

Diversity as an entropic measure derived from entropiclike functionals

Eugenio E. Vogel ^{1,2,*} Constantino Tsallis,^{3,4,5} and Gonzalo Saravia ⁶

¹*Departamento de Ciencias Físicas, Universidad de La Frontera, Temuco, Chile*

²*Cedenna, Facultad de Ingeniería y Arquitectura, Universidad Central de Chile, Santiago, Chile*

³*Centro Brasileiro de Pesquisas Físicas and National Institute of Science and Technology for Complex Systems, Rua Xavier Sigaud 150, Rio de Janeiro 22290-180, Brazil*

⁴*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*

⁵*Complexity Science Hub Vienna, Metternichgasse 8, 1030 Vienna, Austria*

⁶*Los Eucaliptos 1189, Temuco 4812537, Chile*



(Received 13 November 2025; revised 1 February 2026; accepted 9 March 2026; published 20 April 2026)

We present two derivations of an entropy-related function known as diversity in terms of entropy functionals. In the first case, we express diversity as an exponential of the difference between two forms of Shannon entropy. In the second case, the Jensen-Shannon divergence, an entropic function, only depends on diversity. Then, in the second part of this work, we apply diversity and Shannon entropy to a system in equilibrium (Ising model) and also to an out-of-equilibrium system (seismic region in Alaska). It turns out that diversity gives sharper responses at the critical points and more texture in the general trends.

DOI: [10.1103/91n2-p2vr](https://doi.org/10.1103/91n2-p2vr)

I. INTRODUCTION

This paper deals with different entropies and entropic functionals related to them [1]. The main purposes are to provide theoretical foundations to diversity [2,3] as an entropiclike measure or entropy-related function derived from entropic functionals, and to apply diversity to well-known physical systems both in equilibrium and out of equilibrium. Such as the cases of the well-known Ising ferromagnetic system [4] and earthquake sequences in Alaska [5], respectively.

We demonstrate that diversity and Shannon entropy connect via two distinct functionals. They are not qualitatively different functions, but diversity is easy to calculate and offers a sharp response at the critical points. This can be advantageous when determining values for parameters characterizing critical phenomena, as we will show in the applications.

Consider a sequence of successful measurements of an observable stored in a vector file (file with one record per row). The measurements were all taken under the same conditions, so they are equivalent independent of the measured value. So, the probability of any such measurement is $p_i = 1/\rho$, $i = 1, 2, 3, \dots, \rho$.

Then, consider the vector file formed by all the *different* values of such measurements, forming a vector file with ρ^* entries. Regardless of their abundance, all different values have the same probability of belonging to this list, namely,

$p'_j = 1/\rho^*$, $j = 1, 2, 3, \dots, \rho^*$. Since, by construction, $\rho^* \leq \rho$, we can define $p'_j = 0$ for $\rho^* < j \leq \rho$, which is equivalent to expressing these probabilities as

$$p_i = 1/\rho, \quad i = 1, 2, \dots, \rho, \quad (1)$$

and also

$$p'_i = 1/\rho^*, \quad i = 1, 2, \dots, \rho^*, \quad p'_i = 0, \quad i > \rho^*. \quad (2)$$

The next Section presents the derivations of the two entropic functionals. Section III is for the applications of diversity compared to Shannon entropy for data generated from the traditional Ising ferromagnetic model and for the data obtained from a seismic catalog. Finally, Sec. IV is for the conclusions.

II. THEORY

Let us consider any set of data with ρ equivalent measurements for which we can obtain the corresponding histogram: n_M measurements yielding the value M . Then,

$$\sum_{\text{All } M} n_M = \rho. \quad (3)$$

The probability p_M to measure the value M for the observable within the scope of this experiment is then

$$p_M = n_M/\rho, \quad (4)$$

with the normalization condition

$$\sum_{\text{All } M} p_M = 1. \quad (5)$$

This allows to invoke the Shannon entropy based on the values measured for this variable in accordance with the known

*Contact author: ee.vogel@gmail.com

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

expression [6]

$$H = - \sum_{\text{All } M} p_M \ln p_M. \quad (6)$$

Independent of the Shannon entropy associated with the values of the observable, we can formally apply Shannon entropy to the set of original measurements regardless of the abundances. So the Shannon entropy for the set of independent measurements is

$$h_M = - \sum_{i=1}^{\rho} p_i \ln p_i = \ln \rho. \quad (7)$$

By the same token, the Shannon entropy for the different values among the measurements is

$$h_D = - \sum_{i=1}^{\rho} p'_i \ln p'_i = \ln \rho^*. \quad (8)$$

Please note that we use a lowercase symbol to mean that these Shannon entropies come from probabilities different from those used in Eq. (6).

