

Fine-tuning final state interactions model in the NuWro Monte Carlo event generator

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Recent experimental data from MINERvA on transverse kinematics observables across four different nuclear targets—carbon, oxygen, iron, and lead—have been utilized to refine the modeling of final state interaction effects in the NuWro Monte Carlo neutrino event generator. For this purpose, we have developed an event reweighting tool for future applications to adjust the strength of final-state interactions. This study highlights the requirement for stronger nucleon reinteractions than previously assumed, but it still falls within the uncertainty range observed in a study comparing proton transparency measurements. This conclusion has significant implications for both experimental and theoretical work involving NuWro.

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I. INTRODUCTION

Reducing uncertainties in neutrino-nucleus cross sections [1,2] is essential for the precision goals of forthcoming oscillation experiments like DUNE [3,4] and HYPER-K [5]. With the growing event statistics, neutrino cross sections have already become the dominant source of uncertainties in current experiments [6,7]. These experiments rely heavily on Monte Carlo (MC) event generators [8] to simulate neutrino-nucleus interactions and to estimate efficiencies, backgrounds, and systematic uncertainties in experimental results.

The neutrino MC event generators that are widely used, such as GENIE [9], NEUT [10], GIBUU [11], or NuWro [12,13], are based on the factorization scheme. In this context, neutrinos interact with individual bound nucleons, and the overall interaction is separated into two subprocesses. The first is the primary (anti)neutrino-nucleon interaction, and the second is the rescattering of hadrons produced at the primary vertex. GIBUU also employs such a separation at the interaction level; it differs conceptually from the other generators by describing the propagation of all hadrons

using the transport model. During final-state interactions (FSI), where the outgoing hadrons reinteract in various ways with the residual nucleus, they alter the event topology. For instance, a two-nucleon knockout could be due to either a $2p$ - $2h$ interaction or by a quasielastic interaction followed by rescattering of the struck nucleon, or maybe even from a pion-production mechanism followed by pion absorption.

Metropolis *et al.* [14,15], building on an idea of Serber [16], attempted in the early 1950s to describe these processes using what was then a novel MC approach within a cascade model of hadron propagation in the nuclear medium. Subsequently, many significant advancements were made in this direction, rendering the approach more realistic. Within the cascade model, the most critical component is the *microscopic* hadron-nucleon cross section. In generators such as NuWro, NEUT, and GENIE, the cascade is formulated in terms of a local mean-free path, whereas GIBUU instead solves transport equations for phase-space densities, and ACHILLES [17] is based on impact-parameter based parametrizations for modeling an intranuclear cascade. The discussion in this article pertains to the mean-free-path formulation. Within the mean-free-path approach, interacting hadrons experience a nuclear potential, so the free hadron-nucleon cross sections used in the original Metropolis *et al.* papers were modified accordingly. Several approaches exist to model these microscopic cross sections (see, e.g., Refs. [18,19]), but the ultimate test of their effectiveness within a given framework must come from experiments.

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In this paper, we study the NuWro FSI model. NuWro is a widely used MC tool employed in many experimental and comparative studies. We will focus exclusively on the nucleon-FSI part, deferring improvements in pion and hyperon modeling to future studies. Any improvements to neutrino interaction modeling in NuWro can also be applied to other MC generators. We aim to provide a more comprehensive description of the overall strength of nucleon-FSI effects in the NuWro MC generator. In the cascade model, nucleons moving through the nucleus usually undergo only a few interactions. Accurately estimating their interaction probabilities is crucial. Other details, such as the angular distribution of outgoing nucleons after reinteraction, are less significant. The number of reinteractions, proportional to interaction probability, affects nucleon kinetic energy. Since the energy deposited by a detected nucleon in the hadronic calorimeter depends on its kinetic energy, modification of the probability of reinteractions affects the number of reconstructed nucleons and predictions for all exclusive observables.

In the previous comprehensive NuWro nucleon-FSI study, a comparison was done to proton transparency data [20]. The outcome of this study was an estimate of the overall uncertainty in quantifying the strength of the FSI effects, expressed as uncertainty in the value of the nucleon mean-free path. In conclusion, the estimate was $\pm 30\%$. A recent study by W. Filali *et al.* [21] on the comparison of neutrino interaction models (and their implementation in different MC generators) with measurements of *transverse kinematic imbalance* variables [22,23] from the T2K, MicroBooNE, and MINERvA experiments suggests that the FSI models in MCs should be stronger, compared to their current status. In this paper, we examine information contained in the recent MINERvA experimental measurements of transverse kinematic imbalance (TKI) observables [24], in which they performed simultaneous measurements on four different-sized nuclei: carbon, oxygen, iron, and lead. In heavier nuclei, nucleons travel longer distances, therefore are more likely to reinteract, making them more sensitive to FSI. For a nucleon with momentum ~ 0.6 GeV/c, the transparency ranges from 0.77 for carbon to 0.61 for iron and 0.42 for lead.

Recent MINERvA data from [24] discussed in this article are expected to provide valuable information on the strength of FSI effects. The main result of this paper is an estimate of the strength of FSI in NuWro, which is $\sim 24\%$ larger than previously assumed, but remains consistent with the previously determined uncertainty.

To achieve our goal, we implement a reweighting scheme that allows us to modify the cascade's effective strength without requiring costly full-event resampling. The scheme reweights only the nucleon reinteraction part, i.e., nucleon FSI. We reweigh the scaling parameter, which modifies the mean-free path, thereby affecting the probability of nucleon-nucleon reinteraction. We explore different options for the amount of information per event that

must be retained to enable reweighting. Using the proposed method, we then fine-tune NuWro's cascade module to the recent MINERvA's CC1 $p0\pi$ data [24].

The paper is organized as follows. In Sec. II, we describe basic components of the NuWro MC generator, focusing on its FSI module (see Sec. II B). In Sec. III, details about the recent MINERvA CC1 $p0\pi$ measurement [24] used in this study are presented. A separate Sec. IV is devoted to introducing the idea of reweighting in MC generators. In Sec. V we present the main results of this paper, how the MINERvA data can be used to determine the strength of FSI effects. The paper finalizes with a discussion in Sec. VI and conclusions in Sec. VII.

II. NuWro MC EVENT GENERATOR

A. Generalities

NuWro [12,25] is a MC event generator for lepton-nucleus interactions developed at the University of Wrocław since ~ 2005 for accelerator-based neutrino experiments. It covers a broad energy range, from a few hundred MeV up to several hundred GeV. For primary interactions, NuWro simulates interaction mechanisms such as quasielastic (QE), quasielastic hyperon production, meson-exchange currents (MEC), resonance production, and deep inelastic scattering for both charged-current (CC) and neutral-current processes. In the QE channel, for both electron and neutrino scattering, depending on the nucleus, NuWro offers several options to describe the target-bound nucleon: hole spectral function (SF) [26,27], effective density and momentum dependent potential [28], and global and local Fermi gas (LFG). Recently, NuWro has been utilized as the primary MC generator in two significant experimental studies. The first was the KamLAND measurement of the strange axial form factor [29]. The second was the JSNS² measurement of missing energy with KDAR mono-energetic neutrino flux [30].

In this study NuWro simulations are done with the version 25.03 [12]. Compared to earlier distributions, there have been significant improvements in the QE [31], MEC [32], and in single-pion production (SPP) [33]. In the QE channel, the grid SF for argon is updated. In the MEC channel NuWro offers the implementation of a model developed by the Valencia group [34] which has np - nh decomposition of the total MEC cross section, isospin decomposition, and kinematic predictions for the two-nucleon final state. For SPP, NuWro now uses, by default, the Ghent hybrid model [35], which provides a realistic description of the second resonance region, interference with the nonresonant background, and a Regge description at high invariant hadronic masses. For the analysis, we use the LFG model to represent the initial state of the nucleon in NuWro, since spectral-function implementations are available only for carbon, oxygen, argon, and iron, not for lead.

B. Nucleon-FSI module in NuWro

The NuWro's cascade model offers a semiclassical description of hadron propagation in the nuclear medium, including nuclear effects such as in-medium modification of the hadron-nucleon cross section, Pauli blocking, and nucleon-nucleon correlations. Its basic framework is based on the MC approach proposed by Metropolis *et al.* [14,15]. The model assumes that the energies transferred during collisions are much larger than the typical nuclear binding energy, allowing hadrons to be treated as quasifree particles. The wave functions of the propagating hadrons are well defined by their positions and momenta, and their de Broglie wavelengths are smaller than the mean-free path. Additionally, scatterings on different nucleons are considered independent processes, with no interference between them. In Ref. [36], it has been shown that, for nucleons with sufficiently high momentum, these assumptions hold.

In NuWro, the nuclear radius is defined as the distance from the center to the points where the density falls below the maximum density by a factor of 10^5 . The density profiles of the nuclei are sourced from Ref. [37]. The total and elastic free nucleon-nucleon cross sections are fitted to experimental data from Tanabashi *et al.* [38], while the fraction of single-pion production within the inelastic cross section follows the parametrization proposed by Bystricky *et al.* [39]. In-medium modifications to nucleon-nucleon interactions utilize the studies of Pandharipande *et al.* [18], incorporating corrections for Pauli blocking and the effective mass of nucleons. For inelastic interactions, a density-dependent phenomenological correction [40] to the in-medium cross section (σ_{NN}^*) is introduced as

$$\sigma_{\text{NN}}^*(r) = \left(1 - \eta \frac{\rho(r)}{\rho_0}\right) \sigma_{\text{NN}}^{\text{free}}, \quad (1)$$

where ρ_0 is the saturation nuclear density and $\eta = 0.2$.

1. Algorithm

In NuWro, the distance Δx traveled by a propagating hadron between two successive possible interaction points is limited by a fixed step size of 0.2 fm. Over such a short distance, the nuclear density can be assumed to remain approximately constant. Under this assumption, the probability that a hadron travels a distance Δx without undergoing an interaction, hereafter referred to as the *survival probability*, is given by

$$\tilde{p}(\Delta x) = \exp(-\Delta x/\lambda), \quad (2)$$

where $\lambda = (\rho\sigma)^{-1}$ denotes the mean-free path, computed locally from the nuclear density ρ and the microscopic hadron-nucleon cross section σ . We reserve the symbol “ $\tilde{p}$ ” (without tilde) for the interaction probability, defined as $p = 1 - \tilde{p}$. Reducing the step size further does not alter the

physics results, but it significantly increases the computational runtime.

One common approach is to treat the actual travel distance as a random variable determined by the mean-free path. In this formulation, Δx is drawn according to

$$\Delta x = \min(-\lambda \ln \xi, \text{step}), \quad \text{where } \xi \in (0, 1], \quad (3)$$

with ξ a uniformly distributed random number. If the sampled value satisfies $\Delta x < \text{step}$, an interaction is generated at that point; otherwise, the hadron is propagated freely to the next position a distance “step” away.

Alternatively, one may choose to fix $\Delta x = \text{step}$ deterministically. In this case, the decision to generate an interaction at the end of the step is taken by comparing the survival probability with the random number ξ : an interaction occurs if and only if $\tilde{p} < \xi$.

Both approaches lead to practically identical results. Keeping in mind our plan to develop the FSI reweighting tool, in this study, we adopt the second approach where we fix $\Delta x = \text{step}$.

2. Proton transparency constraints on nucleon FSI

To quantify the distortion induced by FSI effects, one can use the concept of *nuclear transparency*, defined as the probability that a struck nucleon escapes the nucleus without undergoing significant reinteractions. Theoretical predictions are compared with experimental measurements by evaluating the MC transparency, defined as the fraction of simulated events in which the outgoing nucleon undergoes no reinteractions. It is important to note, however, that MC transparency is not directly equivalent to nuclear transparency, since experimental measurements cannot distinguish events with no reinteractions from those involving only “soft” FSI; see the discussion in Ref. [20]. A detailed study of the uncertainties associated with modeling nucleon-FSI within the NuWro cascade framework was done under the assumption that the dominant source of uncertainty in the description of nucleon-FSI could be effectively parametrized by scaling the nucleon mean-free path. It was estimated that variations of approximately $\pm 30\%$ (see Fig. 3 of Ref. [20]) covered the experimental uncertainty in the transparency measurements. To incorporate this uncertainty within NuWro, a free parameter was introduced, denoted by s , which linearly scales the nucleon mean-free path as $\lambda \rightarrow \lambda' = s\lambda$ with $s \in [0.7, 1.3]$. A smaller (larger) value of s corresponded to a shorter (longer) mean-free path, leading to a higher (lower) interaction probability within the cascade and thus strengthening (weakening) the overall impact of NuWro's nucleon-FSI.

