

Microscopic features of the effect of vehicle overacceleration on traffic flow

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Through the development of a microscopic deterministic model in the framework of three-phase traffic theory, microscopic features of vehicle overacceleration, which determines the occurrence of the metastability of free traffic flow at a bottleneck, have been revealed: (i) The greater the impact of vehicle overacceleration on free traffic flow at a bottleneck, the higher the maximum flow rate at which free flow can persist at the bottleneck, i.e., the better traffic breakdown can be avoided. (ii) There is vehicle overacceleration in a road lane caused by safety acceleration at the bottleneck. (iii) Overacceleration in a road lane can occur without explicitly introducing it in the model. (iv) Through a microscopic analysis of spatiotemporal competition between speed adaptation and vehicle acceleration behaviors, traffic conditions have been found at which safety acceleration in a road lane and/or vehicle acceleration due to lane-changing on a multilane road become overacceleration. (v) There is spatiotemporal cooperation of different overacceleration mechanisms. (vi) The stronger the overacceleration cooperation, the stronger the maintenance of free flow at the bottleneck due to overacceleration. (vii) On a two-lane road, both speed adaptation and overacceleration in a road lane can affect qualitatively the overacceleration mechanism caused by lane-changing. The microscopic features of the effect of vehicle overacceleration on traffic breakdown are related to traffic flow consisting of human-driving vehicles and/or of automated-driving vehicles.

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I. INTRODUCTION

One of the most well-known and widely accepted theoretical results in vehicular traffic science is the assumption that traffic breakdown results from driver *overreaction* to an unexpected braking of the preceding vehicle: Due to the existence of a driver reaction time, a driver begins to decelerate with a time delay; as a result, the driver decelerates stronger than it needs to avoid the collision with the preceding vehicle. Because of this driver *overdeceleration* caused by overreaction, the vehicle speed becomes less than the speed of the preceding vehicle. If such overdeceleration effect is realized for the following drivers, then traffic flow instability is realized: Due to overdeceleration of the following vehicles, a growing wave of a speed decrease occurs that propagates upstream. This classical traffic flow instability that should explain traffic breakdown was discovered theoretically in 1950s in car-following models by Herman, Gazis, Montroll, Potts, and Rothery [1–3] as well as Kometani and Sasaki [4–7]. Through different mathematical approaches, the classical traffic flow instability has later been incorporated in a diverse variety of mathematical traffic flow models, in particular, well-known and widely used models by Newell [8,9],

Gipps [10,11], Wiedemann [12], macroscopic model of Payne [13,14], cellular automation model of Nagel and Schreckenberg [15], optimal velocity model of Bando *et al.* [16–19], lattice traffic flow model of Nagatani [20,21], intelligent driver model of Treiber [22], and stochastic microscopic model of Krauß [23]. The above-mentioned models as well as many other standard traffic flow models incorporating the classical traffic flow instability (e.g., Refs. [24–26]) are currently one of the main fundamentals of standard traffic science and the basis of the most microscopic traffic simulation tools widely used in traffic engineering [27–44]. It has been found [45] that the development of this classical traffic flow instability lead to the formation of moving traffic jam (J) in free flow (F) ($F \rightarrow J$ transition).¹

¹It should be emphasized that in models of traffic flow consisting of 100% automated-driving vehicles (e.g., Refs. [46–57]) so-called *string instability* can occur (e.g., Refs. [47–52]). As in traffic flow consisting of human-driving vehicles, the cause of string instability is overdeceleration of an automated-driving vehicle, which occurs under particular dynamic characteristics of the automated-driving vehicle, at which the vehicle decelerates stronger than it needs to avoid the collision with a slower moving preceding vehicle. Contrary to human-driving vehicles for which overdeceleration is caused by driver reaction time that cannot be zero, overdeceleration of the automated-driving vehicle and, therefore, string instability of a platoon of such automated-driving vehicles can be avoided through an appropriated choice of dynamic characteristics of the automated-driving vehicles [49]. Despite this physical difference, string instability in traffic flow consisting of automated-driving ve-

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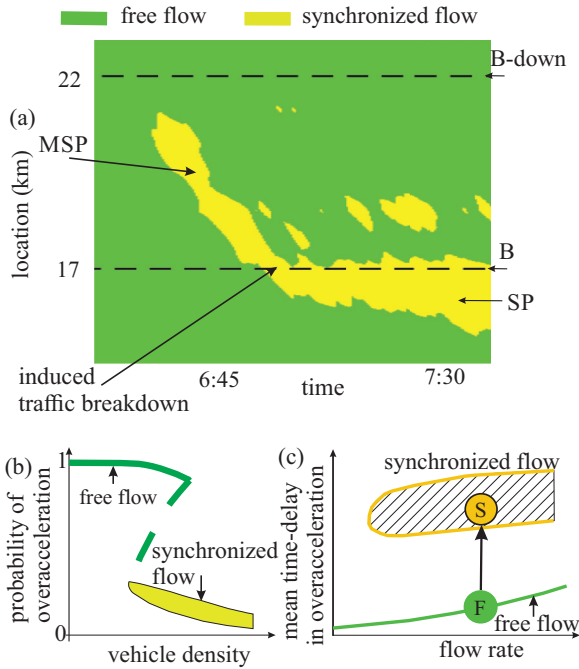


FIG. 1. Hypothesis of three-phase traffic theory about the discontinuous character of driver overacceleration [60]. (a) Empirical nucleation nature of traffic breakdown ($F \rightarrow S$ transition) at bottlenecks. Speed data presented in space and time with averaging method described in Sec. C.2 of Ref. [67] were measured with road detectors installed along road: A moving synchronized flow pattern (MSP) that has emerged at downstream bottleneck (B-down) while propagating upstream induces an $F \rightarrow S$ transition (induced traffic breakdown) leading to emergence of a synchronized flow pattern (SP) at upstream bottleneck (B); adapted from Refs. [62–65]. (b), (c) Qualitative vehicle density-dependence of driver overacceleration probability per a time interval (b) and equivalent presentation of panel (b) as a discontinuous flow-rate dependence of the mean time-delay in overacceleration (c): At given initial traffic parameters (e.g., at given flow rates at traffic network boundaries), during the $F \rightarrow S$ phase transition the probability of overacceleration drops abruptly whereas the mean time delay in overacceleration jumps abruptly; F and S are states of free flow and synchronized flow, respectively.

However, at the end of 1990s it was found that rather than the $F \rightarrow J$ transition of standard traffic flow models, real empirical traffic breakdown in human-driving vehicular traffic is a phase transition from free flow (F) to synchronized flow (S) ($F \rightarrow S$ transition) [58,59]. Empirical studies have shown that the $F \rightarrow S$ transition at a bottleneck exhibits the nucleation nature. None of the standard traffic flow models can explain the empirical nucleation nature of traffic breakdown ($F \rightarrow S$ transition) at the bottleneck. To explain the empirical

nucleation nature of the $F \rightarrow S$ transition [Fig. 1(a)], Kerner introduced three-phase traffic theory [60–66].

As mentioned, in standard traffic theory vehicle overdeceleration caused by driver overreaction in traffic flow consisting of human-driving vehicles should be responsible for traffic breakdown. Contrary to standard traffic flow theory, in three-phase traffic theory it is assumed that the empirical nucleation nature of traffic breakdown ($F \rightarrow S$ transition) is caused by a discontinuity in the probability (per time interval) of driver acceleration that shows up when a driver tries to accelerate from a local speed decrease in traffic flow: The probability of acceleration from a small local speed decrease occurring in free flow drops abruptly when the same local speed decrease occurs in synchronized flow. It is assumed that in free flow drivers can accelerate from a lower speed during car-following to a higher speed with a larger probability than it occurs in synchronized flow [Fig. 1(b)] [60].

A driver acceleration behavior that leads to such a discontinuity in the probability of acceleration to free flow has been called driver *overacceleration* [62–66]. The discontinuity in the overacceleration probability [Fig. 1(b)] can be considered the discontinuous character of overacceleration. An equivalent presentation of the discontinuous character of overacceleration is a discontinuous flow-rate dependence of the mean time-delay in vehicle overacceleration [Fig. 1(c)]. In three-phase traffic theory, the discontinuous character of overacceleration is the cause for the nucleation nature of the $F \rightarrow S$ transition at a bottleneck. The behavioral origin of overacceleration is related to the wish of drivers to move in free flow.

On the microlevel of vehicle motion, there is a finite time-delay in vehicle overacceleration.² In general, the mean time-delay in vehicle overacceleration can considerably depend on traffic situation. In three-phase traffic theory it is assumed that the larger the vehicle density, the longer should be the mean time-delay in vehicle overacceleration: In synchronized flow, the mean time-delay in overacceleration should be considerably longer than it is in free flow. This explains the assumption about the discontinuous character of overacceleration [Fig. 1(c)].

However, the discontinuity of the mean time-delay in vehicle overacceleration is difficult to apply as a definition of overacceleration in mathematical modeling of traffic. This is because the overacceleration effect is in a spatiotemporal competition with the opposite effect of speed adaptation [62–65].

Speed adaptation is a vehicle behavior that tries to equalize the vehicle speed to the speed of the preceding vehicle.³ Contrary to overacceleration that causes a tendency toward free flow, speed adaptation causes a tendency toward synchronized flow: Only if the overacceleration effect is on average stronger than the opposite effect of speed adaptation does free flow

hicles is qualitatively similar to the classical traffic flow instability in traffic flow consisting of human-driving vehicles. This is because in both cases vehicle overdeceleration is the cause of these traffic flow instabilities. For this reason, below we will use the term *vehicle overdeceleration* as a common one for human-driving and automated-driving vehicles.

²The time-delay in vehicle overacceleration should not be confounded with the driver reaction time that is responsible for driver overreaction causing driver overdeceleration.

³An example of speed adaptation is deceleration of the vehicle that approaches a slower moving preceding vehicle that cannot be overtaken.

remain at the bottleneck; otherwise, speed adaptation leads to the $F \rightarrow S$ transition at the bottleneck [62–65].

The discontinuous character of overacceleration is not solely the feature of human-driving vehicles. Indeed, in traffic flow consisting of 100% of automated-driving vehicles there is also a discontinuity in the probability of overacceleration of the automated-driving vehicles that is the cause for the nucleation nature of the $F \rightarrow S$ transition at the bottleneck [68]. Therefore, for both human-driving and automated-driving vehicles we can generalize the definition of the term *overacceleration* as follows:

- Vehicle acceleration behavior that causes the free flow metastability with respect to the $F \rightarrow S$ transition at the bottleneck is called vehicle overacceleration.

Through the analysis of whether an $F \rightarrow S$ transition at the bottleneck exhibits nucleation nature, this macroscopic definition of overacceleration is applicable for a clear distinguishing whether overacceleration occurs or not in both real field traffic data and mathematical modeling: If the $F \rightarrow S$ transition at the bottleneck exhibits the nucleation nature, then overacceleration does occur; otherwise, no overacceleration occurs.⁴ The definition of overacceleration, which can be considered a *sufficient* condition for overacceleration, is made through the macroscopic effect of overacceleration on vehicular traffic: When free flow is in a metastable state with respect to the $F \rightarrow S$ transition at the bottleneck, the overacceleration does occur in vehicular traffic. After we have recognized the occurrence of overacceleration through the nucleation nature of traffic breakdown, in the following microscopic mathematical modeling made in this paper we study overacceleration mechanism(s) on microlevel with the use of a paper methodology explained in Sec. III.

The discontinuous character of overacceleration is a basic feature of overacceleration [Figs. 1(b) and 1(c)]. In general, for both human-driving and automated-driving vehicles moving in traffic flow, the behavioral origin of vehicle overacceleration is related to the vehicle's desire to move in free flow.

⁴As a way for a quantitative mathematical analysis of distinguishing whether overacceleration occurs or not, we use in the paper simulations of a spatiotemporal dynamic behavior of an addition time-limited local speed decrease in free flow: The $F \rightarrow S$ transition exhibits the nucleation nature, i.e., overacceleration occurs, when a small enough local speed decrease decays over time, whereas a large enough local speed decrease whose amplitude exceeds some critical amplitude (critical nucleus) grows over time leading to the $F \rightarrow S$ transition. For the mathematical simulation of the time-limited local speed decrease in free flow, we use usually either a short-time breaking of one of the vehicles or a short-time addition increase in the on-ramp inflow rate at the bottleneck. A study of other possible mathematical ways for the analysis whether the $F \rightarrow S$ transition at the bottleneck exhibits the nucleation nature or not and, therefore, overacceleration occurs or not (like the use of a noise term in the model) that is out of the scope of this paper can be an interesting task for further traffic research.

The term *overacceleration* should distinguish vehicle overacceleration behaviors that do result in the free flow metastability with respect to the $F \rightarrow S$ transition at the bottleneck from usual vehicle acceleration behaviors that do not cause the free flow metastability.

In stochastic three-phase traffic flow models [69–72], overacceleration has been described *on average* through model fluctuations. Although in Refs. [69–72] many empirical spatiotemporal features of traffic breakdown and resulting congested patterns have been explained,⁵ microscopic characteristics of overacceleration is difficult to disclose through a study of the stochastic models. In Ref. [87], a simple deterministic model of overacceleration in a road lane has been introduced that exhibits the discontinuous character. In this model, overacceleration exceeds zero only when the vehicle speed is equal or larger than a threshold synchronized vehicle speed. This overacceleration model explains the empirical nucleation nature of traffic breakdown. However, this simple overacceleration model [87] cannot alone explain a possible diversity of microscopic features of the effect of vehicle overacceleration on traffic flow.

In Ref. [64], the competition between overacceleration and speed adaptation have already been studied. In comparison with results known in the three-phase traffic theory, the following results have been found in the paper:

(i) Through special choices of model parameters up to a transformation of the initial three-phase traffic model into the classical model of adaptive cruise control (ACC) described by Helly's formula [88], the suppression of overacceleration through speed adaptation has been reduced continuously. This has led to the disclosing of microscopic features of competition between speed adaptation and overacceleration in a road lane, i.e., in the absence of lane changing.

(ii) A microscopic overacceleration mechanism on a single-lane road caused by safety acceleration, which occurs when a vehicle accelerates from a local speed decrease to a higher speed, has been revealed. We have found that the overacceleration can occur without explicitly introducing it in the model.

(iii) We have found that there is a microscopic cooperation of different overacceleration mechanisms. The overacceleration cooperation can considerably increase the effect of the self-maintaining of free flow through overacceleration at the bottleneck.

(iv) We have found that overacceleration and speed adaptation in a road lane can affect crucially on overacceleration through lane-changing.

(v) Microscopic features of competition between speed adaptation and overacceleration have been disclosed.

(vi) Microscopic characteristics of a critical nucleus for traffic breakdown ($F \rightarrow S$ transition) have been found.

(vii) Microscopic features of overacceleration cooperation resulting in the $S \rightarrow F$ instability on a two-lane road have been revealed.

⁵These three-phase stochastic traffic flow models [69–72] are currently widely used for simulations of a diverse variety of traffic scenarios (e.g., Refs. [73–86]).

The paper is organized as follows. A deterministic three-phase traffic flow model used for simulations is developed in Sec. II, the methodology of physical modeling used in the paper is presented in Sec. III, overacceleration mechanisms caused by safety acceleration on a single-lane road are studied in Sec. IV, cooperation of different mechanisms of overacceleration is the subject of Sec. V, the effect of overacceleration in a road lane on overacceleration through lane-changing is studied in Sec. VI, a theory of the critical nucleus for spontaneous traffic breakdown is presented in Sec. VII, a comparison of the effect of different overacceleration mechanisms on traffic flow is given in Sec. VIII, and an $S \rightarrow F$ instability on a two-lane road caused by overacceleration cooperation is analyzed in Sec. IX. In Sec. X, we generalize the model for moving jam simulations to study overacceleration versus overdeceleration. A discussion of microscopic features of overacceleration as well as their effect on vehicular traffic is presented in Sec. XI. In Sec. XII, conclusions are formulated.

II. MICROSCOPIC MODEL OF VEHICULAR TRAFFIC

A. Basic three-phase traffic flow model

As a basic model for vehicle motion, we apply Kerner's model of Ref. [87]. In the microscopic deterministic traffic flow model, vehicle acceleration/deceleration a in a road lane is described by a system of equations:

$$a = a_{OA} + K_{\Delta v} \Delta v \quad \text{at } g_{\text{safe}} \leq g \leq G, \quad (1)$$

$$a = a_G \quad \text{at } g > G, \quad (2)$$

$$a = a_{\text{safety}}(g, v, v_\ell) \quad \text{at } g < g_{\text{safe}}, \quad (3)$$

where overacceleration a_{OA} in a road lane is given by the following equation:

$$a_{OA} = \alpha \Theta(v - v_{\text{syn}}); \quad (4)$$

v is the vehicle speed, $0 \leq v \leq v_{\text{free}}$, where v_{free} is a maximum vehicle speed; α is a coefficient of overacceleration; $\Theta(z) = 0$ at $z < 0$ and $\Theta(z) = 1$ at $z \geq 0$; v_{syn} is a given synchronized flow speed ($v_{\text{syn}} < v_{\text{free}}$); g is a space gap to the preceding vehicle, $g = x_\ell - x - d$, where x and x_ℓ are, respectively, the coordinates of the vehicle and the preceding vehicle and d is the vehicle length;

$$\Delta v = v_\ell - v,$$

where v_ℓ is the speed of the preceding vehicle; $K_{\Delta v}$ is a positive dynamic coefficient; G is a synchronization space gap, and g_{safe} is a safe space gap, where $G > g_{\text{safe}}$ at $v > 0$; $a_{\text{safety}}(g, v, v_\ell)$ is a safety vehicle acceleration⁶; a_G is vehicle acceleration at large space gaps; vehicle acceleration and

speed are limited by maximum acceleration $a = a_{\text{max}}$ and $v = v_{\text{free}}$, respectively.⁷

B. Space-gap dependence of overacceleration

In numerical simulations of Eqs. (1)–(4) made in Ref. [87], for simplification the coefficient of overacceleration α in Eq. (4) has been chosen a constant.⁸ However, it is physically more realistic to assume that α in Eq. (4) should depend on the space-gap g . In particular, within the space-gap range $g_{\text{safe}} \leq g \leq G$ overacceleration should be the larger, the larger the space-gap g . Indeed, the less the difference $g - g_{\text{safe}}$ between the space-gap and the safe space-gap, the smaller should be the probability of overacceleration. Therefore, in Eq. (4) we get

$$\alpha(g) = (\alpha_0 - \alpha_1) \left(\frac{g - g_{\text{safe}}}{G - g_{\text{safe}}} \right)^k + \alpha_1 \quad \text{at } g_{\text{safe}} \leq g \leq G, \quad (5)$$

where k , α_0 , and α_1 are positive model parameters, $\alpha_0 > \alpha_1$.

the term “safety vehicle acceleration”. The value $a_{\text{safety}}(g, v, v_\ell)$ in Eq. (3) can be either positive, or negative, or else equal to zero.

⁷Because overacceleration is the key feature distinguishing free flow from synchronized states in traffic flow, in Ref. [89] we have implied that car acceleration has been included in a model among the phase variables, i.e., in addition to velocity and position. The discontinuous character of overacceleration has been simulated in Ref. [89] through a gap between steady states of synchronized flow and free flow: Within the gap, condition $g > G$ is satisfied and, as in Eq. (2), vehicle accelerates to free flow; contrarily, within synchronized flow states, no vehicle acceleration to free flow is possible. The model of Ref. [89] can show the three phases and phase transitions between them as found in real field traffic data. Nevertheless, the modeling of overacceleration through the gap between model steady states as made in Ref. [89] seems to be difficult to use for a study of microscopic features of overacceleration. As we show below, the microscopic analysis of overacceleration can be done through the use of the model of overacceleration a_{OA} (4) [87]. For this reason, rather than the model with explicit consideration of acceleration as the phase variable [89], we have reverted to Newtonian-type dynamics (1)–(4) by the model development. However, a further development of three-phase traffic flow models in which acceleration is included among the phase variables that is out of scope of this paper can be interesting task for future traffic research.

⁸For a study of free flow metastability, in Ref. [87] the following assumptions have been made in the model (1)–(4): (i) coefficient of overacceleration α is a constant, (ii) $a_G = a_{\text{max}}$, and (iii) $a_{\text{safety}}(g, v, v_\ell)$ is given by Helly's formula [88]. Then, *macroscopic* features of overacceleration and the related metastability of free flow at a bottleneck with respect to an $F \rightarrow S$ transition have been found, which explain empirical nucleation nature of the $F \rightarrow S$ transition at the bottleneck satisfactory. However, by a *microscopic* study of overacceleration with the use of this simple model (1)–(4) we have found (not shown) that at some model parameters there can be overacceleration oscillations in the vicinity of the space-gap boundary $g = g_{\text{safe}}$ when the speed $v \geq v_{\text{syn}}$. Because the objective of this paper is to find microscopic features of the effect of overacceleration on traffic, rather than space-gap independent coefficient of overacceleration α used in simulations of Ref. [87], in this paper we have used a more realistic gap-dependent function $\alpha = \alpha(g)$ (5) (see below). It should be emphasized that the use of the function $\alpha = \alpha(g)$ (5) changes qualitatively no results found in Ref. [87].

⁶In Ref. [87], safety vehicle acceleration $a_{\text{safety}}(g, v, v_\ell)$ in Eq. (3) has been called “safety vehicle deceleration”. The choice of this term in Ref. [87] should emphasize that when at small space gaps $g < g_{\text{safe}}$ a vehicle should decelerate, then the value of this vehicle deceleration must prevent collisions between vehicles. However, under condition $g < g_{\text{safe}}$ and at a large enough positive value $\Delta v > 0$ it can also occur that $a_{\text{safety}}(g, v, v_\ell) > 0$, i.e., the vehicle accelerates. For this reason, here and furthermore for $a_{\text{safety}}(g, v, v_\ell)$ in Eq. (3) we use

C. Vehicle acceleration at large space gaps

In Ref. [87], to prove the metastability of free flow at a bottleneck through the effect of overacceleration without vehicle overdeceleration, for simplification of Eq. (2) we have set a_G as constant value: $a_G = a_{\max}$.⁹ However, to develop a microscopic theory of the effect of vehicle overacceleration on traffic flow, the model should be able to capture a variety of real traffic scenarios. Therefore, we get

$$a_G(g, v, v_\ell) = a_{OA} + K_1(g - G) + K_2\Delta v, \quad (6)$$

where a_{OA} is given by Eq. (4) at $\alpha(g) = \alpha_0$; K_1 and K_2 are positive dynamic coefficients. Note that the two last terms on the right-hand side of Eq. (6) are related to the well-known Helly's formula [88]. In particular, through the use of the term $K_2\Delta v$ of Helly's formula we take into account situations at which a vehicle that initial gap $g > G$ approaches a slower moving preceding vehicle.¹⁰

D. Safety vehicle acceleration

Equation (3) describes safety vehicle acceleration that should prevent collisions between vehicles at small space gaps $g < g_{\text{safe}}$. There are many concepts developed in standard traffic flow models [1–12, 15–25, 34–37, 39–45, 88] that can be used for safety vehicle acceleration $a_{\text{safety}}(g, v, v_\ell)$. In Ref. [87], Helly's formula [88] has been used for safety vehicle acceleration. However, from the analysis of Helly's formula made in Ref. [68], we know that in some traffic situations at a large enough negative speed difference Δv , specifically by lane changing at low enough vehicle speeds, vehicle collisions can be possible.¹¹

To avoid vehicle collisions, for safety acceleration $a_{\text{safety}}(g, v, v_\ell)$ in Eq. (3) we apply ideas of dynamic breaking strategies of General Motors car-following model of Herman, Gazis, Montroll, Potts, Rothery, and Chandler [1–3, 33], in which at $\Delta v < 0$ vehicle deceleration is proportional to the term

$$\frac{v\tau_{\text{safe}}}{g}\Delta v, \quad (7)$$

⁹To avoid possible problems with a considerable drop in acceleration at the boundary $g = G$ of the indifferent zone that can occur when space gap decreases continuously beginning from large gaps $g > G$, for simulations of the occurrence of the metastability of free flow at the bottleneck through the effect of overacceleration in Ref. [87] we have chosen a long enough synchronization time-headway τ_G at which no drop-effect in acceleration at the boundary $g = G$ is realized at model parameters used in Ref. [87].

¹⁰It should be noted that in three-phase traffic theory [62, 63] synchronization gap G has been defined as a space-gap, at which a vehicle that approaches a slower moving preceding vehicle begins to decelerate to the speed of the preceding vehicle. For this reason, in stochastic microscopic models of Ref. [69] at $\Delta v < 0$ the value G is the larger, the larger the value $|\Delta v|$. For model simplification, in the paper we use synchronization gap G in Eqs. (1)–(4) that does not depend on Δv . Instead, at a large enough negative value Δv from Eq. (6) it follows that the vehicle can begin to decelerate at $g > G$.

¹¹See footnote 19 of Ref. [68].

where τ_{safe} is a safe-time headway. This idea of the GM model [1–3, 33] insures that the dynamic vehicle breaking should be the stronger, the lower the space gap g between vehicles is. Accordingly, we have used in Helly's model formula (7) as follows:

$$a_{\text{safety}}(g, v, v_\ell) = K_3(g - g_{\text{safe}}) + K_4(g, v, \Delta v)\Delta v, \quad (8)$$

where

$$K_4 = \begin{cases} K_4^{(1)} & \text{at } \Delta v > 0, \\ K_4^{(2)} \frac{v\tau_{\text{safe}}}{g} & \text{at } \Delta v \leq 0. \end{cases} \quad (9)$$

K_3 , $K_4^{(1)}$, and $K_4^{(2)}$ are positive dynamic coefficients.

To make vehicle deceleration without jumps, when the space gap g intersects boundaries G and g_{safe} of the indifferent zone, in Eq. (1) we apply¹²

$$K_{\Delta v} = \begin{cases} K_2 & \text{at } \Delta v > 0, \\ K_{\Delta v}^{(1)} & \text{at } \Delta v \leq 0, \end{cases} \quad (10)$$

where $K_2 = K_4^{(1)}$,

$$K_{\Delta v}^{(1)} = \left(K_2 - K_4^{(2)} \frac{v\tau_{\text{safe}}}{g} \right) f(g, v) + K_4^{(2)} \frac{v\tau_{\text{safe}}}{g}, \quad (11)$$

$$f(g, v) = \begin{cases} \frac{g - g_{\text{safe}}(v)}{G(v) - g_{\text{safe}}(v)} & \text{at } v > 0, \\ 1 & \text{at } v = 0. \end{cases} \quad (12)$$

When a vehicle accelerates from a local speed decrease to a higher speed, Eqs. (8) and (9) ensure safety vehicle acceleration behind the preceding vehicle under condition that time headway $\tau < \tau_{\text{safe}}$. It should be emphasized that in the paper we will use the term “safety acceleration” in a broader sense, i.e., also for safety acceleration from the local speed decrease in traffic flow independent of time headway to the preceding vehicle, i.e., even when $\tau \geq \tau_{\text{safe}}$.