After a straightforward algebra, we can recover the definition of *diversity* [3], expressed as a functional of two Shannon entropies, namely,

$$\delta = \frac{\rho^*}{\rho} = e^{(h_D - h_M)}. \quad (9)$$

As $\rho^* \leq \rho$, $\delta \leq 1.0$. The more repetitive the values M , the smaller is δ . On the contrary, when the measurements span a large manifold (as in the chaotic regime of a phase transition), ρ^* grows and so does δ .

Alternatively, we get a different functional relationship between Shannon entropy and diversity if can recall the definition of the (symmetrized) Jensen-Shannon divergence as follows (see Ref. [7] and references therein),

$$D_{JS}(\{p_i\}, \{p'_i\}) \equiv \frac{1}{2} \left[\sum_{i=1}^{\rho} p_i \ln \frac{m_i}{p_i} + \sum_{i=1}^{\rho} p'_i \ln \frac{m_i}{p'_i} \right], \quad (10)$$

where

$$m_i \equiv \frac{p_i + p'_i}{2}. \quad (11)$$

Let us remark that, if we formally consider $m_i = 1$ and $p_i = p'_i$ ($\forall i$) in definition (10), we precisely obtain the standard Boltzmann-Gibbs-Shannon (BGS) entropy $S_{BG} = - \sum_{i=1}^{\rho} p_i \ln p_i$, which has the same structure given in (6).

Furthermore, if we now calculate $D_{JS}(\{p_i\}, \{p'_i\})$ with (p, p') given respectively in (1) and (2), we straightforwardly obtain

$$D_{JS}(\rho, \rho^*) = \ln \frac{\sqrt{2 + \frac{\rho^*}{\rho} + \frac{\rho}{\rho^*}}}{2}. \quad (12)$$

We verify that, interestingly enough, $D_{JS}(\rho^*, \rho)$ only depends on diversity, namely, on the ratio $\delta = \rho^*/\rho$, $\in (0, 1]$, and monotonically decreases from infinity to zero when ρ^*/ρ increases from zero to unity.

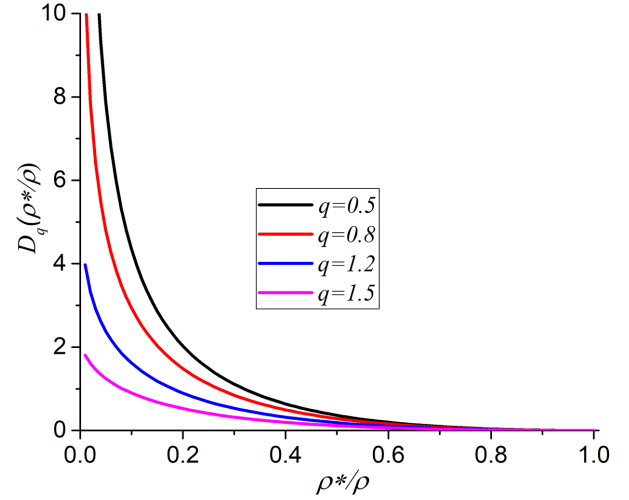


FIG. 1. Divergence $D_q(\rho^*, \rho) = D_q(\rho^*/\rho)$ as a function of $\rho^*/\rho = \delta$ for typical q values.

Definition (10) can be q -generalized (see Refs. [8–10] and references therein) as follows,

$$D_q(\{p_i\}, \{p'_i\}) \equiv \frac{1}{2} \left[\sum_{i=1}^{\rho} p_i \ln_q \frac{m_i}{p_i} + \sum_{i=1}^{\rho} p'_i \ln_q \frac{m_i}{p'_i} \right], \quad (13)$$

where $\ln_q z \equiv \frac{z^{1-q} - 1}{1-q}$ ($\ln_1 z = \ln z$). We can immediately recognize that $D_{JS}(\rho, \rho^*) \equiv D_1(\rho, \rho^*)$, providing a connection to the original Jensen-Shannon divergence.

If we formally consider $m_i = 1$ and $p_i = p'_i$ ($\forall i$) in definition (13), we precisely obtain the q -entropy $S_q = \sum_{i=1}^{\rho} p_i \ln_q \frac{1}{p_i}$ [11,12]. It is relevant mentioning at this point other nonadditive entropic functionals that are focused on as well in the literature, such as Kaniadakis entropy [13,14] and Hanel-Thurner entropy [15,16] to mention but a few.

