range from 0.74 to 0.88 (Table II), well within the theoretical range $(1/2, 1)$ admitted by the symmetric Dirichlet model (see the Appendix), and the Dirichlet model achieves $R_{D,md}^2 \geq 0.865$ for all five surveys. The 1985 dataset is shown in detail in Fig. 2: the optimized Dirichlet model ($c^* = 0.10$) reproduces both $\langle H \rangle$ and $\langle D \rangle$ with high fidelity ($R_{D,md}^2 = 0.942$), and the empirical exponent $\beta_{\text{exp}} = 0.912$ is consistent with the theoretical prediction $\beta \rightarrow 1$ of Eq. (A4). Within this period, c^* decreases monotonically from 0.25 in 1977–1980 to 0.10 in 1985, indicating a gradual loosening of the uniform allocation even before the main transition, while σ^* remains stable at 0.7–0.8 throughout.

B. Structural transition and breakdown: 1995–1999

Between 1985 and 1995, a structural reorganization of the width partition takes place. From 1995 onwards, the Dirichlet model undergoes progressive degradation: $R_{D,md}^2$ drops from 0.942 in 1985 to 0.789 in 1995 and 0.626 in 1996, while the logistic-normal model simultaneously achieves $R_{D,md}^2 = 0.953$ and 0.903 for the same years. The transition culminates in 1999, where $\gamma_{\text{exp}} = 0.431 < 0.5$ triggers the structural breakdown criterion (Sec. II A): no symmetric Dirichlet model of any parametrization can reproduce this scaling, independently of parameter choice. The logistic-normal model with $\sigma^* = 1.1$ recovers $R_{D,md}^2 = 0.968$, demonstrating that the departure from uniform allocation is systematic rather than a sampling artifact (Fig. 3).

In 2005–2007 the Dirichlet model regains superiority in $R_{D,md}^2$ (0.943 and 0.941 respectively), and the empirical exponents $\beta_{\text{exp}} = 0.929$ and $\gamma_{\text{exp}} = 0.684$ return to values consistent with the symmetric state. However, the optimized Dirichlet parameter c^* remains at 0.04–0.05, well below its pretransition values of 0.12–0.25, indicating that the internal

structure of width allocation has not returned to its pre-1985 configuration (Fig. 4).

C. Temporal evolution of σ^* as order parameter

The full temporal evolution of the framework is summarized in Fig. 5. The sequence of independently optimized σ^* values increases monotonically from 0.7 in 1976–1985 to 1.1 in 1999, then partially returns to 1.0 in 2005–2007. This discrete sequence of 11 independent structural estimates is well described by a logistic regression curve [Eq. (11)] with $R^2 = 0.997$, inflection year $t_0 \approx 1988$, and asymptotic values $\sigma^* \rightarrow 0.69$ (pretransition) and $\sigma^* \rightarrow 1.03$ (posttransition). The goodness-of-fit curves of both models cross between 1985 and 1995 [Fig. 5(b)], defining the transition window, while the width concentration exponent $\gamma_{\text{exp}}(t)$ crosses the theoretical threshold $\gamma = 0.5$ between 1995 and 1998 [Fig. 5(c)], marking the peak of the concentration phase.

Crucially, $\sigma^*(2005\text{--}2007) \approx 1.0$ remains substantially above the pretransition asymptote $\sigma^* \rightarrow 0.69$, despite the Dirichlet model regaining model-selection superiority in those years. The two diagnostics, model selection and the continuous order parameter σ^* , are complementary and independent: the former signals the structural recovery of the symmetric partition, while the latter records the persistent memory of the concentration phase on the observational timescale.