In addition to traffic flow consisting of human-driving vehicles, the model (1)–(12) is also applicable for traffic flow consisting of automated-driving vehicles in which automated-driving vehicles move with the use of three-phase adaptive cruise control (TPACC) [72, 91].¹³ The model (1)–(12) is also applicable for mixed traffic flow in which human-driving and automated-driving vehicles are randomly distributed between each other; however, a consideration of mixed traffic flow is out of the scope of this paper.

E. Models of lane-changing and on-ramp bottleneck

We use well-known incentive lane changing rules from the right to left lane $R \rightarrow L$ (13) and from the left to right lane

¹²Contrary to Ref. [90], in model (1)–(12) no different states (like optimistic state or defensive state of Ref. [90]) are assumed in collision-free traffic dynamics governed by safety vehicle braking.

¹³Recall that the basic feature of TPACC model is the existence of the indifferent zone for car-following applied in Eq. (1). In comparison with the TPACC model of Refs. [72, 91], in the TPACC model (1)–(12) vehicle overacceleration a_{OA} (4) of Ref. [87] as well as further developments (6)–(12) of the basic model (1)–(4) are added.

$L \rightarrow R$ (14) as well as well-known safety conditions (15) (see, e.g., Ref. [92])

$$R \rightarrow L : v^+(t) \geq v_\ell(t) + \delta_1 \quad \text{and} \quad v(t) \geq v_\ell(t), \quad (13)$$

$$L \rightarrow R : v^+(t) \geq v_\ell(t) + \delta_2 \quad \text{or} \quad v^+(t) \geq v(t) + \delta_2, \quad (14)$$

$$g^+(t) \geq v(t)\tau_2, \quad g^-(t) \geq v^-(t)\tau_1, \quad (15)$$

at which a vehicle changes to the faster target lane with the objective to pass a slower vehicle in the current lane if time headway to preceding and following vehicles in the target lane are not shorter than some given safety time headway τ_2 and τ_1 . In Eqs. (13)–(15), superscripts $+$ and $-$ denote, respectively, the preceding and the following vehicles in the target lane; τ_1 , τ_2 , δ_1 , δ_2 are positive constants.¹⁴

With the objective explained in Sec. III below, we will consider microscopic features of the effect of overacceleration on traffic flow on a single-lane road and two-lane road independent each other. For this reason, there are two governing sets of model equations in the paper: (i) The model (1)–(12) for a single-lane road and (ii) the model (1)–(15) for a two-lane road, for which Eqs. (13)–(15) specifying lane-changing conditions serve as governing equations together with Eqs. (1)–(12) for vehicle motion in a road lane.

Open boundary conditions are applied. At the beginning of the road $x = 0$ vehicles are generated one after another in each of the lanes of the road at time instants $t^{(k)} = k\tau_{\text{in}}$, $k = 1, 2, \dots$, where $\tau_{\text{in}} = 1/q_{\text{in}}$, q_{in} is a given time-independent flow rate per road lane. The initial vehicle speed is equal to v_{free} . After the vehicle has reached the end of the road $x = L$ it is removed. Before this occurs, the farthest downstream vehicle maintains its speed and lane.

In the on-ramp model, there is a merging region of length L_m in the right lane that begins at location $x = x_{\text{on}}$ within which vehicles can merge from the on-ramp. Vehicles are generated at the on-ramp one after another at time instants $t^{(m)} = m\tau_{\text{on}}$, $m = 1, 2, \dots$, where $\tau_{\text{on}} = 1/q_{\text{on}}$, q_{on} is the on-ramp inflow rate. To reduce a local speed decrease occurring through the vehicle merging at the on-ramp bottleneck, vehicles merge with the speed of the preceding vehicle v^+ at a middle location $x = (x^+ + x^-)/2$ between the preceding and following vehicles in the right lane, when the space gap between the vehicles exceeds some safety value $g_{\text{target}}^{(\min)} = \lambda_b v^+ + d$, i.e., some safety condition $x^+ - x^- - d > g_{\text{target}}^{(\min)}$ should be satisfied, where λ_b is a constant. In accordance with these merging conditions, the space gap for a vehicle merging between each pair of consecutive vehicles in the right lane is checked within merging region L_m , starting from the upstream boundary of the merging region. If there is such a pair of consecutive vehicles, then the vehicle merges onto the right lane; if there is no pair of consecutive vehicles, for which the safety condition is satisfied at the current time step, then the procedure is repeated at the next time step, and so on.

¹⁴It should be noted that in Eqs. (13) and (14) the value v^+ at $g^+ > L_a$ and the value v_ℓ at $g > L_a$ are replaced by ∞ , where L_a is a look-ahead distance; in simulations, we have used $L_a = 80$ m.

Vehicle motion is found from a system of equations $a^* = \min(a, a_{\text{max}})$, $dv^*/dt = a^*$, $v = \max(0, \min(v^*, v_{\text{free}}))$ that results from conditions $0 \leq v \leq v_{\text{free}}$, and $dx/dt = v$ solved with the second-order Runge-Kutta method with time step 10^{-2} s.

In Secs. IV–IX, we use simple speed functions $G(v)$ and $g_{\text{safe}}(v)$:

$$G(v) = v\tau_G, \quad g_{\text{safe}}(v) = v\tau_{\text{safe}}, \quad (16)$$

where τ_G is a synchronization time headway. We choose dynamic coefficients K_1 – K_4 in Eqs. (6), (8), and (9) at which under conditions (16) no classical traffic flow instability and no string instability can occur. Respectively, no wide moving jams (and no cases at which $v \rightarrow 0$) can occur in synchronized flow of the model (1)–(12), (16) for a single-lane road as well as in the model (1)–(16) for a two-lane road. Emergence of wide moving jams in synchronized flow of human-driving vehicles will be considered in Sec. X.

For distinguishing between synchronized flow and wide moving jam traffic phases in simulations presented below, we use microscopic criteria for the traffic phases [63,93]. In particular, within a wide moving jam there is at least one flow interruption interval; in contrast, there is no flow interruption and, therefore, there are no flow interruption intervals within synchronized flow.

In accordance with definitions of congested patterns in three-phase traffic theory [62,63], we use the following acronyms for different synchronized flow patterns (SP): a localized SP (LSP), a moving SP (MSP), and a widening SP (WSP).

III. METHODOLOGY OF MICROSCOPIC PHYSICAL MODELING OF OVERACCELERATION

In the methodology of physical modeling of microscopic features of overacceleration, we analyze different possible overacceleration mechanisms independent of each other as follows:

(i) We study overacceleration on a single-lane road and overacceleration caused by lane-changing on a two-lane road independent of each other.

(ii) To study whether in addition to overacceleration (4) there are other overacceleration mechanisms in a road lane, in the model (1)–(12), (16) for a single-lane road we neglect overacceleration mechanism (4), i.e., in Eq. (4) we get

$$\alpha = 0. \quad (17)$$

(iii) Through different choices of model parameters in the simplified model (1)–(12), (16), (17), a searching for the minimization in the suppression of overacceleration through speed adaptation has been performed. A comparison of simulations of the model (1)–(12), (16), (17) with simulations of Helly's model has led to the disclosing of microscopic features of competition between speed adaptation and overacceleration in a road lane; in this way, we have made the distinguishing between speed adaptation and overacceleration in the absence of lane changing.

(iv) Through a decrease in the suppression of overacceleration due to speed adaptation of item (iii), we have revealed the overacceleration mechanism caused by safety acceleration.

This overacceleration mechanism occurs without explicitly introducing it in the model.

(v) We consider the model (1)–(12), (16) on a single-lane road in which overacceleration (4) is applied. This has led to the revealing of cooperation of overacceleration (4) with overacceleration caused by safety acceleration.

(vi) To study a possible effect of speed adaptation and overacceleration in a road lane on overacceleration due to lane changing, we have applied the methodology of items (ii)–(v) for the model (1)–(16) of vehicle motion on a two-lane road.

(vii) The decrease in the suppression effect of speed adaptation on overacceleration together with the separate consideration of different overacceleration mechanisms [items (ii)–(vi)] have been applied to study the effect of overacceleration on highway capacity at the bottleneck for both single-lane and two-lane roads.

IV. OVERACCELERATION CAUSED BY SAFETY ACCELERATION

In accordance with the methodology of a study of overacceleration (Sec. III), in this section we consider traffic on a single-lane road under condition (17).

A. No nucleation nature of $F \rightarrow S$ transition—No overacceleration

First, we consider a case in which under condition (17) safety acceleration behavior does not act as overacceleration (Fig. 2): There is no acceleration behavior that could cause the free flow metastability with respect to traffic breakdown at the bottleneck. Indeed, in Fig. 2, contrary to empirical data, traffic breakdown does not exhibit the nucleation nature.

To explain this, we consider Fig. 3. After vehicle on-1 has merged from the on-ramp onto the main road [Fig. 2(d)], time-headway of vehicle 1 following vehicle on-1 on the main road becomes shorter than safe-time headway τ_{safe} [Fig. 3(b)]. Therefore, according to Eqs. (8) and (9), vehicle 1 should decelerate strongly [“1, speed adaptation” in Fig. 3(a)]. The speed decrease of vehicle 1 due to its speed adaptation results in deceleration of following vehicles 2 and 3 [Fig. 3(a)]. Deceleration of vehicles 2 and 3 is realized within the indifferent zone for car-following $g_{\text{safe}} \leq g \leq G$ [$\tau_{\text{safe}} \leq \tau \leq \tau_G$ in Fig. 3(b)] in accordance with equation

$$a = K_{\Delta v} \Delta v \quad \text{at } g_{\text{safe}} \leq g \leq G, \quad (18)$$

resulting from Eqs. (1), (4), and (17). Eq. (18) describes vehicle speed adaptation to the speed of the preceding vehicle within the indifferent zone: After vehicle 1 begins to decelerate, following vehicle 2 decelerates while adapting its speed to the speed of vehicle 1; the deceleration of vehicle 2 leads to the deceleration of vehicle 3, and so on. Locations of the minimum speed of vehicles 2, 3, ... (e.g., $v_{\text{min}}^{(2)}$ and $v_{\text{min}}^{(3)}$) are *upstream* of the location of the minimum speed of vehicle 1 [$v_{\text{min}}^{(1)}$ in Fig. 3(a)]. This explains the occurrence of the wave of a speed decrease propagating upstream of the bottleneck [Fig. 2(a)]; at a given q_{in} , the larger the on-ramp inflow rate q_{on} , the lower the speed in this congested traffic [Figs. 2(a)–2(c)]. Such spontaneous propagation congestion upstream of the on-ramp bottleneck is well-known in stan-

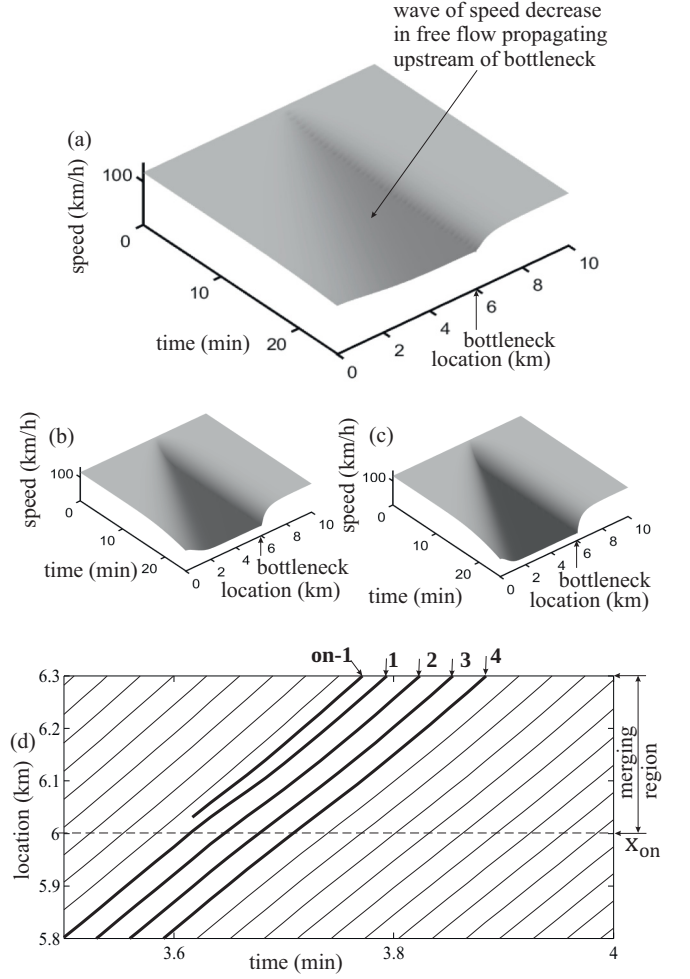


FIG. 2. Explanation to “no nucleation nature of $F \rightarrow S$ transition – no overacceleration.” Simulations of model (1)–(12), (16), (17) on a single-lane road with on-ramp bottleneck at $q_{\text{in}} = 2000$ vehicles/h and different values of on-ramp inflow rate q_{on} at coefficient of overacceleration $\alpha = 0$, i.e., in Eqs. (1) and (6) $a_{\text{OA}} = 0$. (a)–(c) Speed in space and time: (a) $q_{\text{on}} = 100$ vehicles/h, (b) $q_{\text{on}} = 350$ vehicles/h, (c) $q_{\text{on}} = 500$ vehicles/h. (d) Simulated vehicle trajectories in some time interval related to panel (a). Other model parameters: $v_{\text{free}} = 120$ km/h, $\tau_{\text{safe}} = 1$ s, $\tau_G = 2$ s, $v_{\text{syn}} = 80$ km/h, $d = 7.5$ m, $a_{\text{max}} = 2.5$ m/s², $K_1 = 0.3$ s⁻², $K_2 = K_4^{(1)} = 0.6$ s⁻¹, $K_3 = 0.5$ s⁻², $K_4^{(2)} = 1$ s⁻¹, $L = 10$ km, $x_{\text{on}} = 6$ km, $\lambda_b = 0.2$ s, $L_m = 0.3$ km.

dard traffic flow theory.¹⁵ In accordance with overacceleration definition (Sec. I), in the model used in Fig. 2, in which no nucleation nature of traffic breakdown is realized, safety acceleration [labeled by “1, acceleration” in Fig. 3(a)] cannot be considered overacceleration.

¹⁵Although traffic flow model (1)–(12), (16), (17) used in simulations presented in Fig. 2 is qualitatively different from the classical Lighthill-Whitham-Richards (LWR) model of traffic breakdown [94–98], we have qualitative the same theoretical conclusion about the occurrence of traffic congestion at the bottleneck: As the model (1)–(12), (16), (17) at model parameters used in Fig. 2, the LWR model cannot explain the empirical nucleation nature of traffic breakdown at the bottleneck [62–66].

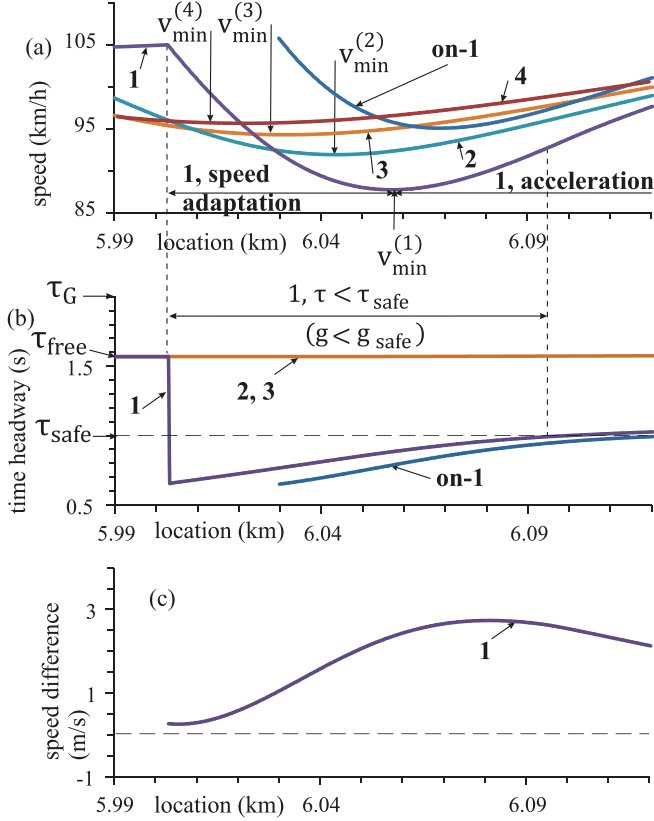


FIG. 3. Continuation of Fig. 2(d). (a), (b) Location functions of speed (a) and time-headway (b) of some vehicles. (c) Location function of speed difference $\Delta v = v^{(\text{on-1})} - v^{(1)}$ between speeds of vehicles on-1 and 1. Vehicle numbers are the same as those in Fig. 2(d), respectively. In (b), $\tau = \tau_{\text{free}}$ is the average time-headway between vehicles in free flow moving at the maximum speed v_{free} .

B. Overacceleration under decrease in time-headway between vehicles in free flow

We consider the same model (1)–(12), (16), (17) as used in Fig. 2. However, we decrease time-headway between vehicles in free flow through an increase in the flow rate on the main road q_{in} . Then, rather than the speed wave propagating *upstream* of the bottleneck [Fig. 2(a)], the wave of a speed decrease propagating *downstream* of the bottleneck occurs [Fig. 4(a)]. As a result, free flow remains upstream of the bottleneck. Moreover, if in simulations shown in Fig. 4(a) at some time instant T_{ind} we apply a short-time impulse of addition large enough on-ramp inflow Δq_{on} , then synchronized flow propagating upstream has been induced in the initial free flow at the bottleneck [Fig. 4(b)]. Thus, contrary to Fig. 2, in Fig. 4 free flow becomes metastable with respect to the $F \rightarrow S$ transition.

To understand the crucial difference between Figs. 2 and 4, first we note that there are common features in behaviors of vehicles on-1 and 1 following each other [Figs. 3 and 5]: After vehicle on-1 has merged from the on-ramp [Figs. 2(d) and 4(c)], space-gap of vehicle 1 becomes smaller than safe space-gap g_{safe} (i.e., time-headway is shorter than τ_{safe}) [Figs. 3(b) and 5(b)]. Therefore, according to Eqs. (8) and (9), vehicle 1 should decelerate strongly [“1, speed adaptation” in Figs. 3(a)

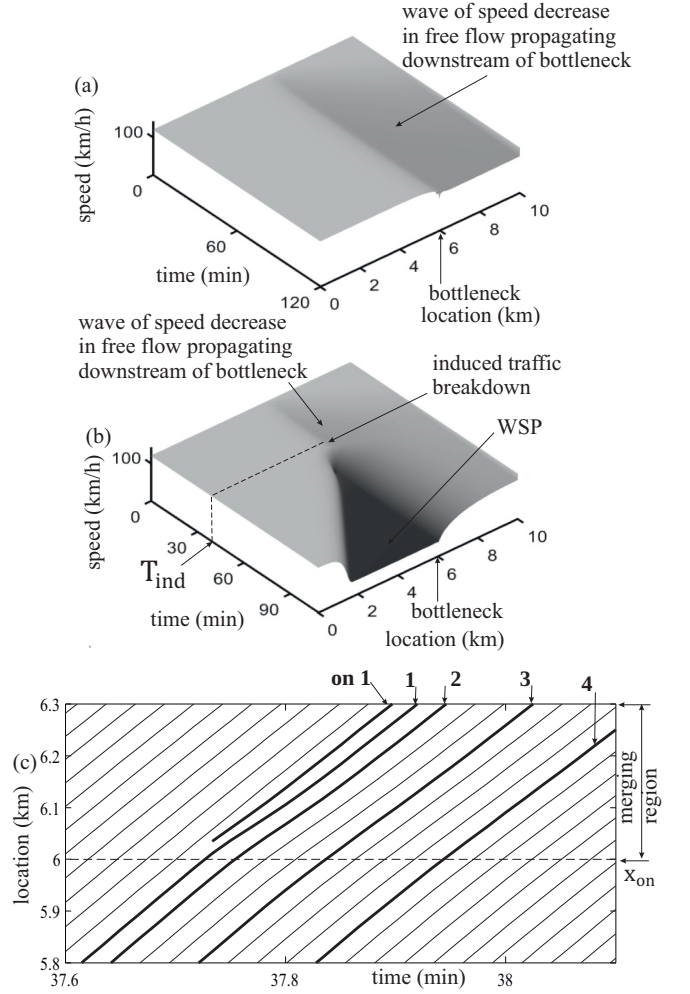


FIG. 4. Overacceleration on a single-lane road with on-ramp bottleneck under decrease in time-headway τ_{free} in free flow upstream of on-ramp bottleneck. Simulations of the model (1)–(12), (16), (17) at model parameters $K_4^{(2)} = 0.8 \text{ s}^{-1}$, $q_{\text{in}} = 2250 \text{ vehicles/h}$, $q_{\text{on}} = 35 \text{ vehicles/h}$. Other model parameters are the same as those in Fig. 2. (a), (b) Speed in space and time. In panel (b), traffic breakdown has been induced in free flow at $T_{\text{ind}} = 40 \text{ min}$ through application of addition on-ramp inflow impulse $\Delta q_{\text{on}} = 465 \text{ vehicles/h}$ of duration $\Delta t = 1 \text{ min}$. (c) Some of simulated vehicle trajectories within local speed decrease in free flow at bottleneck at time $t < T_{\text{ind}}$.

and 5(a)]. Shortly later, although condition $g < g_{\text{safe}}$ is still satisfied, this braking of vehicle 1 changes to its acceleration at the location of the minimum speed $v_{\min}^{(1)}$ of vehicle 1 [Figs. 3(a) and 5(a)]. This safety acceleration of vehicle 1 occurs under condition $g < g_{\text{safe}}$ because the speed difference $\Delta v = v^{(\text{on-1})} - v^{(1)}$ between vehicles on-1 and 1 becomes a large enough positive value [Figs. 3(c) and 5(c)] leading to condition

$$a_{\text{safety}} = K_3(g - g_{\text{safe}}) + K_4^{(1)}\Delta v > 0 \quad \text{at } g < g_{\text{safe}}. \quad (19)$$

The crucial difference between Figs. 2 and 4 is associated with behaviors of vehicles 2, 3, and 4 that follow vehicle 1 on the main road [Figs. 3(a) and 5(a)]. To understand the difference in motion of vehicles 2, 3, and 4 in Fig. 3(a) in comparison with Fig. 5(a), we compare the average space-gap

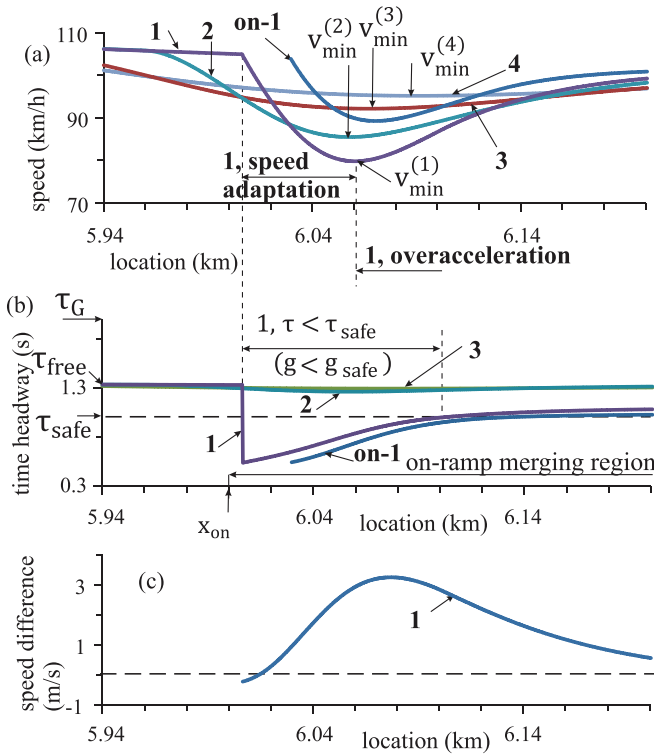


FIG. 5. Continuation of Fig. 4(c). (a), (b) Location functions of speed (a) and time-headway (b) of some vehicles. (c) Location function of speed difference $\Delta v = v^{(on-1)} - v^{(1)}$ between speeds of vehicles on-1 and 1. Vehicle numbers are labeled by the same numbers as those in Fig. 4(c).

$g = g_{free}$ and time-headway $\tau = \tau_{free}$ between vehicles in free flow moving at the maximum speed v_{free} , which are found from formulas $g_{free} = \frac{v_{free}}{q_{in}} - d$ and $\tau_{free} = g_{free}/v_{free}$, respectively.

In Fig. 4, the flow rate q_{in} is greater than it is in Fig. 2. Therefore, time headway τ_{free} in Fig. 5(b) is shorter than it is in Fig. 3(b). When vehicle 1 in Fig. 3(a) brakes sharply, then following vehicle 2 due to a longer time headway τ_{free} finishes its speed adaptation, i.e., it reaches the minimum speed $v_{min}^{(2)}$ [Fig. 3(a)] further upstream from the location of the minimum speed $v_{min}^{(1)}$ of vehicle 1. The same effect is realized for following vehicle 3: In Fig. 3(a), the location of the minimum speed $v_{min}^{(3)}$ is upstream from the location of the minimum speed $v_{min}^{(2)}$ of vehicle 2. Thus, in Fig. 3(a) the locations of the minimum speeds of vehicles 2–4 are upstream of the location of the minimum speed $v_{min}^{(1)}$ of vehicle 1. This leads to upstream congestion propagation discussed in Sec. IV A.