III. MINER ν A CC1p0 π DATA

Recently, the MINER ν A Collaboration published results based on the simultaneous measurement of the ν_μ

quasi-elastic-like (CC1 $p0\pi$) cross section across different nuclear target materials—carbon, oxygen, iron, and lead [24]. These results are obtained from events in which both a muon and a proton are reconstructed in the final state. The MINER ν A study uses the neutrinos-at-main-injector (NuMI) beam in its medium-energy configuration [41], characterized by average neutrino energy $\langle E_\nu \rangle \sim 6$ GeV with a spread of ~ 2 GeV.

A detailed description of the MINER ν A detector appears in Ref. [42]. An important feature for this analysis is the radial variation of the neutrino flux within the detector, which varies by a few percentages. This variation arises because the detector's symmetry axis is not perfectly aligned with the beam axis, resulting in longitudinal variations in flux across the target. As a result, the flux differs for each nuclear target. The variation is most pronounced around 8 GeV, slightly away from the peak of the neutrino flux. The MINER ν A analysis accounts for these variations, and the published data incorporate this information. For consistency, we follow the same procedure in our NuWro simulations by using target-dependent fluxes.

The MINER ν A study [24] focuses on QE-like processes of the form

$$\nu + A \rightarrow \mu^- + p + X, \quad (4)$$

where A is the target nucleus and X is the final-state hadronic system, consisting of the remnant nucleus with possible additional protons or neutrons but with no mesons or heavy baryons. The signal definition selects events with no pions, one muon, and at least one proton, subject to constraints

$$2.0 < |\mathbf{p}_\mu| < 20 (\text{GeV}/c) \quad \theta_\mu < 17^\circ, \quad (5)$$

$$0.5 < |\mathbf{p}_p| < 1.1 (\text{GeV}/c) \quad \theta_p < 70^\circ, \quad (6)$$

where $\mathbf{p}_\mu$ and θ_μ ($\mathbf{p}_p$ and θ_p) denote the outgoing muon (proton) momentum and polar angle relative to the incoming neutrino direction. For events with multiple protons satisfying the selection, the proton with the highest momentum is chosen.

Proton kinematics provide information that enables reconstruction of the transverse momentum imbalance relative to the neutrino direction. The projection of the momentum difference between the outgoing lepton and proton in this plane reflects nuclear effects that modify the proton's kinematics. These quantities—known as TKI variables [22,23,43]—are widely used in experimental analyses to characterize observed nuclear effects. For the process in Eq. (4), where the incident E_ν is unknown, the transverse momentum imbalance is

$$\delta \mathbf{p}_T = \mathbf{p}_T^\mu + \mathbf{p}_T^p, \quad (7)$$

where $\mathbf{p}_T^{\mu(p)}$ is the transverse projection of the outgoing muon (proton) momentum. The imbalance can arise from Fermi motion or FSI. One can further decompose $\delta \mathbf{p}_T$ into x and y components. The standard convention in the neutrino community is to take the z axis along the direction of the incoming neutrino and define

$$\delta p_{T_y} = \hat{y} \cdot \delta \hat{\mathbf{p}}_T, \quad \hat{y} = -\hat{\mathbf{p}}_T^\mu, \quad (8)$$

$$\delta p_{T_x} = \hat{x} \cdot \delta \hat{\mathbf{p}}_T, \quad \hat{x} = \hat{z} \times \hat{y}. \quad (9)$$

Another important observable is the direction of $\delta \mathbf{p}_T$ relative to the transverse projection of the muon momentum ($-\hat{\mathbf{p}}_T^\mu$), denoted as $\delta \alpha_T$:

$$\delta \alpha_T = \arccos(-\hat{\mathbf{p}}_T^\mu \cdot \delta \hat{\mathbf{p}}_T). \quad (10)$$

A further TKI variable commonly used in experimental studies is $\delta \phi_T$, which measures the direction of the outgoing nucleon relative to the momentum-transfer vector $\vec{q}$ in the transverse plane:

$$\delta \phi_T = \arccos(-\hat{\mathbf{p}}_T^\mu \cdot \hat{\mathbf{p}}_T^p). \quad (11)$$

One can also go beyond TKI and explore information contained in longitudinal components of proton and muon, to reconstruct the momentum of the struck neutron [43]. The estimate is very precise for QE interactions without FSI. Using the energy-momentum conservation, we get

$$|\mathbf{p}_n| = \sqrt{|\delta \mathbf{p}_T|^2 + |\delta \mathbf{p}_L|^2}, \quad (12)$$

where

$$|\delta \mathbf{p}_L| = \frac{R^2 - m_{A'}^2 - \delta \mathbf{p}_T^2}{2R} \quad (13)$$

and

$$R = m_A + |\mathbf{p}_L^\mu| + |\mathbf{p}_L^p| - E^\mu - E^p.$$

$m_{A'}$ and $E^{\mu(p)}$ are the mass of the nuclear target (remnant nucleus), and the total energy of the muon (proton). In the MINER ν A study,

$$m_{A'} = m_A - m_n + b,$$

where m_n is the mass of the neutron and b is the excitation energy with the following values: 27.13 MeV for carbon, 24.1 MeV for oxygen, 29.6 MeV for iron, and 22.8 MeV for lead, obtained from the Ref. [43]. Section V C provides a physical interpretation of the TKI variables used in our

analysis when comparing NuWro predictions with the MINERvA's CC1p0 π data.

The MINERvA data release includes measurements of differential cross sections for TKI observables and for the observed muon and proton kinematics, including momenta, transverse, and longitudinal components; angular distributions relative to the beam; and ratios of differential cross sections to the hydrocarbon target. Each observable is accompanied by an overall covariance matrix, which is used to compute the χ^2 per degree of freedom (d.o.f) in comparisons with MC predictions. The collaboration also provides a covariance matrix combined across all nuclear targets for each observable. However, no global covariance matrix combining all nuclear targets and all observables is provided.

IV. REWEIGHTING OF FSI

Any change in the FSI parameters alters the likelihood of events produced by an MC generator. Instead of rerunning the MC with a new configuration, we can achieve the same effect by directly modifying the probability (weight) of each event. This process is known as reweighting. Although full event resampling using MC simulations is relatively fast, the detector simulation is computationally expensive and can take at least a week or longer. Using the reweighting approach, one can quantify the effect of model parameters without rerunning the detector simulation. For example, consider the reweighting procedure for modifying a theoretical parameter such as the QE axial mass M_A in a dipole parametrization. In this case, we compute the axial form factor by evaluating the ratio of differential cross sections for two different values of M_A in each event, using the available information about the four-momentum transfer Q^2 and differential cross section $d\sigma/dQ^2$. However, reweighting FSI parameters operates under different principles. We describe this process below. FSI reweighting has been thoroughly studied before in the context of the NEUT MC generator; see Ref. [44].

A. Exact reweighting

During the runtime of NuWro's cascade module, the MC algorithm checks at each step whether any propagating nucleons interact. This decision is based on the survival probability $\tilde{p}$, which depends on the nuclear density and the effective nucleon-nucleon cross section.

For *exact* reweighting, we classify each cascade step into one of the following categories:

- (1) A step where an interaction occurred.
- (2) A step where an interaction would have occurred but was prevented due to Pauli blocking.
- (3) A step where no interaction occurred.

From the exact-reweighting viewpoint, we treat Pauli-blocked interactions the same way as regular interaction steps.

We can view the FSI part of an event simulation as a chain of interaction and noninteraction steps:

$$\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_j, p_1, \tilde{p}_{j+1}, \dots, p_{N_{s_i}}, \dots, \tilde{p}_{N_{f_i}}.$$

These chains tend to be long because nucleons typically travel through many steps before leaving the nucleus. For example, simulations that use lead as the target can produce chains with ~ 3000 or more steps. Complete information about FSI is contained in all p_k , and $\tilde{p}_j$. However, for the purpose of reweighting it is enough to store all $\tilde{p}_j$ as a single value of the product of $\tilde{p}_j$'s as

$$\tilde{p} = \prod_l \tilde{p}_l.$$

The essence of FSI reweighting is to modify the interaction and noninteraction probabilities in a controlled manner so that the likelihood of a given event changes accordingly.

We define the likelihood of the i^{th} event, observed with N_{s_i} interaction steps and N_{f_i} noninteraction steps, as

$$\mathcal{L}(N_{s_i}, N_{f_i}; \{p_k\}, \{\tilde{p}_l\}) = \underbrace{\prod_{l=1}^{N_{f_i}} \tilde{p}_l}_{\tilde{p}} \prod_{k=1}^{N_{s_i}} p_k. \quad (14)$$

To reweight the strength of the FSI, we scale the mean-free path of the propagating nucleons by a factor of s ; see Eq. (2). This modifies the interaction and noninteraction probabilities as follows:

$$\lambda \rightarrow s\lambda \Rightarrow \begin{aligned} \tilde{p} &\rightarrow \tilde{p}' = (\tilde{p})^{1/s}, \\ p &\rightarrow p' = 1 - (1 - p)^{1/s}. \end{aligned} \quad (15)$$

A larger mean-free path results in a lower interaction probability, while a shorter mean-free path increases the interaction probability.

The modified likelihood becomes

$$\mathcal{L}'(N_{s_i}, N_{f_i}; \{p'_k\}, \{\tilde{p}'_l\}) = \underbrace{\prod_{l=1}^{N_{f_i}} \tilde{p}'_l}_{\tilde{p}' = \tilde{p}^{1/s}} \prod_{k=1}^{N_{s_i}} p'_k. \quad (16)$$

The reweighting factor for the event is therefore

$$w'_i(s) = \frac{\mathcal{L}'}{\mathcal{L}}. \quad (17)$$

Algorithm 1 presents a complete schematic for reweighting FSI.

We use two conditions to assess the consistency of the reweighting scheme:

Algorithm 1. *Exact-reweighting scheme.*

Input: NuWro output `.root` [45] file with M events.Choose the scale parameter s .**for** event $i = 1$ **to** M **do** **for** vertex $j = 1$ **to** N **do** **if** interaction or Pauli-blocked vertex **then** $p_j \rightarrow p'_j = 1 - (1 - p_j)^{1/s}$ **else** $\tilde{p}_j \rightarrow \tilde{p}'_j = \tilde{p}_j^{1/s}$ **end** **end** Assign event with weight: $w'_i(s) = \mathcal{L}'/\mathcal{L}$;**end****Result:** New `.root` [45] file with updated event weights.

-
- (1) Condition 1: FSI is a unitary transformation of final-state particles produced at the primary vertex into the final-state particles detected in the experiments. Therefore, only the relative event weights should change, keeping the original cross section unchanged. The average reweighting factor should satisfy

$$\frac{1}{M} \sum_{i=1}^M w'_i(s) \approx 1. \quad (18)$$

- (2) Condition 2: The outgoing lepton must remain unaffected. Therefore, differential cross sections expressed in lepton kinematic variables must not change.

Both conditions must hold for the reweighting scheme to be reliable. Condition 2 is more difficult to check because it requires a sufficiently large number of events per bin to control statistical fluctuations. We examine this issue shortly in Sec. VA.

To enable exact reweighting, the sequence of interaction probabilities and the product of survival probabilities must be recorded for every event. Storing this information increases the output file size, especially for heavier targets like lead, by up to 60%. To address this issue, we will discuss the introduction of an *approximate* reweighting scheme in Sec. VIB that relies solely on information already stored in the current NuWro version, such as the number of interaction and noninteraction steps.

V. RESULTS

In this section, we discuss the performance of the exact reweighting defined in Sec. IVA and show how it can be used to fine-tune the strength of nucleon FSI using MINER ν A CC1 $p0\pi$ data [24].

A. Consistency of the *exact*-reweighting scheme

We begin by defining a measure $\epsilon(s)$ to quantify how well the overall normalization is preserved under the reweighting scheme. It is given by

$$\epsilon(s) = \frac{1}{M} \sum_{i=1}^M w'_i(s), \quad (19)$$

where M is the total number of events in the sample.