V. DISCUSSION

The parameter σ^* plays the role of an order parameter for the morphodynamic transition. Each value of σ^* is optimized independently from the spatial data of a single survey year, it is not a time series in the dynamical sense, but a sequence of 11 independent structural estimates. Their monotonic increase from $\sigma^* \approx 0.69$ in 1976–1985 to $\sigma^* \approx 1.1$ in 1995–1999, and the persistence of $\sigma^* \approx 1.0$ in 2005–2007, indicate a progressive and sustained growth of channel width concentration throughout the observational window. The logistic regression against survey year ($R^2 = 0.997$) describes this evolution as

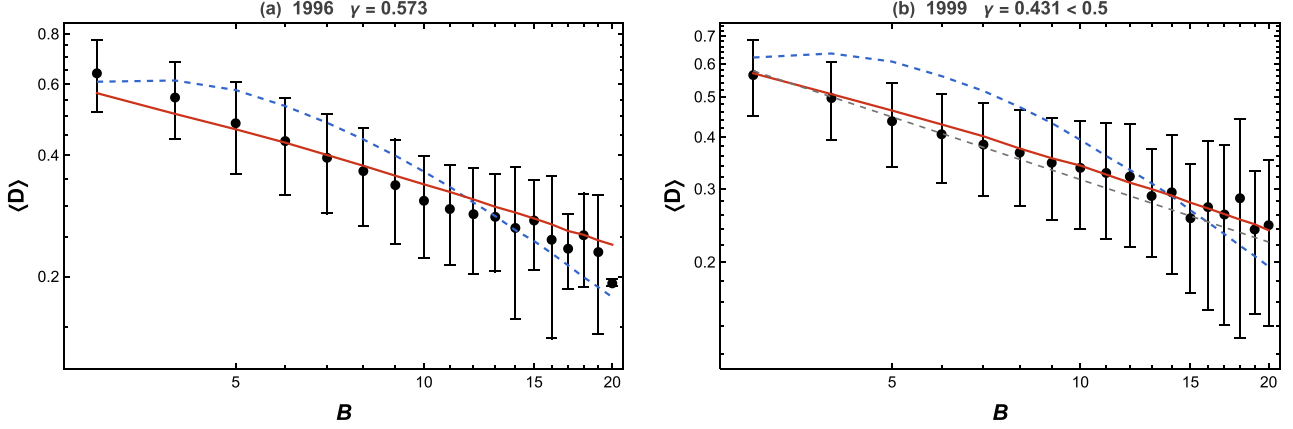


FIG. 3. Width concentration fraction $\langle D \rangle$ vs B during the channel concentration state. (a) 1996; (b) 1999. Points: experimental means $\pm$ SD. Blue dashed curve: Best-fit Dirichlet model ($c^* = 0.05$). Red solid curve: Logistic-normal model ($\sigma^* = 1.1$). Gray dashed line in panel (b): Theoretical lower bound $\langle D \rangle \sim B^{-1/2}$ ($\gamma = 0.5$), below which no symmetric Dirichlet model can reproduce the data (Sec. II A). In 1999 $\gamma_{\text{exp}} = 0.431 < 0.5$, providing a rigorous parameter-free certificate of model failure. Both axes are logarithmic.

a slow, continuous morphodynamic transition rather than an abrupt bifurcation, with inflection year $t_0 \approx 1988$.

Physically, σ^* measures the variance of log-width ratios among channels: a larger σ^* implies greater systematic departure from uniform width allocation, in which one or few channels capture a disproportionate share of the total width. The progressive increase of σ^* therefore corresponds physically to a growing concentration of discharge and sediment transport into a smaller subset of channels. For the Brahmaputra, this is consistent with the documented tendency toward channel network simplification and increased lateral bar stability over the study period [6,8], driven by interannual variability in monsoon peak discharges and gradual vegetation encroachment on exposed bar surfaces. The channel concentration observed between 1995 and 1999 is also consistent with the morphodynamic legacy of the 1950 Assam earthquake, whose associated sand wave is documented to have traversed the Brahmaputra-Jamuna system between the late 1960s and late 1990s, producing rapid widening and increased

braiding intensity followed by progressive channel simplification [27]. Channel width adjustment to discharge in individual threads follows well-established hydraulic geometry relations [28–35], which provide the physical basis for interpreting width redistribution in terms of flow partition among active branches [36].

The framework identifies the morphodynamic transition through two complementary diagnostics: the breakdown of the Dirichlet model (Sec. II A, certified for 1999 where $\gamma_{\text{exp}} = 0.431 < 0.5$) and the monotonic increase of σ^* . The two diagnostics are independent: σ^* tracks the continuous growth of concentration across all 11 surveys, while the breakdown criterion provides a rigorous, parameter-free certificate of structural failure at the peak of concentration. In 2005–2007 the Dirichlet model regains superiority in $R^2_{D,\text{md}}$, but $\sigma^*(2005\text{--}2007) \approx 1.0$ remains above the pretransition level $\sigma^* \rightarrow 0.69$, indicating that concentration has not returned to its pre-1985 level within the observational window.

