

Rolling at right angles: Magnetic anisotropy enables dual-anisotropic active matter

Eavan Fitzgerald¹, Cécile Clavaud^{1,2}, Debasish Das³, Isaac C. D. Lenton¹, and Scott R. Waitukaitis^{1,*}

¹*Institute of Science and Technology Austria, Am Campus 1, Klosterneuburg 3400, Austria*

²*IPR, Université de Rennes, Campus Beaulieu, Allée Jean Perrin Bâtiment 10B, 35042 Rennes, France*

³*Department of Mathematics and Statistics, University of Strathclyde, Livingstone Tower, 26 Richmond Street, Glasgow G1 1XH, United Kingdom*



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We report on an experimental active matter system with motion restricted to four cardinal directions. Our particles are magnetite-doped colloidal spheres driven by the Quincke electrorotational instability. The absence of a magnetic field ($|\mathbf{B}| = 0$) leads to circular trajectories interspersed with short spontaneous runs. Intermediate fields ($|\mathbf{B}| \lesssim 20$ mT) linearize the motion along the axis perpendicular to $\mathbf{B}$. At high magnetic fields, we observe the surprising emergence of a second, distinct linearization along the axis parallel to $\mathbf{B}$. With numerical simulations, we show that this behavior can be explained by anisotropic magnetic susceptibility.

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I. INTRODUCTION

Driven by the ubiquity of systems that are active and inherently nonequilibrium in nature, the field of active matter has emerged as a vibrant area of research in the last few decades. Loosely defined, active matter is composed of autonomous "agents," each capable of consuming energy to perform work and self-propel into motion [1,2]. Remarkably, the violation of equilibrium at the agent level, combined with interactions between agents, can lead to the emergence of global phenomena, in particular flocking and coordinated collective motion [3–10].

Typically the motion of individual agents in active systems can be considered isotropic, i.e., there is no preferred direction of travel. For instance, in biological systems comprised of *Escherichia coli* or microtubule-molecular motors, agents perform isotropic random walks via run-and-tumble dynamics [6,11] or the stochastic binding and unbinding of proteins, which drives extensile sliding or "inching" [12–14], respectively. In synthetic systems, e.g., light-activated colloidal surfers [15] or electric-field activated Quincke rollers [9,16], rotational diffusion randomly reorients particles to result in isotropy. In certain systems, external influences can break symmetry and anisotropize motion, but in most cases this results in linearization along a single axis. Examples include chemotaxis [17,18], phototaxis [19] and magnetotaxis [20] in both biological and biomimetic systems, or otherwise directing motion through magnetic or electric forces in synthetic systems [21–25]. In principle, active matter anisotropized in more complex ways could lead to new ways of directing

or controlling collective motion, but to our knowledge little experimental realization of this principle currently exists.

In this work, we introduce an active matter system that is dual anisotropized, i.e., not along a single linear axis, but instead along two orthogonal axes. Our agents are magnetite-doped spherical particles which achieve motion via Quincke rotation, an electrohydrodynamic instability [26,27]. In the absence of an external magnetic field, the particles' motion is perfectly isotropic, as rotation is induced by a spontaneous symmetry breaking event. By introducing a magnetic field, we first break this isotropy with a linearization perpendicular to the field, which dominates at low to intermediate field strengths. However, as we increase the strength of the magnetic field, we observe the emergence of a second linearization parallel to the field. Using numerical simulations, we show that this is caused by anisotropic magnetic susceptibility within each particle, leading to a complex dynamical system governing its convergence to either the "perpendicular mode" or the "parallel mode" (hereafter meaning either parallel or antiparallel motion). Furthermore, we demonstrate that the steady state dynamics of the perpendicular mode resemble a fixed point attractor, while those of the parallel mode exhibit limit-cycle-like behavior.

II. ROLLING AT RIGHT ANGLES

Quincke rollers have become an important model system for active matter due to their relative simplicity and rich dynamics [9,10,16,29–35]. First observed by Quincke [26], this electrorotational instability has since been well elucidated using a leaky dielectric model [27,36,37], such that we will only briefly outline it here. A dielectric particle suspended in a weak electrolyte will have an antiparallel dipole moment $\mathbf{P}$ when placed in an electric field $\mathbf{E}$ as shown in Fig. 1(a). This antiparallel alignment hinges on the ratio of the electrical permittivity $\epsilon_{f,p}$ and conductivity $\sigma_{f,p}$ of the fluid and particle, respectively, which must fulfill the inequality $\frac{\epsilon_f}{\sigma_f} < \frac{\epsilon_p}{\sigma_p}$. When this condition is met, and the electric field exceeds a

