

Chern-Simons inspired massive gravity

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In this work, we propose a new $2 + 1$ theory of gravity. We start with a modification of Chern-Simons $2 + 1$ gravity. By introducing a vector field Φ to the usual composition of the vierbein and connection, we derive a new action that includes the Einstein-Hilbert term, a cosmological constant, and a polynomial term for the vector field. This allows us to write a Chern-Simons-inspired massive gravity that generalizes new massive gravity. After solving the polynomial function, the action can be rewritten as a Born-Infeld-type action. Moreover, we show that the second-order expansion of this action is equivalent to new massive gravity. We explore solutions to the theory and show that it has a Bañados-Teitelboim-Zanelli solution with an effective cosmological constant in terms of the parameters of the theory, global Lifshitz space-times as well as regular Lifshitz black holes, and asymptotically locally flat black holes. Furthermore, we study the decoupling limit of the theory, showing that it contains the quartic Galileon in addition to the cubic Galileon that is characteristic of new massive gravity.

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I. INTRODUCTION

One of the most successful playgrounds to understand the theoretical intricacies of general relativity (GR) is $2 + 1$ gravity. For example, the Bañados-Teitelboim-Zanelli (BTZ) solution [1] has been fundamental in shedding light on the properties of black holes. Moreover, $2 + 1$ gravity has many remarkable features such as renormalizability, integrability, topological invariance, etc., that guided the quest for a four-dimensional quantum theory of gravity. The origin of these is connected to the fact that $2 + 1$ gravity can be written as a Chern-Simons gauge theory [2], allowing one to treat $2 + 1$ gravity in a similar way to Yang-Mills theory. Following this line of reasoning, in [3] an (anti-)self-dual formulation of $2 + 1$ gravity is proposed, which allows one to write the “standard” and “exotic” actions as linear combinations of the self-dual and anti-self-dual actions. Furthermore, $2 + 1$ gravity has been derived in the context of topological M theory [4,5] and used in understanding the entropy of exotic BTZ black holes [6,7]. Moreover, an interesting feature of the Chern-Simons

action is that its quantization is well known, and, consequently, $2 + 1$ gravity is a quantizable theory.

Interest in $2 + 1$ gravity was reignited by new massive gravity (NMG) [8]. This theory is a generalization of $2 + 1$ GR that includes a propagating massive spin-2 particle with helicity ± 2 . Also, it was argued that topological massive gravity can be considered a special case of NMG, as in the linearized limit has the equivalent field equations to NMG. This can be seen from the factorization of the spin-2 Fierz-Pauli equation in three dimensions, because it is equivalent to linearized topological massive gravity [8–10]. Black hole solutions were explored in NMG [11]; in particular, BTZ and Lifshitz solutions were derived [12].

Extensions to NMG have been proposed, for example, a Born-Infeld generalization of NMG [13] or, more recently, a nonlocal version has been derived from Einstein gravity [14]. Considering that NMG is a generalization of $2 + 1$ GR and that $2 + 1$ GR can be written as a Chern-Simons theory, one can entertain the idea that NMG can have a similar construction. In this work, we push forward this idea by applying a methodology inspired in the developments of [15] for four-dimensional gravity, where it is shown that the MacDowell-Mansouri (MM) [16] construction for GR with a cosmological constant can be modified in such a way that one obtains a theory with additional degrees of freedom, closely related to generalized Proca theories. In particular, the higher-dimensional component of the spin connection used in the (broken) gauge symmetry of MM is shifted by the introduction of a new field which is later related to the vector field of generalized

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Proca. Here, we adapt the idea to $2 + 1$ gravity by shifting the higher-dimensional component of the spin connection used in the Chern-Simons construction of $2 + 1$ GR. It is worth mentioning that, even though the MM theory leads to the same field equations as GR, the fact that it emerges from a Yang-Mills-like Lagrangian allowed us to use this theory, and its self-dual generalization [17], to understand theoretical aspects, like the Immirzi parameter [18,19] and noncommutative gravity [20,21]. See also [22–24] for further applications to unification and supergravity. This underscores the relevance of searching for a generalization of NMG inspired in gauge theories.