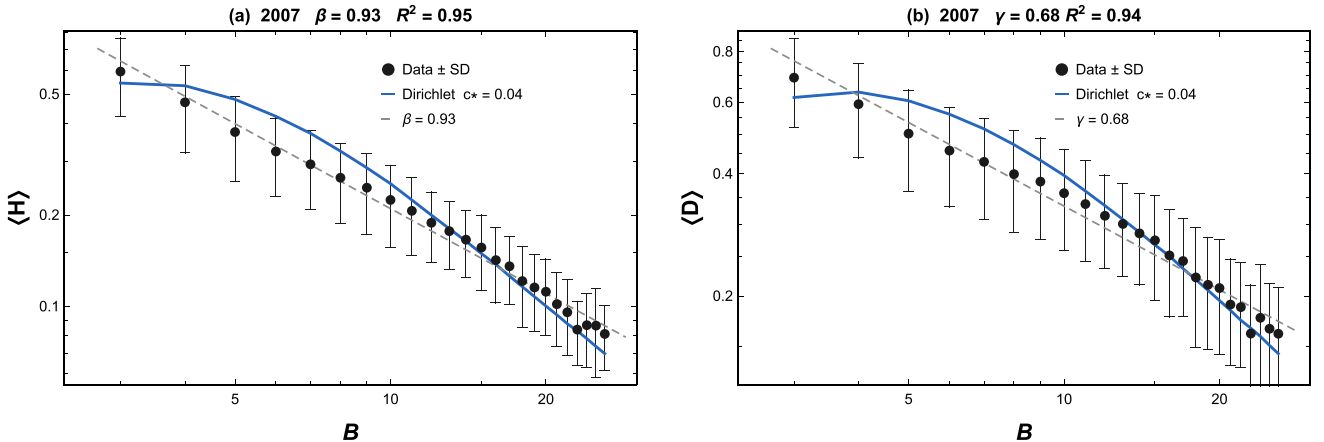


FIG. 4. Scaling of structural indicators for the 2007 dataset. (a) Concentration index $\langle H \rangle$ vs number of channels B ; (b) width concentration fraction $\langle D \rangle$ vs B . Points: Experimental means $\pm$ SD. Solid curve: Dirichlet model with optimized parameter $c^* = 0.04$ ($R^2_{D,\text{md}} = 0.941$). The Dirichlet model regains superiority over the logistic-normal, although $\sigma^*(2007) = 1.0$ remains above the pretransition asymptote $\sigma^* \rightarrow 0.69$, indicating incomplete morphodynamic relaxation. Both axes are logarithmic.

