

Simulating and analyzing the dynamics of collective movement with elastic interactions and leadership influence

A. Eddakoun^{1,*}, A. Hader^{1,2}, I. Tarras³, I. Bakkasi¹, M. Tanasehte¹, R. Moulitif³, M. Eddahby², and A. el Bachiri¹

¹*LBGIM, Ecole Normale Supérieure, Université Hassan II, Casablanca, Morocco*

²*Centre Régional des Métiers de l'Éducation et de la Formation Casablanca-Settat, Morocco*

³*Laboratoire de Physique de la Matière Condensée Faculté des Sciences Ben M'Sik, Casablanca, Morocco*



(Received 7 April 2025; accepted 7 July 2025; published 28 July 2025)

This paper investigates the simulation and analysis of collective self-propelled motion in the presence of a leader, a phenomenon commonly observed in natural and artificial systems, such as bird flocks or robotic swarms. Using a dynamic approach based on the Langevin equation, the study models interactions between particles and a leader through elastic restoring forces and incorporates stochastic noise to simulate random fluctuations. These forces ensure group cohesion, avoid collisions, and mimic real-world conditions where unpredictability plays a significant role. The simulation starts with particles randomly distributed in a two-dimensional space. As the leader moves at a constant velocity, particles adjust their positions and velocities to follow it. The results demonstrate a two-phase evolution: an initial rapid clustering around the leader due to strong elastic forces, followed by a gradual alignment of particle velocities to match the leader. The velocity profile, represented by a linear increase followed by an exponential decay, captures the transition from disorder to order. Key parameters such as the spring constant k_1 , which governs the leader-particle interaction, and the diffusion coefficient D , which regulates stochastic noise, are shown to significantly influence the system dynamics. High interaction strength accelerates synchronization and stabilizes the group, while weaker interactions lead to slower clustering and potential dispersion. Phase diagrams and susceptibility analyses confirm that the leader's role is critical in initiating and maintaining collective behavior. This study highlights the emergence of coordinated motion in a leader-follower context, offering insights into biological systems and inspiring applications in swarm robotics, where efficient and cohesive movement is essential.

DOI: [10.1103/2p2h-vz38](https://doi.org/10.1103/2p2h-vz38)

I. INTRODUCTION

The collective behavior of individuals, whether they are particles or living beings, is one of the most complex phenomena to understand and analyze. This phenomenon manifests at different scales: at the microscopic level in the field of biology [1], and at the macroscopic level in fish [2], birds [3], and in human beings. While heuristic optimization approaches like particle swarm optimization (PSO) [3] and artificial bee colony (ABC) [4] have demonstrated the effectiveness of swarm-inspired algorithms for computational problems, our work focuses specifically on modeling the physical interactions governing natural collective systems. The complexity of studying collective behavior lies in the large number of individuals involved and in the characterization of the interactions between them. For example, birds in a flock fly in an organized and structured manner. This movement is not random but relies on a certain form of communication between the

individuals in the group. The fundamental question that arises is, therefore, how can we predict and model such behavior?

The first researcher to interpret collective behavior was Vicsek, who introduced a pioneering model to describe self-propelled collective motion. In his work, Vicsek [5] studied a two-dimensional system composed of N particles, each moving at a constant speed while adjusting its direction based on the average orientation of its immediate neighbors. His model highlighted a phase transition phenomenon, where the system shifts from a disordered state, with particles moving randomly, to an ordered state where they move coherently. The key parameters influencing this transition are the particle density and the noise disturbing the individuals' orientation.

After Vicsek's work, several researchers continued exploring collective behavior by improving his model. Among them, I. Tarass [3,4] made significant contributions by modifying the original model to better represent the interactions between particles. His work introduced zones of orientation and repulsion, allowing particles to avoid collisions while adjusting their direction. Thanks to these adjustments, Tarass was able to demonstrate greater accuracy in describing group dynamics, particularly how particles maintain cohesion while avoiding destructive interactions. These improvements helped to better understand the mechanism behind the formation of coherent structures, such as bird flight formations, and made the model applicable to a wider range of biological

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

and physical systems. L. Amallah [5,6] took a different approach to simulating collective motion. In her work, she used a Langevin equation, in which she incorporated a random force to model the stochastic behavior of the system. This approach allowed for the exploration of the impact of random fluctuations on collective dynamics, improving the understanding of self-organization processes in noisy environments [7,8], particularly regarding the cohesion and evolution of groups under varying conditions.

