

Inference of thermodynamic conditions in convergent high-energy-density-physics experiments

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Laser-driven spherical implosion experiments create some of the most extreme states of matter accessible in the laboratory. However, experimental observables in implosion experiments are highly integrated, meaning that measurements cannot be straightforwardly inverted to characterize the thermodynamic states or material properties in the implosion. Thus, to understand the extreme states of matter typical of implosions, a new generation of experiments and analyses are needed that leverage modern statistical tools. This work presents a comprehensive Bayesian inference framework that enables the analysis of integrated x-ray self-emission measurements from laser-driven implosions at the OMEGA laser, producing equation-of-state data (pressure, density, and temperature) between 0.1 to 1.5 Gbar. The model is benchmarked against synthetic data from radiation-hydrodynamics simulations, and opens avenues for producing quantitative constraints on physical models relevant to dense astrophysical and laboratory fusion plasmas.

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I. INTRODUCTION

Laser-driven compression experiments [1] assemble matter to extreme energy densities, making them a useful tool for studying physics relevant to many astrophysical systems. Experiments in planar geometry routinely reach pressures on the order of 1 to 10 Mbar (0.1 to 1 TPa), providing laboratory benchmarks for physical models of planetary interiors [2–4]. At these conditions the energy density of the system exceeds those of typical molecular bonds, engaging core electrons in bonding and often leading to complex crystal and electronic structural changes [5,6]. The pressures accessible by laser-driven compression can be increased by several orders of magnitude, well into the Gbar (100-TPa) regime, when convergent geometries are used [7,8]. At Gbar energy densities, a wealth of interesting nuclear and atomic phenomena emerge, including nuclear fusion [9], the distortion of atomic orbitals [10], and pressure-driven electron delocalization [11], all of which push state-of-the-art physics models to their limits.

The quest to control fusion on a laboratory scale has driven enormous investments in large-scale laser facilities around the world [12–16], resulting in the development of multiple robust spherical compression platforms, each with a range

of diagnostics designed to make precise particle, x-ray, and optical measurements of laser-driven systems. The large design space of spherical implosion experiments allows tailored experiments on these platforms to reach a wide range of extreme thermodynamic conditions [Fig. 1], making them a compelling platform to study physics relevant to many dense astrophysical plasmas [17,18] and laboratory-scale thermonuclear fusion [19,20].

While implosion experiments access unrivaled pressures and are well diagnosed by a broad ensemble of diagnostics, the closed geometry of these systems necessarily produces highly integrated measurements [21], which are difficult to interpret and have limited convergent experiments' use as a platform for fundamental physics studies [22,23]. In other words, such data convolve multiple states and processes making the inverse problem [24] intractable. Quantitative interpretation of these large integrated datasets, so as to constrain the implicit physical quantities, requires the development of advanced statistical analysis techniques and remains an area of active research [25]. One such analysis framework that has been quite successful in interpreting implosion data, and the one used in this work, is Bayesian inference [26–29], which typically employs forward modeling and Markov chain Monte Carlo (MCMC) to constrain the parameters of a generative model while providing a straightforward methodology for including prior information with self-consistent uncertainty quantification. This work builds on previous studies [21,27] to construct a comprehensive Bayesian model that simultaneously treats the dynamics, radiation transport, and diagnostics in a set of implosion

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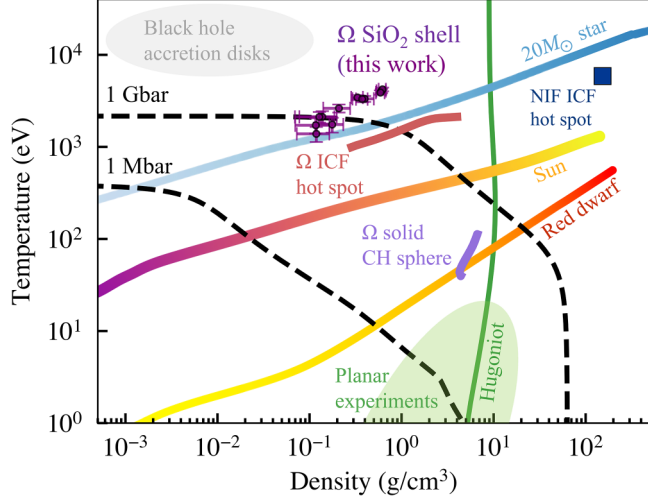


FIG. 1. The temperature and density conditions reached by several implosion designs: the thin SiO_2 shells considered in this work (purple circles); thick, gas-filled plastic (CH) shells on OMEGA (red curve) [30]; converging shocks in solid CH spheres (light purple curve) [31]; and high-yield ICF capsules on the National Ignition Facility (NIF, blue square) [32]. These designs are compared to isobars of SiO_2 at 1 Mbar and 1 Gbar (black dashed lines) and the principal Hugoniot for polycrystalline quartz, calculated from the SESAME 7385 equation of state [33] including a correction term for radiation pressure. Also shown are the predicted conditions in the interior of the sun (orange-yellow curve), a 0.5-solar mass red dwarf star (red-orange curve), and a 20-solar mass main sequence star (blue curve), all calculated using the stellar evolution code MESA [34]. The approximate range of conditions found in black hole accretion disks are shown by the black ellipse, taken from calculations in Ref. [35]. Convergent systems offer access to a broad range of extreme conditions beyond the reach of planar laser-driven compression experiments (green ellipse) and directly relevant to a number of astrophysical systems.

experiments, enabling the joint analysis of multiple integrated measurements in order to constrain the evolution of the thermodynamic states (pressure, temperature, and density) in the system.

The analysis is developed for and applied to a set of implosions of gas-filled glass (SiO_2) capsules from which a suite of x-ray self-emission data was collected at the OMEGA Laser Facility [12]. The self-emission data are modeled directly using a physics-based, reduced-order model that is able to accurately reproduce computationally expensive full-physics models while providing the speed necessary to perform statistical inference of experimentally relevant quantities. The model is evaluated against a synthetic dataset generated using the 1D radiation-hydrodynamics code LILAC [36]. The results of this “synthetic experiment” demonstrate that the analysis provides accurate constraints on the temporal evolution of the peak thermodynamic states in the SiO_2 shell during the expansion phase of the implosion. Applying the analysis to an ensemble of seven experimental datasets provides constraints on the equation of state (EOS) (pressure, temperature, and density) in a Gbar plasma.

TABLE I. Target and laser parameters for the seven implosion experiments analyzed in this work.

Shot no.	SiO_2 thickness (μm)	Outer diameter (μm)	D_2 Fill (atm)	E_{Laser} (kJ)
106536	3.0	857	20.4	16.3
106538	3.0	847	20.4	16.2
106542	3.0	870	20.4	15.1
107850	3.0	821	20.8	15.8
107851	3.0	820	20.8	15.7
107852	3.0	827	20.8	15.6
114517	2.9	887	15.6	15.7

II. EXPERIMENT AND MEASUREMENTS

The experimental systems considered in this work consist of thin (approximately $3\text{-}\mu\text{m}$) SiO_2 shells [37] with a nominal outer diameter of $860\text{-}\mu\text{m}$, filled with deuterium (D_2) gas, shown in Fig. 2(a). These targets were symmetrically illuminated by the 60-UV OMEGA laser beams at the Laboratory for Laser Energetics, which delivered approximately 15 kJ of laser energy in a 600-ps square pulse, shown by the dashed white line in Fig. 2(b). The beams were configured with SG5 distributed phase plates [38] with smoothing by spectral dispersion turned on [39], resulting in a spatially uniform beam profile with diameter of roughly $720\text{ }\mu\text{m}$. The exact target dimensions and delivered laser energy for each of the implosion experiments analyzed in this work are summarized in Table I. The experimental design is similar to the “exploding-pusher” targets used in early laser-fusion experiments [40] and was chosen for its spherical symmetry and hydrodynamic stability. These implosions are “shock-driven,” meaning the evolution of the system is dominated by the converging or expanding shock rather than the compressive work done by the in-falling material [41,42]. The self-symmetrizing properties of spherical shocks smooths out low mode asymmetries, leading to a highly symmetric implosion [43]. Additionally, the modest convergence ratios of such implosions (approximately four to five) are much lower than those of typical high-yield ICF implosions, which can be as large as 30 to 40 [44]. Since instability growth scales with convergence ratio [45], the shock-driven designs used in this work are far more robust against hydrodynamic instabilities, mitigating the role of mix and other 3D effects. Because of these properties, there are many studies in ICF and HEDP which have used exploding-pusher-like designs to test implosion physics and direct-drive ICF concepts, which have demonstrated that such implosions are well described by a 1D model [40,46–48].

The evolution of the implosion system can be divided into two regimes, labeled in Fig. 2(b), which each produce characteristically different emission: the convergence phase and the expansion phase. During the convergence phase, the incident laser energy is absorbed by the electrons in the SiO_2 shell primarily via inverse bremsstrahlung [49], superheating the shell material and causing mass ablation as the outer surface of the shell rapidly expands. This produces ablation pressures of order 100 Mbar, launching a converging shock wave into the target and causing it to implode. The shell continues to accelerate until the laser turns off around $t \approx 0.7\text{ ns}$. The

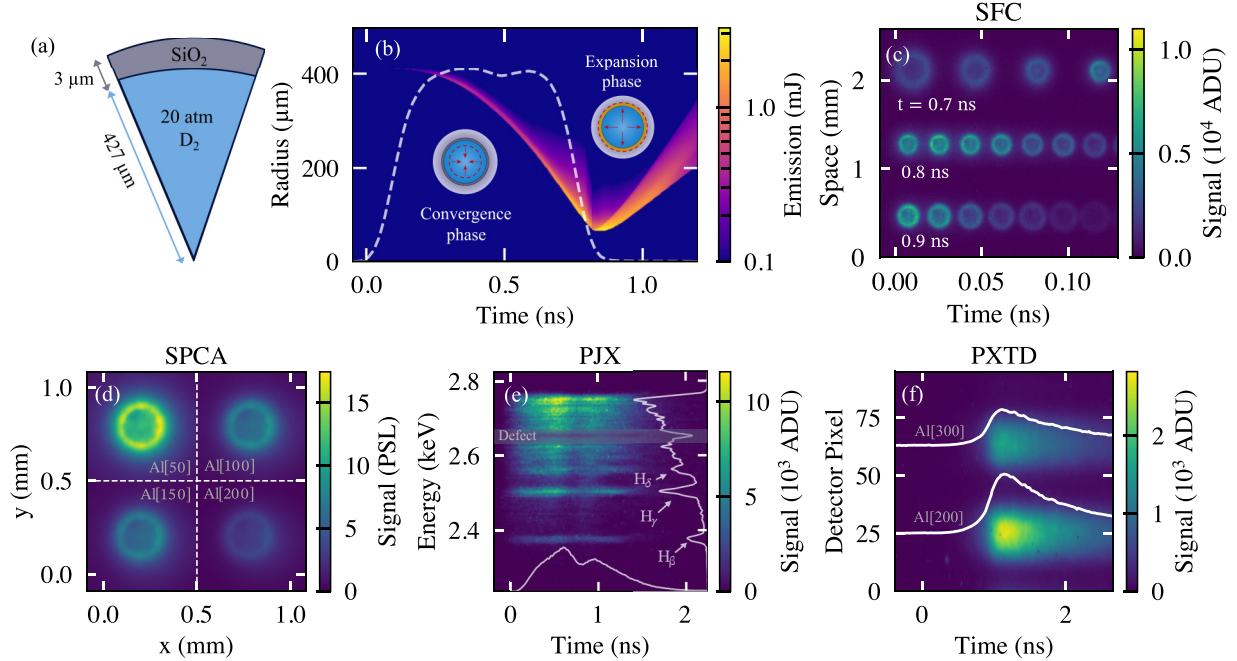


FIG. 2. (a) Nominal target design of the implosion experiments analyzed in this work. (b) Simulated spatiotemporal evolution of the total (energy-integrated) emission as a function of radius and time, calculated by LILAC [36]. The emission from the implosion is dominated by the SiO₂ shell and can be divided into two epochs: the convergence phase and the expansion phase. The emission from both phases is resolved by the suite of x-ray diagnostics described in Sec. II A. (c) Example Sydor framing camera (SFC) data. Each row contains a set of time-gated 2D x-ray images with increasing temporal offset from $t = 0$, denoted by the white labels below each row. The top row shows images during the end of the convergence phase ($t < 0.8$ ns), while the other two capture emission during the expansion phase ($t > 0.8$ ns), which show a symmetric expanding ring of emission corresponding to the shocked SiO₂ ablation plasma. (d) Example time-integrated, differentially filtered images from the static pinhole camera array (SPCA) diagnostic. Each image is filtered by a different thickness of aluminum, denoted in gray. Since the filtration blocks soft x-rays ($E < 5$ keV), the recorded emission is dominated by the hard x-rays emitted by the shocked SiO₂ during the expansion phase. (e) Streaked spectrum of Si line emission recorded by the flat crystal spectrometer coupled to the Paul Jaanimagi x-ray streak camera (PJX). Three lines are observed, corresponding to transitions in H-like silicon ions. The white curves on the x and y axes represent the energy-integrated time history and the time-integrated spectrum measured by PJX, respectively. The dip in signal around 2.65 keV is due to a defect in the streak camera photocathode. (f) Example data from two channels of the particle x-ray temporal diagnostic (PXTD), which measures the emission history of high-energy x-rays in different spectral bands. The long tail of the signal, shown by the white curves, is due to the scintillator's long decay time which must be deconvolved from the signal.

emission during this phase is dominated by soft ($E \lesssim 3$ -keV) x-rays emitted by the laser-heated SiO₂ ablation plasma. This manifests as a thin, converging shell of emission that begins to expand after the laser turns off and the outer surface of the shell begins to release.