Furthermore, it follows that, for generic values of q ,

$$D_q(\rho, \rho^*) = \frac{2}{1-q} \left\{ \left[\frac{1}{2} \left(1 + \frac{\rho^*}{\rho} \right) \right]^{1-q} + \left[\frac{1}{2} \left(1 + \frac{\rho}{\rho^*} \right) \right]^{1-q} - 2 \right\}, \quad (14)$$

which recovers Eq. (12) in the limit $q \rightarrow 1$. We also verify the following particular cases:

$$D_0(\rho, \rho^*) = \frac{\rho}{\rho^*} + \frac{\rho^*}{\rho} - 2, \quad D_2(\rho, \rho^*) = 0. \quad (15)$$

While ρ^*/ρ increases from zero to unity, $D_q(\rho^*/\rho)$ monotonically decreases for $0 < q < 2$, as shown in Fig. 1 for four values of q .

In summary, we have obtained that the Jensen-Shannon divergence as well as its q generalization are monotonic functions of the diversity $\delta = \rho^*/\rho$. It follows that, although not being a formal entropy, diversity can legitimately be interpreted as an entropiclike measure. In the rest of this paper we will explore possible applications of this metric and compare such results with those associated with the Shannon entropy.

III. APPLICATIONS

We begin with the classical Ising ferromagnetic system on a square lattice, used extensively as a benchmark for testing methods and numerical simulations [17]. Then, We will deal with seismic data belonging to the United States Geological Survey (USGS) [18] seismic catalog corresponding to one Alaska region, where important activity has been detected in recent years [19]. The former is an equilibrium system with stochastic processes governed by the Boltzmann-Gibbs statistics. The latter is an out-of-equilibrium system, with a mixture of natural stochastic and interconnected events. Natural data may look disordered but entropy can report clusters of data with definite properties [20].

A. Ising ferromagnetic system

Since the early proposal by Ising [17], this simple possible model of ferromagnetism has been widely used as a reference for modeling magnetic systems. In the case of a square lattice with nearest-neighbor interactions the theoretical derivation of a critical temperature by Onsager [21] has been followed by numerous computer simulations (see, for instance, Refs. [4,22,23] and references therein). Consider a lattice defined on a square array of $L \times L$ sites. Exchange interactions J lead to internal energy given by

$$E = -J \sum_{\langle ij \rangle} S_i S_j, \quad J > 0, \quad (16)$$

where i and j are indices scanning all lattice sites, and $\langle ij \rangle$ means pairs of nearest neighbors with $i \neq j$. S_i refers to the orientation of the i th spin, which can be up or down with respect to an easy direction. Periodic boundary conditions are imposed so that the coordination number is 4 through the lattice. A microstate is defined as the ordered set of $L \times L$ spin orientations.

A Monte Carlo (MC) process is implemented, beginning at very low temperature T , with a ferromagnetic state (all spins pointing “up”). Then a random spin is flipped to the opposite orientation and the energy variation ΔE is calculated. If $\Delta E \leq 0$, then the spin flip is accepted and the current microstate is updated. If $\Delta E > 0$, a fresh random number r between 0 and 1 is generated; next, if $\exp[\Delta E/(k_B T)] \geq r$, then the spin flip is accepted and the MC process continues from the new microstate. This is the well-known Metropolis algorithm. For simplicity, we take $k_B = 1$, which is equivalent to measuring energy in units of temperature.

$L \times L$ spin-flip attempts define a MC step. The code uses the first 20ρ MC steps to equilibrate the system at temperature T , then uses the next 20ρ MC steps to store the energy $E(T)$ of the system every 20 MC steps. So, at the end of the process, we have a vector file with ρ energy entries for each temperature.

Figure 2 presents results for Shannon entropy H and diversity δ for $L = 128$, and Temperature increments in 0.1 units up to 4.0 and in 0.5 units from there on up to 12 . Both curves show a peak near the critical temperature expected in the thermodynamic limit [21]. This backs diversity as able to recognize a critical phenomenon as the Ising ferromagnetic transition. However, there are differences between these curves: (i) The second T derivative of H approaching $T = 2.0$