The theory that we find in this work has an action analogous to the triple master action in [8]. This new action has a cubic polynomial constraint from which a new $2 + 1$ massive gravity is derived. In particular, if we consider up to the quadratic terms on the constraint, the resulting action is equivalent to NMG. This leads to the main result of this paper: a Chern-Simons-inspired new massive gravity (CSIMG). This is achieved by extending the action that results from the Chern-Simons formulation, including a free coupling parameter for every order in the constraint. We study stationary solutions, obtaining a BTZ solution with an effective cosmological constant that can be set to zero for appropriate values of the parameters. Moreover, we show that CSIMG admits asymptotically locally flat BH solutions for the full theory; this is in contrast to NMG where flat solutions are present only in the quadratic theory. We also obtain new exact regular Lifshitz-like black hole solutions. Finally, we discuss the decoupling limit and show that, even in this limit, CSIMG differs from NMG. Importantly, there is a sector of the theory where the decoupling limit does not contain the terms that come from NMG, and in this sector we can find BH solutions that are not present in NMG, such as asymptotically locally flat or regular Lifshitz BHs.

II. CHERN-SIMONS-INSPIRED MASSIVE GRAVITY

NMG is a unitary theory that contains a massive spin-2 particle; it can be derived from the Lagrangian

$$S[g_{\mu\nu}, f_{\mu\nu}] = \int d^3x \sqrt{-g} \left[R + 2\bar{\lambda} + f^{\mu\nu} G_{\mu\nu} + \frac{1}{4} m^2 (f^{\mu\nu} f_{\mu\nu} - f^2) \right], \quad (1)$$

where $f_{\mu\nu}$ is an auxiliary symmetric tensor field with trace $f = g^{\mu\nu} f_{\mu\nu}$. It was shown that, after expanding around a Minkowski space-time, one gets the Fierz-Pauli theory for massive spin-2 particles. Solving for the auxiliary field $f_{\mu\nu}$ and substituting in the action, we get

$$S = \int d^3x \sqrt{-g} \left[R + 2\bar{\lambda} + \frac{1}{m^2} \left(R_{\mu\nu} R^{\mu\nu} - \frac{3}{8} R^2 \right) \right], \quad (2)$$

which is the originally proposed version of NMG [8].

Since $2 + 1$ gravity can be written as a gauge theory with a Chern-Simons action [2], as an alternative to NMG, we can try to introduce degrees of freedom in the $2 + 1$ Chern-Simons construction and relate these additional degrees of freedom with the massive gravity theories (e.g., [8]). Let us start with the Chern-Simons action [3]

$$I_{CS} = \int \epsilon^{\mu\nu\rho} \left(A_\mu^{AB} \partial_\nu A_{\rho AB} + \frac{2}{3} A_\mu^B A_\nu^C A_\rho^A A_C^A \right), \quad (3)$$

where $A, B, C = 0, 1, 2, 3$ and $\eta_{AB} = \text{diag}(-1, 1, 1, 1)$. After the identifications $(A_\mu^{3a}, A_\mu^{ab}) = (\sqrt{\lambda} e_\mu^a, \omega_\mu^{ab})$, one gets the standard action for $2 + 1$ Chern-Simons gravity. It was shown in [2] that this action is equivalent to the $2 + 1$ Einstein-Hilbert action with cosmological constant. For $\lambda > 0$ (de Sitter) one has the symmetry group $SO(3,1)$, and for $\lambda < 0$ (anti-de Sitter) one has $SO(2,2)$. Following the approach in [15], we can introduce the new degrees of freedom to the $2 + 1$ Chern-Simons gravity action. We start by imposing a shift only on the A^{3a} component, $A^{3a} \rightarrow e^a + \Phi^a$, while maintaining the same connection; consequently, the curvature is the same as in Chern-Simons $2 + 1$ gravity.¹ Therefore, the connections are upgraded to $A_\mu^{AB} = (\sqrt{\lambda}(e_\mu^a + \Phi_\mu^a), \epsilon^{abc} \omega_\mu^c)$. When we apply this connection into the action Eq. (3), we obtain a brand new modified action:

$$I_\Phi = I_{CS} + \int \epsilon^{\mu\nu\alpha} [\Phi_{\mu a} (\partial_\nu \omega_\alpha^a - \partial_\alpha \omega_\nu^a + \epsilon^a_{bc} \omega_\nu^b \omega_\alpha^c) + \frac{\lambda}{3} \epsilon_{abc} (3e_\mu^a e_\nu^b \Phi_\alpha^c + 3e_\mu^a \Phi_\nu^b \Phi_\alpha^c + \Phi_\mu^a \Phi_\nu^b \Phi_\alpha^c)]. \quad (4)$$