The converging shock wave reflects off the origin and begins to propagate outward, where it interacts with the infalling SiO₂ at $t \approx 0.8$ ns, beginning the expansion phase. Upon interacting with the rebounded shock wave, the infalling SiO₂ is compressed and shock heated, causing it to radiate x-rays, predominantly in the form of high-energy continuum (free-free and free-bound) emission with some contribution from bound-bound emission due to atomic transitions in the highly ionized silicon. The shock continues to propagate outward, sweeping up material from the ablation plasma and cooling as it expands, forming the expanding ring of emission that can be seen in the series of x-ray images shown in Fig. 2(c). The primary goal of the analysis presented in this work is to take measurements of the x-rays emitted by the shocked material and infer the thermodynamic states present in the SiO₂ as the implosion evolves.

A. Diagnostic suite

The x-ray emission from the SiO₂ plasma is the primary observable from the experiments in this work and was recorded by the suite of x-ray diagnostics shown in Figs. 2(c)–2(f). Assuming spherical symmetry, the emitted spectral radiance, I_ν , can be thought of as a 3D function of space, time, and photon energy, which each diagnostic integrates along one or more axes in order to produce the observed data. Thus, it is important to choose diagnostics that provide complementary coverage of the spatial, temporal, and spectral evolution of I_ν in order to collect as much information as possible. To illustrate the temporal, spatial, and spectral coverage of the diagnostic suite described in this section, the raw data from each diagnostic shown in Fig. 2 are projected along each of the three axes in Fig. 3.

Spatial and temporal information were obtained by two Sydor x-ray framing cameras (SFCs), which captured a series of 2D, time-gated pinhole images of the x-ray emission at $6\times$ magnification [Fig. 2(c)]. The SFCs consist of four gold strips, which convert incident x-rays into photoelectrons, which are amplified by a microchannel plate and read out to a

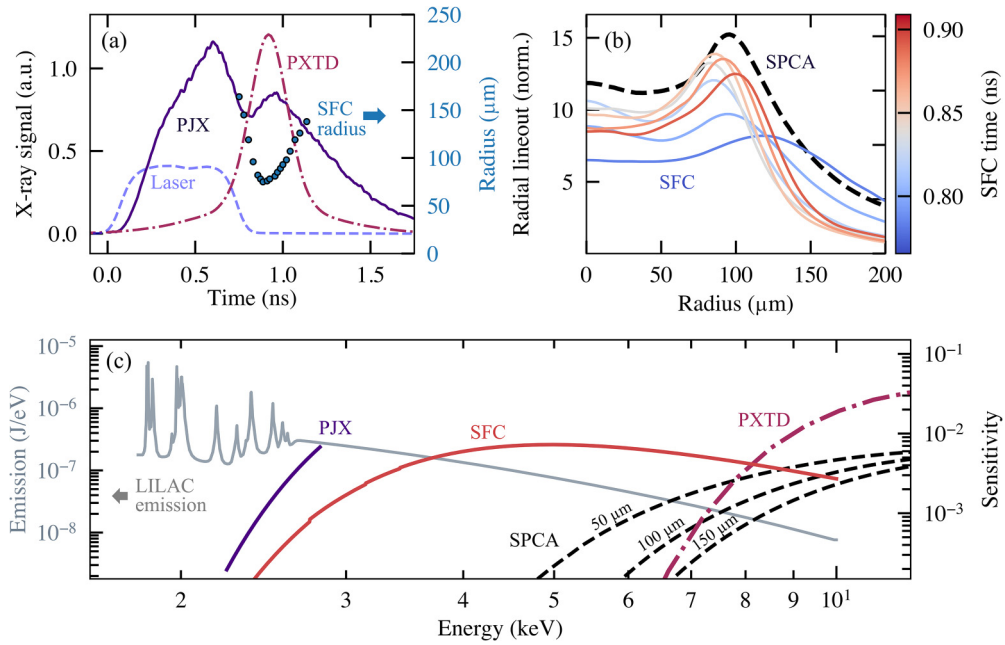


FIG. 3. Reductions of the datasets shown in Fig. 2 that illustrate the information contained therein. (a) Time history of the spectrally integrated PJX data, the deconvolved PXTD time history, and the radius of peak emission from SFC, with the laser pulse shape for context. (b) Azimuthally averaged radial lineouts of the time-integrated SPCA and time-resolved SFC (blue and red) images. The SPCA image is from the 50-μm-Al channel. The SFC images are normalized by the integral of the signal, and are color coded according to increasing time, with the blue curves corresponding to the implosion phase and the red curves corresponding to the expansion phase. (c) The relative spectral sensitivities of all four diagnostics plotted with the simulated spectral radiant energy from LILAC. Three representative SPCA channels are plotted and labeled according to the thickness of the Al filter.

charge-coupled device (CCD) in analog-to-digital units (ADUs) [50]. The camera is coupled to an array of 10-μm-diameter tantalum pinholes, filtered by 203 μm of beryllium. To record the images, a voltage pulse of 50-ps FWHM is run across each of the gold strips at appointed times relative to the start of the laser pulse. While the voltage is “on,” the camera detects x-rays, creating an image at each pinhole with integration time equal to the width of the voltage pulse. Each strip records a series of either seven or 13 images over a 200-ps interval, depending on the number of pinholes in each strip of the array. The two SFCs were fielded along semi-orthogonal lines of sight to confirm that the implosion was symmetric.

The width of the limb-brightened SFC images provide constraint on the evolution of thickness of the shell in time, as can be seen from the radial lineouts in Fig. 3(b), while the location of the peak emission constrains the trajectory of the shell, as shown by the blue points in Fig. 3(a). This latter piece of information allows for the constraint of acceleration and thus the mechanical pressure acting on the shell [21]. The SFC data do not provide reliable photometric information because the absolute gain of framing cameras is difficult to calculate due to effects such as crosstalk and gain droop [50]. Therefore, the signal level measured by the SFC is not necessarily physically meaningful without careful cross-calibration to other diagnostics.

To provide absolute photometrics as well as further spatial and spectral information, a time-integrated, differentially-filtered static pinhole camera array (SPCA) was also fielded, example data from which are shown in Fig. 2(d). The SPCA recorded the absolute signal in different energy bands on a

Fuji BAS-SR image plate [51,52]. Because the conversion from image plate signal (photostimulated luminescence or PSL) to photons is well characterized [53–55], this measurement provides information about the total number of x-rays emitted by the plasma. A total of 16 pairs of pinholes were fielded at a magnification of 12×, and each pair of pinholes was filtered using a different thickness of aluminum, varying from 50 to 200 μm. The SPCA data provide emission-weighted spatial information, as shown by the dashed black radial lineout in Fig. 3(b), which is heavily weighted toward the emission at peak compression (i.e., the SPCA lineout is very similar in shape to the SFC lineout at peak x-ray emission). Additionally, the differential filtering provides spectral information since each channel records only x-rays with energy sufficient to penetrate their respective filtration [56], thus giving relative x-ray yield in each energy band, which depends on the shape of the high-energy continuum spectrum shown in Fig. 3(c).

Further spectral and temporal information was provided by a streaked flat-crystal x-ray spectrometer consisting of a potassium acid phalate (KAP [001]) crystal coupled to a Paul Jaanimagi x-ray streak camera (PJX) [57] with a potassium bromide (KBr) photocathode. Incident x-rays are reflected off the KAP crystal according to Bragg’s law and dispersed onto the photocathode, creating photoelectrons that are accelerated and focused by a set of electron optics onto a phosphor output screen, which is fiber coupled to a CCD [57]. The electron optics provide a time-dependent voltage that sweeps the beam of photoelectrons in time, creating the streaked spectrum shown in Fig. 2(e). The spectrometer was configured to record a

photon energy range of 2.3 to 2.8 keV in order to capture the temporal evolution of the line emission from bound-bound transitions in hydrogen-like silicon ions. The evolution of the intensities and ratios of these lines gives information about the charge state and level populations in the plasma, providing constraint on the thermodynamic states as a function of time.

Lastly, the time history of the total flux of high-energy continuum x-rays was collected using the particle x-ray temporal diagnostic (PXTD) [58]. The PXTD consists of a BC-422 plastic scintillator, which has a well-characterized sharp rise and long decay time [59], coupled to an optical streak camera. The incident x-rays are filtered into several channels by 200 to 400 μm of aluminum before being converted to optical photons via interaction with the scintillator. These optical photons are then read out by an optical streak camera in ADU in a process similar to that described above for the PJX diagnostic. The resulting signal, shown by the white curves in Fig. 2(f) is then the time history of the x-ray emission convolved with the instrument response function (IRF) of the scintillator. The deconvolved time history is plotted in Fig. 3(a), showing that the hard x-ray emission peaks around $t = 0.9$ ns, just after peak compression as measured by SFC, and that the convergence phase is dominated by soft x-rays and line emission (measured by PJX), while the expansion phase is dominated by the harder x-rays measured by PXTD. The combination of these two temporal measurements thus provides time-resolved constraint of a large portion of the x-ray spectrum.

A dozen other diagnostics were also fielded in the experiment, including other x-ray imagers [60], neutron diagnostics [61], and time-integrated x-ray spectrometers [62]. These additional datasets are not included in the analysis presented here for a variety of reasons, but future work could straightforwardly extend the model discussed in Sec. III to include the additional information provided by these secondary diagnostics. The suite of diagnostics described here provides a highly complementary dataset, as can be seen in Fig. 3, demonstrating satisfactory coverage of the total spectral radiance. The Bayesian model outlined in Sec. III self-consistently synthesizes information from all four diagnostics simultaneously, enabling excellent diagnostic access to the implosion plasma.

III. BAYESIAN MODEL

In this section a comprehensive generative model is developed in order to explain the x-ray self-emission data discussed in Sec. II. This involves simultaneously treating the dynamics, radiation transport, and detector properties in the experiment to create synthetic data, which can be compared to the experimental data to construct the posterior distributions for the model parameters θ using Bayes' rule

$$P(\theta|\mathbf{D}) = \frac{\mathcal{L}(\mathbf{D}|\theta)\pi(\theta)}{P(\mathbf{D})}, \quad (1)$$

where $\mathbf{D}$ is the observed data, $\pi(\theta)$ is the prior distribution for the model parameters, $\mathcal{L}(\mathbf{D}|\theta)$ is the likelihood function, and $P(\mathbf{D})$ is the evidence. The posterior distribution $P(\theta|\mathbf{D})$ can be constructed using Markov chain Monte Carlo (MCMC) methods to sample over parameter space. In general, as is the case in this work, it is the unnormalized posterior $\mathcal{L}(\mathbf{D}|\theta)\pi(\theta)$

that is computed, since the evidence $P(\mathbf{D})$ is challenging to calculate explicitly. Various techniques do exist to estimate the evidence either from the unnormalized posterior [63,64] or using specialized sampling algorithms such as nested sampling [65]. This is useful when comparing two or more competing models, since the ratio of the model evidences, or Bayes factor, provides a robust quantitative metric for model selection [66,67].

A common type of generative modeling paradigm used in the analysis of implosion data involves running postexperiment simulations using state-of-the-art radiation-hydrodynamics codes and tuning input parameters to match certain experimental observables [28,46,68]. These codes synthesize the best-available models of the physical processes that occur in implosion experiments, but are often overparameterized and computationally expensive, making detailed comparison to the full suite of experimental data challenging and true uncertainty quantification intractable. Recent work has addressed this by developing neural network-based surrogate models [22,26,29,69] to efficiently map from a subset of input parameters to a subset of experimental observables. This approach encodes sophisticated physics captured by the full-physics code, but the computational cost of producing training data typically limits the output space to a few scalar summary statistics. While these surrogate models have demonstrated excellent explanatory and predictive power [22,29], their use as a tool for physics discovery is currently limited by the reduced input and output spaces in which they operate.

An alternative approach, and the one employed in this work, is the use of physically parameterized, reduced-order models that are able to effectively reproduce the outputs of the full-physics models at much lower computational cost [21,41,70]. While reduced-order models are less physically complete than their full-physics counterparts, their efficiency and flexibility enables modeling of the full dataset directly with minimal data reduction, thereby increasing the amount of information that can be gained from a single experiment. By constructing a model with physically meaningful parameters, the inference can be abstracted beyond the context of the model, enabling powerful inferences from complex datasets.

The development of the reduced-order model used in this work is divided into three parts: a dynamical model, which describes how the thermodynamic states of the system evolve over time; a radiation-transport model, which calculates the emitted spectral radiance given those thermodynamic states; and a series of detector models, which integrate the spectral radiance to produce synthetic data for each diagnostic. The model is completed by the construction of appropriate likelihoods for each dataset and priors for each parameter. The model is constructed entirely in jax [71] to enhance computational efficiency by making use of just-in-time (JIT) compilation and automatic differentiation. This both accelerates the calculation and enables the use of gradient-based MCMC samplers, both of which greatly increase the efficiency of sampling, enabling full uncertainty quantification even in high-dimensional parameter spaces. It should be noted that the use of gradient-based sampling methods is known to introduce some difficulties, especially when the posterior geometry has regions of high curvature such as near a hard

cutoff or constraint. This can lead to biased inference as the sampler cannot efficiently explore these regions. Therefore, care must be taken when constructing differentiable models that the posterior geometry is not too severe for gradient-based MCMC samplers. Difficult geometry can often be diagnosed by the number of divergent MCMC trajectories [72,73] and resolved by an appropriate reparameterization of the posterior, such as simple rescaling [74] or more sophisticated techniques such as autoregressive normalizing flows [75,76].