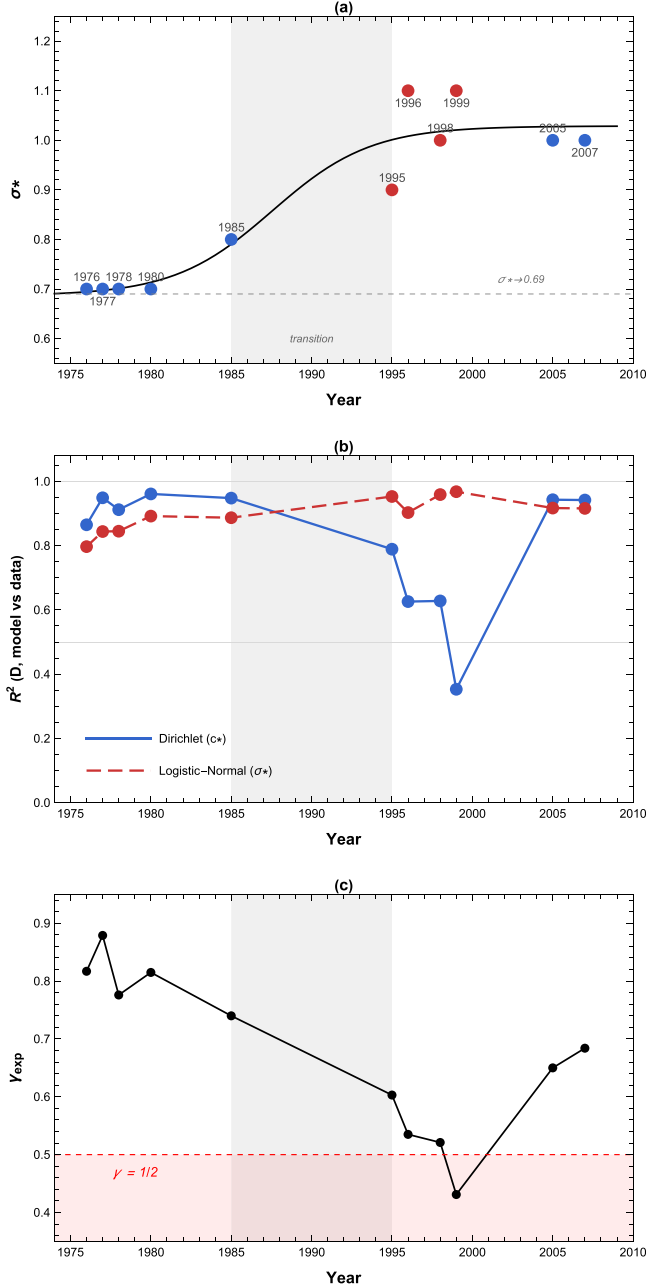


FIG. 5. Temporal evolution of model parameters and fit quality. (a) Sequence of independently optimized σ^* values as a function of survey year; points coloured by best model (blue: Dirichlet; red: logistic-normal); black curve: logistic regression fit ($R^2 = 0.997$, inflection year $t_0 \approx 1988$); gray dashed line: pretransition asymptote $\sigma^* \rightarrow 0.69$; gray band: unobserved transition interval (1985–1995). (b) Goodness-of-fit $R^2_{D,md}$ for the Dirichlet (blue) and logistic-normal (red dashed) models; gray band as in (a). (c) Empirical width concentration exponent $\gamma_{exp}(t)$; red dashed line: theoretical threshold $\gamma = 0.5$ (Sec. II A); red shading: breakdown region ($\gamma < 0.5$); gray band as in (a).

The temporal evolution of the Dirichlet parameter c^* provides a second, independent descriptor of morphodynamic change. Within 1976–1985, c^* decreases monotonically from 0.12–0.25 in 1976–1980 to 0.10 in 1985. In 2005–2007, c^* remains low at 0.04–0.05, well below its pretransition values.

This incomplete recovery of c^* constitutes additional evidence of residual morphodynamic change, independent of σ^* , and suggests that the internal organization of width allocation retains memory of the concentration phase on the observational timescale.

These findings have three broader implications. First, the concentration index H and the width concentration fraction D , analyzed as functions of the braiding index B , provide quantitative descriptors of internal channel organization that are complementary to existing scalar braiding metrics and entropy-based indices [20]. Second, the parameter-free breakdown criterion offers a practical diagnostic tool for identifying structural transitions in braided rivers directly from remote sensing data, without requiring discharge measurements. Third, the logistic temporal evolution of σ^* suggests that morphodynamic transitions in braided rivers are gradual on decadal timescales, with implications for long-term river management and climate change adaptation [6,27].

Future work should extend this framework to other large braided systems and investigate whether the identified transitions correspond to distinct hydraulic states detectable from discharge records or sediment transport proxies.

VI. CONCLUSIONS

We have introduced a stochastic simplex framework for width partitioning in braided river cross sections, grounded in compositional data analysis [21]. Two probability models on Δ^{B-1} are developed with independent physical content: the symmetric Dir $_B(\alpha)$, which characterizes morphodynamic states in which all channels are statistically equivalent and any width concentration is a purely stochastic fluctuation around a uniform baseline, and the isotropic logistic-normal LN $_B(\sigma)$, which identifies and quantifies organized concentration regimes through a single scale parameter. Exact closed-form expressions are derived for all relevant moments, and a parameter-free breakdown criterion (Sec. II A) provides a rigorous, model-independent signature of structural failure of the Dirichlet model.