*Contact author: scott.waitukaitis@ist.ac.at

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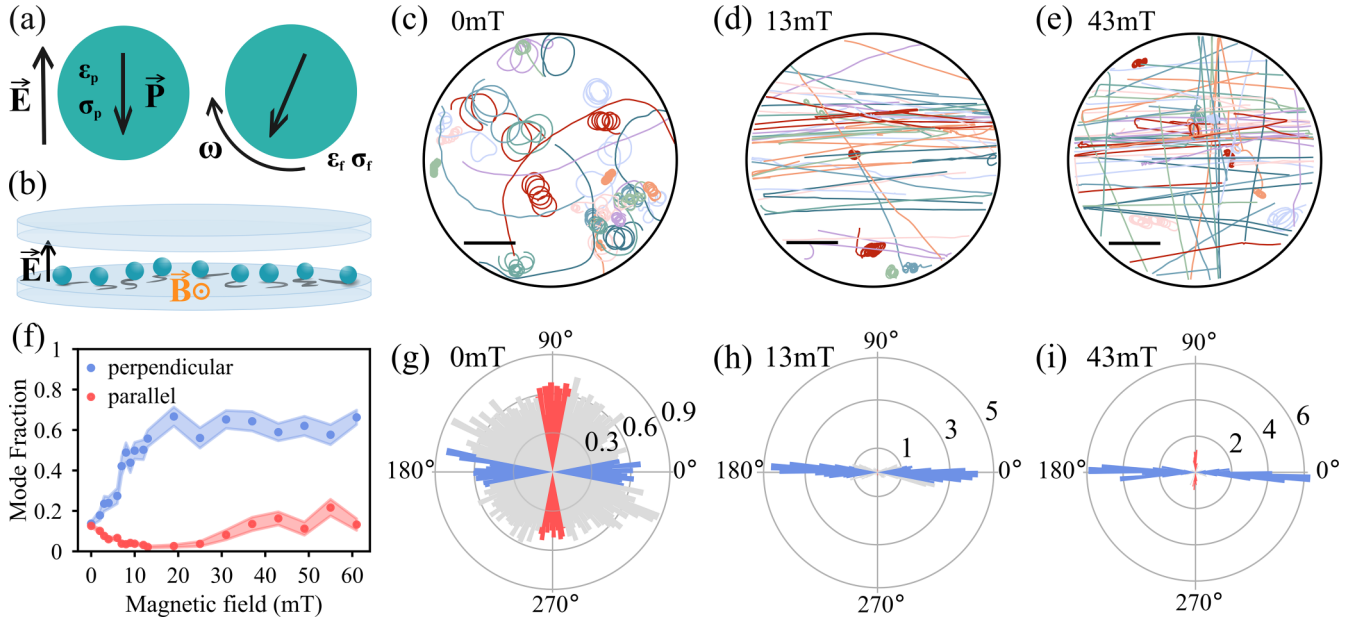


FIG. 1. (a) Intrinsically unstable dipole $\mathbf{P}$ induced in the particle by an applied electric field $\mathbf{E}$ when the condition $\frac{\epsilon_f}{\sigma_f} < \frac{\epsilon_p}{\sigma_p}$ is met [28]. For $E > E_c$ any small perturbation leads to persistent "Quincke" rotation, or rolling in the vicinity of a surface. (b) Schematic of the experimental setup: particles lie on the bottom surface of our ITO-coated glass cell. A potential is applied between the glass electrodes to drive a DC electric field out of the plane of motion, while two electromagnets (not shown) drive a constant in-plane magnetic field $\mathbf{B}$. (c)–(e) The trajectories of an ensemble of rollers confined to a circular region (1 mm in diameter) at different magnetic field intensities and fixed potential $|\mathbf{E}| \approx 2E_c$ (scale bar is 200 μm , see Supplemental Movies 1–3). The figures show a top-down view of the schematic shown in (b), with $\mathbf{B}$ aligned with the vertical axis and $\mathbf{E}$ coming out of the page. (c) $\mathbf{B} = 0$ mT: isotropic motion consisting of tight circular orbits interspersed with runs. (d) $\mathbf{B} = 13$ mT: first linearization of motion when a magnetic field is applied. A magnetic torque aligns the induced moment with the field, fixing the Quincke rotation axis to give rolling $\perp$ to $\mathbf{B}$. (e) $\mathbf{B} = 43$ mT: emergence of the second linearization, with motion parallel to $\mathbf{B}$ above 20 mT. The magnetic moment must have a nonzero component in the vertical plane, to drive motion collinear with $\mathbf{B}$. (f–h) Normalized distributions of the angular displacements computed over ten-frame segments of the trajectories shown in (c)–(e). A 12° tolerance window is defined and highlighted in blue (red) for characterization of the perpendicular (parallel) mode, with remaining displacements colored in gray. (h) Homogeneous distribution in the absence of $\mathbf{B}$, corresponding to the trajectories in (c). (h) Collapse in the distribution of angular displacements of trajectories shown in (d), as roller motion is driven perpendicular to the magnetic field axis when a 13 mT magnetic field is applied. (i) The emergence of dual anisotropy with the coexistence of the perpendicular and parallel modes at higher field intensities, evident from the trajectories in (e). (f) The probability of observing a roller in a given mode at a specific field strength. This is estimated by computing the fraction of time each roller spends in the perpendicular or parallel mode, and the ensemble average of this fraction is plotted as a function of the magnetic field intensity.