In this work, we will use a dynamic approach based on the Langevin equation [9–12] to simulate collective motion. The situation we aim to simulate is one where a group of particles seeks to follow a leader, a scenario frequently encountered in real-life cases, such as during animal migrations or in robotic swarms. The originality of our approach lies in modeling the interactions between particles and with the leader using an elastic restoring force. This force simulates both collisions between the particles and attraction to the leader, ensuring group cohesion while avoiding destructive interactions. By also integrating stochastic forces, we explore the effect of random fluctuations on collective movement, providing a deeper understanding of the emerging dynamics in noisy environments.

Building upon prior research, this work introduces an avenue for investigating collective movement, enabling the exploration of dynamic aspects previously overlooked in predominantly kinematic models. Moreover, integrating this dynamic framework with systems like machine learning can enhance the prediction of collective motion, with practical applications in everyday scenarios such as the evacuation of sports stadiums or densely populated areas, and also in forecasting the movement of certain animals.

This paper includes four sections organized as follows: A general introduction in the first section; the model used in this study is presented in Sec. II. The obtained results are presented and discussed in Sec. III. Finally, we summarize our numerical results in the last section.

II. THE MATHEMATICAL MODEL

In our model, we consider N particles initially at rest, randomly dispersed in a two-dimensional space. A leader, moving at a constant speed, enters the system. Once the leader arrives, the particles instantaneously adjust their positions to follow it, resulting in an abrupt change in their velocities. However, in biological systems, both at microscopic scales and at larger scales, particles do not simply follow a leader; they communicate with each other.

To model this interaction, we introduce an elastic restoring force between each particle and its nearest neighbors. This force prevents the particles from getting too close to each other. If two particles are closer than a threshold distance r_0 , a repulsive force is applied to push them apart. This force is described by the following expression:

$$\vec{F}_{re} = k_2(r_0 - d) \frac{\vec{r}_i - \vec{r}_j}{d}, \quad (1)$$

where d is the distance between particles i and j , and k_2 is the repulsion constant.

Similarly, the connection between each particle and the leader is modeled by another elastic restoring force, which pulls each particle toward the leader. The magnitude of this force is proportional to the distance between the particle and the leader and is adjusted by a spring constant, k_1 . This force is written as

$$\vec{F}_{Le} = k_1(\vec{r}_{le} - \vec{r}_i), \quad (2)$$

where $\vec{r}_{le}$ leader is the position of the leader and $\vec{r}_i$ is the position of particle i .

In addition to these two forces, we must consider the stochastic noise in the collective motion. Stochastic noise is a random term added to the equations of motion to represent unpredictable or disordered influences. This is common in complex dynamic systems, such as particles suspended in a fluid, where they are subject to thermal fluctuations (as in Brownian motion). The noise adds variability to the system and alters the particles' trajectories in a nondeterministic way. In our simulation, the stochastic noise is represented by the following expression:

$$F_{st} = \sqrt{2D\Delta t}N(0, 1)\eta_{max}A, \quad (3)$$

with

$$A = \min(1, \frac{1}{10}). \quad (4)$$

The term A is an attenuation factor that gradually reduces the effect of the stochastic noise during the first few iterations to avoid abrupt changes at the start of the simulation. Specifically,

- (i) during the first few iterations (frame < 10), the stochastic force is progressively increased;
- (ii) at the tenth iteration (frame $= 10$), the noise reaches its maximum intensity; and
- (iii) after the tenth iteration, the attenuation factor equals 1, and the stochastic noise becomes fully active.

D is the diffusion coefficient, which controls the intensity of the stochastic fluctuations, Δt is the time step of the simulation, $N(0,1)$ is a randomly drawn number from a Gaussian distribution with a mean of 0 and variance of 1, and η_{max} is the maximum amplitude of the stochastic noise.

The goal of this study is to predict the collective behavior of particles in the presence of a leader. In particular, our simulation allows us to interpret the formation of groups, such as animals or bacteria, using these physical interactions. To achieve this, we must calculate the position and velocity of each particle at every iteration, which requires solving the Langevin equation that incorporates the three aforementioned forces:

$$\frac{d\vec{v}_i}{dt} = \vec{F}_{Le} + \vec{F}_{re} + \vec{F}_{st}. \quad (5)$$

Then, the velocities and positions are updated using the Euler method:

$$\vec{v}_i(t + dt) = \vec{v}_i(t) + \frac{\vec{F}^{(total)}}{m} \Delta t, \quad (6)$$

$$\vec{r}_i(t + dt) = \vec{r}_i(t) + \vec{v}_i(t) \Delta t. \quad (7)$$

The large number of particles, as well as the multiple interactions between particles and the leader, make manual