Contrarily, due to shorter τ_{free} , vehicle 3 in Fig. 5(a) finishes its speed adaptation, i.e., it reaches the minimum speed $v_{min}^{(3)}$ even slightly downstream of the location of the minimum speed $v_{min}^{(2)}$ of vehicle 2. The same conclusion is valid for vehicle 4. For this reason, in Fig. 5(a) the locations of the minimum speed $v_{min}^{(3)}$ and $v_{min}^{(4)}$ of vehicles 3 and 4, respectively, are *downstream* of the location of the minimum speed $v_{min}^{(1)}$ of vehicle 1. Moreover, the distances between the location of the minimum speed $v_{min}^{(1)}$ of vehicle 1 [Fig. 5(a)] and locations of the minimum speeds of the following vehicles increase with

the vehicle number [$v_{min}^{(3)}$ and $v_{min}^{(4)}$ in Fig. 5(a)]. This explains the occurrence of the wave of a speed decrease propagating downstream of the bottleneck [Fig. 4(a)]. This wave results from safety acceleration of vehicle 1 described by Eq. (19).

There is no free flow metastability in Fig. 2. In Sec. IV A we have already stated that in Fig. 2, in which no nucleation nature of traffic breakdown is realized, safety acceleration [“1, acceleration” in Fig. 3(a)] cannot be considered overacceleration. Contrarily, in Fig. 4(b) free flow is in a metastable state with respect to the $F \rightarrow S$ transition at the bottleneck. Thus, in accordance with the overacceleration definition, safety acceleration should be considered overacceleration [“1, overacceleration” in Fig. 5(a)].

Thus, the qualitative difference between the two cases [Figs. 2 and 4(b)] is the different suppression effect of speed adaptation on the tendency to free flow through safety acceleration: In Fig. 2, the speed wave caused by speed adaptation moves upstream of the bottleneck. In this case, speed adaptation suppresses fully the tendency to free flow through safety acceleration. The effect of speed adaptation is that safety acceleration is not able to maintain free flow at the bottleneck: No free flow metastability and, therefore, no overacceleration occur.

In contrast with Fig. 2(a), in Fig. 4(a) the speed wave caused by speed adaptation moves downstream of the bottleneck. In this case, safety acceleration is able to maintain free flow causing free flow metastability with respect to the $F \rightarrow S$ transition at the bottleneck; therefore, safety acceleration should be considered overacceleration. The question about a flow rate-range within which safety acceleration is able to maintain free flow at the bottleneck will be answered in Sec. VIII A.

C. Overacceleration under increase in time-headway between vehicles in free flow above synchronization time-headway

Under condition (17), in the model (1)–(12), (16) at the same flow rates q_{in} and q_{on} , respectively, as those in Fig. 2(b), the overacceleration mechanism caused by safety acceleration has also been found (Fig. 6). This overacceleration mechanism is realized under condition

$$g_{free} > G, \text{ i.e., } \tau_{free} > \tau_G. \quad (20)$$

Indeed, under condition (20) rather than congestion propagation upstream of the bottleneck [Fig. 2(b)], free flow remains at the bottleneck. In this free flow, only a small speed decrease localized at the bottleneck is realized [Fig. 6(a)]. This free flow is in a metastable state with respect to an $F \rightarrow S$ transition at the bottleneck [Fig. 6(b)]. To understand the physics of this overacceleration effect, first we should note that there are common features in behaviors of vehicles on-1 and 1 in Figs. 3 and 7. These common features are exactly the same as those explained above in Sec. IV B when we have compared Figs. 3 and 5. In particular, under condition $g < g_{safe}$ safety acceleration of vehicle 1 [Figs. 7(a) and 7(b)] occurs in accordance with Eq. (19).

There is the following crucial difference between the case under consideration (Fig. 6) and the cases studied in Secs. IV A and IV B [Figs. 2 and 4]: In Figs. 2 and 4, deceleration and acceleration of the vehicles 2 and 3 following vehicle

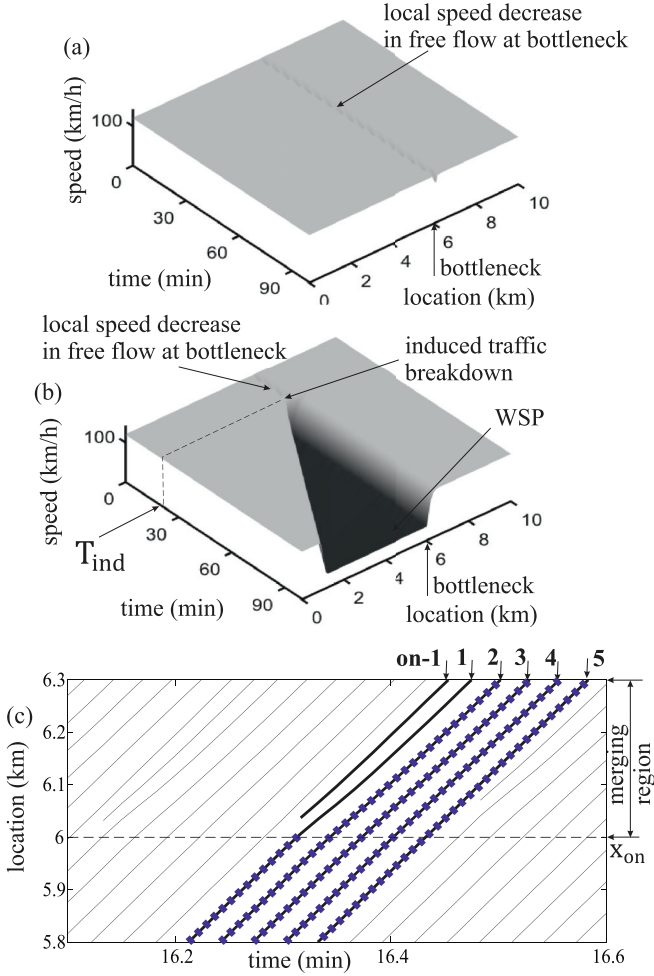


FIG. 6. Induced traffic breakdown on a single-lane road with on-ramp bottleneck under condition (20). Simulations of model (1)–(12), (16), (17) at $\tau_G = 1.4$ s, $q_{in} = 2000$ vehicles/h, and $q_{on} = 350$ vehicles/h. Other model parameters are the same as those in Fig. 2. (a), (b) Speed in space and time: Local speed decrease at bottleneck in free flow (a) and WSP (b) induced in free flow in panel (a) at $T_{ind} = 20$ min through application of addition on-ramp inflow impulse $\Delta q_{on} = 250$ vehicles/h of duration $\Delta t = 1$ min. (c) Some simulated vehicle trajectories at bottleneck at time $t < T_{ind}$; parts of trajectories within which condition $g > G$ is satisfied are marked through colored blue rectangles.

1 are realized within the indifferent zone for car-following $g_{safe} \leq g \leq G$ [region $\tau_{safe} \leq \tau \leq \tau_G$ in Figs. 3(b) and 5(b)], i.e., it corresponds to solutions of Eq. (18). Contrarily, under condition (20) for which Fig. 6 is valid, vehicles 2–5 following vehicle 1 [Figs. 6(c) and 7(b)] decelerate and accelerate in accordance with Eqs. (2), (6), (16), and (17) corresponding to solutions of equation

$$a_G = K_1(g - v\tau_G) + K_2\Delta v \quad \text{at } g > v\tau_G. \quad (21)$$

When vehicle 1 decelerates [“1, speed adaptation” in Fig. 7(a)], then both Eqs. (18) and (21) describe speed adaptation of following vehicles to the speed of vehicle 1. However, as long as condition $g > v\tau_G$ is satisfied, due to positive term $K_1(g - v\tau_G)$ in Eq. (21), at the same negative speed difference Δv in Eqs. (18) and (21) the deceleration and respective speed

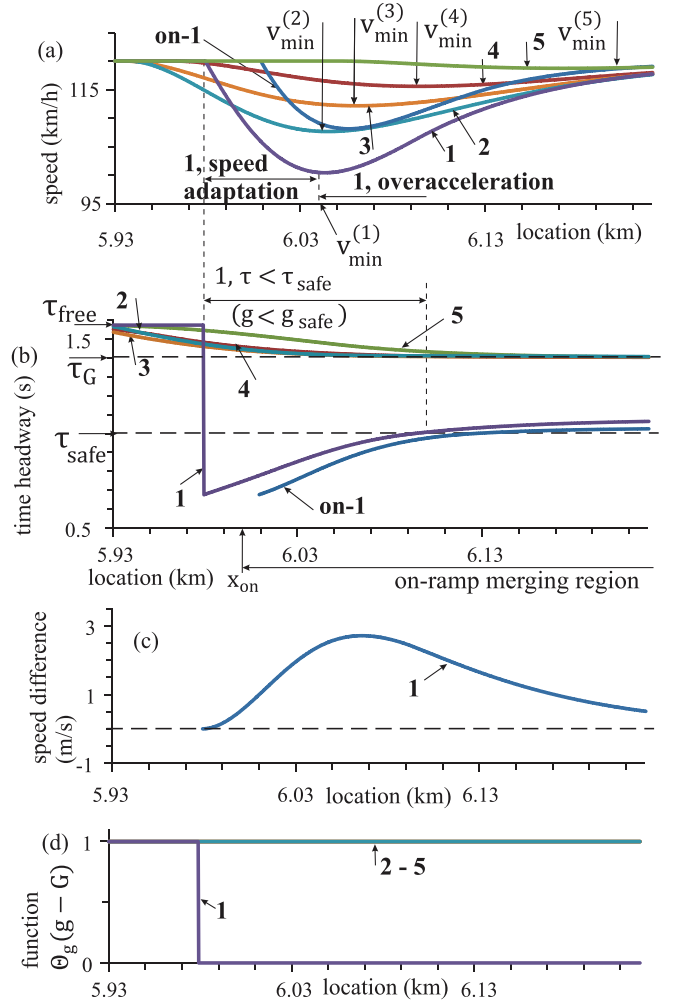


FIG. 7. Continuation of Fig. 6(c). (a), (b) Location functions of speed (a) and time-headway (b) of some vehicles. (c) Location function of speed difference $\Delta v = v^{(on-1)} - v^{(1)}$ between speeds of vehicles on-1 and 1. (d) Dependencies of function $\Theta_g(g - G)$ on road location for vehicles 1–5, where $\Theta_g(z) = 0$ at $z < 0$ and $\Theta_g(z) = 1$ at $z \geq 0$. Vehicle numbers are labeled by the same numbers as those in Fig. 6(c).

decrease of vehicles 2–5 in Fig. 7(a) are noticeably lower than those of vehicles 2–4 in Fig. 3(a).

As mentioned in Sec. IV A, a larger speed decrease of vehicles 2–4 in Fig. 3(a) leads to the wave of a speed decrease propagating *upstream* of the bottleneck [Fig. 2(a)]. Contrarily, due to a weaker deceleration (21), vehicles 2–5 in Fig. 7(a) reach the bottleneck while moving still in free flow, i.e., free flow remains at the bottleneck. Deceleration of vehicles 2–5 causes only a wave of a low speed decrease propagating *downstream* of the bottleneck. Indeed, the locations of the minimum speed of vehicles 2–5 [$v_{min}^{(2)}$, $v_{min}^{(3)}$, $v_{min}^{(4)}$, and $v_{min}^{(5)}$ in Fig. 7(a), respectively] following vehicle 1 are downstream of the location of the minimum speed $v_{min}^{(1)}$ of vehicle 1 [Fig. 7(a)]; however, contrary to Fig. 4(a), due to the weak deceleration of vehicles 2–5 the speed wave disappears almost fully at a short distance downstream of the beginning of the on-ramp [Fig. 7(a)]. For this reason, a speed decrease caused by vehicle merging from the on-ramp can be considered as a

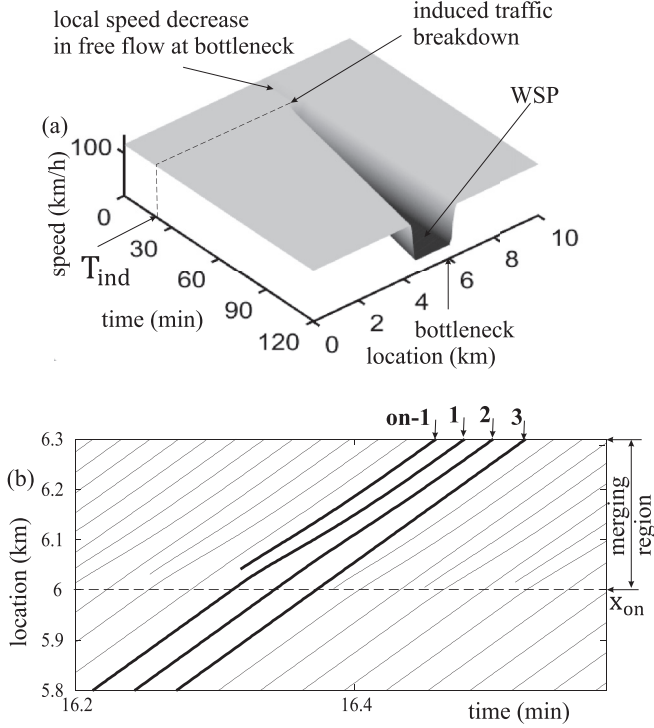


FIG. 8. Overacceleration through safely acceleration in ACC model (22), (23) on a single-lane road with on-ramp bottleneck at model parameters $\tau_d = 1$ s; $q_{in} = 2000$ vehicles/h, $q_{on} = 920$ vehicles/h; $K_{ACC,1} = 0.3$ s $^{-2}$ and $K_{ACC,2}^{(0)} = 0.9$ s $^{-1}$. (a) Speed in space and time: Local speed decrease at bottleneck in free flow and WSP induced in free flow at $T_{ind} = 20$ min through application of addition on-ramp inflow impulse $\Delta q_{on} = 280$ vehicles/h of duration $\Delta t = 2$ min. (b) Some of simulated vehicle trajectories at the bottleneck at time $t < T_{ind}$. Other model parameters are the same as those in Fig. 2.

local speed decrease at the bottleneck [“local speed decrease in free flow at bottleneck” in Fig. 6(a)].

In Fig. 6, contrary to Fig. 2, safety acceleration does cause the metastability of free flow with respect of $F \rightarrow S$ transition at the bottleneck. Thus, due to condition (20), safety acceleration becomes overacceleration [“1, overacceleration” in Fig. 7(a)].

D. Safety acceleration as origin of overacceleration in traffic consisting of ACC vehicles moving on a single-lane road

In Ref. [68], it has been found that in traffic consisting of 100% automated-driving ACC vehicles moving on a single-lane road in accordance with Helly’s model there is the free flow metastability with respect to the $F \rightarrow S$ transition at the bottleneck (Sec. IV.C.2 and Fig. 30 of Ref. [68]). This means that should be some overacceleration mechanism in a road lane for the ACC model. However, in Ref. [68] the origin of the overacceleration mechanism has not been disclosed. Based on results of Sec. IV C, here we reveal that the cause of this ACC vehicle overacceleration is safety acceleration [Figs. 8 and 9].

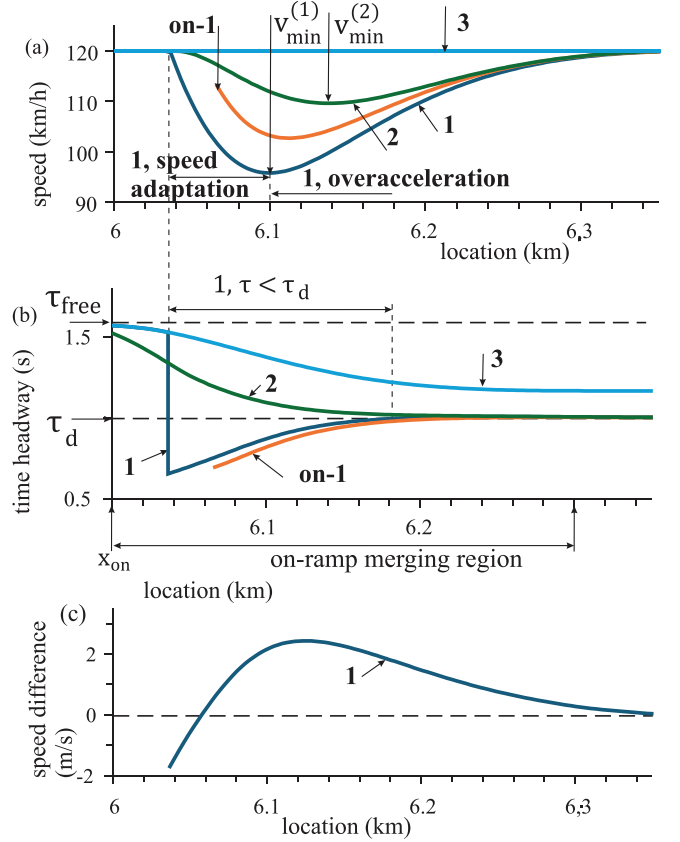


FIG. 9. Continuation of Fig. 8(b). (a), (b) Location functions of speed (a) and time-headway (b) of some vehicles that numbers are, respectively, the same as those in Fig. 8(b). (c) Location function of speed difference $\Delta v = v^{(on-1)} - v^{(1)}$ between speeds of vehicles on-1 and 1.

The well-known classical ACC model (see, e.g., Refs. [46–57]) is described by Helly’s formula [88]:

$$a(g, v, v_\ell) = K_{ACC,1}(g - v\tau_d) + K_{ACC,2}\Delta v, \quad (22)$$

where τ_d is a desired time headway of the ACC vehicle to the preceding vehicle; $K_{ACC,1}$ and $K_{ACC,2}$ are dynamic coefficients of ACC vehicle adaptation. Following formula (9), to avoid possible vehicle collisions at very low speeds, we choose $K_{ACC,2} = K_{ACC,2}(g, v, \Delta v)$:

$$K_{ACC,2}(g, v, \Delta v) = \begin{cases} K_{ACC,2}^{(0)} \frac{v\tau_d}{g} & \text{at } \Delta v \leq 0 \text{ and } g < v\tau_d, \\ K_{ACC,2}^{(0)} & \text{otherwise,} \end{cases} \quad (23)$$

$K_{ACC,2}^{(0)}$ is a constant (Fig. 8).¹⁶ In all simulations of ACC vehicles presented in the paper, we have chosen values of

¹⁶We have also simulated the ACC model (22) with the same constant values $K_{ACC,1} = 0.3$ s $^{-2}$ and $K_{ACC,2} = 0.9$ s $^{-1}$ as those in Ref. [68] at which condition for string stability [49] is satisfied. We have found that at chosen model parameters no moving jams occur in synchronized flow and the term $v\tau_d/g$ in Eq. (23) does not influence results noticeably. For this reason, simulations of the ACC model (22), (23) presented in the paper can be applied for a comparison with those found in Ref. [68] for both single-lane and two-lane roads.

$K_{\text{ACC},1}$ and $K_{\text{ACC},2}^{(0)}$ at which no string instability can occur (Fig. 8).

The ACC model (22), (23) can be considered the result of a transformation of the three-phase traffic flow model (1)–(12) in which vehicle motion within the indifferent zone for car-following, i.e., Eq. (1) is removed, overacceleration $a_{\text{OA}} = 0$ (17), and $\tau_{\text{safe}} = \tau_d$.

For the ACC model (22), (23), condition (20) of Sec. IV C transforms to $\tau_{\text{free}} > \tau_d$ that is satisfied. As in Fig. 7(a), after vehicle 1 has braked strongly due to on-ramp merging vehicle on-1 [“1, speed adaptation” in Fig. 9(a)], the tendency to free flow is realized through safety acceleration of vehicle 1 [Figs. 9(a) and 9(c)]. As in the model (1)–(12), (16), (17) [Fig. 6(b)], in the ACC model (22), (23) safety acceleration of vehicle 1 does cause the metastability of free flow with respect of the $F \rightarrow S$ transition at the bottleneck [Fig. 8(a)]. Thus, safety acceleration in ACC model (22), (23) becomes overacceleration [“1, overacceleration” in Fig. 9(a)].

However, there is the following important difference between the ACC model (22), (23) and the model (1)–(12), (16), (17). In the ACC model (22), (23), the indifference zone for car-following is absent. Therefore, in the ACC model (22), (23) there is also no speed adaptation caused by the indifference zone of the model (1)–(12), (16), (17). For this reason, the effect of the suppression of overacceleration through speed adaptation in the ACC model (22), (23) is considerably weaker than that in the model (1)–(12), (16), (17). Indeed, after vehicle 1 has braked strongly [“1, speed adaptation” in Fig. 9(a)], contrary to results of Figs. 3(a), 5(a), and 7(a), for vehicles following vehicle 1 speed adaptation is realized only for vehicle 2 [Fig. 9(a)]: Already next vehicle 3 following vehicle 2 continues to move at the maximum speed $v = v_{\text{free}}$ through the bottleneck location. The weak suppression effect of speed adaptation on overacceleration in the ACC model (22), (23) leads to a large value of highway capacity at the bottleneck in traffic consisting of ACC vehicles (Sec. VIII B).

E. Microscopic behaviors of vehicle motion explaining competition of speed adaptation with safety acceleration

In Figs. 3, 5, 7, and 9 we have shown that vehicle 1, which must break while approaching slower on-ramp vehicle on-1, accelerates later through safety acceleration. This reflects the vehicle’s desire to move in free flow. However, the breaking of vehicle 1 forces vehicles following vehicle 1 to decelerate; this speed adaptation is the opposite effect to safety acceleration.

When for any vehicle speed within the local speed decrease at the bottleneck speed adaptation is on average stronger than safety acceleration, then safety acceleration cannot prevent congested traffic at the bottleneck. This is independent of the speed within the local speed decrease at the bottleneck (Fig. 2). In this case, safety acceleration does not lead to a nucleation character of traffic breakdown at the bottleneck and, therefore, safety acceleration cannot be considered overacceleration (Sec. IV A).

At other traffic parameters (Secs. IV B–IV D) speed adaptation is on average weaker than safety acceleration at high

enough speeds only, whereas speed adaptation becomes on average stronger than safety acceleration in synchronized flow.

In particular, in Fig. 8(a) after the $F \rightarrow S$ transition has occurred at the bottleneck the WSP has occurred that is localized at the bottleneck. In synchronized flow of the WSP, as found in simulations, ACC vehicles move on average at time headway $\tau = \tau_d$. This time headway between vehicles is considerably shorter than $\tau = \tau_{\text{free}}$ in free flow [Fig. 9(b)]. For this reason, contrary to free flow, speed adaptation within the WSP is strong. The strong speed adaptation suppresses the tendency to free flow through safety acceleration. This explains why in traffic flow of classical ACC vehicles at a given flow rate q_{in} either free flow with long time headway $\tau = \tau_{\text{free}}$ between ACC vehicles, or the WSP with short time headway $\tau = \tau_d$ between ACC vehicles can exist at the bottleneck at the same on-ramp inflow rate q_{on} [Fig. 8(a)].

Thus, in the cases considered in Secs. IV B–IV D safety acceleration does prevent congested traffic at the bottleneck at high enough speeds within the local speed decrease at the bottleneck. Safety acceleration does lead to the nucleation character of traffic breakdown ($F \rightarrow S$ transition) at the bottleneck. In these cases, in accordance with overacceleration definition, safety acceleration is overacceleration. Contrarily, at low synchronized flow speeds within the WSP at the bottleneck speed adaptation is on average considerably stronger than safety acceleration. This suppresses the tendency to free flow through safety acceleration within the WSP [Figs. 4(b), 6(b), and 8(a)].

V. COOPERATION OF DIFFERENT MECHANISMS OF OVERACCELERATION

We consider the model (1)–(12), (16) on a single-lane road when condition (17) used in Sec. IV is not applied. As in Ref. [87], we have found that the application of overacceleration mechanism given by Eqs. (4) and (5) in the model (1)–(12), (16) leads to the free flow metastability with respect to an $F \rightarrow S$ transition (traffic breakdown) at the bottleneck. The behavioral origin of the discontinuous character of overacceleration a_{OA} (4) has already been explained in Ref. [87]. Here we have found that there can be a cooperation of the overacceleration mechanism (4) of Ref. [87] with the overacceleration mechanism due to safety acceleration revealed in Sec. IV C [Figs. 10 and 11].

A. Cooperation of overacceleration a_{OA} with overacceleration caused by safety acceleration

First, overacceleration due to safety acceleration of vehicle 1 [“1, overacceleration” in Fig. 11(a)] following on-ramp vehicle on-1 occurs. This overacceleration mechanism is the same as explained in Sec. IV C. Later, as can be seen from Figs. 11(b) and 11(c), overacceleration cooperation is realized due to addition overacceleration mechanism a_{OA} (4), (5) [labeled by “1, change of overacceleration” in Fig. 11(a)]. Overacceleration a_{OA} is particular large for vehicles 2 and 3 that follow vehicle 1 [Figs. 11(b) and 11(c)]. In other words, overacceleration cooperation can occur over time [time-sequence of the effects of different overacceleration mechanisms on motion of vehicle 1 and the effect of

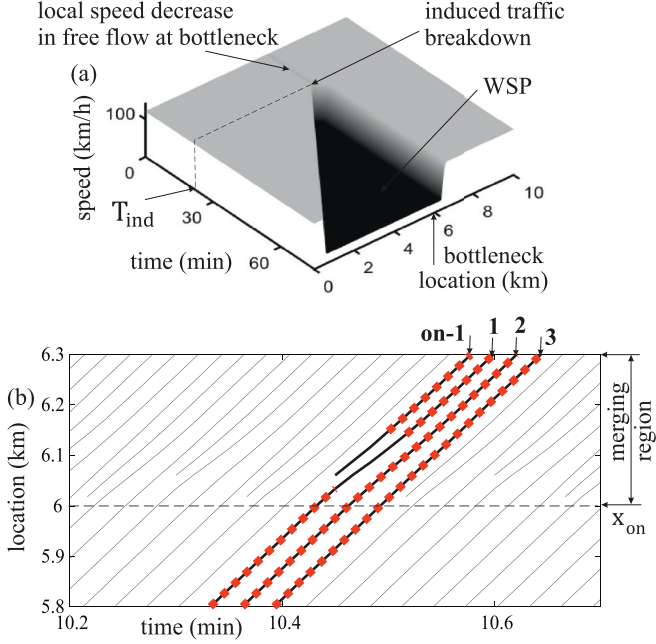


FIG. 10. Simulations of cooperation of different mechanisms of overacceleration on a single-lane road. Induced traffic breakdown on a single-lane road with on-ramp bottleneck. Simulations of model (1)–(12), (16) with model parameters: $\alpha_0 = 2 \text{ m/s}^2$, $\alpha_1 = 0.1 \text{ m/s}^2$, $k = 1$, $\tau_G = 1.4 \text{ s}$, $q_{in} = 2000 \text{ vehicles/h}$, $q_{on} = 800 \text{ vehicles/h}$; other model parameters are the same as those in Fig. 2. (a) Speed in space and time: Local speed decrease at bottleneck in free flow and WSP induced in the free flow at $T_{ind} = 20 \text{ min}$ through application of addition on-ramp inflow impulse $\Delta q_{on} = 100 \text{ vehicles/h}$ of duration $\Delta t = 2 \text{ min}$. (b) Some of simulated vehicle trajectories within local speed decrease in free flow at bottleneck at time $t < T_{ind}$ before traffic breakdown has been induced; parts of trajectories within which condition $a_{OA} > 0$ is satisfied are marked through colored red squares.

overacceleration a_{OA} on motion of vehicles 2 and 3 shown in Fig. 11].

