

RETRACTED: Schwinger-Keldysh formulation of electromagnetic leptogenesis in an EFT framework

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Can the electromagnetic dipole interactions probed in precision experiments be the same interactions that generated the baryon asymmetry of the Universe? We address this question by formulating low-scale electromagnetic leptogenesis (EMLG) as a gauge-invariant effective field theory (EFT), rather than as a broken-phase dipole ansatz. The point is not only to connect leptogenesis to laboratory dipole observables, but first to test whether the dipole-dominated dynamics itself can satisfy the cosmological viability requirement. Starting from a softly broken Z_2 UV completion, we match onto the ν SMEFT dipole operators $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$, run their Wilson coefficients to the electroweak scale, and construct the broken-phase dipole couplings to γ , Z , and $W^\pm$. The soft breaking keeps the ordinary Yukawa-mediated sector subleading, thereby isolating the electromagnetic mechanism. We find that the nonresonant hierarchical regime is structurally inefficient, while in the quasidegenerate regime the self-energy contribution is resonantly enhanced and dipole-dominated EMLG can reproduce the observed baryon asymmetry for viable TeV-scale parameters. The same EFT pipeline maps the successful EMLG parameter space onto low-energy dipole observables, including charged-lepton flavor violation, electric dipole moments, and anomalous magnetic moments. Precision dipole searches can therefore constrain, discriminate, and potentially falsify an electromagnetic origin of baryogenesis.

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I. INTRODUCTION

The observed baryon asymmetry of the Universe (BAU) points to dynamics that satisfy the three Sakharov conditions [1]: baryon-number violation, C and CP violation, and a departure from thermal equilibrium. In the Standard Model (SM), electroweak anomalous transitions, or sphaleron processes [2,3], violate $B + L$ while conserving $B - L$, where B and L denote baryon and lepton number, respectively. They can therefore convert a preexisting lepton asymmetry into a baryon asymmetry. Fukugita and Yanagida showed that such a lepton asymmetry can be generated by CP -violating out-of-equilibrium decays of heavy right-handed neutrinos [4], establishing the framework of leptogenesis. This framework is naturally compatible with neutrino-mass mechanisms such as the type-I [5–8], type-II [9–13], and type-III [14–16] seesaws; see Refs. [17,18] for reviews.

In this paper we study electromagnetic leptogenesis (EMLG), a variant of leptogenesis in which the CP -violating source is generated by gauge-invariant dipole operators rather than by the renormalizable neutrino Yukawa interaction. After electroweak symmetry breaking, these operators induce effective couplings of the right-handed neutrinos to γ , Z , and $W^\pm$, so that the relevant decay channels contain gauge bosons in the final state and differ qualitatively from the standard Yukawa-mediated modes. The original proposal of Bell *et al.* [19] treated the corresponding dipole couplings as free parameters at the decay scale. Our aim is different: we seek a formulation that is consistent with effective field theory (EFT) and gauge invariance, in which the couplings entering the kinetic equations are predicted from a UV completion.

The particular interest of low-scale EMLG is that, once formulated in a gauge-invariant EFT and matched further onto lower-energy theories, it opens an observational portal that is largely absent in standard high-scale leptogenesis. In principle, the same dipole sector can be probed by low-energy observables such as charged-lepton flavor violation, electric dipole moments, anomalous magnetic moments, and collider or intensity-frontier signatures [20–24]. Before using such low-energy observables to constrain the model, we first address a more basic dynamical question: can the

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gauge-invariant dipole sector, with the ordinary renormalizable Yukawa interactions kept subleading, generate the observed baryon asymmetry in the electroweak crossover window? The purpose of this paper is to answer this question by deriving the EFT matching, the electroweak-scale dipole couplings, the resonantly regulated CP -odd source, and the density-matrix kinetic equations needed to compute the frozen-out baryon asymmetry.

A gauge-invariant realization of EMLG is highly non-trivial. Gauge invariance forces a Higgs insertion in the dipole operators, which suppresses the broken-phase dipole couplings and, with them, both the CP -odd source and the dipole-induced washout. As a result, the nonresonant hierarchical regime is structurally inefficient [25]. This is not a statement about TeV-scale leptogenesis in general: successful TeV-scale constructions are known in the standard Yukawa framework, including inverse-seesaw, linear-seesaw, and resonant-leptogenesis realizations [26–28]. Rather, it is a distinctive feature of the gauge-invariant, dipole-dominated mechanism considered here.

A second difficulty is that a generic UV completion also allows the renormalizable interaction $\bar{L}_\alpha \tilde{H} N_i$. If this Yukawa channel is not suppressed, it contributes to the total decay width, broadens the resonant regulator, and induces additional washout through inverse decays, thereby obscuring the dipole dynamics [29]. For this reason, we adopt the softly broken Z_2 completion proposed by Choudhury *et al.* [30]. In the present work this UV completion serves two related purposes. First, it provides a gauge-invariant high-energy origin for the dipole operators entering the kinetic equations, so that $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$ are obtained by matching rather than introduced as independent broken-phase interactions. Second, its softly broken Z_2 structure keeps the ordinary renormalizable Yukawa sector subleading, which is necessary for testing whether electromagnetic leptogenesis itself can account for the observed baryon asymmetry. In the exact- Z_2 limit, the interaction $\bar{L}_\alpha \tilde{H} N_i$ is forbidden. Soft breaking is introduced by the dimension-two scalar mixing term $\mu_{\text{soft}}^2 H^\dagger D + \text{H.c.}$, where D denotes a heavy $SU(2)_L$ scalar doublet with the same electroweak quantum numbers as the SM Higgs doublet H but with odd Z_2 parity. After integrating out the heavy doublet, this mixing induces the small effective Yukawa coupling

$$(y_{\text{eff}})_{ai} := (h_D)_{ai} \sin \theta \simeq -(h_D)_{ai} \frac{\mu_{\text{soft}}^2}{m_D^2}, \quad (1)$$

while the same soft insertion converts the one-loop heavy-sector dipole amplitudes with $\tilde{D}$ into the ν SMEFT operators containing $\tilde{H}$. This construction makes it possible to keep the Yukawa-mediated contribution to production, washout, and the resonant width subleading, thereby isolating a genuinely dipole-dominated regime.

The Sakharov conditions are realized in this scenario as follows. Lepton number is broken by the Majorana masses of the heavy neutrinos, which allow the same state N_i to participate in channels carrying opposite lepton number. CP violation arises from irreducible complex phases in the dipole couplings and from the interference of the tree-level decay amplitudes with loop amplitudes, in particular the self-energy contribution in the quasidegenerate regime [26,27,31]. The departure from equilibrium is encoded in the time-dependent heavy-neutrino density matrix: during the electroweak window the expansion of the Universe and the finite reaction rates drive Y_N away from Y_N^{eq} , and in the quasidegenerate regime the off-diagonal coherences themselves become dynamical [28,32,33]. The CP -odd asymmetry is generated by the $\Delta L = 1$ decays and inverse decays, while the leading $\Delta L = 0$ scatterings do not create lepton number directly; instead they modify relaxation, washout, and the damping of coherence, and hence affect the final efficiency only indirectly.

Above the electroweak scale we work in the Standard Model effective field theory extended by right-handed neutrinos, often denoted ν SMEFT [34,35] or SMNEFT [36,37]. Among the dimension-six operators in this EFT, the interactions relevant for electromagnetic leptogenesis are the dipole operators $\mathcal{O}_{NB,ai}$ and $\mathcal{O}_{NW,ai}$. Their explicit gauge-invariant definitions and normalization conventions are given below in Sec. II A. In the Introduction we only use these symbols to identify the dipole sector whose Wilson coefficients control the electromagnetic contribution to production, washout, and the CP -odd source. These operators enter the EFT Lagrangian as

$$-\mathcal{L}_{\text{EFT}} \supset \frac{C_{NB,ai}}{\Lambda^2} \mathcal{O}_{NB,ai} + \frac{C_{NW,ai}}{\Lambda^2} \mathcal{O}_{NW,ai} + \text{H.c.}, \quad (2)$$

where Λ denotes the EFT cutoff. We determine the Wilson coefficients at the matching scale Q_{match} and evolve them to an electroweak reference scale Q_{ref} , where the broken-phase dipole couplings relevant for leptogenesis are defined.

Our main results are the following.

- (i) We formulate low-scale electromagnetic leptogenesis as a gauge-invariant EFT calculation, starting from a UV completion, performing the one-loop matching onto $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$, and evolving the corresponding Wilson coefficients to the electroweak scale.
- (ii) We show that the nonresonant hierarchical regime is structurally insufficient in the gauge-invariant dipole-dominated scenario, whereas the quasidegenerate regime can evade this suppression through the resonantly enhanced self-energy contribution.
- (iii) In the quasidegenerate regime we derive density-matrix quantum kinetic equations (QKEs) on the closed time path. Their collision term incorporates

decays, inverse decays, and the leading $\Delta L = 0$ scatterings without double counting and therefore without real-intermediate-state subtraction.

- (iv) Solving these QKEs, we identify proof-of-principle TeV-scale parameter regions in which dipole-dominated electromagnetic leptogenesis reproduces or exceeds the observed BAU. For our benchmarks the leading $\Delta L = 0$ scatterings satisfy $S_i/D_i \sim 10^{-5}$ and therefore change the final efficiency only at the expected tiny level, confirming that the unified collision term reproduces the correct hierarchy between the $\Delta L = 1$ and leading $\Delta L = 0$ contributions in the dipole-dominated regime.

The paper is organized as follows. Section II introduces the softly broken Z_2 UV completion, explains how the effective Yukawa interaction is suppressed, and derives the matching conditions for the gauge-invariant dipole operators O_{NB} and O_{NW} . Section III presents the renormalization-group evolution of the corresponding Wilson coefficients from Q_{match} to Q_{ref} . Section IV derives the photon-mode decay width and CP asymmetry and explains the resonant enhancement in the quasidegenerate regime. Section V extends the analysis to the O_{NW} -inclusive γ , Z , and $W^\pm$ channels. Section VI summarizes the closed-time-path/Schwinger-Keldysh formalism used in the nonequilibrium treatment. Section VII constructs the collision kernels for decays, inverse decays, and the leading $\Delta L = 0$ scatterings. Section VIII assembles the density-matrix QKE system used in the numerical analysis. Section IX presents the numerical results, and Sec. X discusses their physical interpretation, phenomenological implications, and conclusions. Technical derivations of the one-loop matching, anomalous dimensions, CP -asymmetry algebra, nonequilibrium formalism, QKE projection, and scattering kernels are collected in the Appendixes.

II. UV MODEL

In this section, we specify the UV completion used for the matching calculation. The motivation for adopting the softly broken Z_2 construction, and in particular its role in suppressing the ordinary Yukawa sector relative to the dipole sector, has been explained in Sec. I. Here we therefore focus on the technical ingredients needed below: the gauge-invariant operator basis above the electroweak scale, the field content and interactions of the UV model, the one-loop matching onto the dipole Wilson coefficients, and the benchmark choice of heavy-sector parameters. These ingredients define the EFT input that is subsequently evolved to the electroweak scale and used in the CP -asymmetry and kinetic analyses.

A. EFT setup and the electroweak window

In this subsection we specify the gauge-invariant EFT framework and the temperature regime relevant for

low-scale electromagnetic leptogenesis. Above the electroweak scale we work in the Standard Model effective field theory extended by right-handed neutrinos, commonly denoted νSMEFT [34,35]. The leading interactions responsible for electromagnetic leptogenesis are the gauge-invariant dimension-six dipole operators

$$O_{NB,ai} = (\bar{L}_a \sigma^{\mu\nu} P_R N_i) \tilde{H} B_{\mu\nu}, \quad (3)$$

$$O_{NW,ai} = (\bar{L}_a \sigma^{\mu\nu} \tau^a P_R N_i) \tilde{H} W_{\mu\nu}^a, \quad (4)$$

with

$$\sigma^{\mu\nu} := \frac{i}{2} [\gamma^\mu, \gamma^\nu], \quad \tilde{H} := i\sigma_2 H^*. \quad (5)$$

The corresponding EFT Lagrangian is

$$-\mathcal{L}_{\text{EFT}} \supset \frac{C_{NB,ai}}{\Lambda^2} O_{NB,ai} + \frac{C_{NW,ai}}{\Lambda^2} O_{NW,ai} + \text{H.c.}, \quad (6)$$

where Λ denotes the νSMEFT cutoff scale. Here $\alpha = e, \mu, \tau$ labels the charged-lepton flavor, and i labels the mass eigenstates of the right-handed neutrinos. In a given UV completion, the Wilson coefficients $C_{NX,ai}$ with $X = B, W$ are fixed at the matching scale $Q_{\text{match}} = 10$ TeV and are subsequently evolved down to an electroweak reference scale $Q_{\text{ref}} = 150$ GeV by renormalization-group running.

We focus on low-scale electromagnetic leptogenesis, for which the relevant epoch lies in the electroweak crossover window [38,39]

$$T_{\text{EW}} \geq T \geq T_{\text{sph}}, \quad T_{\text{EW}} \simeq 160 \text{ GeV}, \\ T_{\text{sph}} \simeq 130 \text{ GeV}, \quad (7)$$

where T_{sph} denotes the temperature at which electroweak sphaleron transitions freeze out. In this temperature range the charged-lepton Yukawa interactions are in equilibrium [40,41], so flavor coherences among e, μ , and τ are damped and a fully flavor-resolved treatment is appropriate.

We therefore track the three flavor charges

$$Y_{\Delta_\alpha} := \frac{1}{3} Y_B - Y_{L_\alpha}, \quad \sum_\alpha Y_{\Delta_\alpha} = Y_{B-L}, \quad (8)$$

where $Y_X := n_X/s$ denotes the yield of species X , n_X is the corresponding number density, and s is the entropy density. Electroweak sphaleron transitions continue to convert part of $B - L$ into baryon number until they freeze out at $T = T_{\text{sph}}$. We therefore evaluate the baryon asymmetry at sphaleron decoupling as

$$Y_B^{\text{FO}} = c_{\text{sph}}(T_{\text{sph}}) \sum_{\alpha} Y_{\Delta_{\alpha}}(T_{\text{sph}}), \quad (9)$$

where $c_{\text{sph}}(T)$ denotes the sphaleron conversion coefficient.

A central goal of this paper is to formulate, within the closed-time-path (CTP)/Schwinger-Keldysh framework, the time evolution of the heavy-neutrino ensemble and of the flavor charges introduced above in a plasma where dipole interactions are operative. The relevant dipole-induced processes fall into two classes.

First, the decays and inverse decays

$$N_i \leftrightarrow \nu_{\alpha} \gamma, \quad N_i \leftrightarrow \nu_{\alpha} Z, \quad N_i \leftrightarrow \ell_{\alpha} W \quad (10)$$

are $\Delta L = 1$ processes. In the dipole-dominated scenario studied here, they provide the direct source of the lepton asymmetry and also the leading inverse-decay washout.

Second, combining one dipole vertex with one Standard Model gauge interaction yields $\Delta L = 0$ $2 \leftrightarrow 2$ scatterings. Representative channels are the neutral-current processes

$$N_i f \leftrightarrow \nu_{\alpha} f \quad (11)$$

and, when $\mathcal{O}_{NW}$ is present, the charged-current processes

$$N_i f \leftrightarrow \ell_{\alpha} f', \quad (12)$$

where f and f' are Standard Model fermions belonging to the same $\text{SU}(2)_L$ doublet. These scatterings do not generate lepton number directly. Rather, they modify relaxation, washout, and the damping of flavor coherence, and hence affect the final efficiency only indirectly.

In the dipole-dominated regime considered here, a simple power counting shows that these $\Delta L = 0$ $2 \leftrightarrow 2$ scatterings provide the leading scattering contribution to the damping and washout kernels. Here $\mu_{\alpha i}^V$, with $V = \gamma, Z, W$, denotes the broken-phase effective dipole coupling of N_i to the corresponding gauge boson and to the lepton flavor α ; its relation to the gauge-invariant Wilson coefficients $C_{NB, \alpha i}$ and $C_{NW, \alpha i}$ is given explicitly after electroweak symmetry breaking in Sec. IV. At the amplitude level the scatterings contain one dipole vertex and one SM gauge interaction on the plasma target. Their rates therefore scale schematically as $\mathcal{O}(|\mu^V|^2 g^2)$, where g denotes the relevant SM gauge coupling and $|\mu^V|^2$ stands for the appropriate flavor and gauge-channel combination. By contrast, genuine lepton-number-violating scattering reactions, $|\Delta L| = 2$, would require at least two additional dipole insertions or higher-loop topologies and are therefore parametrically suppressed as $\mathcal{O}(|\mu^V|^4)$ or higher in the regime of interest. We thus neglect such subleading $|\Delta L| \neq 0$ scatterings and construct the scattering contribution entirely from the $\Delta L = 0$ self-energies $\Sigma_{\text{scatt}}^{<, >}$.

If only $\mathcal{O}_{NB}$ is present, photon exchange provides the dominant neutral-current contribution. In the presence of

$\mathcal{O}_{NW}$, the neutral-current sector is generalized to include Z exchange and γZ interference, and charged-current channels mediated by $W^{\pm}$ must also be included. In practice, these contributions are incorporated into the density-matrix quantum kinetic equations through the heavy-neutrino Wightman self-energies.

Although integrating out N also induces the Weinberg operator $\mathcal{O}_5$ at tree level, the associated $\Delta L = 2$ washout becomes efficient only at much higher temperatures. For $\mathcal{O}_5$ normalized to the observed light-neutrino masses, the relevant scale is typically $T \sim 10^{11} - 10^{12}$ GeV [42]. By contrast, the asymmetry in the present scenario is generated in the electroweak crossover window,

$$T \simeq 130 - 160 \text{ GeV}, \quad (13)$$

where the $\mathcal{O}_5$ -induced $\Delta L = 2$ rate is negligible compared with the Hubble expansion for neutrino masses in the oscillation range. Moreover, the UV completion considered here does not generate semileptonic $\Delta L = 2$ operators at the matching scale at the order we work. We therefore neglect low-scale $\Delta L = 2$ washout in the present analysis.

B. UV completion and field content

In this subsection we give the technical implementation of the UV completion whose motivation was summarized in Sec. I. We work in the mass-eigenstate basis of the right-handed Majorana neutrinos. The Majorana mass matrix M_N is Autonne-Takagi diagonalized [43,44] by a unitary matrix U_N as

$$U_N^{\dagger} M_N U_N = \text{diag}(M_1, \dots, M_{n_N}), \quad (14)$$

and we formulate the discussion in terms of the corresponding real, diagonal Majorana fields N_i ($i = 1, \dots, n_N$). All Yukawa matrices and effective couplings appearing below are expressed in this basis. We now specify the Z_2 assignments, the additional field content, the renormalizable interactions, and the soft-breaking insertion used in the matching calculation.

With only $G_{\text{SM}} := \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$ and renormalizability, the operator $\bar{L} \tilde{H} N$ is allowed as a dimension-four interaction. Simply omitting it would therefore leave the definition of the theory incomplete and would require a fine-tuning against radiatively regenerated contributions order by order in perturbation theory. We thus impose a minimal discrete symmetry Z_2 that forbids $\bar{L} \tilde{H} N$ exactly and close the UV completion so that this prohibition is stable under radiative corrections.

The same exact Z_2 symmetry, however, also forbids the dipole operators introduced in Sec. II A. Exact Z_2 alone is therefore insufficient: it suppresses the unwanted Yukawa sector, but it also removes the dipole sector required for electromagnetic leptogenesis. Following [30], we resolve this tension by keeping the renormalizable Lagrangian Z_2

TABLE I. Gauge quantum numbers and Z_2 parities of the additional fields. All SM fields are taken to be Z_2 even. Bold numerals in the $SU(3)_c$ and $SU(2)_L$ columns denote the dimensions of the corresponding gauge representations.

Type	Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	Z_2
Fermions	$\begin{Bmatrix} N_i \\ E_{L,R} \end{Bmatrix}$	$\begin{Bmatrix} \mathbf{1} \\ \mathbf{1} \end{Bmatrix}$	$\begin{Bmatrix} \mathbf{1} \\ \mathbf{1} \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ -1 \end{Bmatrix}$	$\begin{Bmatrix} - \\ - \end{Bmatrix}$
	$\begin{Bmatrix} \Sigma \\ D \\ \Phi^+ \end{Bmatrix}$	$\begin{Bmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \end{Bmatrix}$	$\begin{Bmatrix} \mathbf{2} \\ \mathbf{2} \\ \mathbf{1} \end{Bmatrix}$	$\begin{Bmatrix} +1/2 \\ +1/2 \\ +1 \end{Bmatrix}$	$\begin{Bmatrix} - \\ - \\ + \end{Bmatrix}$

symmetric and breaking Z_2 explicitly only through a dimension-two soft term. In this way, the dipole interactions and the parametrically suppressed effective Yukawa interaction originate from the same controlled source of symmetry breaking.

All SM fields are taken to be Z_2 even. The additional fields, together with their gauge quantum numbers and Z_2 parities, are listed in Table I. The only Z_2 -even new field is Φ^+ , whereas N_i , $E_{L,R}$, Σ , and D are Z_2 odd. This assignment forbids $\bar{L} \tilde{H} N$ exactly while preserving the renormalizable interactions needed for the one-loop generation of the dipole operators. For an $SU(2)_L$ doublet X , we define $\tilde{X} := i\sigma_2 X^*$, and $\Sigma \cdot D := \Sigma^\dagger i\sigma_2 D$ denotes the $SU(2)_L$ -invariant inner product.