To write the action in terms of the three-dimensional space-time Riemann curvature tensor, we consider a three-dimensional space-time M with metric $g_{\mu\nu}$ and an orthonormal frame field e . In this frame, the standard action takes the form

$$I_\Phi = \int d^3x \sqrt{-g} \left[2\lambda + R - 2G^{\alpha\mu} \Phi_{\alpha\mu} + \lambda \left(2\Phi - \epsilon^{\mu\nu\alpha} \epsilon_{\gamma\delta\alpha} \Phi_\mu^\gamma \Phi_\nu^\delta - \frac{1}{3} \epsilon^{\mu\nu\alpha} \epsilon_{\gamma\delta\beta} \Phi_\mu^\gamma \Phi_\nu^\delta \Phi_\alpha^\beta \right) \right], \quad (5)$$

where $\Phi_{\mu\nu}$ is an auxiliary symmetric tensor field with trace $\Phi = g^{\mu\nu} \Phi_{\mu\nu}$. Because this action is the analog of Eq. (1),

¹We assume that the triad is torsionless, and, consequently, Φ^a cannot be removed by a redefinition.

one can consider the Chern-Simons result as the starting point to propose a more general action. In some sense, this is similar to the case of four-dimensional massive gravity, where, for particular values of the coupling constants, one can have theories with different behaviors [25]. Therefore, let us propose a Chern-Simons-inspired massive gravity, introducing independent couplings to each of the terms that depend only on Φ :

$$S_\Phi = \int d^3x \sqrt{-g} \left[2\lambda + R - 2G^{\alpha\mu} \Phi_{\alpha\mu} + 2\zeta \Phi - \sigma \epsilon^{\mu\nu\alpha} \epsilon_{\gamma\delta\alpha} \Phi_\mu^\gamma \Phi_\nu^\delta - \frac{1}{3} \xi \epsilon^{\mu\nu\alpha} \epsilon_{\gamma\delta\beta} \Phi_\mu^\gamma \Phi_\nu^\delta \Phi_\alpha^\beta \right], \quad (6)$$

which we will refer to as CSIMG. We can establish the connection with NMG by considering up to quadratic terms in Φ ,

$$I_\Phi^{(2)} = \int d^3x \sqrt{-g} \left[2\lambda + R - 2G^{\alpha\mu} \Phi_{\alpha\mu} + \sigma \left(2\frac{\zeta}{\sigma} \Phi + \Phi^2 - \Phi_{\alpha\mu} \Phi^{\alpha\mu} \right) \right], \quad (7)$$

and solving for Φ we can rewrite the action in terms of the curvature. After an appropriate reparametrization of the constants, the action takes the form of Eq. (2), the action of NMG, with the effective cosmological constant Λ written in terms of λ and ξ .

Now we consider the full CSIMG action with up to cubic terms. Its variation with respect to Φ gives

$$G_{\mu\nu} = \zeta g_{\mu\nu} + \sigma (g_{\mu\nu} \Phi^\alpha_\alpha - \Phi_{\mu\nu}) - \xi \left(\frac{1}{2} g_{\mu\nu} \Phi_{\alpha\beta} \Phi^{\alpha\beta} - \frac{1}{2} g_{\mu\nu} \Phi^\alpha_\alpha \Phi^\beta_\beta + \Phi^\alpha_\alpha \Phi_{\mu\nu} - \Phi_\mu^\alpha \Phi_{\nu\alpha} \right). \quad (8)$$

In order to get rid of the linear terms of Φ , we make $\Phi_{\mu\nu} = -\sigma g_{\mu\nu}/\xi + \Omega_{\mu\nu}/\xi$, and after the substitution in (8) one gets

$$G_{\mu\nu} + \left(\frac{\sigma^2}{\xi} - \zeta \right) g_{\mu\nu} = -\frac{1}{2\xi} \epsilon_{\mu\alpha\beta} \epsilon_{\nu\delta\gamma} \Omega^{\alpha\delta} \Omega^{\beta\gamma}. \quad (9)$$

Note that the right-hand side in the last line is the adjoint of the tensor Ω . Solving the previous equation, one gets

$$\Omega_{\mu\nu} = \pm \sqrt{\det(\xi G^\mu_\nu + (\sigma^2 - \zeta\xi) g^\mu_\nu)} \times [\xi G_{\mu\nu} + (\sigma^2 - \zeta\xi) g_{\mu\nu}]^{-1}. \quad (10)$$

The solution for Φ is found by plugging this into $\Phi_{\mu\nu} = -\sigma g_{\mu\nu}/\xi + \Omega_{\mu\nu}/\xi$. After substituting in the original

action, we obtain a Born-Infeld-type action [26]:

$$I = \int d^3x \sqrt{-g} \left[\left(1 - \frac{\sigma}{\xi} \right) R + 2\lambda + \frac{4\sigma^3}{\xi^2} - \frac{6\zeta\sigma}{\xi} \pm 4\xi^{-2} \sqrt{(\sigma^2 - \zeta\xi)^3} \sqrt{\det \left(g^\mu_\nu + \frac{\xi G^\mu_\nu}{\sigma^2 - \zeta\xi} \right)} \right]. \quad (11)$$

If we consider the expansion of the determinant to second order and after redefining the constants, we recover the NMG action in Eq. (1).