Contrary to Fig. 7(a), due to overacceleration cooperation, the speed of vehicle 3 that follows vehicle 2 reaches its maximum value v_{free} already within the local speed decrease at the bottleneck [Fig. 11(a)]. As we will show in Sec. VIII A, overacceleration cooperation increases the tendency to free flow.

B. Microscopic behaviors of vehicle motion as the cause of overacceleration in a road lane

1. Suppression of speed adaptation within indifferent zone through overacceleration a_{OA}

We consider a vehicle that approaches the local speed decrease at the bottleneck while moving still upstream of the bottleneck within the indifferent zone $g_{safe} \leq g \leq G$. When condition $\alpha = 0$ (17) has been assumed (Sec. IV), then the speed adaptation effect (18) can be stronger than safety acceleration at any speed leading to the absence of overacceleration (Sec. IV A).

Contrarily, under opposite condition $\alpha \neq 0$ due to overacceleration a_{OA} (4), (5) vehicle tries to escape from the local

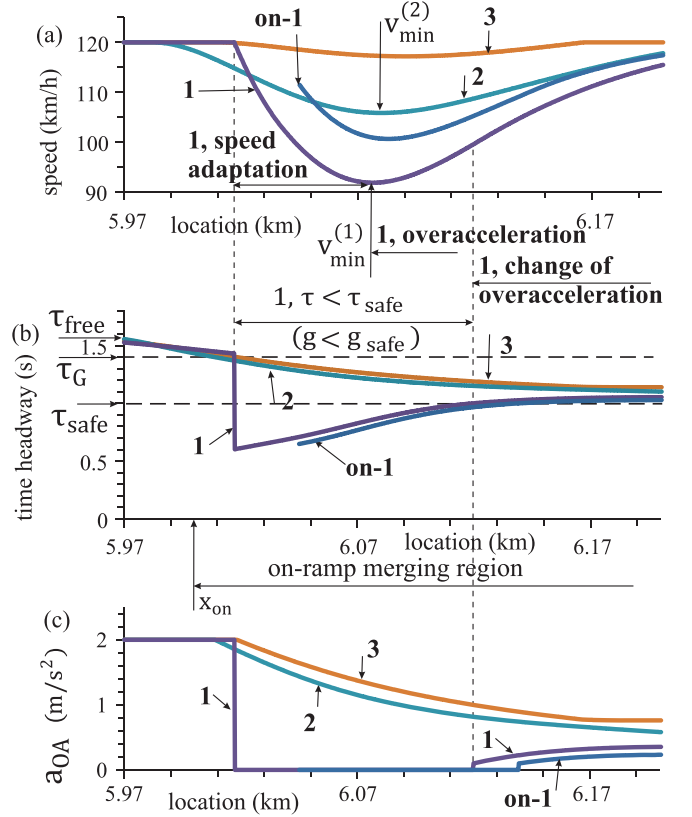


FIG. 11. Continuation of Fig. 10(b). (a)–(c) Location-dependencies of microscopic characteristics for some of the vehicles whose numbers are the same as those in Fig. 10(b), respectively: (a) vehicle speeds; (b) vehicle time-headway; (c) overacceleration a_{OA} (4), (5).

speed decrease at the bottleneck. This overacceleration can suppress speed adaptation within the indifferent zone $g_{safe} \leq g \leq G$ as long as in Eq. (4) speed $v \geq v_{syn}$.

2. Overacceleration cooperation in a road lane

Microscopic vehicle behaviors leading to overacceleration cooperation in a road lane are as follows: (i) Overacceleration a_{OA} (4), (5) suppresses speed adaptation as explained in Sec. VB 1. (ii) In addition to this effect of the suppression of speed adaptation due to overacceleration a_{OA} (4), (5), there is overacceleration caused by safety acceleration [“1, overacceleration” in Fig. 11]. (iii) Over time, overacceleration of vehicle 1 increases due to cooperation of overacceleration caused by safety acceleration with overacceleration a_{OA} [“1, change of overacceleration” in Fig. 11].

VI. EFFECT OF OVERACCELERATION AND SPEED ADAPTATION IN A ROAD LANE ON OVERACCELERATION THROUGH LANE-CHANGING

A. Prevention of overacceleration through lane-changing due to speed adaptation within the indifferent zone

First, we consider traffic breakdown on a two-lane road under assumption $\alpha = 0$ (17) and at the same model parameters for vehicle motion in a road lane as those used in Fig. 2,

at which no overacceleration occurs on a single-lane road. Then, contrary to Helly's model for motion of automated-driving vehicles used in Ref. [68], in model (1)–(17) we find that no overacceleration through lane-changing is realized [Figs. 12(a) and 12(b)]. This result is explained as follows. As shown in Sec. IV A for a single-lane road (Fig. 2), vehicle 1 following on-ramp vehicle on-1 should decelerate strongly resulting in deceleration of following vehicles [vehicles 2 and 3 in Fig. 3(a)] due to speed adaptation (18). The same effect we observe on a two-lane road: In the right lane, a wave of a speed decrease in free flow propagating upstream of the bottleneck occurs [Figs. 12(a) and 12(b)].

The speed adaptation effect (18) is also realized for vehicles in the left lane that follow a slower moving vehicle that has just changed from the right lane to the left lane. For this reason, in the left lane we also observe a wave of the speed decrease in free flow propagating upstream of the bottleneck [Figs. 12(a) and 12(b)]. Under used parameters in model (1)–(17), i.e., when no overacceleration exists in a road lane, there is also no overacceleration through lane-changing and, respectively, traffic breakdown does not exhibit the nucleation character. This result can be explained as follows: Speed adaptation within the indifferent zone (18) is on average stronger than vehicle acceleration to free flow downstream. Therefore, the acceleration cannot maintain free flow upstream of the bottleneck.

B. Effect of safety acceleration on overacceleration due to lane-changing through decrease in time headway between vehicles

Following Sec. IV B, we use a shorter time headway $\tau = \tau_{\text{free}}$ between vehicles in free flow through the same increase in q_{in} as that in Fig. 4. Then, as in Fig. 4(a), rather than upstream propagation of congestion found in Fig. 12(a), we observe the wave of a speed decrease in free flow propagating downstream of the bottleneck [Fig. 12(c)]. As in Fig. 4(b), we have found that this free flow is in a metastable state with respect to the $F \rightarrow S$ transition at the bottleneck [Fig. 12(d)]. The physics of this result is the same as that explained in details in Sec. IV B. In Sec. VID 2, we will show that this free flow metastability results from cooperation of overacceleration due to safety acceleration found in Sec. IV B with overacceleration through lane-changing.

C. Effect of safety acceleration on overacceleration due to lane-changing through decrease in synchronization time headway

The situation considered in Figs. 12(a) and 12(b) changes also basically when following Sec. IV C we decrease the synchronization time-headway from $\tau_G = 2$ s to shorter value $\tau_G = 1.4$ s that satisfies condition (20). As found Sec. IV C, in this case there is overacceleration in a road lane through safety acceleration. Then, traffic breakdown does exhibit the nucleation character [Fig. 12(e)]. As in the case considered in Sec. VIB, this free flow metastability results from cooperation of overacceleration due to safety acceleration with overacceleration through lane-changing (Sec. VID 2).

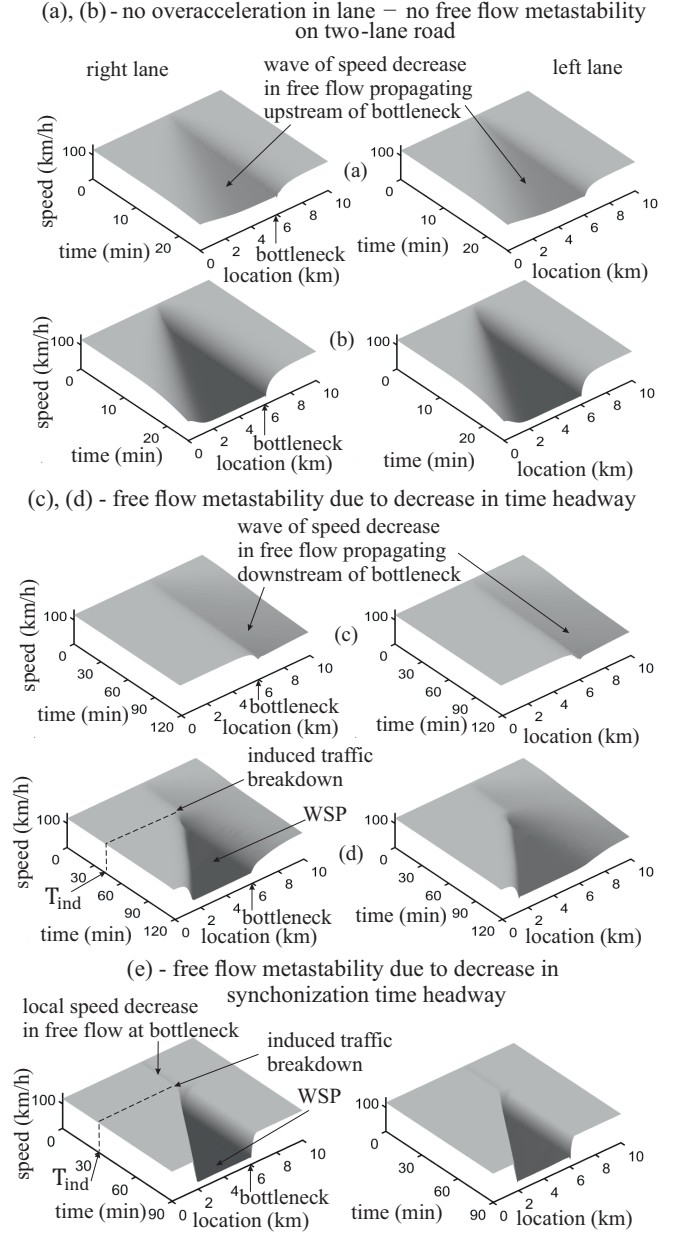


FIG. 12. Simulations of the effect of overacceleration in a road lane on free flow metastability on a two-lane road with on-ramp bottleneck with model (1)–(17), i.e., at $a_{OA} = 0$ in Eqs. (1) and (6). Speed in space and time in the right lane (left column) and in the left lane (right column). (a), (b) No overacceleration in a road lane—no free flow metastability; simulations at $q_{\text{in}} = 2000$ (vehicles/h)/lane, $\tau_G = 2$ s; (a) $q_{\text{on}} = 300$ vehicles/h, (b) $q_{\text{on}} = 900$ vehicles/h. (c)–(e) Overacceleration through safety acceleration in a road lane—free flow metastability on a two-lane road. In panels (c), (d), $q_{\text{in}} = 2250$ (vehicles/h)/lane, $\tau_G = 2$ s, $q_{\text{on}} = 60$ vehicles/h. In panel (e), $q_{\text{in}} = 2000$ (vehicles/h)/lane, $\tau_G = 1.4$ s, $q_{\text{on}} = 700$ vehicles/h. $F \rightarrow S$ transition has been induced at $T_{\text{ind}} = 40$ min in panel (d) and 30 min in panel (e) through addition on-ramp inflow-rate impulse $\Delta q_{\text{on}} = 440$ in panel (d) and 200 vehicles/h in panel (e) of duration $\Delta t = 2$ min. Parameters of lane-changing: $\tau_1 = 0.5$ s, $\tau_2 = 0.3$ s, $\delta_1 = 1$ m/s, $\delta_2 = 5$ m/s. Other parameters are the same as those in Fig. 2.

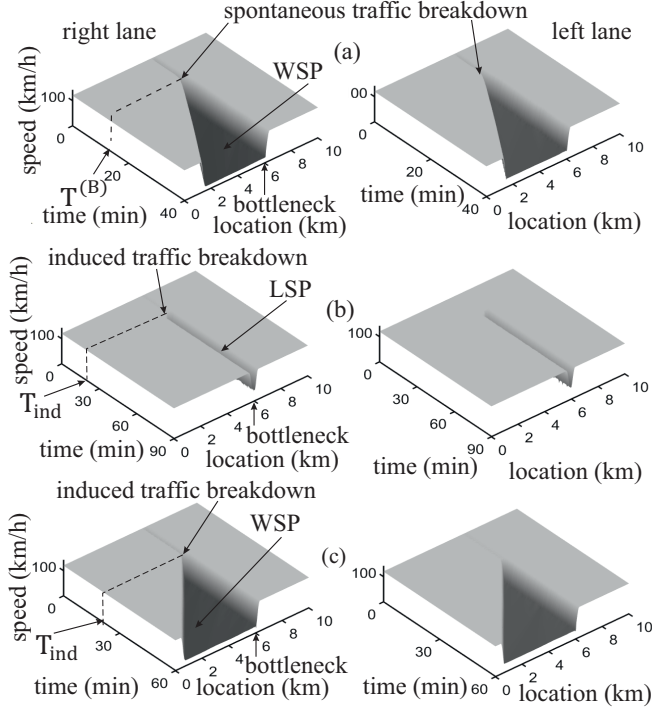


FIG. 13. Simulations of the effect of overacceleration a_{OA} in a road lane on overacceleration through lane-changing on a two-lane road with on-ramp bottleneck at $q_{in} = 2000$ (vehicles/h)/lane, $\tau_G = 2$ s. Speed in space and time in the right lane (left column) and in the left lane (right column). (a)–(c) Traffic breakdown in model (1)–(16); parameters of overacceleration in a road lane: $\alpha_0 = 2$ m/s², $\alpha_1 = 0.1$ m/s², $k = 1$. (a) Spontaneous traffic breakdown with WSP formation at $q_{on} = 1144$ vehicles/h that exceeds slightly the maximum flow rate $q_{on, max} = 1142$ vehicles/h in free flow: At $q_{on} > q_{on, max}$, spontaneous traffic breakdown occurs (see explanations in Sec. VIII). (b) LSP induced at bottleneck at the minimum flow rate $q_{on} = q_{on, min} = 517$ vehicles/h at which SP can still be induced in free flow. (c) Induced traffic breakdown at $q_{on} = 980$ vehicles/h. In panels (b), (c), $F \rightarrow S$ transition has been induced at $T_{ind} = 20$ min through addition on-ramp inflow-rate impulse $\Delta q_{on} = 883$ (b) and 420 vehicles/h (c) of duration $\Delta t = 2$ min. Other parameters are the same as those in Fig. 12.

D. Microscopic features of overacceleration cooperation on a two-lane road

1. Cooperation of overacceleration a_{OA} with overacceleration through lane-changing

If assumption $\alpha = 0$ (17) is not applied, then contrary to Figs. 12(a) and 12(b), due to the existence of overacceleration in a road lane a_{OA} (4) traffic on a two-lane road becomes metastable with respect to an $F \rightarrow S$ transition (Fig. 13). There is the following common feature of vehicle overacceleration found in Ref. [68] for automated-driving vehicles moving based on Helly's model and in the model (1)–(16): When free flow [Figs. 14(a) and 14(b)] transforms into synchronized flow [Figs. 14(c) and 14(d)], there is a discontinuity in the rate of $R \rightarrow L$ lane-changing denoted by R_{RL} (Fig. 15). This causes the discontinuity in the overacceleration rate R_{OA}

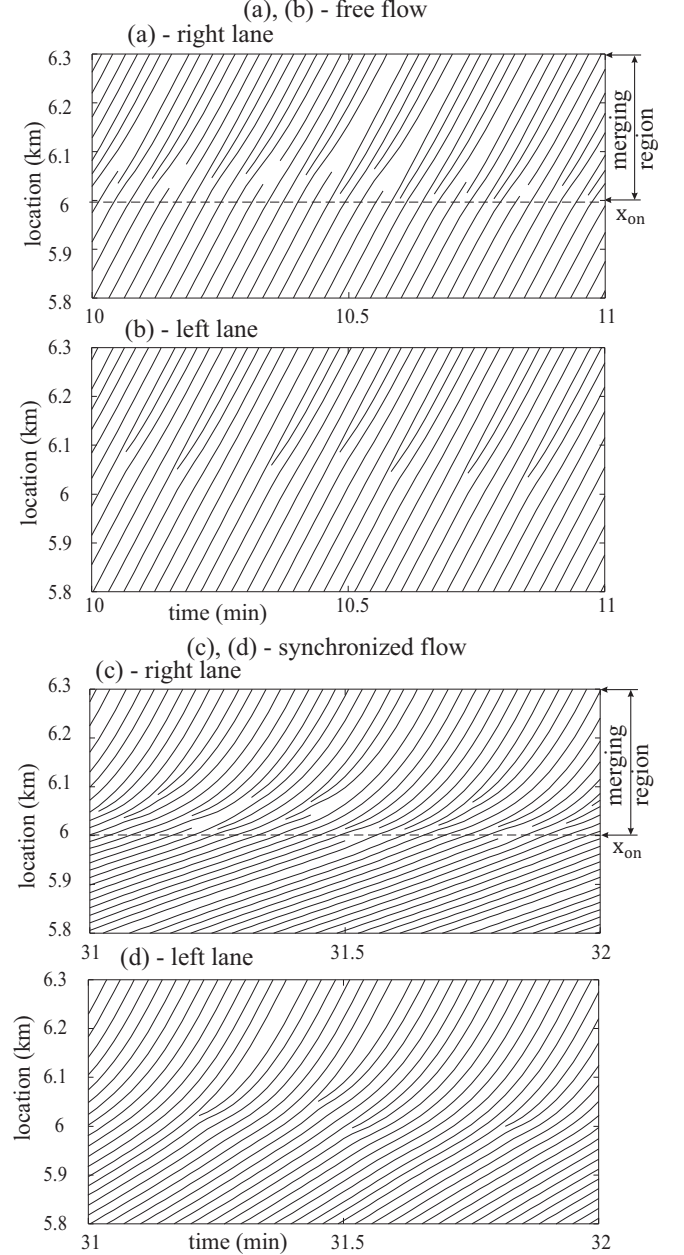


FIG. 14. Continuation of Fig. 13(c). (a), (b) Some of simulated vehicle trajectories within local speed decrease in free flow at bottleneck at time $t < T_{ind} = 20$ min in the right (a) and left (b) lanes. (c), (d) Some of simulated vehicle trajectories within synchronized flow at bottleneck at time $t > T_{ind} + \Delta t$.

in accordance with equation $R_{OA} = R_{RL}$ (Fig. 15). There are the following differences in features of overacceleration in Helly's model for automated-driving vehicles [68] and model (1)–(16) [Figs. 16–18]:

(i) When due to lane-changing of vehicle 2 (Fig. 16) space gap g between vehicle 1 and 3 becomes larger than synchronization gap G [i.e., $\tau > \tau_G$ in Fig. 17(b)], acceleration of vehicle 3 given by Eq. (6) increases considerably due to overacceleration in a road lane a_{OA} (4), (5) [Fig. 17(c)]. As a result, in model (1)–(16) due to cooperation of overaccelera-

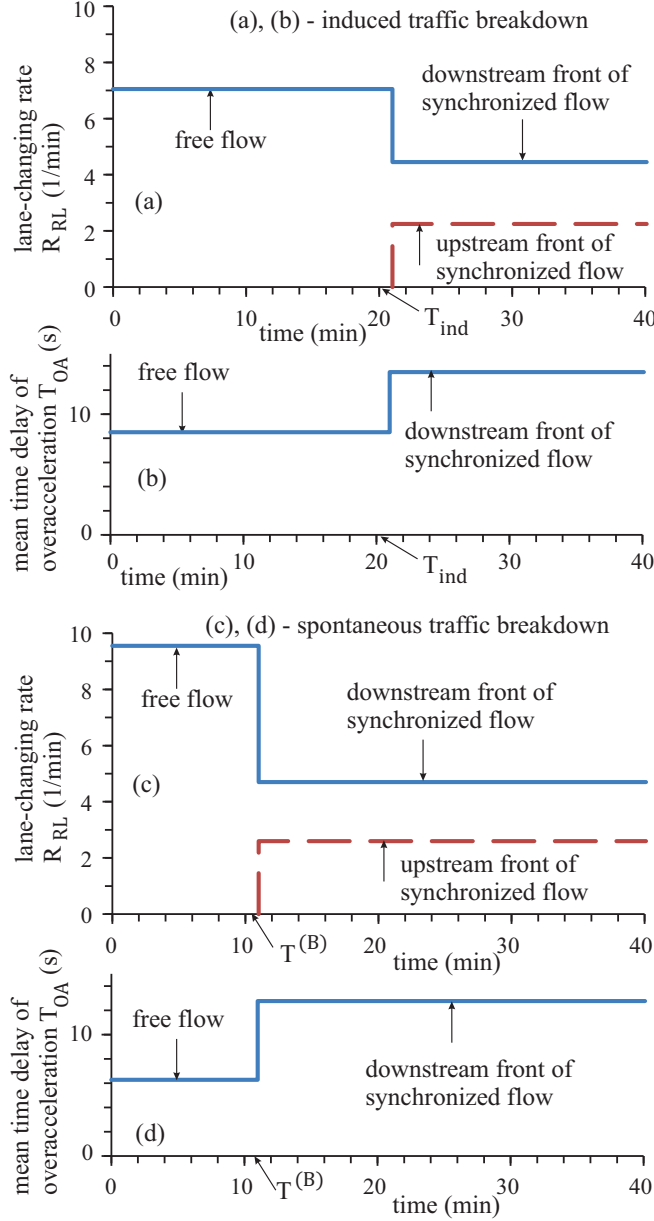


FIG. 15. Continuation of Figs. 13(a) and 13(c). Time-dependencies of the lane-changing rate R_{RL} (a), (c) and the mean time delay in overacceleration due to lane-changing $T_{OA} = 1/R_{OA}$ (b), (d). Solid blue colored curves are related to cases in which $R_{OA} = R_{RL}$, where R_{OA} is the rate of overacceleration due to lane-changing. In panels (a), (c), dashed red colored curves are time-dependencies of the lane-changing rate $R_{RL}^{(up)}$ at the upstream front of synchronized flow. (a), (b) For induced traffic breakdown shown in Fig. 13(c). (c), (d) For spontaneous traffic breakdown shown in Fig. 13(a). Values R_{OA} , $R_{RL}^{(up)}$, and T_{OA} have been averaged in free flow (during time intervals $0 \leq t < T_{ind}$, $T_{ind} = 20$ min and $0 \leq t < T^{(B)}$, $T^{(B)} \approx 10.8$ min for induced and spontaneous breakdowns, respectively) and in synchronized flow (during time intervals $T_{ind} + \Delta t < t \leq 40$ min and $T^{(B)} < t \leq 40$ min for induced and spontaneous breakdowns, respectively).

tion due to lane-changing with overacceleration a_{OA} vehicle 3 accelerates strongly while reaching the maximum speed v_{free} quickly [“3, overacceleration” in Fig. 17(a)].

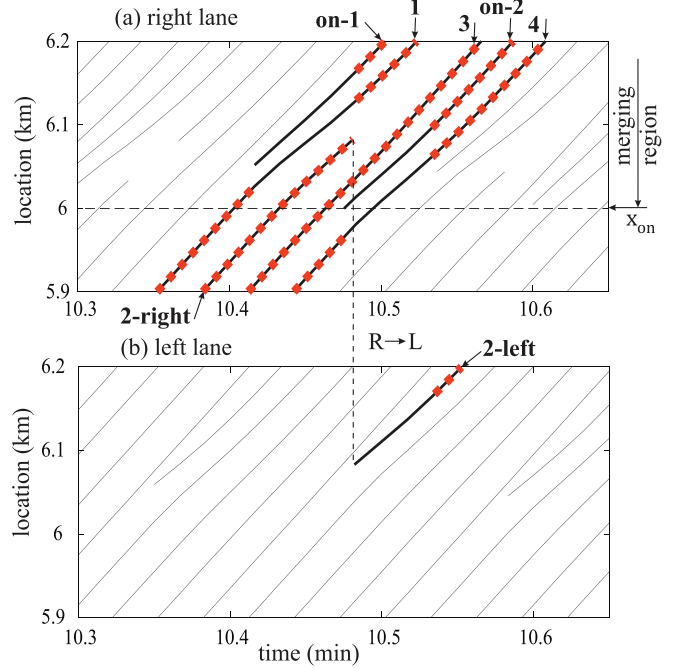


FIG. 16. Continuation of Fig. 13(c). (a), (b) Simulated vehicle trajectories within local speed decrease in free flow at bottleneck in the right lane (a) and left lane (b) at time $t < T_{ind} = 20$ min. R → L lane-changing of vehicle 2 is marked by dashed vertical lines R → L. Part of trajectories within which condition $a_{OA} > 0$ is satisfied are marked through colored red squares.

(ii) After time-headway of vehicle 4 in Fig. 16 following the on-ramp vehicle on-2 satisfies condition $\tau \geq \tau_{safe}$, vehicle acceleration increases through overacceleration mechanism a_{OA} [Figs. 18(b) and 18(c)]. Overacceleration of vehicle 4 leads to recovering of free flow already within on-ramp merging region [“4, overacceleration” in Fig. 18(a)].

2. Cooperation of overacceleration caused by safety acceleration with overacceleration through lane-changing

In accordance with Sec. VIB, when overacceleration $a_{OA} = 0$ (17) and at the same $\tau_G = 2$ s as that in Fig. 13, due to the decrease in τ_{free} , free flow metastability is realized at the bottleneck [Fig. 12(d)]. Microscopic analysis presented in Fig. 19 shows that the cause of this free flow metastability is cooperation of overacceleration through safety acceleration with overacceleration due to lane-changing.