Given the field content and the Z_2 assignments in Table I, the renormalizable interactions relevant for the present construction are uniquely restricted. We denote by $\mathcal{L}_{\text{SM}}$ the Standard Model Lagrangian, including the usual gauge, kinetic, and Yukawa terms, and by $V(H, \Sigma, D, \Phi^+)$ the scalar potential. Working in the mass-eigenstate basis of the right-handed neutrinos introduced above, we write the UV Lagrangian as

$$\begin{aligned} \mathcal{L}_{\text{UV}} = & \mathcal{L}_{\text{SM}} + \frac{1}{2} \bar{N}_i (i\not{\partial} - M_i) N_i + \bar{E} (i\not{\partial} - M_E) E \\ & + (D_\mu \Sigma)^\dagger (D^\mu \Sigma) + (D_\mu D)^\dagger (D^\mu D) + (D_\mu \Phi^+)^\dagger (D^\mu \Phi^+) \\ & - [(y_H)_i \bar{N}_i P_L E \Phi^+ + (y_\Sigma)_\alpha \bar{L}_\alpha \Sigma P_R E + (y_D)_\alpha \bar{L}_\alpha D P_R E \\ & + (h_\Sigma)_{\alpha i} \bar{L}_\alpha \tilde{\Sigma} P_R N_i + (h_D)_{\alpha i} \bar{L}_\alpha \tilde{D} P_R N_i + \text{H.c.}] \\ & - V(H, \Sigma, D, \Phi^+). \end{aligned} \quad (15)$$

Here H is the SM Higgs doublet, and the covariant derivative is

$$D_\mu = \partial_\mu - ig \frac{\sigma^a}{2} W_\mu^a - ig' Y B_\mu \quad (16)$$

with the hypercharge Y chosen according to the representation of the field on which D_μ acts. The $SU(3)_c$ term is absent because, as shown in Table I, all additional fields are color singlets. Equation (15) therefore collects all renormalizable interactions involving the new fields that are allowed by $G_{\text{SM}} \times Z_2$. In particular, the renormalizable

neutrino Yukawa interaction $\bar{L} \tilde{H} N$ is absent because it is Z_2 odd. Since $[L] = [E] = [N] = 3/2$ and $[H] = [\Sigma] = [D] = [\Phi^+] = 1$, every operator in Eq. (15) has mass dimension four, and the UV theory is manifestly renormalizable.

For an exact Z_2 symmetry, the scalar sector is described by the most general renormalizable potential that is both G_{SM} invariant and Z_2 even. Following [30], we parametrize it as

$$\begin{aligned} V(H, \Sigma, D, \Phi^+) = & -\mu_H^2 (H^\dagger H) + m_\Sigma^2 (\Sigma^\dagger \Sigma) + m_D^2 (D^\dagger D) + m_+^2 |\Phi^+|^2 \\ & + \lambda_H (H^\dagger H)^2 + \lambda_\Sigma (\Sigma^\dagger \Sigma)^2 + \lambda_D (D^\dagger D)^2 + \lambda_+ |\Phi^+|^4 \\ & + \lambda_{H+} (H^\dagger H) |\Phi^+|^2 + \lambda_{D+} (D^\dagger D) |\Phi^+|^2 \\ & + \lambda_{\Sigma+} (\Sigma^\dagger \Sigma) |\Phi^+|^2 + \lambda_{D\Sigma+} (D^\dagger D) (\Sigma^\dagger \Sigma) |\Phi^+|^2 \\ & + \frac{\lambda_{H\Sigma}}{2} [(H^\dagger \Sigma)^2 + \text{H.c.}] + \lambda_{DH} (D^\dagger \Sigma) (H^\dagger H) \\ & + f_1 (H^\dagger H) (D^\dagger D) + f_2 (H^\dagger H) (\Sigma^\dagger \Sigma) \\ & + f_3 |H^\dagger D|^2 + f_4 |H^\dagger \Sigma|^2 + f_5 (D^\dagger D) (\Sigma^\dagger \Sigma) + f_6 |D^\dagger \Sigma|^2 \\ & + [\mu_s (\Sigma \cdot D) (\Phi^+)^* + \text{H.c.}]. \end{aligned} \quad (17)$$

We take $m_\Sigma^2 > 0$ and $m_D^2 > 0$, so that in the exact- Z_2 limit one has $\langle \Sigma \rangle = \langle D \rangle = 0$. We furthermore assume that m_Σ and m_D are sufficiently larger than the heavy-neutrino masses M_i , so that the decays $N \rightarrow \nu + \Sigma$ and $N \rightarrow \nu + D$ are kinematically forbidden.

An exact Z_2 symmetry, however, also forbids the dipole operators required for electromagnetic leptogenesis. The symmetry must therefore be broken. In order to keep all Z_2 -violating effects under parametric control, and at the same time to ensure that the induced neutrino Yukawa interaction remains technically natural, we choose to break Z_2 only through a soft dimension-two term,

$$V_{\text{soft}} = \mu_{\text{soft}}^2 H^\dagger D + \text{H.c.} \quad (18)$$

This is the minimal source of Z_2 breaking directly required for the present analysis. Other soft terms, such as $H^\dagger \Sigma$, could also be introduced, but they are not needed here. Since the exact Z_2 symmetry is restored in the limit $\mu_{\text{soft}}^2 \rightarrow 0$, the smallness of μ_{soft}^2 is technically natural in the 't Hooft sense [45].

C. Soft Z_2 breaking and induced small parameters

The soft term in Eq. (18) mixes the doublets H and D , and therefore induces a small vacuum expectation value for the neutral component of D even when $m_D^2 > 0$. To make this explicit, we retain only the neutral components and write

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad D = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_D + d \end{pmatrix}, \quad (19)$$

where $v := \sqrt{2}\langle H^0 \rangle \simeq 246$ GeV and $v_D := \sqrt{2}\langle D^0 \rangle$. For the purpose of extracting the parametric dependence of v_D and of the H - D mixing angle, it is sufficient to consider the minimal H - D potential

$$V(H, D) = -m_H^2 |H|^2 + \frac{\lambda_H}{2} |H|^4 + m_D^2 |D|^2 + \frac{\lambda_D}{2} |D|^4 + \lambda_{HD} |H|^2 |D|^2 + (\mu_{\text{soft}}^2 H^\dagger D + \text{H.c.}), \quad (20)$$

with $m_D^2 > 0$, so that in the exact- Z_2 limit the vacuum $v_D = 0$ is naturally realized even if the coupling $\bar{L} \tilde{D} N$ is present.

Substituting Eq. (19) into Eq. (20), we obtain

$$V(v, v_D) = -\frac{1}{2} m_H^2 v^2 + \frac{\lambda_H}{8} v^4 + \frac{1}{2} m_D^2 v_D^2 + \frac{\lambda_D}{8} v_D^4 + \frac{\lambda_{HD}}{4} v^2 v_D^2 + \mu_{\text{soft}}^2 v v_D, \quad (21)$$

where we have taken μ_{soft}^2 real. The stationarity condition along the v_D direction is then

$$0 = \frac{\partial V}{\partial v_D} = m_D^2 v_D + \frac{\lambda_D}{2} v_D^3 + \frac{\lambda_{HD}}{2} v^2 v_D + \mu_{\text{soft}}^2 v. \quad (22)$$

In the technically natural regime $\mu_{\text{soft}}^2 \ll m_D^2$, one expects self-consistently that $v_D \ll v$, so the cubic term in v_D may be neglected. If, in addition, $m_D^2 \gg \lambda_{HD} v^2$, one finds

$$v_D \simeq -\frac{\mu_{\text{soft}}^2}{m_D^2 + \frac{\lambda_{HD}}{2} v^2} v. \quad (23)$$

Using $m_D^2 \gg \lambda_{HD} v^2$ then gives

$$v_D \simeq -v \frac{\mu_{\text{soft}}^2}{m_D^2}. \quad (24)$$

The same small parameter controls the mixing angle between the CP -even neutral scalars. Expanding Eq. (20) around the vacuum, the mass-squared matrix in the basis (h, d) becomes

$$M^2 \simeq \begin{pmatrix} m_h^2 & \mu_{\text{soft}}^2 \\ \mu_{\text{soft}}^2 & m_D^2 \end{pmatrix}, \quad m_h^2 \simeq \lambda_H v^2, \quad (25)$$

up to corrections suppressed by v_D/v . For $m_D^2 \gg m_h^2$, the mixing angle satisfies

$$\tan 2\theta \simeq -\frac{2\mu_{\text{soft}}^2}{m_D^2 - m_h^2} \simeq -\frac{2\mu_{\text{soft}}^2}{m_D^2} \Rightarrow \theta \simeq -\frac{\mu_{\text{soft}}^2}{m_D^2}. \quad (26)$$

Hence, both the induced VEV and the scalar mixing are controlled by the same small ratio,

$$v_D \sim v\theta, \quad \theta \sim -\frac{\mu_{\text{soft}}^2}{m_D^2}. \quad (27)$$

This relation is the key ingredient for the induced effective Yukawa coupling discussed next.

In the exact- Z_2 limit, the interaction

$$-\mathcal{L} \supset (h_D)_{ai} \bar{L}_a \tilde{D} P_R N_i + \text{H.c.} \quad (28)$$

is allowed, whereas $\bar{L} \tilde{H} N$ is forbidden. Once the soft term in Eq. (18) induces H - D mixing, the SM-like Higgs direction and its orthogonal heavy-doublet direction can be written as

$$H_1 = H \cos \theta + D \sin \theta, \quad H_2 = -H \sin \theta + D \cos \theta. \quad (29)$$

For $v_D \ll v$, the field H_1 is the SM-like doublet that acquires the vacuum expectation value, whereas H_2 is heavy. Since

$$D = H_1 \sin \theta + H_2 \cos \theta, \quad (30)$$

Equation (28) becomes

$$-\mathcal{L} \supset (h_D)_{ai} \bar{L}_a \tilde{H}_1 P_R N_i \sin \theta + (h_D)_{ai} \bar{L}_a \tilde{H}_2 P_R N_i \cos \theta + \text{H.c.} \quad (31)$$

This motivates the definition of the effective Yukawa coupling to the SM-like Higgs direction,

$$(y_{\text{eff}})_{ai} := (h_D)_{ai} \sin \theta \simeq (h_D)_{ai} \theta, \quad (32)$$

and using Eq. (26) we obtain

$$(y_{\text{eff}})_{ai} \simeq -(h_D)_{ai} \frac{\mu_{\text{soft}}^2}{m_D^2}. \quad (33)$$

Since H_1 carries the electroweak vacuum expectation value, the induced Dirac mass matrix is

$$(M_D)_{ai} = \frac{v}{\sqrt{2}} (y_{\text{eff}})_{ai} = \frac{v_D}{\sqrt{2}} (h_D)_{ai}. \quad (34)$$

Thus, y_{eff} is not merely a shorthand notation: it is the effective Yukawa coupling along the SM-like Higgs direction selected by electroweak symmetry breaking.

To maintain a genuinely dipole-dominated regime, the Yukawa-induced contribution to the heavy-neutrino widths must remain subleading to the dipole-induced one. Parametrically, the vacuum width of the Yukawa channel $N_i \rightarrow L_\alpha H_1$ is

$$\Gamma_Y(N_i) \simeq \frac{M_i}{8\pi} (y_{\text{eff}}^\dagger y_{\text{eff}})_{ii}, \quad (35)$$

whereas the dipole-induced width, summed over the γ , Z , and W channels, scales as

$$\Gamma_{\text{EM}}(N_i) \simeq \frac{M_i^3}{2\pi} \sum_\alpha |\mu_{ai}|^2. \quad (36)$$

Introducing the effective electromagnetic neutrino mass [46],

$$\tilde{m}_i^{\text{EM}} := v^2 M_i \sum_\alpha |\mu_{ai}|^2, \quad (37)$$

one may rewrite the dipole width as

$$\Gamma_{\text{EM}}(N_i) \simeq \frac{M_i^2}{2\pi v^2} \tilde{m}_i^{\text{EM}}. \quad (38)$$

A convenient sufficient condition for dipole dominance is then

$$\Gamma_Y(N_i) \lesssim \epsilon \Gamma_{\text{EM}}(N_i) \quad (\epsilon \ll 1), \quad (39)$$

which implies

$$(y_{\text{eff}}^\dagger y_{\text{eff}})_{ii} \lesssim 4\epsilon \frac{M_i}{v^2} \tilde{m}_i^{\text{EM}}. \quad (40)$$

Thus, as a typical bound,

$$|y_{\text{eff}}| \lesssim 2\sqrt{\epsilon} \frac{\sqrt{M_i \tilde{m}_i^{\text{EM}}}}{v}. \quad (41)$$

For the representative values

$$M_i \sim 1 \text{ TeV}, \quad \tilde{m}_i^{\text{EM}} \sim 10^{-3} \text{ eV}, \quad v = 246 \text{ GeV}, \quad (42)$$

one finds

$$\frac{\sqrt{M_i \tilde{m}_i^{\text{EM}}}}{v} \sim 1.3 \times 10^{-7}. \quad (43)$$

Demanding that the Yukawa contribution remain below the percent level, $\epsilon \sim 10^{-2}$, yields the useful benchmark criterion

$$|y_{\text{eff}}| \lesssim 10^{-8}. \quad (44)$$

Using Eq. (33), this translates into

$$\frac{\mu_{\text{soft}}}{m_D} \lesssim \sqrt{\frac{10^{-8}}{|h_D|}} \Rightarrow \mu_{\text{soft}} \lesssim 3 \times 10^{-3} m_D, \quad (45)$$

for $|h_D| \sim \mathcal{O}(10^{-3})$.

In our benchmark analysis we take $m_D \sim 10 \text{ TeV}$ and $\mu_{\text{soft}} = 10 \text{ GeV}$, for which $\mu_{\text{soft}}^2/m_D^2 \sim 10^{-6}$. Since the exact Z_2 symmetry is restored in the limit $\mu_{\text{soft}}^2 \rightarrow 0$, the smallness of both $\mu_{\text{soft}}^2/m_D^2$ and y_{eff} is technically natural in the 't Hooft sense [45]. The important physical point is that y_{eff} is suppressed by $\mu_{\text{soft}}^2/m_D^2$, whereas the dipole Wilson coefficients are generated at one loop and scale with inverse powers of the heavy masses. The two sectors therefore scale differently, and there exists a parameter region in which the electromagnetic channels dominate the heavy-neutrino dynamics.

D. Light neutrino masses and the seesaw mechanism

Here we address the question of whether forbidding the renormalizable interaction $\bar{L} \tilde{H} N$ eliminates the type-I seesaw mechanism altogether. The answer is no, but the relevant point for the present paper is more specific: in the dipole-dominated regime required for electromagnetic leptogenesis, the induced type-I contribution is parametrically too small to account for the observed light-neutrino masses.

In the exact- Z_2 limit one has $\langle D \rangle = 0$ for $m_D^2 > 0$, and therefore no Dirac mass is generated even though the coupling $\bar{L} \tilde{D} N$ is allowed. Once the soft term

$$V_{\text{soft}} = \mu_{\text{soft}}^2 H^\dagger D + \text{H.c.} \quad (46)$$

is turned on, however, the H - D mixing induces

$$\theta \sim -\frac{\mu_{\text{soft}}^2}{m_D^2}, \quad v_D := \langle D^0 \rangle \simeq v\theta, \quad (47)$$

and therefore the Dirac mass matrix

$$(M_{\text{Dirac}})_{ai} = \frac{v_D}{\sqrt{2}} (h_D)_{ai}. \quad (48)$$

Formally, the standard type-I seesaw relation then holds,

$$(m_\nu^{(1)})_{\alpha\beta} \simeq (M_{\text{Dirac}})_{ai} (M_N^{-1})_{ij} (M_{\text{Dirac}}^\dagger)_{j\beta}. \quad (49)$$

Equivalently, since

$$(y_{\text{eff}})_{ai} \simeq (h_D)_{ai} \theta \simeq -(h_D)_{ai} \frac{\mu_{\text{soft}}^2}{m_D^2}, \quad (50)$$

one may write

$$M_{\text{Dirac}} = \frac{v}{\sqrt{2}} y_{\text{eff}}. \quad (51)$$

Neutrino oscillation data imply characteristic light-neutrino mass scales of order

$$m_{\text{osc}} \sim \sqrt{\Delta m_{21}^2}, \quad \sqrt{|\Delta m_{3\ell}^2|} = \mathcal{O}(10^{-2}) \text{ eV}. \quad (52)$$

For definiteness, we take

$$m_{\text{osc}} := \sqrt{|\Delta m_{3\ell}^2|} \simeq \sqrt{2.5 \times 10^{-3} \text{ eV}^2} \simeq 0.05 \text{ eV}, \quad (53)$$

as a representative scale inferred from oscillation data [47] (NuFit-6.0). For a TeV-scale type-I seesaw to reproduce this scale, one typically needs

$$|y_{\text{eff}}| \simeq \frac{\sqrt{m_\nu M}}{v} \sim 10^{-6} \quad (M \sim 1 \text{ TeV}). \quad (54)$$

By contrast, the dipole-dominated condition derived in Sec. II C requires

$$|y_{\text{eff}}| \lesssim 10^{-8}, \quad (55)$$

which implies

$$m_\nu^{(1)} \sim \frac{v^2}{M} |y_{\text{eff}}|^2 \lesssim 6 \times 10^{-6} \text{ eV}, \quad (M \sim 1 \text{ TeV}). \quad (56)$$

This is far below the atmospheric-neutrino scale. Thus, in the Yukawa-suppressed benchmark relevant for the present work, a type-I seesaw structure is formally present but is phenomenologically negligible. We therefore treat the observed light-neutrino masses as independent low-energy input encoded in the Weinberg-operator coefficient C_5 .

A simple high-scale realization of this low-energy input is collected in Appendix A, where we give a type-II-seesaw illustration. Since that sector is assumed to lie far above the matching scale $\mathcal{Q}_{\text{match}} = 10 \text{ TeV}$, it enters the low-energy theory only through $\mathcal{O}_5$ and does not modify the dipole matching discussed in the present section.

E. One-loop matching from the UV theory to electromagnetic dipole operators

The one-loop matching amounts to computing the 1PI four-point amplitudes with external legs

$$N_i, \quad L_\alpha, \quad \tilde{H}, \quad X_\mu,$$

where $X = B, W^a$, expanding them at low external momenta, and projecting onto the antisymmetric dipole structure. We use the normalization of the dipole operators introduced in Sec. II A and write the relevant process as

$$N_i(p) \rightarrow L_\alpha(p') + \tilde{H}(k) + X_\mu(q), \quad X = B, W^a, \quad (57)$$

with $q = p - p' - k$. For an outgoing gauge boson with polarization vector $\varepsilon_\mu(q)$, the field strength gives

$$X_{\mu\nu}(q) = i(q_\mu \varepsilon_\nu - q_\nu \varepsilon_\mu), \quad (58)$$

and therefore the EFT dipole amplitude is proportional to

$$\bar{u}_{L_\alpha} \sigma^{\mu\nu} q_\nu P_R u_{N_i} \varepsilon_\mu. \quad (59)$$

Thus, the Wilson coefficient $C_{NX,ai}/\Lambda^2$ is obtained by extracting the coefficient of this spinor structure from the UV one-loop amplitude. The detailed vertex-function decomposition and projection formula are collected in Appendix B; in the main text we keep only the matching strategy and the resulting Wilson coefficients.

We first integrate out D at tree level to generate an effective $\Sigma - H - \Phi^+$ vertex. Since the potentials in Eqs. (17) and (18) contain

$$V \supset m_D^2 D^\dagger D + \mu_{\text{soft}}^2 H^\dagger D + \mu_s \Sigma \cdot D(\Phi^+)^* + \text{H.c.}, \quad (60)$$

for sufficiently heavy m_D the equation of motion for D implies

$$D \sim -\frac{\mu_{\text{soft}}^2}{m_D^2} H + \dots. \quad (61)$$

The ellipsis denotes terms generated by the other linear sources for D , such as the $\mu_s^* \tilde{\Sigma} \Phi^+$ source, possible quartic sources involving additional light fields, and derivative corrections suppressed by p^2/m_D^2 . These terms either generate operators with extra fields or contribute only at higher order in the $1/m_D^2$ expansion, and therefore do not affect the dimension-six dipole matching retained here. Substituting the leading soft-mixing term into the μ_s interaction yields the gauge-invariant effective three-point interaction

$$V_{\text{eff}} \supset \mu_{\text{eff}} \Sigma \cdot H(\Phi^+)^* + \text{H.c.}, \quad \mu_{\text{eff}} := -\mu_s \frac{\mu_{\text{soft}}^2}{m_D^2}. \quad (62)$$

Next, we generate $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$ at one loop with E, Σ , and Φ^+ . We consider one-loop diagrams with external legs N_i (incoming), L_α (outgoing), $\tilde{H}$ (outgoing), and one gauge boson (either B_μ or W_μ^a , outgoing).

Here we derive the electromagnetic dipole operators $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$ from the minimal field content listed in Table I. The purpose here is to make explicit, from the viewpoint of symmetries and scale separation, that in the decays of the right-handed neutrinos N_i the dipole-induced channels can dominate over the standard Yukawa decay $N \rightarrow LH$.

The heavy fields that are integrated out when constructing the EFT are the charged states E, Σ, D, Φ^+ , which we take to lie at a characteristic scale of $\mathcal{O}(10)$ TeV. By contrast, we keep the N_i as dynamical degrees of freedom in the EFT, since their decays, inverse decays, scatterings, and CP asymmetries are the central ingredients of leptogenesis. The EFT constructed in this section is therefore the ν SMEFT with the SM fields and the dynamical right-handed neutrinos N_i as low-energy degrees of freedom; among the dimension-six operators in this EFT, we retain $\mathcal{O}_{NB,ai}$ and $\mathcal{O}_{NW,ai}$ as the operators relevant for the present EMLG analysis.

Keeping only the Yukawa-type interaction terms in the UV Lagrangian, we have

$$-\mathcal{L}_{\text{Yukawa}} \supset y_H \bar{N}_R E_L \Phi^+ + y_\Sigma \bar{L} \Sigma E_R + h_D \bar{L} D \tilde{N}_R + \text{H.c.}, \quad (63)$$

where we have suppressed matrix indices. Furthermore, the scalar potential contains

$$V \supset \mu_s \Sigma \cdot D (\Phi^+)^* + \text{H.c.}, \quad (64)$$

where $\Sigma \cdot D := \Sigma^\dagger i \sigma_2 D$ denotes the $SU(2)_L$ -invariant inner product.

Since the spontaneous breaking of an exact discrete symmetry can lead to the cosmological domain-wall problem [48–50], we break Z_2 softly with the soft term in this model

$$V_{\text{soft}} = \mu_{\text{soft}}^2 H^\dagger D + \text{H.c.} \quad (65)$$

This term allows D to acquire a small VEV even for $m_D^2 > 0$, thereby inducing a Dirac mass $M_D = h_D v_D$. At the same time, it permits magnetic-moment-type interactions once the heavy states are integrated out. The crucial point is that, when the soft breaking is small, the induced effects are parametrically suppressed by $\theta \sim \mu_{\text{soft}}^2 / m_D^2$.