Figure 1 shows the behavior of $\epsilon(s)$ for different nuclear targets. As seen in Fig. 1(a), the normalization is maintained within 1% for all targets in a wide range of the parameter s . For carbon (red curve) and oxygen (green curve), the curves are almost flat with a minimal deviation. The normalization for iron (blue curve) is slightly overestimated, but remains within 1% for the whole range of s . For lead (orange curve), the normalization remains stable in the interval $s \sim [0.79, 1.3]$, but begins to deviate by more than 1% for $s \lesssim 0.78$. This deterioration can be attributed to the limited statistics used for the reweighting. This is demonstrated in Fig. 1(b), where we show the effect of an increased number of events from 10^5 to 5×10^6 . With the higher-statistics sample [blue curve in Fig. 1(b)], for $s > 1$ the curve is flat, while for $s < 1$ the range over which normalization is preserved up to 1% expands to $s \gtrsim 0.74$.

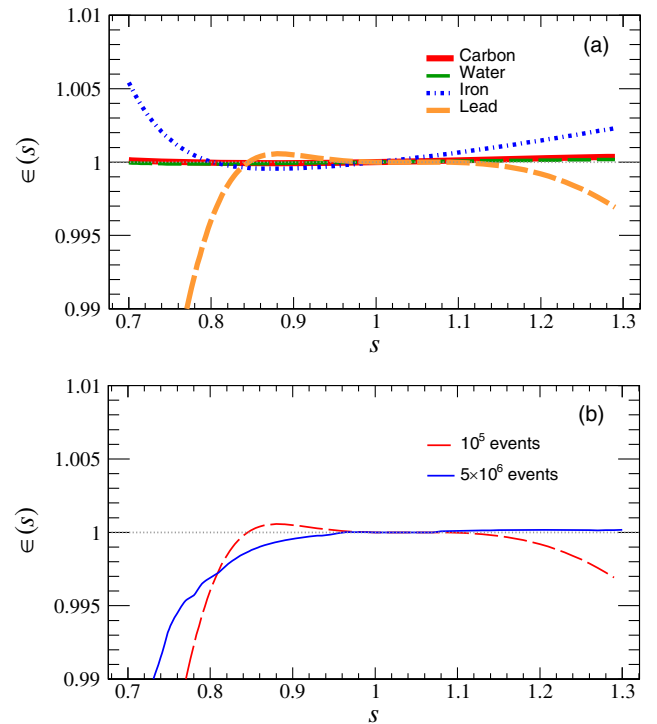


FIG. 1. Preservation of the overall normalization, as defined in Eq. (19), for the method summarized in Algorithm 1. (a) for different targets with 10^5 events and (b) for the lead target but with different statistics.

Similarly, we examined whether the differential cross section expressed in terms of lepton kinematic variables remains stable while reweighting within statistical fluctuations. To assess this, we compared the intervals in the scaling parameter s over which the kinematic distributions remain shape preserving across the two statistical samples considered above. Using a moderately fine binning, we checked whether distributions in variables such as $|\mathbf{p}_\mu|$ and θ_μ agree to within 1%; see Appendix A.

For the sample with 10^5 events, the shape invariance condition is typically satisfied over the range $s \sim [0.82, 1.18]$, which is smaller than the interval in which normalization is retained. For the higher-statistics sample, the range over which the kinematic distributions remain stable broadens to $[0.78, 1.30]$. This demonstrates that the reweighting scheme's performance improves as the number of statistics increases. In the limit of infinite statistics, one expects the exact scheme to preserve both normalization and shape of kinematic distributions.

It is possible to overcome limitations caused by finite statistics in the reweighting range through an iterative process. To get results for smaller s values, we run NuWro with $s = 0.76$ and reweight downward relative to this value, repeating the procedure if necessary.

B. Fine-tuning NuWro's cascade module

To fine-tune NuWro's cascade module, we applied the *exact*-reweighting scheme (discussed in Sec. IV A), which allows reweighting of FSI events without the need to rerun NuWro. In this section, we describe the methodology used to optimize NuWro's cascade module using the recent MINERvA CC1 $p0\pi$ data [24].

1. Finding the optimal value of “ s ”

Our goal is to determine the value of the A -independent cascade parameter s . To achieve this, we compare NuWro predictions across all four nuclear targets simultaneously. First, NuWro output `.root` [45] files are generated using the default cascade parameter value, $s = 1$, with the corresponding neutrino flux and the LFG nuclear model. We then select observables sensitive to FSI modeling. Our choice of experimental observables is based on the following observations:

- (1) FSI reweighting only affects the kinematics of outgoing hadrons, keeping the lepton kinematics unaffected; therefore, observables based only on lepton kinematics cannot be used.
- (2) Transverse projections such as TKI variables, as described in Ref. [23], and $|\mathbf{p}_{p_T}|$ provide direct measurements of nuclear effects and are included while optimizing s .
- (3) More information about FSI is contained in observables, which also involve the longitudinal components of final state particles, like reconstructed neutron momentum $|\mathbf{p}_n|$.

Based on these facts, we decided to select the following observables:

- (1) TKI variables $\delta\alpha_T, \delta\mathbf{p}_T, \delta\mathbf{p}_{Ty}, \delta\phi_T$.
- (2) Reconstructed neutron momentum $|\mathbf{p}_n|$ and transverse projection of the outgoing proton's momentum $|\mathbf{p}_{p_T}|$.

We then adopt the *exact*-reweighting procedure to fine-tune the cascade parameter to these observables.

To determine the optimal cascade parameter, we scan s in a large interval $[0.3, 1.3]$ with a step of $\Delta s = 0.01$, calculating χ^2 for all selected observables. In our analysis, we use the covariance matrix combined over all nuclear targets to compute the $\chi^2/\text{d.o.f}$ for each observable and to fine-tune NuWro's cascade module with respect to the parameter s . In the absence of a global covariance matrix covering all targets and observables, the analysis assumes that there are no correlations between the observables. For each observable, the fitting process follows the algorithm outlined in Algorithm 2. This procedure is then repeated for all observables considered. For each observable, we get a best fit value of s .

To estimate the best fit value of s globally across all observables, along with its uncertainty, we examine the likelihood function inferred from the $\chi^2(s)$ functions associated with each observable. In statistics, the goodness of fit expressed through χ^2 can be related to the likelihood ratio of two competing models. For a model with parameters $\hat{\theta}$, having likelihood $L(\hat{\theta})$, compared with a reference (null) model with likelihood L_0 , the likelihood ratio test gives

$$\chi^2 \sim -2 \ln \left(\frac{L(\hat{\theta})}{L_0} \right).$$

Inverting the above relation yields a likelihood function $L(\hat{\theta})$ for each observable based on its corresponding χ^2 . In our case, we compute the likelihood $L(s)$ of observing s for a given observable, using the distribution of $\chi^2(s)$ for that observable. The likelihood distributions for different observables used in our analysis are shown in the top panel of Fig. 2. By comparing results across six observables, we obtain a consistent picture: the parameter s should be much smaller than its default value. Differences between

Algorithm 2. Fine-tuning of s .

Input NuWro output `.root` [45] file for all targets with $s = 1$
for $s \in [0.3, 1.3]$ **do**
 1. Reweight NUWRO events using the scheme given in Algorithm 1.
 2. Apply event selection cuts [24] to reweighted events.
 3. Produce distributions of the desired observable and compute $\chi^2/\text{d.o.f}$ using the combined covariance matrix over all targets (see Sec. III).
end

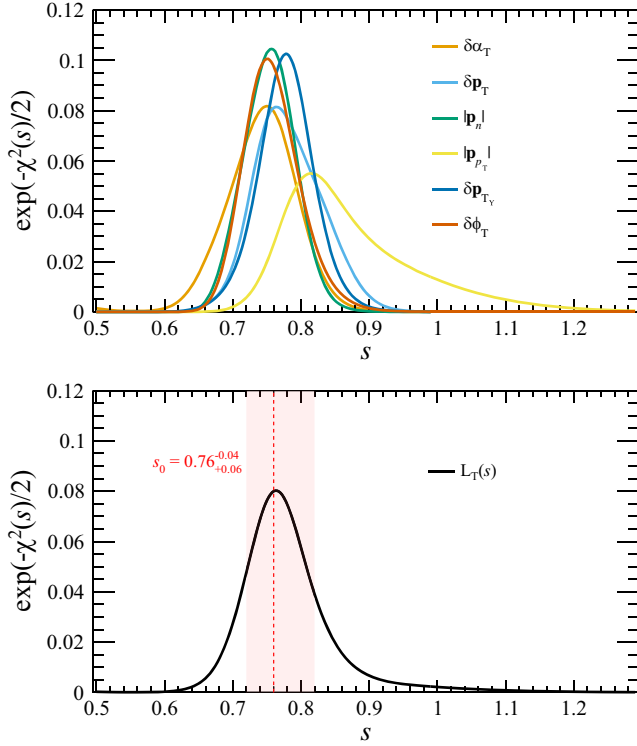


FIG. 2. Top: likelihood $\hat{L}(s)$ inferred using $\chi^2(s)$ for each observable. Bottom: overall likelihood $L_T(s)$ by summing over all observables. The red dashed line represents the peak. The red band represents the minimum length for which the enclosed area is 68.27%.

observables are not surprising, keeping in mind the complexity of physics mechanisms giving rise to the experimental signal measured by the MINERvA, and the fact that we focus on only one aspect of the FSI model—the overall strength.

To obtain a global constraint on s , we define the overall likelihood L_T as the (normalized) average of the individual likelihoods L_o from all observables:

$$L_T = \frac{1}{6} \cdot \sum_{o \in \left\{ \delta\alpha_T, \delta p_T, \delta p_{T_y}, \delta\phi_T, |\mathbf{p}_n|, |\mathbf{p}_p| \right\}} \hat{L}_o, \quad (20)$$

where the hat denotes that each likelihood has been normalized such that the area under the curve is unity. The resulting overall likelihood distribution is shown in the bottom panel of Fig. 2. The peak of the likelihood is located at

$$s_0 = 0.76. \quad (21)$$

The red band represents the 1σ uncertainty interval around $s_0 = 0.76$, defined as the section of minimal length such that the enclosed area is 68.27% of the total, which is given as

$$s \in [0.72, 0.82]. \quad (22)$$

This will be the new uncertainty interval in the upcoming version of NuWro. We emphasize that the new uncertainty interval is fully contained within the previous one, which was determined as $s \in [0.7, 1.3]$, as reported in Ref. [20].

C. Performance of NuWro FSI with $s = s_0$

The NuWro performance with $s = s_0$ for all the observables is summarized in Table I. The calculations were performed using individual covariance matrices for each target and each observable, separately. Table I presents information about six observables used in the fitting process, as well as two additional ones ($|\mathbf{p}_p|$) and θ_p). The table supports our statement that the results are fully consistent. In nearly all cases, changing from $s = 1$ to $s = s_0$ produces significantly better agreement with the data. The only exceptions occur when the $\chi^2/\text{d.o.f.}$ ratio is below 1, which makes it problematic to draw firm conclusions.

Figures 3(a)–3(d) show the differential cross section measurements reported in [24] as a function of $|\mathbf{p}_n|$ for carbon, water, iron, and lead. The peak at low values of $|\mathbf{p}_n|$ (0–200 MeV/c) reflects Fermi motion for CCQE events without FSI. The region above 200 MeV/c includes contributions from the multinucleon knockout mechanism, pion absorption, and CCQE with FSI. In all the cases, we observe that increasing the strength of the nucleon-FSI module improves the $\chi^2/\text{d.o.f.}$ For carbon, improvements occur primarily in the first two bins, indicating fewer nucleon-nucleon reinteractions due to a light nucleus. Water shows a similar trend with improvements in the first three bins. As we go to heavier nuclei, the improvement in $\chi^2/\text{d.o.f.}$ becomes more pronounced. Lead exhibits the largest improvement, and the shapes produced by the two NuWro configurations differ significantly. For lead, the peak height and tail thickness are comparable, indicating strong contributions from both Fermi motion and FSI processes.

Figures 3(a)–3(d) demonstrate that increasing the FSI strength reduces the overall $|\mathbf{p}_n|$ cross section. This can be understood by analyzing how event-selection cuts affect the overall normalization. As the mean-free path of propagating nucleons decreases, reinteractions become more frequent, shifting the momentum distribution of the leading proton toward lower values. Consequently, many events are removed to satisfy the muon and proton cut [see Eqs. (5) and (6)], because they fall below the proton-momentum threshold of $|\mathbf{p}_p| = 0.5$ GeV/c, which leads to the lowering of the overall normalization. A measure to quantify the dilution of signal events caused by experimental cuts after increasing FSI-strength is discussed in Ref. [21] for various experiments.