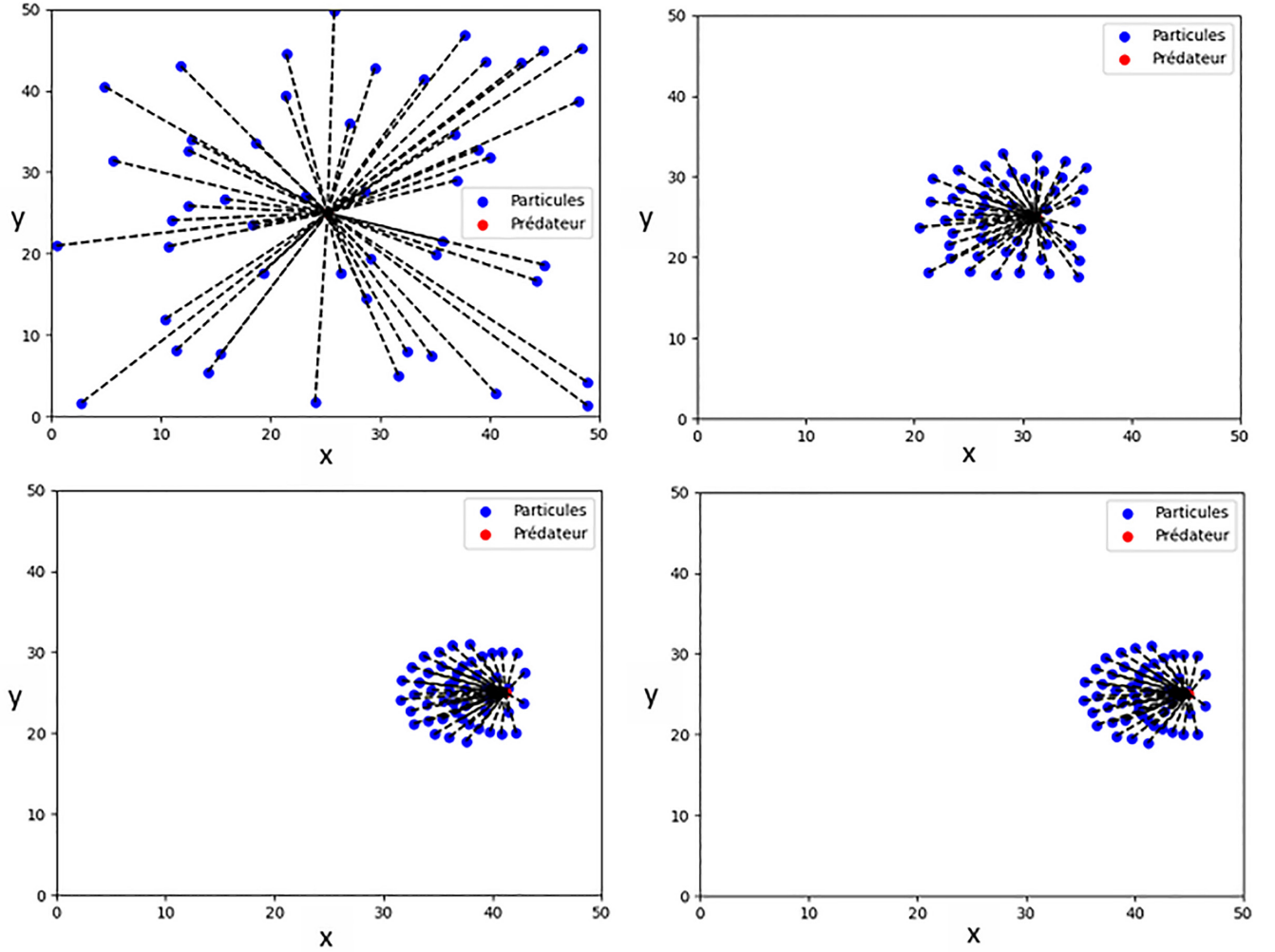


FIG. 1. Representation of the system image at four times: $t = 1$; $t = 30$; $t = 80$; $t = 90$.

resolution of the equation impossible, thus requiring the use of a programming language such as Python to implement the simulation.

III. SIMULATION RESULTS AND DISCUSSION

To discuss the evolution of the system and compare the results to a real case of collective movement observed in certain living organisms, we begin with a visual representation of the system, showing the evolution of particle positions at different moments, from the initial state to the final state. This representation gives us a general overview of the results of our simulation and allows us to check whether the particle evolution is similar to that encountered in nature, either from a macroscopic perspective, such as in the movement of larger organisms, or at a microscopic level, such as bacteria. Figure 1 presents an image of the system at four distinct time points.