critical value E_c , any perturbation to the dipole moment $\mathbf{P}$ drives the system toward steady rotation at a constant rate. At the critical field strength, the angular velocity ω undergoes a pitchfork bifurcation, reaching a steady state amplitude $|\omega| = \frac{1}{\tau} \sqrt{(\frac{E}{E_c})^2 - 1}$, inversely proportional to the Maxwell-Wagner relaxation time $\tau = \frac{\epsilon_p + 2\epsilon_f}{\sigma_p + 2\sigma_f}$ and oriented in the plane perpendicular to $\mathbf{E}$. In the vicinity of a surface, rotation couples to friction, resulting in rolling at a fixed speed on the order of millimeters per second for a micron sized object. Given the stochastic nature of the perturbation which induces rolling (and the fact $\omega \cdot \mathbf{E} = 0$), the initial direction of motion is random, rendering dilute ensemble or time averages isotropic.

Our setup is shown in Fig. 1(b). The particles are polymeric colloidal spheres doped with superparamagnetic nanoparticles—single magnetic domains—of iron oxide (8 μm diameter, $\sim 3 - 3.5\%$ magnetite content; UMDG003 COMPELTM from Bangs Laboratories). Nominally, the embedded nanoparticles render the spheres paramagnetic with essentially no remnant magnetization. We disperse

the particles in an AOT-hexadecane solution [30 mM bis(2-ethylhexyl) sulfosuccinate sodium salt with 0.03% (w/v) H_2O , and a conductivity $\sigma_f \sim 10^{-8} \text{ Sm}^{-1}$]. The particles roll in a custom-made glass cell, confined between two ITO coated microscope slides (Sigma Aldrich, $75 \times 25 \text{ mm}$, $70 - 100 \Omega/\text{sq}$). Circular or square conductive regions of varying sizes are defined by patterned deposition of SU-8 photoresist on the bottom slide via UV lithography. Once the colloids have sedimented, a vertical electric field is applied between the electrodes to initiate Quincke rolling. A pair of electromagnets, with their cores oriented along the minor axis of the rectangular cell, generate a magnetic field in the plane of motion, which is nearly perfectly homogeneous across the domain. The rollers are observed from above with a Basler ace camera (acA2040) and recorded at framerates of 150–180 fps. Individual trajectories are extracted using the trackpy Python package [38] based on Crocker and Grier's [39] tracking algorithm, and analyzed using custom scripts.

Observing the dynamics of our particles in the absence of a magnetic field yields trajectories characteristic of those