The correspondence to the Chern-Simons formulation is found by setting $\sigma = \zeta = \xi = \lambda$, following the same steps as before; the resulting action is

$$I = \frac{4}{\sqrt{\lambda}} \int d^3x \sqrt{-g} [\sqrt{\det(G^\mu_\nu)}]. \quad (12)$$

III. STATIC SOLUTIONS

To find solutions, we can follow two approaches: solve the equations of motion in the determinant formulation Eq. (11), or we can solve for the action in Eq. (6), where we solve for Φ and the metric. As Lifshitz solutions are not too common and are one of the interesting features of NMG, we will focus our interest not only on BTZ black holes, but more importantly on Lifshitz black hole solutions.

For 2 + 1 gravity, the first solution was found in [1]; this kind of solution is known as a BTZ black hole. To explore BTZ-like solutions, we propose the line element

$$ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{N(r)^2} + r^2 (d\phi + N_\phi dt)^2, \quad (13)$$

where the angular shift is $N_\phi = -J/2r^2$. Substituting the metric into the equation of motion for the action Eq. (11), we find two solutions for the function $N(r)$:

$$N_\pm^2 = -M + \Lambda_\pm r^2 + \frac{J^2}{4r^2}, \quad (14)$$

with

$$\Lambda_\pm = \frac{2\zeta^2\xi - \zeta\sigma(3\xi + \sigma) + \lambda\xi^2 - \lambda\xi\sigma + 2\sigma^3}{(\sigma - \xi)^2} \pm \frac{2\sqrt{(\sigma^2 - \zeta\xi)^2(\zeta^2 - \zeta(\xi + \sigma) + \lambda\xi - \lambda\sigma + \sigma^2)}}{(\sigma - \xi)^2}. \quad (15)$$

For the BTZ solution, $\Lambda_\pm$ is an effective cosmological constant given in terms of the parameters ζ , σ , and ξ . Also, notice that the extremal condition is the same as for the usual BTZ black hole, with the only difference that it is given in terms of $\Lambda_\pm$.

IV. ASYMPTOTICALLY LOCALLY FLAT SOLUTIONS

The Chern-Simons case has the static (nonrotating, $J = 0$) asymptotically locally flat black hole solution²

$$ds^2 = -(\mu + br)dt^2 + \frac{dr^2}{\mu + br} + r^2 d\phi^2. \quad (16)$$

For the Chern-Simons case, the Ricci scalar is not present in the action; this is the same situation as in NMG, where Eq. (16) is a solution to only the quadratic term. In the full CSIMG, for the nonrotating black hole, the special case $0 = \zeta^2 - \zeta(\xi + \sigma) + \lambda(\xi - \sigma) + \sigma^2$ implies $\Lambda_+ = \Lambda_- = \Lambda$ and allows the existence of a solution given by

$$ds^2 = -(\mu + br + \Lambda r^2)dt^2 + \frac{dr^2}{\mu + br + \Lambda r^2} + r^2 d\phi^2, \quad (17)$$

where

$$\Lambda = \frac{(2\zeta - \lambda - \sigma)(\zeta^2 - \lambda\sigma)}{(\zeta - \sigma)^2}. \quad (18)$$

This solution is not valid for arbitrary values of the free parameters. After substituting Eq. (17) in the field equations, we get the condition

$$B(r) \pm \sqrt{B(r)^2} = 0, \quad (19)$$

where

$$B(r) = \frac{(\zeta^2 - \lambda\sigma)}{(\zeta - \lambda)(\zeta - \sigma)^3} [(\zeta - \sigma)^2(\zeta^2 - \zeta\sigma + \sigma(\sigma - \lambda)) + b + 2r(\zeta - \lambda)(\zeta^2 - \lambda\sigma)^2]. \quad (20)$$

Hence, in general, the solution exists for $B(r) < 0$ for the positive branch or $B(r) > 0$ for the negative one. In particular, if $b > 0$, the solution exists if $0 < \zeta < \sigma$ and $0 < \lambda \leq \frac{\zeta^2}{\sigma}$ for the positive³ branch of $\Phi_{\mu\nu}$.

The Ricci curvature scalar is

$$R = -6\Lambda - \frac{2b}{r}, \quad (21)$$

meaning that the scalar curvature is asymptotically constant and has a singularity at the origin (the same applies for $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$, the other curvature scalars). We get an asymptotically locally flat solution provided that $2\zeta = \lambda + \sigma$. This condition does not remove the Ricci term

²The interpretation of Eq. (16) as an asymptotically locally flat black hole is justified in [27]; see also [28].