Even though the $\chi^2/\text{d.o.f.}$ improves after strengthening NuWro's nucleon-FSI effects the overall $\chi^2/\text{d.o.f.}$ is still

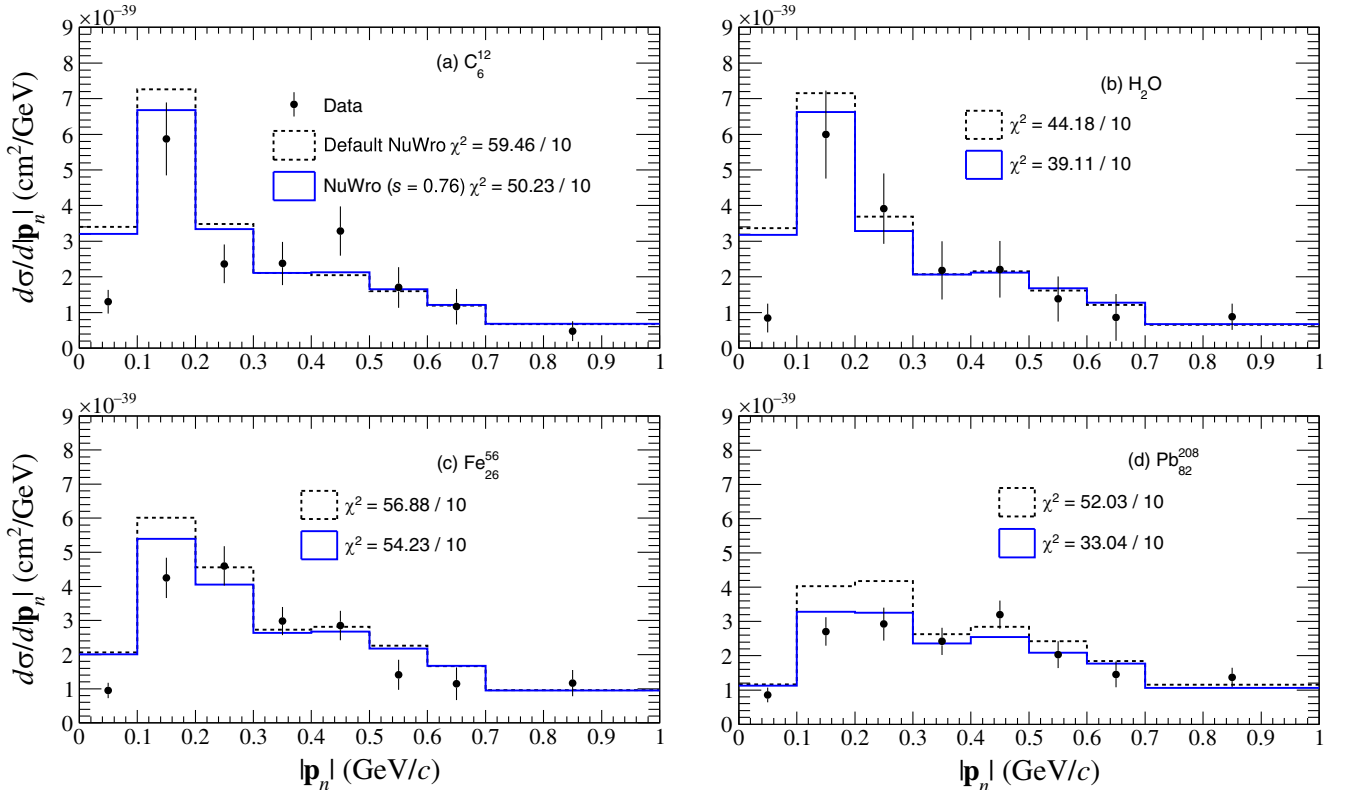
TABLE I. $\chi^2/\text{d.o.f}$ comparing NuWro with MINER ν A's CC1 $p0\pi$ data [24] for different nuclear targets.

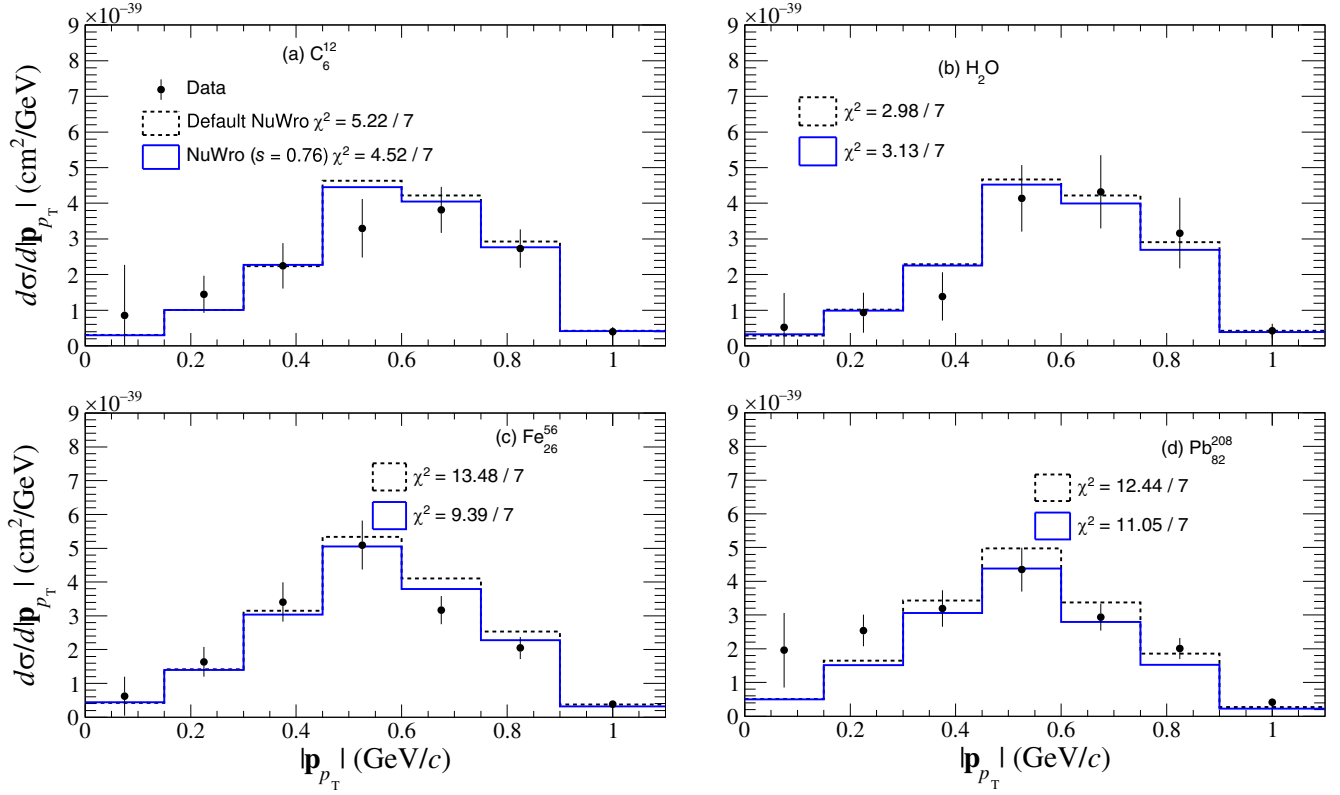
Observable	NuWro ($s = 1$)				NuWro ($s = s_0$)			
	Carbon	Water	Iron	Lead	Carbon	Water	Iron	Lead
1. $\delta\alpha_T$	14.29/5	12.35/5	19.11/5	52.56/5	13.05/5	8.85/5	8.89/5	30.14/5
2. $\delta\mathbf{p}_T$	77.32/9	21.91/9	92.46/9	53.02/9	62.95/9	19.82/9	85.12/9	37.16/9
3. $\delta\mathbf{p}_{T_y}$	16.80/9	10.22/9	20.11/9	39.45/9	14.82/9	7.27/9	10.45/9	23.32/9
4. $ \mathbf{p}_n $	59.46/10	44.18/10	56.88/10	52.03/10	50.23/10	39.11/10	54.23/10	33.04/10
5. $\delta\phi_T$	39.56/6	11.25/6	25.03/6	20.14/6	29.11/6	8.68/6	14.11/6	13.28/6
6. $ \mathbf{p}_{pT} $	5.22/7	2.98/7	13.48/7	12.44/7	4.52/10	3.13/7	9.39/7	11.05/7
Not used while optimizing $L_T(s)$								
Observable	NuWro ($s = 1$)				NuWro ($s = s_0$)			
	Carbon	Water	Iron	Lead	Carbon	Water	Iron	Lead
7. $ \mathbf{p}_p $	12.75/5	1.59/5	19.62/5	29.52/5	12.18/5	1.18/5	15.16/5	19.51/5
8. θ_p	4.07/10	5.55/10	8.81/10	21.94/10	4.68/10	5.80/10	6.11/10	17.19/10

rather poor. For carbon, oxygen, and iron, the primary contribution to the large values of χ^2 originates from overestimation of the cross section in the first two bins. This is mainly due to our choice to model the initial nuclear state using the LFG model. This will be discussed in detail in Sec. VI A.

Figures 4(a)–4(d) show the differential cross section measurements as a function of $|\mathbf{p}_{pT}|$ for all nuclear targets. In Fig. 4(a), for carbon, we find that increasing the cascade

strength changes the shape of the distribution only minimally. This occurs because most nucleons produced inside the carbon nucleus escape without significant reinteraction (see Sec. I) due to its smaller size. We observe a similar trend for water. Both carbon and water show good agreement with the data in terms of $\chi^2/\text{d.o.f.}$, both before and after increasing the cascade strength. For iron, in Fig. 4(c), increasing the cascade strength reduces the number of events with $|\mathbf{p}_{pT}| \gtrsim 0.5$ GeV, leading to improved agreement with

FIG. 3. Comparison of $|\mathbf{p}_n|$ with the MINER ν A CC1 $p0\pi$ data. (a) For carbon, (b) for water, (c) for iron, and (d) for lead target.

FIG. 4. Same as Fig. 3 but for $|\mathbf{p}_T|$.

the data. For lead, we observe that the cross section decreases in all bins except the first. The reason for this overall reduction is the same as in the case of $|\mathbf{p}_n|$.

A review of the remaining observables used in our analysis are moved to Appendix B.

VI. DISCUSSION

A. Sensitivity to nuclear modeling

In NuWro, to describe nuclear effects in QE scattering on carbon, oxygen, argon, and iron targets, one can choose (among others) between the LFG and SF approaches. The SF approach is more realistic, but it is not available for lead. In the LFG model, nuclear shell structure and nucleon correlations are absent. Nucleons are assumed to occupy all momenta up to the Fermi momentum $|\mathbf{p}_F(r)|$, which depends on the radial distance r and the nuclear density, approximated in NuWro using the charge distribution. In contrast, the SF approach incorporates shell structure and nucleon correlations [26,27]. It combines information from electron-scattering measurements of nuclear shells with theoretical calculations for nuclear matter at various densities.

Since the MINERνA Collaboration released correlation matrices for individual observables and four different nuclear targets together, the only consistent approach was to calculate NuWro results with the same nuclear model, i.e., with LFG.

It is essential to check if our findings are not biased by the use of an oversimplified nuclear model.

Figures 5(a)–5(c) show a comparison of MINERνA measurements for iron with NuWro predictions for QE interactions simulated using the SF model at two cascade-strength values, along with predictions obtained using the LFG model at $s = 1$. In all cases, the LFG model overestimates the cross section. In the upper panel of Fig. 5, the LFG model significantly overestimates the first two bins—corresponding to the onset of the Fermi-motion peak—leading to poor $\chi^2/\text{d.o.f}$, whereas SF-based predictions show better agreement. Increasing the cascade strength further improves the agreement with data.

In the middle panel of Fig. 5, although the shape of the $\delta\alpha_T$ distribution is similar for all approaches, SF produces a lower overall normalization, yielding improved $\chi^2/\text{d.o.f}$. Increasing the nucleon-FSI strength provides a slight additional benefit.