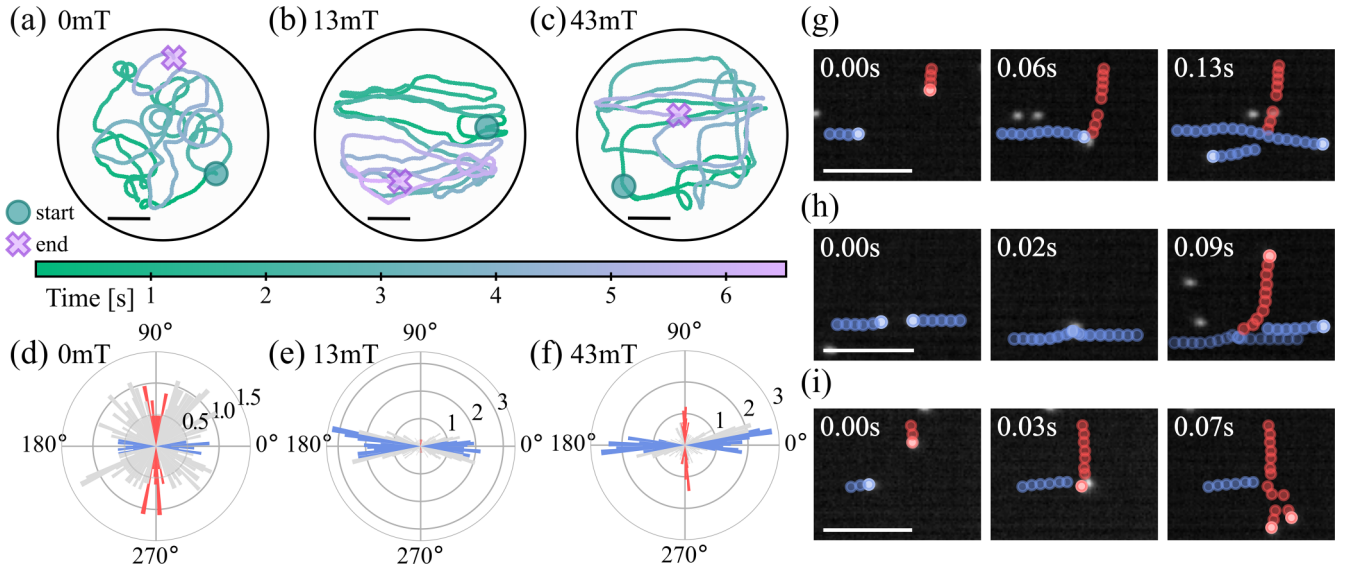


FIG. 2. (a)–(c) The trajectory of a single roller confined to a 0.25 mm circular region (scale bar is 50 μ m, see Supplemental Movies 4–6), under the same field conditions as in Fig. 1, with B along the vertical axis and E coming out of the page. The trajectory is colored by time, with a circle (cross) denoting the start (end) point, while (d)–(f) show the corresponding angular distributions. (a), (d) Circular motion and a homogeneous distribution at 0 mT. (b) Motion perpendicular to the field dominates the trajectory, evident in the collapse of the angular distribution in (e) at intermediate field strengths. (c), (f) Emergence of motion parallel to the field at higher field strengths, with the roller switching spontaneously between parallel and perpendicular modes. This points to the dual mode capacity of rollers, rather than an intrinsic preference for a particular mode at high fields. (g)–(i) Stills from raw videos (see Supplemental Movie 7) capturing nonconservative interactions between rollers at $|E| = 2E_c$ and $B = 61$ mT, demonstrate switching back and forth between perpendicular and parallel rolling. Sequential columns show the rollers before, during, and after a collision, respectively, as indicated by the annotated time. (g) Two rollers, initially in orthogonal modes [perpendicular (blue) and parallel (red)], interact and both emerge in the perpendicular mode. (h) A roller switches from perpendicular to parallel rolling, following a head-on collision with another perpendicular roller. (i) A collision between orthogonal rollers triggers a switch from perpendicular to parallel rolling. We emphasize the two-way switching between modes, despite the perpendicular motion being more stable and energetically favorable. Scale bar is 50 μ m.

in Fig. 1(c). These tight circular orbits, occasionally interspersed with "runs," are in contrast to the standard Quincke roller trajectories which are similarly isotropic for equivalent confinement, exhibit much larger persistence lengths. Previous studies using nonspherical active rollers have produced circular or chiral trajectories [40–42], however they are much more ordered across the population and across experiments than what we observe in our system. While we suspect an underlying asymmetry to be behind these anomalous dynamics, in this work we simply wish to highlight the homogeneous distribution of the angular displacements in Fig. 1(g), in the absence of a magnetic field. Angular displacements (relative to the horizontal axis) over ten frames are extracted from overlapping segments of the individual trajectories, and the normalized distribution of the displacements from all trajectories are plotted in Figs. 1(g)–1(i).

What we focus on in the remainder of this work are the dynamics observed under the influence of a uniform magnetic field, with flux densities up to 62 mT. At intermediate field strengths (10–30 mT), we observe a striking linearization of the roller trajectories in the direction perpendicular to B —the perpendicular mode—captured in Fig. 1(d). The coherence of the perpendicular mode is quantified in Fig. 1(g), with the collapse in the distribution of the angular displacements within a 12° tolerance window, highlighted in blue. The saturation of this coherence around 15 mT is shown in the blue curve in Fig. 1(i). This curve represents the ensemble mean of the

fraction of a trajectory spent in the perpendicular mode, but serves as an effective probability of observing perpendicular rolling as a function of the magnetic field strength.