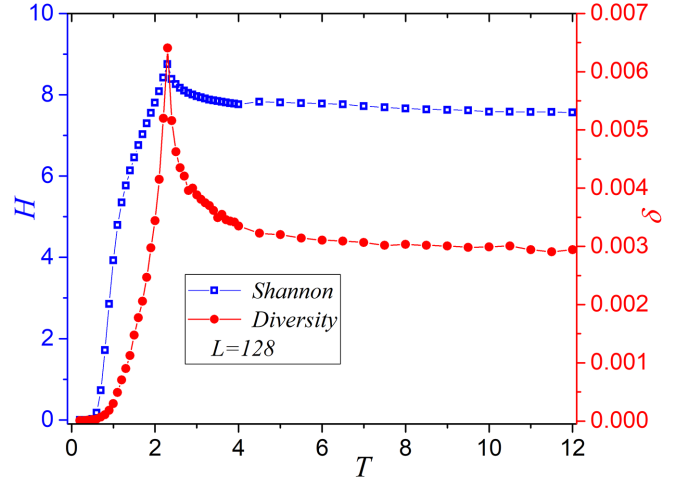


FIG. 2. Shannon entropy and diversity for an Ising ferromagnetic lattice for $L = 128$ and periodic boundary conditions ($\rho = 120\,000$).

is negative (downward concavity) while it is positive for δ (upward concavity); (ii) H presents an inflection point just before the maximum while δ does not; (iii) Both curves were obtained from the same MC calculation but the decrease from the maximum to the high-temperature tendency is approximately 10% in the case of H , while it is 53% for δ . This makes the maximum easier to appreciate in the case of diversity.

Figure 3 presents the results with $\Delta T = 0.02$ around the cusp of both curves, namely in the interval 1.0 – 3.0 . Both curves maximize near the theoretical critical temperature $2.269\,185\dots$, derived by Onsager [21], which is marked with an downward triangle at the top of this plot. Figure 3 clearly shows that diversity approaches the maximum with upward concavity presenting a sharp maximum, while Shannon entropy changes concavity just before the maximum. This different response by diversity allows for an accurate determination of the critical temperature when the temperature steps

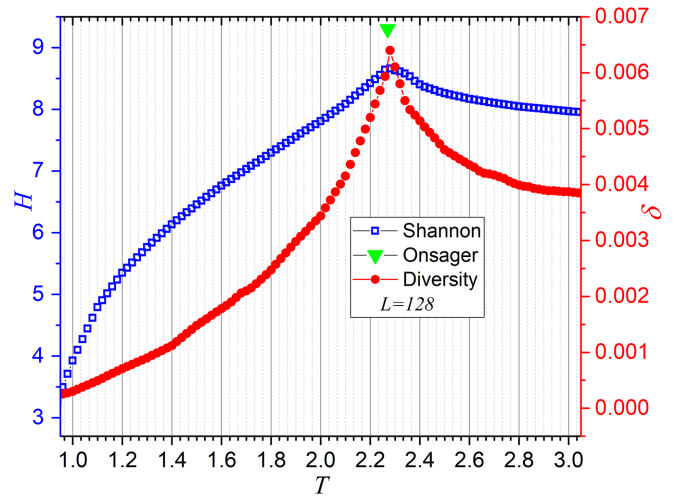


FIG. 3. Detailed maxima for Shannon entropy and diversity in connection with Fig. 2. Temperature variations are now of $0.02J$, and $\rho = 120\,000$. The theoretical value of the critical temperature by Onsager [21] is marked by an downward (green) triangle at the top.

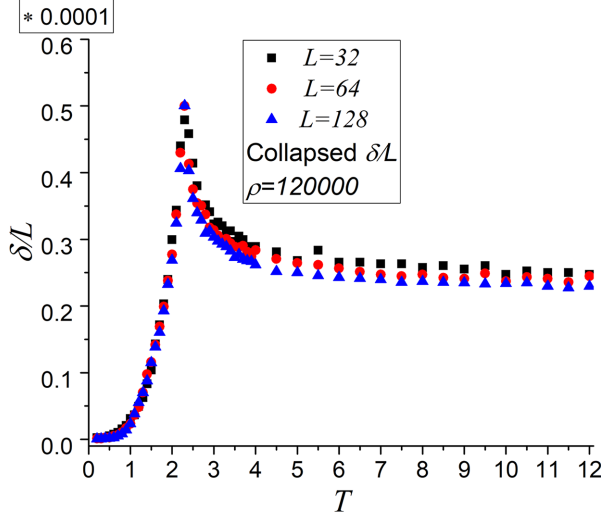


FIG. 4. Collapsed diversity for Ising systems. Periodic boundaries are used. Please notice that ordinates have been amplified by a factor 10^4 .

are shorter. Then, the temperature intervals are divided in smaller steps allowing to find the temperature of the maximum with more precision: in the present case, the interval $\Delta T = 0.1$ was broken into five steps with $\Delta T = 0.02$, allowing to get a better reading of the next decimal position and thus estimating the critical temperature at $T = 2.28$, which is close to the already quoted theoretical value valid for the thermodynamic limit ($L \rightarrow \infty$).