A. Dynamical model

The dynamical model of the system must describe the evolution of the thermodynamic states (temperature T , density ρ , and pressure P) in the implosion as a function of space and time. The treatment can be simplified by assuming that the spatial variation of the thermodynamic states are only functions of radius r and considering only the states in the SiO_2 shell since it dominates the x-ray emission. The model constructed here is an extension of the thin shell mechanical model of Ref. [21], which relates the radius of the shell, taken to be the radius of peak emission measured by the framing cameras, to the pressures acting on the shell during convergence and expansion. The trajectory of the shell evolves according to the second-order ordinary differential equation,

$$\frac{d^2 R}{dt^2} = \frac{4\pi R^2 P(t)}{M(t)}, \quad (2)$$

where $P(t)$ is the parameterized pressure at the shell interface, and $M(t)$ is the mass of the shell as it is ablated by the laser and subsequently reaccreted upon expansion. The complete details of the parametrization and implementation of the mechanical model can be found in Refs. [21] and [27].

The mechanical model is extended to include parameterized temperature and density profiles in the SiO_2 shell that evolve over time. The system is treated as an isolated shell of SiO_2 plasma that extends from the D_2 interface radius R_0 to a time-varying shock-front radius $R_0 + dR$, with the prescribed density and temperature profiles interior, visualized in Fig. 4(a). Any emission from the D_2 or the residual SiO_2 ablation plasma is treated as negligible, and the temperature and density in those regions are set to zero.

The density profile inside the shell has peak value, ρ_0 , at the D_2 interface and decreases linearly until it reaches the shock-front value ρ_s ,

$$\rho(r) = \begin{cases} \rho_0 - \frac{\rho_s}{dR}(r - R_0), & R_0 \leq r \leq R_0 + dR \\ 0, & \text{otherwise} \end{cases}. \quad (3)$$

The temperature in the shell is assumed to be spatially uniform with temperature T_0 ,

$$T(r) = \begin{cases} T_0, & R_0 \leq r \leq R_0 + dR \\ 0, & \text{otherwise} \end{cases}. \quad (4)$$

The choice of a flat temperature profile in the shell agrees well with LILAC [36] simulations, as shown in Fig. 4(a), but warrants further discussion. As the shock expands, it preferentially heats the ions in the plasma, which collisionally transfer energy to the electrons, creating a positive temperature gradient in the shell ($\frac{\partial T}{\partial r} > 0$). However, since the ion-electron equilibration time is short (~ 10 ps) [77], this thermal gradient

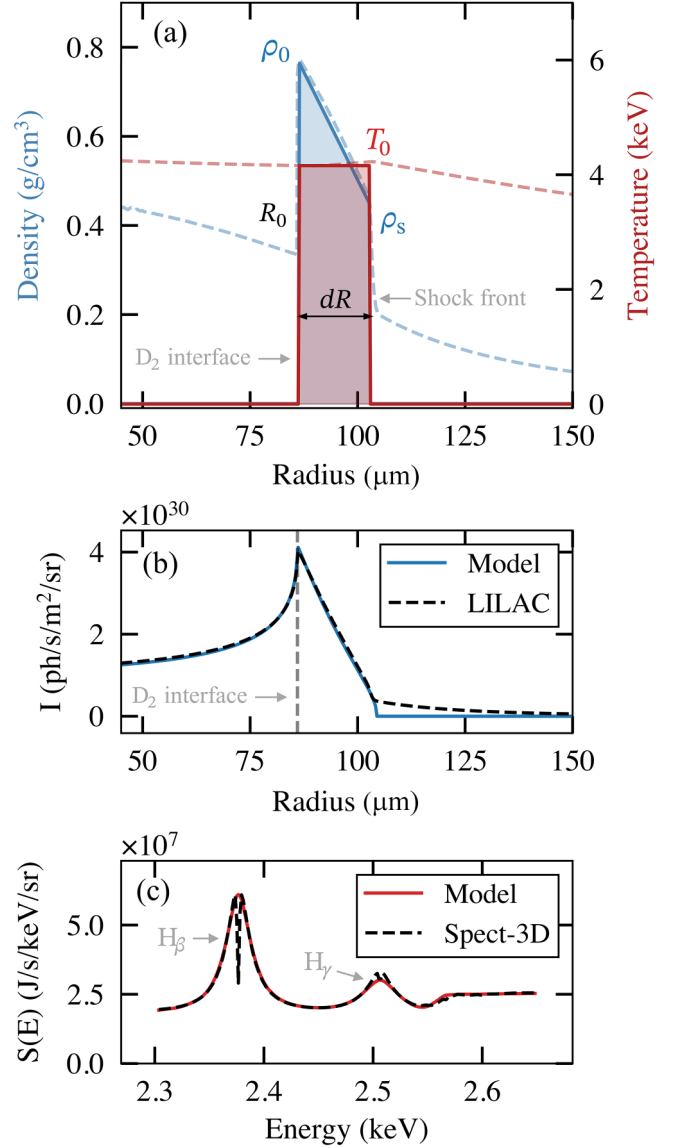


FIG. 4. An example instantiation of thermodynamic and radiative models for a given set of the dynamical parameters $\theta_d = \{R_0, dR, \rho_0, \rho_s, T_0\}$. The solid curves represent outputs from the model while the dashed curves show relevant output from LILAC, demonstrating the plausibility of the reduced-order model. (a) The thermodynamic profiles from the parameterized profiles in Eqs. (3) and (4) with the relevant parameters labeled. The shaded region corresponds to the shocked shell of emitting SiO_2 . (b) The spectrally integrated, spatially resolved emission calculated from Eq. (16). The radius of peak emission (dashed line) corresponds to the interface position, R_0 . (c) The spatially integrated emission from the bound-bound transitions of silicon calculated by PRISMSPECT for the thermodynamic profiles in (a). The doublet H_β line in the SPECT3D calculation arises due to resonant absorption in the low-density ablation plasma, which is not included in the model.

is quickly dissipated, leading to a relatively small spatial temperature gradient. LILAC estimates the temperature gradient to be less than 10% over the width of the shell during the expansion phase, motivating the flat temperature profile in Eq. (4).

The temperature and density, and therefore the x-ray emission, of the SiO₂ shell at a given time are defined by five parameters, $\theta_d = \{R_0, dR, \rho_0, \rho_s, T_0\}$, each of which evolve as a function of time. Since the radius of peak emission is collocated with the interface position [Fig. 4(b)], the solution to Eq. (2) can be used to determine the temporal evolution of the parameter R_0 . The time histories of the other four parameters in θ_d evolve nonparametrically to increase the flexibility of the model. The shell thickness dR , and the shock front density ρ_s are allowed to vary freely at each time step with no assumed temporal correlations, while the temperature T_0 and density ρ_0 are modeled using Gaussian processes (GPs) [78]

$$\rho_0(t) \sim \mathcal{GP}[\mu_{\rho_0}(t), K_{\rho_0}(t, t')], \quad (5)$$

$$T_0(t) \sim \mathcal{GP}[\mu_{T_0}(t), K_{T_0}(t, t')], \quad (6)$$

with kernels K_i and mean functions μ_i . The covariance kernel for both GPs is chosen to be a squared exponential,

$$K_i(t, t') = \tau_i e^{-\frac{(t-t')^2}{2l_i^2}}, \quad (7)$$

where τ_i and l_i are hyperparameters that set the scale and correlation length of the GP, respectively. This choice of kernel encodes the belief that the temperature and density histories should be continuous, smoothly varying functions of time. The mean functions, $\mu_{\rho_0}(t)$, and $\mu_{T_0}(t)$ are chosen to encode physical intuition about the qualitative behavior of each quantity. The GP is able to capture any deviations from this mean behavior, resulting in a flexible but physically informed model. For the density history, the functional form is chosen to be bilinear in time

$$\mu_{\rho_0}(t) = \begin{cases} \rho_{\max} + \dot{\rho}_1(t - t_\rho) & t < t_\rho \\ \rho_{\max} - \dot{\rho}_2(t - t_\rho) & t > t_\rho \end{cases} \quad (8)$$

with free parameters $\rho_{\max}$, $\dot{\rho}_1$, $\dot{\rho}_2$, and t_ρ , where $\dot{\rho}_1$ and $\dot{\rho}_2$ are strictly positive. Similarly, the temperature mean is parametrized as

$$\mu_{T_0}(t) = \begin{cases} T_{\max} + \dot{T}_1(t - t_T) & t < t_T \\ T_{\max} - \dot{T}_2(t - t_T) & t > t_T \end{cases} \quad (9)$$

with free parameters $T_{\max}$, $\dot{T}_1$, $\dot{T}_2$, and t_T , with $\dot{T}_1$ and $\dot{T}_2$ strictly positive. These parametrizations for the mean functions are not unique, but they encode important physical intuition about the system. Specifically, they encode the expectation that the temperature and density histories should be unimodal in time, increasing as the shell converges, and decreasing as the shell expands. Furthermore, the rates of change of the thermodynamic states should be different before and after the time of their peak values since the implosion dynamics are asymmetric about peak compression. This is reflected by the different slopes on either side of the peak state in Eqs. (8) and (9). Multiple mean functions satisfying the same criteria were explored, but the results of the inference were identical to the few-percent level for any reasonable choice of mean function.

The dynamical model described above forms the backbone of the Bayesian analysis developed in this work. It creates a direct mapping from the thermodynamic states in the plasma to the observed emission, enabling simultaneous constraint of the evolution of pressure, temperature, and density in the

shocked SiO₂. Furthermore, the model does not explicitly assume an equation of state (EOS) for the plasma, causing the results of the inference to be largely EOS-agnostic except through weak constraints via the choice of radiative properties in Sec. III B [79]. Therefore, within the context of the assumptions of the model, the thermal pressure implied from the measured T and ρ can be directly compared to the mechanical pressure P inferred from the trajectory of the shell, providing quantitative constraint on the EOS of the system.

B. Radiation transport

The spatiotemporal temperature and density profiles generated by the dynamical model can be used to calculate the emitted spectral radiance, $I_\nu(r, t)$. In general, this involves calculating the radiative properties emissivity, j_ν , and opacity, κ_ν , as a function of space, time, and photon energy, and solving the equation of radiative transfer neglecting scattering,

$$\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \hat{\Omega} \cdot \nabla I_\nu = j_\nu - \rho \kappa_\nu I_\nu, \quad (10)$$

where c is the speed of light, and $\hat{\Omega}$ is the direction of propagation along a line of sight. Under the steady state and parallel line of sight approximations, this reduces to the form [80]

$$\frac{dI_\nu}{dz} = j_\nu - \rho \kappa_\nu I_\nu, \quad (11)$$

where z is the axis along the line of sight to the detector.

The calculation is simplified by splitting it into two parts: calculation of the high-energy ($E \gtrsim 4$ -keV) continuum emission from free electrons, and calculation of lower energy ($E \sim 2$ to 3 keV) line emission from bound electrons in the Si ions. The former is measured by SFC, SPCA, and PXTD, while the latter is measured by the streaked spectrometer (PJX) as shown in Fig. 3(c).

The continuum emission can be calculated analytically as the sum of the free-free and free-bound (radiative recombination) emissivities [81]

$$j_\nu = j_\nu^{\text{ff}} + j_\nu^{\text{fb}}. \quad (12)$$

The free-free emissivity is calculated as

$$j_\nu^{\text{ff}} = \frac{2.15 \times 10^{35}}{h\nu} g_v^{\text{ff}} Z^2 \frac{\bar{Z} \rho^2}{T^{1/2}} e^{-\frac{h\nu}{T}} \left[\frac{\text{ph}}{\text{m}^3 \text{s sr eV}} \right], \quad (13)$$

where h is Planck's constant, Z is the average (unscreened) nuclear charge of the ions, $\bar{Z}$ is the average ionization state, ρ and T are the density and temperature in $\frac{\text{g}}{\text{cm}^3}$ and eV, respectively, and g_v^{ff} is the free-free gaunt factor as calculated by Ref. [82]. The bound-free emissivity is calculated considering only recombination to H-like silicon because the lack of He-like lines in the PJX record implies the dominant charge states are Si XIV and Si XV. The free-bound emissivity is then given by [83]

$$j_\nu^{\text{fb}} = 1.6 \times 10^{58} \frac{\bar{Z} \rho^2 (h\nu - \chi)}{T^{3/2}} e^{-\frac{h\nu - \chi}{T}} \sigma^{\text{fb}} \left[\frac{\text{ph}}{\text{m}^3 \text{s sr eV}} \right], \quad (14)$$

where χ is the binding energy of the ground state of the single electron in hydrogenic Si, taken to be the isolated atom value of 2673 eV [84]. The cross section of radiative recombination

$\sigma^{fb}(h\nu)$ is calculated as the sum over the relevant Milne cross sections for principle quantum number n ,

$$\sigma_n^{fb}(h\nu) = g_v^{fb} \frac{32\pi}{3\sqrt{3}} \frac{\eta^4}{[n(\eta^2 + n^2)]\alpha^3 a_B^2}, \quad (15)$$

where α is the fine-structure constant, a_B is the Bohr radius, and $\eta = \sqrt{\frac{\chi}{h\nu - \chi}}$. The free-bound Gaunt factor g_v^{fb} is calculated using the angular momentum-averaged analytical formula provided in Ref. [85]. The cross sections are summed up to a maximum principle quantum number of $n = 4$.