We observe that the system evolves from an initial state where particles are randomly distributed (at $t = 0$) to a final state where all particles follow the leader at a constant speed. Moreover, the particles organize themselves into a specific structure around the leader. This configuration is reminiscent of the V-shaped formations observed in groups of birds, where

an individual guides the group. An examination of Fig. 1 reveals collision-free movement between particles and a smooth transition from the initial to the final state. These characteristics are attributable to the choice of model parameters, notably the spring constant of the elastic forces, the diffusion constant D , and the noise intensity. These parameters respectively control the interaction between particles, the tendency toward diffusion, and the randomness of the movement.

We will now focus on the impact of control parameters on the system dynamics. Recall that the system undergoes a phase transition, moving from an initial random state to a final equilibrium state. To characterize this transition, we will analyze the time evolution of the system's average velocity. Figure 2 represents the time evolution of the average velocity $V_m(t)$ of the system, defined by the equation

$$V_m(t) = \frac{1}{N} \sum_{i=1}^N V_i(t). \quad (8)$$

We observe a two-phase evolution of the average particle velocity. Initially, the average velocity increases rapidly. This phenomenon can be explained by the fact that the particles, initially dispersed, suddenly reorganize to follow the leader's motion. This initial acceleration, which can even exceed the

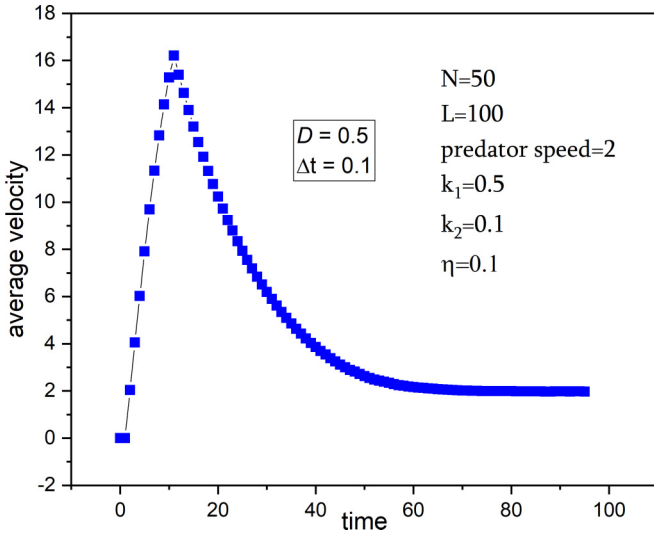


FIG. 2. Time evolution of the mean particle velocity.

leader's velocity, is due to the strong interactions between the leader and the particles, modeled by intense forces applied at the beginning of the simulation.

The second phase is characterized by an exponential decay of the average velocity. This decay can be interpreted as follows: After the initial acceleration phase, where particles quickly cluster together, they enter an adjustment phase. Each particle then tries to align its velocity with that of the leader. This gradual adjustment, which results in an exponential decay, shows that the system is seeking to reach an equilibrium state where all particles move at the same speed as the leader. To further detail the mathematical expression of velocity in both phases, we propose a linear regression of the two curves. The results of the regression are shown in Fig. 3.

Initially, the velocity evolves linearly with time, following the equation $v(t) = 1.59t - 0.41$. This confirms the observed linear growth. In the second phase, the decrease in velocity is

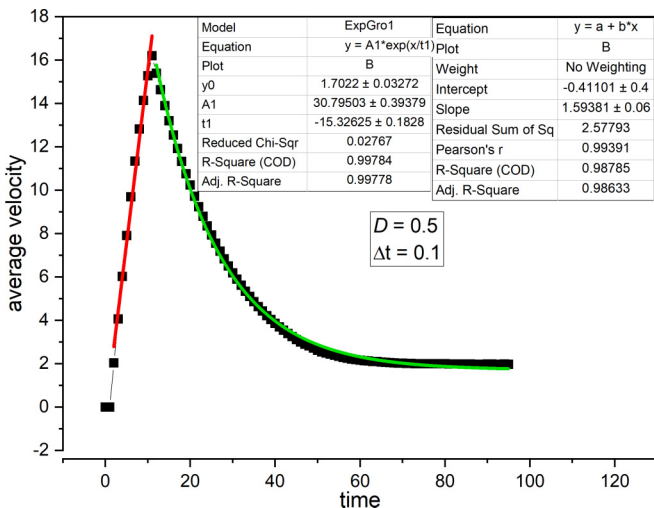
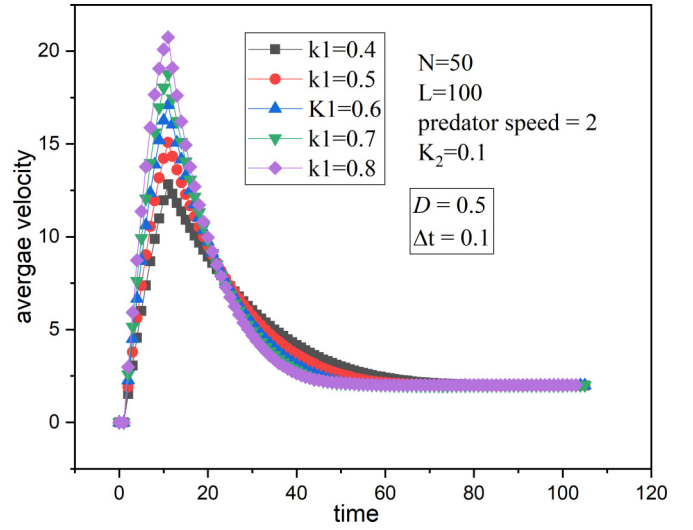