Figure 4 presents the diversity results for $L = 32, 64, 128$, and 256 using $\rho = 120\,000$ records. The temperature increased as in Fig. 4. Diversity, as defined by Eq. (9), scales with the “diameter” of the two-dimensional (2D) system. To compare different lattice sizes, in Fig. 4 we have plotted δ/L . The collapse is good for $\rho = 120\,000$ records, which can be improved by considering larger ρ values. To have a more complete picture of the collapse, Fig. 5 plots δ/L with respect

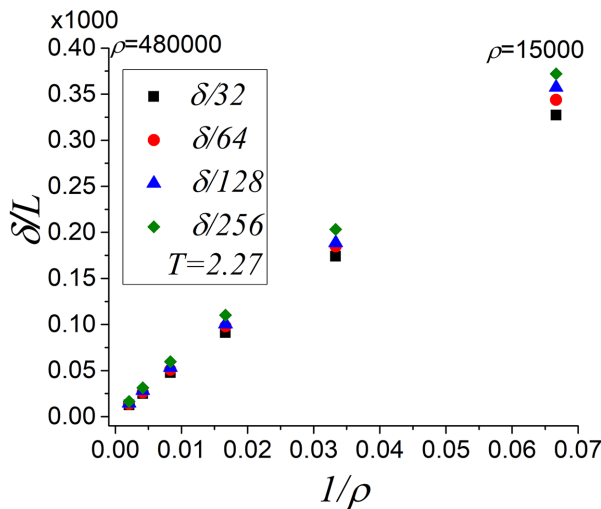


FIG. 5. Ratios of diversity over the lattice size L for four different sizes are plotted with respect to the reciprocal number of records at the theoretical critical temperature (2.27).

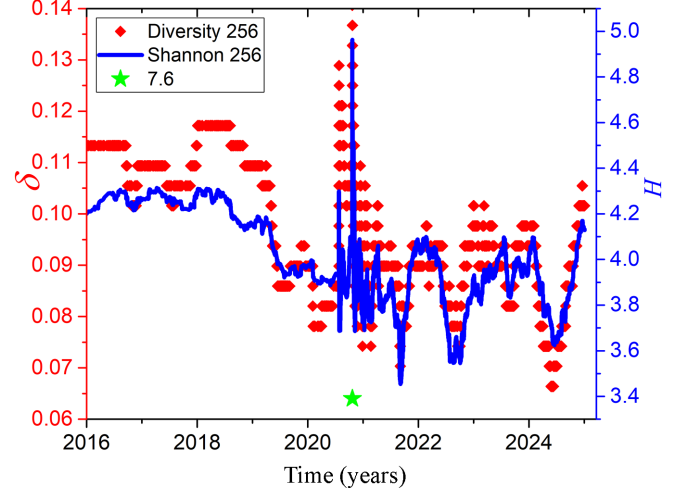


FIG. 6. Diversity and Shannon entropy using a mobile time window with $\rho = 256$ consecutive seisms for the selected Alaska region. The Vertical span is nearly the same for both curves to allow for a comparison of the dynamical variations. The time of the 7.6 earthquake is marked by a green star.

to $1/\rho$, for four lattice sizes using the critical temperature $T = 2.27$. The dispersion in values of δ/L at different values of ρ is minimal, and larger sampling ρ can diminish it.

B. Seismicity in Alaska

Entropy has been used in the analysis of different seismic regions worldwide [24–29]. In particular, several recent papers made use of Shannon entropy to characterize the seismic dynamics of different data [30–33]. This is encouraging since we just saw in the previous application that diversity can provide a means of more accurate determination of critical instances.

From the United States Geophysical Survey (USGS) catalog [18] we extracted all seisms with magnitude ≥ 1.8 , with depth up to 60 km, latitudes in the interval $[-160.6, -158.6]$ and longitudes within $[53.6, 55.6]$. This means 86 331 seisms, whose highest magnitude was 7.6, for the earthquake of 2020.10.19 (year.month.day). A vector file collects all the magnitudes M , including the 7.6 earthquake whose epicenter is nearly at the center of this geographical region.

We now do a dynamical approach, calculating diversity and Shannon entropy for windows with ρ consecutive events [30,34]. Such windows overlap on one seism sliding along the entire vector file storing the sequential magnitudes. Actually, we have considered $\rho = 128, 256$, and 512 . The former is noisier, while the latter needs considerable time (months or even years) to accumulate the 512 consecutive seismic events [19]. So we report here the results for $W = 256$, as presented in Fig. 6. The time of the 7.6 earthquake is marked with a green star.