Applied to 11 annual surveys of the Brahmaputra-Jamuna River (1976–2007), the framework reveals a slow decadal-scale morphodynamic transition characterized by a progressive growth of channel width concentration from 1995 onwards. The scale parameter σ^* , optimized independently for each survey year, increases monotonically and follows a logistic trajectory ($R^2 = 0.997$), linking the statistical structure of width partitioning to decadal-scale morphodynamic reorganization driven by discharge variability, vegetation encroachment, and the morphodynamic legacy of the 1950 Assam earthquake [27]. Both σ^* and c^* retain memory of the concentration phase on the observational timescale: $\sigma^*(2005\text{--}2007) \approx 1.0$ remains above the pretransition level $\sigma^* \rightarrow 0.69$, and c^* remains at 0.04–0.05, well below its pretransition values of 0.12–0.25.

DATA AVAILABILITY

The data that support the findings of this article are publicly available [25,26].

APPENDIX: MATHEMATICAL FRAMEWORK

1. Notation

Throughout this paper, $\langle \cdot \rangle$ denotes the ensemble mean over the model distribution (theoretical expectation $\mathbb{E}[\cdot]$) or, when applied to empirical data, the sample mean across all cross sections sharing the same braiding index B for a given survey year. The two usages are distinguished by context: theoretical scaling relations such as $\langle H \rangle \sim B^{-\beta}$ refer to model predictions, while quantities labeled with subscript exp (e.g., $\langle D \rangle_{\text{exp}}$) refer to empirical sample means.

2. Moments of the symmetric Dirichlet model

The moment structure of $\text{Dir}_B(\alpha)$ follows directly from the Dirichlet integral (3) (Sec. II). By the density (2), for every i and integer $k \geq 1$:

$$\begin{aligned} \mathbb{E}[p_i^k] &= \frac{\Gamma(B\alpha)}{\Gamma(\alpha)^B} \int_{\Delta^{B-1}} p_i^{\alpha+k-1} \prod_{j \neq i} p_j^{\alpha-1} d\mathbf{p} \\ &= \frac{\Gamma(B\alpha)}{\Gamma(\alpha)} \frac{\Gamma(\alpha+k)}{\Gamma(B\alpha+k)}, \end{aligned}$$

where the second equality applies (3) with exponents $\alpha+k$ in place of α_i and α in place of α_j for $j \neq i$. Repeated use of $\Gamma(x+1) = x\Gamma(x)$ then gives the fractional moment formula

$$\mathbb{E}[p_i^k] = \prod_{j=0}^{k-1} \frac{\alpha+j}{B\alpha+j}, \quad (\text{A1})$$

from which all moments used in our analysis are obtained as special cases.

Under $\text{Dir}_B(\alpha)$, setting $k=1, 2$ in (A1) gives

$$\mathbb{E}[p_i] = \frac{1}{B}, \quad \mathbb{E}[p_i^2] = \frac{\alpha+1}{B(B\alpha+1)}, \quad (\text{A2})$$

and by linearity, $H = \sum_i p_i^2$ has expectation

$$\mathbb{E}[H] = B \mathbb{E}[p_i^2] = \frac{\alpha+1}{B\alpha+1}. \quad (\text{A3})$$

Setting $\alpha = cB$ ($c > 0$) yields the scaling law

$$\mathbb{E}[H] = \frac{cB+1}{cB^2+1} = \frac{1}{B}(1 + O(B^{-1})) \sim \frac{1}{B}, \quad B \rightarrow \infty, \quad (\text{A4})$$

which is the key scaling result used throughout our analysis. Because this asymptotic scaling is only exact as $B \rightarrow \infty$, it is useful to quantify the deviation at finite B through the local log-log slope of (A4), denoted $\beta_{\text{eff}}(B)$:

$$\beta_{\text{eff}}(B) = -\frac{d \ln \mathbb{E}[H]}{d \ln B} = \frac{2cB^2}{cB^2+1} - \frac{cB}{cB+1}, \quad (\text{A5})$$

which satisfies $\beta_{\text{eff}}(B) < 1$ for all finite B , approaching 1 only as $B \rightarrow \infty$. This finite-size correction explains the systematic observation $\beta_{\text{exp}} < 1$ over the finite scaling ranges accessible in the data.