As we increase the field strength beyond 20 mT and the saturation of the perpendicular mode, something quite unexpected happens. Unlike the uniaxial anisotropy also observed in recent work by Reyes-Garza *et al.* [25], we see the emergence of a second, distinct linear mode. A striking fraction of rollers execute stable paths parallel to B , i.e., demonstrating simultaneous right-angled rolling. This dual-axis anisotropy is clearly evidenced in the trajectories at 43 mT in Fig. 1(e), and in the corresponding distribution of angular displacements in Fig. 1(h), where the displacements associated with the parallel mode are highlighted in red. The probability of observing parallel rolling as a function of the magnetic field strength is indicated by the red curve in Fig. 1(f), capturing its emergence once the field exceeds the saturation value of the perpendicular mode, and reaching a plateau around 40 mT.

A natural question that arises is whether certain rollers have an intrinsic preference for this parallel mode, or if all rollers have a dual mode capacity. To address this we refer to Fig. 2, first showing the dynamics of a single roller more tightly confined (domain size is 0.25 mm) than the previously discussed ensemble, but otherwise subject to the same experimental conditions. Figure 2(a) shows the similarly isotropic motion when no magnetic field is applied, with the trajectory in this case colored by time. This isotropy is represented in

Fig. 2(f) by the relatively homogeneous distribution of angular displacements. Introducing a magnetic field, the linearization of motion and dominance of the perpendicular mode at intermediate field strengths is duly replicated—see Figs. 2(b) and (e). Moving to higher field strengths, in Figs. 2(c) and (f) we finally observe right-angled rolling, clearly showing the bistability of orthogonal motion, with 2(c) in particular illustrating a single roller switching back and forth between modes.

Returning to the ensemble case, the coexistence of both modes is also facilitated by interactions and collisions between rollers. In Figs. 2(g)–2(i), we highlight three different cases of such collisions which demonstrate two-way switching between modes. Frames are extracted from raw videos and a time lapse of the paths of colliding rollers are colored in blue or red, according to their respective mode. As can be seen in the figure, collisions between rollers may trigger a switch in its motional mode, but the result is not possible to predict *a priori*. We do not observe a prevalence of alignment outcomes, as is typically exhibited by polar active matter via hydrodynamic effects [1]. Hence, returning to the original question posed, we confirm the bistability of the modes for each roller, as demonstrated both by their individual capacity for right-angled rolling and, in the ensemble case, by observation of collision-mediated turns and the coexistence of parallel and perpendicular rolling.

III. MODELING RIGHT ANGLES

Tackling the question of how this right-angled rolling emerges, and in particular how the parallel mode is sustained, begs an understanding of a roller's magnetic moment during motion. The first and most simple model we might consider is an ideal paramagnetic response, as bulk (i.e., not single-particle) data from the particle supplier would suggest. In this case, the induced moment of a roller would be $\mathbf{m} = \chi \mathbf{B}$, where the susceptibility, χ , is a scalar constant. This model quickly collapses by the preclusion of a magnetic torque, since $\mathbf{T}_m = \mathbf{m} \times \mathbf{B} = \chi(\mathbf{B} \times \mathbf{B}) = 0$. Therefore, pure paramagnetism cannot account for any of our observations (i.e., the perpendicular or parallel modes).

A second possibility to consider, and one that would sustain a steering magnetic torque, is the existence of a permanent dipole moment in each sphere. If this were the case, it is easy to see that the most energetically favorable state would have $\mathbf{m} \parallel \mathbf{B}$ at all times, with any deviation triggering a restoring magnetic torque. This enforces stable motion perpendicular to $\mathbf{B}$, fixed by $\mathbf{m}$ serving as an "axle" to the rolling sphere. This mechanism is consistent with Garza *et al.* [25], who worked with 20 μm SiO₂ magnetic Quincke rollers and confirmed the alignment of the rotational axis using high-speed videography. However, while this simple model can explain the perpendicular mode, it cannot account for field-parallel motion. Qualitatively, persistent parallel motion requires an angular velocity (and hence a sustaining torque) orthogonal to the field. In such a configuration, $\mathbf{m}$ cannot function as a stable axle, as it occupies a torque-maximum orientation, and is continuously driven toward realignment with $\mathbf{B}$. To maintain rolling parallel to the field, $\mathbf{m}$ must have a component directed out of the rolling plane. Hence, for $\mathbf{m}$ to be at least