The lower panel of Fig. 5 also shows that the SF approach yields better $\chi^2/\text{d.o.f}$ when compared to NuWro (with QE LFG) predictions, demonstrating the sensitivity of MINERνA -exclusive data to nuclear-state modeling. Only in the case of $|\mathbf{p}_n|$ we observe that NuWro with $s = s_0$ and QE SF (histogram with green outline) the $\chi^2/\text{d.o.f}$ is slightly worse compared to the $s = 1$ case (histogram with red outline). However, the difference between them is small, making it difficult to determine whether it reflects statistical fluctuations or indicates that the best-fit value of s for this observable under the SF model is different. A tension between the second and third bins may indicate that the discrepancy is attributable to other

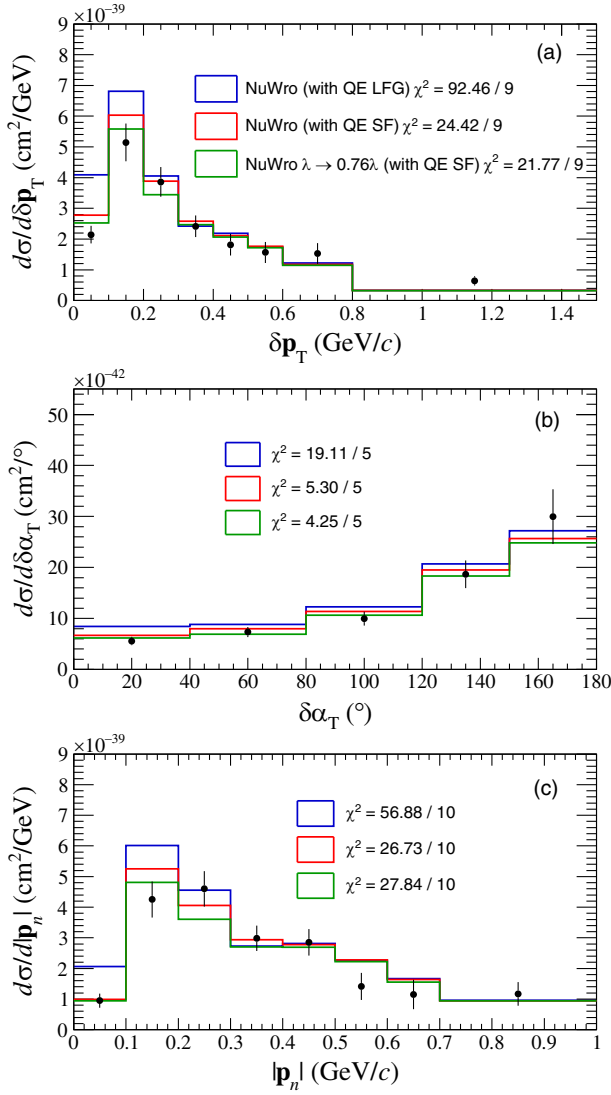


FIG. 5. Same as Fig. 3 but for the iron target, comparing LFG and SF models for the initial nucleon state using $s = s_0$. (a) For $|\delta\mathbf{p}_T|$ (b) for $\delta\alpha_T$ and (c) for $|\mathbf{p}_n|$.

components of the theoretical model (MEC and pion absorption).

Table II lists the $\chi^2/\text{d.o.f.}$ for various observables for iron. Except for $|\mathbf{p}_n|$, all cases show reduced χ^2 when the SF model is used. In instances where $\chi^2 < \text{d.o.f.}$, increasing the cascade strength either improves agreement further or sometimes produces slightly worse χ^2 ; the $\chi^2/\text{d.o.f.}$ remains below 1. Overall, we conclude that increasing the strength of the nucleon-FSI module in NuWro's cascade improves agreement with data, regardless of the nuclear model employed.

B. Approximate reweighting

To perform the exact reweighting described in Sec. IV A, it is necessary to increase significantly the amount of information stored in each event. In this section, we explore

TABLE II. $\chi^2/\text{d.o.f.}$ values comparing NuWro predictions with the MINERvA CC1 $p0\pi$ data [24] for iron, using the SF model to simulate QE interactions.

Observable	NuWro ($s = 1$)	NuWro ($s = s_0$)
1. $\delta\alpha_T$	5.30/5	4.25/5
2. $\delta\mathbf{p}_T$	24.42/9	21.77/9
3. $\delta\mathbf{p}_{T_y}$	20.11/9	9.07/9
4. $ \mathbf{p}_n $	26.73/10	27.84/10
5. $\delta\phi_T$	3.65/6	1.83/6
6. $ \mathbf{p}_{p_T} $	5.52/7	3.34/7
7. $ \mathbf{p}_p $	17.51/5	14.57/5
8. θ_p	3.98/10	4.72/10

a possibility to define an approximate reweighting method that yields a satisfactory approximation of the exact process, while requiring less storage. The goal is to develop a tool that could be easily used in future MC FSI studies.

The main idea in this approach is to use approximate values of (non)interaction probability. In a typical event simulated by NuWro's FSI module, the total number of steps ($N = N_S + N_f$), for an intermediate size nucleus, is about ~ 1000 , while the probability of interaction (p) is around ~ 0.01 . We can estimate the average interaction probability ($\hat{p}$) using pregenerated samples from NuWro as

$$\hat{p} = \left(\sum_{i=1}^M N_{s_i} \right) / \left(\sum_{i=1}^M N_i \right), \quad (23)$$

where M is the number of events, and $N_i = N_{s_i} + N_{f_i}$ is the overall number of steps for the i^{th} event. Table III shows the values of $\hat{p}$ obtained for different nuclear targets using the corresponding NuMI ME flux.

The estimator $\hat{p}$ represents the probability of interaction at each step, within the nucleus. In the approximate approach, the problem is to express the scaling of the average interaction probability, which differs across nuclear targets, in terms of the *universal* rescaling of the mean-free path, which is the same for all targets. With this information, we scale the estimator $\hat{p}$, as

$$\hat{p} \rightarrow \hat{p}' = 1 - (1 - \hat{p})^{1/r}, \quad (24)$$

where r will be the scaling parameter in the *approximate* scheme, analogous to the s parameter in the case of Algorithm 1.

TABLE III. Average interaction probability $\hat{p}$ for different nuclear targets using the corresponding NuMI ME flux [24] using 10^5 events.

Nuclear target	$\hat{p}$
1. Carbon	8.04×10^{-3}
2. Water	8.05×10^{-3}
3. Iron	1.38×10^{-2}
4. Lead	2.20×10^{-2}

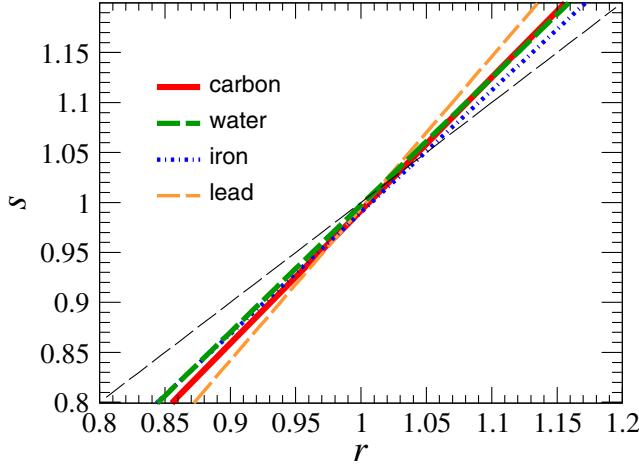


FIG. 6. A relationship between the scaling parameter r for interaction probability $\hat{p}$ and the scaling parameter s for mean-free path.

We attempted to relate r to the *universal* rescaling parameter s . To do this, we fixed a nuclear target and produced a NuWro output file with the mean-free path scaled by a value $s = s'$. We then reweighted events from a default NuWro output file ($s = 1$) using the reweight parameter r until the momentum distribution of the most energetic proton matched the distribution generated by NuWro with $s = s'$ within a 1% tolerance; we denote this value of r by r' . We then say that scaling the average interaction probability $\hat{p}$ by r' produces similar effects to scaling the mean-free path by s . We then repeat this procedure for a sufficient number of s' values and obtain a linear fit of s versus r . Figure 6 shows the resulting relation between r and s obtained with this method for different nuclear targets.

In this scheme, we define the likelihood of the i^{th} event as

$$\mathcal{L}(N_{s_i}, N_{f_i}; \hat{p}) \approx (1 - \hat{p})^{N_{f_i}} \hat{p}^{N_{s_i}}. \quad (25)$$

After scaling the interaction probability, $\hat{p}$, the new likelihood of the event is $\mathcal{L}(N_{s_i}, N_{f_i}; \hat{p}')$ where $\hat{p}'$ is the scaled interaction probability. The reweighting factor $w_i(r)$ is given as

$$w_i(r) = \frac{\mathcal{L}(N_{s_i}, N_{f_i}; \hat{p}')}{\mathcal{L}(N_{s_i}, N_{f_i}; \hat{p})}. \quad (26)$$

We quantify the performance of the *approximate*-reweighting scheme using the quantitative measure $\epsilon(r)$

Algorithm 3. *Approximate*-reweighting scheme.

Input NuWro output `.root` [45] file with M events.
 Fix the value of the *reweight* parameter r .
 Scale the average probability of interaction: $\hat{p}$ using Eq. (24).
for event $i = 1$ **to** M **do**
 Assign new weight $w_i(r) = \frac{\mathcal{L}(N_{s_i}, N_{f_i}; \hat{p}')}{\mathcal{L}(N_{s_i}, N_{f_i}; \hat{p})}$
end
Result: New `.root` [45] file with weighted events.

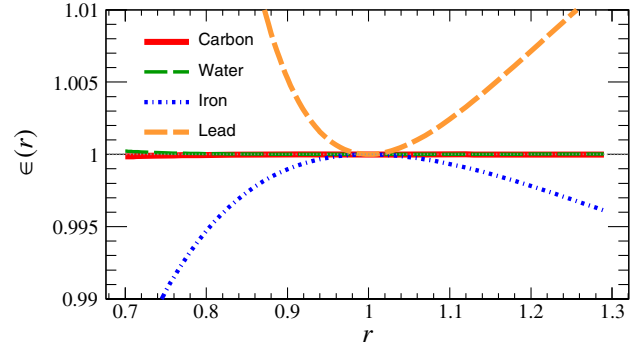


FIG. 7. Effect on the overall normalization using the measure $\epsilon(r)$ for the method given in Algorithm 3 for various nuclear targets.

[same as $\epsilon(s)$ in Eq. (19)]. For this, we produced a sample of 10^5 events using the NuWro MC generator (with $s = 1$) and reweighted events for a range of r . In Fig. 7, we quantify the performance of the *approximate* scheme in terms of preservation of the overall normalization. The normalization is maintained with perfect precision for carbon and oxygen. For iron, the normalization remains stable within 1% in the interval $[0.75, 1.3]$. For the lead, the situation is worse, as normalization is preserved at most up to 1% within the interval $[0.87, 1.26]$.

We then examined whether the differential cross sections, expressed in terms of lepton kinematic variables, remain stable when reweighting within statistical fluctuations. For the sample with 10^5 events, this condition is typically satisfied over the range of $[0.90, 1.11]$.

We qualitatively compare the *exact* and *approximate* reweighting schemes in Fig. 8. We choose $r = 0.9$, which lies within the region of applicability of both the *approximate* and *exact* schemes. The shapes produced by the two schemes are very similar, demonstrating the robustness of the *approximate* scheme, which uses only a fraction of the information compared with the *exact* scheme. The $\chi^2/\text{d.o.f}$ obtained with the *exact* and *approximate* reweighting scheme are similar, which proves that the *approximate* scheme may be a useful tool that helps to get a general orientation on how modification of the strength of FSI modifies NuWro predictions.