3. Variance of the concentration index

The variance of H requires the fourth and mixed second moments of the Dirichlet components. Setting

$k=4$ in (A1) gives $\mathbb{E}[p_i^4] = (\alpha+1)(\alpha+2)(\alpha+3)/[B(B\alpha+1)(B\alpha+2)(B\alpha+3)]$; applying the Dirichlet integral (3) with exponents $\alpha+2$ in place of both α_i and α_j for the mixed term and simplifying via $\Gamma(\alpha+2) = (\alpha+1)\alpha\Gamma(\alpha)$ gives, for $i \neq j$,

$$\mathbb{E}[p_i^2 p_j^2] = \frac{(\alpha+1)^2 \alpha}{B(B\alpha+1)(B\alpha+2)(B\alpha+3)}. \quad (\text{A6})$$

Combining these moments through $\text{Var}(H) = \mathbb{E}[H^2] - \mathbb{E}[H]^2$, where $\mathbb{E}[H^2] = \sum_i \mathbb{E}[p_i^4] + \sum_{i \neq j} \mathbb{E}[p_i^2 p_j^2]$, yields, under $\text{Dir}_B(\alpha)$,

$$\text{Var}(H) = \frac{2B(B-1)(\alpha+1)^2 \alpha}{(B\alpha+1)^2(B\alpha+2)(B\alpha+3)}, \quad (\text{A7})$$

and under $\alpha = cB$, as $B \rightarrow \infty$,

$$\text{Var}(H) \sim \frac{2}{cB^3}, \quad \text{SD}(H) \sim \sqrt{\frac{2}{c}} B^{-3/2}, \quad (\text{A8})$$

where $\text{SD}(H) = \sqrt{\text{Var}(H)}$ is the standard deviation of H . The coefficient of variation, defined as $\text{CV}(H) = \text{SD}(H)/\mathbb{E}[H]$, then satisfies $\text{CV}(H) \sim \sqrt{2/c} B^{-1/2} \rightarrow 0$.

The concentration index is therefore *self-averaging*: its relative fluctuations vanish for large B . This justifies the use of $\langle H \rangle$ as a reliable summary statistic and explains the reduction of error bars with increasing B observed in the data.

4. Bounds on the width concentration fraction

For any $\mathbf{p} \in \Delta^{B-1}$, the two structural indicators satisfy the pointwise inequality $D^2 \leq H \leq D$. Indeed, writing $D = \max_i p_i$ and letting i^* denote a maximizing index, the upper bound follows from $H = \sum_i p_i^2 \leq D \sum_i p_i = D$, while the lower bound follows from $H \geq p_{i^*}^2 = D^2$.

Under $\text{Dir}_B(\alpha)$, the expected width concentration fraction is bounded as

$$\frac{1}{B} \leq \mathbb{E}[D] \leq \sqrt{\mathbb{E}[H]} = \left(\frac{\alpha+1}{B\alpha+1} \right)^{1/2}. \quad (\text{A9})$$

The lower bound follows by symmetry: $\mathbb{E}[p_i] = 1/B$ and $D \geq p_i$ for all i , so $\mathbb{E}[D] \geq 1/B$, with equality approached as $\alpha \rightarrow \infty$. The upper bound follows from $D \leq \sqrt{H}$ together with Jensen's inequality ($\sqrt{\cdot}$ concave), $\mathbb{E}[D] \leq \mathbb{E}[\sqrt{H}] \leq \sqrt{\mathbb{E}[H]}$. Under $\alpha = cB$, as $B \rightarrow \infty$, substituting the scaling law (A4) gives $\mathbb{E}[D] \leq \sqrt{(cB+1)/(cB^2+1)} \sim (cB)^{-1/2}$.