The electromagnetic dipole operators $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$ are gauge-invariant dimension-six operators that necessarily contain one Higgs doublet insertion. Their Wilson coefficients are therefore generated at leading order by one-loop Feynman diagrams involving the heavy fields. In the present model, renormalizable couplings directly connecting N and L are forbidden by Z_2 , so the shortest renormalizable path connecting N to L is, from Eq. (15),

$$N \xrightarrow{\Phi^+} E \xrightarrow{\Sigma} L, \quad (66)$$

and closing the loop further requires the vertex $\mu_s \Sigma \cdot D (\Phi^+)^*$ that connects Φ^+ and Σ . Equivalently, to form a loop that connects N and L , the fields E, Σ , and Φ^+ are simultaneously required, while D enters as an external leg

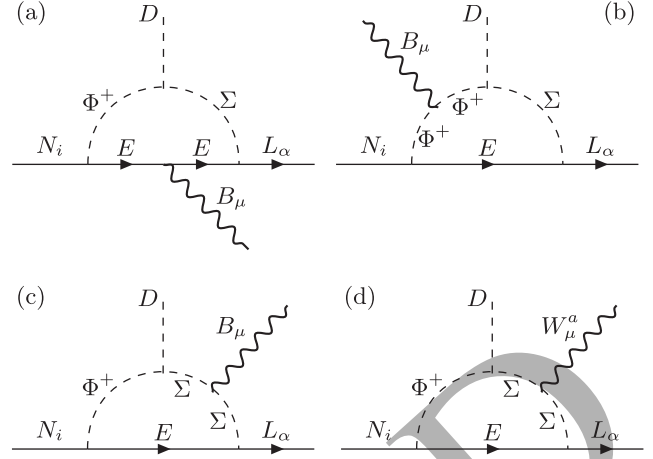


FIG. 1. Representative one-loop matching diagrams generating the gauge-invariant electromagnetic dipole operators $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$. These diagrams make explicit the EFT origin of the interactions that enter the Kadanoff-Baym equations.

attached to the μ_s vertex. This is precisely the structure of the one-loop matching diagrams shown in Fig. 1.

Moreover, in order to obtain $B_{\mu\nu}$ or $W_{\mu\nu}^a$ one must attach a gauge boson to a charged line in the loop. Since W^a can couple only to lines carrying $SU(2)_L$ charge, attaching it to the Σ line is essential for generating $\mathcal{O}_{NW}$. For the hypercharge gauge boson B , in addition to the attachment to the Σ line there are contributions from attaching it to the Φ^+ and E lines; these contributions must be summed to obtain a gauge-invariant result.

To streamline the calculation, we split the matching into two steps.

First, we integrate out E, Σ, Φ^+ while treating D as an external heavy doublet. This gives the intermediate D -based dipole operators

$$\mathcal{O}_{NB,ai}^{(D)} := (\bar{L}_a \sigma^{\mu\nu} P_R N_i) \tilde{D} B_{\mu\nu}, \quad (67)$$

$$\mathcal{O}_{NW,ai}^{(D)} := (\bar{L}_a \sigma^{\mu\nu} \tau^a P_R N_i) \tilde{D} W_{\mu\nu}^a. \quad (68)$$

The corresponding Wilson coefficients are denoted by $C_{NB,ai}^{(D)}$ and $C_{NW,ai}^{(D)}$.

Second, at scales $p^2 \ll m_D^2$, we integrate out the heavy doublet D . Solving its classical equation of motion in a $1/m_D^2$ expansion gives, to leading order in the soft-breaking source,

$$D \simeq -\frac{(\mu_{\text{soft}}^2)^*}{m_D^2} H, \quad \tilde{D} \simeq -\frac{\mu_{\text{soft}}^2}{m_D^2} \tilde{H}. \quad (69)$$

Therefore,

$$\mathcal{O}_{NB,ai}^{(D)} \rightarrow -\frac{\mu_{\text{soft}}^2}{m_D^2} \mathcal{O}_{NB,ai}, \quad \mathcal{O}_{NW,ai}^{(D)} \rightarrow -\frac{\mu_{\text{soft}}^2}{m_D^2} \mathcal{O}_{NW,ai}. \quad (70)$$

Equivalently, the ν SMEFT Wilson coefficients are related to the intermediate coefficients by

$$\frac{C_{NB,ai}}{\Lambda^2} \simeq -\frac{\mu_{\text{soft}}^2}{m_D^2} \frac{C_{NB,ai}^{(D)}}{\Lambda^2}, \quad \frac{C_{NW,ai}}{\Lambda^2} \simeq -\frac{\mu_{\text{soft}}^2}{m_D^2} \frac{C_{NW,ai}^{(D)}}{\Lambda^2}. \quad (71)$$

This is the tree-level matching step by which the D -based dipole operators generated at one loop are converted into the gauge-invariant ν SMEFT operators $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$. The detailed derivation of the D equation of motion, including the classification of linear source terms and $D^\dagger D$ -type corrections, is given in Appendix B.

F. Summary and benchmark parametrization

For simplicity, and in order to display the matching coefficients in a compact analytic form, we use the degenerate heavy-spectrum benchmark

$$M_E = m_{\Phi^+} = m_\Sigma = m_D := M_{\text{eq}} = 10.0 \text{ TeV}. \quad (72)$$

This degeneracy is not an essential assumption of the mechanism. For a nondegenerate heavy spectrum, the common factor M_{eq}^{-3} in the expressions below is replaced by the corresponding mass-dependent loop functions of M_E , m_{Φ^+} , m_Σ , and m_D . The qualitative conclusions are unchanged as long as all heavy masses remain above the matching scale relevant for the EFT description and no additional threshold is introduced in the electroweak-scale kinetic theory. In particular, the soft-mixing suppression of the induced Yukawa coupling, the loop origin of the dipole Wilson coefficients, and the need for resonant enhancement in the right-handed-neutrino sector do not rely on exact degeneracy among the heavy UV states. Large hierarchies or accidental cancellations in the loop functions would, however, have to be treated separately in a dedicated nondegenerate parameter scan.

Using the results in Appendix B yields

$$\begin{aligned} \frac{C_{NB,ai}^{(D)}}{\Lambda^2} &= \frac{g' y_{\Sigma,\alpha} y_{H,i}^* \mu_s^*}{16\pi^2} \frac{1}{M_{\text{eq}}^3} \times \frac{1}{6} \times (Y_E + Y_{\Phi^+} + Y_\Sigma) \\ &= \frac{g' y_{\Sigma,\alpha} y_{H,i}^* \mu_s^*}{192\pi^2} \frac{1}{M_{\text{eq}}^3}, \end{aligned} \quad (73)$$

$$\frac{C_{NW,ai}^{(D)}}{\Lambda^2} = \frac{g y_{\Sigma,\alpha} y_{H,i}^* \mu_s^*}{96\pi^2} \frac{1}{M_{\text{eq}}^3}. \quad (74)$$

Eliminating D via the equation of motion according to Eq. (69), we obtain

$$\frac{C_{NB,ai}}{\Lambda^2} \simeq -\frac{\mu_{\text{soft}}^2}{m_D^2} \frac{g' y_{\Sigma,\alpha} y_{H,i}^* \mu_s^*}{192\pi^2} \frac{1}{M_{\text{eq}}^3}, \quad (75)$$

$$\frac{C_{NW,ai}}{\Lambda^2} \simeq -\frac{\mu_{\text{soft}}^2}{m_D^2} \frac{g y_{\Sigma,\alpha} y_{H,i}^* \mu_s^*}{96\pi^2} \frac{1}{M_{\text{eq}}^3}. \quad (76)$$

Rotating $B_{\mu\nu}$ and $W_{\mu\nu}^3$ into $F_{\mu\nu}$ and $Z_{\mu\nu}$, the electromagnetic coupling is obtained as

$$\begin{aligned} \frac{C_{N\gamma,ai}}{\Lambda^2} &= \frac{C_{NB,ai}}{\Lambda^2} \cos \theta_W + \frac{C_{NW,ai}}{2\Lambda^2} \sin \theta_W \\ &= -\frac{\mu_{\text{soft}}^2}{m_D^2} \frac{y_{\Sigma,\alpha} y_{H,i}^* \mu_s^*}{192\pi^2} \frac{1}{M_{\text{eq}}^3} (g' \cos \theta_W + g \sin \theta_W) \\ &\simeq -\frac{\mu_{\text{soft}}^2}{m_D^2} \frac{e y_{\Sigma,\alpha} y_{H,i}^* \mu_s^*}{96\pi^2} \frac{1}{M_{\text{eq}}^3}, \end{aligned} \quad (77)$$

where in the third line we used $e = g \sin \theta_W = g' \cos \theta_W$.

III. RENORMALIZATION-GROUP EVOLUTION OF C_{NB} , C_{NW}

We evolve the Wilson coefficients C_{NX} ($X = B, W$) obtained in the previous section from the matching scale $Q_{\text{match}} = 10 \text{ TeV}$ —where the additional UV degrees of freedom have been integrated out—down to an electroweak reference scale $Q_{\text{ref}} = 150 \text{ GeV}$ using renormalization-group equations. We denote the Standard Model gauge couplings by g' for $U(1)_Y$, g for $SU(2)_L$, and g_s for $SU(3)_c$, where g' is not normalized according to grand unified theory (GUT) conventions. Since Q_{ref} lies close to the top-quark threshold, we neglect threshold matching effects at this stage, which are beyond the one-loop accuracy pursued here and introduce only small logarithms over the interval of interest. The resulting low-scale coefficients serve as the input for the broken-phase effective dipole couplings and for the CP -asymmetry calculation across the electroweak crossover window, $T \simeq 130\text{--}160 \text{ GeV}$, presented in the next section.

A. Effective Lagrangian and renormalization

We work in the ν SMEFT and restrict ourselves to the dipole sector defined in Sec. II A. Throughout this subsection, a subscript 0 denotes a bare quantity. The effective Lagrangian relevant for the running of the Wilson coefficients is

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \mathcal{L}_{\text{SM}}^{\text{kin}}(g', g, g_s, y_{u,d,e}) \\ &\quad - \frac{C_{NB,ai,0}}{\Lambda^2} \mathcal{O}_{NB,ai} - \frac{C_{NW,ai,0}}{\Lambda^2} \mathcal{O}_{NW,ai} + \text{H.c.}, \end{aligned} \quad (78)$$

where $\mathcal{L}_{\text{SM}}^{\text{kin}}$ denotes the SM kinetic terms and the renormalizable SM interactions, parametrized by the gauge couplings g' , g , g_s and the Yukawa couplings $y_{u,d,e}$.

We define the field renormalizations by

$$B_{0\mu} = Z_B^{1/2} B_\mu, \quad W_{0\mu}^a = Z_W^{1/2} W_\mu^a, \quad (79)$$

$$L_0 = Z_L^{1/2} L, \quad \tilde{H}_0 = Z_H^{1/2} \tilde{H}. \quad (80)$$

The singlet neutrino N_i has no SM gauge interaction in the EFT and hence does not acquire an additional SM wave-function counterterm at the order considered here. We renormalize the Wilson coefficients as

$$(C_{NX,ai})_0 = Q^{2\epsilon} Z_{NX} C_{NX,ai}(Q), \quad X = B, W, \quad (81)$$

where Q denotes the renormalization scale. We work in dimensional regularization with

$$n = 4 - 2\epsilon. \quad (82)$$

Here ϵ is the dimensional-regularization parameter. This notation is used only for UV regularization and is distinct from the CP -asymmetry parameters ϵ_{ai} introduced later. In the modified minimal subtraction scheme we retain only the UV-divergent poles,

$$\bar{\epsilon}^{-1} := \frac{2}{4-n} - \gamma + \ln 4\pi, \quad \epsilon^{-1} := \frac{2}{4-n}. \quad (83)$$

The factors Z_{NX} are defined so that Green's functions with one insertion of $\mathcal{O}_{NX}$ are UV finite.

In the unbroken $SU(2)_L \times U(1)_Y$ theory, the B - and W -dipole operators do not mix with each other, and we may therefore treat Z_{NX} as diagonal in X . Operator mixing into the remaining ν SMEFT basis is neglected in this section; given our matching conditions and the one-loop accuracy of the present running, such effects would modify C_{NB} and C_{NW} only beyond one-loop order.

We employ the all-legs renormalization scheme: $Z_{NX}^{(1)}$ is fixed by cancelling the remaining $\bar{\epsilon}^{-1}$ pole in the amputated 1PI Green's function with one insertion of $\mathcal{O}_{NX}$, after including the wave-function renormalization factors for all external legs ($L_\alpha, N_i, H, B_\mu/W_\mu^I$).

B. $B-W^3$ mixing

A possible source of mixing between C_{NB} and C_{NW} is the mixed vacuum polarization between B_μ and W_μ^3 . In the unbroken electroweak phase, the divergent part has the form

$$\Pi_{BW}^{\mu\nu}(q)|_{\text{div}} \propto \frac{g'g}{16\pi^2} \bar{\epsilon}^{-1} (g^{\mu\nu} q^2 - q^\mu q^\nu) \sum_{\text{light}} YT^3. \quad (84)$$

For the light SM field content, the sum over each complete $SU(2)_L$ multiplet vanishes,

$$\sum_{\text{light}} YT^3 = 0, \quad (85)$$

and hence

$$Z_{BW}^{(1)} = 0. \quad (86)$$

Thus, there is no one-loop $B-W^3$ wave-function mixing in the electroweak-symmetric phase, and the running of C_{NB} and C_{NW} is diagonal at the order considered here. The explicit vacuum-polarization calculation and the light-field sums are collected in Appendix C.

C. Renormalization-group equations for the Wilson coefficients

From the one-loop calculation summarized in Appendix C, the anomalous dimensions relevant for the dipole-operator running are

$$\gamma_{BW}^{(1)} = 0, \quad \gamma_B^{(1)} = \frac{41}{6} \times \frac{g^2}{16\pi^2}, \quad \gamma_W^{(1)} = \frac{5}{6} \times \frac{g^2}{16\pi^2}, \quad (87)$$

$$\gamma_L^{(1)} = \frac{1}{16\pi^2} \left(\frac{1}{4} g^2 + \frac{3}{4} g'^2 \right), \quad (88)$$

$$\gamma_H^{(1)} = \frac{1}{16\pi^2} \left(\frac{1}{2} g^2 + \frac{3}{2} g'^2 - 3y_t^2 \right). \quad (89)$$

Since $Z_{BW}^{(1)} = 0$, the running is diagonal in the basis $\{\mathcal{O}_{NB}, \mathcal{O}_{NW}\}$. Combining the external-leg renormalization factors gives

$$Z_{NB} = (Z_L Z_H Z_B)^{-1/2}, \quad Z_{NW} = (Z_L Z_H Z_W)^{-1/2}. \quad (90)$$

The resulting one-loop RGEs in the electroweak-symmetric phase are

$$\begin{aligned} Q \frac{d}{dQ} C_{NB,ai}(Q) &= -\frac{1}{16\pi^2} \left(\frac{91}{12} g^2 + \frac{9}{4} g'^2 - 3y_t^2 \right) C_{NB,ai}(Q), \end{aligned} \quad (91)$$

$$\begin{aligned} Q \frac{d}{dQ} C_{NW,ai}(Q) &= -\frac{1}{16\pi^2} \left(\frac{3}{4} g^2 + \frac{37}{12} g'^2 - 3y_t^2 \right) C_{NW,ai}(Q). \end{aligned} \quad (92)$$

Equivalently,

$$C_{NX,ai}(Q_{\text{ref}}) = C_{NX,ai}(Q_{\text{match}}) \times \exp \left[\frac{1}{16\pi^2} \int_{\ln Q_{\text{ref}}}^{\ln Q_{\text{match}}} F_X(Q) d \ln Q \right],$$

$$X = B, W, \quad (93)$$

with

$$F_B(Q) := \frac{91}{12} g^2(Q) + \frac{9}{4} g^2(Q) - 3y_i^2(Q), \quad (94)$$

$$F_W(Q) := \frac{3}{4} g^2(Q) + \frac{37}{12} g^2(Q) - 3y_i^2(Q). \quad (95)$$

IV. CP ASYMMETRIES AND RESONANT ENHANCEMENT

In this section we compute the flavor-dependent CP asymmetries generated in the decays of heavy Majorana neutrinos N_i . The UV completion constructed in Sec. II generates two dimension-six dipole operators in νSMEFT , $\mathcal{O}_{NB}$, and $\mathcal{O}_{NW}$. After electroweak symmetry breaking (EWSB), these operators induce effective dipole couplings to the photon, the Z boson, and (through the charged components of $\mathcal{O}_{NW}$) the $W^\pm$ bosons. Consequently, the dipole interaction allows the two-body decay modes

$$N_i \rightarrow \nu_\alpha \gamma, \quad N_i \rightarrow \nu_\alpha Z, \quad N_i \rightarrow \ell_\alpha^\mp W^\pm, \quad (96)$$

as well as their CP -conjugate channels, which are summarized in Fig. 2. To set up the CP -asymmetry calculation

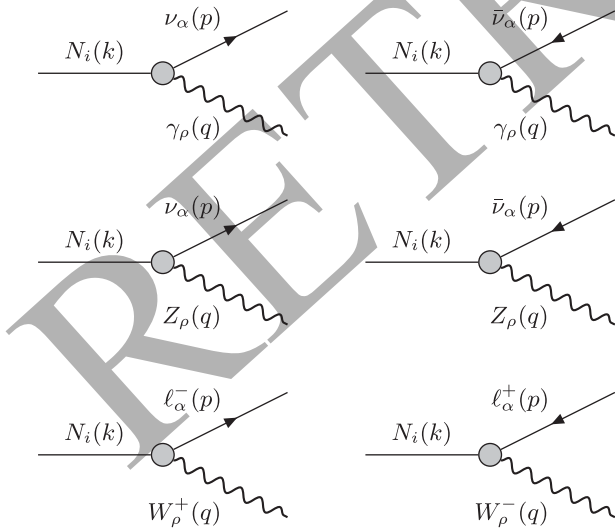


FIG. 2. Feynman diagrams for the two-body decays of N_i induced by the dipole operators $\mathcal{O}_{NB}, \mathcal{O}_{NW}$. The gray blob indicates that each effective vertex incorporates the UV information of the heavy fields E, Σ, D, Φ^+ through the Wilson coefficients C_{NB} and C_{NW} .

in a transparent manner, we first consider a fictitious limit in which only $\mathcal{O}_{NB}$ is present and only the photon channel contributes. This isolates the essential interference structure between the tree and the one-loop amplitudes. We then generalize the result to include the Z channel and the $W^\pm$ channel induced by $\mathcal{O}_{NW}$, reusing the photon-channel building blocks whenever possible.

For later reference, the Wilson coefficients are matched at the heavy scale $Q_{\text{match}} = 10$ TeV in our UV completion. For example, at leading order in the soft-breaking expansion one finds parametrically

$$\frac{C_{NB,ai}(\mu)}{\Lambda^2} \simeq -\frac{\mu_{\text{soft}}^2}{m_D^2} \frac{g' y_{\Sigma,\alpha} y_{H,i}^* \mu_s^*}{192\pi^2} \frac{1}{M_{\text{eq}}^3},$$

$$\frac{C_{NW,ai}(\mu)}{\Lambda^2} \simeq -\frac{\mu_{\text{soft}}^2}{m_D^2} \frac{g y_{\Sigma,\alpha} y_{H,i}^* \mu_s^*}{96\pi^2} \frac{1}{M_{\text{eq}}^3}, \quad (97)$$

and the low-scale coefficients $C_{NX}(Q_{\text{ref}})$ ($X = B, W$) are obtained by RG evolution from Q_{match} down to $Q_{\text{ref}} = 150$ GeV as discussed in Sec. III. In the remainder of this section we use these low-scale coefficients as fixed inputs for the CP -asymmetry calculation, i.e., we evaluate the effective dipole couplings induced after EWSB at the reference scale $Q = Q_{\text{ref}}$. Any residual Q dependence beyond this choice is higher order than our one-loop, RGE-improved accuracy and will be neglected.

A. Effective dipole interaction in the photon channel

In Sec. IV, we consider, as a toy limit, the effective interaction that induces only the photon mode after electroweak symmetry breaking. The weak mixing angle θ_W appears because the hypercharge field strength $B_{\mu\nu}$ contained in $\mathcal{O}_{NB}$ is rotated into the photon and Z field strengths in the mass eigenbasis,

$$B_{\mu\nu} = F_{\mu\nu} \cos \theta_W - Z_{\mu\nu} \sin \theta_W, \quad (98)$$

$$W_{\mu\nu}^3 = F_{\mu\nu} \sin \theta_W + Z_{\mu\nu} \cos \theta_W, \quad (99)$$

where the weak mixing angle is defined by

$$\cos \theta_W := \frac{g}{\sqrt{g'^2 + g^2}}, \quad \sin \theta_W := \frac{g'}{\sqrt{g'^2 + g^2}}, \quad (100)$$

with $\sin^2 \theta_W \simeq 0.23$. Keeping only the photon component in this toy limit, we obtain the effective interaction after EWSB by inserting the Higgs VEV into $\tilde{H}$ and projecting the lepton doublet onto its neutrino component. In unitary gauge, one may write

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}, \quad \tilde{H} = i\sigma^2 H^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v+h \\ 0 \end{pmatrix}, \quad (101)$$

so that

$$\begin{aligned}\mathcal{O}_{NB} &= (\bar{L}_\alpha \sigma^{\mu\nu} P_R N_i) \tilde{H} B_{\mu\nu} \\ &\rightarrow \frac{v+h}{\sqrt{2}} (\bar{\nu}_\alpha \sigma^{\mu\nu} P_R N_i) F_{\mu\nu} \cos \theta_W, \quad (102)\end{aligned}$$

where we used Eq. (98) and kept only the photon piece. The dipole operator $\mathcal{O}_{NB}$ then yields

$$\begin{aligned}-\mathcal{L}_{\text{eff}} &\supset \frac{(v+h) \cos \theta_W}{\sqrt{2} \Lambda^2} C_{NB,ai}(Q_{\text{ref}}) \\ &\times (\bar{\nu}_\alpha \sigma^{\mu\nu} P_R N_i) F_{\mu\nu} + \text{H.c.} \quad (103)\end{aligned}$$

It is convenient to define the effective dipole coupling

$$\mu_{ai} := \frac{v \cos \theta_W}{\sqrt{2} \Lambda^2} C_{NB,ai}(Q_{\text{ref}}), \quad (104)$$

where μ_{ai} has mass dimension -1 , and

$$\begin{aligned}-\mathcal{L}_{\text{eff}} &\supset \mu_{ai} (\bar{\nu}_\alpha \sigma^{\mu\nu} P_R N_i) F_{\mu\nu} \\ &+ \mu_{ai} \frac{h}{v} (\bar{\nu}_\alpha \sigma^{\mu\nu} P_R N_i) F_{\mu\nu} + \text{H.c.} \quad (105)\end{aligned}$$

The first term mediates the two-body decays $N_i \rightarrow \nu_\alpha \gamma$ and $N_i \rightarrow \bar{\nu}_\alpha \gamma$ that define the photon-mode CP asymmetry in this subsection. The second term contains one physical Higgs field and therefore induces three-body decays such as $N_i \rightarrow \nu_\alpha \gamma h$ (and the CP -conjugate channel). These do not interfere with the two-body decay rates at leading order, and their contribution to ε_{ai}^γ is phase-space suppressed compared to the two-body channel. We therefore neglect the Higgs-emission term proportional to h in what follows and keep only the first term in Eq. (105) for the photon-mode analysis [51,52]:

$$-\mathcal{L}_{\text{eff}} \supset \mu_{ai} (\bar{\nu}_\alpha \sigma^{\mu\nu} P_R N_i) F_{\mu\nu} + \text{H.c.} \quad (106)$$

In this section we compute the decay widths and CP asymmetries in the photon channel (cf. [53]). The Z channel and the W channel induced by $\mathcal{O}_{NW}$ are discussed in Sec. V and below.