FIG. 3. Linear regression of the two phases of the motion.

FIG. 4. Effect of spring constant k_1 on velocity over time.

well modeled by a decreasing exponential function:

$$v(t) = 30.79 \exp\left(-\frac{t}{15, 32}\right) + 1.70. \quad (9)$$

This equation indicates that the velocity asymptotically tends toward a constant value (1.70) over time. It is interesting to note that the profile observed during the second phase is in agreement with the results obtained by Tarras [4,13,14] and Bakir [15] in their work on leaderless systems. This agreement suggests that the mechanisms at work in our system are similar to those identified in their studies.

Now that we have identified the characteristic velocity profile during this phase transition, let us focus on the different parameters that influence this phenomenon. We will start by studying the effect of the interaction between the leader and the particles, characterized by the spring constant k_1 . Figure 4 clearly illustrates the impact of this constant on the velocity profile.

By examining the graph, we can see that the strength of the interaction between the particles and the leader, represented by the spring constant k_1 , has a significant impact on the time evolution of the average velocity.

(i) Initial phase: A higher spring constant results in a steeper slope at the beginning of the linear growth phase, indicating a faster acceleration of the particles due to the stronger attraction from the leader.

(ii) Final phase: The time it takes for the exponential decay of the velocity to occur decreases as k_1 increases. This implies that the particles reach their steady-state velocity, where they follow the leader stably, more quickly. In other words, a stronger interaction between the leader and the particles (higher k_1) promotes faster synchronization of the particles with the leader's motion and stabilizes the system.

To better understand the influence of the spring constant k_1 on the behavior of particles under elastic interaction, we studied the evolution of the average inter-particle distance over time for different values of k_1 and the result is shown in Fig. 5. By varying k_1 , we essentially adjust the strength of the attractive force exerted by the leader on each particle.

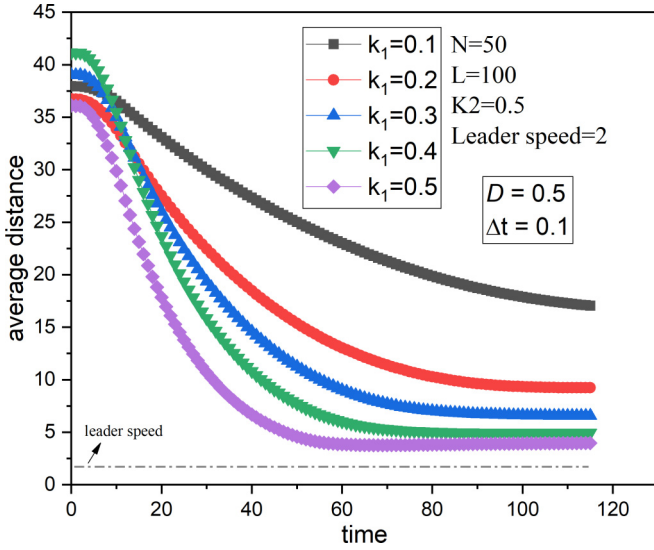


FIG. 5. Effect of the spring constant k_1 on the evolution of the average interparticle distance over time.

The results clearly show that the higher the spring constant k_1 , the shorter the time it takes for the particles to reach an equilibrium configuration. This observation is entirely consistent with the physical interpretation of the phenomenon: A higher value of k_1 corresponds to a stronger restoring force, which accelerates the process of particles clustering around the leader. The exponential decay of the average interparticle distance over time is also as expected.