A sharper estimate follows from extreme-value theory: for large B with $\alpha = cB$, each component has variance $\text{Var}(p_i) \sim (cB^2)^{-1}$, and classical extreme-value theory for exchangeable nearly-Gaussian variables gives

$$\mathbb{E}[D] \approx \frac{1}{B} + \frac{\sqrt{2 \ln B}}{\sqrt{cB}}, \quad (\text{A10})$$

consistent with the empirical exponents $\gamma_{\text{exp}} \in (0.62, 0.87)$ observed in the symmetric state, which lie between $1/2$ and 1.

- [1] P. Ashmore, Morphology and dynamics of braided rivers, in *Treatise on Geomorphology*, Vol. 9, Fluvial Geomorphology, edited by E. Wohl (Academic Press, New York, 2013), pp. 289–312.
- [2] P. Ashmore, Intensity and characteristic length of braided channel patterns, *Can. J. Civ. Eng.* **36**, 1656 (2009).
- [3] W. Bertoldi, L. Zanoni, and M. Tubino, Planform dynamics of braided streams, *Earth Surf. Proc. Landf.* **34**, 547 (2009).
- [4] J. L. Best and C. S. Bristow, Braided Rivers: Perspectives and Problems, in *Braided Rivers*, edited by C. S. Bristow and J. L. Best (Geological Society, London, 1993), pp. 1–11.
- [5] E. J. Hundey and P. E. Ashmore, Length scale of braided river morphology, *Water Resour. Res.* **45**, 2008WR007521 (2009).
- [6] R. Egozi and P. Ashmore, Experimental analysis of braided channel pattern response to increased discharge, *J. Geophys. Res.* **114**, 2008JF001099 (2009).
- [7] A. D. Howard, M. E. Keetch, and C. L. Vincent, Topological and geometrical properties of braided streams, *Water Resour. Res.* **6**, 1674 (1970).
- [8] A. M. Gurnell, G. E. Petts, D. M. Hannah, B. P. G. Smith, P. J. Edwards, J. Kollmann, J. V. Ward, and K. Tockner, Riparian vegetation and island formation along the gravel-bed Fiume Tagliamento, Italy, *Earth Surf. Process. Landf.* **26**, 31 (2001).
- [9] S. De Bartolo, C. Fallico, and E. Ferrari, Simple scaling analysis of active channel patterns in fiumara environment, *Geomorphology* **232**, 94 (2015).
- [10] S. De Bartolo, S. Rizzello, E. Ferrari, F. Frega, G. Napoli, R. Vitolo, M. Scaraggi, N. Fallico, and G. Severino, Scaling behaviour of braided active channels: A Taylor's power law approach, *Eur. Phys. J. Plus* **137**, 622 (2022).
- [11] S. De Bartolo and C. De Michele, Braiding as branching at the channel scale: A direct approach to network scaling, *Eur. Phys. J. Plus* **140**, 668 (2025).
- [12] S. De Bartolo, C. De Michele, and L. Primavera, Spatial persistence in the Brahmaputra river: Rescaled range and multiscaling analyses, *Stoch. Environ. Res. Risk Assess.* **40**, 78 (2026).
- [13] C. De Michele, L. Primavera, G. R. Tomasicchio, A. Lauria, A. Francone, E. Leone, G. Salvadori, and S. De Bartolo, Multifractal analysis of braided channel networks using structure functions and fixed-mass measures, *Physica A* **675**, 130831 (2025).
- [14] P. Meakin, *Fractals, Scaling and Growth Far from Equilibrium* (Cambridge University Press, Cambridge, 1998).
- [15] I. Rodríguez-Iturbe and A. Rinaldo, *Fractal River Basins: Chance and Self-Organization* (Cambridge University Press, Cambridge, 1997).
- [16] P. S. Dodds and D. H. Rothman, Scaling, universality, and geomorphology, *Annu. Rev. Earth Planet. Sci.* **28**, 571 (2000).
- [17] O. Malcai, D. A. Lidar, O. Biham, and D. Avnir, Scaling range and cutoffs in empirical fractals, *Phys. Rev. E* **56**, 2817 (1997).