Evaluation of Eqs. (13) and (14) is simplified by assuming a constant value for the charge state $\bar{Z}$ corresponding to the fully ionized limit, $\bar{Z} = 10$. This assumption is justified by charge state calculations from PRISMSPECT [86], which predicts $\bar{Z} > 9.5$ for $T > 1$ keV for all densities $\rho < 10$ g/cm³. This high degree of ionization is supported by the lack of observed He-like emission lines in the spectral record, which implies the dominant charge states of Si in the plasma are H-like and fully ionized. Since the charge state enters Eqs. (13) and (14) as a multiplicative prefactor, the uncertainty in $\bar{Z}$ can be wrapped up into the total uncertainty in the signal, as discussed in Sec. III C.

The radiative transfer of the continuum emission is handled by taking the optically thin limit of Eq. (11) since the extinction length is much greater than the shell thickness for the relevant x-ray energies. Under this assumption, the solution to the Eq. (11) simplifies to the Abel transform of the emissivity

$$I_\nu(r, t) = \int_r^\infty \frac{2z j_\nu(z, t)}{\sqrt{z^2 - r^2}} dz, \quad (16)$$

which has a computationally stable analytic form for the emissivity profile calculated from Eqs. (3), (4), and (12). The emission profile calculated from Eq. (16) is compared to the full solution to Eq. (10) from LILAC postprocessed using SPECT3D in Fig. 4(b). The agreement between the two profiles is nearly exact, demonstrating that the reduced-order model captures the necessary physics to explain the x-ray emission recorded by the SPCA, SFC, and PXTD.

The bound-bound line emission is more complicated because the predicted peak electron densities ($\sim 2 \times 10^{23}$ cm⁻³) are on the edge of the region where local thermodynamic equilibrium (LTE) population kinetics can safely be applied to calculate the level populations [87]. Therefore, the line emission is calculated using PRISMSPECT, which computes the non-LTE emissivity and opacity coefficients as a function of temperature and density over the relevant energy range. Equation (11) is then solved numerically for the geometry specified by Eqs. (3) and (4). Since this calculation can be computationally intensive, a neural network (NN) was trained to find an efficient mapping between the five input parameters θ_d and the resultant spatially integrated source function, $S(E)$. The NN architecture is based on the one described in Ref. [88], and was built using the jax-based equinox library [89]. An example output of the NN for the model parameters shown in Fig. 4(a) is shown in Fig. 4(c) and compared to the output from LILAC and SPECT3D. Once again, the agreement is excellent, with some deviations near the peak of each line due to resonant absorption by the ablation plasma, which is not captured by the isolated shell model in Sec. III A.

Overall, the agreement between the reduced-order model and the full radiation-hydrodynamics and radiative-transfer calculation demonstrate that the emission at a given time can be described by the parameters θ_d and that the parameters are physically meaningful with respect to the full-physics model.

C. Detector models

Once the spectral radiance has been calculated, it must be fed through a model of each detector so that it can be compared to the observed data. The signal on each detector D_i is cast as a series of convolutions of the spectral radiance with the spectral, spatial, and temporal responses of the detector [90],

$$D_i = \iiint I_\nu(r', t', E') f_i(r - r') g_i(t - t') \times h_i(E - E') G_i(r', t', h\nu') dr' dt' dE', \quad (17)$$

where f_i , g_i , and h_i are the spatial, temporal, and spectral responses of detector i , respectively, and G_i is the “geometric dilution” of the detector. For PJX, this is given by the energy-dependent sensitivity as described in Refs. [91] and [92],

$$G_{PJX}(E) = \frac{d\Omega}{dA} R(E) \frac{dE}{d\theta} \xi_{PJX}(E), \quad (18)$$

where $R(E)$ is the integrated reflectivity of the KAP crystal, $\frac{d\Omega}{dA}$ is the solid angle seen by the detector per unit area on the crystal, $\frac{dE}{d\theta}$ is the derivative of Bragg’s law, and $\xi_{PJX}(E)$ is the spectral sensitivity of the detector, given as the product of the transmission through its filtration and the absorptivity of the KBr photocathode. The integrated reflectivity is calculated using PYTTE [93], $\xi_{PJX}(E)$ is calculated from the NIST XCOM database [94], and $\frac{d\Omega}{dA}$ and $\frac{dE}{d\theta}$ are computed using a numerical ray trace of the spectrometer geometry, which is also used to compute the dispersion relation of the spectrometer [95].

For the SFC, SPCA, and PXTD models, the geometric dilution is approximated as

$$G_i(r, t, E) = \frac{\Omega_i}{d_i^2} \xi_i(E), \quad (19)$$

where Ω_i is the solid angle seen by a resolution element on the detector, assumed to be independent of position, d_i is the distance from source to detector plane, and $\xi_i(E)$ is once again the product of the filter transmission and the detector sensitivity.

The response functions, f_i , g_i , and h_i , are parameterized as Gaussians of variable width δ_i ,

$$f_i(r) = \frac{1}{\sqrt{2\pi\delta_i^2}} e^{-\frac{r^2}{2\delta_i^2}}. \quad (20)$$

The choice of a Gaussian was made because it provides a reasonable approximation to the true point spread function (PSF) of a pinhole imager [96], the resolution broadening of a flat crystal spectrometer [92], and the gain pulse of a framing camera [97]. The simple parametrization also allows the widths of the response functions to be included as nuisance parameters, meaning the posterior distribution thereof is marginalized over when considering the posteriors of the physically meaningful parameters defined in Sec. III A. This

TABLE II. Values and uncertainties for the Gaussian widths used in the diagnostic models. Spatial values are in the object plane. Where the uncertainty is neither quoted in the literature nor able to be derived from known quantities, it is assumed to be 10% of the nominal value.

Diagnostic	Time (ps)	Space (μm)	Energy (eV)	Reference
SFC	21 ± 2	5.3 ± 0.4	None	[97,98]
PXTD	53 ± 5	None	None	[58,59,99]
PJX	34 ± 3	None	3.2 ± 0.4	[57,100]
SPCA	None	4.6 ± 0.2	None	[52]

allows systematic uncertainties in the diagnostic responses to be naturally propagated to the final inference, providing more accurate estimates of the total uncertainty in the physical quantities of interest. The uncertainties in the diagnostic responses are set either by experimental measurements or, if existing measurements are insufficient, by Monte Carlo error propagation of numerically calculated response functions with uncertain inputs. For example, the PSF of the SPCA diagnostic is given as the convolution of the geometric PSF of a pinhole with the PSFs of the image plate and Airy diffraction. Each of these components is a function with some uncertain parameters (e.g., the diameter of the pinholes, which have a quoted uncertainty of around 5%). A family of PSFs can be constructed by sampling from these uncertain inputs, convolving the component PSFs, and fitting the resulting curves with a Gaussian to obtain an estimate for the uncertainty in the spatial resolution. This allows for the construction of a suitable prior distribution for the width of the PSF as a nuisance parameter in the model, enabling inclusion of uncertainty in the instrument response into the analysis, which is a major advantage of a Bayesian analysis framework. The relative resolutions of the diagnostics considered in this work as well as their uncertainties are summarized in Table II.

In addition to the widths of the response functions, two other types of nuisance parameters are introduced to account for systematic errors in model. The first is a set of multiplicative prefactors, $\mathbf{A}$, distinct for each diagnostic, which coarsely account for uncertainty in both the calculated emission in Sec. III B and the total detector sensitivity (e.g., misspecification of the filter transmission) for each diagnostic. Second is a vector of systematic timing offsets to account for error in the absolute timing of each diagnostic. Although the diagnostics are all timed to the rise of the laser pulse [101], the absolute timing can only be known to a few tens of picoseconds [102], and the absolute timing of each diagnostic is known to drift over the course of standard operations. To account for this, a pair of systematic temporal offsets, $\mathbf{t}_0$, are added to the time axes of the PJX and PXTD data to allow them to move relative to each other and the SFC data. For simplicity, the SFC time axis is assumed to be fixed since it serves as the time basis for the mechanical analysis used to constrain the shell trajectory $R_0(t)$ and the interface pressure $P(t)$.

D. Likelihood functions

With the models described in Secs. III A–III C, synthetic data for all diagnostics can be produced for a given set of

parameters, which can be compared to experimental data via an appropriately chosen likelihood function. For simplicity, the measurements are treated as independent, allowing for the construction of the total likelihood as the product of the individual likelihoods for each dataset

$$\mathcal{L}(\mathbf{D}|\boldsymbol{\theta}) = \prod_i \mathcal{L}_i(\mathbf{D}_i|\boldsymbol{\theta}), \quad (21)$$

where $\mathcal{L}_i$ is the probability of observing dataset $\mathbf{D}_i$ given the vector of model parameters $\boldsymbol{\theta}$.

In this section the reasoning used to construct the likelihood function for each diagnostic is discussed, carefully considering the statistical properties of each respective measurement. It is common practice to assume that an uncorrelated Gaussian likelihood is sufficient [23–25,41,51]

$$\mathcal{L}_i(\mathbf{D}_i|\boldsymbol{\theta}) = \prod_{j=1}^N \frac{1}{\sqrt{2\pi}\sigma_{ij}^2} \exp \left\{ -\frac{[D_{ij} - F(\boldsymbol{\theta})_{ij}]^2}{2\sigma_{ij}^2} \right\}, \quad (22)$$

where σ_{ij} is the uncertainty in data point D_{ij} and $F(\boldsymbol{\theta})$ is the output of the model developed in Secs. III A–III C. This is not necessarily bad practice since the Gaussian likelihood represents the maximum entropy (i.e., the least informed) distribution for a known variance [103]. Indeed, this work will largely end up using some form of a Gaussian likelihood for computational convenience, but for the sake of transparency, the use of this likelihood or any other should be justified using statistical arguments.

1. PJX likelihood: From the pulse-height distribution

The gold standard for the computation of the likelihood function is to rigorously characterize the photostatistics of the detector and thereby construct an accurate likelihood function including correlations and non-Gaussian effects. The use of the “true” likelihood function enables direct comparison to the raw data with minimal data processing, enabling extraction of the maximal information from a given measurement. A simplified version of this methodology is used here, adapted from previous works studying the statistics of streak cameras [57,95,104–106].

The likelihood function of a streak camera can be constructed using Poisson statistics and the camera’s pulse-height distribution (PHD). The former governs the number of photons absorbed by the photocathode for a given irradiance, n_{ph} , and the latter gives the probability of observing a signal, in ADU, given a single absorbed photon. As shown by Ref. [106], the combination of these statistical processes is well described by a truncated normal distribution

$$\mathcal{L}(\text{ADU}|\bar{n}_{\text{ph}}) = \text{TruncNorm}[\mu(\bar{n}_{\text{ph}}), \sigma(\bar{n}_{\text{ph}})], \quad (23)$$

where μ and σ are the untruncated mean and standard deviation of the distribution, parameterized in terms of the expected number of absorbed photons $\bar{n}_{\text{ph}}$.

The PHD used in this work, shown in Fig. 5(a), was measured in Ref. [57] for the PJX streak camera with a KBr photocathode using a broadband electron bombardment x-ray source with a gold anode foil. The spectrum of the x-ray source was a broadband continuum from 1-to-6-keV x-rays with a strong contribution from the gold $M - \alpha$ line (2.12

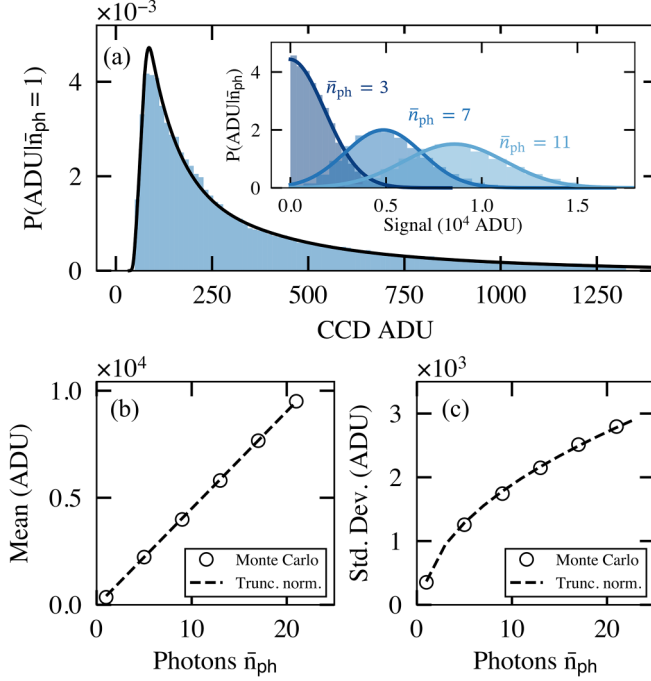


FIG. 5. (a) The measured pulse-height distribution for the PJX streak camera with KBr photocathode from Ref. [57]. This distribution is used to construct probability distributions of observing a number of ADU given an expected number of absorbed photons, $\bar{n}_{ph}$, using staged Monte Carlo sampling as shown in the inset. Each distribution is fit with a truncated normal (trunc. norm.) distribution, and the values of the untruncated mean and variance are fit as linear functions of $\bar{n}_{ph}$ to produce an analytic form for the likelihood. The mean and standard deviation of the parameterized likelihood are compared to those obtained from Monte Carlo sampling are shown in (b) and (c), respectively, showing excellent agreement.

keV). Since the photon energies observed by the PJX spectrometer are comparable to those used in this calibration, it is justifiable to assume the measured PHD is applicable to the energy range considered in the model.