³In the case of the negative branch for the solution of $\Phi_{\mu\nu}$, the parameters satisfy $0 < \zeta < \sigma$ and $\zeta^2/\sigma \leq \lambda < \zeta$.

from the action, and, therefore, unlike NMG, there are asymptotically locally flat solutions for the complete theory. These solutions have only one horizon, located at $r = -\mu/b$.⁴ The present analysis started by requiring $\Lambda_+ = \Lambda_- = 0$; we do not discard that other asymptotically locally flat solutions may be found by independently requiring either $\Lambda_+ = 0$ or $\Lambda_- = 0$.

V. REGULAR LIFSHITZ BLACK HOLES

For new massive gravity, given by Eq. (2), Lifshitz black hole solutions have been studied in [29]. These solutions emerge by proposing the line element

$$ds^2 = -\left(\frac{r}{l}\right)^{2z} (1 - H(r))dt^2 + \frac{l^2}{r^2} \frac{dr^2}{(1 - H(r))} + \frac{r^2}{l^2} d\phi^2, \quad (22)$$

where z is a dynamic exponent and l is a scale parameter. A broad analysis of the spectra of z values is shown in [12], and the only valid solutions meeting the boundary conditions occur for $z = 1$ and $z = 3$.

For the action Eq. (6), the situation is different. Studying first the case of a global Lifshitz metric [$H(r) = 0$], one notices that the equations of motion are considerably simpler if one fixes the constants $\zeta = \xi$ and $\sigma = -\xi$. Then, one finds that the Lifshitz solution exists for any z under the conditions⁵

$$\begin{aligned} \lambda + \xi - \frac{z^{3/2}}{l^3 \sqrt{\xi}} &= 0, \\ (1 - z + z^2)\xi - 2lz^{1/2}\xi^{3/2} &= 0. \end{aligned} \quad (23)$$

Solutions without fixing ζ and σ do exist as well; however, the resulting expressions for λ and ξ are too cumbersome. These solutions will be presented and analyzed in detail elsewhere. Here, we keep $\zeta = \xi$ and $\sigma = -\xi$ and proceed to study Lifshitz-like black hole solutions in the special case $z = 3$, which is of interest for the purpose of comparing with the NMG $z = 3$ Lifshitz black hole. Unlike NMG, for the action Eq. (6) an asymptotic ($r \rightarrow \infty$) analysis shows that we need to relax the metric ansatz Eq. (22) to the more general form

$$ds^2 = -\left(\frac{r}{l}\right)^{2z} (1 - F(r))dt^2 + \frac{l^2}{r^2} \frac{dr^2}{(1 - F(r))} + \frac{r^2}{l^2} d\phi^2, \quad (24)$$

⁴In the case of Eq. (17), we have up to two horizons, given by the roots of $\mu + br + \Lambda r^2 = 0$. Assuming, as above, $b > 0$, the two horizons exist if $\Lambda < 0$, $\mu < 0$, and $b^2 > 4\Lambda\mu$, and the extremal case (when the two horizons coincide) corresponds to $b^2 = 4\Lambda\mu$.

⁵Taking the negative sign in Eq. (10); otherwise, one would need $l < 0$.

where both H and F are required to vanish as $r \rightarrow \infty$. Then, an exact solution is found as

$$1 - F(r) = \left(1 - \frac{l^2 \mu}{r^2}\right) \left(1 + \frac{2\mu l^2}{r^2}\right)^2, \\ 1 - H(r) = 1 - \frac{l^2 \mu}{r^2}. \quad (25)$$

The function $H(r)$ —and, therefore, the black hole horizon—coincides with the solution for the $z = 3$ Lifshitz black hole in NMG. Assuming $\mu > 0$, the additional factor in $F(r)$ does not lead to new roots of g_{tt} , however; it has important consequences for the Ricci curvature, which turns out to be everywhere regular (assuming again $\mu > 0$):

$$R = -\frac{26}{l^2} + \frac{48\mu}{2\mu l^2 + r^2}. \quad (26)$$

As $r \rightarrow 0$ we have $R \rightarrow -26/l^2$, while for $r \rightarrow \infty$ we get $R \rightarrow -26/l^2$. Similarly, the curvature scalars $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ are everywhere regular.