- [18] S. Rizzello, M. Scaraggi, A. D. Nelson, L. Primavera, G. Napoli, G. Stecca, R. Vitolo, and S. De Bartolo, Multiscaling behavior of braided channel networks: An alternative formulation of Taylor's law variate transformations, *Phys. Rev. E* **109**, 034306 (2024).
- [19] M. Hiatt, W. Sonke, E. A. Addink, W. M. van Dijk, M. van Kreveld, T. Ophelders, K. Verbeek, J. Vlaming, B. Speckmann, and M. G. Kleinmans, Geometry and topology of estuary and braided river channel networks automatically extracted from topographic data, *J. Geophys. Res.: Earth Surf.* **125**, e2019JF005206 (2020).
- [20] A. Tejedor, J. Schwenk, M. Kleinmans, A. B. Limaye, L. Vulis, P. Carling, H. Kantz, and E. Foufoula-Georgiou, The entropic braiding index (eBI): A robust metric to account for the diversity of channel scales in multi-thread rivers, *Geophys. Res. Lett.* **49**, e2022GL099681 (2022).
- [21] J. Aitchison, *The Statistical Analysis of Compositional Data* (Chapman and Hall, London, 1986).
- [22] J. Aitchison, The statistical analysis of compositional data, *J. R. Stat. Soc. B* **44**, 139 (1982).
- [23] E. H. Simpson, Measurement of diversity, *Nature (London)* **163**, 688 (1949).
- [24] J. Aitchison and S. M. Shen, Logistic-normal distributions: Some properties and uses, *Biometrika* **67**, 261 (1980).
- [25] S. Rizzello, M. Scaraggi, A. D. Nelson, L. Primavera, G. Napoli, G. Stecca, R. Vitolo, and S. De Bartolo, Software and dataset (aggregation, variance and mean in braided channel networks) [Computer software], Zenodo, 2023, doi: 10.5281/zenodo.10104612.
- [26] S. De Bartolo, S. Rizzello, R. Vitolo, M. Scaraggi, G. Napoli, A. D. Nelson, and G. Stecca, Dataset (aggregation, variance and mean in braided channel networks) [Dataset], Zenodo, 2023, doi: 10.5281/zenodo.8019973.
- [27] M. H. Sarker, C. R. Thorne, M. N. Aktar, and M. R. Ferdous, Morpho-dynamics of the Brahmaputra–Jamuna River, Bangladesh, *Geomorphology* **215**, 45 (2014).
- [28] K. Gaurav, F. Métivier, O. Devauchelle, R. Sinha, H. Chauvet, M. Houssais, and H. Bouquerel, Morphology of the Kosi megafan channels, *Earth Surf. Dynam.* **3**, 321 (2015).
- [29] K. Gaurav, S. K. Tandon, O. Devauchelle, R. Sinha, and F. Métivier, A single width–discharge regime relationship for individual threads of braided and meandering rivers from the Himalayan Foreland, *Geomorphology* **295**, 126 (2017).
- [30] G. Lacey, Stable channels in alluvium, *Minutes Proc. Inst. Civ. Eng.* **229**, 259 (1930).
- [31] L. B. Leopold and T. Maddock, Jr., The hydraulic geometry of stream channels and some physiographic implications, in *U.S. Geological Survey Professional Paper* Vol. 252 (U.S. Government Printing Office, Washington, DC, 1953).
- [32] L. B. Leopold, M. G. Wolman, and J. P. Miller, *Fluvial Processes in Geomorphology* (W. H. Freeman, San Francisco, 1964).
- [33] F. Métivier, O. Devauchelle, H. Chauvet, E. Lajeunesse, P. Meunier, K. Blanckaert, P. Ashmore, Z. Zhang, Y. Fan, Y. Liu, Z. Dong, and B. Ye, Geometry of meandering and braided gravel-bed threads from the Bayanbulak Grassland, Tianshan, and P. R. China, *Earth Surf. Dynam.* **4**, 273 (2016).
- [34] F. Métivier, E. Lajeunesse, and O. Devauchelle, Laboratory rivers: Lacey's law, threshold theory, and channel stability, *Earth Surf. Dynam.* **5**, 187 (2017).
- [35] M. P. Mosley, Semi-determinate hydraulic geometry of river channels, South Island, New Zealand, *Earth Surf. Proc. Landf.* **6**, 127 (1981).
- [36] A. M. F. da Silva and M. S. Yalin, *Fluvial Processes*, IAHR Monographs, 2nd ed. (Routledge, London, 2017).