The physics of overacceleration through safety acceleration is the same as explained in Sec. IV B: The decrease in τ_{free} leads to a decrease in the suppression effect of speed adaptation on the tendency to free flow through safety acceleration. As a result, safety acceleration of vehicle 1 becomes overacceleration [“1, overacceleration” in Fig. 19(c)].¹⁷

¹⁷In Ref. [68], overacceleration through lane-changing in traffic of classical ACC vehicles on a two-lane road has been revealed. The result found in Sec. IV D that safety acceleration in ACC model (22) becomes overacceleration means that the cause of free flow metastability in traffic of classical ACC vehicles on a two-lane road

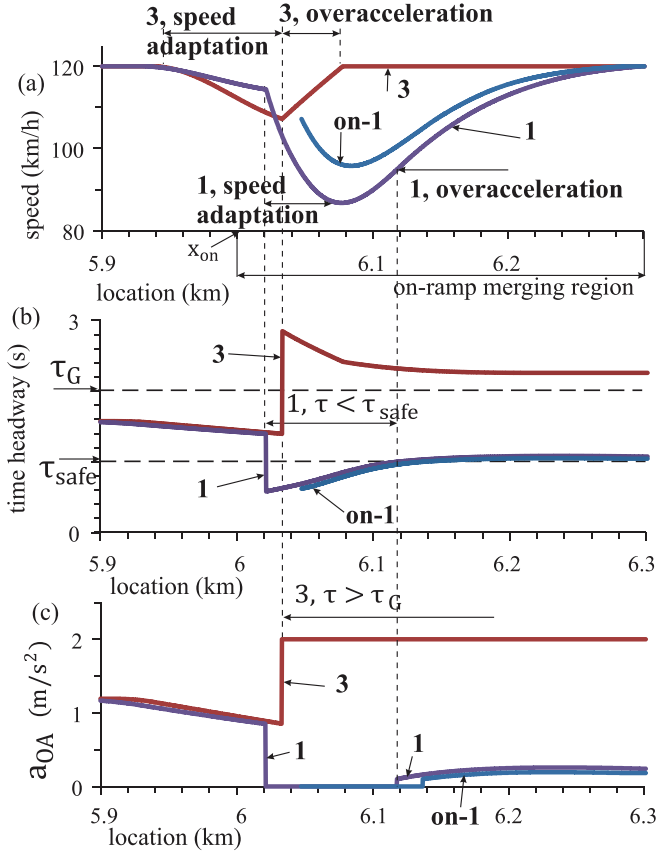


FIG. 17. Continuation of Fig. 16. (a)–(c) Location-dependencies of microscopic characteristics for some of the vehicles whose numbers are the same as those in Fig. 16, respectively: (a) vehicle speeds; (b) vehicle time-headway; (c) overacceleration a_{OA} (4), (5).

We have found (not shown) that as in the case shown in Fig. 15, a discontinuous character of the overacceleration rate R_{OA} is realized. This means that overacceleration due to lane-changing occurs. As in Figs. 16(a), 17(a), and 17(b), after vehicle 2 changes from the right lane to the left lane [Figs. 19(a) and 19(b)], vehicle 3 in the right lane accelerates due to the considerable increase in time headway [“3, overacceleration” in Figs. 19(c) and 19(d)].¹⁸

Microscopic features of cooperation of overacceleration caused by safety acceleration with overacceleration through

is cooperation of overacceleration through lane-changing with overacceleration through safety acceleration. Thus, as made in Fig. 9(a), in Fig. 8(c) of Ref. [68] the labeling “1, overacceleration” should be added beginning at the point of the minimum speed of vehicle 1 (about at $t \approx 20.56$ min in Fig. 8(c) of Ref. [68]).

¹⁸This acceleration of vehicle 3 is realized *only* after vehicle 2 has changed to the left lane. The rate of lane-changing exhibits the discontinuous character (Fig. 15). For this reason, we can also call the acceleration of vehicle 3 as overacceleration [labeled as “3, overacceleration” in Fig. 17(a)]. This conclusion is also related to traffic flow consisting of automated-driving vehicles studied in Ref. [68]. This means that label “3, acceleration” in Fig. 8(a) of Ref. [68] should be replaced by “3, overacceleration.”

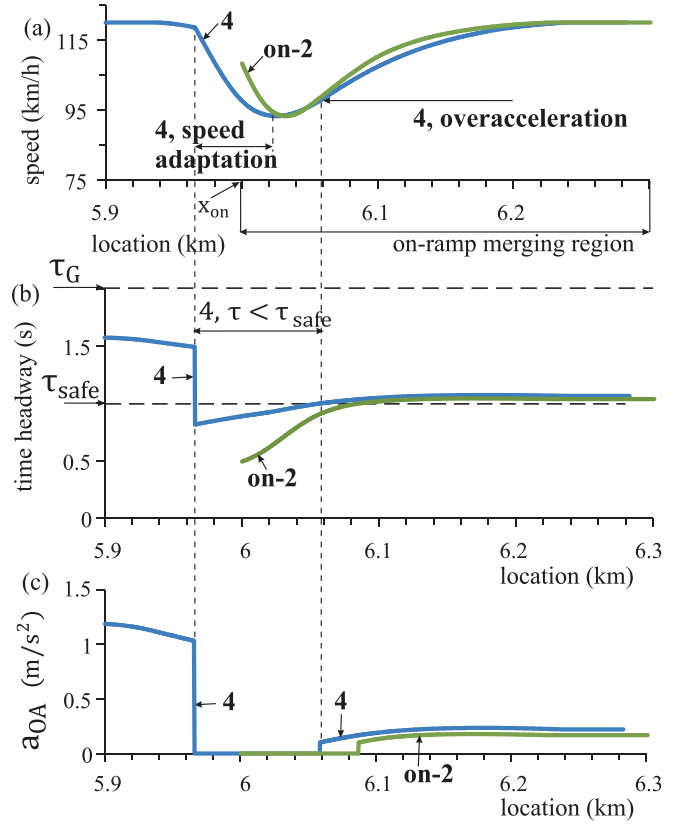


FIG. 18. Continuation of Fig. 16. (a)–(c) Location-dependencies of microscopic characteristics for some of the vehicles whose numbers are the same as those in Fig. 16, respectively: (a) vehicle speeds; (b) vehicle time-headway; (c) overacceleration a_{OA} (4), (5).

lane-changing shown in Fig. 19 remain qualitatively the same for free flow metastability studied in Sec. VIC.

VII. CRITICAL NUCLEUS FOR SPONTANEOUS TRAFFIC BREAKDOWN

As for Helly’s model used for automated-driving vehicles in Ref. [87], in the model of Sec. II at a given q_{in} when the on-ramp inflow rate q_{on} increases, then at some $q_{on} > q_{on, max}$ the minimum speed within a local speed decrease at the bottleneck decreases over time and after a time-delay $T^{(B)}$ spontaneous traffic breakdown (spontaneous $F \rightarrow S$ transition) occurs [Figs. 13(a) and 20(a)]. Here we reveal microscopic features of the critical nucleus for spontaneous traffic breakdown.

A. Microscopic characteristics of critical nucleus

On a single-lane road [Figs. 20 and 21] and on a two-lane road [Figs. 22 and 23], during interval $0 < t < T^{(B)}$ a minimum speed v_{min} within the local speed decrease at the bottleneck is a decreasing time function [minimum speeds of vehicles 1–3 in Fig. 21(a) for a single-lane road and minimum speeds of vehicles 1–3 and 7–9 in Fig. 23 for a two-lane road]. When v_{min} becomes less than v_{syn} (4), a state within the local speed decrease can be considered synchronized flow. Thus, free flow persists upstream of the bottleneck, while

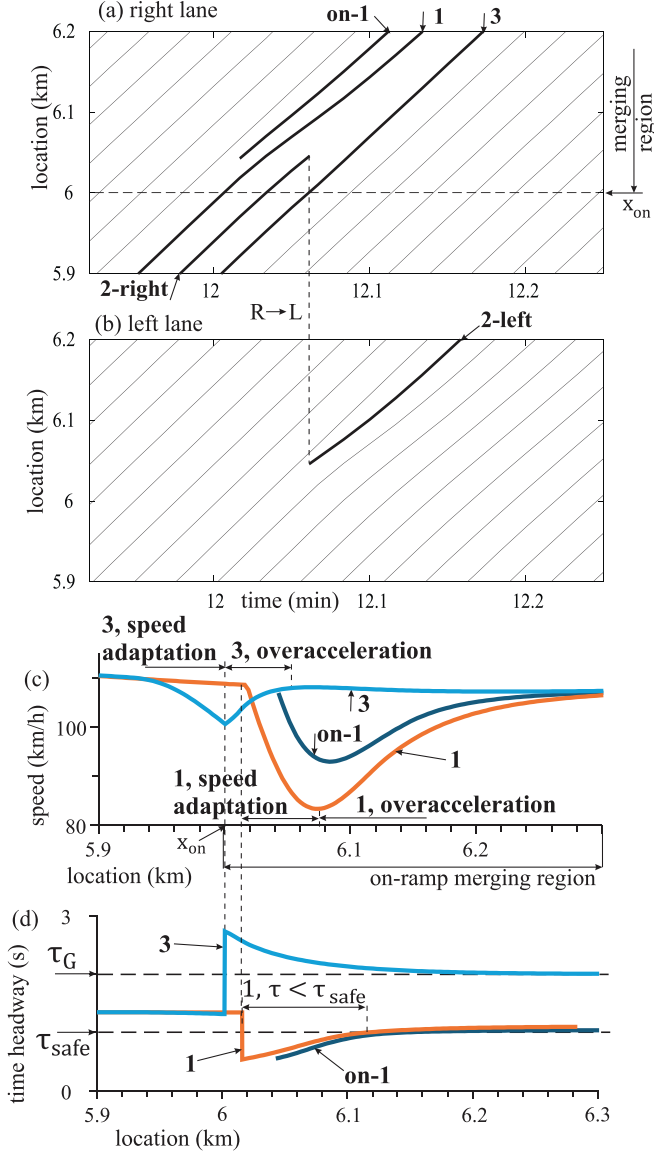


FIG. 19. Continuation of Fig. 12(d). (a), (b) Some of simulated vehicle trajectories within local speed decrease in free flow at bottleneck at time $t < T_{\text{ind}} = 40$ min in the right (a) and left (b) lanes. (c), (d) Location-dependencies of vehicle speeds (c) and time-headway (d) for some of the vehicles whose numbers are the same as those in panel (a).

synchronized flow is only within the local speed decrease at the bottleneck.

We have found that time-delay of spontaneous traffic breakdown $T^{(B)}$ is determined by a saturation in the decrease of v_{min} over time: At $t > T^{(B)}$ without further noticeable decrease in v_{min} , the upstream front of synchronized flow moves upstream of the bottleneck, i.e., free flow upstream of the bottleneck transforms over time into synchronized flow propagating upstream [vehicles 4–6 in Fig. 21(a) and vehicles 4–6 and 10–12 in Figs. 23(a) and 23(b)]. The beginning of the upstream propagation of synchronized flow can be seen from a comparison of microscopic speeds along trajectories 8 and 13 in Fig. 21(b) for a single-lane road as well as a comparison of

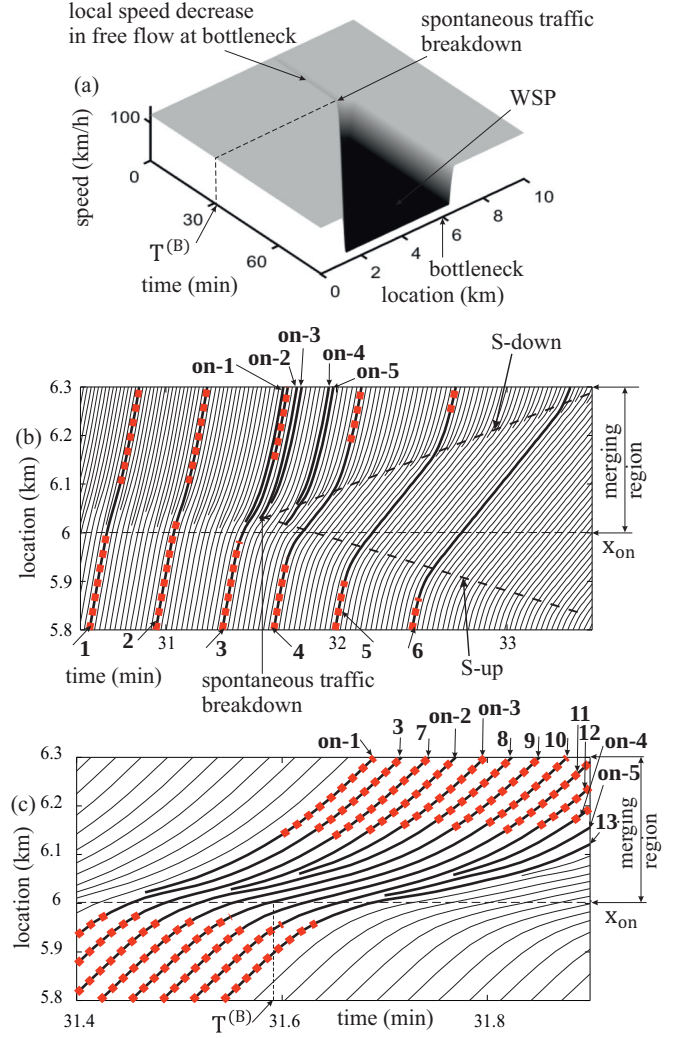


FIG. 20. Spontaneous traffic breakdown on a single-lane road with on-ramp bottleneck. Simulations of the same model (1)–(12), (16) at $\tau_G = 1.4$ s and $q_{\text{in}} = 2000$ vehicles/h used in Fig. 10, however, at another value of $q_{\text{on}} = 809$ vehicles/h that is only slightly higher than $q_{\text{on}} = q_{\text{on, max}} = 807$ vehicles/h. Other parameters are the same as those in Fig. 10. (a) Speed in space and time: Local speed decrease in free flow at bottleneck and WSP that occurs spontaneously in free flow after time-delay $T^{(B)} \approx 31.6$ min. (b) Some of simulated vehicle trajectories at the bottleneck in a vicinity of $t = T^{(B)}$. (c) Some of the trajectories in panel (b) shown during in a shorter time-scale. In panels (b), (c), parts of the trajectories within which condition $a_{\text{OA}} > 0$ is satisfied are marked by colored red squares. In panel (b), dashed lines labeled by S-down and S-up mark, respectively, downstream and upstream fronts of synchronized flow.

microscopic speeds along trajectories 16 and 20 for a two-lane road in Fig. 23(c).

Thus, the critical nucleus at the bottleneck is a local speed decrease at the bottleneck caused by the last on-ramp vehicle or the last sequence on-ramp vehicles merging from the on-ramp at $t < T^{(B)}$ at which the speed decrease is still localized at the bottleneck. This means that the next on-ramp vehicle or sequence of on-ramp vehicles that have merged from the on-ramp at $t = T^{(B)}$ cause another local speed decrease that

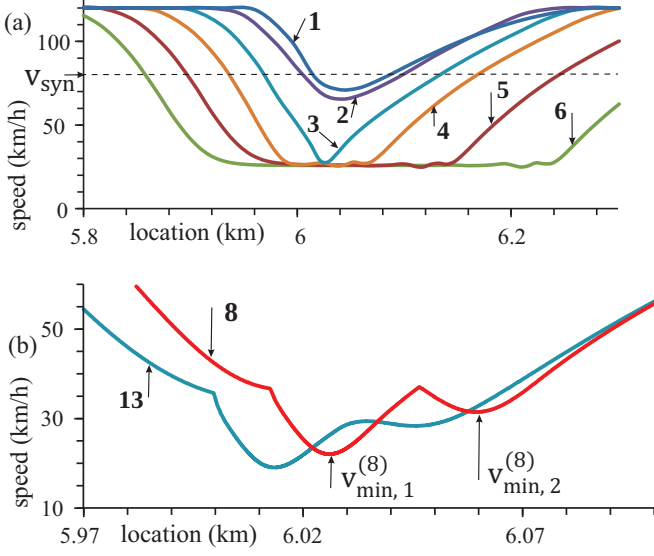


FIG. 21. Continuation of Figs. 20(b) and 20(c). Location-dependence of vehicle speeds those trajectories are shown by the same vehicle numbers in Figs. 20(b) and 20(c): (a) Averaged speed along trajectories (1-s averaged data) shown in Fig. 20(b); dashed line $v = v_{syn}$ separates road regions on vehicle trajectories within which overacceleration a_{OA} is either more than zero (on and above the line) or it is equal to zero (below the line). (b) Microscopic (not averaged data) speeds along trajectories 8 and 13 shown in Fig. 20(c).

cannot be localized at the bottleneck any more: At $t > T^{(B)}$ the irreversible upstream propagation of synchronized flow occurs.

There can be one, two, or more minimums of the synchronized flow speed within the nucleus. In Fig. 20(c), two following each other on-ramp vehicles have merged onto the main road [vehicles on-2 and on-3]. This results in two speed minimums for vehicle 8 [$v_{min,1}^{(8)}$ and $v_{min,2}^{(8)}$ in Fig. 21(b)] that follows vehicles on-2 and on-3. A similar result is realized on a two-lane road for vehicle 16 following two on-ramp vehicles [vehicles on-2 and on-3 in Fig. 22(c)]. This leads to two speed minimums for vehicle 16 [$v_{min,1}^{(16)}$ and $v_{min,2}^{(16)}$ in Fig. 23(c)]. The lowest speed minimum within the critical nucleus can be considered a microscopic *critical speed* for traffic breakdown ($F \rightarrow S$ transition).

B. Effect of overacceleration in road lane a_{OA} on critical nucleus development

When time t is not very close to $T^{(B)}$, the minimum speed v_{min} within the local speed decrease at the bottleneck is not considerably less than v_{syn} [e.g., $v_{min}^{(1)} \approx 72.2$ km/h for vehicle 1 on a single-lane road in Fig. 21(a) and $v_{min}^{(1)} \approx 53.3$ km/h for vehicle 1 on a two-lane road in Fig. 23(a)]. As a result, at the bottleneck overacceleration $a_{OA} > 0$ with the exception of a narrow road region at the bottleneck within which $v < v_{syn}$ [e.g., vehicle 1 in Fig. 21(a), vehicle 1 in Fig. 23(a), and vehicle 7 in Fig. 23(b)] and, therefore, $a_{OA} = 0$.

When $t \rightarrow T^{(B)}$, then the effect of overacceleration a_{OA} on the maintenance of free flow at the bottleneck becomes weaker. Indeed, due to a decrease in v_{min} , the size of a road

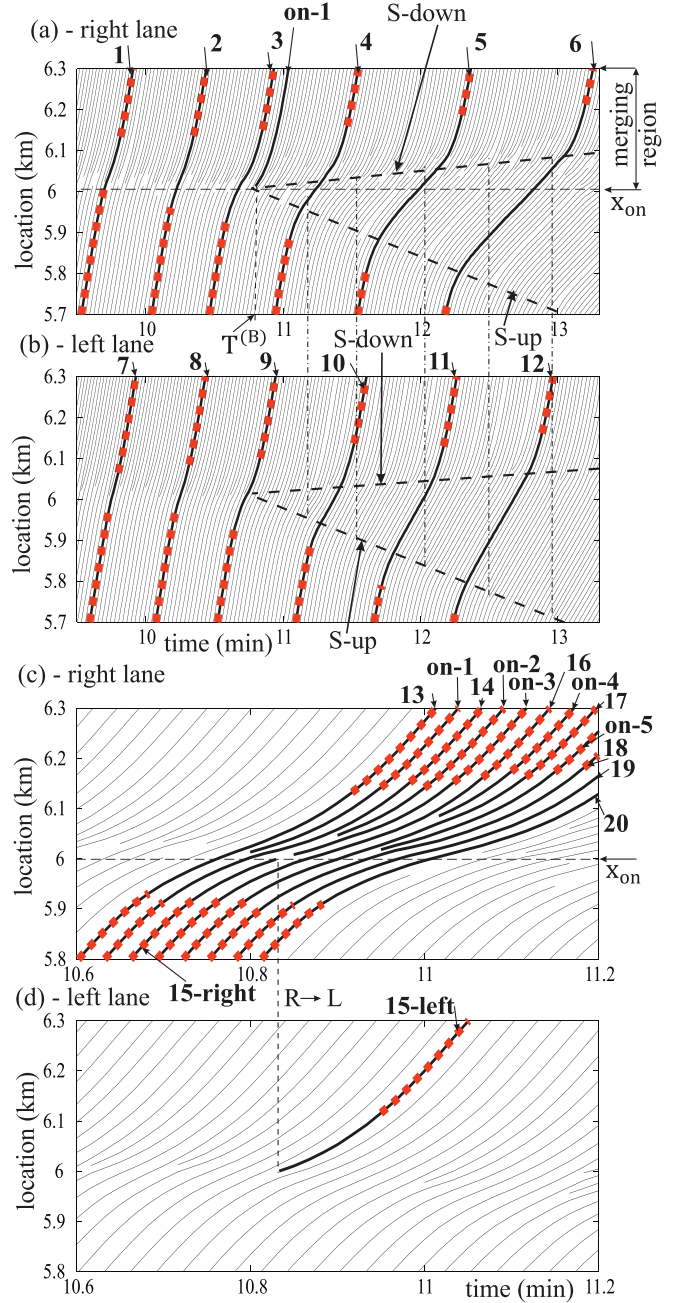


FIG. 22. Continuation of Fig. 13(a). (a), (b) Simulated vehicle trajectories within local speed decrease in free flow on a two-lane road with bottleneck showing spontaneous traffic breakdown occurring after time-delay at $T^{(B)} \approx 10.8$ min. (c), (d) Some of the simulated vehicle trajectories of panels (a), (b) shown in a shorter time-scale. Parts of trajectories within which condition $a_{OA} > 0$ is satisfied are marked through colored red squares. In panels (a), (b), dashed lines labeled by S-down and S-up mark, respectively, downstream and upstream fronts of synchronized flow.

region in the bottleneck vicinity within which the speed $v < v_{syn}$, i.e., overacceleration $a_{OA} = 0$, increases [this can be seen on trajectories 2, 3 for a single-lane road in Fig. 21(a) and on trajectories 2, 3 and 8, 9 for a two-lane road in Figs. 23(a) and 23(b)]. Thus, at $t \rightarrow T^{(B)}$ the effect of overacceleration a_{OA} on the critical nucleus development is caused by the increase

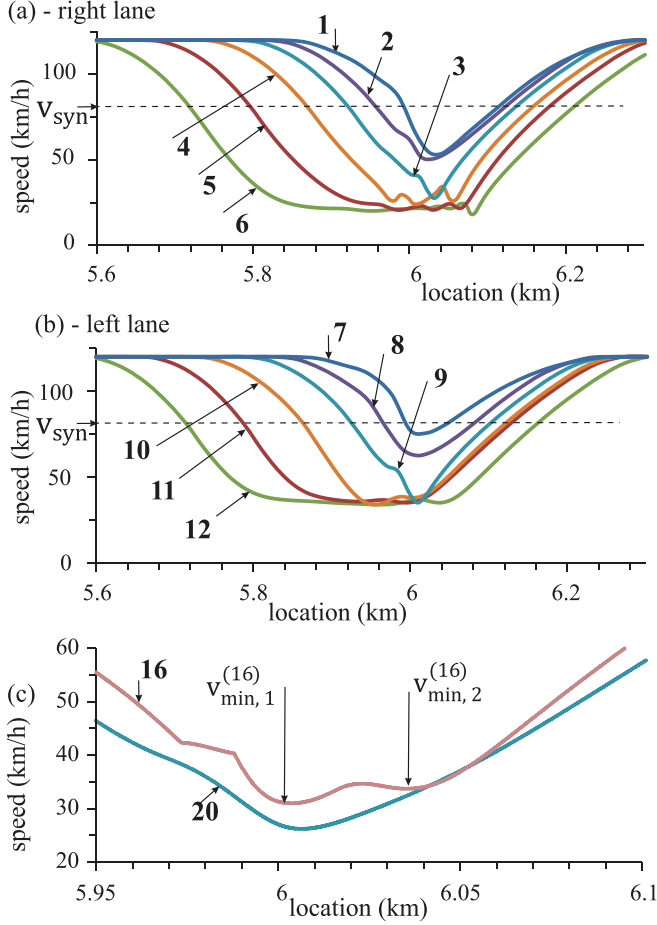


FIG. 23. Continuation of Figs. 22(a)–22(c). (a), (b) Location-dependence of averaged vehicle speeds (1-s averaged data); dashed lines $v = v_{syn}$ separate road regions on vehicle trajectories within which overacceleration a_{OA} is either more than zero (on and above the lines) or it is equal to zero (below the lines). (c) Location-dependence of speeds (not averaged data) of vehicles 16 and 20. Trajectories are shown with the same vehicle numbers as those in Figs. 22(a)–22(c), respectively.

in the road region size in the bottleneck vicinity within which $a_{OA} = 0$.

C. Vehicle breaking capability as a reason for critical speed

The suppression of speed adaptation through overacceleration a_{OA} discussed in Sec. VB 1 can be realized only when in Eq. (4) condition $v \geq v_{syn}$ is satisfied. This can explain the existence of a critical speed for traffic breakdown (Sec. VII A) as follows. There is a finite breaking capability of a vehicle: The lower the minimum speed v_{min} within the local speed decrease at the bottleneck, the longer the vehicle breaking distance, therefore, the longer the width of the upstream front of the local speed decrease within which the vehicle decelerates. For this reason, the road location, at which condition $v < v_{syn}$ has been just satisfied and, therefore, overacceleration $a_{OA} = 0$, moves upstream, when v_{min} decreases over time.

This effect is realized both on a single-lane road [Figs. 20(b) and 21(a)] and a two-lane road [Figs. 22(a), 22(b), 23(a), and 23(b)]. Thus, the suppression effect of speed adap-

tation through overacceleration a_{OA} (4) becomes the weaker, the lower the minimum speed v_{min} within the local speed decrease at the bottleneck. For this reason, there should be a critical minimum speed within the local speed decrease at which speed adaptation becomes on average stronger than overacceleration. This leads to traffic breakdown ($F \rightarrow S$ transition).

VIII. EFFECT OF DIFFERENT OVERACCELERATION MECHANISMS ON MAXIMUM HIGHWAY CAPACITY

- The main effect of overacceleration on vehicular traffic at a bottleneck is the maintenance of free flow upstream of the bottleneck, i.e., the prevention of traffic breakdown ($F \rightarrow S$ transition).