The parametrizations of the untruncated mean and variance were constructed using the method of Ref. [106]. For each value of $\bar{n}_{ph}$, a staged Monte Carlo sampling was performed to construct a family of distributions as shown in the inset of Fig. 5(a). These distributions were fit with a truncated normal distribution to obtain point estimates of $\mu(\bar{n}_{ph})$ and $\sigma^2(\bar{n}_{ph})$. A linear fit was performed for both sets of values to obtain the parametrizations $\mu(\bar{n}_{ph}) = 453.9\bar{n}_{ph} - 38.9$ and $\sigma^2(\bar{n}_{ph}) = (3.81\bar{n}_{ph} - 0.25) \times 10^5$, respectively. The mean and standard deviation of the parameterized likelihood vs the exact value from the Monte Carlo sampling are shown in Figs. 5(b) and 5(c), showing excellent agreement and validating the approximation especially at higher values of $\bar{n}_{ph}$.

Streak cameras also have intrinsic correlations between pixels that make the use of an uncorrelated likelihood like the one described above incorrect [104,107]. However, this can be mitigated by rebinning the data such that correlations are reduced. The average spread of a single-photon event for the PJX streak camera was measured in Ref. [57] to have a $1/e$ width of 0.56 pixels. Therefore, for the purposes of

deriving the likelihood function, the effects of correlations in the PJX data can be neglected by adopting a 3×3 rebinning of the streaked spectrum in time and energy. This reduces the spectral and temporal resolution of the instrument by throwing away information in the rebinning process, but it greatly simplifies the calculation of the likelihood by diagonalizing the covariance matrix.

2. PXTD likelihood: From deconvolution

The observed PXTD signal is a convolution of the x-ray time history of the implosion with the light-decay curve of the BC-422 plastic scintillator, meaning the measured signal contains a non-negligible contribution due to x-ray emission from the ablation plasma during the convergence phase of the implosion, which is not described by the dynamical model in Sec. III A. Therefore, a full forward model of the raw PXTD signal shown in Fig. 2(f) is not possible given the model, and the scintillator response must be deconvolved in order to isolate the emission from the SiO_2 near peak convergence. This is done using Bayesian deconvolution based on the method of Ref. [108]. The quantity of interest is the energy deposited in the scintillator by incident x-rays $X(t)$, which is related to the observed PXTD signal $S(t)$ by

$$S(t) = L_s T_{\text{opt}} G_{\text{CCD}} [X(t) \otimes \text{IRF}(t)], \quad (24)$$

where L_s is the scintillator luminosity, or the number of optical photons per unit absorbed energy, T_{opt} is the transmission efficiency of the optical relay, G_{CCD} is the gain of the CCD, and $\text{IRF}(t)$ is the measured scintillator impulse response function, which is parameterized as a double exponential,

$$\text{IRF}(t) = (1 - e^{-t/\tau_1})e^{-t/\tau_2}\Theta(t), \quad (25)$$

where $\tau_1 = 0.089$ ns, $\tau_2 = 1.4$ ns, and Θ is the Heaviside theta function [59].

The deconvolution is performed by modeling the x-ray time history as the sum of two Gaussians, one for the ablation plasma and one for the emission from the shocked SiO_2 , and using Bayesian inference to calculate the posterior distribution for $X(t)$ given $S(t)$. This parametrization is appropriate since $X(t)$ is proportional to the time history of x-rays emitted by the plasma convolved with the temporal resolution of PXTD. The temporal resolution of 53 ps is comparable to the timescales of the implosion dynamics, meaning the broadened time history should be dominated by the temporal resolution kernel g_{PXTD} . Therefore, the assertion of the Gaussian model in the deconvolution does not contradict the primary model from Sec. III A, and the staged inference is valid. The likelihood function is assumed to be normal with covariance matrix set by Poisson statistics of the photoelectrons measured by the optical streak camera and the PSF of the optical streak-camera setup. An example deconvolved x-ray history is shown in Fig. 6(a).

The posterior distribution for the deconvolved x-ray time history $P[X(t)|S(t)]$ can then be used to construct the likelihood function for the PXTD data. The posterior is parameterized as a multivariate normal (MVN) distribution with mean and covariance calculated using the generalized maximum product of spacings method [109]. A comparison of the true posterior to the parameterized fit at two different times

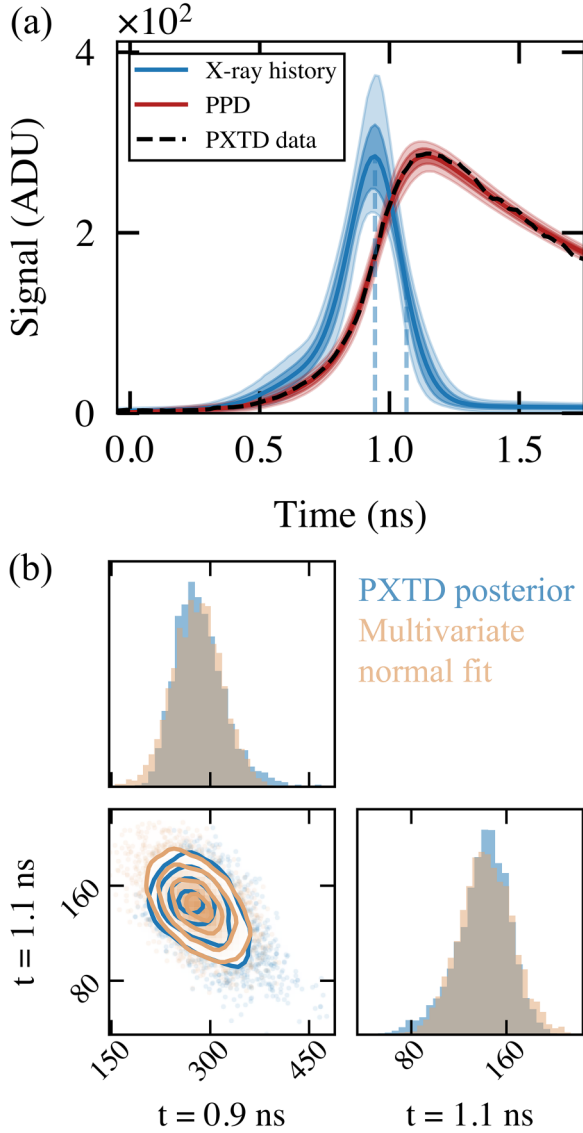


FIG. 6. (a) The PXTD trace (dashed black) is fit using Bayesian deconvolution to obtain the posterior distribution for the x-ray time history (blue). The posterior predictive distribution (PPD) from the fit is shown in red, indicating that the inferred time history appropriately describes the observed signal. The shaded contours represent 68% and 95% highest density intervals (HDIs) of the PPD. (b) The posterior for the x-ray time history $P[X(t)|S(t)]$ is fit with a multivariate normal distribution to construct the likelihood function for the PXTD data. The blue histograms show the posterior of the inferred x-ray history at two times, corresponding to the blue dashed lines in (a). The orange histograms show the multivariate normal fit to the posterior. The fit approximately captures the structure of the posterior including the mean, width, and correlations. Because of the long decay time of the scintillator response, the posterior for the signal at various times can be highly correlated, necessitating the use of a nondiagonal covariance in the likelihood to capture the statistical structure of the original data.

is plotted in Fig. 6(b), demonstrating that the fit is able to capture the essential features of the deconvolved posterior, including correlations, which can be quite large due to the long decay time of the scintillator response. The accuracy

of the MVN parametrization was tested by comparing the posterior predictive distributions for $S(t)$ calculated from the exact and MVN posteriors for $X(t)$. The PPDs were identical to better than 1%, demonstrating that the multivariate normal distribution preserves the probabilistic structure of the raw PXTD data and justifying its use as the likelihood function for the reduced PXTD data.

3. SFC and SPCA likelihoods: From the central limit theorem

The likelihood functions for both of the imagers (SFC and SPCA) follow from reducing the 2D raw images to a series of azimuthally averaged radial lineouts and invoking the central limit theorem. The central limit theorem dictates that the sampling distribution of the mean of a series of identical independently distributed (IID) random variables will converge to a normal distribution as long as the sample size is large (usually $\gtrsim 30$) [110]. Under the assumption of spherical symmetry, the projected 2D image will be uniform as a function of azimuthal angle at a given radius. Thus, the ensemble of pixels at a given radius can be thought of as IID measurements of the same quantity, and the central limit theorem can be applied. Therefore, a Gaussian likelihood with parameters set by the means and standard deviations of the radial lineouts was used for the image datasets. The statistical properties of this method of data reduction are visualized in Fig. 7. The raw 2D image shown in Fig. 7(a) is reduced to the ensemble of radial lineouts shown by the red curves in Fig. 7(b). Taking histograms of the ensemble at different radii, the distribution of signals converges to a distribution that is approximately normal as shown in Fig. 7(c).

Two caveats should be noted here: first, the central limit requires a sufficiently large sample size to be valid, meaning only radii for which a sufficient number of pixels are included in the average should be included in the calculation. Calculating this for a uniform grid of pixels averaged over the full 2π azimuthal coverage gives a minimum radius of seven pixels for a minimum sample size of 30. The second is that there are regions of the image for which deviations from spherical symmetry are apparent due to interaction of the expanding plasma with the stalk on which the target was initially mounted. This produces a noticeable bright spot on the image, violating the assumption of spherically isotropic emission. While this does not appear to alter the dynamics of the system as a whole, the azimuthal region of the images in which the stalk appears must be excluded from the ensemble of radial lineouts, which extends the central limit theorem sample size criterion to 15 pixels, shown by the gray region in Fig. 7(b).

E. Priors

To complete the model, a set of appropriate prior distributions, $\pi(\theta)$, must be chosen to quantify prior knowledge of the parameter values [103]. The set of all parameters includes the time-varying dynamical parameters $\theta_d(t) = \{R_0, dR, \rho_0, \rho_s, T_0\}$, the Gaussian process hyperparameters and mean function parameters $\theta_{GP} = \{\rho_{max}, \dot{\rho}_1, \dot{\rho}_2, t_\rho, \ell_\rho, \tau_\rho, T_{max}, \dot{T}_1, \dot{T}_2, t_T, \ell_T, \tau_T\}$, the parameters of the mechanical model θ_M , and the detector parameters $\theta_D = \{A, \delta, t_0\}$. In general, each parameter is assigned a

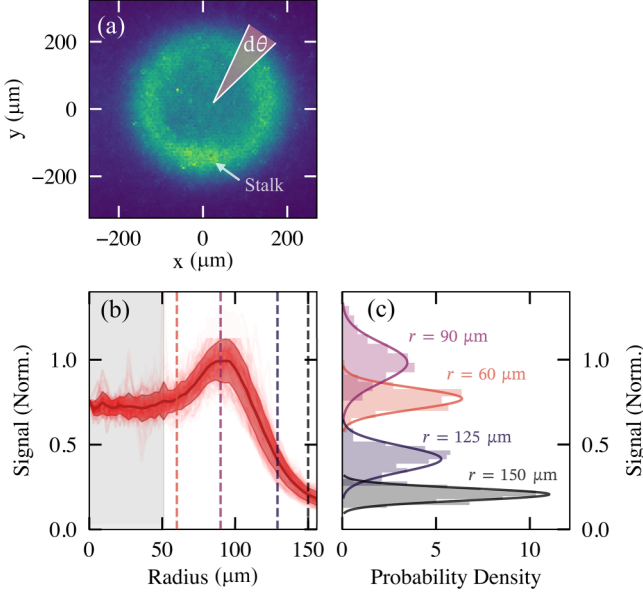


FIG. 7. The 2D x-ray images are reduced to a series of azimuthally averaged radial lineouts. Under the assumption of a spherically symmetric emitter, the distribution for the mean expected emission at a given radius should approach a standard normal distribution according to the central limit theorem. (a) A representative framing camera image during the expansion phase of shot 106536. (b) The image is reduced to an ensemble of radial lineouts (red traces). The solid and shaded lines show the median and standard deviation of the ensemble of lineouts, respectively. The shaded gray region represents radii for which there are not enough pixels to apply the central limit theorem. (c) The distribution of signal levels at each dashed line in (b) fit with a normal distribution (solid lines), demonstrating that the observed signal at a given radius is approximately normally distributed along the azimuth.

normal prior distribution with physically motivated bounds where appropriate. Where limited prior or physical constraint is available, such as for the thermodynamic parameters θ_{GP} , the priors were chosen to be weakly informative [111]. That is, they span as broad a range of parameter space as possible while still being consistent with physical constraints and physical intuition. This was done to avoid biasing the inference while excluding regions of parameter space corresponding to unphysical values of the thermodynamic states such as negative or rapidly oscillating temperatures or densities. The parameters and prior distributions for θ_{M} are the same as those used in Ref. [21], and the complete accounting of all other model parameters and their priors is presented in Table III.