Indeed, at the initial time, the particles are randomly distributed in two-dimensional space, resulting in a high average distance. When the leader starts moving, the particles are subjected to an attractive force that encourages them to come closer together and follow the leader. This clustering dynamics is the origin of the observed exponential decay. We can also observe that when the interaction between the leader and the particles is weak, the group of particles is unable

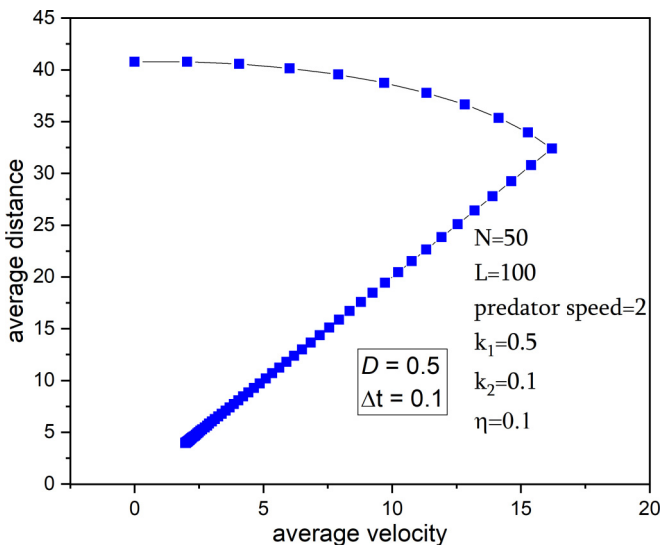


FIG. 6. Phase diagram.

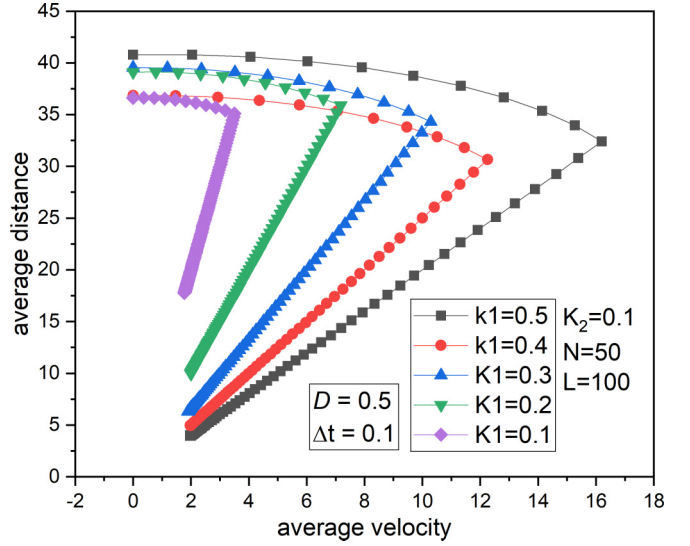


FIG. 7. Impact of interaction intensity on the phase diagram.

to effectively follow the leader. Indeed, the attractive force exerted by the leader is then insufficient to induce a rapid and coherent clustering of the particles.

Consequently, the particles cannot reach the leader's speed, and the average interparticle distance decreases more slowly, or even plateaus. Our results corroborate the findings of Carine Beatrice [13,16] and Tarass [17–19] regarding the exponential decay of the average inter-particle distance. Furthermore, our analyses reveal that this decrease is primarily attributed to leader-follower attraction. This interaction plays a central role in the formation of clusters and group cohesion. To delve deeper into the relationship between particle movement mechanisms and average distance, let us examine the phase diagram where we represent the average particle velocity as a function of their average distance from an origin. This representation allows us to highlight different movement

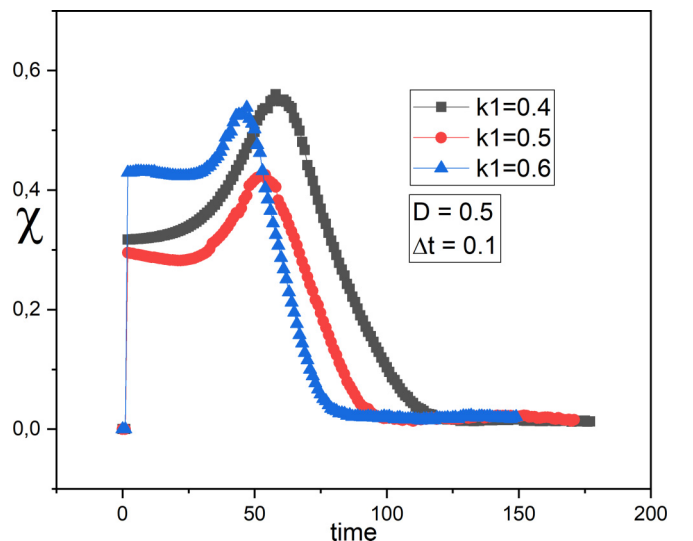


FIG. 8. Time evolution of susceptibility for different particle-leader interaction strengths.

regimes and to study the evolution of particle motion as a function of system conditions.