The two curves are similar. The energy accumulation prior to the 7.6 earthquake led to a decrease in both functions during the year before the major quake. Immediately after the 7.6 earthquake, sudden increases in both diversity and Shannon entropy are observed. Then oscillations evidenced the large

seismic activity during the aftershock regime which extended for a few years.

Although similar and having almost parallel development, diversity and Shannon entropy present important differences. Since magnitudes are discrete, we can clearly appreciate this in the diversity curve. As ρ is constant (256) throughout this figure, the only difference comes from ρ^* the number of different magnitudes present in each time window, which is just a few dozen different values in the magnitude expressed with one integer and one decimal.

Although the scales were adjusted to have a similar vertical span, diversity holds more dynamical information than Shannon entropy. A more quantitative way to see this difference is by comparing percent variations on these curves. We compare the high values for both curves at the beginning of 2016 (4.30 for H and 0.113 for δ) to the low values of both curves at the beginning of 2020 (3.85 for H and 0.080 for δ). We find decreases of 11% and 29% for H and δ , respectively. This makes diversity a sensitive parameter that can be useful in the foreshock analysis in seismology. The appreciations done by the naked eye in Fig. 6 also confirm that diversity presents texture variations distinguishing different seismic regimes: high relative values and long period oscillations in the foreshock and lower values and short period oscillations in the aftershock regime.

In the year before the 7.6 seism, both curves decrease in a phenomenon that entropy has shown in other seismic regions [35]. The minimum shown by the diversity curve is notorious. Days afterwards, both curves present a sharp maximum in coincidence with the 7.6 earthquake. This is an entropic manifestation: a large earthquake generates a response of seisms of different magnitudes, thus enlarging the configuration space and increasing the values of entropy and associated functions.

A careful analysis of the seismic activity previous to the strongest earthquake shows that diversity produces a sharp maximum for the 5.9 earthquake dated on 2020.10.19, almost similar to the 7.6 earthquake occurring 13 days later. The response of Shannon entropy shows two peaks with different maximum values. This can be an advantage for diversity when examining the foreshock regimes: earthquakes of moderate magnitude can produce an appreciable response.

Both curves present a fast recovery during the aftershock activity, with diversity presenting large oscillations for almost a year. During the aftershock period, large seisms (with magnitude over 5.5, say) and intermediate seisms (with magnitude over 4.0, say) are progressively less frequent. So, the configuration space “shrinks,” leading to a decrease in entropy and associated functions. This slow process is clearly captured by the red curve after the maximum in Fig. 6.

After 2022 both functions oscillate in rather low values evidencing the prolonged subduction process, but again variations between maxima and minima are a bit more pronounced in the case of diversity. Then diversity shows an absolute minimum towards the middle of 2024, followed by a rise not as sharp as for the main earthquake; Shannon entropy follows this variation in a less pronounced way. These variations are a consequence of the 5.6 earthquake occurring in this area on 2024.08.01.

Entropy functions do not render a sequence of magnitudes but the results of the combination of periods of seismic calm, large earthquakes with prolonged aftershock regimes, isolated quakes, and swarms of local seisms. All of these phenomena have a signature in the entropy-related functions and diversity has a possibility of contributing to the understanding of the dynamics of seismicity if applied to different regions on the globe. For the time being, let us say that Fig. 6 (mostly due to diversity) indicates that the present activity is still under the average level of 2016–2018, probably evidencing that the subduction process continues with no large accumulation of energy between the colliding plates. From this point of view no large earthquakes (with a magnitude over 7.0, say) is likely to occur in this zone for the next few years.

Using Shannon entropy to deal with seismic data has been previously available in the literature [30,36]. The introduction of diversity can not only confirm this behavior but, with more and better data available, precursor signs could be established. Eventually, this could have been the case of the large contrast of diversity between the years 2018 and 2020 in the Alaska data treated in Fig. 6. However, more work is necessary in different seismic zones before a firm hypothesis on precursors can be proposed.

IV. CONCLUDING REMARKS

Diversity can be related to current entropic functionals, which opens the possibility of describing phenomena in a statistical physics framework complementary to those existing.

In one scheme, an exponential functional, which depends on the difference of two Shannon entropies, directly gives diversity. In the other scheme, we can express the Jensen-Shannon divergence using a functional that depends only on the diversity. Moreover, this expression can be q generalized, which can offer several other possibilities of analysis.