B. General formula for the flavored CP asymmetry

We denote the partial decay width

$$\Gamma_{ai} := \Gamma(N_i \rightarrow \nu_\alpha \gamma), \quad (107)$$

and its CP conjugate width

$$\bar{\Gamma}_{ai} := \Gamma(N_i \rightarrow \bar{\nu}_\alpha \gamma). \quad (108)$$

The flavor-dependent CP asymmetry in the photon channel is defined by

$$\varepsilon_{ai}^\gamma := \frac{\Gamma_{ai} - \bar{\Gamma}_{ai}}{\Gamma_{\text{tot}}}, \quad (109)$$

where Γ_{tot} is the total decay width of N_i (including all open channels).

For the effective Lagrangian in Eq. (106), the tree-level amplitude is

$$\mathcal{M}_\gamma^{(0)} = \mu_{ai} \bar{u}_{\nu_\alpha}(p) \sigma^{\mu\nu} P_R u_i(k) \varepsilon_\mu^*(q) q_\nu. \quad (110)$$

Since the spinor and momentum structure will remain the same throughout, we will henceforth display only the coupling dependence explicitly.

Summing over a different heavy-neutrino generation N_m ($m \neq i$), the one-loop amplitude receives vertex and self-energy contributions. Schematically,

$$\mathcal{M}_\gamma^{(1)} = \sum_{m \neq i} \mu_{am} \mu_{\beta m} \mu_{\beta i}^* I_{\text{loop}}(x_{mi}) \cdots, \quad (111)$$

where $I_{\text{loop}}(x)$ is a loop factor whose imaginary part becomes nonzero in the quasidegenerate limit $x \simeq 1$ (after the appropriate width regularization), and $x_{mi} := M_m^2/M_i^2$ ($m \neq i$) denotes the squared mass ratio of the heavy neutrinos. The dots indicate the same external spinor and momentum structure as in Eq. (110).

Separating the tree squared, the interference terms, and the one-loop squared, and neglecting $\mathcal{O}(\mu^6)$, it is convenient to factor out the universal two-body phase-space factor

$$\Phi_2 := \int d\Phi_2 = \frac{\|\mathbf{q}\|}{8\pi M_i^2} = \frac{1}{16\pi M_i}, \quad (112)$$

where $\mathbf{q}$ is the three-momentum of the photon in the rest frame of N_i , and $m_\nu = m_\gamma = 0$. We then write the phase-space integrated partial width as

$$\Gamma_{ai} = \Phi_2 \left\{ |\mu_{ai}|^2 I_{\text{tree}} + \sum_{m \neq i} [A_{\mu,aim}^{(5)} I_{\text{loop}}(x_{mi}) + \text{c.c.}] \right\}, \quad (113)$$

where I_{tree} denotes the (real) kinematic factor obtained from the spin-summed tree-level matrix element, and $I_{\text{loop}}(x)$ is the corresponding loop factor entering the tree-loop interference. Here $x_{mi} := M_m^2/M_i^2$. The coefficient $A_{\mu,aim}^{(5)}$ collects the product of effective dipole couplings appearing in the tree-loop interference term,

$$A_{\mu,aim}^{(5)} := \mu_{ai}^* \mu_{am} \sum_\beta \mu_{\beta m} \mu_{\beta i}^* = \mu_{ai}^* \mu_{am} (\mu^\dagger \mu)_{im}. \quad (114)$$

The superscript “(5)” is a mnemonic indicating that these couplings originate from the dimension-five dipole interaction after EWSB; correspondingly, the interference term in Eq. (113) is of order μ^4 .

The CP -conjugate width is

$$\bar{\Gamma}_{ai} = \Phi_2 \left\{ |\mu_{ai}|^2 I_{\text{tree}} + \sum_{m \neq i} [A_{\mu,aim}^{(5)*} I_{\text{loop}}(x_{mi}) + \text{c.c.}] \right\}. \quad (115)$$

Subtracting Eqs. (113) and (115), the tree-level contribution cancels and we obtain

$$\Gamma_{ai} - \bar{\Gamma}_{ai} = -4\Phi_2 \sum_{m \neq i} \text{Im}(A_{\mu,aim}^{(5)}) \text{Im}[I_{\text{loop}}(x_{mi})]. \quad (116)$$

Dividing by Γ_{tot} yields the general formula for the flavored CP asymmetry [53],

$$\varepsilon_{ai}^{\gamma} = -\frac{4\Phi_2}{\Gamma_{\text{tot}}} \sum_{m \neq i} \text{Im}(A_{\mu,aim}^{(5)}) \text{Im}[I_{\text{loop}}(x_{mi})]. \quad (117)$$

C. Tree-level total width in the photon channel

The Feynman rule for the photon channel follows from the dimension-five effective interaction (106) and the effective dipole coupling (104). Using $q_{\mu} = -i\partial_{\mu}$, the three-point vertex factor is

$$N_i(k) \rightarrow \nu_{\alpha}(p) + \gamma_{\rho}(q): 2\mu_{ai}\sigma^{\rho\lambda}q_{\lambda}P_R, \quad (118)$$

where we used the antisymmetry of $\sigma^{\mu\nu}$ and $F_{\mu\nu}$ to write $\sigma^{\mu\nu}F_{\mu\nu} = 2\sigma^{\mu\nu}\partial_{\mu}A_{\nu}$.

The tree-level amplitude for $N_i \rightarrow \nu_{\alpha}\gamma_{\rho}$ is therefore

$$i\mathcal{M}_{\gamma}^{(0)} = \mu_{ai}\bar{u}_{\alpha}(p)(2\sigma^{\rho\lambda}q_{\lambda}P_R)u_i(k)\varepsilon_{\rho}^{*}(q). \quad (119)$$

Squaring the amplitude, averaging over the initial spin, and summing over final-state spins and photon polarizations, we obtain

$$\begin{aligned} \langle |\mathcal{M}_{\gamma}^{(0)}|^2 \rangle &= \frac{1}{2} \sum_{s,s'} \sum_{\text{pol.}} |\mathcal{M}_{\gamma}^{(0)}(s,s')|^2 \\ &= -2|\mu_{ai}|^2 \text{tr}[\sigma^{\rho\lambda}q_{\lambda}P_R(k+M_i)P_L\sigma_{\rho}^{\beta}q_{\beta}\not{p}]. \end{aligned} \quad (120)$$

In the following we neglect the tiny light-neutrino mass $m_{\nu} \simeq 0.05$ eV compared to $M_i = 1$ TeV. Using $p^2 = 0$, $q^2 = 0$, $k = p + q$, and the identities

$$\sigma^{\mu\nu}q_{\nu} = \frac{i}{2}(\gamma^{\mu}\not{q} - \not{q}\gamma^{\mu}), \quad (121)$$

$$\sigma^{\mu\nu}\gamma^{\rho} = i(g^{\nu\rho}\gamma^{\mu} - g^{\mu\rho}\gamma^{\nu}) - \epsilon^{\mu\nu\rho\sigma}\gamma_{\sigma}\gamma_5, \quad (122)$$

we obtain

$$\text{tr}[\sigma^{\rho\lambda}q_{\lambda}P_R(k+M_i)P_L\sigma_{\rho}^{\beta}q_{\beta}\not{p}] = -8(k \cdot q)(p \cdot q). \quad (123)$$

Here we dropped the $\epsilon^{\mu\nu\rho\sigma}$ term using the antisymmetry of $\epsilon^{\mu\nu\rho\sigma}$ and the symmetry of $q_{\mu}q_{\rho}$, and we used $q^2 = 0$ for the photon mode. From momentum conservation $k = p + q$ and $p^2 = q^2 = 0$, we have

$$k \cdot p = (p + q) \cdot p = p^2 + p \cdot q = p \cdot q, \quad (124)$$

$$k \cdot q = (p + q) \cdot q = p \cdot q + q^2 = p \cdot q, \quad (125)$$

and $k^2 = M_i^2$ implies

$$M_i^2 = (p + q)^2 = 2p \cdot q \Rightarrow p \cdot q = \frac{M_i^2}{2}. \quad (126)$$

Collecting these results,

$$p \cdot q = \frac{M_i^2}{2}, \quad k \cdot p = \frac{M_i^2}{2}, \quad k \cdot q = \frac{M_i^2}{2}. \quad (127)$$

Substituting into Eq. (123), we obtain

$$\text{tr}[\sigma^{\rho\lambda}q_{\lambda}P_R(k+M_i)P_L\sigma_{\rho}^{\beta}q_{\beta}\not{p}] = -2M_i^4. \quad (128)$$

Inserting this into Eq. (120), we find

$$\langle |\mathcal{M}_{\gamma}^{(0)}|^2 \rangle = 4|\mu_{ai}|^2 M_i^4, \quad (129)$$

for both neutrino and antineutrino final states. For massless light neutrinos, the spin-summed squared amplitude is identical for $N_i \rightarrow \nu_{\alpha}\gamma_{\rho}$ and $N_i \rightarrow \bar{\nu}_{\alpha}\gamma_{\rho}$, so we include both channels by an overall factor of 2.

Including both contributions, the differential decay rate is

$$d\Gamma = 2 \cdot \frac{\|\mathbf{p}\|}{8\pi M_i^2} \langle |\mathcal{M}_{\gamma}^{(0)}|^2 \rangle \frac{d\Omega}{4\pi}, \quad (130)$$

where $\mathbf{p}_{\nu} = -\mathbf{p}_{\gamma} =: \mathbf{p}$ in the rest frame of N_i and

$$\|\mathbf{p}\| = \frac{\sqrt{\lambda(M_i^2, m_{\nu}^2, m_{\gamma}^2)}}{2M_i},$$

$$\lambda(a, b, c) := a^2 + b^2 + c^2 - 2ab - 2ac - 2bc, \quad (131)$$

where $\lambda(a, b, c)$ is the Källén function [54] defined in the second line of Eq. (131). The same expression will be used below for the Z and W channels by the replacements $(m_{\nu}, m_{\gamma}) \rightarrow (m_{\nu}, m_Z)$ and $(m_{\nu}, m_{\gamma}) \rightarrow (m_{\ell}, m_W)$, respectively. Neglecting the light-neutrino mass and using $m_{\gamma} = 0$, this reduces to

$$\|\mathbf{p}\| = \frac{\sqrt{\lambda(M_i^2, 0, 0)}}{2M_i} = \frac{M_i}{2}. \quad (132)$$

Integrating over the solid angle, the partial width for a fixed flavor α is [19,20,53]

$$\Gamma_{\gamma,ai}^{(0)} = 2 \times \frac{\|\mathbf{p}\|}{8\pi M_i^2} \langle |\mathcal{M}_\gamma^{(0)}|^2 \rangle = \frac{|\mu_{ai}|^2 M_i^3}{2\pi}. \quad (133)$$

Summing over $\alpha = e, \mu, \tau$, we obtain the total photon-channel width

$$\begin{aligned} \Gamma_{\gamma,i}^{(0)} &= \sum_\alpha \Gamma_{\gamma,ai}^{(0)} = \frac{M_i^3}{2\pi} \sum_\alpha |\mu_{ai}|^2 \\ &= \frac{v^2 \cos^2 \theta_W}{4\pi} \cdot \frac{M_i^3}{\Lambda^4} \sum_\alpha |C_{NB,ai}(Q_{\text{ref}})|^2. \end{aligned} \quad (134)$$

D. Photon-mode CP asymmetry away from quasidegeneracy

The loop-level derivation of the nonresonant photon-mode CP asymmetry is given in Appendix D. In the main text we quote only the result needed for the kinetic analysis. For the photon mode $N_i \rightarrow \nu_a \gamma$ and its CP -conjugate channel, the flavor-dependent CP asymmetry can be written as

$$\begin{aligned} \varepsilon_{ai}^\gamma &= -\frac{M_i^2}{2\pi \sum_\beta |\mu_{\beta i}|^2} \sum_{m \neq i} \text{Im}[\mu_{ai}^* \mu_{am} \\ &\quad \times \{(\mu^\dagger \mu)_{im} [f_{V_a}(x) + f_{S_a}(x)] + (\mu^\dagger \mu)_{mi} f_{S_b}(x)\}], \end{aligned} \quad (135)$$

with $x := M_m^2/M_i^2$. The real loop functions are

$$\begin{aligned} f_{V_a}(x) &= \sqrt{x} \left\{ 1 + 2x \left[1 - (x+1) \ln \frac{x+1}{x} \right] \right\}, \\ f_{S_a}(x) &= \frac{\sqrt{x}}{1-x}, \quad f_{S_b}(x) = \frac{1}{1-x}. \end{aligned} \quad (136)$$

For later use it is also useful to separate the vertex and self-energy pieces:

$$\varepsilon_{ai,\text{vert}}^\gamma = -\frac{M_i^2}{2\pi \sum_\beta |\mu_{\beta i}|^2} \sum_{m \neq i} \text{Im}[\mu_{ai}^* \mu_{am} (\mu^\dagger \mu)_{im} f_{V_a}(x)], \quad (137)$$

$$\begin{aligned} \varepsilon_{ai,\text{self}}^\gamma &= -\frac{M_i^2}{2\pi \sum_\beta |\mu_{\beta i}|^2} \\ &\quad \times \sum_{m \neq i} \text{Im} \left\{ \mu_{ai}^* \mu_{am} \left[(\mu^\dagger \mu)_{im} \frac{\sqrt{x}}{1-x} + (\mu^\dagger \mu)_{mi} \frac{1}{1-x} \right] \right\}. \end{aligned} \quad (138)$$

The singular behavior of the self-energy functions as $x \rightarrow 1$ is regulated in the quasidegenerate regime, as discussed next.

E. CP source in the quasidegenerate limit of the photon mode

The nonresonant CP asymmetry is given by Eq. (135). The self-energy loop functions $f_{S_a}(x) = \sqrt{x}/(1-x)$ and $f_{S_b} = 1/(1-x)$ defined in Eq. (136) diverge in the limit $x \rightarrow 1$, indicating a resonant enhancement [26,55–58]. The behavior of these loop functions is shown in Fig. 3.

In the quasidegenerate regime $M_i \simeq M_m$ ($m \neq i$), overlapping poles dominate the heavy-neutrino two-point function and the naive perturbative expansion breaks down [26,27,55–60]. In this section, we therefore distinguish the nonresonant expressions displaying the singular behavior from their counterparts regulated by Dyson-resummed propagators in the sense of Pilaftsis-Underwood (PU) [27]. We denote the Breit-Wigner (BW) regulated loop functions by $\tilde{f}_{S_{a,b}}$.

When the resonance occurs, the only modification of the naive Feynman rules is the Dyson resummation of the internal N_m propagator, which regulates the would-be pole appearing in the self-energy diagrams. Using the Dyson-resummed propagator

$$S_{N_m}(k) = \frac{i(k + M_m)}{k^2 - M_m^2 + iM_m \Gamma_m}, \quad (139)$$

the loop functions are regulated as

$$\begin{aligned} f_{S_a}(x) &\rightarrow \tilde{f}_{S_a}(x) = \frac{\sqrt{x}(1-x)}{(1-x)^2 + (\Gamma_m/M_i)^2}, \\ f_{S_b}(x) &\rightarrow \tilde{f}_{S_b}(x) = \frac{1-x}{(1-x)^2 + (\Gamma_m/M_i)^2}, \end{aligned} \quad (140)$$

where we use the vacuum width Γ_m in the regulator functions $\tilde{f}_{S_{a,b}}$. Defining the physical mass splitting

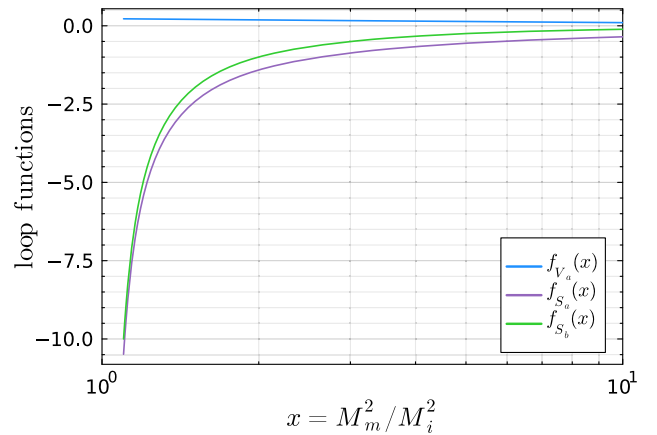


FIG. 3. Loop functions $f_{V_a}(x)$, $f_{S_a}(x)$, and $f_{S_b}(x)$. The self-energy functions $f_{S_a}(x)$ and $f_{S_b}(x)$ diverge as $x \rightarrow 1$ (i.e. in the quasidegenerate limit $M_m \simeq M_i$).

$$\begin{aligned}\Delta M &:= M_m - M_i, \\ \Delta M^2 &:= M_m^2 - M_i^2 = (M_m + M_i)\Delta M \\ &\simeq 2M_i\Delta M,\end{aligned}\quad (141)$$

we can rewrite Eq. (140) in terms of ΔM as

$$\begin{aligned}\tilde{f}_{S_a} &= -\frac{M_i M_m \Delta M^2}{(\Delta M^2)^2 + (M_i \Gamma_m)^2}, \\ \tilde{f}_{S_b} &= -\frac{M_i^2 \Delta M^2}{(\Delta M^2)^2 + (M_i \Gamma_m)^2},\end{aligned}\quad (142)$$

making the Breit-Wigner structure explicit. Schematically, the BW-type function

$$\mathcal{R}_{im}(\Delta M) \propto \frac{(\Delta M^2) M_i \Gamma_m}{(\Delta M^2)^2 + (M_i \Gamma_m)^2} \quad (m \neq i), \quad (143)$$

is maximized when the mass splitting balances the total width, yielding [28,61]

$$\Delta M \simeq \Gamma_m/2. \quad (144)$$

The derivation of Eq. (140) from the Dyson-resummed self-energy diagrams is given in Appendix E. In the main text we only use the resulting replacement

$$f_{S_a}(x) \rightarrow \tilde{f}_{S_a}(x), \quad f_{S_b}(x) \rightarrow \tilde{f}_{S_b}(x), \quad (145)$$

inside the self-energy part of the photon-mode CP asymmetry. Thus, in the quasidegenerate regime the resonantly regulated photon-mode CP asymmetry is obtained from the nonresonant expression by keeping the vertex-function $f_{V_a}(x)$ unchanged and replacing the singular self-energy loop functions by their Breit-Wigner-regulated counterparts.

Combining these ingredients, the photon-mode CP asymmetry used in the resonant analysis is

$$\begin{aligned}\epsilon_{ai}^{\gamma, \text{res}} &= -\frac{M_i^2}{2\pi \sum_{\beta} |\mu_{\beta i}|^2} \sum_{m \neq i} \text{Im}[\mu_{ai}^* \mu_{am}] \\ &\times \{(\mu^\dagger \mu)_{im} [f_{V_a}(x) + \tilde{f}_{S_a}(x)] + (\mu^\dagger \mu)_{mi} \tilde{f}_{S_b}(x)\},\end{aligned}\quad (146)$$

with $x := M_m^2/M_i^2$. This expression makes explicit that only the self-energy loop functions are resonantly regulated, while the vertex contribution remains nonsingular as $x \rightarrow 1$.

V. EFFECTS OF $\mathcal{O}_{NW}$ ON DECAY WIDTHS AND CP ASYMMETRIES

In this section we extend the photon-mode analysis of Sec. IV to the realistic broken-phase case in which both $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$ are present. The extension has three physical

consequences. First, the neutral component of $\mathcal{O}_{NW}$ contributes to the $W_{\mu\nu}^3$ dipole and therefore makes the photon and Z-boson dipole matrices independent in general. Second, the charged components of $\mathcal{O}_{NW}$ open the two-body channels $N_i \rightarrow \ell_\alpha^- W^+$ and $N_i \rightarrow \ell_\alpha^+ W^-$. Third, the on-shell cuts in the self-energy diagrams now include the intermediate states (ν, γ) , (ν, Z) , and (ℓ, W) . The detailed Cutkosky cuts, angular integrations, and finite-gauge-boson-mass-loop functions are collected in Appendix F. Here we quote only the broken-phase couplings, the tree-level widths, and the final CP asymmetries used in the QKEs.

After electroweak symmetry breaking, we define

$$\mu_{ai}^B := \frac{v}{\sqrt{2}\Lambda^2} C_{NB,ai}, \quad (147)$$

$$\mu_{ai}^3 := \frac{v}{2\sqrt{2}\Lambda^2} C_{NW,ai}, \quad (148)$$

$$\mu_{ai}^W := \frac{v}{2\Lambda^2} C_{NW,ai}. \quad (149)$$

The neutral broken-phase dipole matrices are

$$\mu_{ai}^\gamma = \mu_{ai}^B \cos \theta_W + \mu_{ai}^3 \sin \theta_W, \quad (150)$$

$$\mu_{ai}^Z = -\mu_{ai}^B \sin \theta_W + \mu_{ai}^3 \cos \theta_W. \quad (151)$$

Thus, the broken-phase interaction relevant for the two-body decays can be written as

$$\begin{aligned}-\mathcal{L}_{\text{int}} &\supset \mu_{ai}^\gamma \bar{\nu}_\alpha \sigma^{\mu\nu} P_R N_i F_{\mu\nu} + \mu_{ai}^Z \bar{\nu}_\alpha \sigma^{\mu\nu} P_R N_i Z_{\mu\nu} \\ &+ \mu_{ai}^W \bar{\ell}_\alpha \sigma^{\mu\nu} P_R N_i W_{\mu\nu}^- + \text{H.c.}\end{aligned}\quad (152)$$

In the limit $C_{NW} \rightarrow 0$, one recovers $\mu^W = 0$ and $\mu^Z = -\mu^\gamma \tan \theta_W$, so the neutral channels are aligned. With $\mathcal{O}_{NW}$ present, this alignment is lost.