The phase diagram in Fig. 6 reveals two distinct phases of particle movement. Initially, there is an acceleration phase associated with a decrease in the average distance between particles. This decrease continues until the particles reach a maximum velocity. This first phase corresponds to an adjustment period where particles move closer together, forming increasingly dense groups.

Second, there is a deceleration phase, characterized by a decrease in the average velocity of the particles. This regime change suggests that the particles have reached a dynamic equilibrium state, where attractive and repulsive forces balance each other out. The continued decrease in velocity can be explained by an optimization of group formation, as particles seek to minimize their potential energy by adopting a more stable configuration. To validate our previous findings regarding the influence of the leader-particle interaction, we reconstructed the phase diagram by adjusting the strength of this interaction through the spring constant K_1 and the result is shown in Fig. 7.

The analysis of Fig. 7 shows that as the interaction intensity decreases, the inflection point shifts to the left. This implies that particles tend to cluster at lower velocities before accelerating. However, it is important to note that only the black curve, corresponding to a high interaction intensity, reaches a velocity close to that of the 'leader'. Therefore, we can conclude that: a strong interaction between the "leader" and the particles promotes group cohesion and allows particles to reach high velocities by following the leader's motion efficiently. Conversely, a weak interaction leads to particle dispersion and clustering at lower velocities.

To conclude on the possibility of observing a phase transition from an initial configuration where particles are randomly distributed in a two-dimensional system to an ordered state where all particles coherently follow a leader, we can study the time evolution of the susceptibility. Recall that a susceptibility curve, commonly used in such studies (for example, in the works of Najem Moussa [15,20] and Iliass Tarras [10,20]), is a valuable tool for detecting phase transitions. By following the time evolution of this susceptibility, we can identify potential peaks or abrupt changes that would signal a transition to a highly correlated state, where all particles move in a coordinated manner.

The susceptibility $\chi(t)$ is defined as

$$\chi(t) = N^2(\langle v^2(t) \rangle - \langle v(t) \rangle^2), \quad (10)$$

$\langle v^2(t) \rangle - \langle v(t) \rangle^2$: This difference represents the velocity variance, a measure of how spread out the velocities are around

the average. A larger value indicates greater differences in particle velocities.

The susceptibility curves in Fig. 8 exhibit a pronounced peak for all three values of k_1 , indicating a maximum system response to a perturbation at a given time. However, it is observed that this peak is reached more quickly for higher values of k_1 . This evolution can be explained by the fact that a strong particle-leader interaction (high k_1) leads to a rapid synchronization of the particles with the leader. Indeed, particles strongly coupled to the leader tend to quickly adopt its velocity and direction, thus reducing velocity fluctuations at the system scale. Once this synchronization is achieved, the susceptibility decreases and tends toward zero, indicating a loss of system reactivity to any further perturbation.

The shape of our susceptibility curve agrees with the results obtained by Tarass [9] and Bo Li [21,22], where a peak followed by a decay to zero is observed. This suggests the establishment of an ordered state, where particles are aligned and stable in the absence of perturbations. However, our study, conducted in the presence of a leader, highlights the crucial role of the latter in the emergence and maintenance of this order. The results confirm that many collective motion systems require a leading entity to initiate and coordinate the movement of the other individuals.

IV. CONCLUSION

The study conclusively demonstrates that leader-particle interactions play a pivotal role in the emergence of order and synchronization within collective motion systems. Stronger interactions, characterized by higher spring constants, lead to faster clustering and velocity alignment, stabilizing the system into a dynamic equilibrium. Additionally, the inclusion of stochastic forces enhances the realism of the model, illustrating how random fluctuations influence collective dynamics.

These findings have broad implications for understanding the mechanisms' underlying collective behavior in both natural and artificial systems. They offer a robust framework for designing efficient leader-follower strategies in swarm robotics, biological research, and other applications requiring coordinated movement in noisy environments. Future studies could extend this work by exploring more complex interactions or multileader scenarios to deepen our understanding of collective dynamics.

DATA AVAILABILITY

The data that support the findings of this article are openly available [23].