To explain this statement, we show that for the same chosen flow rate in free flow q_{in} upstream of the bottleneck, the stronger the effect of overacceleration on vehicular traffic, the larger the maximum on-ramp inflow rate $q_{on} = q_{on, max}$, i.e., the larger the maximum highway capacity at which free flow upstream of the bottleneck is still maintained. Through the revealing of different overacceleration mechanisms as well as their cooperation (Secs. V and VID), here we make a comparison of the effect of different overacceleration mechanisms on maximum highway capacity of vehicular traffic at the bottleneck.

A. Traffic flow consisting of human-driving vehicles or automated-driving TPACC vehicles

In Figs. 24(a) and 25(a), there is no overacceleration (Secs. IV A and VI A). Then, vehicles on the main road moving initially at free flow speed v_{free} upstream of the bottleneck must decelerate due to vehicles merging from the on-ramp. Through this speed adaptation effect (18), at the given q_{in} the larger the on-ramp inflow-rate q_{on} , the lower the average vehicle speed upstream of the bottleneck. This means that speed adaptation is the cause for the emergence of congested traffic at the bottleneck.

Overacceleration is the opposite effect to speed adaptation. Indeed, contrary to Figs. 24(a) and 25(a), in Figs. 24(b)–24(d) and 25(b)–25(d) when overacceleration is realized at the bottleneck, free flow can be self-maintained upstream of the bottleneck up to some maximum on-ramp inflow rate $q_{on} = q_{on, max}$, i.e., up to the maximum highway capacity C_{max} that is equal to $C_{max} = q_{in} + q_{on, max}$ for a single-lane road and $C_{max} = 2q_{in} + q_{on, max}$ for a two-lane road, respectively. Thus, as long as $q_{on} < q_{on, max}$, i.e., $q_{sum} < C_{max}$ (where q_{sum} is the total flow rate across the road in free flow, $q_{sum} = q_{in} + q_{on}$ for a single-lane road and $q_{sum} = 2q_{in} + q_{on}$ for a two-lane road, respectively), overacceleration occurring in Figs. 24(b)–24(d) and 25(b)–25(d) prevents traffic congestion at the bottleneck caused by speed adaptation in Figs. 24(a) and 25(a).

On a single-lane road, when overacceleration is caused by safety acceleration at $g_{free} > G$ (20) only (Sec. IV C), then the maximum on-ramp inflow-rate $q_{on, max}$ [Fig. 24(b)] and, therefore, the maximum capacity C_{max} are, respectively, less than these values are when overacceleration is caused by the term a_{OA} (4), (5) [Fig. 24(c)]. This means that at chosen model parameters the effect of overacceleration a_{OA} (4), (5)

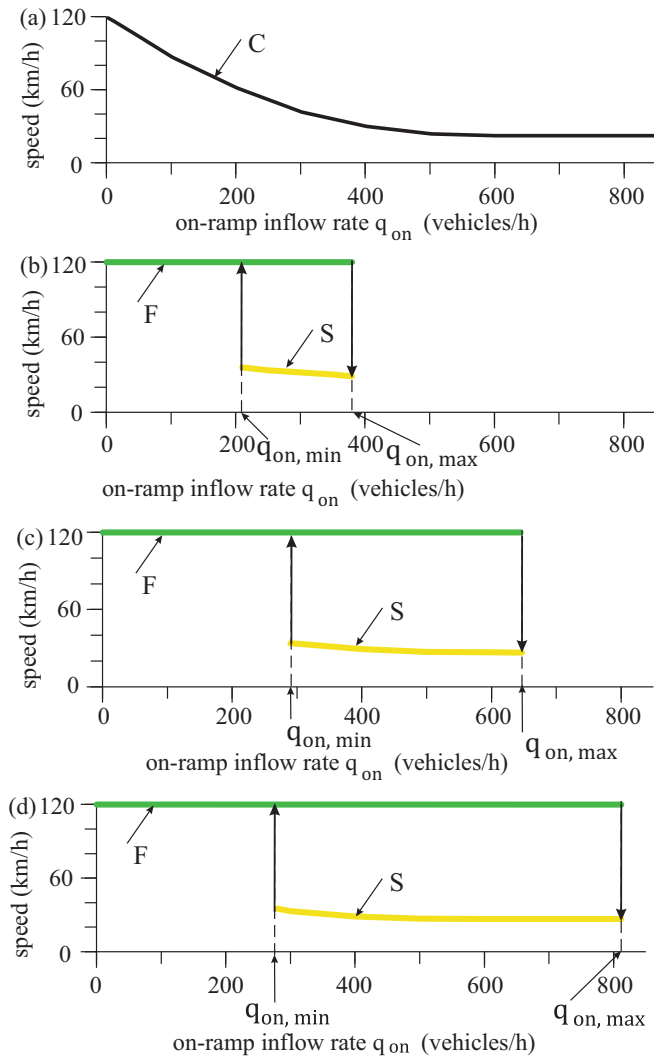


FIG. 24. Effect of different overacceleration mechanisms on maximum highway capacity of vehicular traffic on a single-lane road with the use of speed-on-ramp-inflow characteristics calculated at $q_{in} = 2000$ vehicles/h: (a) No overacceleration, model and parameters of Fig. 2. (b) Overacceleration mechanism through safety acceleration only, model and parameters of Fig. 6. (c) Overacceleration due to a_{OA} (4), (5) in model (1)–(12), (16) with parameters: $\alpha_0 = 2 \text{ m/s}^2$, $\alpha_1 = 0.1 \text{ m/s}^2$, $k = 1$; other model parameters are the same as those in Fig. 2. (d) Cooperation of overacceleration mechanisms through safety acceleration at $g_{free} > G$ (20) and due to overacceleration a_{OA} (4), (5); model and parameters of Fig. 10. Contrary to panel (d), in panel (c) there is only overacceleration mechanism a_{OA} ; this is because due to $\tau_G = 2 \text{ s}$ used in panel (c) condition (20) is not satisfied and, therefore, safety acceleration cannot be considered overacceleration at the given q_{in} . C—congested traffic, F—free flow, S—synchronized flow. $q_{on} = q_{on, min}$ is a minimum on-ramp flow rate related to a minimum highway capacity $C_{min} = q_{in} + q_{on, min}$, whereas $q_{on} = q_{on, max}$ is related to a maximum highway capacity $C_{max} = q_{in} + q_{on, max}$. Within the range $q_{on, min} \leq q_{on} < q_{on, max}$ either free flow or an SP can exist at the bottleneck; at $q_{on} < q_{on, min}$ no congested patterns can be induced in free flow, whereas at $q_{on} > q_{on, max}$ the SP occurs spontaneously at the bottleneck after a time delay $T^{(B)}$. Calculated parameters: $(q_{on, min}, q_{on, max}) = (217, 372)$ vehicles/h (b), $(293, 648)$ vehicles/h (c), $(280, 807)$ vehicles/h (d).

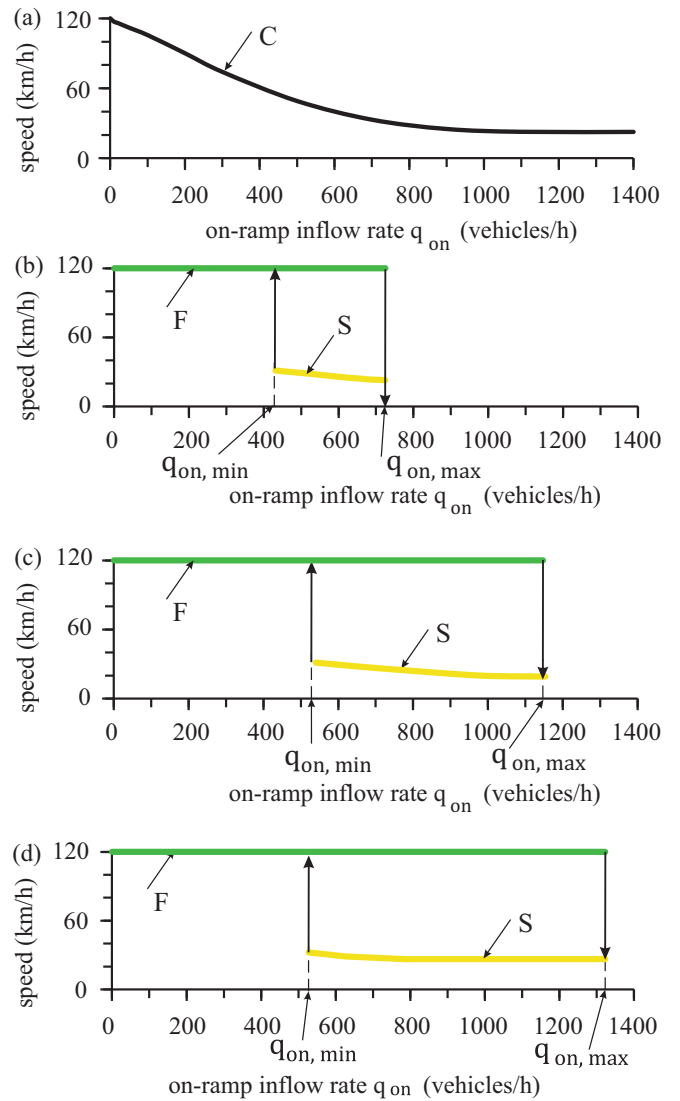


FIG. 25. Effect of different overacceleration mechanisms on maximum highway capacity of vehicular traffic on a two-lane road with the use of speed-on-ramp-inflow characteristics calculated at $q_{in} = 2000$ (vehicles/h)/lane: (a) No overacceleration, model and parameters of Figs. 12(a) and 12(b). (b) Model and parameters of Fig. 12(e). (c) Model and parameters of Fig. 13. (d) Model (1)–(16) at $\tau_G = 1.4 \text{ s}$; other parameters are the same as those in Fig. 13. C—congested traffic, F—free flow, S—synchronized flow. Minimum and maximum highway capacities, respectively, are: $C_{min} = 2q_{in} + q_{on, min}$ and $C_{max} = 2q_{in} + q_{on, max}$. Calculated parameters: $(q_{on, min}, q_{on, max}) = (430, 721)$ vehicles/h (b), $(517, 1142)$ vehicles/h (c), $(525, 1324)$ vehicles/h (d).

on the maintenance of free flow at the bottleneck is stronger than the overacceleration effect due to safety acceleration. The stronger the overacceleration effect on free flow is, the larger the on-ramp inflow rate $q_{on, max}$ at which free flow is still self-maintained at the bottleneck. This statement is confirmed by the result that the greatest values $q_{on, max}$ and C_{max} are realized when overacceleration is further increased through cooperation of overacceleration a_{OA} (4), (5) and overacceleration caused by safety acceleration at $g_{free} > G$ (20) [Fig. 24(d)].

As on a single-lane road, the stronger the overacceleration effect on free flow on a two-lane road is, the larger the on-ramp inflow rate $q_{on, \max}$ at which free flow is still self-maintained at the bottleneck. Indeed, in Fig. 25(b) overacceleration in a road lane is determined by safety acceleration at $g_{free} > G$ (20) only; as explained above [Fig. 24(b)], this overacceleration mechanism is weaker than that determined by overacceleration a_{OA} (4), (5). Therefore, we have found that cooperation of overacceleration due to safety acceleration at $g_{free} > G$ (20) and overacceleration through lane-changing can self-maintain free flow at the bottleneck only up to a relative small on-ramp inflow rate $q_{on, \max}$ [Fig. 25(b)]. In Fig. 25(c), overacceleration in a road lane is determined by a_{OA} (4), (5) that, as above-mentioned, is stronger than overacceleration due to safety acceleration at $g_{free} > G$ (20). As a result, we have found that cooperation of two overacceleration mechanisms caused by a_{OA} and through lane-changing maintains free flow at the bottleneck up to a considerably larger value $q_{on, \max}$ [Fig. 25(c)] than that in Fig. 25(b).

Finally, in Fig. 25(d) related to model (1)–(16) at $\tau_G = 1.4$ s there is cooperation of three mechanisms of overacceleration (due to a_{OA} (4), (5), through safety acceleration at $g_{free} > G$ (20), and through lane-changing). Then, we have found the greatest maximum on-ramp inflow-rate $q_{on, \max}$ (and, therefore, the greatest maximum highway capacity $C_{\max}$ at the given q_{in}). This shows that the cooperation of three mechanisms of overacceleration exhibits the strongest effect on the self-maintaining of free flow at the bottleneck.

B. Traffic flow consisting of automated-driving ACC vehicles moving on a single-lane road with bottleneck

In traffic consisting of ACC vehicles (22), (23), we have found the strongest effect of overacceleration on traffic flow [Fig. 26(a)]: The maximum highway capacity $C_{\max} = q_{in} + q_{on, \max} = 2936$ vehicles/h is only a few vehicles less than the maximum value of the total flow rate $q_{sum} = q_{in} + q_{on}$ that is possible in traffic flow on a single-lane road $q_{sum, \max} = 3600/(\tau_d + d/v_{free}) \approx 2939$ vehicles/h.¹⁹

In accordance with results of Sec. IV D, this large value of maximum highway capacity $C_{\max}$ at the bottleneck in traffic consisting of ACC vehicles can be explained by a very weak suppression of overacceleration through speed adaptation in the ACC model (22), (23).²⁰

¹⁹Note that the maximum highway capacity found in Ref. [68] for a two-lane road with the bottleneck $C_{\max} = 2q_{in} + q_{on, \max} = 5868$ vehicles/h is nearly equal to the doubled value of the maximum capacity of single-lane road that is 5872 vehicles/h. Respectively, the maximum capacity for a two-lane road is also only slightly less than the maximum value of the total flow rate 5878 vehicles/h that is possible in traffic flow on a two-lane road. This result is explained by a very weak suppression of overacceleration through speed adaptation in the ACC model (22), (23) found in Sec. IV D.

²⁰However, as shown in Ref. [72], in mixed traffic flow consisting of human-driving vehicles with a low share of randomly distributed ACC vehicles, a single ACC vehicle can lead to traffic breakdown at the bottleneck at a significantly lower traffic flow rate. This is because the ACC vehicle can cause a large local speed decrease at the bottleneck.

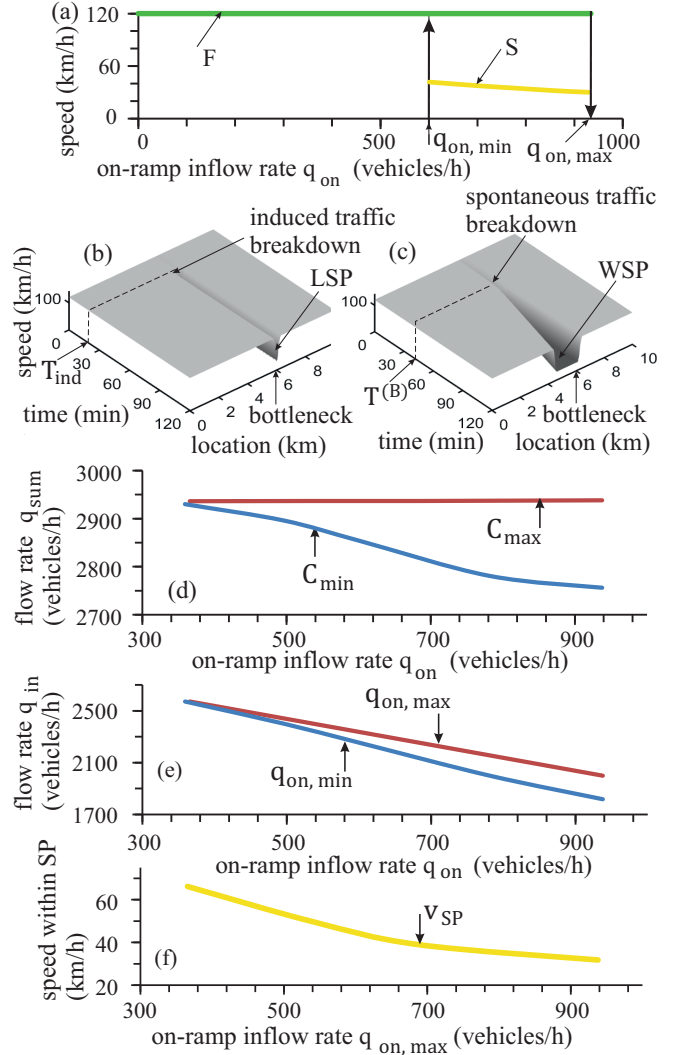


FIG. 26. Characteristics of highway capacity of traffic consisting of ACC vehicles moving on a single lane-road with on-ramp bottleneck simulated with model (22), (23). (a) Speed–on-ramp-inflow characteristic at $q_{in} = 2000$ vehicles/h. (b), (c) Speed in space and time related to (a) at $q_{on} = q_{on, \min} = 781$ vehicles/h (b) and at $q_{on} = q_{on, \max} = 936$ vehicles/h (c) that is close to $q_{on} = q_{on, \max} = 936$ vehicles/h. In panel (b), traffic breakdown has been induced in free flow at $T_{ind} = 20$ min through application of addition on-ramp inflow impulse $\Delta q_{on} = 619$ vehicles/h of duration $\Delta t = 2$ min. In panel (c), $T^{(B)} = 43$ min. (d) Dependencies of minimum highway capacity $C_{on, \min} = q_{in} + q_{on, \min}$ and maximum highway capacity $C_{on, \max} = q_{in} + q_{on, \max}$ on q_{on} . (e) Dependencies $q_{in}(q_{on})$ related to $C_{on, \min}(q_{on})$ (curve denoted by $q_{on, \min}$) and to $C_{on, \max}(q_{on})$ (curve denoted by $q_{on, \max}$), respectively. (f) Dependence of the average speed in synchronized flow v_{SP} within SP resulting from spontaneous traffic breakdown at the bottleneck as function of $q_{on, \max}$. Other model parameters are the same as those in Fig. 8.

In traffic consisting of ACC vehicles (22), (23), at $q_{on} = q_{on, \min}$ an LSP occurs [Fig. 26(b)]. It has been found that the result of the occurrence of the LSP at $q_{on} = q_{on, \min}$ is independent of the chosen value q_{in} used in simulations presented in Figs. 26(d) and 26(e). We have also found that when

at a given q_{in} the on-ramp inflow rate q_{on} decreases below the threshold value $q_{on} = q_{on, min}$, overacceleration overcomes speed adaptation within the LSP leading to the return $S \rightarrow F$ transition at the bottleneck [up-arrow at $q_{on} = q_{on, min}$ in Fig. 26(a)].

It should be emphasized that the larger the speed within the LSP, the easier the return $S \rightarrow F$ transition occurs at the bottleneck. This feature of the return $S \rightarrow F$ transition can explain why the difference between the maximum and minimum on-ramp inflow rates $q_{on, max} - q_{on, min}$, which is equal to 155 vehicles/h at $q_{in} = 2000$ vehicles/h [Fig. 26(a)], decreases drastically to 5 vehicles/h found for a single-lane road in Ref. [68] at $q_{in} = 2571$ vehicles/h. First we note that as for a two-lane road (Fig. 24(c) of Ref. [68]), the maximum capacity C_{max} of a single-lane road with the bottleneck does not depend on the flow rate q_{in} at chosen model parameters [Fig. 26(d)]. Therefore, the larger the flow rate q_{in} , the lower should be the value of $q_{on, max}$ [Fig. 26(e)]. In turn, the lower the value of $q_{on, max}$, the larger the mean synchronized flow speed v_{SP} within an SP occurring at the bottleneck at $q_{on} = q_{on, max}$ [Fig. 26(f)]. The mean synchronized flow speed within the LSP at the threshold value $q_{on} = q_{on, min}$ is even larger than the speed v_{SP} shown in Fig. 26(f) for a larger on-ramp inflow rate $q_{on} = q_{on, max}$. The larger the speed v_{SP} within the LSP, the easier the return $S \rightarrow F$ transition occurs. Therefore, when q_{in} increases, the return $S \rightarrow F$ transition occurs at the bottleneck already at a small decrease in q_{on} below $q_{on, max}$, i.e., at a small difference $q_{on, max} - q_{on, min}$ [Fig. 26(e)].

IX. MICROSCOPIC EFFECT OF OVERACCELERATION COOPERATION ON $S \rightarrow F$ INSTABILITY

We show that overacceleration cooperation has a considerable impact on spatiotemporal dynamics of an $S \rightarrow F$ instability on a two-lane road in traffic flow consisting of either human-driving vehicles or automated-driving TPACC vehicles described by the model (1)–(16).

A. Overacceleration cooperation and $S \rightarrow F$ instability on a two-lane road without bottlenecks

In Ref. [68], we have shown that overacceleration a_{OA} (4), (5) on a single-lane road without bottlenecks can cause an $S \rightarrow F$ instability that exhibits the nucleation nature. It can be expected that on a two-lane road overacceleration a_{OA} (4), (5) can also cause the $S \rightarrow F$ instability (Fig. 27). Indeed, a local disturbance in initial homogeneous synchronized flow caused by an addition acceleration of one of the vehicles during $\delta t = 5$ s initiates the $S \rightarrow F$ instability [Figs. 27(a) and 27(b)]. Contrarily, if the same vehicle accelerates during $\delta t = 4.5$ s (i.e., only 0.5 s shorter), then no $S \rightarrow F$ instability occurs [Fig. 27(c)]. Here we show that cooperation of overacceleration in a road lane a_{OA} (4), (5) with overacceleration caused by lane-changing (Sec. VI) leads to spatiotemporal lane-changing effects that qualitatively change the dynamics of the $S \rightarrow F$ instability in comparison with that found in Ref. [68].

Acceleration of vehicle 1 [Figs. 28(a) and 29(a)] causes a continuous increase in the speed of following vehicles 2–7 in the right lane [Fig. 29(a)]. As in Ref. [68], this $S \rightarrow F$

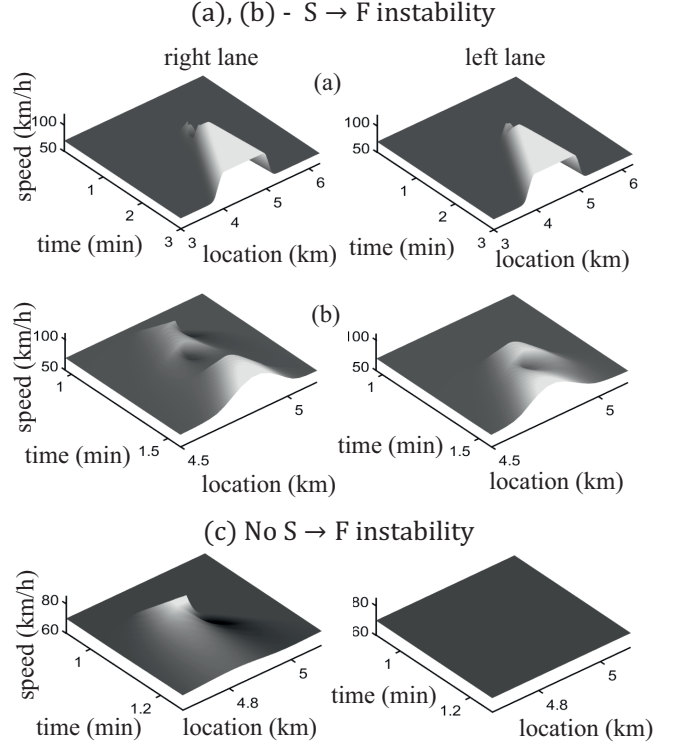


FIG. 27. Nucleation nature of $S \rightarrow F$ instability on a two-lane road caused by cooperation of overacceleration in a road lane a_{OA} (4), (5) with overacceleration caused by lane-changing. Simulations with model (1)–(16) at $\tau_G = 1.4$ s made on a two-lane road of length 8 km without bottlenecks with initial steady synchronized flow state at speed $v = 70$ km/h and space gap between vehicles $g = 33$ m. Vehicle speed in space and time in the right lane (left subplots) and in left lane (right subplots): (a) $S \rightarrow F$ instability is caused by initial local speed increase of a vehicle [vehicle 1 in Fig. 28(a)] moving in the right lane simulated through short-time acceleration of the vehicle with $a = 1$ m/s² during $\delta t = 5$ s. (b) The beginning of the $S \rightarrow F$ instability in panel (a) shown in a larger scale. (c) $S \rightarrow F$ instability does not occur if the same vehicle as that in panels (a), (b) accelerates with the same acceleration, however, during $\delta t = 4.5$ s, i.e., 0.5 s shorter. Other model parameters are the same as those in Fig. 13.

instability causing the avalanche-like local speed increase in the initial synchronized flow is due to overacceleration a_{OA} in the right lane: Indeed, regions on trajectories, in which condition $a_{OA} > 0$ is satisfied, become broader along the successive sequence of vehicles 2–7 [Fig. 28(a)].