sometimes aligned with $\mathbf{B}$ while the sphere is in motion, it must undergo rotation in the vertical plane. However, for a permanent moment, this necessarily means being driven toward antialignment with the $\mathbf{B}$ for half of its rotation cycle, a clearly unfavorable state. In reality, what occurs is a restoring torque which accelerates the sphere to stable motion perpendicular to $\mathbf{B}$, with $\mathbf{m}$ stationary and aligned with the field. Hence, neither an intrinsic permanent magnetic moment, nor the paramagnetic-response model, can explain our dual-anisotropized system. In principle, to sustain perpendicular and parallel modes we need a body-fixed magnetic coupling that (a) leads to stable rolling when some aspect is aligned with $\mathbf{B}$, as in the permanent moment case, and (b) also permits a secondary stable mode when it is misaligned. These considerations lead us to explore a third possibility: anisotropic magnetic susceptibility. Mathematically, this corresponds to an induced magnetic moment of the form

$$\mathbf{m} = \mathbf{X} \cdot \mathbf{B}, \quad (1)$$

where $\mathbf{X}$ is a 3×3 susceptibility tensor that rotates with the particle. This model differs from the ideal paramagnetic case in that $\mathbf{m}$ is not generally aligned with $\mathbf{B}$, allowing for sustained magnetic torques. It also differs from the permanent dipole model in that $\mathbf{m}$ is a linear response to the applied field, with magnetic energy $U = -\mathbf{B}^T \mathbf{X} \mathbf{B}$ that is always negative and bounded from below, eliminating the possibility of unstable antialigned configurations. It also permits a three-dimensional magnetic response that can stabilize the effective magnetic moment throughout rotation. In this way, anisotropic susceptibility enables field-responsive torques with complex orientational dependencies, potentially opening the door for richer dynamical effects.

We explore such effects using numerical simulations of an isolated Quincke roller with an anisotropic magnetic susceptibility. The leaky dielectric model explicitly describes the evolution of the electric dipole moment $\mathbf{P}$, while the magnetic susceptibility tensor $\mathbf{X}$ of our virtual sphere simply rotates with the angular velocity $\boldsymbol{\omega}$. The interaction between the components of the rotating moments not aligned with their respective driving field leads to sustained electric and magnetic torques acting on the roller, and determines the instantaneous angular velocity. Finally, in the absence of inertia, we balance electric and magnetic torque by the viscous torque acting on the roller, yielding [9,27,43–45]

$$4\pi\epsilon_f \mathbf{P} \times \mathbf{E} + \frac{4\pi}{\mu} (\mathbf{X} \cdot \mathbf{B}) \times \mathbf{B} = 8\pi\eta a^3 \boldsymbol{\omega}, \quad (2)$$

where μ is the magnetic permeability, a is the radius of our spherical roller, η is the fluid viscosity, and all other symbols are unchanged. This leads to a closed system of equations that can be numerically integrated in time with a given set of initial conditions for the electric dipole and susceptibility tensor—see the Supplemental Material for more details [46]. We consider a simplified case where the susceptibility tensor in its eigenbasis has just two elements, χ_s and χ_h , such that the soft axis susceptibility $\chi_s > \chi_h$ defines the direction of greatest response. The tensor we consider can be written explicitly