Some parameters, such as the resolutions of the detectors, δ , have well-defined measurement uncertainties that make natural priors (Table II). For the others, LILAC simulations are used to assign reasonable Gaussian means for each parameter. Upper and lower bounds are chosen to enforce both physical and computational constraints. For R_0 , the prior distribution was constructed from the posterior distribution of the mechanical analysis, Ref. [21]. Since the mechanical model requires only the framing camera trajectory data, it can be performed outside of the context of the full model to reduce computational cost of the inference. The posteriors for the equations of

TABLE III. Summary of the prior distributions for all model parameters. All parameters are assigned normal priors, truncated at upper, and lower bounds where appropriate due to physical constraints. The prior mean and covariance for the parameter R_0 are constructed from the posterior distribution of the mechanical analysis. The covariances of the parameters ρ_0 and T_0 are handled by the kernel of the GPs, Eq. (7).

Parameter	Units	Location	Scale	Lower	Upper
R_0	μm	$\mu_{R_0}(t)$	$\Sigma_{R_0}(t, t')$		
dR	μm	100.0	200.0	0.0	300.0
ρ_0	g/cm^3	Eq. (8)			
T_0	keV	Eq. (9)			
ρ_s	g/cm^3	$\rho_0/2$	$\rho_0/2$	0.0	ρ_0
t_T	ns	0.84	0.1	0.7	
$\dot{T}_1$	keV/ns	36.6	18.3	0.0	
$\dot{T}_2$	keV/ns	16.7	8.4	0.0	
T_{max}	keV	5.0	3.0	0.0	
τ_T	keV	0.0	1.0	0.0	
ℓ_T	ns	0.1	0.05	0.0	
t_ρ	ns	0.84	0.1	0.7	
$\dot{\rho}_1$	$\text{g}/\text{cm}^3/\text{ns}$	13.4	6.7	0.0	
$\dot{\rho}_2$	$\text{g}/\text{cm}^3/\text{ns}$	9.7	4.9	0.0	
ρ_{max}	g/cm^3	1.0	0.5	0.0	5.0
τ_ρ	g/cm^3	0.0	1.0	0.0	
ℓ_ρ	ns	0.1	0.05	0.0	
$\bar{A}$	None	1.0	0.1	0.0	
$\bar{\sigma}$	Misc.	See Table II			
$\dot{t}_0$	ns	0.0	0.05	-0.1	0.1

motion of the shell can be used to construct the prior mean $\mu_{R_0}(t)$ and covariance $\Sigma_{R_0}(t, t')$ for R_0 which is fed into the self-emission model described in Sec. III.

Finally, various physical constraints are applied to the model as potentials (additive terms to the log-likelihood) to enforce prior knowledge. These potentials consist of a test function $w_i(\theta)$, which is compared to some boundary value w_0 . The potential $p_i(\theta)$ is then added to the log-likelihood as

$$p_i(\theta) = -\ln \{1 + e^{\gamma[w_0 - w_i(\theta)]}\}, \quad (26)$$

where γ is a scale factor that controls the sharpness and direction of the potential boundary. For example, a potential restricting the mass of the SiO_2 shell to be less than the initial shell mass M_0 has $w(\theta) = \frac{4\pi}{3} \int_{R_0}^{R_0+dR} \rho(r)r^2 dr$, $w_0 = M_0$, and $\gamma = 10$. The sigmoid form of Eq. (26) was chosen because it and its derivatives are continuous, making the posterior geometry near the constraint easier to sample compared to a discontinuous hard cutoff and allowing the sampler to explore all regions of parameter space while enforcing physical consistency. The absolute value of γ is chosen based on the value of the test function to provide a smooth transition across the boundary without introducing numerical instability, which can be assessed by the number of divergent MCMC trajectories that occur while sampling from the prior and posterior distributions.

Further potentials enforce positivity of the GP outputs for ρ_0 and T_0 and regularize the time derivative of $dR(t)$ to prevent it from exceeding the peak implosion velocity of the system. These potentials limit parameter space to only values of the

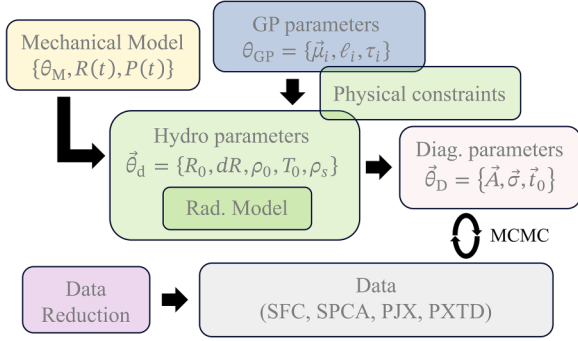


FIG. 8. A schematic of the complete model framework described in Sec. III showing the flow of information. The prior distribution for the trajectory parameters $R_0(t)$ is determined from mechanical analysis of the framing camera trajectory data, while the priors for the thermodynamic state variables, ρ_0 and T_0 , are set by the parameters and hyperparameters of the GP models. The model as a whole consists of modular pieces which can be interchanged to test and improve the overall quality of the model. The `jax`-based model architecture enables JIT compilation and automatic differentiation, allowing for rapid evaluation of the model and its gradients for efficient inference over the large parameter space.

parameters that are physically plausible, improving both the quality and numerical stability of the inference.

The models discussed in Secs. III A–III C in conjunction with the likelihoods developed in Sec. III D and the priors discussed in this section completely define a Bayesian model of the experimental system. A schematic breakdown of the components and parameters of the model is shown in Fig. 8. In total, the model is computationally efficient, able to reproduce the self-emission from the full-physics models, and parameterized in terms of relevant physical quantities. These qualities make it well suited for the detailed analysis necessary to interpret the highly integrated measurements obtained from the implosions considered in this work.

IV. MODEL VALIDATION

Before the model constructed in Sec. III can be applied to experimental data, it is important to assess its accuracy and utility by performing inference on a simulated dataset where the underlying values of the quantities of interest are known [21, 103]. The purpose of this process is twofold. First, it determines whether the simple model is able to describe the data produced by more-sophisticated simulations containing the best available physical models of the system. Second, it demonstrates whether the model and suite of measurements are informative of the physical quantities of interest. The metric of success for the first test is the quality of fit to the data, while the second is the ability of the model to accurately recover the ground-truth values for the thermodynamic quantities of interest (i.e., temperature, density, and pressure in the SiO_2 shell).

To this end, a “synthetic experiment” was performed using the 1D Lagrangian radiation-hydrodynamics code LILAC [36]. The LILAC simulation was run for the nominal target and laser parameters described in Sec. II using experimentally measured laser profiles, SESAME equation of state tables [33],

and NLTE average ion opacity tables [112]. The simulation was run with a flux-limited conduction model, where the value of the flux limiter was chosen to be 0.08 to match the experimentally observed time of peak neutron emission.

The LILAC simulation was postprocessed using SPECT3D [113] to calculate the emitted spectral radiance, which was fed through the detector models described in Sec. III C to generate synthetic data. For the PJX detector, realistic noise was added using the pulse-height distribution, while uncertainties for the images and PXTD were modeled heuristically using power-law fits to the experimental uncertainties as a function of signal level.

The posterior distribution for the synthetic data was sampled using the No-U-Turn Sampler [114] with four parallel chains. Convergence was assessed using the Gelman-Rubin statistic $\hat{r}$, which estimates the ratio of the within-chain variance to the interchain variance for each parameter [115]. As the Markov chains converge to their stationary distribution, $\hat{r}$ approaches unity, making it a popular convergence diagnostic for MCMC samplers. Following a burn-in period, the MCMC was terminated once it reached a Gelman-Rubin statistic of less than 1.01 for all parameters with no divergent trajectories.

To assess the quality of the fit to the data, the posterior predictive distributions (PPDs) can be examined for the various datasets. The PPD is defined as the distribution of potential unobserved values conditioned on the observed values

$$P(\mathbf{D}'|\mathbf{D}) = \int d\theta P(\theta|\mathbf{D})P(\mathbf{D}'|\theta), \quad (27)$$

where $\mathbf{D}$ is the dataset on which the model is conditioned, and $\mathbf{D}'$ is the “unobserved” dataset. The PPDs are constructed by sampling from the posterior distributions for each parameter and pushing those samples through the full model to create an ensemble of datasets given the inferred distributions on the parameters. If the data are well represented by the model, the PPDs should closely reflect the input dataset. This is visualized in Figs. 9(a)–9(e), which show a representative subset of the PPDs from the synthetic experiment, and demonstrate the agreement across several different axes.

Because the dataset is large and multidimensional, it is difficult to visualize model performance as a whole using a reasonable number of plots. Therefore, it is useful to summarize the performance of the model using an accuracy plot as shown in Fig. 9(f). The data are placed on equal footing by dividing by the data uncertainty so that all datasets lie on the same scale. The strong correlation between the observed and predicted means for the dataset as a whole indicates that the data are well represented by the model.

In order to be useful, the model must not only be able to describe the data, but it must also be able to constrain physical quantities of interest. Figures 9(g)–9(i) compare the marginal posteriors for the density, temperature, and pressure from the model to the underlying ground-truth values from LILAC, taken to be the values of each quantity at the D_2 interface. Figure 10 shows the posterior correlation structure between these quantities to ensure that they are uniquely constrained and identifiable within the context of the model. This process also provides interesting insight into the physics of the system. Evidently, the temperature and density at a given time are strongly anticorrelated. This is consistent with the functional

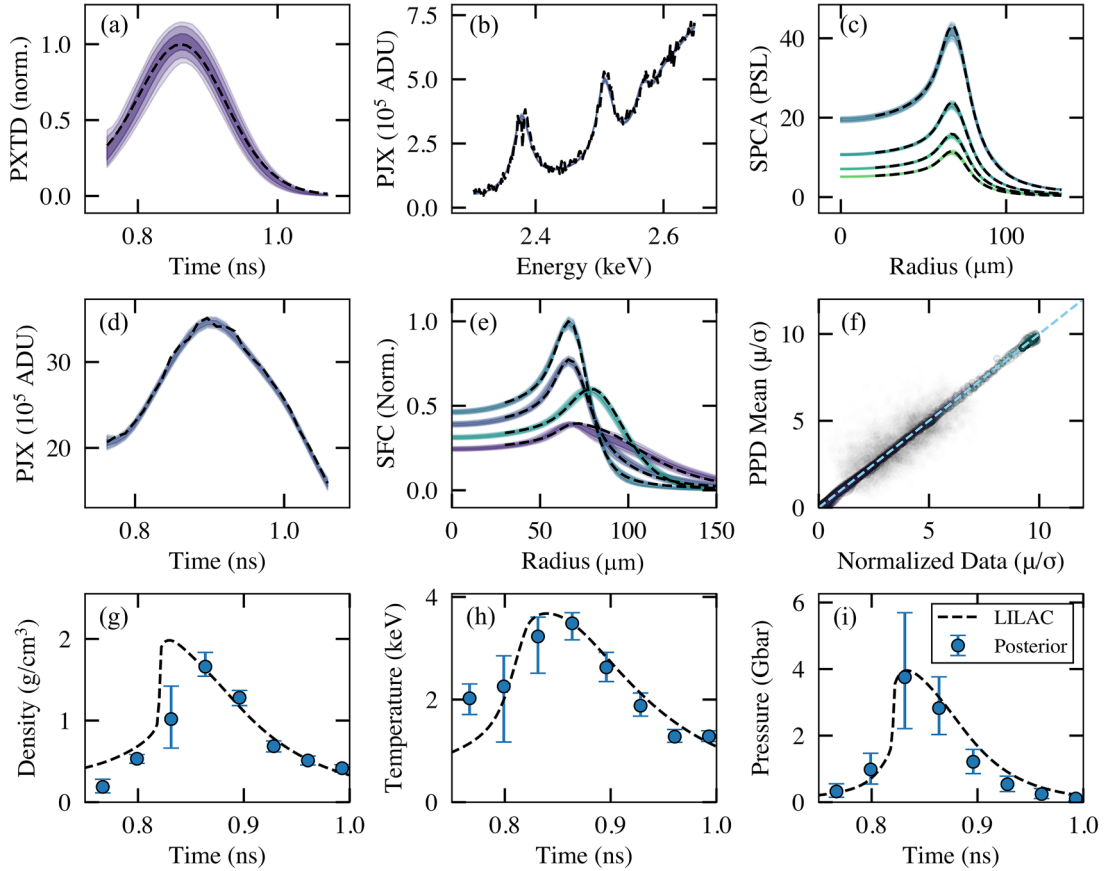


FIG. 9. A representative selection of the posterior predictive distributions (PPDs) for a few relevant integrated quantities for each diagnostic: (a) the x-ray time history from PXTD, (b) the time-integrated PJX spectrum, (c) four of the SPCA channels, (d) the energy-integrated PJX time-history, and (e) representative framing camera images, normalized by the integrated signal. Excellent agreement between synthetic data (dashed curves) and the posterior predictive distributions (shaded curves, showing the 68% HDIs) demonstrates that the model is able to capture the relevant physics to reproduce the self-emission data. (f) The synthetic dataset normalized by the data uncertainty is plotted against the mean of the PPD scaled by the same factor, showing excellent agreement across the entire dataset. The blue dashed line represents perfect agreement between the model and the data. The posterior distributions for the time histories of the (g) density, (h) temperature, and (i) pressure recover the ground-truth values from lilac (black dashed lines), although the density during the end of the convergence phase is systematically underestimated. The error bars show the marginal 68% HDI of the posterior at each time point. The full posterior structure of the thermodynamic quantities is shown in Fig. 10. Ground-truth values are taken to be the values at the $\text{SiO}_2\text{-D}_2$ interface from the LILAC simulation. The ability of the model to both reproduce the synthetic data and recover the thermodynamic quantities validates it as a useful analysis tool for interpreting the experimental data.

forms of the emission terms in Eqs. (13) and (14), which show that the total (energy-integrated) emission increases with both temperature and density. Therefore, as one increases, the other must decrease to compensate. Similar correlation analysis could be used to identify which measurements or information are required to tighten the constraint on various quantities of interest.