Another case of interest for comparison with NMG is $z = 1$. In this case, the solution for the metric becomes the static ($J = 0$) BTZ black hole but with different constraints on the parameters of the model than those arising for the same solution NMG. This traces back to the assumptions $\zeta = \xi$, $\sigma = -\xi$, which makes the theory necessarily different from NMG, where we would need $\xi = 0$ but $\sigma \neq 0$. In our case, the constraint is $\xi = -\lambda + 2/l^2$, while in NMG one needs $l^2\lambda = 1 - 1/(4l^2\sigma)$. Notice that $\xi = 0$ implies $\lambda = 2/l^2$, which is the condition for obtaining the BTZ solution in $2 + 1$ Einstein gravity. Finally and as expected, this solution is the special case $J = 0$, $\zeta = \xi$, and $\sigma = -\xi$ in Eqs. (14) and (15).

VI. DECOUPLING LIMIT

One of the remarkable features of NMG is the description by a cubic Galileon of linearized interactions in the decoupling limit. In order to have a better understanding of the theory and unambiguously establish the inequivalence with NMG, we explore this limit in CSIMG. First, we introduce the Planck mass as a global factor in Eq. (6), as well as the graviton mass m via the redefinition⁶ $\lambda = \bar{\lambda}m^2$, $\zeta = \bar{\zeta}m^2$, $\sigma = \bar{\sigma}m^2$, and $\xi = \bar{\xi}m^2$. Following [30], we take $M_P \rightarrow \infty$ and $m \rightarrow 0$ and we fix $\Lambda_{5/2} \equiv (\sqrt{M_P}m^2)^{2/5}$ to get the linearized decoupling limit. Furthermore, we use

$$h_{\mu\nu} = \frac{\bar{h}_{\mu\nu}}{\sqrt{M_P}}, \\ \Phi_{\mu\nu} = s_0 g_{\mu\nu} + \frac{\bar{f}_{\mu\nu}}{\sqrt{M_P}} + \nabla_\mu V_\nu + \nabla_\nu V_\mu, \quad (27)$$

⁶The quantities $\bar{\lambda}$, $\bar{\sigma}$, and $\bar{\xi}$ are dimensionless constants.

where the constant s_0 satisfies $s_0\xi = -\sigma + \sqrt{\sigma^2 - \zeta\xi}$ and is introduced in order to simplify the analysis of the decoupling limit by removing the explicit contribution from the term $2\zeta\Phi$ in the CSIMG action. We can rewrite the Lagrangian in terms of π and A_μ (the helicity 0 and helicity 1 modes) by decomposing V_μ (see the Appendix). Then, the field equation obtained from the variation of the Lagrangian with respect to the auxiliary field $\bar{f}^{\mu\nu}$ has the solution

$$\bar{h}_{\mu\nu} = 4\bar{s}_1 g_{\mu\nu} \pi - 4 \frac{\bar{\xi} \partial_\mu \pi \partial_\nu \pi}{\Lambda_{5/2}^{5/2}} + \text{gauge transformation}. \quad (28)$$

After substitution of this solution into the Lagrangian and integration by parts, we get

$$\mathcal{L} = \left(2(1 + \bar{s}_0)\pi\Box\pi + \frac{\Box\pi\partial_\alpha\pi\partial^\alpha\pi}{\Lambda_{5/2}^{5/2}}\right)\bar{s}_1^{-2} + \frac{1}{4}\bar{s}_1 F_{\alpha\gamma}F^{\alpha\gamma} \\ + \frac{\bar{\xi}}{\Lambda_{5/2}^{5/2}}\mathcal{L}_1 - \frac{(1 + \bar{s}_0)\bar{\xi}^2}{\Lambda_{5/2}^5}\mathcal{L}_2 \\ + \bar{s}_1\bar{\xi}\left(-\frac{2(1 + \bar{s}_0)}{\Lambda_{5/2}^{5/2}}\mathcal{L}_3 + \frac{6}{\Lambda_{5/2}^5}\Box\pi\partial_\alpha\pi\partial^\alpha\pi\right), \quad (29)$$

where $s_1 = s_0\xi + \sigma$ and $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ are defined in the Appendix. The term $\mathcal{L}_2$ is precisely the quartic Galileon (see, e.g., [31]). There are two special cases worth mentioning: (i) If we assume $\xi \gg \sigma$, $\xi \gg \zeta$, the dominant contribution coming from the cubic CSIMG terms is the quartic Galileon, and (ii) for $\bar{\zeta} = \bar{\sigma}^2/\bar{\xi}$, the Lagrangian reduces to

$$\mathcal{L} = \frac{\bar{\xi}}{\Lambda_{5/2}^{5/2}}\mathcal{L}_1 - \frac{\left(1 - \frac{\bar{\sigma}}{\bar{\xi}}\right)\bar{\xi}^2}{\Lambda_{5/2}^5}\mathcal{L}_2. \quad (30)$$

Thus, in this case the decoupling limit is clearly different from that of NMG. If $\bar{\xi}$ is dominant, this decoupling limit is given only by the quartic Galileon. Interestingly, the choice $\bar{\zeta} = \bar{\sigma}^2/\bar{\xi}$ is compatible with the CS formulation given, equivalently, in Eqs. (5) and (12), which admits asymptotically locally flat black holes. It is also compatible with the conditions leading to regular black holes Eq. (25). These types of solutions do not exist in NMG; it is remarkable that they do exist in a sector of CSIMG that differs from NMG even in the decoupling limit.