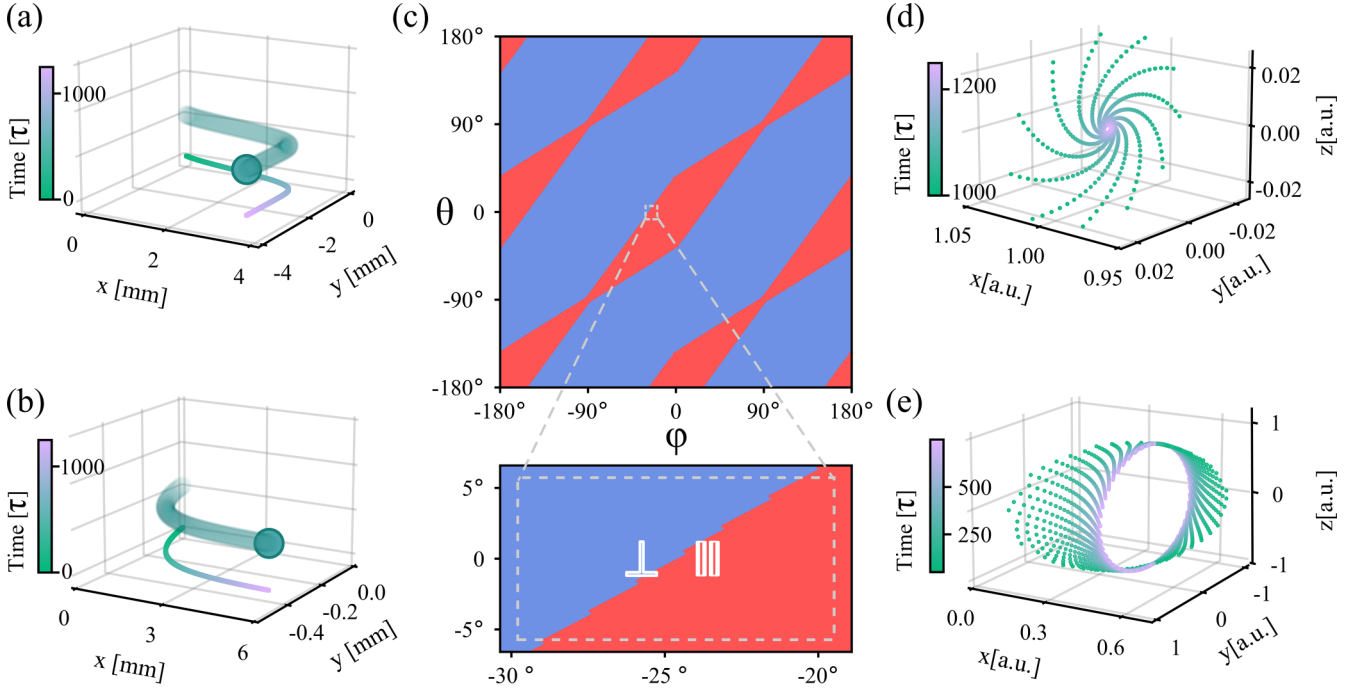


FIG. 3. Results from dynamic numerical simulations of an isolated sphere with an anisotropic magnetic susceptibility under the influence of a driving electric and steering magnetic field. Simulations are carried out for a range of initial conditions given by ϕ —the direction of motion—and θ —the rotation of the soft axis in the xy plane. (a) First 1250τ segment of a 5000τ trajectory resolving to perpendicular motion: $\phi = 25.5^\circ$, $\theta = 0^\circ$. (b) First 1250τ segment of a trajectory resolving to parallel motion: $\phi = 23.5^\circ$, $\theta = 0^\circ$. (c) Phase diagram showing the converged mode from a 5000τ sweep over pairwise values for ϕ and θ from -180° to 180° , with resolution 400×400 . Blue (red) denotes the roller converging to the perpendicular (parallel) mode. Enlarged region shows the proximity of the initial conditions corresponding to the trajectories shown in (a), (b): “ $\perp$ ” (“ $\parallel$ ”) is the perpendicular (parallel) trajectory. (d) The dynamics of the soft axis as a function of time, in the frame of the virtual roller for the final 250τ of the trajectory in (a). The soft axis spirals toward a stable fixed point, located at the spiral center. Every 100th point is plotted, as for a 250τ segment at this scale, the spiral generated from the inclusion of every point would be overly dense. (e) The dynamics of the soft axis as function of time, in the frame of the roller body from $50 - 750\tau$ of the trajectory in (b), with every 100th point shown once again. The soft axis enters a steadily oscillating state, strongly reminiscent of a limit cycle in the xz plane.

in its eigenbasis as

$$\mathbf{X} = \begin{bmatrix} \chi_s & 0 & 0 \\ 0 & \chi_h & 0 \\ 0 & 0 & \chi_h \end{bmatrix},$$

and we set the ratio of the susceptibility coefficients to $\chi_s/\chi_h = 2$. We note that this ratio is merely chosen as a representative case, as a midpoint between perfect isotropy and uniaxial anisotropy, but is not a fixed property of the system, and does not qualitatively impact our results. We define the initial conditions of a simulated trajectory by the orientation ϕ of the velocity $\mathbf{v}$ relative to the magnetic field, and an initial orientation of the soft axis of $\mathbf{X}$ by some angle θ in the xy plane (again relative to $\mathbf{B}$). We again refer the reader to the Supplemental Material [46] for additional information on the parameters and methods used.

Two example trajectories are presented in Figs. 3(a) and (b), which immediately illustrate that anisotropic susceptibility is indeed capable of driving both perpendicular and parallel motion. Figure 3(a) shows a sphere with initial conditions $\phi = 25.5^\circ$ and $\theta = 0^\circ$, which quickly turns to roll perpendicular to $\mathbf{B}$ in the steady state. By contrast, in Fig. 3(b) where the roller has initial conditions $\phi = 23.5^\circ$ and $\theta = 0^\circ$, the roller turns to demonstrate stable motion parallel to $\mathbf{B}$ in the steady state. These examples have initial ϕ values only 2° apart,

highlighting the sharp cutoff between initial conditions which result in orthogonal steady states. To make this point even clearer, Fig. 3(c) presents a phase diagram for a full sweep of pairwise combinations of ϕ and θ , where color indicates the resolved mode. The initial conditions of the trajectories shown in 3(a) and 3(b) are plotted in the highlighted region.