1. Interruption of $S \rightarrow F$ instability

However, contrary to a single-lane road studied in Ref. [68], vehicles in the left lane that move initially in synchronized flow at $v = 70$ km/h search for the opportunity to change to the higher speed region caused by the $S \rightarrow F$ instability in the right lane. After vehicle 8 in the left lane [vehicle 8-left in Fig. 28(b)] has changed to the right lane [vehicle 8-right in Figs. 28(a) and 29(a)], the situation changes basically: Vehicles 9–11 in the right lane that follow vehicle 8-right begin to decelerate [Fig. 29(a)]. Because the speeds of vehicles 10 and 11 become less than $v_{syn} = 80$ km/h in Eq. (4), along

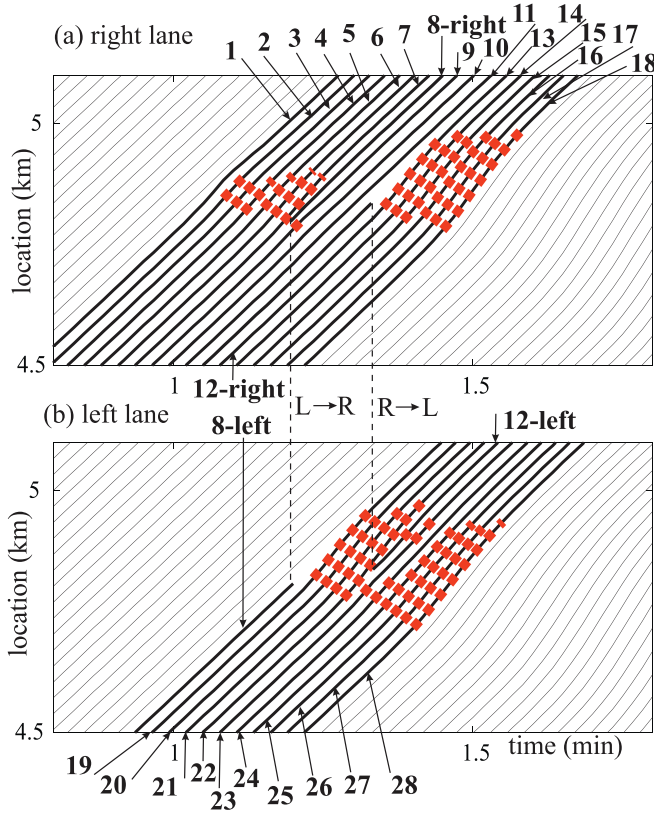


FIG. 28. Continuation of Figs. 27(a) and 27(b). (a), (b) Simulated vehicle trajectories within local speed increase in synchronized flow in the right lane (a) and left lane (b). $L \rightarrow R$ lane-changing of vehicle 8 is marked by dashed vertical line $L \rightarrow R$. $R \rightarrow L$ lane-changing of vehicle 12 is marked by dashed vertical line $R \rightarrow L$. Part of trajectories within which condition $a_{OA} > 0$ is satisfied are marked through colored red squares.

trajectories of vehicles 10 and 11 overacceleration $a_{OA} = 0$ [Fig. 28(a)]. This means that the $S \rightarrow F$ instability in the right lane is abruptly interrupted through $L \rightarrow R$ lane-changing. The interruption of the development of the $S \rightarrow F$ instability in the right lane occurs because just after lane-changing the speed of vehicle 8-right is considerably lower than the speed of vehicle 9 whereas time headway of vehicle 9 to preceding vehicle 8-right is considerably shorter than the safe-time headway τ_{safe} ; therefore, vehicle 9 must decelerate strongly [Figs. 29(a) and 29(b)].

2. $S \rightarrow F$ instability in the left lane causing repetition of $S \rightarrow F$ instability in the right lane

There are three dynamic effects following $L \rightarrow R$ lane-changing: (i) The interruption of the $S \rightarrow F$ instability in the right lane (Sec. IX A 1), (ii) the beginning of the $S \rightarrow F$ instability in the left lane, and (iii) the repetition of the $S \rightarrow F$ instability in the right lane.

The $S \rightarrow F$ instability in the left lane is caused by overacceleration through $L \rightarrow R$ lane-changing of vehicle 8: Time headway of vehicle 19 in the left lane increases considerably [Figs. 28(b), 29(c), and 29(d)]. This results in overacceleration of vehicle 19 [$a_{OA} > 0$ on trajectory 19 in Fig. 28(b)]. This

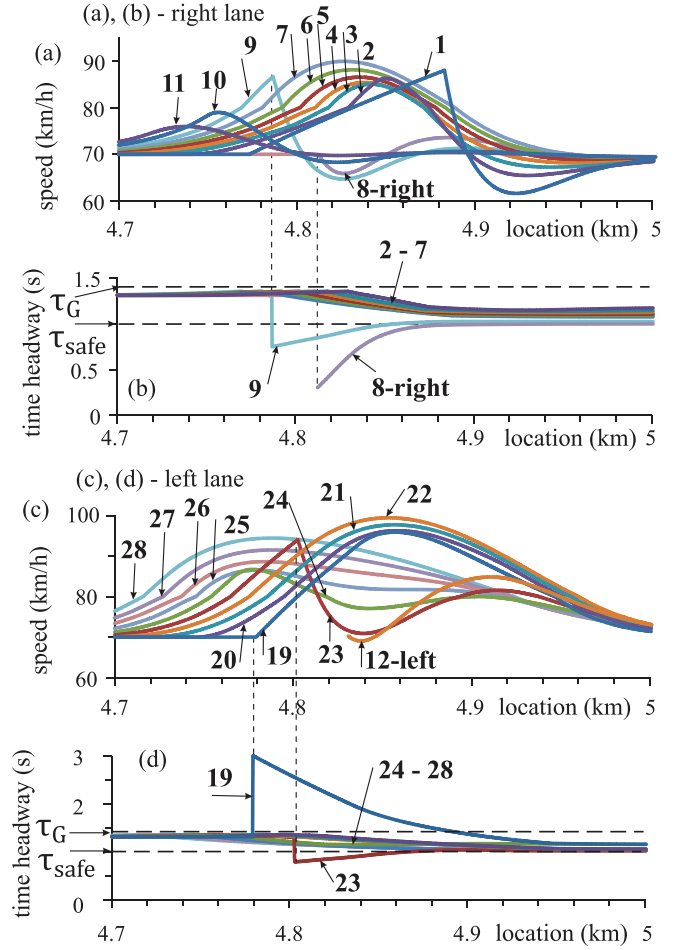


FIG. 29. Continuation of Fig. 28. Microscopic vehicle speeds (a), (c) and time headway (b), (d) for the right lane (a), (b) and the left lane (c), (d) for some of the vehicles whose trajectories are labeled by the same numbers as those in Fig. 28, respectively.

overacceleration causes the $S \rightarrow F$ instability in the left lane [vehicles 20–22 in Figs. 28(b) and 29(c)].

Now vehicles following vehicle 8-right in the right lane, which due to the interruption of the $S \rightarrow F$ instability in the right lane should move in synchronized flow, search for the opportunity to change to a higher speed region caused by the $S \rightarrow F$ instability in the left lane: After vehicle 12 in the right lane [vehicle 12-right in Fig. 28(a)] has changed to the left lane [vehicle 12-left in Figs. 28(b) and 29(c)], two effects occur [Figs. 29 and 30]: (i) Because vehicle 12-right changes the lane, time headway of vehicle 13 in the right lane increases considerably [Fig. 30(b)]. (ii) Vehicle 23 that follows vehicle 12-left in the left lane decelerates strongly [Fig. 29(c)].

The first effect of $R \rightarrow L$ lane-changing causes repetition of the $S \rightarrow F$ instability in the right lane (Fig. 30). Indeed, time headway of vehicle 13 increases [Fig. 30(b)]; this leads to overacceleration of vehicle 13 [$a_{OA} > 0$ on trajectory 13 in Fig. 28(a)]. This causes the repetition of the $S \rightarrow F$ instability in the right lane [vehicles 14–18 in Fig. 30(a)].

The second effect of $R \rightarrow L$ lane-changing, which leads to deceleration of vehicle 23 following vehicle 12-left, does not cause interruption of the $S \rightarrow F$ instability in the left

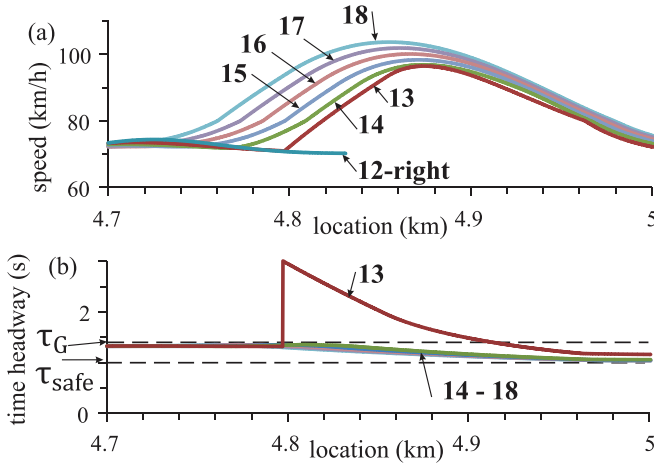


FIG. 30. Continuation of Fig. 28. Repetition of $S \rightarrow F$ instability in the right lane. Microscopic vehicle speeds (a) and time headway (b) in the right lane for some of the vehicles whose trajectories are labeled by the same numbers as those in Fig. 28, respectively.

lane. This is in contrast to $L \rightarrow R$ lane-changing of vehicle 8 causing the interruption of the $S \rightarrow F$ instability in the right lane (Sec. IX A 1). To explain it, we note that along trajectory of vehicle 24 following vehicle 23 in the left lane overacceleration a_{OA} does not disappear [trajectory 24 in Fig. 28(b)]. For this reason, even due to speed decrease of vehicle 23, the $S \rightarrow F$ instability in the left lane continues to develop [Fig. 28(b)]. Contrarily, after $L \rightarrow R$ lane-changing of vehicle 8 has occurred, along trajectory of vehicle 10 overacceleration $a_{OA} = 0$ [vehicle 10 in Fig. 28(a)]; this causes the interruption of the $S \rightarrow F$ instability in the right lane. Finally, due to the $S \rightarrow F$ instability in the left and right lanes, local regions of free flow are built in both lanes [Fig. 27(a)].

B. Microscopic characteristics of overacceleration cooperation under $S \rightarrow F$ instability at bottleneck

As found from stochastic three-phase traffic flow models [71], simulations of deterministic model (1)–(16) show also that there can be a spontaneous $S \rightarrow F$ transition at the bottleneck (Fig. 31), which is governed by an $S \rightarrow F$ instability [Figs. 32–34]. The uninterrupted development of the $S \rightarrow F$ instability leads to an $S \rightarrow F$ transition at the bottleneck, i.e., free flow is recovered at the bottleneck. There are two possibilities: (i) After a short time interval, an $F \rightarrow S$ transition occurs spontaneously at the bottleneck, i.e., the recovered free flow persists only the short time interval [Fig. 31(a)]. (ii) The recovered free flow persists during a long enough time interval at the bottleneck [Fig. 31(b)].

As in stochastic models [71], the $S \rightarrow F$ instability at the bottleneck is initiated spontaneously by a “speed peak” [Fig. 33(a)]. After vehicle 1 moving in the right lane has reached the downstream boundary of synchronized flow at the bottleneck, it accelerates from synchronized flow to free flow downstream of the bottleneck. Vehicle 2 that is upstream of vehicle 1 also starts to accelerate. However, vehicle LR-1 that has changed from the left lane [Fig. 32(a)] forces vehicle 2 to decelerate, creating the speed peak [“speed peak due to $L \rightarrow R$ ” in Fig. 33(a)].

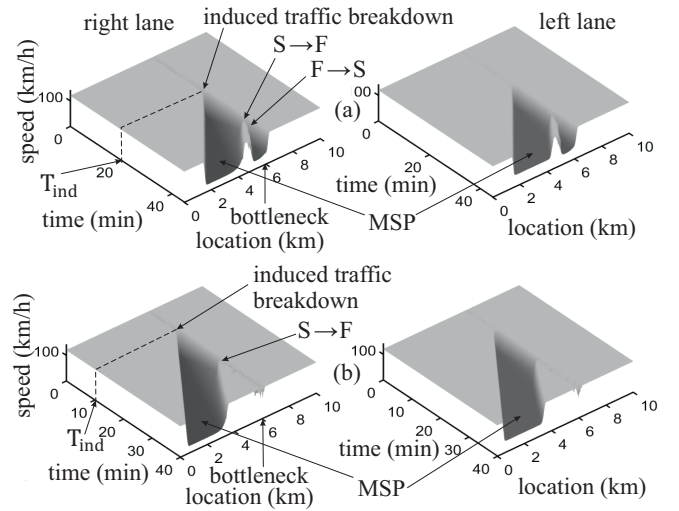


FIG. 31. Cooperation of overacceleration in road lane a_{OA} with overacceleration due to lane-changing leading to $S \rightarrow F$ instability on a two-lane road with on-ramp bottleneck at $x_{on} = 6$ km. Simulations with model (1)–(16) at $\tau_G = 1.4$ s, $q_{in} = 2400$ (vehicles/h)/lane made with $q_{on} = 495$ vehicles/h (a) and $q_{on} = 500$ vehicles/h (b). Vehicle speed is space and time in the right lane (left subplots) and in the left lane (right subplots). Traffic breakdown has been induced in free flow at $T_{ind} = 20$ min (a) and $T_{ind} = 10$ min (b) through application of addition on-ramp inflow impulse $\Delta q_{on} = 505$ (a) and 500 vehicles/h (b) of duration $\Delta t = 1$ min. Spontaneous $S \rightarrow F$ and $F \rightarrow S$ transitions at the bottleneck are labeled by arrows $S \rightarrow F$ and $F \rightarrow S$, respectively. MSP emerges at the bottleneck due to the development of the $S \rightarrow F$ instability. Other model parameters are the same as those in Fig. 13.

In this paper, we have revealed that overacceleration cooperation (Secs. V and VI D) effects crucially on microscopic traffic dynamics of the $S \rightarrow F$ instability. We limit a further consideration of the case shown in Fig. 31(b).

1. Effect of lane-changing and merging of on-ramp vehicles in the right lane

In the right lane, the $S \rightarrow F$ instability is caused by overacceleration cooperation through $R \rightarrow L$ lane-changing and overacceleration a_{OA} in a road lane. Contrarily, speed adaptation that occurs through the merging of on-ramp vehicles together with $L \rightarrow R$ lane-changing tries to prevent the $S \rightarrow F$ instability development [Figs. 32–34].

Indeed, after $L \rightarrow R$ lane-changing [vehicle LR-3 in Fig. 32(a)] vehicles 3 and RL-3 following vehicle LR-3 in the right lane should decelerate adapting their speeds to the speed of vehicle LR-3. This causes $R \rightarrow L$ lane-changing of vehicle RL-3 [Fig. 32(a)]. In its turn, this $R \rightarrow L$ lane-changing results in time headway increase for vehicle 4 in the right lane [Fig. 33(b)] and, therefore, in overacceleration of vehicle 4 [“4, overacceleration” in Figs. 33(a) and 33(b)]. Overacceleration of vehicle 4 causes a local speed increase upstream of the on-ramp merging region [“speed increase due to overacceleration” in Fig. 33(a)]. However, later due to on-ramp vehicle merging [vehicle on-1 in Fig. 32(a)], vehicle 4 must decelerate, i.e., speed adaptation effect is realized [“speed adaptation due to on-ramp vehicle” in Figs. 33(a) and 33(b)].

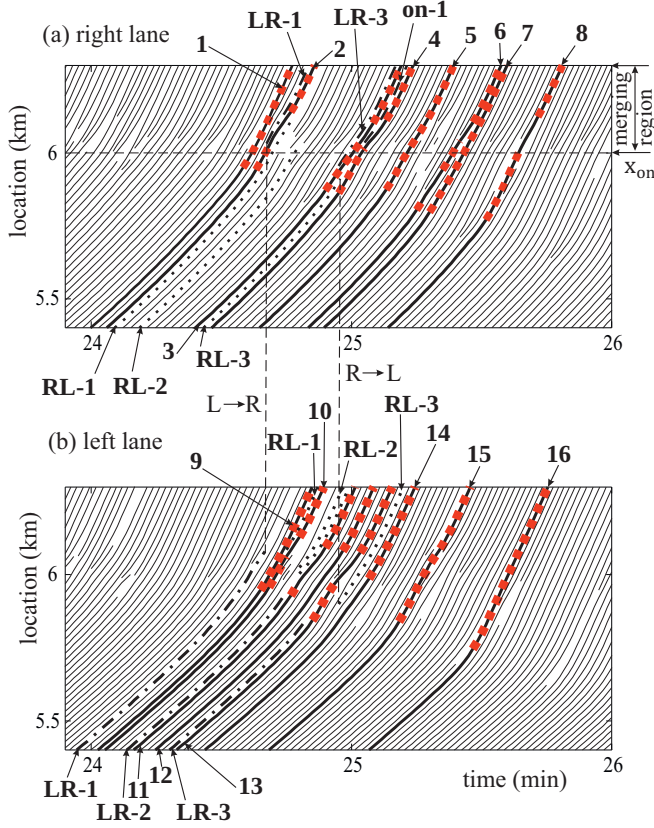


FIG. 32. Continuation of Fig. 31(b). (a), (b) Simulated vehicle trajectories in synchronized flow in the right lane (a) and left lane (b). One of the $L \rightarrow R$ lane-changing effects (for vehicle LR-1) is marked by dashed vertical line $L \rightarrow R$. One of the $R \rightarrow L$ lane-changing effects (for vehicle RL-3) is marked by dashed vertical line $R \rightarrow L$. Part of trajectories within which condition $a_{0A} > 0$ is satisfied are marked through colored red squares. Some of the vehicle trajectories used for explanations in text have been highlighted by bold as follows: solid trajectories—vehicles that do not change lane, dotted trajectories—vehicles that change from the right lane to left lane ($R \rightarrow L$ lane-changing), dashed-dotted trajectories—vehicles that change from the left lane to right lane ($L \rightarrow R$ lane-changing), dashed trajectory—vehicle on-1 that has merged from the on-ramp.

For this reason, the local speed increase upstream of the on-ramp merging region changes to a local speed decrease within the on-ramp merging region. Rather than the interruption of the $S \rightarrow F$ instability, this competition of overacceleration and speed adaptation leads only to a decrease in the growth of the $S \rightarrow F$ instability.²¹

The competition between overacceleration and speed adaptation leading to the sequence of the local speed increase and local speed decrease on trajectory 4 explained above can occur several times for some of the following vehicles [vehicles 5–7

²¹However, in some other cases we have found (not shown) that a sequence of $L \rightarrow R$ lane-changing and vehicle merging from the on-ramp can lead to a short-time interruption of the development of the $S \rightarrow F$ instability in the right lane; this interruption effect is qualitatively similar to $S \rightarrow F$ instability interruption explained in Sec. IX A 1 [Fig. 29(a)].

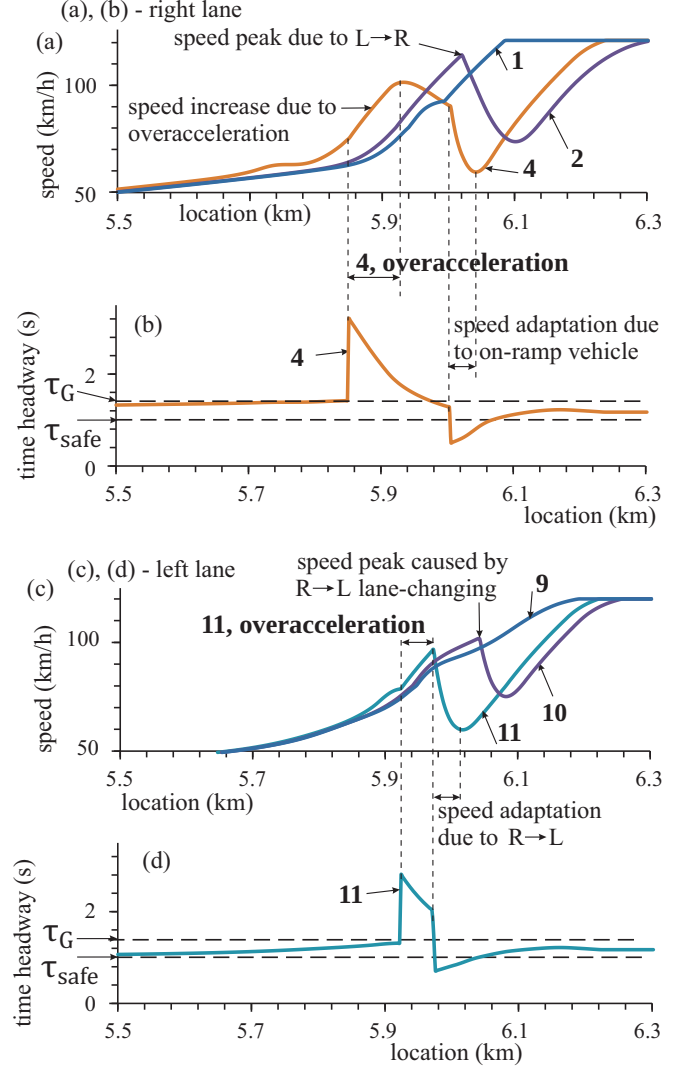


FIG. 33. Continuation of Fig. 32. Speed peak in the right lane initiating $S \rightarrow F$ instability in the right lane and the beginning of $S \rightarrow F$ instability in the left lane. Microscopic vehicle speeds (a), (c) and time headway (b), (d) for the right lane (a), (b) and the left lane (c), (d) for some of the vehicles whose trajectories are labeled by the same numbers as those in Fig. 32, respectively.

in Fig. 34(a)]. Finally, the $S \rightarrow F$ instability is finished by the $S \rightarrow F$ transition at the bottleneck [vehicle 8 in Fig. 34(a)] and the formation of a local speed decrease in free flow at the bottleneck caused mostly by on-ramp vehicles [“local speed decrease at bottleneck” in Fig. 34(a)].

2. Emergence of speed peak in the left lane

Vehicle 9 accelerates within the downstream boundary of the synchronized flow to free flow downstream [Fig. 33(c)]. Vehicle 10 that follows vehicle 9 also starts to accelerate within the downstream boundary of the synchronized flow. However, a slower moving vehicle RL-1 that has changed from the right lane [vehicle RL-1 in Fig. 32(b)] forces vehicle 10 to decelerate, creating the speed peak on vehicle trajectory 10 [“speed peak caused by $R \rightarrow L$ lane-changing” in Fig. 33(c)].

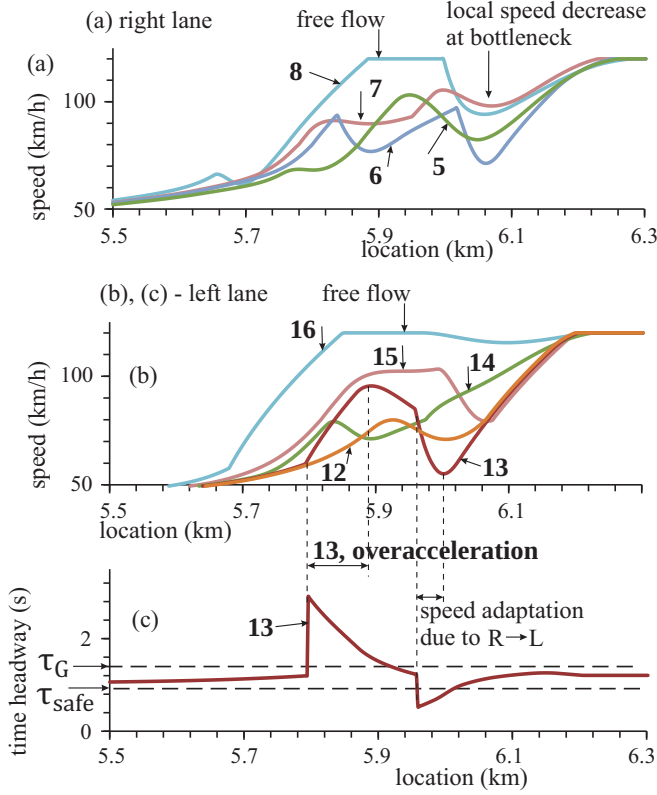


FIG. 34. Continuation of Fig. 32. Development of $S \rightarrow F$ instability in the right lane (a) and the left lane (b), (c): Microscopic vehicle speeds (a), (b) and time headway (c) for some of the vehicles whose trajectories are labeled by the same numbers as those in Fig. 32, respectively.

3. Effect of lane-changing on $S \rightarrow F$ instability in the left lane

In the left lane, the $S \rightarrow F$ instability is caused by overacceleration cooperation through $L \rightarrow R$ lane-changing and overacceleration a_{OA} in a road lane. Contrarily, speed adaptation through $R \rightarrow L$ lane-changing tries to interrupt the $S \rightarrow F$ instability development.

Indeed, because upstream of the speed peak in the left lane the speed is low, $L \rightarrow R$ lane-changing is realized [vehicle LR-2 in Fig. 32(b)]. This results in a time-headway increase for vehicle 11 [Fig. 33(d)] and, therefore, in overacceleration of vehicle 11 [“11, overacceleration” in Fig. 33(c)]. Shortly afterward, however, two vehicles change from the right lane to the left lane in front of vehicle 11 [the first of these two vehicles in front of vehicle 11 is vehicle RL-2 marked in Fig. 32]. These $R \rightarrow L$ lane-changing cause speed adaptation of vehicle 11 [“speed adaptation due to $R \rightarrow L$ ” in Figs. 33(c) and 33(d)].

Thus, there is a sequence of the local speed increase due to overacceleration with the following speed decrease caused by speed adaptation. As in the right lane (Sec. IX B 1), the local speed increase caused by overacceleration propagates upstream and it can become lower for some of the following vehicles moving upstream of vehicle 11 [e.g., vehicles 12 and 14 in Fig. 34(b)]. Nevertheless, for other vehicles overacceleration leads to the growth of the local speed increase [vehicle 13 in Fig. 34(b)]. This occurs when due to $L \rightarrow R$

lane-changing [vehicle LR-3 in Fig. 32(b)] time-headway of vehicle 13 increases considerably [vehicle 13 in Fig. 34(c)] leading to a greater overacceleration [“13, overacceleration” in Figs. 34(b) and 34(c)]. On average, due to overacceleration the local speed increase grows upstream [vehicles 13 and 15 in Fig. 34(b)]. Finally, the $S \rightarrow F$ instability is finished by the $S \rightarrow F$ transition in the left lane [vehicle 16 in Fig. 34(b)].

X. TRAFFIC BREAKDOWN: OVERACCELERATION VERSUS OVERDECELERATION

Up to now in all simulations of the deterministic three-phase traffic flow model of Sec. II, we have used values of model parameters at which there is no vehicle overdeceleration and, therefore, no classical traffic flow instability exists. For this reason, moving jams occur in none of the simulations even when the synchronized flow speed is very low. However, empirical data have shown that moving jams do emerge in synchronized flow when the synchronized flow speed is low enough [62]. For simplification of moving jam simulations in Sec. X, we limit a consideration of traffic flow of human-driving vehicles moving on a single-lane road with the bottleneck.

A. Generalization of deterministic model for moving jam simulations in human-driving vehicular traffic

To simulate moving jam emergence in deterministic model (1)–(12), as already made in some stochastic three-phase traffic flow models (e.g., Refs. [69,70]), we assume that dynamic model parameters (at least some of the model parameters) can change at a synchronized flow speed that is less than some characteristic speed denoted by v_{pinch} :

$$\mathcal{P} = \begin{cases} \mathcal{P} & \text{at } v \geq v_{pinch}, \\ \mathcal{P}_{pinch} & \text{at } v < v_{pinch}, \end{cases} \quad (24)$$

where $\mathcal{P}$ denotes one of the model parameters τ_G , τ_{safe} , K_1 , K_2 , K_3 , $K_4^{(1)}$, $K_4^{(2)}$, etc., used in model (1)–(12), whereas the subscript *pinch* in $\mathcal{P}_{pinch}$ is used to distinguish values of the same model parameters for low speeds $v < v_{pinch}$. We assume also that in Eq. (4) condition

$$v_{pinch} < v_{syn} \quad (25)$$

is satisfied. In the generalized model, rather than functions (16), we use formulations for the speed functions $G(v)$ and $g_{safe}(v)$ ²² in which at $v \rightarrow 0$ the space gap tends to some minimum space-gap g_{min} between vehicles.²³ Respectively,

²²In general, synchronization space gap G and safe space gap g_{safe} can also be formulated as functions of different vehicle variables; for example, in Ref. [69] we have used the synchronization space gap G as a function of both the speed v and speed difference Δv .

