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In this paper, in the section on the fluctuation theorem, an error was made by intuitively assuming that the probability distribution \( p(x) \) is stationary. The error is corrected by including the ratio of the probabilities of initial and final points of the path to the definition of the entropy production variable \( W_x(k) \). In a per path notation it becomes

\[
W(\gamma) = \ln \frac{p(\gamma_i)w(\gamma)}{p(\gamma_f)w(\gamma_f)}.
\]  

This variable still is constant on each of the classes \( C(x,k) \). However, its value has an additional contribution \( \ln[p(\gamma_i)] - \ln[p(\gamma_f)] \). The fluctuation theorem then becomes as follows.

**Theorem 2.** In any Markov chain with finite state space the entropy production variable satisfies

\[
\frac{\text{Prob}(W = K)}{\text{Prob}(W = -K)} = e^K. \tag{26}
\]

**Proof.** One has

\[
\text{Prob}(W = K) = \sum_{\gamma} p(\gamma_i)w(\gamma) \delta_{[w(\gamma),K]} = e^K \sum_{\gamma} p(\gamma_f)w(\gamma_f) \delta_{[w(\gamma_f),K]} = e^K \sum_{\gamma} p(\gamma_f)w(\gamma) \delta_{[w(\gamma_f),K]} = e^K \text{Prob}(W = -K). \tag{27}
\]

This ends the proof.

The average value of the entropy production variable satisfies

\[
\langle W \rangle = \frac{1}{2} \sum_{\gamma} \frac{p(\gamma_i)w(\gamma) - p(\gamma_f)w(\gamma_f)}{p(\gamma_f)w(\gamma_f)} \ln \frac{p(\gamma_i)w(\gamma)}{p(\gamma_f)w(\gamma_f)}.
\]

It is immediately clear that it is positive on a per term basis. However, the positivity also follows from the fluctuation theorem, with the argument given in the paper

\[
\langle W \rangle = \sum_K K \text{Prob}(W = K) = \frac{1}{2} \sum_K \text{Prob}(W = K)K(1 - e^{-K}) \geq 0. \tag{29}
\]

The correct relation between the entropy production \( \langle W \rangle \) and the dynamical entropies is

\[
\overline{S}_0^{(o)} - S_0^{(o)} = \langle \Psi_0 \rangle - \langle \Psi_\phi \rangle = \langle W \rangle = \sum_{\gamma} p(\gamma_i)w(\gamma) \ln \frac{p(\gamma_i)}{p(\gamma_f)}. \tag{28}
\]

If \( p \) is stationary then the latter term vanishes and Eq. (28) of the paper is recovered.