The process of comparing the inferred parameters to the ground-truth values from a physically consistent simulation is helpful for identifying under what circumstances the values of the model parameters are physically meaningful and can be abstracted beyond the context of the model. For example, even though the model is able to fit the self-emission data during the end of the convergence phase, the inferred density (i.e., the posterior distribution for the parameter ρ_0) is systematically lower than the ground-truth interface value. This is possibly due to misspecification of the density profile during the late convergence phase, where the outer surface releases

because it is no longer supported by the ablation pressure. This creates a superlinear density gradient that introduces a bias in the relationship between the imaged emission from Eq. (16) and the temperature and density values in the plasma. In principle, this could be addressed by iterating on the model and introducing a different density profile for the convergence phase, but since the principle quantities of interest in these experiments are the thermodynamic states in the shocked SiO_2 during the expansion phase, the accuracy of the inference shown in Figs. 9(g)-9(i) is sufficient.

It is important to note that the process of benchmarking the model against LILAC simulations implicitly assumes that a 1D hydrodynamic model appropriately describes the experimental system. As discussed in Sec. II, the implosions studied in this work are known to be highly symmetric and hydrodynamically stable, and the data qualitatively supports these assumptions. Regardless, future work should seek to quantify the systematic effects of deviations from 1D

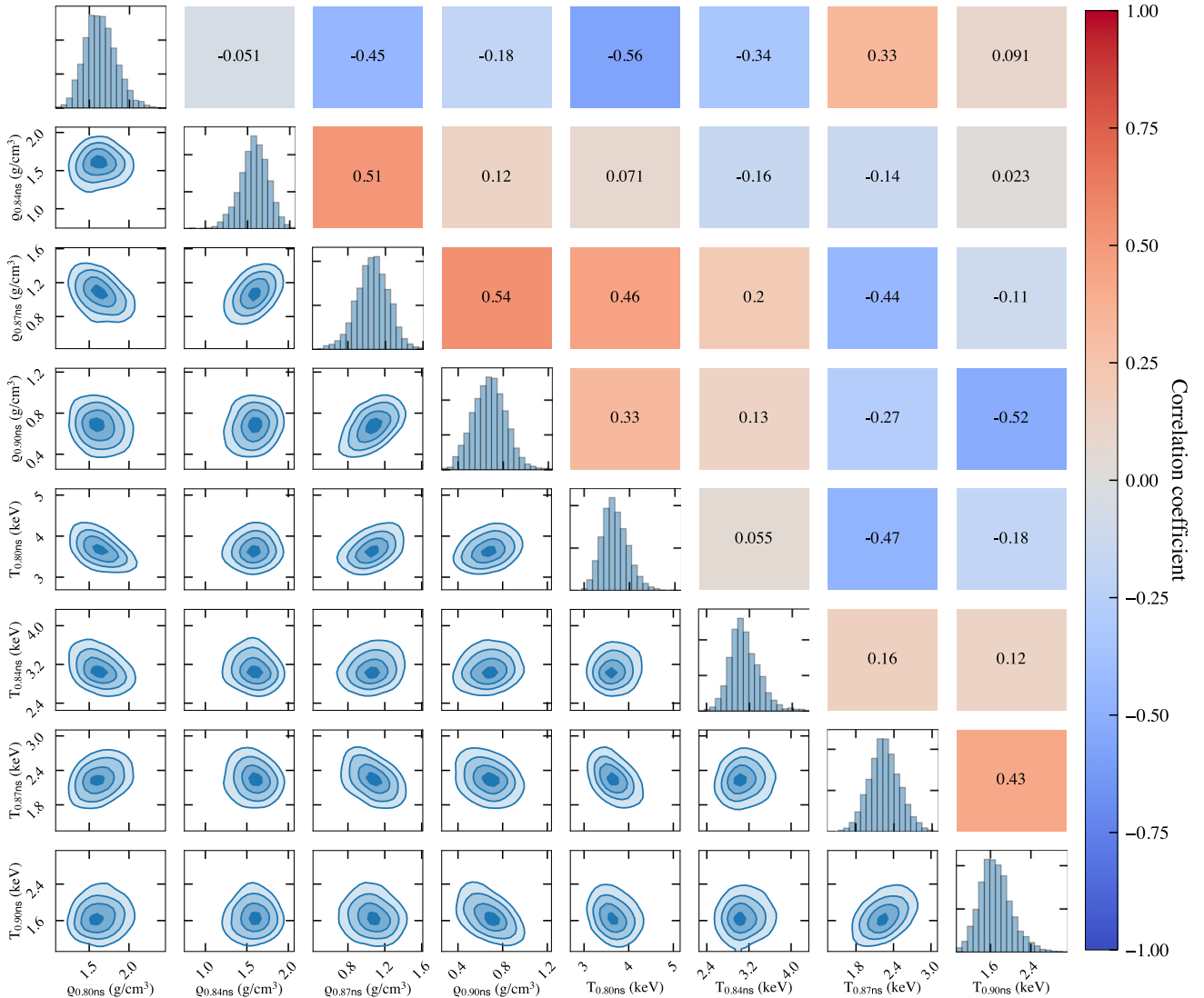


FIG. 10. Pairwise distribution plots for a subset of the inferred temperatures and densities from the synthetic model test shown in Figs. 9(g) and 9(h). The colored squares in the upper right triangle are the Pearson correlation coefficient between the marginal posteriors plotted on the diagonal. The posterior exhibits interesting correlation structure, with the temperature and density at a given time exhibiting strong negative correlation. The temperatures and densities at adjacent times are also correlated, as expected given the GP priors defined by Eqs. (5)–(9). Examination of the posterior structure shows that all thermodynamic parameters are identifiable and uniquely constrained.

hydrodynamic behavior. The validation framework outlined in this section can straightforwardly be adapted to include other high-fidelity models that explicitly treat things like kinetic effects, low-mode asymmetries and mix.

In total, the results of the synthetic experiment show that the reduced-order model described in Sec. III is able to describe the complete self-emission dataset generated by the full-physics simulation while accurately recovering the ground-truth values of the thermodynamic states in the shocked SiO_2 during the expansion phase. This demonstrates the model's utility as an analysis tool for interpreting the suite of integrated measurements obtained in these experiments.

A. Information content of measurements

Another advantage of testing the model against synthetic data is that it enables detailed quantification of the information

content of each diagnostic [41]. Given the complex, integrated nature of the datasets in implosion experiments, it is not always intuitive how a measurement maps onto the inference of quantities of interest. This is helpful for experimental design because it allows the experimentalist to decide which diagnostics to field in order to accomplish the inferential goals of the experiment, it also helps understand from an analysis point of view which datasets contain the most useful information.

To this end, a series of synthetic experiments was considered in which one or more of the diagnostics were omitted in order to better understand the information content of each measurement with respect to the inferred thermodynamic states. The synthetic analysis described in Sec. IV was repeated for five cases: no PJX, no SPCA, no PJX or SPCA, and no PJX with a normalized version of the SPCA data (i.e., containing relative but not absolute photometric information), corresponding to datasets that were collected experimentally

TABLE IV. The completeness of the diagnostic suite for each for each implosion experiment analyzed in this work.

Shot no.	SFC	SPCA	PXTD	PJX
106536	✓	✓	✓	
106538	✓	✓	✓	
106542	✓	✓		
107850	✓	✓	✓	✓
107851	✓	✓	✓	✓
107852	✓	✓	✓	✓
114517	✓	✓		✓

with incomplete diagnostic suites, cataloged in Table IV. To assess the information gained from the experiment, the posterior distributions for the inferred peak temperature and density are plotted in Figs. 11(a) and 11(b). These two quantities can be thought of as proxies for success in the inference. As expected, the complete dataset is the most constraining of both values and the dataset without any sort of photometric information (no PJX or SPCA) is completely unconstrained for both. Interestingly, the information provided by PJX appears to be somewhat degenerate with SPCA in this space since

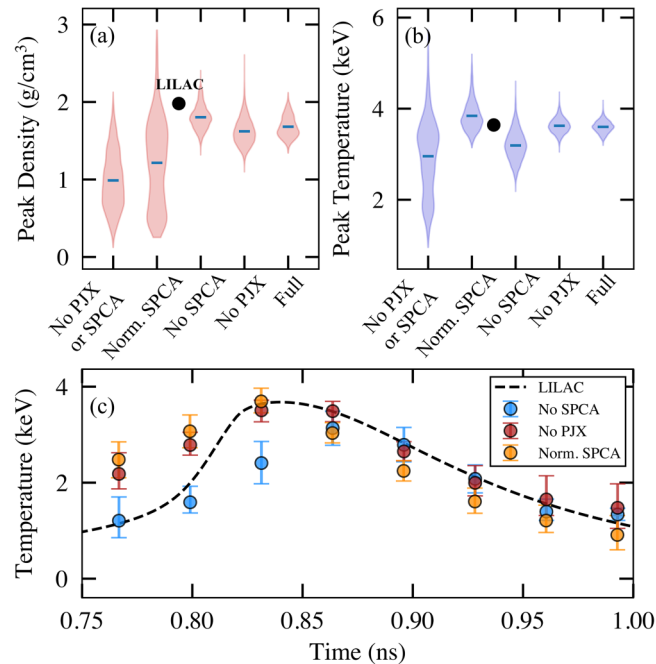


FIG. 11. Violin plots of the inferred posterior distributions for the peak (a) density and (b) temperature when various diagnostics are omitted from the analysis. The black circle represents the ground-truth value from the underlying LILAC simulation. (c) The inferred temperature histories for the synthetic dataset without SPCA (blue), without PJX (red), and with a normalized version of the SPCA data (orange) compared to the ground-truth value from LILAC (dashed black). The SPCA data provide superior temporal constraint on the temperature history despite being a time-integrated measurement. This demonstrates that the information provided by the PJX data is largely redundant with respect to the inferential goals of the analysis, although its inclusion does reduce the uncertainty in the temperature late in time.

its inclusion only marginally improves constraint on the peak quantities.

The main takeaway from Fig. 11 is that absolute photometric information is essential to constrain the density of the plasma, since it provides constraint on the total emitting mass of the plasma. This means the constraint on density is only as accurate as the diagnostic photometric models. This uncertainty is handled by the priors on A , but the leverage of the photometrics provides clear motivation for further characterization of the instrument sensitivities [106]. Conversely, the temperature is well constrained by the spectral content (i.e., the slope of the continuum) contained in the relative intensities of the SPCA channels even without absolute photometric information. Figure 11(c) shows that the SPCA dataset alone provides accurate constraint on the temporal evolution of the temperature, despite the fact that it is a time-integrated diagnostic. The exact reason behind this is unclear, and an example of how the mutual information contained in a set of integrated measurements is often counterintuitive. It could be a result of the diminished time resolution of the PJX data due to the rebinning procedure discussed in Sec. IIID 1, or it could be that the line ratios of hydrogenic silicon ions are not a sensitive temperature diagnostic at the conditions reached in these experiments. Regardless of the underlying cause, this is an important result for two reasons. First, it suggests that the PJX diagnostic is not essential to the inferential goals of the experiment and that the implosion experiments in Table IV without PJX data should still be included in the dataset. Second, it opens a window to perform detailed atomic physics studies at Gbar pressures using streaked spectroscopy as an auxiliary diagnostic to test atomic physics models. If the other diagnostics are able to provide quasi-independent temperature and density measurements, PJX can be used to study how linewidths and ratios vary with temperature and density at extreme conditions. A similar implosion capsule with a mid-Z dopant added to the SiO_2 , such as iron [116] or germanium [117] would enable detailed testing of more interesting atomic physics at known conditions via streaked spectroscopy.