We also sought to relax certain assumptions while retaining the information required to reweight an FSI event, thereby improving the performance of the *approximate* scheme. In this scheme, it is assumed that the nucleon interaction probability (success rate) is constant throughout the nuclear volume. However, this is valid only if the overall nuclear density is constant. In reality, the success rate varies with the positions of reinteraction points within the nucleus. There is a relatively high success rate when the interaction point is deep within the nucleus, and much lower at the outskirts of the nucleus, where the density is minimal. Based on the nuclear density profile of a given nucleus, we tried to divide the total volume of the nucleus into three different zones. We then counted successes and failures in each zone using Eq. (23) and computed three

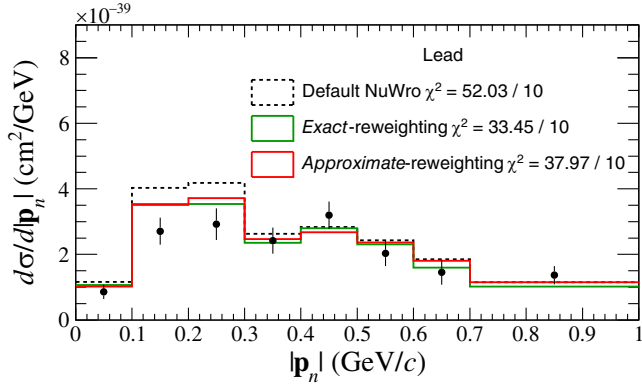


FIG. 8. A comparison between the *exact*- and *approximate*-reweighting schemes for $r = 0.90$. Data points are from [24].

separate average interaction probabilities. In this case, we defined the likelihood of an event as

$$\mathcal{L}(N_{s_i}, N_{f_i}; \hat{p}_I, \hat{p}_{II}, \hat{p}_{III}) \approx \prod_{\text{zone}}^{\{I, II, III\}} (1 - \hat{p}_{\text{zone}})^{N_{f_i}^{\text{zone}}} \hat{p}_{\text{zone}}^{N_{s_i}^{\text{zone}}}, \quad (27)$$

where N_{s_i}, N_{f_i} are the total number of interaction steps and noninteraction steps in three zones. We then scaled the interaction probability in each zone using Eq. (24) and assigned the additional weight to the event, defined as the ratio of the likelihood $W_i(r) = \frac{\mathcal{L}'}{\mathcal{L}}$. We concluded that the performance gain was smaller than expected.

VII. CONCLUSIONS

In this article, we fine-tune the strength of NuWro's nucleon-FSI part of the cascade module using recent CC1 $p0\pi$ data [24] from the MINERvA experiment, performed simultaneously on carbon, oxygen, iron, and lead. For this, we developed an *exact*-reweighting framework to reweigh events with nucleon FSI in NuWro without rerunning the whole simulation. The procedure is based on scaling the probability of (non)interactions of the propagating nucleon(s) in a well-defined manner and quantifying the occurrence of each event by computing its likelihood.

Using MINERvA's CC1 $p0\pi$ measurements on carbon, water, iron, and lead, we performed a global fit of several exclusive observables which are sensitive to nuclear effects like FSI, including TKI variables, reconstructed neutron momentum, and proton transverse momentum. The joint-likelihood combinations of all observables used in our analysis indicated a consistently preferred increase in FSI strength by

$$s_0 = 0.76_{-0.04}^{+0.06}. \quad (28)$$

The new uncertainty band around s_0 is around $\sim 10\%$. The tuned FSI parameter s and the reweighting implementation presented here will be incorporated into the next NuWro release, providing a more realistic estimate of FSI uncertainties and improving generator predictions for current and future oscillation experiments.

ACKNOWLEDGMENTS

We would like to thank Clarence Wret and Kamil Skwarczynski for providing us with details of the implementation of FSI reweighting in NEUT. This work was partly (A. M. A., K. M. G., B. E. K., and J. T. S.) or fully (R. D. B., J. L. B., and H. P.) supported by the National Science Centre under Grant No. UMO-2021/41/B/ST2/02778. K. M. G. is also partly supported by the Excellence Initiative—Research University, 2020–2026 at the University of Wrocław.

DATA AVAILABILITY

The data is produced using NuWro Monte Carlo event generator which is openly available at Ref. [46]. The experimental data used in our study is provided by the MINERvA Collaboration upon request.

APPENDIX A: DISTRIBUTION OF LEPTON KINEMATICS

As mentioned in Sec. VA, we demonstrate the shape preservation of the kinematic variable distributions of the

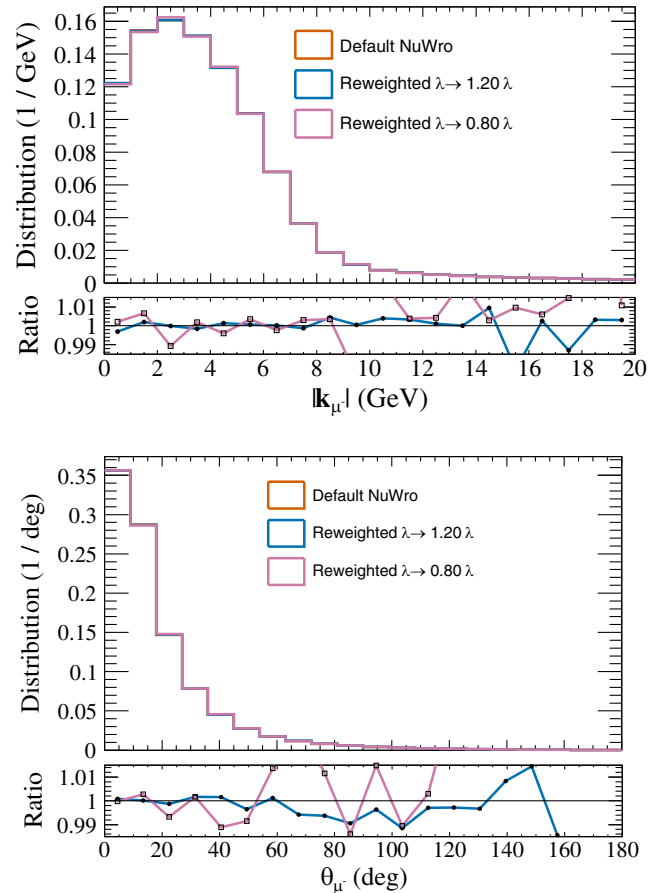


FIG. 9. Top: default NuWro vs NuWro reweighted to $s \in \{0.8, 1.2\}$ for muon momentum. Bottom: same as above but for angle of the outgoing muon.

outgoing lepton in Fig. 9 for the lead target. The differences remain within 1%, up to statistical fluctuations.

APPENDIX B: IMPACT OF REWEIGHTING FSI ON DIFFERENTIAL CROSS SECTIONS

Here we assess NuWro’s performance for six observables not discussed in Sec. VI A. Figures 10–15 show the differential cross section measurements as functions of $\delta\alpha_T$, $\delta\mathbf{p}_T$, $\delta\mathbf{p}_{Ty}$, $\delta\phi_T$, $|\mathbf{p}_p|$, and θ_p , respectively. For almost all observables, the $\chi^2/\text{d.o.f}$ improves after strengthening the FSI model, with the most significant improvement observed for lead.

Figures 11(a)–11(d) show NuWro’s performance for the observable $\delta\mathbf{p}_T$. In the absence of nuclear effects, $\delta\mathbf{p}_T$ would

approximate the initial nucleon momentum and peak around ~ 0.2 GeV, indicating the Fermi motion. Figures 13(a)–13(d) present the differential cross section measurements as functions of $\delta\phi_T$. The persistently poor χ^2 , even after increasing the FSI strength, arises mainly from the first bin and reflects limitations of our chosen nuclear model.

Figures 14 and 15 show NuWro’s comparison with the MINER ν A data [24] for the two observables that were not used while fine-tuning the s parameter. For both observables, the $\chi^2/\text{d.o.f}$ improves across all nuclear targets when using the best-fit value s_0 obtained from the six fitted observables.