VII. DISCUSSION AND FINAL REMARKS

Starting from the Chern-Simons action, with an appropriate identification of the gauge fields (a vector is introduced together with the tetrad) and the introduction of free parameters, a generalization of NMG is proposed. A similar idea was presented in [32], where they introduce an antisymmetric contorsion to the spin connection. In our

case, the formulation is based on the algebraic structure of the Lagrangian of the original theory in the version involving an auxiliary field. This construction ensures that the scalar and vector sectors of the theory are nondynamical; indeed, the scalar sector of the new terms in the Lagrangian reduces to a total derivative. The equations of motion for the full Lagrangian involve at most second-order derivatives of both the metric and the auxiliary field, and the dynamics of the auxiliary field come purely from the coupling to the Einstein tensor already present in NMG. No new degrees of freedom are added with respect to NMG; the resulting theory can be considered as an extension of Fierz-Pauli for a massive spin-2 field in $2 + 1$ dimensions. By integrating out the auxiliary field, we obtain a Born-Infeld-type action for the cubic theory, and if we expand the determinant to second order, we get NMG.

One of the interesting features of NMG is the existence of the asymptotically anti-de Sitter black hole as well as the Lifshitz black hole solution. Asymptotically locally flat solutions in $2 + 1$ gravity are not common; these are not present in $2 + 1$ GR and for NMG are present only for the pure quadratic theory [27]. Moreover, in nonlocal massive gravity [14], locally flat solutions are only conjectured. The theory presented in this work, as in NMG, admits a BTZ solution with a cosmological constant. This cosmological constant is written in terms of the parameters of the theory and can be made to vanish; thus, unlike NMG, in CSIMG we found asymptotically locally flat solutions for the full theory and not only for the pure quadratic theory. Also, this theory admits Lifshitz solutions. The simplest solution is for global Lifshitz space-times, which exist for any z that satisfies Eq. (23). Regarding Lifshitz black holes, we find a novel solution in the particular case $z = 3$ and with a specific choice of the coupling constants that represents an asymptotically anti-de Sitter black hole. The horizon of this BH is the same as for the Lifshitz BH solution in NMG with $z = 3$, but, unlike the NMG case, for this solution $g_{rr} \neq g_{tt}^{-1}$; the extra term in g_{tt} introduces modifications such that the curvature scalars are regular at $r = 0$. Therefore, CSIMG has a regular Lifshitz black hole solution for $z = 3$. Solutions with other choices of the coupling constants will be presented in detail elsewhere. The question of solutions with other values of z remains open.

The startling differences between the solutions of CSIMG and NMG motivate the search of other solutions that have such an interesting behavior. We identified that the choices of parameters that allow for asymptotically locally flat and regular black holes are such that the decoupling limit of CSIMG is distinct from that of NMG. Furthermore, we can ask ourselves if there are gravitational solitons or wormholes. These ideas are being explored and will be reported elsewhere. It would also be interesting to explore the implications for the AdS/CFT duality. For instance, NMG has been rederived from a holographic c theorem and a Lagrangian involving up to

four derivative curvature invariants, and it has been shown that a sixth-order Lagrangian is also compatible with such a theorem [33]. Given that CSIMG admits a closed form in terms of curvatures [Eq. (11)], one could analyze the existence of a c theorem for this theory; this would also call for the inclusion of a matter term.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX

After taking the linearized decoupling limit, removing the $2\zeta\Phi$ contribution, and decomposing V_μ as

$$V_\mu = \frac{A_\mu}{\sqrt{M_P m}} + \frac{\nabla_\mu \pi}{\sqrt{M_P m^2}}, \quad (\text{A1})$$