For $V = \gamma, Z, W$, define

$$r_V := \frac{m_V^2}{M_i^2}, \quad R_V := (1 - r_V)^2 (1 + r_V). \quad (153)$$

The tree-level width in channel V , summed over the CP -conjugate final states within the same channel, is

$$\Gamma_{V,ai}^{(0)} = \frac{M_i^3}{2\pi} R_V |\mu_{ai}^V|^2, \quad V = \gamma, Z, W, \quad (154)$$

and

$$\Gamma_{V,i}^{(0)} = \sum_\alpha \Gamma_{V,ai}^{(0)}, \quad \Gamma_i^{(0)} = \sum_{V=\gamma,Z,W} \Gamma_{V,i}^{(0)}. \quad (155)$$

We denote by

$$x_{mi} := \frac{M_m^2}{M_i^2}, \quad r_U := \frac{m_U^2}{M_i^2}, \quad U = \gamma, Z, W \quad (156)$$

the mass ratios entering the loop functions. The finite-mass vertex-function $F_{V_a}(x; r_V, r_U)$ and the resonantly regulated self-energy functions $\tilde{F}_{S_a}(x; r_U)$, $\tilde{F}_{S_b}(x; r_U)$ are derived in Appendix F. They reduce to the photon-mode functions of Sec. IV in the massless limit,

$$F_{V_a}(x; 0, 0) = f_{V_a}(x), \quad (157)$$

$$\tilde{F}_{S_a}(x; 0) = \tilde{f}_{S_a}(x), \quad (158)$$

$$\tilde{F}_{S_b}(x; 0) = \tilde{f}_{S_b}(x). \quad (159)$$

We define the channel-normalized flavor-dependent CP asymmetry by

$$\hat{\varepsilon}_{ai}^V := \frac{\Gamma_{V,ai} - \bar{\Gamma}_{V,ai}}{\Gamma_{V,i}^{(0)}}. \quad (160)$$

The vertex contribution is

$$\begin{aligned} \hat{\varepsilon}_{\text{vert},ai}^{V,(a)} = & -\frac{M_i^2}{2\pi R_V \sum_{\beta} |\mu_{\beta i}^V|^2} \sum_{m \neq i} \sum_{U=\gamma,Z,W} \\ & \times [(1 - \delta_{VW})(1 - \delta_{UW}) + \delta_{VW}\delta_{UW}] \\ & \times \text{Im} \left[\mu_{ai}^{V*} \mu_{am}^U (\mu^{U\dagger} \mu^V)_{im} \right] F_{V_a}(x_{mi}; r_V, r_U). \end{aligned} \quad (161)$$

The factor in square brackets implements the channel-selection rule: for $V = \gamma, Z$, the internal boson can be $U = \gamma, Z$, whereas for $V = W$ one must take $U = W$.

The two self-energy contributions are

$$\begin{aligned} \hat{\varepsilon}_{\text{self},ai}^{V,(a)} = & -\frac{M_i^2}{2\pi R_V \sum_{\beta} |\mu_{\beta i}^V|^2} \sum_{m \neq i} \sum_{U=\gamma,Z,W} \\ & \times \text{Im} \left[\mu_{ai}^{V*} \mu_{am}^U (\mu^{U\dagger} \mu^U)_{im} \right] \tilde{F}_{S_a}(x_{mi}; r_U), \end{aligned} \quad (162)$$

$$\begin{aligned} \hat{\varepsilon}_{\text{self},ai}^{V,(b)} = & -\frac{M_i^2}{2\pi R_V \sum_{\beta} |\mu_{\beta i}^V|^2} \sum_{m \neq i} \sum_{U=\gamma,Z,W} \\ & \times \text{Im} \left[\mu_{ai}^{V*} \mu_{am}^U (\mu^{U\dagger} \mu^U)_{mi} \right] \tilde{F}_{S_b}(x_{mi}; r_U). \end{aligned} \quad (163)$$

Combining these terms, the CP asymmetry for the channel V is

$$\hat{\varepsilon}_{ai}^V = \hat{\varepsilon}_{\text{vert},ai}^{V,(a)} + \hat{\varepsilon}_{\text{self},ai}^{V,(a)} + \hat{\varepsilon}_{\text{self},ai}^{V,(b)}. \quad (164)$$

For the transport equations, we use the CP asymmetry normalized by the total width,

$$\varepsilon_{ai}^{\text{tot}} = \sum_{V=\gamma,Z,W} \frac{\Gamma_{V,i}^{(0)}}{\Gamma_i^{(0)}} \hat{\varepsilon}_{ai}^V. \quad (165)$$

This is the CP source used in the QKEs below.

VI. SCHWINGER-KELDYSH FORMALISM FOR NONEQUILIBRIUM QUANTUM FIELDS

In this work we track the real-time evolution of the right-handed-neutrino ensemble and of the associated flavor charges in the electroweak plasma. In the quasidegenerate regime, a standard Boltzmann description in terms of independent on-shell number densities is not sufficient, because the off-diagonal correlations of the nearly degenerate right-handed neutrinos, their damping, and the CP -odd source must be treated in a common framework. We therefore use the closed-time-path, or Schwinger-Keldysh, formalism [62–66]. In leptogenesis, this approach is particularly useful because it organizes the CP source, washout, and scattering contributions at the level of Wightman self-energies and avoids the real-intermediate-state subtraction required in a purely Boltzmann treatment [67–71].

For a generic fermion field Ψ , we define the contour-ordered propagator on the closed time path $\mathcal{C}$ by

$$\Delta(x, y) = \langle T_{\mathcal{C}} \Psi(x) \bar{\Psi}(y) \rangle. \quad (166)$$

Its lesser and greater components are

$$\Delta^>(x, y) = \langle \Psi(x) \bar{\Psi}(y) \rangle, \quad \Delta^<(x, y) = -\langle \bar{\Psi}(y) \Psi(x) \rangle. \quad (167)$$

The corresponding gauge-boson Wightman functions are defined analogously. Our sign conventions follow the detailed construction summarized in Appendix G.

The contour Schwinger-Dyson equation leads to the Kadanoff-Baym equations. In the Wigner representation [72],

$$X := \frac{x+y}{2}, \quad Y := x-y, \quad (168)$$

$$\Delta^{<,>}(K, X) := \int d^4 Y e^{iK \cdot Y} \Delta^{<,>} \left(X + \frac{Y}{2}, X - \frac{Y}{2} \right), \quad (169)$$

and at leading order in the gradient expansion, the collision term has the gain-loss structure

$$\mathcal{C}[\Delta] = \Sigma^>(K, X) \Delta^<(K, X) - \Sigma^<(K, X) \Delta^>(K, X), \quad (170)$$

where $\Sigma^{<,>}$ are the Wightman components of the self-energy. For gauge fields one obtains the analogous structure

$$C[D] = \Pi^>(K, X)D^<(K, X) - \Pi^<(K, X)D^>(K, X), \quad (171)$$

with $\Pi^{<>}$ the gauge-boson self-energies.

The density-matrix QKEs used below are obtained by applying this structure to the right-handed-neutrino two-point functions and projecting the Kadanoff-Baym equations onto the on-shell heavy-neutrino quasiparticle modes. The matrix structure in the heavy-neutrino flavor space is retained. This is essential in the quasidegenerate regime, where coherent oscillations and damping are of the same order as the CP -odd source.

A central advantage of the CTP formulation is that the CP source, inverse-decay washout, relaxation terms, and the leading $\Delta L = 0$ scattering contributions arise from the same Wightman self-energies. As a result, the kinetic equations are organized without double counting and do not require an additional real-intermediate-state subtraction.

The detailed construction of the 1PI/2PI effective actions, the contour generating functional, the Kadanoff-Baym equations, the Wigner transform, the leading-gradient expansion, and the quasiparticle limit is collected in Appendix G. In the following sections we use only the Wightman self-energies and their physical cuts to construct the collision terms.

VII. QUANTUM COLLISION TERMS: $1 \leftrightarrow 2$ DECAYS, INVERSE DECAYS, AND $2 \leftrightarrow 2$ SCATTERINGS

In this section we organize the collision kernels that enter the density-matrix QKEs. The $1 \leftrightarrow 2$ pole contribution is fixed by the decay widths and CP asymmetries derived in Secs. IV and V. The new ingredient constructed here is the leading $\Delta L = 0$ scattering contribution, which arises from the Landau-damping part of the gauge-boson Wightman functions. In the Schwinger-Keldysh formulation, the pole and Landau-damping pieces originate from the same heavy-neutrino self-energies, so they can be separated at the spectral level without introducing an additional real-intermediate-state subtraction [67–69,73].

We write the heavy-neutrino damping matrix as

$$\Gamma_N(z) = \Gamma_N^{\text{dec}}(z) + \Gamma_N^{\text{scatt}}(z), \quad (172)$$

where Γ_N^{dec} is the pole contribution associated with decays and inverse decays, while Γ_N^{scatt} is the Landau-damping contribution associated with $2 \leftrightarrow 2$ scatterings. Equivalently, in the dimensionless QKE notation used below, we introduce

$$D_i(z) := \frac{\gamma_{D,i}(T)}{zH(z)n_{N_i}^{\text{eq}}(T)}, \quad S_i(z) := \frac{\gamma_{S,i}(T)}{zH(z)n_{N_i}^{\text{eq}}(T)}. \quad (173)$$

Here $\gamma_{D,i}$ is the thermally averaged decay/inverse-decay reaction density, and $\gamma_{S,i}$ is the total $\Delta L = 0$ scattering

reaction density. The latter is the quantity computed in this section.

A. Pole and Landau-damping contributions

The lesser/greater self-energies $\Sigma_N^{<>}$ contain both pole and Landau-damping contributions. The pole part corresponds to the on-shell $1 \leftrightarrow 2$ processes

$$N_i \leftrightarrow \nu_a \gamma, \quad N_i \leftrightarrow \nu_a Z, \quad N_i \leftrightarrow \ell_a W, \quad (174)$$

and is already incorporated in the decay widths and CP asymmetries of Secs. IV and V. The Landau-damping part corresponds to scattering off plasma constituents and gives the leading $\Delta L = 0$ contribution at order $\mathcal{O}(|\mu^V|^2 g^2)$.

In this work we do not numerically solve the full Kadanoff-Baym equations. Instead, we use the density-matrix QKE obtained after the quasiparticle projection. The effect of scatterings is then summarized by Γ_N^{scatt} , or equivalently by $S_i(z)$. Concretely, we extract the scattering contribution from the Landau-damping part of the gauge-boson spectral function,

$$D^{<>}(K) = D_{\text{pole}}^{<>}(K) + D_{\text{LD}}^{<>}(K), \quad (175)$$

and define

$$\Sigma_{\text{scatt}}^{<>} := \Sigma^{<>}[D_{\text{LD}}^{<>}]. \quad (176)$$

This organization cleanly separates decays/inverse decays from genuine scatterings at the level of Wightman self-energies. It is the field-theoretic origin of the statement that no separate real-intermediate-state (RIS) subtraction is required in the kinetic equations.

B. Effective interactions after electroweak symmetry breaking

For the scattering calculation, we use the broken-phase dipole interactions already defined in Sec. V. We repeat them here only to fix the notation for the external channel $V = \gamma, Z, W$ and for the gauge boson exchanged with the plasma:

$$\begin{aligned} -\mathcal{L}_{\text{int}} \supset & \mu_{ai}^{\gamma} \bar{\nu}_a \sigma^{\mu\nu} P_R N_i F_{\mu\nu} + \mu_{ai}^Z \bar{\nu}_a \sigma^{\mu\nu} P_R N_i Z_{\mu\nu} \\ & + \mu_{ai}^W \bar{\ell}_a \sigma^{\mu\nu} P_R N_i W_{\mu\nu}^{-} + \text{H.c.} \end{aligned} \quad (177)$$

The relation between the Wilson coefficients C_{NB}, C_{NW} and the effective dipole couplings μ_{ai} is defined in Sec. IV as

$$\begin{aligned} \mu_{ai}^B &= \frac{v}{\sqrt{2}\Lambda^2} C_{NB}^{ai}, & \mu_{ai}^3 &= \frac{v}{2\sqrt{2}\Lambda^2} C_{NW}^{ai}, \\ \mu_{ai}^W &= \frac{v}{2\Lambda^2} C_{NW}^{ai}, \end{aligned} \quad (178)$$

and

$$\begin{aligned}\mu_{ai}^Y &= \mu_{ai}^B \cos \theta_W + \mu_{ai}^3 \sin \theta_W, \\ \mu_{ai}^Z &= -\mu_{ai}^B \sin \theta_W + \mu_{ai}^3 \cos \theta_W.\end{aligned}\quad (179)$$

Hence, the two genuinely new elements to be included are (i) μ^Y and μ^Z can be independent (due to $\mathcal{O}_{NW}$), and (ii) μ^W generates charged-current channels.

C. Why the leading scattering is $\Delta L = 0$

The scattering contribution Γ_N^{scatt} is extracted from the heavy-neutrino self-energy $\Sigma^{<,>}$. The lowest order is a one-loop diagram with two dipole insertions, giving $\mu\mu^\dagger \sim \mathcal{O}(\mu^2)$. By contrast, to generate a genuine $\Delta L = 2$ process (e.g. of $LL \leftrightarrow XX$ type, or $N \leftrightarrow \bar{L} + X$ type), one needs at least additional dipole insertions or a higher-loop structure, yielding at least $\mathcal{O}(\mu^4)$. Therefore, in the dipole-dominated regime the leading washout from scatterings is controlled by $\Delta L = 0$ channels, and we compute the $\mathcal{O}(\mu^2 g^2)$ $\Delta L = 0$ scatterings.

A key viewpoint is that, rather than enumerating and computing all individual $2 \rightarrow 2$ processes by hand, the polarization tensor $\Pi^{<,>}$ that appears in $\Sigma_{\text{scatt}}^{<,>}$ already contains all scattering channels present in the plasma. In practice, one may either (i) use the hard-thermal-loop (HTL) approximation for the finite-temperature gauge-boson polarization tensor $\Pi^{<,>}$, or (ii) use a bottom-up approximation by computing representative $2 \rightarrow 2$ cross sections and summing over plasma species.

D. List of $\Delta L = 0$ channels at $\mathcal{O}(\mu^2 g^2)$

We enumerate t -channel processes in which a heavy neutrino N_i scatters on a light plasma particle and converts into a lepton-doublet component. Neglecting chemical potentials, particles and antiparticles contribute equally, so it suffices to compute particle channels and add CP -conjugate channels.

(1) Neutral-current type (ν_α production):

$$N_i f \leftrightarrow \nu_\alpha f. \quad (180)$$

(a) Exchange boson: $V = \gamma, Z$ (and γZ interference).

(b) Target f : light SM fermions.

(i) γ exchange: all charged fermions (charged leptons and quarks).

(ii) Z exchange: all fermions (including neutrinos).

(c) Representative process:

$$N_i(p) + f(k) \rightarrow \nu_\alpha(p') + f(k'). \quad (181)$$

(2) Charged-current type (ℓ_α production):

$$N_i f \leftrightarrow \ell_\alpha f'. \quad (182)$$

(a) Exchange boson: $V = W$.

(b) The target transitions to its $SU(2)_L$ partner:

- (i) Leptons: $f = \ell_\beta, f' = \nu_\beta$ (or CP conjugate).
- (ii) Quarks: $f = d_b, f' = u_a$ (including CKM matrix element V_{ab} [74,75]), or CP conjugate.
- (c) Representative charge assignments:

$$N_i(p) + \ell_\beta^-(k) \rightarrow \ell_\alpha^-(p') + \nu_\beta(k'),$$

$$N_i(p) + d_b(k) \rightarrow \ell_\alpha^-(p') + u_a(k'), \quad (183)$$

and their CP conjugates (with W^- exchange).

These $\Delta L = 0$ scatterings are shown in Fig. 4.

E. Kadanoff-Baym definition: From $\Sigma_N^{<,>}$ to Γ_N

The scattering part of the heavy-neutrino self-energy (for a gauge boson V) is

$$\begin{aligned}\Sigma_{\text{scatt},ij}^{<,>}(K) &= -\sum_\alpha \int \frac{d^4 Q}{(2\pi)^4} (\Gamma^V)_{ai}^\mu(Q) S_{L,\alpha}^{<,>}(K-Q) \\ &\quad \times (\bar{\Gamma}^V)_{aj}^\nu(Q) D_{V,\mu\nu}^{<,>}(Q),\end{aligned}\quad (184)$$

where $S_{L,\alpha}^{<,>}$ is the internal lepton line (ν_α for NC, ℓ_α for CC), and $D_{V,\mu\nu}^{<,>}$ is the gauge-boson Wightman function. The scattering contribution corresponds to the Landau-damping part of $D^{<,>}$ and is given by the Keldysh relation

$$D_{V,\mu\nu}^{<,>}(Q) = D_{V,\mu\rho}^R(Q) \Pi_{V,\rho\sigma}^{<,>}(Q) D_{V,\sigma\nu}^A(Q), \quad (185)$$

where $\Pi_V^{<,>}$ is the polarization tensor and contains all scattering channels of plasma constituents.

Under the quasiparticle approximation, the collision term has the gain-loss structure

$$C_N \sim \Sigma_N^{>} S_N^{<} - \Sigma_N^{<} S_N^{>}. \quad (186)$$

The scattering width Γ_N^{scatt} is read off from this structure. In the remainder, we make this extraction explicit by reducing it to $2 \rightarrow 2$ cross sections.

F. Scattering equilibration function $S_i(z)$

The detailed derivation of the neutral-current and charged-current cross sections is given in Appendix H.

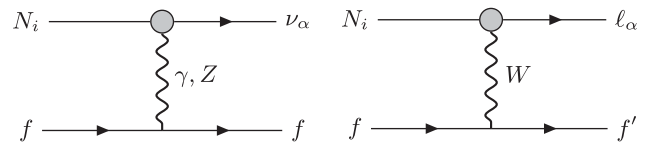


FIG. 4. Representative t -channel $\Delta L = 0$ scatterings at $\mathcal{O}(\mu^2 g^2)$, in which a heavy neutrino N_i scatters off a light plasma fermion f and converts into a lepton-doublet component. Left: neutral-current processes $N_i f \leftrightarrow \nu_\alpha f$ mediated by γ and Z exchange (including γZ interference). Right: charged-current processes $N_i f \leftrightarrow \ell_\alpha f'$ mediated by W exchange, where f' denotes the $SU(2)_L$ partner of f . CP -conjugate channels are implied.

Here we keep only the reaction-density definitions needed for the QKEs.

We decompose the total $\Delta L = 0$ scattering reaction density as

$$\gamma_{S,i}(T) = \gamma_{S,i}^{\text{NC}}(T) + \gamma_{S,i}^{\text{CC}}(T), \quad (187)$$

and define the scattering equilibration function by

$$S_i(z) := \frac{\gamma_{S,i}(T)}{zH(T)n_{N_i}^{\text{eq}}(T)}, \quad z := \frac{M_i}{T}. \quad (188)$$

Equivalently, introducing the scattering rate

$$\Gamma_{\text{scatt},i}(T) := \frac{\gamma_{S,i}(T)}{n_{N_i}^{\text{eq}}(T)}, \quad (189)$$

one may write

$$S_i(z) = \frac{\Gamma_{\text{scatt},i}(T)}{zH(T)}. \quad (190)$$

For the QKEs below, we only need the diagonal scattering equilibration functions $S_i(z)$. The channel-resolved cross sections are collected in Appendix H; here we keep only the minimal decomposition required for the numerical implementation.

We define

$$(\mu^{V\dagger}\mu^U)_{ii} := \sum_a \mu_{ai}^{V*}\mu_{ai}^U, \quad V, U = \gamma, Z, W. \quad (191)$$

Then the total $\Delta L = 0$ scattering reaction density can be written as

$$\gamma_{S,i}(T) = \hat{\gamma}_i^\gamma(T)(\mu^{\gamma\dagger}\mu^\gamma)_{ii} + \hat{\gamma}_i^Z(T)(\mu^{Z\dagger}\mu^Z)_{ii} + \hat{\gamma}_i^{\gamma Z}(T)\text{Re}(\mu^{\gamma\dagger}\mu^Z)_{ii} + \hat{\gamma}_i^W(T)(\mu^{W\dagger}\mu^W)_{ii}. \quad (192)$$

Here $\hat{\gamma}_i^Y$ denotes the coefficient reaction density with the dipole-coupling dependence stripped off. Explicitly,

$$\hat{\gamma}_i^Y(T) := g_N \frac{T}{32\pi^4} \int_{M_i^2}^{\infty} ds \frac{\Delta^2}{\sqrt{s}} K_1\left(\frac{\sqrt{s}}{T}\right) \hat{\sigma}_Y(s; T), \quad (193)$$

$$\Delta := s - M_i^2, \quad Y = \gamma, Z, \gamma Z, W. \quad (194)$$

The explicit expressions for $\hat{\sigma}_\gamma$, $\hat{\sigma}_Z$, $\hat{\sigma}_{\gamma Z}$, and $\hat{\sigma}_W$, including Debye screening for photon exchange and the sums over plasma species, are given in Appendix H.

Using

$$n_{N_i}^{\text{eq}}(T) = g_N \frac{M_i^2 T}{2\pi^2} K_2\left(\frac{M_i}{T}\right), \quad (195)$$

we define

$$\Gamma_{\text{scatt},i}(T) := \frac{\gamma_{S,i}(T)}{n_{N_i}^{\text{eq}}(T)}. \quad (196)$$

The scattering equilibration function entering the heavy-neutrino QKE is therefore

$$S_i(z) = \frac{\Gamma_{\text{scatt},i}(T)}{zH(T)} = \frac{\gamma_{S,i}(T)}{zH(T)n_{N_i}^{\text{eq}}(T)}. \quad (197)$$

These are the only scattering inputs required in the numerical QKEs below.

VIII. FROM KADANOFF-BAYM EQUATIONS TO DENSITY-MATRIX QUANTUM KINETIC EQUATIONS

In this section we assemble the density-matrix quantum kinetic equations used in the numerical analysis. The detailed projection from the Kadanoff-Baym equations, including the Wigner transform, the leading-gradient expansion, the quasiparticle ansatz, the Hermitization of the collision term, the index convention relating Wightman functions to density matrices, and the derivation of the flavor-charge source and washout terms, is given in Appendix I. Here we keep only the physical structure and the final QKE system.