Using Shannon entropy and diversity with the classical 2D Ising ferromagnetic model gives the expected behavior, including the critical temperature near 2.27 in units of the exchange interaction. However, diversity produces the sharpest response at the critical temperature. So, diversity appears to a suitable candidate for detecting critical phenomena.

The application of diversity and Shannon entropy to the seismic data of a region in Alaska gives similar results characterized by higher values of the entropic function before the large earthquake, a clear descent in their values prior to the earthquake, a sharp rise coinciding with the main quakes followed by oscillating lower values afterwards in the aftershock regime. It is clear that the level of these functions is still under their respective averages reached a couple of years before the large earthquake, evidencing the present instability of the region that continues to report seisms with magnitudes near or over 6.0. The details of the response to diversity make it a suitable candidate to compare seismic regions and eventually establish common patterns characterizing seismic risk.

We find less difference between Shannon entropy and diversity in the seismic case, as compared with the Ising model, since the density of seismic data limits ρ . In the magnetic simulations, the number of measurements is a matter of computer

time only, but in the seismic data, nature and instruments limit this.

Shannon entropy (or any other form of entropy) measures the number of states available in the configuration space; diversity is a complementary function related to entropy which can determine critical behavior directly and with accuracy.

The calculation of diversity requires just counting the total number of measurements ρ and the number of different measurements ρ^* among them. Not even a histogram is necessary, which makes the diversity calculations easier and their interpretation more direct.

ACKNOWLEDGMENTS

We acknowledge E. M. F. Curado for useful remarks. We acknowledge funding from Fondecyt (Chile) Under Contract No. 1230055 and CEDENNA (Chile) under Project No. ANID CIA250002, and also from CNPq and Faperj (Brazilian agencies).

DATA AVAILABILITY

The data that support the findings of this article are openly available [18].