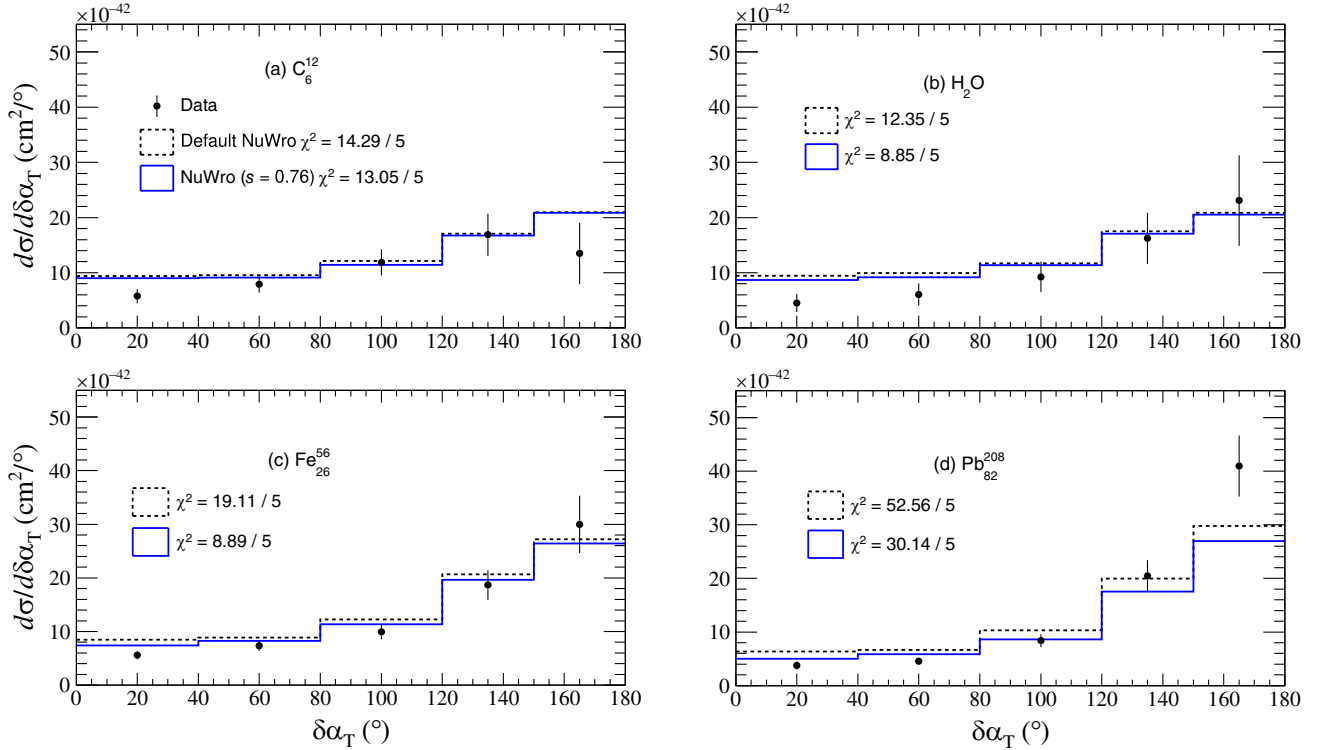
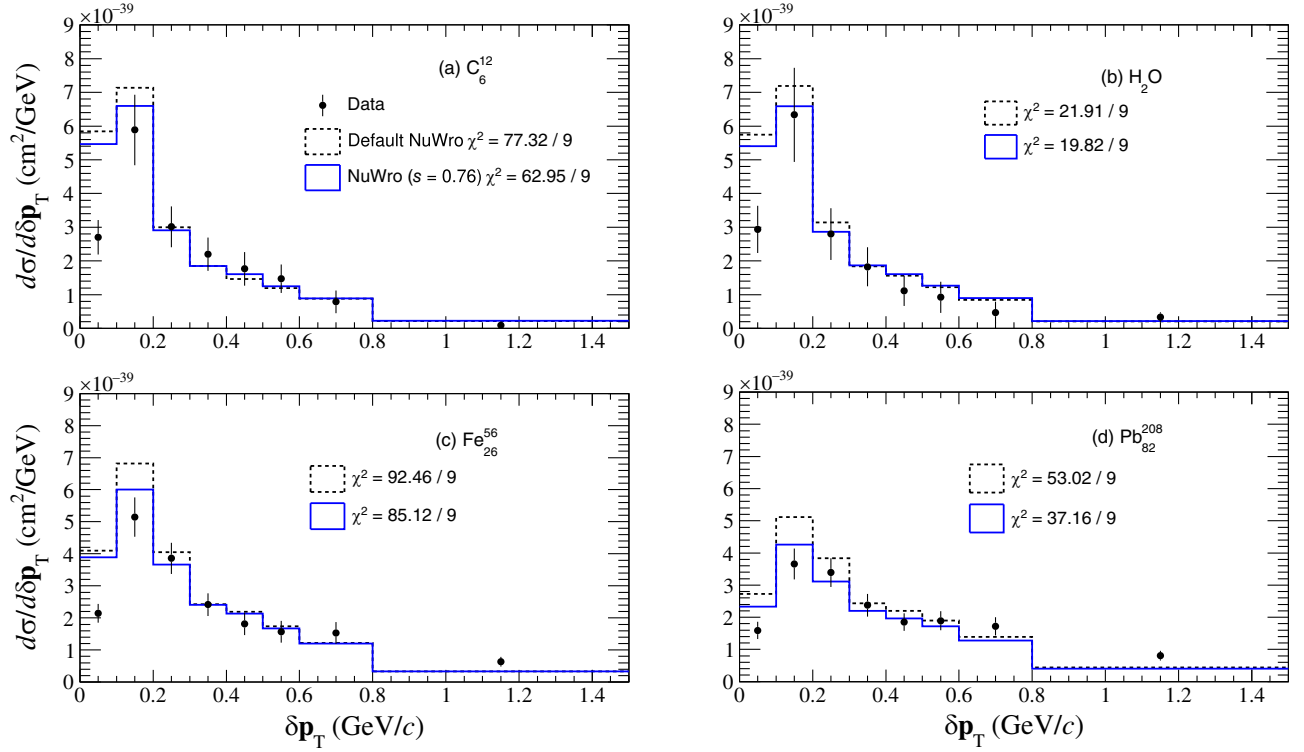
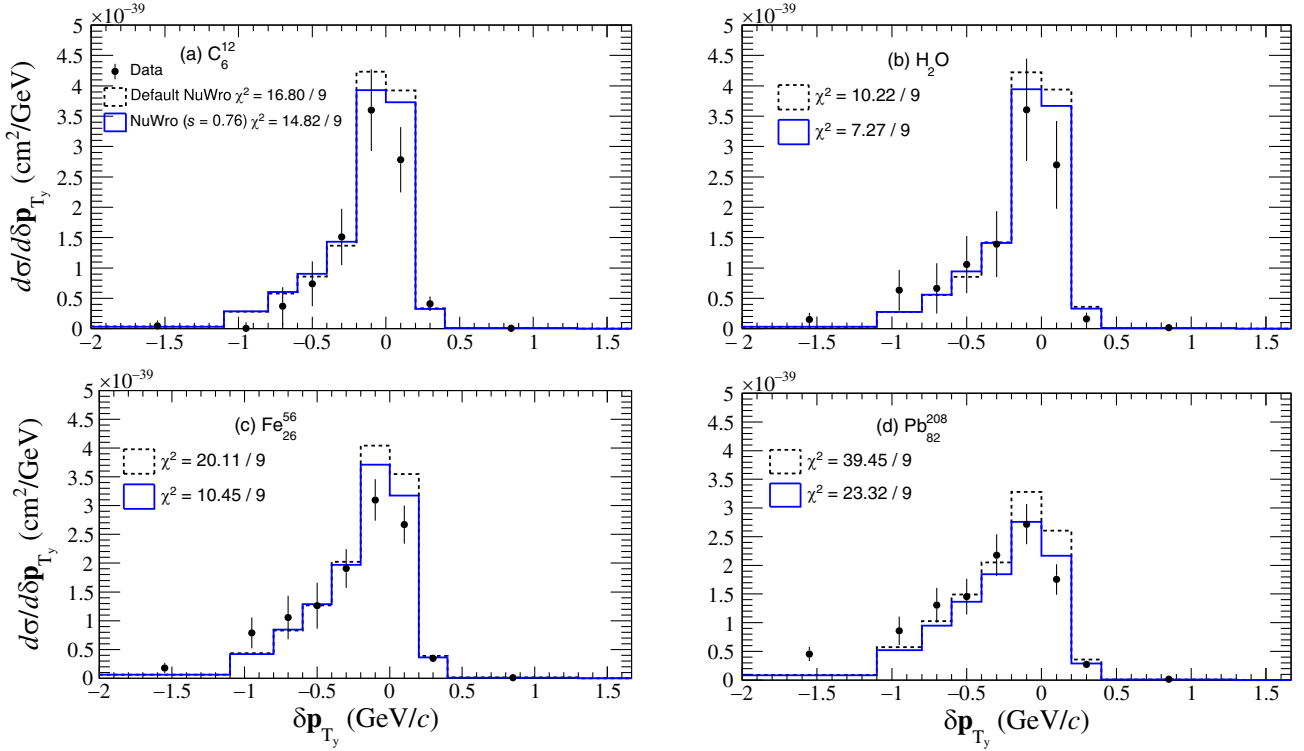
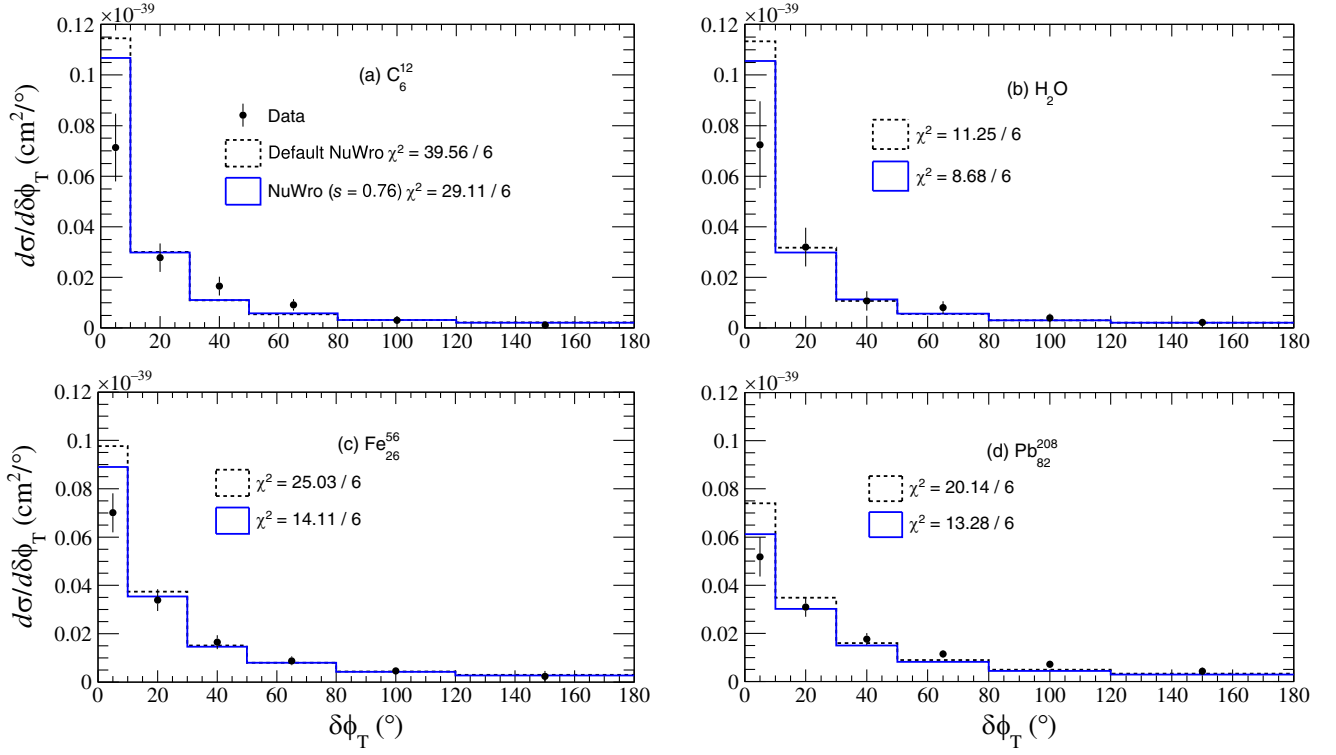
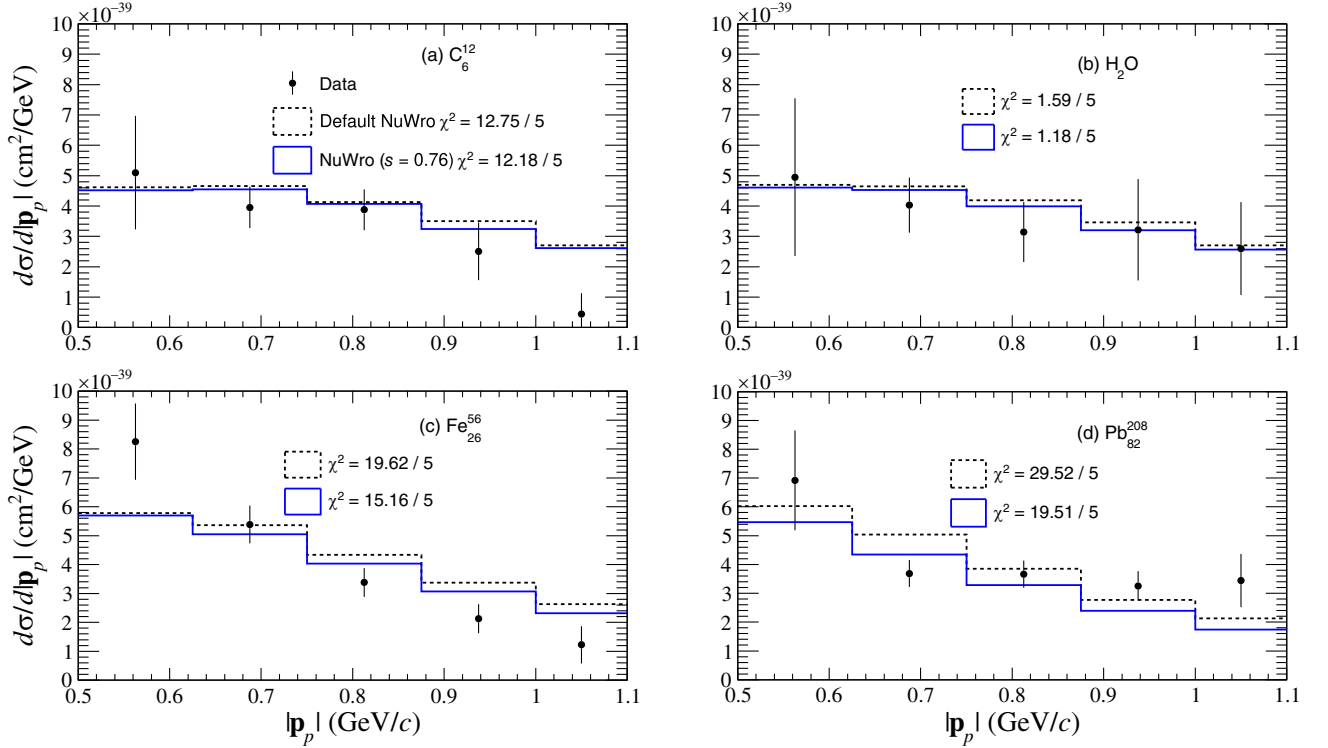
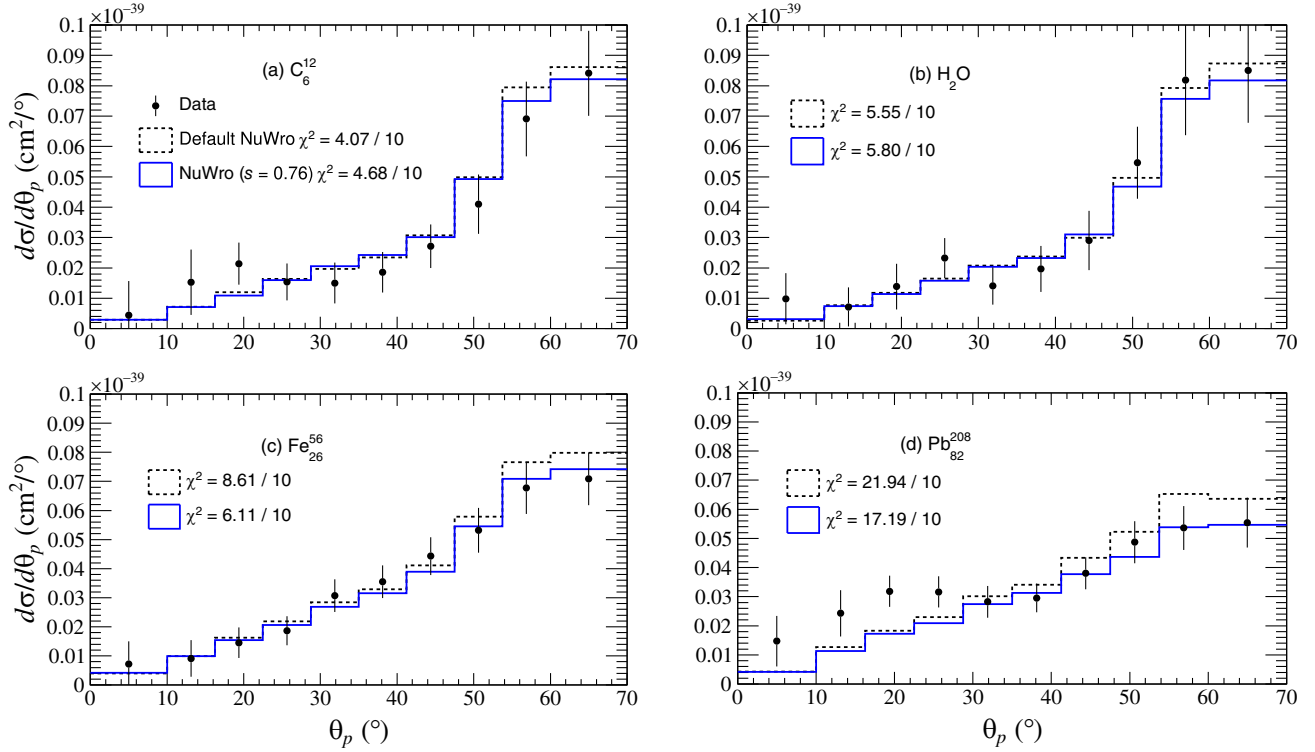