²³The use of the minimum space-gap g_{min} is well-known for many standard deterministic traffic flow models (see, e.g., Ref. [39]). Note that the incorporation of the minimum gap g_{min} in the model is physically equivalent to *slow-to-start-rule* used in cellular automata and other stochastic traffic flow models [25,99]. Indeed, when a vehicle is in a standstill within a wide moving jam and the preceding vehicle begins to accelerate from the jam, then in accordance with

in Eqs. (1)–(12), rather than $g_{\text{safe}} = v\tau_{\text{safe}}$ and $G = v\tau_G$ (16), we get

$$g_{\text{safe}} = \begin{cases} v\tau_{\text{safe}} & \text{at } v \geq v_{\text{pinch}}, \\ g_{\text{min}} + v(\tau_{\text{safe}} - \tau_{\text{min}}) & \text{at } v < v_{\text{pinch}}, \end{cases} \quad (26)$$

$$G = \begin{cases} v\tau_G & \text{at } v \geq v_{\text{pinch}}, \\ g_{\text{min}} + v(\tau_G - \tau_{\text{min}}) & \text{at } v < v_{\text{pinch}}, \end{cases} \quad (27)$$

where $\tau_{\text{min}} = g_{\text{min}}/v_{\text{pinch}}$, g_{min} is a parameter.

B. Can vehicle overdeceleration impact on the occurrence of traffic breakdown?

In real empirical traffic data, traffic breakdown at a bottleneck is an $F \rightarrow S$ transition that exhibits the empirical nucleation nature [62]. To study whether vehicle overdeceleration caused by driver overreaction in traffic flow consisting of human-driving vehicles (Sec. I) effects on the $F \rightarrow S$ transition, we make the following physical modeling.

1. Simulations of spontaneous $F \rightarrow S$ transition

In model (1)–(12), (16), in which no vehicle overdeceleration occurs, spontaneous traffic breakdown ($F \rightarrow S$ transition) is realized after a time delay $T^{(B)}$ [Fig. 35(a)] (see Sec. VII). When we use generalized model (1)–(12), (26), (27) of Sec. X A, in which at speeds $v < v_{\text{pinch}}$ vehicle overdeceleration does occur, we find that after traffic breakdown has occurred wide moving jams appear almost immediately [Fig. 35(b)]; in this case, at $t > T^{(B)}$ the spatiotemporal distribution of traffic congestion is qualitatively the same as that well-known from standard traffic flow models (e.g., Refs. [22,39,42,43]), in which traffic breakdown is explained by the classical traffic flow instability resulting from vehicle overdeceleration.

To demonstrate that vehicle overdeceleration does not affect features of the $F \rightarrow S$ transition at the bottleneck in Fig. 35(b), we note that traffic breakdown and its time delay $T^{(B)}$ do not depend whether there is vehicle overdeceleration or not. Indeed, we have found that both the microscopic spatiotemporal development of a local speed decrease in free flow at the bottleneck during time interval $t < T^{(B)}$ and time delay $T^{(B)}$ remain exactly the same in simulations presented in Fig. 35(a) when vehicle overdeceleration does not exist and in Fig. 35(b) when vehicle overdeceleration does exist: Vehicle overdeceleration in the generalized model (1)–(12), (26), (27) applied in Fig. 35(b) does not influence on the microscopic features of the development of the $F \rightarrow S$ transition (traffic breakdown) at all.

Thus, the microscopic features of traffic breakdown do not depend on whether vehicle overdeceleration is incorporated in the three-phase traffic flow model or not: The physics of the $F \rightarrow S$ transition is solely determined by vehicle overacceleration. The $F \rightarrow S$ transition leads to the emergence of synchronized flow at the bottleneck. Vehicle overdeceleration

occurring in this synchronized flow [Fig. 35(b)] causes the emergence of wide moving jams (called as $S \rightarrow J$ transitions): Wide moving jams result from a sequence of $F \rightarrow S \rightarrow J$ transitions.

2. Simulations of empirical induced $F \rightarrow S$ transition

For a deeper understanding of the effect of vehicle overdeceleration on real (empirical) vehicular traffic, we simulate empirical induced $F \rightarrow S$ transition shown in Fig. 1(a) with either model (1)–(12), (16) [Fig. 35(c)] or with generalized model (1)–(12), (26), (27) [Fig. 35(d)].

For physical modeling of empirical data in Fig. 1(a) we use the fact that there is a long time delay $T^{(B)} \approx 24$ min of the $F \rightarrow S$ transition in both Figs. 35(a) and 35(b). Therefore, free flow at bottleneck B is in metastable state with respect to the $F \rightarrow S$ transition at $t < T^{(B)}$. With the use of a short on-ramp inflow impulse applied to downstream on-ramp (B-down) we induce an MSP that reaches upstream bottleneck B at $t < T^{(B)}$. The MSP induces traffic breakdown at bottleneck B [Figs. 35(c) and 35(d)]. Rather than on dynamics of the $F \rightarrow S$ transition, vehicle overdeceleration affects only the congested pattern resulting from the $F \rightarrow S$ transition: When model (1)–(12), (16) is used, in which no vehicle overdeceleration is realized, then either spontaneous or induced $F \rightarrow S$ transition causes the occurrence of the WSP in which no wide moving jams occur [Figs. 35(a) and 35(c)]; this is independent of how low synchronized flow speed is within the WSP. In contrast, when model (1)–(12), (26), (27) is used, in which vehicle overdeceleration can occur, then wide moving jams spontaneously emerge in low-speed synchronized flow with the speed $v < v_{\text{pinch}}$; this synchronized flow, in turn, originally resulted from either a spontaneous or induced $F \rightarrow S$ transition at bottleneck B [Figs. 35(b) and 35(d), respectively].

We can conclude that the occurrence of the $F \rightarrow S$ transition is solely determined by vehicle overacceleration: Independent of whether spontaneous [Figs. 35(a) and 35(b)] or induced $F \rightarrow S$ transition [Figs. 35(c) and 35(d)] is realized, vehicle overdeceleration does not affect features of the $F \rightarrow S$ transition at the bottleneck.

C. Traffic breakdown at bottleneck: Driver overacceleration, not driver overreaction

To emphasize a possible confusion by simulations of traffic congestion in human-driving vehicular traffic, we consider Fig. 36. The time delay of the spontaneous $F \rightarrow S$ transition $T^{(B)}$ decreases rapidly as the on-ramp inflow rate q_{on} , which is larger than $q_{\text{on, max}}$, continues to increase [Fig. 36(a)]. Therefore, at a large enough flow rate q_{on} , time-delay $T^{(B)}$ can be very short.

When model (1)–(12), (16) is used, in which there is no vehicle overdeceleration, the result of traffic breakdown [Fig. 36(b)] is qualitatively different from well-known results of the standard traffic flow models in which the classical traffic flow instability caused by driver overdeceleration leads to the emergence of moving jams (e.g., Refs. [18,22,23,39,43,45]).

Contrarily, when overdeceleration is realized in the generalized model (1)–(12), (26), (27), then the result of traffic breakdown is a congested pattern in which a sequence of wide moving jams emerge (such a congested pattern is

Eqs. (1)–(12), (26), (27) the vehicle waits to accelerate behind the preceding vehicle as long as the space gap is less than the minimum gap g_{min} .

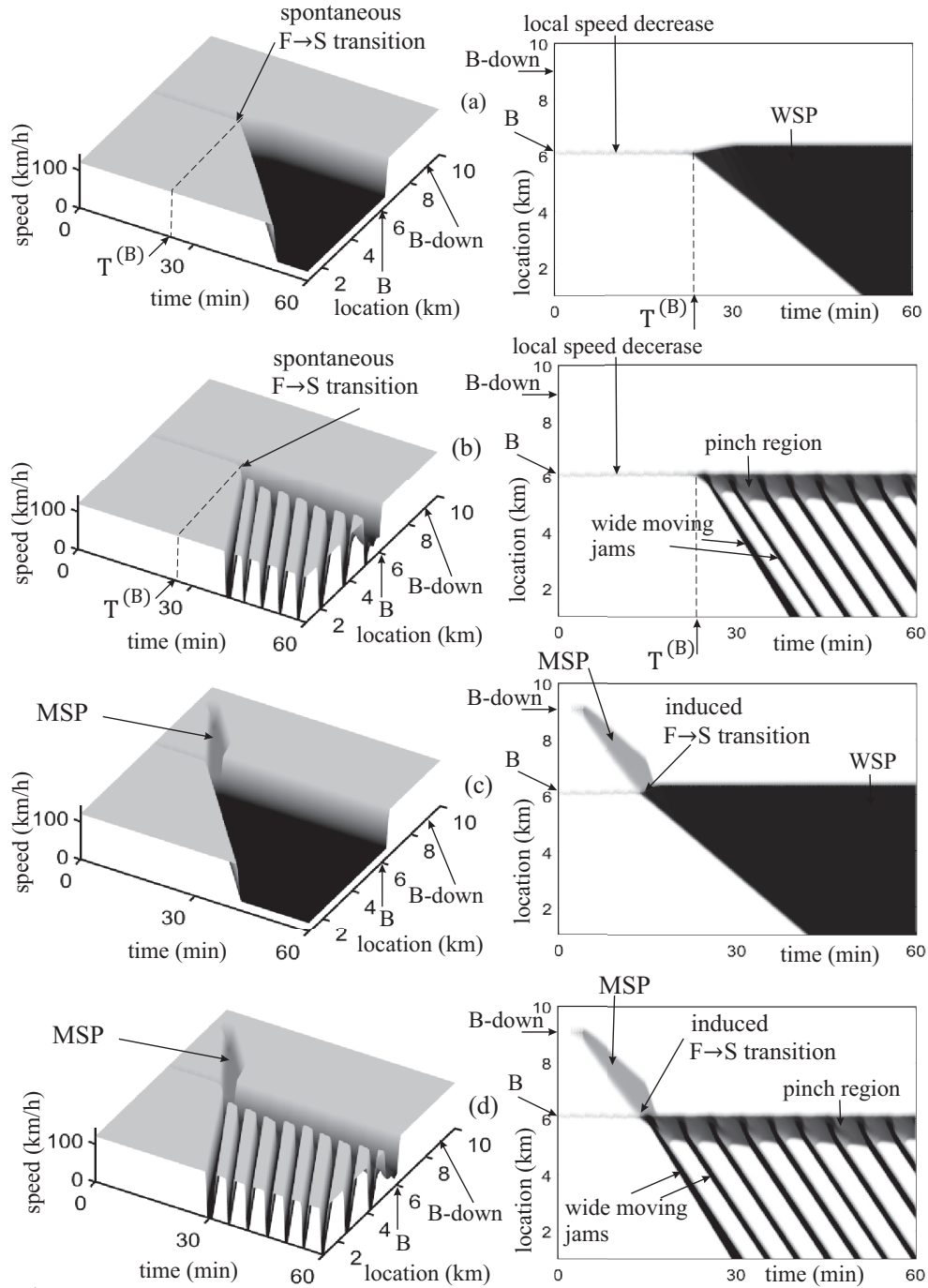


FIG. 35. Vehicle overacceleration versus vehicle overdeceleration: What is the cause of real (empirical) traffic breakdown at bottleneck? (a)–(d) Left subplots—simulations of speed in space and time on a single-lane road with two identical on-ramp bottlenecks denoted as “B” and “B-down” located at $x_{on} = 6$ km and $x_{on2} = 9$ km, respectively; right subplots—the same vehicle speed data as those left, respectively, presented by regions with variable shades of gray [shades of gray vary from white to black when the speed decreases from 120 km/h (white) to 0 km/h (black)]. In panels (a), (c), simulations of model (1)–(12), (16) at $q_{in} = 2250$ vehicles/h, $q_{on} = 568.5$ vehicles/h at bottleneck B and on-ramp inflow rate $q_{on2} = 0$ at bottleneck B-down; other model parameters are the same as those in Fig. 10 at which no vehicle overdeceleration is realized. In panels (b), (d), simulations of model (1)–(12), (26), (27), in which vehicle overdeceleration is realized at $v < v_{pinch}$ with model parameters $g_{min} = 5$ m, $v_{pinch} = 36$ km/h, at $v < v_{pinch}$ parameters $K_{3,pinch} = 0.1$ s⁻² and $K_{4,pinch}^{(2)} = 0.8$ s⁻¹; other model parameters are the same as those in panels (a), (c). In panels (c), (d), MSP propagating upstream has been induced at downstream bottleneck B-down at $t = 2$ min through application of on-ramp inflow impulse $\Delta q_{on2} = 122$ vehicles/h of duration 2 min; while reaching upstream bottleneck B, the MSP induces $F \rightarrow S$ transition (traffic breakdown) at upstream bottleneck B. In panels (b), (d), “pinch region” labels a road area within synchronized flow where several growing moving traffic jams merge.

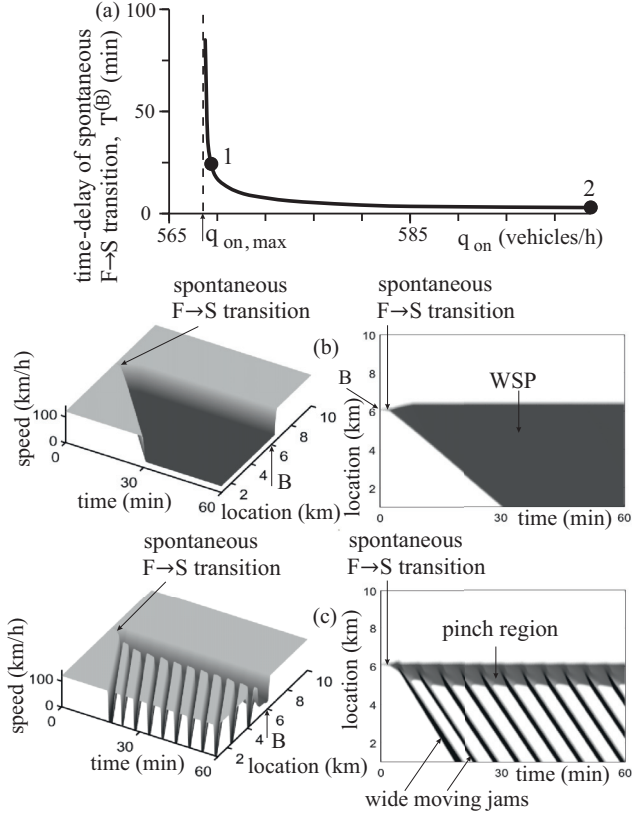


FIG. 36. Explanation of a possible confusion by simulations of traffic congestion. Simulations of the same models as those in Figs. 35(a) and 35(b) made on the same single-lane road in which only on-ramp bottleneck B is used as well as the same flow rate $q_{in} = 2250$ vehicles/h. (a) Time-delay of spontaneous traffic breakdown ($F \rightarrow S$ transition) $T^{(B)}$ as function of on-ramp inflow rate q_{on} ; function $T^{(B)}(q_{on})$ does not depend on whether the model used in Fig. 35(a) or the model used in Fig. 35(b) has been applied; $q_{on, max} = 568$ vehicles/h. (b), (c) Simulations of speed in space and time (left subplots) at $q_{on} = 600$ vehicles/h with the model used in Fig. 35(a) for panel (b) and with the model used in Fig. 35(b) for panel (c); right subplots—the same vehicle speed data as those left, respectively, presented by regions with variable shades of gray [shades of gray vary from white to black when the speed decreases from 120 km/h (white) to 0 km/h (black)]. In panel (a), point 1 is related to $q_{on} = 568.5$ vehicles/h used in Figs. 35(a) and 35(b) for which $T^{(B)} \approx 24$ min, whereas point 2 is related to panels (b), (c) for which $T^{(B)} \approx 3$ min.

called “general congested pattern” (GP) in three-phase traffic theory) [Fig. 36(c)]. The GP in Fig. 36(c) is qualitatively the same as well-known one for the standard traffic flow models (e.g., Refs. [22,39,42,43]).

To explain this similarity between the standard traffic models and three-phase traffic theory, we should recall that there are some common features of moving jam emergence due to vehicle overdeceleration in the standard traffic theory [1–45] and the three-phase traffic theory [62–65]. In both theories, when a large enough local speed decrease occurs in traffic flow, then vehicle overdeceleration causes a growing moving jam (Sec. I). Due to a large on-ramp inflow rate q_{on} in Figs. 35(b), 35(d), and 36(c), a sequence of vehicles merging

from the on-ramp causes a sequence of growing moving jams. Many of these jams interact during upstream jam propagation within a so-called pinch region within which the moving jams merge [“pinch region” in Figs. 35(b), 35(d), and 36(c)]. The jam interaction and jam merging result in a rare sequence of wide moving jams.²⁴ This spatiotemporal nonlinear jam dynamics, which is well-known since middle of 1990s [17,22,23,45], is qualitatively the same in the standard traffic models and three-phase traffic theory [62–65].

However, the origin of the emergence of the GP [Fig. 36(c)] is understood qualitatively different in standard traffic flow theory [35–44] and three-phase traffic theory [62–65]:

- In standard traffic flow theory [35–44], it is assumed that the origin of traffic breakdown in free flow at the bottleneck leading to the GP [Fig. 36(c)] is the classical traffic flow instability caused by vehicle overdeceleration due to driver overreaction. Contrary to the standard traffic flow theory, in three-phase traffic theory [62–65] it is assumed that the origin of traffic breakdown in free flow at the bottleneck is the $F \rightarrow S$ transition caused by a competition between speed adaptation and overacceleration, not overdeceleration, i.e., not the classical traffic flow instability.

In three-phase traffic theory [62–65], the explanation of the GP shown in Fig. 36(c) is as follows: There is almost no time-delay between the $F \rightarrow S$ transition and $S \rightarrow J$ transition within the sequence of the $F \rightarrow S \rightarrow J$ transitions presented in Fig. 36(c): At the chosen model parameters of the three-phase traffic model (1)–(12), (26), (27), moving jams occurs due to overdeceleration already in the emergent synchronized flow. However, as shown and explained in Sec. X B, the emergence of this synchronized flow caused by the $F \rightarrow S$ transition does not depend on whether there is vehicle overdeceleration or not.

XI. DISCUSSION

To disclose microscopic features of overacceleration, we have applied a methodology (Sec. III) in which overacceleration a_{OA} (4), which has been explicitly introduced in the model of Sec. II, has firstly been removed. Then, the change in model characteristics has been performed with the objective to search for possible traffic situations in which free flow metastability with respect to the $F \rightarrow S$ transition nevertheless occurs. The application of this methodology has led to disclose of microscopic features of competition between speed adaptation and overacceleration. Results found in the paper are as follows:

- (i) At a large enough synchronization time headway τ_G and long enough time headway τ_{free} between vehicles in free flow on a single-lane road with the bottleneck, speed adaptation causes a speed wave of the speed decrease propagating up-

²⁴Indeed, whereas the frequency of local speed decreases in traffic flow caused by vehicles merging from the on-ramp is determined by $q_{on} = 600$ vehicles/h in Fig. 36(c), the frequency of the wide moving jams is about 11 per hour.

stream. In this case, no free flow metastability with respect to the $F \rightarrow S$ transition is realized. Therefore, no overacceleration occurs (Sec. IV A).

(ii) When we have decreased τ_{free} , then at the same remaining model parameters free flow metastability with respect to the $F \rightarrow S$ transition is realized in free flow at the bottleneck and, therefore, overacceleration occurs. We have found that on microlevel of vehicle motion this overacceleration is caused by safety acceleration (Sec. IV B). At a given flow rate on the main road q_{in} , the overacceleration maintains free flow at the bottleneck: Up to some maximum inflow rate at the bottleneck $q_{\text{on, max}}$, no congestion propagates upstream (Sec. VIII A).

(iii) When instead of the decrease in τ_{free} , synchronization time headway decreases so that condition $\tau_G < \tau_{\text{free}}$ is satisfied, then speed adaptation cannot also fully suppress overacceleration: Free flow metastability is realized at the bottleneck (Sec. IV C). It turns out that overacceleration caused by safety acceleration maintains free flow at a larger value $q_{\text{on, max}}$ than it has been in the case of item (ii), i.e., the suppression of overacceleration through speed adaptation decreases (Sec. VIII A).

(iv) Finally, we have disclosed that overacceleration in traffic consisting of classical ACC vehicles (Helly's model) moving on a single-lane road on microlevel of vehicle motion is caused by safety acceleration (Sec. IV D). In this case, the suppression of overacceleration through speed adaptation is the lowest one: Due to strong overacceleration, free flow is maintained at the largest on-ramp inflow rate $q_{\text{on, max}}$ (Sec. VIII B). Comparison of simulations of Helly's model with simulations of the model of Sec. II under condition $a_{\text{OA}} = 0$ (17) made in Sec. IV have helped us to disclose microscopic features of competition between speed adaptation and overacceleration in a road lane; in this way, we have made the distinguishing between speed adaptation and overacceleration in the absence of lane changing.

Overacceleration caused by safety acceleration [items (ii)–(iv)] results from usual vehicle motion: No special features of overacceleration leading to its discontinuous character has been introduced in rules of vehicle motion that describe safety acceleration. Therefore, this overacceleration mechanism realizes without explicitly introducing it in the model.

When overacceleration mechanism a_{OA} (4) has been applied in addition in the model, then cooperation of this overacceleration with overacceleration caused by safety acceleration has been revealed (Sec. V). We have found that in traffic consisting of human-driving vehicles or TPACC vehicles overacceleration cooperation increases the inflow rate $q_{\text{on, max}}$ at which free flow is maintained at the bottleneck (Sec. VIII A).

The above methodology has been applied for traffic of human-driving vehicles and TPACC vehicles moving on a two-lane road with the bottleneck. Respectively, overacceleration mechanism a_{OA} (4) has been firstly removed. Then, we have found that at long enough values τ_G and τ_{free} overacceleration through lane-changing does not occur in traffic of human-driving vehicles and TPACC vehicles: Speed adaptation suppresses overacceleration both in a road lane and through lane-changing (Sec. VI A). Through a decrease in τ_{free} , cooperation of overacceleration caused by safety acceleration in a road lane with overacceleration due to lane-changing appears (Sec. VIB). The same effect is realized when instead of τ_{free} , value τ_G becomes shorter (Sec. VIC). In

these cases (Secs. VIB and VIC), cooperation of overacceleration caused by safety acceleration with overacceleration due to lane-changing occurs even without overacceleration a_{OA} (4). When overacceleration a_{OA} (4) is used in addition, then cooperation of three overacceleration mechanisms [due to safety acceleration, through lane-changing, and formula (4)] has been found [Fig. 25(d)]. This overacceleration cooperation leads to the largest possible on-ramp inflow rate $q_{\text{on, max}}$ at which for a given q_{in} free flow is still self-maintained at the bottleneck (Sec. VIII A). Overacceleration cooperation also affects characteristics of a critical nucleus for traffic breakdown (Sec. VII).

Microscopic features of overacceleration found in the paper have also been used for a study of the $S \rightarrow F$ instability on a two-lane road with the bottleneck (Sec. IX). We have revealed that cooperation of different overacceleration mechanisms and their competition with speed adaptation occurring on microlevel of vehicle motion lead to complex spatiotemporal development of the $S \rightarrow F$ instability in which both lane-changing and the merging of on-ramp vehicles are essential.

Microscopic physics of vehicle overacceleration disclosed in the paper shows that overacceleration is related to a totally different behavior of microscopic vehicle motion in comparison with overdeceleration caused by driver overreaction in traffic flow of human-driving vehicles (Sec. X). This explains why features of traffic breakdown and highway capacity simulated with three-phase traffic flow models [62–65] are qualitatively different from features of traffic breakdown and highway capacity simulated with the standard traffic flow models [1–45].

XII. CONCLUSIONS

(1) For a given flow rate in free flow upstream of an on-ramp bottleneck, the stronger the effect of vehicle overacceleration on traffic flow, the larger the maximum on-ramp inflow rate at which free flow can persist at the bottleneck, i.e., the better traffic breakdown can be avoided.

(2) In addition to known overacceleration a_{OA} (4) [87], there can be vehicle overacceleration in a road lane caused by safety acceleration at the bottleneck.

(3) Overacceleration in a road lane can occur without explicitly introducing it in the model.

(4) Whether an acceleration behavior becomes overacceleration or not depends on traffic characteristics and model parameters. This conclusion is valid for both safety acceleration in a road lane and for vehicle acceleration through lane-changing.

(5) The microscopic effect of vehicle overacceleration on traffic flow is determined by a spatiotemporal cooperation of different overacceleration mechanisms. The overacceleration cooperation increases the effect of overacceleration on the maintaining of free flow at the bottleneck.

(6) Both speed adaptation and overacceleration in a road lane can affect qualitatively the overacceleration mechanism caused by lane-changing.

(7) The $S \rightarrow F$ instability on a two-lane road without bottlenecks occurs through cooperation of different overacceleration mechanisms.

(8) The $S \rightarrow F$ instability on a two-lane road with the bottleneck exhibits the following spatiotemporal dynamics.

(i) In the right lane, the $S \rightarrow F$ instability is caused by overacceleration cooperation through $R \rightarrow L$ lane-changing and overacceleration a_{OA} in a road lane. Contrarily, speed adaptation that occurs through the merging of on-ramp vehicles together with $L \rightarrow R$ lane-changing tries to prevent the $S \rightarrow F$ instability development.

(ii) In the left lane, the $S \rightarrow F$ instability is caused by overacceleration cooperation through $L \rightarrow R$ lane-changing

and overacceleration a_{OA} in a road lane. Contrarily, speed adaptation through $R \rightarrow L$ lane-changing tries to prevent the $S \rightarrow F$ instability development.

(9) The microscopic features of the effect of vehicle overacceleration on traffic breakdown are related to traffic flow consisting of human-driving and/or automated-driving vehicles.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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