The heavy-neutrino collision term is expressed in terms of the Wightman self-energies $\Sigma_N^{\leq, >}$. With HTL-resummed gauge propagators, the gauge-boson spectral density separates into quasiparticle poles and a Landau-damping cut. In the one-loop heavy-neutrino collision term, these two domains map respectively onto the $1 \leftrightarrow 2$ decay/inverse-decay contribution and the leading $\Delta L = 0$ scattering contribution. Since the two contributions have disjoint spectral support, there is no double counting between decays and scatterings, and no additional real-intermediate-state subtraction is required [67–69,73].

The gauge-parameter independence of the final scattering kernels follows from current conservation and transverse projection in the scattering amplitudes, as shown explicitly in Appendix H. This ensures that the computed Y_B^{FO} is a physical quantity within the adopted quasiparticle and leading-gradient truncation.

A. Heavy-neutrino density-matrix equation

The projection of the heavy-neutrino Kadanoff-Baym equation gives, for on-shell quasiparticles in an Friedmann-Lemaître-Robertson-Walker (FLRW) background,

$$\begin{aligned} & (\partial_t - H\mathbf{k} \cdot \nabla_{\mathbf{k}})f_N(\mathbf{k}, t) + i[\Omega_N(\mathbf{k}, t), f_N(\mathbf{k}, t)] \\ & = -\frac{1}{2}\{\Gamma_N(\mathbf{k}, t), f_N(\mathbf{k}, t) - f_N^{\text{eq}}(\mathbf{k}, t)\}. \end{aligned} \quad (198)$$

Here f_N is a matrix-valued distribution function in the heavy-neutrino generation space. The Hermitian matrix Ω_N describes coherent evolution, including the vacuum mass splitting and dispersive medium effects, while Γ_N is the damping matrix.

After momentum averaging, the pole part of Γ_N defines the decay/inverse-decay equilibration matrix $D(z)$, while the Landau-damping part defines the scattering equilibration matrix $S(z)$. In the numerical implementation we work in the heavy-neutrino mass basis and take these matrices to be diagonal,

$$D_{ij}(z) = D_i(z_i)\delta_{ij}, \quad S_{ij}(z) = S_i(z_i)\delta_{ij}, \quad z_i := \frac{M_i}{T}. \quad (199)$$

The decay kernel is defined from the total tree-level width of N_i ,

$$\Gamma_i^{\text{dec}} := \sum_{\alpha=e,\mu,\tau} \sum_{V=\gamma,Z,W} \Gamma_{V,ai}^{(0)}, \quad (200)$$

where the channel widths $\Gamma_{V,ai}^{(0)}$ are derived in Secs. IV and V. In the Maxwell-Boltzmann approximation, the thermally averaged decay rate is

$$\bar{\Gamma}_{D,i}(T) = \frac{K_1(z_i)}{K_2(z_i)} \Gamma_i^{\text{dec}}, \quad (201)$$

and hence

$$D_i(z_i) := \frac{\bar{\Gamma}_{D,i}(T)}{z_i H(T)} = \frac{\Gamma_i^{\text{dec}}}{z_i H(T)} \frac{K_1(z_i)}{K_2(z_i)}. \quad (202)$$

The scattering kernel is defined from the $\Delta L = 0$ scattering rate constructed in Sec. VII,

$$S_i(z_i) := \frac{\Gamma_{\text{scatt},i}(T)}{z_i H(T)}. \quad (203)$$

With these definitions, the yield-level heavy-neutrino QKE is

$$\frac{dY_N}{dz} = -i[\Omega_N(z), Y_N] - \frac{1}{2}\{D(z) + S(z), Y_N - Y_N^{\text{eq}}\}. \quad (204)$$

Here Y_N is the heavy-neutrino yield matrix. The equilibrium quantity Y_N^{eq} denotes the diagonal equilibrium yield matrix in the heavy-neutrino mass basis. We do not assume a common equilibrium abundance for all heavy-neutrino generations.

B. Flavor-charge equation

The CP -odd source for the flavor charge Y_{Δ_α} is generated by the $\Delta L = 1$ decays and inverse decays. The detailed index bookkeeping between the heavy-neutrino

Wightman function and the density matrix is given in Appendix I. With our conventions, the source term can be written as

$$\left. \frac{dY_{\Delta_\alpha}}{dz} \right|_{\text{source}} = \text{tr}[\varepsilon_\alpha D(z)(Y_N - Y_N^{\text{eq}})], \quad (205)$$

where ε_α is the CP -asymmetry matrix in the heavy-neutrino generation space. For diagonal Y_N , this reduces to the familiar Boltzmann form

$$\left. \frac{dY_{\Delta_\alpha}}{dz} \right|_{\text{source}} = \sum_i \varepsilon_{ai} D_i(z_i)(Y_{N_i} - Y_{N_i}^{\text{eq}}). \quad (206)$$

The same lepton Wightman self-energies determine the washout. Their pole part gives inverse-decay washout, while their Landau-damping part gives the leading $\Delta L = 0$ scattering washout. We parametrize the washout contribution as

$$\left. \frac{dY_{\Delta_\alpha}}{dz} \right|_{\text{washout}} = - \sum_\beta [W_{\alpha\beta}^{\text{ID}}(z) + W_{\alpha\beta}^{\text{scatt}}(z)] Y_{\Delta_\beta}. \quad (207)$$

Here $W_{\alpha\beta}^{\text{ID}}$ is determined by the inverse-decay kernels and flavor projectors, while $W_{\alpha\beta}^{\text{scatt}}$ is determined by the $\Delta L = 0$ scattering kernels of Sec. VII. Their detailed derivation from the lepton collision term is given in Appendix I.

C. Final density-matrix QKE system

Combining the heavy-neutrino equation with the source and washout terms, the QKE system solved in Sec. IX is

$$\begin{aligned} \frac{dY_N}{dz} &= -i[\Omega_N(z), Y_N] \\ &\quad - \frac{1}{2}\{D(z) + S(z), Y_N - Y_N^{\text{eq}}\}, \end{aligned} \quad (208)$$

$$\begin{aligned} \frac{dY_{\Delta_\alpha}}{dz} &= \text{tr}[\varepsilon_\alpha D(z)(Y_N - Y_N^{\text{eq}})] \\ &\quad - \sum_\beta [W_{\alpha\beta}^{\text{ID}}(z) + W_{\alpha\beta}^{\text{scatt}}(z)] Y_{\Delta_\beta}. \end{aligned} \quad (209)$$

The first equation describes coherent heavy-neutrino evolution, relaxation by decays and inverse decays, and the additional damping induced by the leading $\Delta L = 0$ scatterings. The second equation describes the CP -odd source and the flavor-resolved washout of the charges Y_{Δ_α} .

Finally, electroweak sphalerons convert $B - L$ into B until sphaleron freeze-out. We therefore evaluate the frozen-out baryon asymmetry as

$$Y_B^{\text{FO}} = c_{\text{sph}}(T_{\text{sph}}) \sum_{\alpha=e,\mu,\tau} Y_{\Delta_\alpha}(T_{\text{sph}}). \quad (210)$$

The sphaleron conversion factor $c_{\text{sph}}(T_{\text{sph}})$ was defined in Sec. II A. Equations (208)–(210) are the starting point for the numerical analysis below.

IX. RESULTS

Hereafter, we focus on the quasidegenerate pair N_1, N_2 and set the generation indices as $i = 1$ and $m = 2$. In this section we solve the density-matrix QKE system assembled in Sec. VIII. The CP -odd source is built from the decay CP asymmetries derived in Secs. IV and V, while the inverse-decay and $\Delta L = 0$ scattering kernels are constructed in Sec. VII and incorporated into the QKEs in Sec. VIII.

The purpose of the numerical analysis is threefold. First, we exhibit representative time evolutions of the heavy-neutrino density matrix and the flavor charges across the electroweak window. Second, we scan the effective electromagnetic neutrino mass $\tilde{m}_1^{\text{EM}}$ to identify the region where the resonantly enhanced dipole source can reproduce or exceed the observed BAU. Third, we compare the full result with a computation in which the leading $\Delta L = 0$ scatterings are switched off, thereby quantifying the numerical impact of the scattering kernel derived in Sec. VII.

The frozen-out baryon asymmetry is evaluated using Eq. (210).

A. Conventions and definitions

We consider two right-handed neutrinos N_1 and N_2 and denote the temperature variable by

$$z := \frac{M_1}{T}. \quad (211)$$

The yield of N_1 is defined as $Y_{N_1} := n_{N_1}/s$, where s is the entropy density. For the equilibrium yield we use the Maxwell-Boltzmann approximation,

$$Y_{N_1}^{\text{eq}}(z) = \frac{45g_N}{4\pi^4 g_{\star s}} z^2 K_2(z), \quad g_N = 2, \quad (212)$$

with K_2 the modified Bessel function of the second kind.

To parametrize the overall strength of the dipole-induced interactions we define the effective electromagnetic neutrino mass [46] by the combination that controls the total decay width,

$$\tilde{m}_1^{\text{EM}} := v^2 M_1 \sum_{\alpha} \sum_{V=\gamma, Z, W} R_V(M_1) |\mu_{\alpha 1}^V|^2. \quad (213)$$

The phase-space factors are

$$R_V(M_1) = (1 - r_V)^2 (1 + r_V), \quad r_V = \frac{m_V^2}{M_1^2}. \quad (214)$$

With these definitions the partial widths read

$$\Gamma_{V,1} = \frac{M_1^3}{2\pi} R_V(M_1) \sum_{\alpha} |\mu_{\alpha 1}^V|^2, \quad (215)$$

so that the total width satisfies

$$\Gamma_1 = \Gamma_{\gamma,1} + \Gamma_{Z,1} + \Gamma_{W,1} = \frac{M_1^2}{2\pi v^2} \tilde{m}_1^{\text{EM}}. \quad (216)$$

The decay parameter is then defined as

$$K_1 := \frac{\Gamma_1}{H(T=M_1)}, \quad H(T=M_1) = 1.66 \sqrt{g_{\star}} \frac{M_1^2}{M_{\text{Pl}}}, \quad (217)$$

and can be written in the standard form

$$K_1 = \frac{\tilde{m}_1^{\text{EM}}}{m_{\star}^{\text{EM}}}, \quad m_{\star}^{\text{EM}} := \frac{2\pi \times 1.66 \sqrt{g_{\star}} v^2}{M_{\text{Pl}}} \simeq 5.35 \times 10^{-4} \text{ eV}, \quad (218)$$

with $g_{\star} = 106.75$. Accordingly, $K_1 = 1$ corresponds to $\tilde{m}_1^{\text{EM}} = m_{\star}^{\text{EM}} \simeq 5.35 \times 10^{-4} \text{ eV}$ and marks the approximate boundary between the weak- and strong-washout regimes. We use the standard terminology in which $K_1 \ll 1$ denotes the weak-washout regime, where inverse decays and scatterings are inefficient compared with the Hubble expansion, whereas $K_1 \gg 1$ denotes the strong-washout regime, where these processes efficiently drive the system toward the quasiequilibrium trajectory. Thus, the value $K_1 = 1$ should be understood as a conventional crossover scale rather than as a sharp dynamical threshold.

We integrate the QKEs across the electroweak window and use the dimensionless temperature variable $z := M_1/T$, as already defined in Eq. (211). Furthermore, we denote

$$z_{\text{EW}} := \frac{M_1}{T_{\text{EW}}}, \quad z_{\text{sph}} := \frac{M_1}{T_{\text{sph}}}, \quad (219)$$

with $T_{\text{EW}} \simeq 160 \text{ GeV}$ and $T_{\text{sph}} \simeq 130 \text{ GeV}$. As an initial condition at $z = z_{\text{EW}}$ we take the heavy-neutrino density matrix to be diagonal and close to thermal equilibrium or zero,

$$Y_N(z_{\text{EW}}) = Y_N^{\text{eq}}(z_{\text{EW}}), \quad (220)$$

$$Y_N(z_{\text{EW}}) = 0, \quad (221)$$

and set the flavor charges to zero,

$$Y_{\Delta\alpha}(z_{\text{EW}}) = 0. \quad (222)$$

To probe the resonant regime, we adopt a quasidegenerate spectrum

$$M_2 = M_1 + \Delta M, \quad \Delta M = 5 \times 10^{-12} \text{ GeV}. \quad (223)$$

In our parameter scan we keep ΔM fixed for simplicity. Since the total width Γ_2 depends on the effective dipole coupling, the exact resonance condition $\Delta M \simeq \Gamma_2/2$ is realized only in a band of parameter space; fixing ΔM therefore allows us to track continuously how the Breit-Wigner enhancement competes with washout as the effective dipole strength is varied.

B. Benchmark input

For the UV completion we take the heavy masses to be degenerate at

$$M_E = m_{\Phi^+} = m_{\Sigma} = m_D =: M_{\text{eq}} = 10 \text{ TeV}. \quad (224)$$

For the right-handed neutrinos we choose

$$M_1 = 1 \text{ TeV}, \quad M_2 = (1 + 5 \times 10^{-15}) \text{ TeV}. \quad (225)$$

At the electroweak reference scale $Q_{\text{ref}} = 150 \text{ GeV}$ we use

$$g' = 0.357, \quad g = 0.652, \quad g_s = 1.17, \quad y_t = 0.93. \quad (226)$$

We parametrize the soft-breaking term as $V_{\text{soft}} = \mu_{\text{soft}}^2 H^\dagger D + \text{H.c.}$, so that μ_{soft} has mass dimension one; we quote μ_{soft} itself in the benchmark. For the Yukawa and dimensionful couplings in the UV model we adopt the representative values

$$\begin{aligned} y_H &= (1, e^{-0.7i})^T \times 10^{-3}, \\ y_\Sigma &= (10, 8e^{-0.3i}, 6e^{-1.1i})^T \times 10^{-4}, \\ \mu_{\text{soft}} &= 10 \text{ GeV}, \quad \mu_s = 10e^{-0.4i} \text{ GeV}, \end{aligned} \quad (227)$$

which realize a suppressed effective Yukawa coupling to the SM-like Higgs direction (see Sec. II) while generating the dipole operators at one loop. All other renormalizable couplings in the UV Lagrangian that do not enter the one-loop matching onto $\mathcal{O}_{NB}$, $\mathcal{O}_{NW}$ at the order considered (e.g. h_Σ) are irrelevant for the present numerical analysis and are left unspecified.

C. Numerical analysis

For the numerical analysis we adopt a minimal set of standard approximations. We integrate the density-matrix QKEs only over the electroweak crossover window, $T_{\text{EW}} \gtrsim T \gtrsim T_{\text{sph}}$, and evaluate the baryon asymmetry at sphaleron freeze-out. The kinetic description is based on the leading-gradient (first-order) truncation, with a quasi-particle/narrow-width treatment of the relevant excitations. In the benchmark regime inverse decays dominate the washout, while the leading $\Delta L = 0$ scatterings are included mainly as a consistency check and are found to induce only a small correction to the efficiency.

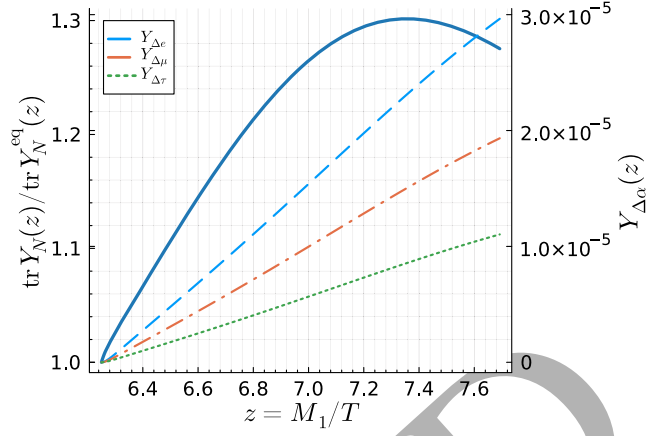


FIG. 5. Time evolution of the QKE solution for $\tilde{m}_1^{\text{EM}} = 10^{-3} \text{ eV}$ with thermal initial heavy-neutrino abundance. The solid curve, read from the left axis, shows the normalized trace of the heavy-neutrino density matrix, $\text{tr} Y_N(z)/\text{tr} Y_N^{\text{eq}}(z)$. The dashed, dash-dotted, and dotted curves, read from the right axis, show the flavor charges $Y_{\Delta\alpha}(z)$ for $\alpha = e, \mu, \tau$. The growth of the flavor charges over the electroweak window reflects the resonantly enhanced CP -odd source in this benchmark.

The numerical solutions of the density-matrix QKEs are shown in Figs. 5–7.

Figures 5 and 6 display the time evolution for a representative value $\tilde{m}_1^{\text{EM}} = 10^{-3} \text{ eV}$. The solid curve (left axis) shows

$$\frac{\text{tr} Y_N(z)}{\text{tr} Y_N^{\text{eq}}(z)} = \frac{Y_{11}(z) + Y_{22}(z)}{Y_{11}^{\text{eq}}(z) + Y_{22}^{\text{eq}}(z)}. \quad (228)$$

The dashed, dash-dotted, and dotted curves (right axis) show the flavor charges $Y_{\Delta\alpha}(z)$ for $\alpha = e, \mu, \tau$, respectively.

Since we take $M_1 = 1 \text{ TeV}$, the temperature variable $z = M_1/T$ evolves from

$$z = z_{\text{EW}} = 6.25 \rightarrow z = z_{\text{sph}} \simeq 7.69. \quad (229)$$

The normalized trace of the heavy-neutrino density matrix, $\text{tr} Y_N(z)/\text{tr} Y_N^{\text{eq}}(z)$, first increases and then decreases; note, however, that this does not imply that $\text{tr} Y_N(z)$ itself increases and then decreases in the same manner. In the interval $6.25 \leq z \lesssim 7.35$, the equilibrium trace $\text{tr} Y_N^{\text{eq}}(z)$ drops more rapidly than $\text{tr} Y_N(z)$, while $\text{tr} Y_N(z)$ still decreases, indicating that the heavy-neutrino decay reactions are operative. As a consequence, the flavor charges $Y_{\Delta\alpha}$ shown by the dashed curves increase nearly linearly with z over this short interval (cf. Sec. IX D).

Figure 6 shows the time evolution obtained with the zero initial abundance in Eq. (221), plotted in the same format as Fig. 5. The trace ratio $\text{tr} Y_N(z)/\text{tr} Y_N^{\text{eq}}(z)$ rises rapidly just after the onset of the electroweak window, $z_{\text{EW}} \leq z \lesssim 6.3$, as inverse decays populate the heavy-neutrino ensemble starting from zero abundance. It then continues to increase

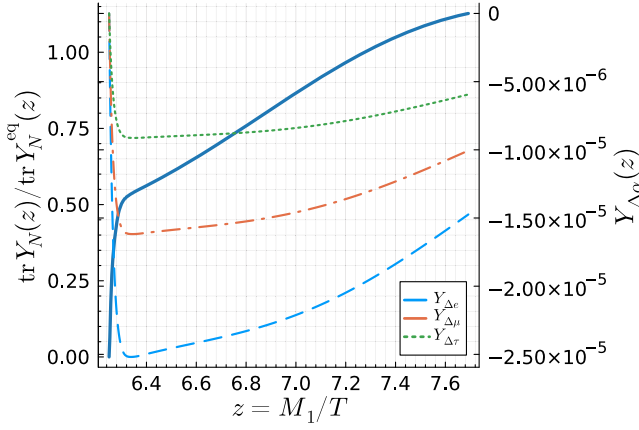


FIG. 6. Same as Fig. 5, but with zero initial heavy-neutrino abundance. Inverse decays first populate the heavy-neutrino ensemble, leading to a rapid rise of $\text{tr} Y_N(z)/\text{tr} Y_N^{\text{eq}}(z)$. For the illustrative point $\tilde{m}_1^{\text{EM}} = 10^{-3}$ eV, the flavor charges remain negative at sphaleron freeze-out, so this benchmark gives a negative Y_B^{FO} . Positive-BAU zero-initial-abundance points are found at larger $\tilde{m}_1^{\text{EM}}$, as shown in Fig. 7.

monotonically and can exceed unity before sphaleron freeze-out at $z = z_{\text{sph}} \simeq 7.69$, reflecting the fact that $\text{tr} Y_N^{\text{eq}}(z)$ decreases rapidly with z while $\text{tr} Y_N(z)$ follows with a finite relaxation rate. In contrast, the flavor charges $Y_{\Delta\alpha}(z)$ ($\alpha = e, \mu, \tau$) drop sharply from zero to negative values in the same early interval and subsequently approach the origin from below. For the benchmark point $\tilde{m}_1^{\text{EM}} = 10^{-3}$ eV, the charge densities remain negative at $z = z_{\text{sph}}$ with the CP -phase convention used in this scan. The corresponding signed frozen-out baryon asymmetry is therefore

$$Y_B^{\text{FO}} < 0, \quad (230)$$

as obtained from Eq. (210). We keep this signed value in order to display the phase convention and the sign change of the zero-initial-abundance branch explicitly. The sign itself is controlled by the CP -odd phase of the dipole Wilson coefficients: reversing the relevant CP -odd phase reverses the sign of Y_B^{FO} without changing the efficiency of the production mechanism. Thus, the benchmark $\tilde{m}_1^{\text{EM}} = 10^{-3}$ eV is used only to illustrate the time evolution of the density-matrix dynamics, and with the phase convention chosen here it does not reproduce the observed positive BAU.

For the fixed CP -phase convention used in the scan, the underlying signed zero-initial-abundance solution changes sign within the neutrino-oscillation-motivated green band around $\tilde{m}_1^{\text{EM}} \simeq 3 \times 10^{-2}$ eV. Since Fig. 7 displays the magnitude $|Y_B^{\text{FO}}|$ on a logarithmic scale, this sign change appears there as a sharp dip of the dashed blue curve rather than as a literal zero crossing of the plotted quantity.