FIG. 10. Same as Fig. 3 but for $\delta\alpha_T$.

FIG. 11. Same as Fig. 3 but for $|\delta p_T|$.FIG. 12. Same as Fig. 3 but for δp_{Ty} .

FIG. 13. Same as Fig. 3 but for $\delta\phi_T$.FIG. 14. Same as Fig. 3 but for $|p_p|$.

FIG. 15. Same as Fig. 3 but for θ_p .

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- [1] L. Alvarez-Ruso, M. S. Athar, M. Barbaro *et al.*, Nustec1 Neutrino scattering theory experiment collaboration <http://nustec.fnal.gov>. white paper: Status and challenges of neutrino–nucleus scattering, *Prog. Part. Nucl. Phys.* **100**, 1 (2018).
- [2] T. Katori and M. Martini, Neutrino–nucleus cross sections for oscillation experiments, [arXiv:1611.07770](https://arxiv.org/abs/1611.07770).
- [3] B. Abi, R. Acciarri, M. Acero *et al.*, Volume I. Introduction to dune, *J. Instrum.* **15**, T08008 (2020).
- [4] B. Abi, R. Acciarri, M. A. Acero *et al.*, Deep underground neutrino experiment (DUNE), far detector technical design report, volume II: Dune physics, [arXiv:2002.03005](https://arxiv.org/abs/2002.03005).
- [5] K. Abe, N. Akhlaq, R. Akutsu *et al.*, Measurements of neutrino oscillation parameters from the T2K experiment using 3.6×10^{21} protons on target, *Eur. Phys. J. C* **83**, 782 (2023).
- [6] M. A. Acero, P. Adamson, L. Aliaga *et al.* (The NOvA Collaboration), Improved measurement of neutrino oscillation parameters by the nova experiment, *Phys. Rev. D* **106**, 032004 (2022).
- [7] P. Abratenko, D. Andrade Aldana, J. Anthony *et al.* (Micro-BooNE Collaboration), First constraints on light sterile neutrino oscillations from combined appearance and disappearance searches with the MicroBooNE detector, *Phys. Rev. Lett.* **130**, 011801 (2023).
- [8] H. Gallagher and Y. Hayato (Particle Data Group), Monte Carlo neutrino generators, *Phys. Rev. D* **110**, 030001 (2024).
- [9] C. Andreopoulos, A. Bell, D. Bhattacharya *et al.*, The genie neutrino monte carlo generator, *Nucl. Instrum. Methods Phys. Res., Sect. A* **614**, 87 (2010).
- [10] Y. Hayato and L. Pickering, The neut neutrino interaction simulation program library, *Eur. Phys. J. Spec. Top.* **230**, 4469 (2021).
- [11] O. Buss, T. Gaitanos, K. Gallmeister, H. van Hees, M. Kaskulov, O. Lalakulich, A. Larionov, T. Leitner, J. Weil, and U. Mosel, Transport-theoretical description of nuclear reactions, *Phys. Rep.* **512**, 1 (2012).
- [12] J. Sobczyk, NuWro: A Monte Carlo Neutrino Event Generator, <https://github.com/NuWro/nuwro> (2025), gitHub repository. Accessed: 2026-01-22.
- [13] T. Golan, C. Juszczak, and J. T. Sobczyk, Final state interactions effects in neutrino-nucleus interactions, *Phys. Rev. C* **86**, 015505 (2012).
- [14] N. Metropolis, R. Bivins, M. Storm, A. Turkevich, J. M. Miller, and G. Friedlander, Monte Carlo calculations on intranuclear cascades. I. Low-energy studies, *Phys. Rev.* **110**, 185 (1958).
- [15] N. Metropolis, R. Bivins, M. Storm, J. M. Miller, G. Friedlander, and A. Turkevich, Monte Carlo calculations

- on intranuclear cascades. II. High-energy studies and pion processes, *Phys. Rev.* **110**, 204 (1958).
- [16] R. Serber, Nuclear reactions at high energies, *Phys. Rev.* **72**, 1114 (1947).
- [17] J. Isaacson, W. I. Jay, A. Lovato, P. A. N. Machado, and N. Rocco, Introducing a novel event generator for electron-nucleus and neutrino-nucleus scattering, *Phys. Rev. D* **107**, 033007 (2023).
- [18] V. R. Pandharipande and S. C. Pieper, Nuclear transparency to intermediate-energy nucleons from (e,e'p) reactions, *Phys. Rev. C* **45**, 791 (1992).
- [19] G.-Q. Li and R. Machleidt, Microscopic calculation of in-medium nucleon-nucleon cross-sections, *Phys. Rev. C* **48**, 1702 (1993).
- [20] K. Niewczas and J. T. Sobczyk, Nuclear transparency in Monte Carlo neutrino event generators, *Phys. Rev. C* **100**, 015505 (2019).
- [21] W. Filali, L. Munteanu, and S. Dolan, Benchmarking neutrino interaction models via a comparative analysis of kinematic imbalance measurements from the T2K, MicroBooNE, and MINERvA experiments, *Phys. Rev. D* **111**, 032009 (2025).
- [22] X.-G. Lu, D. Coplowe, R. Shah, G. Barr, D. Wark, and A. Weber, Reconstruction of energy spectra of neutrino beams independent of nuclear effects, *Phys. Rev. D* **92**, 051302 (2015).
- [23] X.-G. Lu, L. Pickering, S. Dolan, G. Barr, D. Coplowe, Y. Uchida, D. Wark, M. O. Wascko, A. Weber, and T. Yuan, Measurement of nuclear effects in neutrino interactions with minimal dependence on neutrino energy, *Phys. Rev. C* **94**, 015503 (2016).
- [24] J. Kleykamp, S. Akhter, Z. Ahmad Dar *et al.* (MINERvA Collaboration), Measurement of the a dependence of the ν_μ charged-current quasielasticlike cross section as a function of muon and proton kinematics at $\langle E_\nu \rangle \sim 6$ GeV, *Phys. Rev. D* **112**, 052005 (2025).
- [25] C. Juszczak, J. A. Nowak, and J. T. Sobczyk, Simulations from a new neutrino event generator, *Nucl. Phys. B, Proc. Suppl.* **159**, 211 (2006).
- [26] O. Benhar, A. Fabrocini, and S. Fantoni, The nucleon spectral function in nuclear matter, *Nucl. Phys.* **A505**, 267 (1989).
- [27] O. Benhar, A. Fabrocini, S. Fantoni, and I. Sick, Spectral function of finite nuclei and scattering of GeV electrons, *Nucl. Phys.* **A579**, 493 (1994).
- [28] C. Juszczak, J. A. Nowak, and J. T. Sobczyk, Spectrum of recoil nucleons in quasi-elastic neutrino nucleus interactions, *Eur. Phys. J. C* **39**, 195 (2005).
- [29] S. Abe *et al.* (KamLAND Collaboration), First measurement of the strange axial coupling constant using neutral-current quasielastic interactions of atmospheric neutrinos at KamLAND, *Phys. Rev. D* **107**, 072006 (2023).
- [30] E. Marzec *et al.* (JSNS² Collaboration), First measurement of missing energy due to nuclear effects in monoenergetic neutrino charged-current interactions, *Phys. Rev. Lett.* **134**, 081801 (2025).
- [31] R. D. Banerjee, A. M. Ankowski, K. M. Graczyk, B. E. Kowal, H. Prasad, and J. T. Sobczyk, Jlab spectral functions of argon in NuWro and their implications for MicroBooNE, *Phys. Rev. D* **109**, 073004 (2024).
- [32] H. Prasad, J. T. Sobczyk, A. M. Ankowski, J. L. Bonilla, R. D. Banerjee, K. M. Graczyk, and B. E. Kowal, New multinucleon knockout model in the NuWro Monte Carlo generator, *Phys. Rev. D* **111**, 036032 (2025).
- [33] Q. Yan, K. Niewczas, A. Nikolakopoulos *et al.*, The Ghent hybrid model in NuWro: A new neutrino single-pion production model in the GeV regime, *J. High Energy Phys.* **12** (2024) 141.
- [34] J. E. Sobczyk, J. Nieves, and F. Sánchez, Exclusive-final-state hadron observables from neutrino-nucleus multinucleon knockout, *Phys. Rev. C* **102**, 024601 (2020).
- [35] González-Jiménez *et al.*, Electroweak single-pion production off the nucleon: from threshold to high invariant masses, *Phys. Rev. D* **95**, 113007 (2017).
- [36] Y. Yariv *et al.*, Intra-nuclear cascade models at low energy?, in *Proceedings of the International Conference on Nuclear Data for Science and Technology*, edited by O. Bersillon *et al.* (EDP Sciences, Paris, 2008), p. 1125.
- [37] H. De Vries, C. De Jager, and C. De Vries, Nuclear charge-density-distribution parameters from elastic electron scattering, *At. Data Nucl. Data Tables* **36**, 495 (1987).
- [38] M. Tanabashi, K. Hagiwara, K. Hikasa *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **98**, 030001 (2018).
- [39] J. Bystricky, P. La France, F. Lehar, F. Perrot, T. Siemiarzczuk, and P. Winternitz, Energy dependence of nucleon-nucleon inelastic total cross-sections, *J. Phys. (Les Ulis, Fr.)* **48**, 1901 (1987).
- [40] D. Klakow, G. Welke, and W. Bauer, Nuclear flow excitation function, *Phys. Rev. C* **48**, 1982 (1993).
- [41] L. Zazueta, S. Akhter, Z. Ahmad Dar *et al.* (MINERvA Collaboration), Improved constraint on the MINERvA medium energy neutrino flux using $\bar{\nu}e^- \rightarrow \bar{\nu}e^-$ data, *Phys. Rev. D* **107**, 012001 (2023).
- [42] L. Aliaga, L. Bagby, B. Baldin *et al.*, Design, calibration, and performance of the MINERvA detector, *Nucl. Instrum. Methods Phys. Res., Sect. A* **743**, 130 (2014).
- [43] A. P. Furmanski and J. T. Sobczyk, Neutrino energy reconstruction from one-muon and one-proton events, *Phys. Rev. C* **95**, 065501 (2017).
- [44] T. Nonnenmacher, Development of detectors and simulation method for measurement of hadrons from neutrino interactions, Ph.D. thesis, (Imperial College London, 2020), 10.25560/85985.
- [45] R. Brun and F. Rademakers, ROOT: An object oriented data analysis framework, *Nucl. Instrum. Methods Phys. Res., Sect. A* **389**, 81 (1997).
- [46] <https://github.com/NuWro/nuwro.git>.