the Lagrangian takes the form

$$\begin{aligned} \mathcal{L} = & 2\bar{f}^{\alpha\gamma} G^{(1)}_{\alpha\gamma} + \frac{1}{2}(1 + \bar{s}_0)\bar{h}^{\alpha\gamma} G^{(1)}_{\alpha\gamma} \\ & - 4\bar{s}_1 \left(-\frac{1}{4} F_{\alpha\gamma} F^{\alpha\gamma} - \bar{f}^{\alpha\gamma} (\partial_\gamma \partial_\alpha \pi - \eta_{\alpha\gamma} \square \pi) \right. \\ & \left. + \frac{1}{\Lambda_{5/2}^{5/2}} R^{(1)}_{\alpha\gamma} \nabla^\alpha \pi \nabla^\gamma \pi \right) + \frac{4\bar{\xi}}{\Lambda_{5/2}^{5/2}} \Xi_{-2}, \end{aligned} \quad (\text{A2})$$

where $F_{\mu\nu}$ is the curvature of A_μ and

$$\begin{aligned} \Xi_{-2} = & \frac{1}{2} F_{\gamma\nu} F^{\gamma\nu} \square \pi - \square \pi^2 \bar{f} + 2\bar{f}_{\gamma\nu} \square \pi \nabla^\nu \nabla^\gamma \pi \\ & + \bar{f} \nabla_\gamma \nabla_\alpha \pi \nabla^\gamma \nabla^\alpha \pi - 2\bar{f}_{\alpha\nu} \nabla_\gamma \nabla^\nu \pi \nabla^\gamma \nabla^\alpha \pi \\ & + 2\square \pi \nabla_\alpha A_\gamma \nabla^\gamma A^\alpha - \nabla_\gamma A^\nu \nabla^\gamma A^\alpha \nabla_\nu \nabla_\alpha \pi \\ & - \nabla^\gamma A^\alpha \nabla_\nu \nabla_\gamma \pi \nabla^\nu A_\alpha - 2\nabla_\alpha \nabla_\nu \pi \nabla^\gamma A^\alpha \nabla^\nu A_\gamma \\ & + 4\nabla_\alpha A^\alpha \nabla_\gamma \nabla_\nu \pi \nabla^\nu A^\gamma - 2\square \pi \nabla_\alpha A^\alpha \nabla_\gamma A^\gamma. \end{aligned} \quad (\text{A3})$$

From the variation with respect to the auxiliary field $\bar{f}^{\mu\nu}$, we find

$$\begin{aligned} G^{(1)}_{\mu\nu} = & -2\bar{s}_1 (\partial_\mu \partial_\nu \pi - \eta_{\mu\nu} \square \pi) \\ & + \frac{4\bar{\xi}}{\Lambda_{5/2}^{5/2}} \left(\frac{1}{2} g_{\mu\nu} \partial_\beta \partial_\alpha \pi \partial^\beta \partial^\alpha \pi - \frac{1}{2} g_{\mu\nu} (\square \pi)^2 \right. \\ & \left. + \square \pi \partial_\mu \partial_\nu \pi - \partial_\mu \partial^\alpha \pi \partial_\nu \partial_\alpha \pi \right). \end{aligned} \quad (\text{A4})$$

After substitution Eq. (28) into the Lagrangian and integrating by parts, we get Eq. (29), where

$$\begin{aligned}
 \mathcal{L}_1 &= -2\Box\pi\partial_\alpha A^\alpha\partial_\gamma A^\gamma + \Box\pi\partial_\alpha A_\gamma\partial^\gamma A^\alpha + \Box\pi\partial_\gamma A_\alpha\partial^\alpha A^\alpha - \partial_\gamma A^\nu\partial^\gamma A^\alpha\partial_\nu\partial_\alpha\pi \\
 &\quad - \partial^\gamma A^\alpha\partial_\nu\partial_\gamma\pi\partial^\nu A_\alpha - 2\partial^\gamma A^\alpha\partial_\nu\partial_\alpha\pi\partial^\nu A_\gamma + 4\partial_\alpha A^\alpha\partial_\gamma\partial_\nu\pi\partial^\nu A^\gamma, \\
 \mathcal{L}_2 &= (\Box\pi)^2\partial_\alpha\pi\partial^\alpha\pi - 2\Box\pi\partial^\alpha\pi\partial_\gamma\partial_\alpha\pi\partial^\gamma\pi + 2\partial_\alpha\partial^\nu\pi\partial^\alpha\pi\partial_\gamma\partial_\nu\pi\partial^\gamma\pi - \partial_\alpha\pi\partial^\alpha\pi\partial_\nu\partial_\gamma\pi\partial^\nu\partial^\gamma\pi, \\
 \mathcal{L}_3 &= \Box\pi\partial^\alpha\pi\partial_\gamma\partial_\alpha\pi\partial^\gamma\pi - \partial_\alpha\partial^\nu\pi\partial^\alpha\pi\partial_\gamma\partial_\nu\pi\partial^\gamma\pi.
 \end{aligned} \tag{A5}$$

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