-
- [1] P. Rahmani, F. Peruani, and P. Romanczuk, Flocking in complex environments—Attention trade-offs in collective information processing, *PLoS Comput. Biol.* **16**, e1007697 (2020).
 - [2] J. K. Parrish, S. V. Viscido, and D. Grünbaum, Self-Organized Fish Schools: An Examination of Emergent Properties, *Biol. Bull.* **202**, 296 (2002).

- [3] W. Bialek *et al.*, Statistical mechanics for natural flocks of birds, *Proc. Natl. Acad. Sci. USA* **109**, 4786 (2012).
- [4] J. Kennedy and R. Eberhart, Particle swarm optimization, in *Proceedings of ICNN'95 - International Conference on Neural Networks* (IEEE, New York, NY, 1995), Vol. 4, pp. 1942–1948.
- [5] D. Karaboga, An Idea Based on Honey Bee Swarm for Numerical Optimization, Erciyes University, Kayseri, Turkey,

- Technical Report TR06 (2005), https://abc.erciyes.edu.tr/pub/tr06_2005.pdf.
- [6] T. Vicsek, A. Czirok, E. Ben-Jacob, I. Cohen, and O. Sochet, Novel type of phase transition in a system of self-driven particles, *Phys. Rev. Lett.* **75**, 1226 (1995).
 - [7] B. Li, Z.-X. Wu, and J.-Y. Guan, Collective motion patterns of self-propelled agents with both velocity alignment and aggregation interactions, *Phys. Rev. E* **99**, 022609 (2019).
 - [8] Q. Chen and Y. Ma, Collective motion of self-propelled particles with density-dependent switching effect, *AIP Adv.* **6**, 055024 (2016).
 - [9] I. Tarras, N. Moussa, M. M. Mazroui, Y. Boughaleb, and A. Hajjaji, Collective behavior of interacting particles: Radius-dependent phase transition, *Mod. Phys. Lett. B* **27**, 1350028 (2013).
 - [10] I. Tarras, N. Moussa, and Y. Boughaleb, The study of results simulation of collective motion, *Int. J. Comput. Appl.* **46**, 21 (2012).
 - [11] L. Amallah *et al.*, Correlations and diffusions process in self-propelled particles dynamic: Langevin equation treatment, *Sens. Lett.* **16**, 688 (2018).
 - [12] F. Nedelec and D. Foethke, Collective Langevin dynamics of flexible cytoskeletal fibers, *New J. Phys.* **9**, 427 (2007).
 - [13] L. Amallah, A. Hader, R. Bakir, H. Sbiaaia, I. Tarras, and Y. Boughaleb, Langevin Dynamic Simulation of self-propelled particles in two-dimensional systems, *IOP Conf. Ser.: Mater. Sci. Eng.* **948**, 012002 (2020).
 - [14] I. Tarras *et al.*, Effect of repulsive interaction and initial velocity on collective motion process, *Eur. Phys. J. E* **47**, 62 (2024).
 - [15] T. Iliass, D. Cambui, and L. Youssef, Non-universality of critical exponents of a self-propelled particles system, *Chin. J. Phys.* **54**, 108 (2016).
 - [16] A. Eddakoun *et al.*, Study of elastic interaction in collective motion phenomenon, *Mod. Phys. Lett. B* **38**, 2350230 (2024).
 - [17] C. Beatrice, C. Kirch, S. Henkes, F. Graner, and L. Brunnet, Comparing individual-based models of collective cell motion in a benchmark flow geometry, *Soft Matter* **19**, 5583 (2023).
 - [18] N. Moussa, I. Tarras, M. Mazroui, and Y. Boughaleb, Effects of agent's repulsion in 2D flocking models, *Int. J. Mod. Phys. C* **22**, 661 (2011).
 - [19] N. Moussa, Dangerous situations in two-lane traffic flow models, *Int. J. Mod. Phys. C* **16**, 1133 (2005).
 - [20] L. Caprini, D. Breoni, A. Ldov, C. Scholz, and H. Löwen, Dynamical clustering and wetting phenomena in inertial active matter, *Commun. Phys.* **7**, 343 (2024).
 - [21] R. Bakir *et al.*, Effects of agent's orientation in 2D flocking models, *Chin. J. Phys.* **55**, 692 (2017).
 - [22] D. Gu and Z. Wang, Leader-follower flocking: Algorithms and experiments, *IEEE Trans. Control Syst. Technol.* **17**, 1211 (2009).
 - [23] A. Eddakoun, Numerical simulation data of the Langevin equation dynamics for the article: Simulating and analyzing the dynamics of collective movement with elastic interactions and leadership influence, Zenodo (2025), doi:10.5281/zenodo.16184150.