- [1] E. Zanchini and G. P. Beretta, Recent progress in the definition of thermodynamic entropy, *Entropy* **16**, 1547 (2014).
- [2] E. E. Vogel and G. Saravia, Information theory applied to econophysics: Stock market behaviors, *Eur. Phys. J. B* **87**, 177 (2014).
- [3] O. A. Negrete, P. Vargas, F. J. Peña, G. Saravia, and E. E. Vogel, Short-range Berezinskii-Kosterlitz-Thouless phase characterization for the q -state clock model, *Entropy* **23**, 1019 (2021).
- [4] L. Pierre, B. Bernu, and L. Messio, Derivation of free energy, entropy and specific heat for planar Ising models: Application to Archimedean lattices and their duals, *SciPost Phys.* **19**, 025 (2025).
- [5] M. Sedghizadeh and R. Shcherbakov, The analysis of the after-shock sequences of the recent mainshocks in Alaska, *Appl. Sci.* **12**, 1809 (2022).
- [6] C. E. Shannon, A mathematical theory of communication, *Bell Syst. Tech. J.* **27**, 379 (1948).
- [7] J. Lin, Divergence measures based on the Shannon entropy, *IEEE Trans. Inf. Theory* **37**, 145 (1991).
- [8] P. Lamberti and A. Majtey, Non-logarithmic Jensen-Shannon divergence, *Physica A* **329**, 81 (2003).
- [9] J. Antolín, S. Lopez-Rosa, J. Angulo, and R. Esquivel, Jensen-Tsallis divergence and atomic dissimilarity for position and momentum space electron densities, *J. Chem. Phys.* **132**, 044105 (2010).
- [10] J. Angulo, J. Antolin, S. Lopez-Rosa, and R. Esquivel, Jensen-Tsallis divergence and atomic dissimilarity for ionized systems in conjugated spaces, *Physica A* **390**, 769 (2011).
- [11] C. Tsallis, Possible generalization of Boltzmann-Gibbs statistics, *J. Stat. Phys.* **52**, 479 (1988).
- [12] C. Tsallis, Generalized entropy-based criterion for consistent testing, *Phys. Rev. E* **58**, 1442 (1998).
- [13] G. Kaniadakis, Non-linear kinetics underlying generalized statistics, *Physica A* **296**, 405 (2001).
- [14] G. Kaniadakis, Statistical mechanics in the context of special relativity, *Phys. Rev. E* **66**, 056125 (2002).
- [15] S. T. R. Hanel, A comprehensive classification of complex statistical systems and an axiomatic derivation of their entropy and distribution functions, *Europhys. Lett.* **93**, 20006 (2011).
- [16] S. T. R. Hanel, When do generalised entropies apply? How phase space volume determines entropy, *Europhys. Lett.* **96**, 50003 (2011).
- [17] E. Ising, Beitrag zur theorie des ferromagnetismus, *Z. Phys.* **31**, 253 (1925).
- [18] United States Geological Survey, <https://earthquake.usgs.gov/earthquakes/search/>.
- [19] E. E. Vogel, D. Pastén, G. Saravia, M. Aguilera, and A. Posadas, 2021 Alaska earthquake: Entropy approach to its precursors and aftershock regimes, *Nat. Hazards Earth Syst. Sci.* **24**, 3895 (2024).
- [20] J. V. V. Flauzino, T. L. Prado, N. Marwan, J. Kurths, and S. R. Lopes, Quantifying disorder in data, *Phys. Rev. Lett.* **135**, 097401 (2025).
- [21] L. Onsager, Crystal statistics. I. A two-dimensional model with an order-disorder transition, *Phys. Rev.* **65**, 117 (1944).
- [22] M. Metra, L. Zorrilla, M. Zani, E. Puppini, and P. Biscari, Temperature-dependent criticality in random 2D Ising models, *Eur. Phys. J. Plus* **136**, 939 (2021).
- [23] D. Wordofa Tola and M. Bekele, Nonequilibrium phase transitions in a 2D ferromagnetic spins with effective interactions, *J. Phys. A: Math. Theor.* **57**, 395001 (2024).
- [24] E. Anyfadi, S. Avgerinou, G. Michas, and F. Vallianatos, Universal non-extensive statistical physics temporal pattern of major subduction zone aftershock sequences, *Entropy* **24**, 1850 (2022).
- [25] F. Vallianatos, G. Michas, G. Papadakis, and P. Sammonds, A non-extensive statistical physics view to the spatiotemporal properties of the June 1995, aigion earthquake (M6.2) after-shock sequence (West Corinth rift, Greece), *Acta Geophys.* **60**, 758 (2012).
- [26] O. Sotolongo-Costa and A. Posadas, Fragment-asperity interaction model for earthquakes, *Phys. Rev. Lett.* **92**, 048501 (2004).
- [27] A. Posadas and O. Sotolongo-Costa, Non-extensive entropy and fragment-asperity interaction model for earthquakes, *Commun. Nonlinear Sci. Numer. Simul.* **117**, 106906 (2023).
- [28] E. Sardeli, G. Michas, K. Pavlou, D. Zaccagnino, and F. Vallianatos, Spatiotemporal properties of the 2020–2021 Petrinja (Croatia) earthquake sequence, *J. Seismol.* **28**, 899 (2024).
- [29] J. B. Rundle, I. Baughman, and T. Zhang, Nowcasting earthquakes with stochastic simulations: Information theory of earthquake catalogs, *Earth Space Sci.* **11**, e2023EA003367 (2024).
- [30] J. Rundle, A. Giguere, D. L. Turcotte, J. P. Crutchfield, and A. Donnellan, Global seismic nowcasting with Shannon information entropy, *Earth Space Sci.* **6**, 191 (2019).
- [31] L. Telesca, A. Thai, M. Lovallo, D. Cao, and L. Nguyen, Shannon entropy analysis of reservoir-triggered seismicity at Song Tranh 2 hydropower plant, Vietnam, *Appl. Sci.* **12**, 8873 (2022).

- [32] F. A. Nava, Seismic magnitudes, entropy and b -value, [Acta Geophys.](#) **73**, 1147 (2025).
- [33] A. Skabylov, A. Agishev, D. Zhexebay, M. Ibraimov, S. Khokhlov, and A. Maksutova, Application of spectral entropy to P-wave detection in continuous seismogram analysis, [Appl. Sci.](#) **15**, 8718 (2025).
- [34] W. Huang, X. Chuai, Y. Rao, and B. Li, Shannon entropy-based seismic local correlation measure and enhancement, [IEEE Geosci. Remote Sens. Lett.](#) **19**, 3000905 (2022).
- [35] D. Pasten, E. E. Vogel, G. Saravia, A. Posadas, and O. Sotolongo, Tsallis entropy and mutability to characterize seismic sequences: The case of 2007–2014 Northern Chile earthquakes, [Entropy](#) **25**, 1417 (2023).
- [36] E. E. Vogel, F. G. Brevis, D. Pastén, V. Muñoz, R. A. Miranda, and A. C.-L. Chian, Measuring the seismic risk along the Nazca–South American subduction front: Shannon entropy and mutability, [Nat. Hazards Earth Syst. Sci.](#) **20**, 2943 (2020).