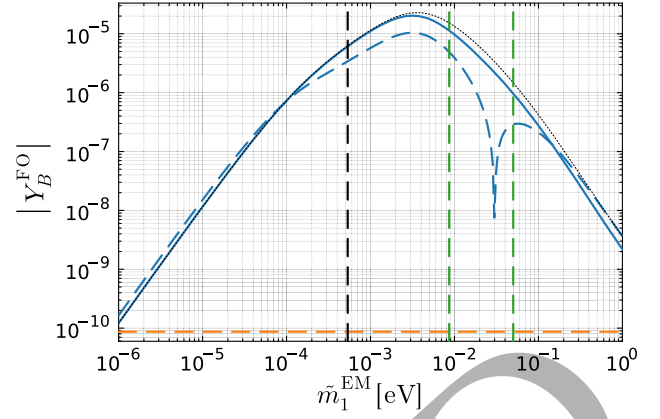


FIG. 7. Magnitude of the frozen-out baryon asymmetry, $|Y_B^{\text{FO}}|$, as a function of the effective electromagnetic neutrino mass $\tilde{m}_1^{\text{EM}}$. The solid blue curve assumes thermal initial heavy-neutrino abundance, while the dashed blue curve assumes zero initial abundance. The sharp dip of the dashed blue curve near $\tilde{m}_1^{\text{EM}} \simeq 3 \times 10^{-2}$ eV reflects the sign change of the underlying signed zero-initial-abundance solution. The dotted black curve shows the thermal-initial-abundance result with the leading $\Delta L = 0$ scattering terms switched off, i.e. with inverse-decay washout only. The horizontal dashed orange line indicates $Y_B^{\text{obs}} \simeq 8.7 \times 10^{-11}$. The green band shows the characteristic light-neutrino mass scales suggested by oscillation data in normal ordering. The near overlap of the solid blue and dotted black curves shows that the leading $\Delta L = 0$ scatterings produce only a tiny efficiency correction in this benchmark.

Together with Fig. 6, this shows that the signed zero-initial-abundance branch changes from negative to positive across this point. Hence, within the scanned range and for the same fixed CP -phase convention, a positive BAU is obtained only for

$$3 \times 10^{-2} \text{ eV} \lesssim \tilde{m}_1^{\text{EM}} \leq 1 \text{ eV}, \quad (231)$$

which overlaps with the neutrino-oscillation-motivated green band. When comparing only the size of the generated asymmetry, the relevant quantity is the magnitude $|Y_B^{\text{FO}}|$, as plotted in Fig. 7; the signed solution is nevertheless used in the text to identify which side of the zero produces the observed positive baryon asymmetry for the chosen CP -phase convention.

Figure 7 summarizes the main numerical result. We show the magnitude of the frozen-out baryon asymmetry, $|Y_B^{\text{FO}}|$, as a function of the effective electromagnetic neutrino mass $\tilde{m}_1^{\text{EM}}$. The solid blue curve corresponds to thermal initial abundance, while the dashed blue curve corresponds to zero initial abundance. The dotted black curve is obtained by switching off the leading $\Delta L = 0$ scattering terms for the thermal initial condition, keeping only the inverse-decay washout. The horizontal dashed orange line indicates the observed baryon asymmetry,

$$Y_B^{\text{obs}} \simeq 8.7 \times 10^{-11}, \quad (232)$$

and the green band indicates the characteristic light-neutrino mass scales suggested by oscillation data in normal ordering.

The thermal-initial-abundance branch exhibits the expected rise-and-fall behavior. In the weak-washout regime, increasing $\tilde{m}_1^{\text{EM}}$ enhances the dipole-induced decay rate and the CP -odd source, so $|Y_B^{\text{FO}}|$ grows. In the strong-washout regime, larger $\tilde{m}_1^{\text{EM}}$ strengthens inverse decays and also increases the total width entering the resonant regulator. Since the mass splitting is held fixed in the scan, the Breit-Wigner enhancement is gradually reduced at large $\tilde{m}_1^{\text{EM}}$, and the final asymmetry decreases.

For thermal initial abundance, the frozen-out asymmetry exceeds the observed baryon asymmetry in part of the scanned parameter space. This should be interpreted as an undiluted freeze-out yield. If additional late-time entropy production occurs after leptogenesis, the final baryon-to-entropy ratio would be reduced by a dilution factor,

$$Y_B = \frac{Y_B^{\text{FO}}}{\mathcal{D}}, \quad \mathcal{D} > 1. \quad (233)$$

We do not model such a dilution mechanism here. The point is that the resonant dipole source is strong enough to generate an asymmetry of the observed order.

For zero initial abundance, the final asymmetry changes sign as $\tilde{m}_1^{\text{EM}}$ is increased. Below the sign-changing point the frozen-out asymmetry is negative, as illustrated in Fig. 5. Above it, the asymmetry becomes positive and can reach the observed order within the scanned interval.

The effect of the leading $\Delta L = 0$ scatterings is small throughout the benchmark scan. The dotted black curve is nearly parallel to the solid blue curve, showing that the scatterings mainly induce a tiny overall rescaling of the efficiency rather than a qualitative change in the $\tilde{m}_1^{\text{EM}}$ dependence. This confirms that, in the dipole-dominated benchmark considered here, the baryon asymmetry is generated by the $\Delta L = 1$ decays and inverse decays, while the leading $\Delta L = 0$ scatterings only modify the relaxation and washout efficiency.

The behavior displayed in Fig. 7 admits three simple analytic interpretations, which we spell out in the following subsections. First, the nearly linear behavior of the flavor charges in the thermal-initial-abundance example follows from the short width of the electroweak window and the mild variation of the CP source. Second, the weak-washout side of the scan exhibits an approximately quadratic dependence on $\tilde{m}_1^{\text{EM}}$, as expected from the common dipole scaling of the CP source. Third, the leading $\Delta L = 0$ scatterings remain numerically small because $S_1/D_1 \ll 1$ over the relevant window.

D. On the behavior of the flavor charges $Y_{\Delta\alpha}$ for thermal initial abundance

The linear dependence of the flavor charges $Y_{\Delta\alpha}$ on $z = M_1/T$ shown in Fig. 5 can be understood directly from the structure of the flavor-charge QKE (209). Denoting the source term by

$$\mathcal{S}_\alpha(z) := \text{tr}[\epsilon_\alpha D(z)(Y_N(z) - Y_N^{\text{eq}}(z))], \quad (234)$$

the evolution equation takes the schematic form

$$\frac{dY_{\Delta\alpha}}{dz} = \mathcal{S}_\alpha(z) - \sum_\beta W_{\alpha\beta}(z)Y_{\Delta\beta}(z), \quad (235)$$

where $\mathcal{S}_\alpha$ and $W_{\alpha\beta}$ are in general z dependent and the full system is matrix valued. In the narrow window $z_{\text{EW}} = 6.25 \leq z \leq 7.69 \simeq z_{\text{sph}}$; however, the asymmetries are still small and the washout term is subdominant, while the source term varies only mildly. Approximating $\mathcal{S}_\alpha(z) \simeq \mathcal{S}_\alpha(z_{\text{EW}})$ and neglecting the washout term at leading order then yields

$$Y_{\Delta\alpha}(z) \simeq Y_{\Delta\alpha}(z_{\text{EW}}) + \mathcal{S}_\alpha(z_{\text{EW}})(z - z_{\text{EW}}), \quad (236)$$

which explains the quasilinear rise of the flavor charges.

E. Origin of the $(\tilde{m}_1^{\text{EM}})^2$ scaling

In the weak-washout regime ($K_1 \leq 1$), i.e., the left part of Fig. 7 exhibits an approximately quadratic scaling,

$$|Y_B^{\text{FO}}| \propto (\tilde{m}_1^{\text{EM}})^2. \quad (237)$$

This follows from the common dipole scaling in the CP -odd source term of the QKEs: for $\Gamma_2 \ll \Delta M$ (so that the resonant denominator is effectively constant at fixed ΔM), the flavored CP asymmetries scale as $\epsilon_{\alpha 1} \propto \mu^2$, while the decay factor obeys

$$D(z) \propto \frac{\Gamma_1}{H} \propto \tilde{m}_1^{\text{EM}} \propto \sum_\alpha \sum_V |\mu_{\alpha 1}^V|^2. \quad (238)$$

Since the source entering $dY_{\Delta\alpha}/dz$ is

$$\mathcal{S}_\alpha(z) = \text{tr}[\epsilon_\alpha D(z)(Y_N(z) - Y_N^{\text{eq}}(z))], \quad (239)$$

it carries an overall μ^4 dependence, and therefore $|Y_B^{\text{FO}}|$ inherits

$$|Y_B^{\text{FO}}| \propto \mu^4 \propto (\tilde{m}_1^{\text{EM}})^2, \quad (240)$$

consistent with a slope of 2 on log-log axes.

F. Origin of the approximately constant contribution of the $\Delta L = 0$ scatterings

In the z interval shown in Fig. 7 we find that the scattering kernel is parametrically tiny,

$$\Xi_1(z) := \frac{S_1(z)}{D_1(z)} \simeq 1.5 \times 10^{-5}. \quad (241)$$

Accordingly, scatterings influence the evolution in two logically distinct places, but in both cases only at the 10^{-5} level. On the heavy-neutrino side, the deviation from equilibrium obeys

$$\delta Y_{N_1}(z) := Y_{N_1}(z) - Y_{N_1}^{\text{eq}}(z), \quad \frac{dY_{N_1}}{dz} = -(D_1 + S_1)\delta Y_{N_1}. \quad (242)$$

In the regime where δY_{N_1} adjusts on a z scale short compared to the variation of $Y_{N_1}^{\text{eq}}(z)$, one may use the quasistatic estimate

$$\delta Y_{N_1}(z) \simeq -\frac{1}{D_1(z) + S_1(z)} \frac{dY_{N_1}^{\text{eq}}(z)}{dz}, \quad (243)$$

where we use $Y_{N_1} = Y_{N_1}^{\text{eq}} + \delta Y_{N_1}$. Since the production term is proportional to $D_1 \delta Y_{N_1}$, scatterings enter the source through

$$\begin{aligned} D_1(z) \delta Y_{N_1}(z) &\simeq -\frac{D_1(z)}{D_1(z) + S_1(z)} \frac{dY_{N_1}^{\text{eq}}(z)}{dz} \\ &= -\frac{1}{1 + [S_1(z)/D_1(z)]} \frac{dY_{N_1}^{\text{eq}}(z)}{dz}, \end{aligned} \quad (244)$$

so that in this range the source is reduced only by a relative factor

$$\begin{aligned} \frac{1}{1 + (S_1/D_1)} &= \frac{1}{1 + \Xi_1} \simeq 1 - \Xi_1 \\ &\simeq 1 - 1.5 \times 10^{-5}. \end{aligned} \quad (245)$$

On the flavor-charge side, scatterings contribute directly to the washout through the additional kernel W^{scatt} , while inverse decays contribute through W^{ID} ; in our normalization these are proportional to the corresponding reaction densities, $W^{\text{ID}} \propto \gamma_D$ and $W^{\text{scatt}} \propto \gamma_S$, so that

$$\frac{W_1^{\text{scatt}}}{W_1^{\text{ID}}} = \frac{\gamma_{S,1}}{\gamma_{D,1}} = \frac{S_1}{D_1} = \Xi_1 = 1.5 \times 10^{-5}, \quad (246)$$

and

$$\begin{aligned} W_1^{\text{tot}} &= W_1^{\text{ID}} + W_1^{\text{scatt}} = W_1^{\text{ID}}[1 + (S_1/D_1)] \\ &= W_1^{\text{ID}}(1 + \Xi_1) \simeq W_1^{\text{ID}}(1 + 1.5 \times 10^{-5}). \end{aligned} \quad (247)$$

Hence, for the benchmark parameters and within the electroweak window considered here, $\Delta L = 0$ scatterings are parametrically tiny in the strong-washout regime and modify both the effective source term and the total washout only at the $\mathcal{O}(10^{-5})$ level in our normalization.

The numerical analysis therefore leads to two main conclusions. First, resonant dipole-dominated EMLG can generate an asymmetry of the observed order, and can even overproduce it, in the quasidegenerate regime. Second, for the present benchmark the leading $\Delta L = 0$ scatterings are not the origin of the asymmetry; their role is to provide a small correction to the relaxation and washout kernels. The dominant source is the CP -violating $\Delta L = 1$ decay/inverse-decay sector, resonantly enhanced by the quasidegenerate heavy-neutrino spectrum.

X. DISCUSSION AND CONCLUSIONS

This section summarizes what has been achieved in the present paper. Building on the electromagnetic leptogenesis proposal of Ref. [19], we provide a fully gauge-invariant EFT realization and an Schwinger-Keldysh (SK)-based density-matrix treatment appropriate for the quasidegenerate (resonant) regime. Our central new ingredient is to derive density-matrix quantum kinetic equations on the Schwinger-Keldysh contour whose collision term incorporates decays, inverse decays, and the leading $\Delta L = 0$ scatterings consistently from the same effective interactions, avoiding double counting and therefore eliminating the need for RIS subtraction. This construction enables a quantitative mapping from UV input to the frozen-out baryon asymmetry in the electroweak window, and it clarifies which physical conditions control the viability of dipole-dominated leptogenesis.

A. Overcoming the intrinsic dipole suppression via resonance

Our results show that a genuinely dipole-dominated scenario is viable only if the structural suppression away from resonance is evaded. For dipole-induced decays the tree-level amplitude is proportional to the effective dipole coupling $\mu_{\alpha 1}$, and the total width scales as

$$\Gamma_1 \propto M_1^3 \sum_{\alpha} |\mu_{\alpha 1}|^2. \quad (248)$$

Away from the resonant regime, both the CP -odd source and the washout rates scale with the same small combination $\mu_{\alpha 1}^2$, so the final asymmetry is parametrically suppressed. In the quasidegenerate regime, by contrast, the self-energy contribution provides a Breit-Wigner-type enhancement, and the CP source is controlled by the relative sizes of ΔM and the width(s) entering the regulator

[27,31]. Since our scan keeps ΔM fixed, increasing $\tilde{m}_1^{\text{EM}}$ enhances Γ_2 and therefore drives the regulator away from its peak once $\Gamma_2 \gg 2\Delta M$, which directly explains the turnover of Y_B^{FO} observed in Fig. 7. More generally, the enhancement is maximal when the splitting is comparable to the width that regulates the resummed self-energy pole, so scanning over ΔM as well as $\tilde{m}_1^{\text{EM}}$ provides a direct handle on where the Breit-Wigner envelope peaks.

B. Suppressing Yukawa-driven effects to isolate a dipole-dominated regime

We also find that controlling Yukawa-induced channels is essential for testing a genuinely dipole-dominated mechanism. In gauge-invariant UV completions, the renormalizable operator $\bar{L}\tilde{H}N$ is generically allowed; if its effective coupling is not suppressed, Yukawa-mediated decays and inverse decays dominate both the total width and the washout, enlarging the width entering the resonant regulator and obscuring the dipole dynamics. To isolate electromagnetic leptogenesis, we therefore restrict to the regime where Yukawa channels remain subdominant [30]. Quantitatively, as shown in Sec. II, this can be enforced by the conservative bound

$$|y_{\text{eff}}| \lesssim 10^{-8}. \quad (249)$$

Our benchmark satisfies this condition, so Yukawa-mediated processes do not materially affect either the resonant regulator or the washout kernels in the electroweak window. In the present UV completion this hierarchy is technically natural because y_{eff} is controlled by a soft Z_2 -breaking parameter, so the dipole-dominated limit corresponds to a stable parametric regime rather than a tuned choice of couplings.

C. Applicability of the electroweak-window analysis

We first comment on the range of applicability of the present analysis. Our study is designed to isolate the leptogenesis dynamics in the vicinity of sphaleron decoupling, and we therefore evolve the QKEs only over the electroweak window $T_{\text{EW}} \geq T \geq T_{\text{sph}}$. In this range, we describe the dipole sector in a broken-phase effective theory, in which the gauge-invariant νSMEFT operators are matched onto effective dimension-five dipole interactions parametrized by μ_{ai}^V . The dominant processes are the two-body channels $N_i \rightarrow \nu_a \gamma$, $N_i \rightarrow \nu_a Z$, and $N_i \rightarrow \ell_a W$ (and their inverse decays), supplemented by the leading $\Delta L = 0$ scatterings.

This treatment should be regarded as an approximation to the smooth electroweak crossover. Near T_{EW} the Higgs background and thermal masses vary with temperature, and in the electroweak-symmetric phase ($T > T_{\text{EW}}$) the gauge-invariant dipole operators induce three-body decays $N_i \rightarrow \ell_a \tilde{H} X$ with $X = B, W$, for which the leading nonvanishing

CP asymmetries arise only at two loops [19]. A fully continuous analysis would require matching the kinetic description across the crossover while incorporating the temperature dependence of the Higgs background and the symmetric-phase reaction network. We leave this extension to future work; the present results demonstrate that, once the dipole sector is treated consistently in the broken phase with SK-based density-matrix kinetics, the electroweak-window contribution can reproduce the observed BAU in a resonant, dipole-dominated setup.

D. Phenomenology and testability

Complementary probes of the dipole sector were discussed in [25,29]. On the direct-search side, transition dipoles of heavy neutral leptons (HNLs) can be tested at colliders and intensity-frontier experiments. In particular, LHC sensitivity studies of dipole-portal signatures (including channels involving τ leptons) have been presented in [21,22]. A mono- γ plus missing-energy strategy at NA62, targeting radiative decays $N \rightarrow \nu \gamma$, has been proposed in [76].

On the precision side, electroweak matching and RG evolution induce the charged-lepton dipole operator in the low-energy effective theory. In terms of the corresponding low-energy effective field theory (LEFT) Wilson coefficient $C_{e\gamma}$, one has the parametric relations

$$\text{BR}(\mu \rightarrow e\gamma) \propto |(C_{e\gamma})_{e\mu}(m_\mu)|^2 + |(C_{e\gamma})_{\mu e}(m_\mu)|^2, \quad (250)$$

$$d_e \propto \text{Im}(C_{e\gamma})_{ee}(m_e), \quad (251)$$

$$\Delta a_\mu \propto \text{Re}(C_{e\gamma})_{\mu\mu}(m_\mu), \quad (252)$$

with the QED evolution from $Q_{\text{ref}} = 150 \text{ GeV}$ to $\mu = m_\mu, m_e$ discussed in [23,24]. For current experimental limits, see MEG II for $\mu \rightarrow e\gamma$ [77], JILA for the electron electric dipole moment (EDM) [78], and the Muon $g - 2$ Collaboration for $a_\mu = (g - 2)_\mu/2$ [79].

Light-neutrino masses can be accommodated independently of the dipole sector in the Yukawa-suppressed benchmark considered here. As discussed in Sec. II D, requiring $|y_{\text{eff}}| \lesssim 10^{-8}$ renders the type-I seesaw contribution from the TeV-scale states $N_{1,2}$ phenomenologically negligible, so the observed masses are naturally attributed to an additional lepton-number-violating sector, for instance a high-scale type-II seesaw completion. At low energies such a completion enters only through the coefficient of the Weinberg operator (equivalently, through m_ν) and can be confronted with neutrino data, including cosmological limits on $\sum m_\nu$ [80], oscillation measurements [47] (NuFit-6.0), and direct bounds from KATRIN [81].

Finally, we emphasize that the resonant enhancement relevant for the CP -odd sources arises from width-regularized heavy-neutrino self-energies in the baryogenesis

dynamics [27,60], whereas low-energy dipole observables are controlled by the on-shell ($q^2 = 0$) matching onto the LEFT dipole operator and its QED running [23,24,51,82,83]; the BAU requirement and low-energy dipole searches therefore provide complementary information.

This complementarity is the main phenomenological motivation for pursuing the low-scale formulation developed in this paper. The baryon asymmetry fixes a cosmological viability criterion for the dipole sector, whereas charged-lepton flavor violation, EDMs, anomalous magnetic moments, and direct HNL searches probe the same sector from the low-energy and laboratory side. A full SMEFT-to-LEFT matching analysis of the dipole coefficients, including flavor structure and correlations among $\mu \rightarrow e\gamma$, d_e , Δa_μ , and collider observables, is beyond the scope of the present work and will be addressed separately. The results obtained here provide the necessary prerequisite: they identify the resonant, dipole-dominated region in which such phenomenological tests are meaningful because the mechanism can generate the observed BAU.

E. Conclusions

In this paper, we studied low-scale electromagnetic leptogenesis in a gauge-invariant EFT framework and asked whether the dipole-dominated mechanism can generate the observed baryon asymmetry in the electroweak crossover window. Starting from a softly broken Z_2 UV completion, we matched onto the gauge-invariant dipole operators $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$, evolved their Wilson coefficients to an electroweak reference scale, and constructed the broken-phase dipole couplings to γ , Z , and $W^\pm$.

The central physical result is that the nonresonant, hierarchical regime is structurally inefficient in gauge-invariant dipole-dominated EMLG, while the quasidegenerate regime can generate a sufficient baryon asymmetry through the resonantly enhanced self-energy contribution. In the benchmark studied here, the QKE solutions reproduce, and in part of parameter space overproduce, the observed BAU. This establishes that the low-scale dipole-dominated mechanism is viable at least as a proof of principle once the resonant regime is included.

We also derived density-matrix QKEs in the Schwinger-Keldysh formalism. In this formulation, the CP -odd source, inverse decays, and the leading $\Delta L = 0$ scatterings arise from a common self-energy structure. This organization avoids double counting and does not require a separate real-intermediate-state subtraction. Numerically, the leading $\Delta L = 0$ scatterings are not the origin of the asymmetry in our benchmark; they mainly provide a small correction to the relaxation and washout kernels, with $S_i/D_i \sim 10^{-5}$.

The motivation for formulating EMLG at low scale is that the same dipole sector can eventually be confronted with laboratory observables after SMEFT/LEFT matching. The present work provides the required cosmological prerequisite: it identifies the resonant, dipole-dominated

parameter region in which the mechanism can generate the observed BAU. A full analysis of the resulting correlations with charged-lepton flavor violation, electric dipole moments, anomalous magnetic moments, and direct HNL searches will be pursued separately.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: A TYPE-II-SEESAW ILLUSTRATION FOR THE LIGHT-NEUTRINO MASSES

As a minimal and standard realization of such an alternative origin, one may introduce a type-II seesaw [9–13] sector with a scalar triplet $\Delta \sim (\mathbf{1}, \mathbf{3}, 1)$. The corresponding UV interactions can be written as

$$-\mathcal{L} \supset \frac{1}{2} (y_\Delta)_{\alpha\beta} L_\alpha^\dagger C i \sigma^2 \Delta L_\beta + \mu_\Delta H^\dagger i \sigma^2 \Delta^\dagger H + \text{H.c.} + M_\Delta^2 \text{tr}(\Delta^\dagger \Delta), \quad (\text{A1})$$

and integrating out Δ generates the Weinberg operator,

$$-\mathcal{L}_{\text{eff}} \supset \frac{1}{2} (C_5)_{\alpha\beta} (\bar{L}_\alpha^c \tilde{H}^*) (\tilde{H}^\dagger L_\beta) + \text{H.c.}, \quad (C_5)_{\alpha\beta} \simeq (y_\Delta)_{\alpha\beta} \frac{\mu_\Delta}{M_\Delta^2}. \quad (\text{A2})$$

After electroweak symmetry breaking,

$$(m_\nu^{(\text{II})})_{\alpha\beta} = (C_5)_{\alpha\beta} v^2 \simeq (y_\Delta)_{\alpha\beta} \mu_\Delta \frac{v^2}{M_\Delta^2}. \quad (\text{A3})$$

If $|(y_\Delta)_{\alpha\beta}| \sim \mathcal{O}(1)$ and $\mu_\Delta \sim M_\Delta \simeq 1 \times 10^{15}$ GeV, one obtains

$$m_\nu^{(\text{II})} \sim \frac{v^2}{M_\Delta} \simeq \frac{246^2}{1 \times 10^{15}} \text{ GeV} \simeq 0.06 \text{ eV}. \quad (\text{A4})$$

Assuming a negligible lightest mass $m_1 \rightarrow 0$, the heaviest eigenvalue is $m_3 \simeq \sqrt{\Delta m_{31}^2} \simeq 0.05 \text{ eV}$ [47] (NuFit-6.0), comparable to our estimate $m_\nu^{(\text{II})} \sim 0.06 \text{ eV}$.