V. RESULTS AND DISCUSSION

The analysis described in Sec. IV was applied to the experimental datasets obtained from the seven implosion experiments at the OMEGA Laser Facility in Table I. Each shot was analyzed independently using the model constructed in Sec. III. Figures 12(a)–12(e) show the results of the analysis along the same axes as in Fig. 9 for a representative implosion dataset (shot 107851). As in the synthetic case, the strong correlation between the observed data and the predicted mean of the PPDs in Fig. 12(f) demonstrate a satisfactory fit to the data. Unlike in the synthetic case, Fig. 9(f), there is noticeable structure in the accuracy plot, especially at the lower end of the dataset (corresponding to regions of lower signal to noise), which can be attributed to aspects of the experimental dataset that are not captured by the model. The posterior distributions of the inferred thermodynamic state variables for shot 107851 and two nominally identical experiments, shots 107850 and 107852, are shown in Figs. 12(g)–12(i). The correlations between thermodynamic parameters for shot 107851 are shown

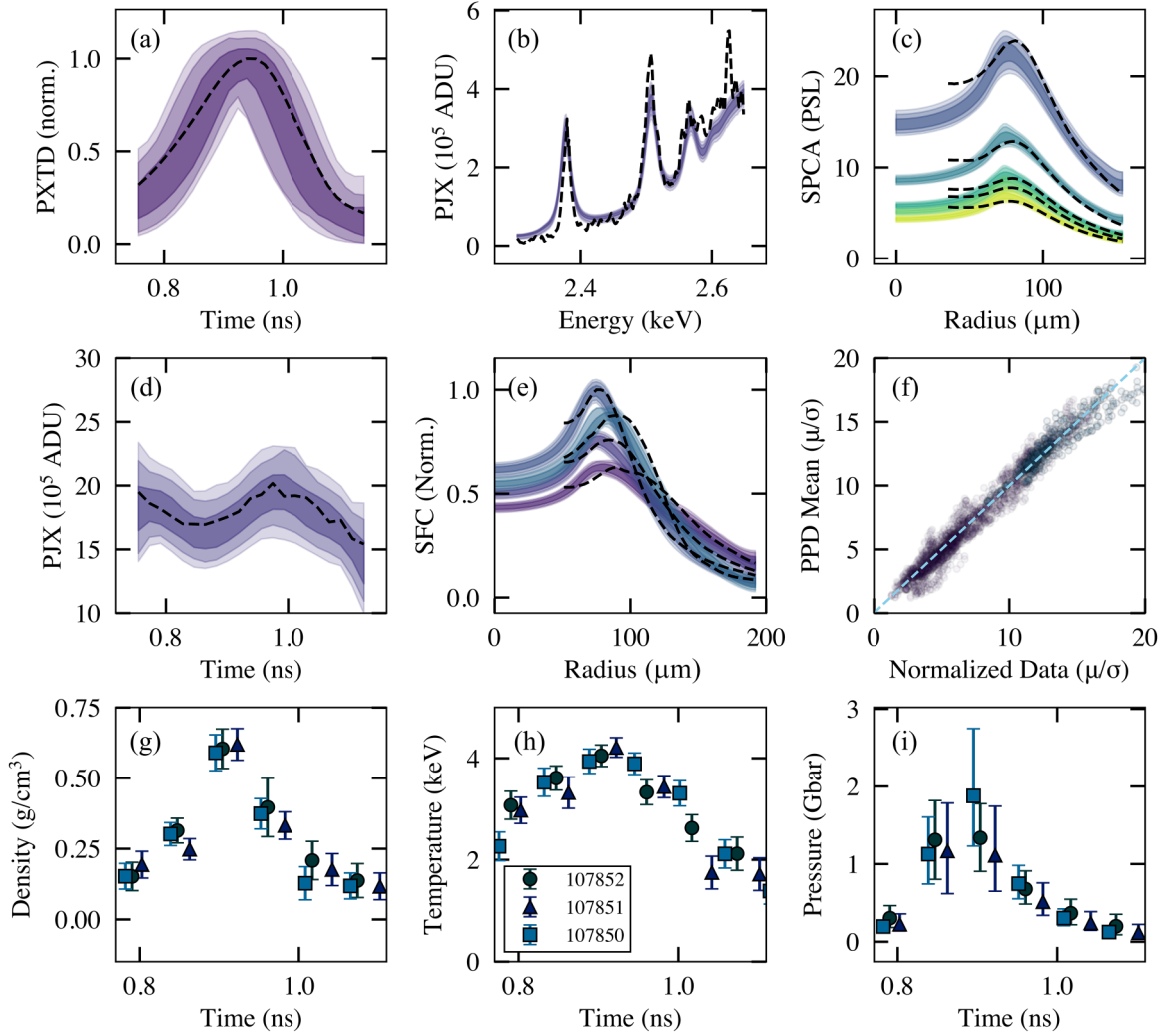


FIG. 12. Representative data PPDs for the fit to shot 107851. The posterior predictive distributions show excellent agreement with experimental data for (a) the PXTD history, (b) the time-integrated PJX signal, (c) the SPCA channels, (d) the PJX time history, and (e) SFC lineouts. (f) There is noticeably more scatter in the accuracy plot than is seen in the synthetic case in Fig. 9(f), but the overall agreement is excellent, and the model is able to satisfactorily fit all four diagnostics simultaneously. The thermodynamic posteriors for (g) density, (h) temperature, and (i) pressure from three identical shots (shots 107850–107852). The scatter between the shots gives a representative idea of the systematic error incurred by shot-to-shot variation and by data-processing choices.

in Fig. 13, which shows qualitatively similar posterior structure to the synthetic case, shown in Fig. 10. The comparison between identical implosions shows that the analysis is repeatable and gives an idea of the scatter that results from data processing choices and shot-to-shot variation.

The complete set of inferred thermodynamic conditions across all seven experiments from Table I is shown in Fig. 14. While each implosion reaches comparable peak pressures of roughly 1.5 Gbar, the peak densities and temperatures vary significantly with input parameters. For example, shot 114517, which has a lower gas-fill pressure of 15 atm reaches a significantly higher peak temperature and lower density than the other shots that have higher gas-fill pressures. Likewise, the set of shots with smaller outer diameters (ODs), shots 107850–107852, reach higher peak densities than the targets with larger ODs. More experiments are needed to explore these trends, but they do imply that modest changes in input parameters can lead to measurable changes in the

thermodynamic states of the implosion. This potentially provides a lever for tuning these experiments to map out high-energy-density phase space and test physical models over a broader range of thermodynamic conditions. For example, the simultaneous inference of all three state variables provides constraint, conditioned on the assumptions of the model, on the EOS of plasmas at conditions relevant to high-mass stars and ICF plasmas. The variation in Fig. 14 implies that this constraint could be improved by tuning target parameters to map out a broader swath of thermodynamic phase space, enabling better understanding of the physical processes that occur in these extreme systems. Alternatively, coarser adjustments to the conditions probed in these experiments could be achieved by drastically changing the experimental setup and taking advantage of the wide design space of convergent laser-driven compression experiments shown in Fig. 1. While the dynamic model of Sec. III is clearly only suited for a small subset of possible implosion designs, it can easily be

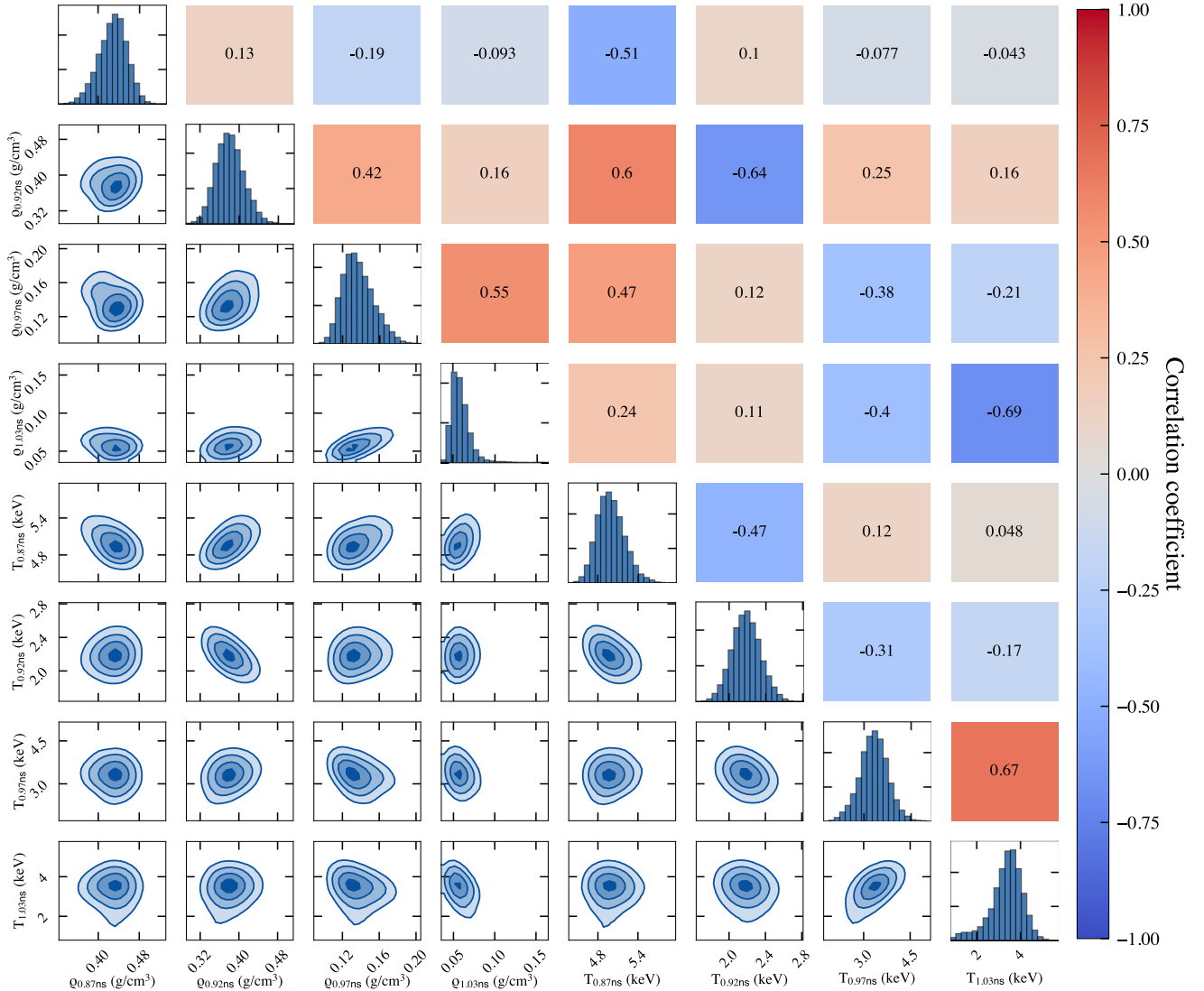


FIG. 13. Representative pairwise posterior distributions for a subset of the thermodynamic conditions inferred from shot 107851. The correlation structure is similar to that in the synthetic case, shown in Fig. 10, with the strongest correlations being between the temperature and density at a given time and the conditions at adjacent time points. This correlation structure must be explicitly considered when using the inferred conditions to test physical models.

replaced with a more sophisticated model that considers, for example, the finite inertia of a thicker imploding shell [118], the dynamics and nuclear reactions of deuterium [32], or the role of ion-electron equilibration [41].

VI. CONCLUSIONS

With the practical and engineering aspects of performing implosion experiments more or less solved, the final piece of the puzzle in harnessing the potential of convergent experiments as a driver for fundamental high-energy-density physics lies in the interpretation of the integrated datasets obtained in these experiments. The amount of data collected in a single implosion experiment is vast, enabling a large amount of information to be extracted, given properly constructed models of the physical system and detectors. This work demonstrates that Bayesian inference along with simplified physical models

provide a promising solution to the problem of diagnostic access. The Bayesian paradigm effects an information-centered approach to experimental analysis and design [21,119], as construction of the full posterior enables quantitative assessment of both the absolute and relative information gain of a particular experiment and analysis. The model presented in this work is not unique, but it defines a physically informed prior over the space of models, informed by and benchmarked to high-fidelity simulations. Future work should seek to explore the impact and validity of the assumptions involved in the construction of the model, for example, by exploring alternative parametrizations or benchmarking against 2D and 3D simulations. The machinery of Bayesian inference provides a straightforward framework for model comparison and combination [21,66,120,121], allowing data-driven quantitative testing of physical assumptions and models. Building on the foundation laid out in this work by developing more sophisticated and generalized

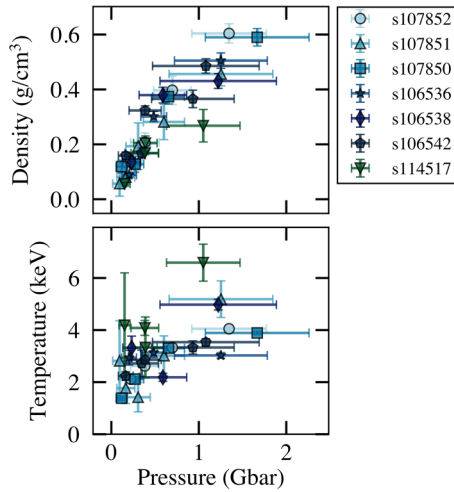


FIG. 14. The complete set of thermodynamic conditions inferred for the seven implosion experiments in Table I. Since the synthetic experiment suggests the inferred conditions during the convergence phase are not accurate, these points display only the inferred conditions during the expansion phase. These data provide the highest-pressure EOS constraint in a laboratory experiment to date, providing benchmark data at conditions that are essential to modeling high-mass stars and ICF plasmas.

models will enable increasingly precise, data-driven inference of the physics of implosions, spanning greater regions of parameter space and opening new windows to directly study materials at Gbar pressures.

Just as planar shock experiments changed our understanding of matter at planetary interior conditions [2–4], convergent experiments are uniquely positioned to reshape our understanding of the Gbar plasmas found in the interiors of stars and other astrophysical objects. There are many outstanding questions about these objects, including about our own sun [122,123], which could potentially be answered by the appropriate set of implosion experiments. Even the most precise helioseismological measurements rely on models of the material properties deep inside the sun [124], which would benefit greatly from laboratory validation. The ability of implosion experiments to access these extreme pressures, combined with

the rich integrated datasets collected in these experiments, make them a compelling platform to study some of the most extreme plasmas in the universe.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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