To gain insight into how this dual-anisotropized motion is permitted, in Figs. 3(d) and 3(e) we plot the components of the soft axis in the frame of the roller as a function of time. Looking first at the case of perpendicular motion in Fig. 3(d), initially displaced from its $\theta = 0^\circ$ position, we see a transient regime of the soft axis spiraling inward as a function of time to ultimately align with the magnetic field. Hence, much like the case of the permanent moment, this mode corresponds to an energetic minimum of the magnetic energy at all times, where the soft axis is an “axle” parallel to $\mathbf{B}$. In the language of dynamical systems, the perpendicular mode can be considered a fixed point attractor, and all blue regions of the phase diagram in the Fig. 3(c) are its basins of attraction.

Shifting our focus to the case demonstrating parallel motion, we see that the soft axis resolves to an oscillatory state strongly reminiscent of a limit cycle. Figure 3(e) shows the soft axis coordinates converging to a closed circular path in the xz plane, traced by the 90° phase lag between χ_s^x and χ_s^z , while χ_s^y oscillates with a comparatively negligible

amplitude. This limit-cycle-like behavior is replicated in the dynamics of the induced magnetic moment, as well as the angular velocity, as ω_x and ω_z precess around the driving component ω_y . This drives motion collinear with $\mathbf{B}$, but not perfectly so, as the roller "wobbles" back and forth over a single cycle. This characteristic "wobbling" is crucial for stability of the parallel mode, and a key distinction between the anisotropic susceptibility and permanent moment models. For the permanent moment, the sign of the magnetic torque is invariant under reflection about $\hat{\omega}$. Consequently, the direction of the angular velocity does not change over a rotation cycle, rendering the oscillatory dynamics associated with the parallel motion impossible. Conversely, the magnetic torque induced by a reflection of the tensor $\mathbf{X}$ about $\hat{\omega}$ in the anisotropy model is antisymmetric. Therefore, as $\mathbf{X}$ corotates with the body of the roller, this antisymmetry triggers a change in the sign of the magnetic torque during a single cycle of rotation. This in turn leads to the periodic "wobbling" about $\hat{\omega}$, to ultimately sustain motion collinear with $\mathbf{B}$ and the oscillatory tumbling of the magnetic moment in the vertical plane, giving us the parallel mode. The result is oscillating steady state solution for the angular velocity, distinct from the steady fixed solution for the perpendicular mode—see the Supplemental Material and Figs. S1 and S2 for additional details [46].

IV. DISCUSSION AND CONCLUSIONS

A physical basis for the source of magnetic anisotropy in our particles is readily motivated by an inhomogeneous distribution of magnetite nanoparticles within the body of each roller, as well as a distribution of anisotropy axis orientations [51,52]. Moreover, as discussed in [52], a clustering of nanoparticles which leads to dipole-dipole interactions invalidates the assumption of single isolated magnetic domains, culminating in a more complex magnetic response within the particle. Accordingly, the combined variability of these factors suggest a distribution of anisotropy within a population. In combination with other randomized or isotropic forces present in the absence of a magnetic field—and in particular those that cause the circular trajectories in Fig. 1(c)—this may account for the field dependence of the modes in Fig. 1(i), as well as the observation that some rollers exhibit no response to the magnetic field whatsoever. Additionally, varying degrees of roller asphericity would not only contribute to an anisotropic magnetic response, but could also lead to the anomalous Quincke-rolling behavior observed when no magnetic field is applied.

Our results are consistent with a distinct class of active matter in which an anisotropic response is not constrained to a single axis. Most known polar systems are either isotropic or become anisotropic along a single axis when subject to external fields. With our "paramagnetic" colloidal particles and the Quincke instability, we observe striking dynamics consistent with a dually anisotropized system of two orthogonal linear modes. This behavior cannot be explained by either simple paramagnetism or fixed dipole moments. Instead, it emerges from an anisotropic paramagnetic response. The result is a rich single-particle phase space, where particles can spontaneously switch between modes, even under steady driving. These findings show that anisotropic properties at the agent level can expand the dynamical regimes accessible to active systems, and in doing so may open the door to unforeseen behaviors and new forms of control over both single-particle and collective dynamics.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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