Placing this type-II seesaw sector at a scale far above $Q_{\text{match}} = 10 \text{ TeV}$ implies that, at low energies, it appears only through $\mathcal{O}_5$ (i.e., m_ν) and can be treated independently of the UV completion that generates the electromagnetic

dipole operators $\mathcal{O}_{NX}$ ($X = B, W$). In particular, as long as Δ does not couple directly to N , it does not induce additional contributions to $\mathcal{O}_{NX}$, and one can realize $m_\nu = \mathcal{O}(10^{-2})$ eV while remaining consistent with the EFT analysis of electromagnetic leptogenesis.

APPENDIX B: DETAILS OF THE UV COMPLETION AND ONE-LOOP MATCHING ONTO THE DIPOLE OPERATORS

1. Classical reduction of $\tilde{D}$ to $\tilde{H}$

Here we give the derivation of the tree-level matching step used in Sec. II. At low energies, where the typical external momenta satisfy $p^2 \ll m_D^2$, the heavy scalar doublet D can be integrated out by solving its classical equation of motion.

We consider the part of the Lagrangian involving the kinetic term of D ,

$$\mathcal{L} \supset (D_\mu D)^\dagger (D^\mu D) - V(H, \Sigma, D, \Phi^+). \quad (\text{B1})$$

Treating D and $D^\dagger$ as independent fields, the Euler-Lagrange equation gives

$$(D_\mu D^\mu D)_a + \frac{\partial V(H, \Sigma, D, \Phi^+)}{\partial D_a^\dagger} = 0. \quad (\text{B2})$$

The terms in the scalar potential that act as leading sources for D are those linear in D or $D^\dagger$. From the soft-breaking term and the D -dependent interactions in the scalar potential, we obtain

$$\begin{aligned} V_{\text{linear}} \supset & \mu_{\text{soft}}^2 H^\dagger D + (\mu_{\text{soft}}^2)^* D^\dagger H \\ & + \mu_s (\Sigma \cdot D) (\Phi^+)^* + \mu_s^* (D^\dagger \cdot \Sigma) \Phi^+ \\ & + \lambda_{DH} (D^\dagger \Sigma) (H^\dagger H) + \lambda_{D\Sigma+} (D^\dagger \Sigma) |\Phi^+|^2. \end{aligned} \quad (\text{B3})$$

Therefore,

$$\begin{aligned} \frac{\partial V_{\text{linear}}}{\partial D^\dagger} = & (\mu_{\text{soft}}^2)^* H + \mu_s^* \tilde{\Sigma} \Phi^+ \\ & + \lambda_{DH} \Sigma (H^\dagger H) + \lambda_{D\Sigma+} \Sigma |\Phi^+|^2. \end{aligned} \quad (\text{B4})$$

On the other hand, the terms proportional to $D^\dagger D$, such as $m_D^2 D^\dagger D$, $f_1 (H^\dagger H) (D^\dagger D)$, and $f_3 |H^\dagger D|^2$, contribute terms proportional to $\tilde{D}$ itself. Thus the equation of motion can be organized as

$$[m_D^2 + \mathcal{O}(H^\dagger H, \Sigma^\dagger \Sigma, |\Phi^+|^2)] D = -\frac{\partial V_{\text{linear}}}{\partial D^\dagger} - D_\mu D^\mu D. \quad (\text{B5})$$

Since $D_\mu D^\mu / m_D^2 = \mathcal{O}(p^2 / m_D^2)$, the derivative term is subleading in the low-energy expansion. Keeping only the leading $1/m_D^2$ contribution gives

$$\begin{aligned} D \simeq & -\frac{1}{m_D^2} \left[(\mu_{\text{soft}}^2)^* H + \mu_s^* \tilde{\Sigma} \Phi^+ \right. \\ & \left. + \lambda_{DH} \Sigma (H^\dagger H) + \lambda_{D\Sigma+} \Sigma |\Phi^+|^2 \right] + \mathcal{O}\left(\frac{1}{m_D^4}\right). \end{aligned} \quad (\text{B6})$$

For the conversion of the D -based dipole operators into the ν SMEFT dipole operators, it is sufficient to retain the soft-breaking source alone:

$$D \simeq -\frac{(\mu_{\text{soft}}^2)^*}{m_D^2} H, \quad \tilde{D} \simeq -\frac{\mu_{\text{soft}}^2}{m_D^2} \tilde{H}. \quad (\text{B7})$$

Hence,

$$\mathcal{O}_{NB,ai}^{(D)} \rightarrow -\frac{\mu_{\text{soft}}^2}{m_D^2} \mathcal{O}_{NB,ai}, \quad \mathcal{O}_{NW,ai}^{(D)} \rightarrow -\frac{\mu_{\text{soft}}^2}{m_D^2} \mathcal{O}_{NW,ai}, \quad (\text{B8})$$

which yields the matching relation quoted in Sec. II.

2. General form of the vertex-function decomposition

The quantity computed in this section is the 1PI amplitude of the four-point vertex function associated with the gauge-invariant operators in the symmetric phase,

$$\begin{aligned} \mathcal{O}_{NB,ai} &= (\bar{L}_a \sigma^{\mu\nu} P_R N_i) \tilde{H} B_{\mu\nu}, \\ \mathcal{O}_{NW,ai} &= (\bar{L}_a \sigma^{\mu\nu} \tau^a P_R N_i) \tilde{H} W_{\mu\nu}^a. \end{aligned} \quad (\text{B9})$$

That is, taking the external legs to be N_i , L_a , $\tilde{H}$, and X_μ , we define the amputated vertex-function Γ_X^μ through

$$\mathcal{M}_X^\mu = \bar{u}_{L_a}(p') \Gamma_X^\mu(p, p', k) P_R u_{N_i}(p) \varepsilon_\mu(q). \quad (\text{B10})$$

At the stage where D is treated as an external field (the first-step matching), one simply replaces $\tilde{H} \rightarrow \tilde{D}$, while the Lorentz structure discussed below remains unchanged. For $X = W^a$, the $\text{SU}(2)_L$ generator τ^a is understood to be factored out on both the UV and EFT sides.

By Lorentz covariance, the amputated 1PI vertex-function Γ_X^μ admits a tensor decomposition. Focusing on the dependence on the external gauge-boson momentum q and using a minimal basis, we write

$$\begin{aligned} \Gamma_X^\mu(q) = & F_{V,X}(q^2) \gamma^\mu \\ & + F_{D,X}(q^2) i \sigma^{\mu\nu} q_\nu \\ & + F_{q,X}(q^2) q^\mu + \dots \end{aligned} \quad (\text{B11})$$

The q^2 dependence of the coefficients $F_{n,X}(q^2)$ is the usual loop form-factor dependence. In EFT matching we work in the regime $q^2 \ll M_{\text{heavy}}^2$ and expand

$$F_{n,X}(q^2) = F_{n,X}(0) + \mathcal{O}\left(\frac{q^2}{M_{\text{heavy}}^2}\right), \quad (\text{B12})$$

retaining only the leading terms that correspond to local operators.

The crucial observation is that the dimension-six dipole operators contain the field strength $X_{\mu\nu}$, so the corresponding EFT amplitude necessarily carries the antisymmetric tensor structure $\sigma^{\mu\nu}q_\nu$ at first order in q [cf. Eq. (59)]. Indeed, from the EFT interaction (2) one obtains the tree-level amplitude

$$i\mathcal{M}_X^{(\text{EFT})} = i\bar{u}_{L\alpha} i\sigma^{\mu\nu} q_\nu P_R u_{N_i \epsilon_\mu} \frac{2C_{NX,ai}}{\Lambda^2}, \quad (\text{B13})$$

where the factor 2 follows from contracting the antisymmetric tensors, $X_{\mu\nu}(q) = i(q_\mu \epsilon_\nu - q_\nu \epsilon_\mu)$ and $\sigma^{\mu\nu} = -\sigma^{\nu\mu}$.

On the UV side, inserting the decomposition (B11) into the definition (B10) and expanding at small q shows that the dipole structure in the one-loop amplitude is governed by $F_{D,X}(0)$. Comparing the coefficients of $\bar{u}i\sigma^{\mu\nu}q_\nu P_R u \epsilon_\mu$ between the UV and EFT amplitudes therefore yields the matching condition

$$\frac{C_{NX,ai}}{\Lambda^2} = \frac{1}{2} F_{D,X}(0), \quad (\text{B14})$$

up to higher-order corrections of $\mathcal{O}(q^2/M_{\text{heavy}}^2)$ corresponding to higher-dimensional operators. In this sense, extracting the antisymmetric part linear in q from the UV one-loop result and reading off $F_{D,X}(0)$ is precisely the projection onto the electromagnetic dipole operators; it is not an *ad hoc* assumption but follows from organizing the spinor chains using Clifford-algebra identities such as $\gamma^\mu \not{q} = \not{q} - i\sigma^{\mu\nu}q_\nu$.

Gauge invariance constrains the remaining structures. It is convenient to reorganize the nondipole terms into a transverse (anapole-type) structure and a longitudinal piece,

$$\begin{aligned} \Gamma_X^\mu(q) &= F_{D,X}(q^2) i\sigma^{\mu\nu} q_\nu \\ &\quad + F_{A,X}(q^2) (q^2 \gamma^\mu - q^\mu \not{q}) \\ &\quad + F_{L,X}(q^2) q^\mu + \dots \end{aligned} \quad (\text{B15})$$

The transverse γ^μ contribution can appear only through the anapole-type combination $(q^2 \gamma^\mu - q^\mu \not{q})$, and thus starts at $\mathcal{O}(q^2)$; it corresponds to local operators of dimension eight and higher and does not affect the leading extraction of the dimension-six dipole coefficient in Eq. (B14). The longitudinal term proportional to q^μ does not contribute to physical amplitudes with transverse polarization, $\epsilon \cdot q = 0$.

Here the relevant constraints are provided by the Ward-Takahashi identity for $U(1)_Y$ [84,85] and, in the non-Abelian case, by the Slavnov-Taylor identities [86–88], which are functional identities for the effective action Γ implied by Becchi-Rouet-Stora-Tyutin (BRST) symmetry of the gauge-fixed theory [89–92]. In particular,

longitudinal components (terms proportional to q_μ) are not independent but are related to other Green's functions (including ghost and would-be Goldstone contributions); physically, only the transverse part contributes to observables.

In summary,

- (i) it is natural for vertex-function coefficients to depend on q^2 as loop form factors, and the EFT matching is organized as an expansion in q^2/M_{heavy}^2 ;
- (ii) the dimension-six dipole is extracted by projecting onto the $i\sigma^{\mu\nu}q_\nu$ structure, yielding the matching relation $C_{NX,ai}/\Lambda^2 = \frac{1}{2} F_{D,X}(0)$ in Eq. (B14); and
- (iii) the “vertex function” here is the amputated four-point amplitude with external legs N_i , L_α , $\tilde{H}$, and X_μ ; it is not a three-point function involving N_i , L_α , and D .

3. Setup for one-loop matching onto $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$

In this section, we outline the two-step matching strategy and fix our conventions for extracting the Wilson coefficients of $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$ from the UV theory with N_i , E , Σ , D , and Φ^+ .

Among the additional fields in Table I, we take E , Σ , D , and Φ^+ to be heavy, while assuming that N_i are much lighter:

$$\begin{aligned} M_E, m_\Sigma, m_{\Phi^+}, m_D &= \mathcal{O}(M_{\text{heavy}}), \\ M_i &\ll M_{\text{heavy}}. \end{aligned} \quad (\text{B16})$$

Under this hierarchy, the appropriate low-energy description is ν SMEFT, obtained by integrating out E , Σ , D , and Φ^+ . In particular, the leptogenesis degrees of freedom N_i remain dynamical in the ν SMEFT.

We take the minimal set of UV interactions needed in this section to be

$$\begin{aligned} -\mathcal{L}_{\text{int}} &= y_{H,i} \bar{N}_i E_L \Phi^+ + y_{\Sigma,\alpha} \bar{L}_\alpha \Sigma E_R \\ &\quad + \mu_s \Sigma \cdot D (\Phi^+)^\dagger + \text{H.c.}, \end{aligned} \quad (\text{B17})$$

where $\Sigma \cdot D := \Sigma^\dagger i\sigma_2 D$ is the $SU(2)_L$ -antisymmetric inner product. The gauge couplings are fixed uniquely by the covariant derivatives in the kinetic terms. Specifically, from

$$\bar{E} i \not{D} E, \quad (D_\mu \Sigma)^\dagger (D^\mu \Sigma), \quad (D_\mu \Phi^+)^\dagger (D^\mu \Phi^+), \quad (\text{B18})$$

one obtains the couplings to B_μ and W_μ^a . An $SU(2)_L$ singlet does not couple to W_μ^a since its generators satisfy $T^a = 0$, whereas an $SU(2)_L$ doublet couples through $T^a = \tau^a$. The hypercharge coupling is proportional to the $U(1)_Y$ generator Y , and fields with the same hypercharge couple to B_μ with the same strength.

For the heavy fields, the hypercharges in Table I are

$$Y_E = -1, \quad Y_{\Phi^+} = +1, \quad Y_\Sigma = Y_D = +\frac{1}{2}, \quad (\text{B19})$$

and the $SU(2)_L$ generators for Σ are $T^a = \tau^a$.

The relevant external legs for matching onto $\mathcal{O}_{NB}$ and $\mathcal{O}_{NW}$ are N_i , L_α , $\tilde{H}$, and $X_{\mu\nu}$. However, since the UV model contains no $\tilde{L}\tilde{H}N$ interaction, we proceed in two steps: we first generate the gauge-invariant operators involving D ,

$$\begin{aligned} \mathcal{O}_{NB,ai}^{(D)} &= (\bar{L}_\alpha \sigma^{\mu\nu} P_R N_i) \tilde{D} B_{\mu\nu}, \\ \mathcal{O}_{NW,ai}^{(D)} &= (\bar{L}_\alpha \sigma^{\mu\nu} \tau^a P_R N_i) \tilde{D} W_{\mu\nu}^a, \end{aligned} \quad (\text{B20})$$

and then use the mixing in the scalar sector (soft breaking) to reduce $\tilde{D} \rightarrow \tilde{H}$. The basic topology is a one-loop diagram with an internal E fermion line and (Φ^+, Σ) scalar lines: the external legs N_i and L_α are attached via the vertices $y_{H,i}$ and $y_{\Sigma,\alpha}$, while the insertion of D is provided by the vertex $\mu_s \Sigma \cdot D (\Phi^+)^\dagger$ (see Fig. 1). Since the gauge boson attaches to a charged internal line, the minimal set of diagrams consists of

- (i) three independent diagrams for B_μ , where B_μ is attached to the E line, the Φ^+ line, or the Σ line, and
- (ii) a single diagram for W_μ^a , where W_μ^a is attached only to the Σ line, since Σ is an $SU(2)_L$ doublet.

We compute these diagrams in dimensional regularization by analytically continuing to $n = 4 - 2\epsilon$ spacetime dimensions.

Denoting the external gauge-boson momentum by q and the polarization vector by ϵ_μ , we decompose the contributions of the individual diagrams as

$$\mathcal{M}_B = \mathcal{M}_B^{(E)} + \mathcal{M}_B^{(\Phi^+)} + \mathcal{M}_B^{(\Sigma)}, \quad \mathcal{M}_W = \mathcal{M}_W^{(\Sigma)}. \quad (\text{B21})$$

Here the superscripts indicate the internal particle line to which the gauge boson is attached.

In what follows, we take the external gauge boson to be outgoing, with momentum q defined accordingly. If one writes the EFT Feynman rules using the “all momenta incoming” convention, one must replace $q \rightarrow -q$; however, as long as the same convention is used on both the UV and EFT sides, the Wilson coefficients themselves are unchanged.

4. Exact evaluation of the one-loop diagram with B_μ attached to the E line

In this section, we isolate the one-loop diagram in which the hypercharge gauge boson B_μ is attached to the internal fermion line E . Our goal is to compute directly in the UV theory the IPI-amputated four-point vertex function with external legs L_α , N_i , $\tilde{D}$, and B_μ , to extract at $\mathcal{O}(q)$ the part proportional to $\sigma^{\mu\nu} q_\nu$, and thereby to obtain the contribution of diagram (a) to the Wilson coefficient C_{NB} in the

symmetric phase. Since the external spinors satisfy that L_α is left-handed and N_i is right-handed, we note

$$\begin{aligned} u_{L_\alpha} &= P_L u, & \bar{u}_{L_\alpha} &= \bar{u} P_R, \\ u_{N_i} &= P_R u, & \bar{u}_{N_i} &= \bar{u} P_L. \end{aligned} \quad (\text{B22})$$

The interactions required for Fig. 1(a) are

$$\begin{aligned} -\mathcal{L}_{\text{int}} &\supset y_{\Sigma,\alpha} \bar{L}_\alpha \Sigma P_R E + y_{H,i} \bar{N}_i P_L E \Phi^+ \\ &\quad + \mu_s (\Sigma \cdot D) (\Phi^+)^\dagger. \end{aligned} \quad (\text{B23})$$

However, to match the external chiralities and the fermion flow, we will compute using the following Hermitian-conjugate form:

$$\begin{aligned} -\mathcal{L}_{\text{int}} &\supset y_{\Sigma,\alpha}^* \bar{E} P_L \Sigma^\dagger L_\alpha + y_{H,i}^* \bar{E} P_R N_i (\Phi^+)^\dagger \\ &\quad + \mu_s^* (D^\dagger \cdot \Sigma^\dagger) \Phi^+, \end{aligned} \quad (\text{B24})$$

which simply selects the terms that yield a nonvanishing spinor chain for the given external legs $(\bar{u}_{L_\alpha}, u_{N_i})$. Using Eq. (B23) instead would amount to computing the Hermitian-conjugate process (with complex-conjugated couplings). Therefore, the final effective Lagrangian must always be accompanied by H.c.

The gauge interaction is read off from the covariant derivative,

$$D_\mu = \partial_\mu - ig' Y B_\mu, \quad (\text{B25})$$

so that the B_μ coupling of E follows from

$$\mathcal{L}_{\text{UV}} \supset \bar{E} i \not{D} E \Rightarrow \mathcal{L}_{\text{int}} \supset g' Y_E \bar{E} \gamma^\mu E B_\mu. \quad (\text{B26})$$

Hence, the $B - E - E$ vertex factor is $ig' Y_E \gamma^\mu$.

Let q be the outgoing momentum of the external gauge field and ϵ_μ its polarization vector. We define the contribution of Fig. 1(a) as

$$\begin{aligned} i\mathcal{M}_B^{(E)} &= (ig' Y_E) (-iy_{\Sigma,\alpha}) (-iy_{H,i}^*) (-i\mu_s^*) \\ &\quad \times \bar{u}_{L_\alpha}(p') I_B^{(E)\mu}(q) u_{N_i}(p) \epsilon_\mu(q), \end{aligned} \quad (\text{B27})$$

where the loop integral (vertex function), including the chiral projectors, is

$$\begin{aligned} I_B^{(E)\mu}(q) &:= \int \frac{d^n k}{(2\pi)^n} \frac{P_R(k + M_E) \gamma^\mu (k - \not{q} + M_E) P_R}{(k^2 - M_E^2)[(k - q)^2 - M_E^2]} \\ &\quad \times \frac{1}{(k^2 - m_{\Phi^+}^2)(k^2 - m_\Sigma^2)}. \end{aligned} \quad (\text{B28})$$

Here the scalar propagators do not carry q . This reflects the approximation appropriate for extracting the dipole at $\mathcal{O}(q)$: external momenta are taken to be small compared

to the heavy masses, and the denominators are evaluated at k^2 ,

$$p^2, p'^2 \ll M_E^2, m_{\Phi^+}^2, m_\Sigma^2, \quad (\text{B29})$$

which is sufficient for the leading $1/M^2$ matching contribution to C_{NB} .

Starting from Eq. (B28), we first simplify the Dirac numerator

$$\begin{aligned} P_R(k + M_E)\gamma^\mu(k - q + M_E)P_R \\ = M_E(2k^\mu - q^\mu + i\sigma^{\mu\nu}q_\nu)P_R. \end{aligned} \quad (\text{B30})$$

We now substitute Eq. (B30) into Eq. (B28). Since we are interested in the dipole piece, which is linear in q and carries the antisymmetric tensor $i\sigma^{\mu\nu}q_\nu$, we may set the shifted fermion propagator to its leading term,

$$\frac{1}{(k-q)^2 - M_E^2} = \frac{1}{k^2 - M_E^2} + \frac{2k \cdot q}{(k^2 - M_E^2)^2} + \mathcal{O}(q^2), \quad (\text{B31})$$

because any $\mathcal{O}(q)$ correction from the denominator would multiply the already $\mathcal{O}(q)$ dipole numerator and hence contribute only at $\mathcal{O}(q^2)$ to the dipole projection. The same reasoning justifies evaluating the scalar propagators at k^2 in this step. We therefore obtain, up to $\mathcal{O}(q^2)$,

$$\begin{aligned} I_B^{(E)\mu}(q) = \int \frac{d^n k}{(2\pi)^n} \frac{M_E(2k^\mu - q^\mu + i\sigma^{\mu\nu}q_\nu)P_R}{(k^2 - M_E^2)^2(k^2 - m_{\Phi^+}^2)(k^2 - m_\Sigma^2)} \\ + \mathcal{O}(q^2). \end{aligned} \quad (\text{B32})$$

The terms proportional to $2k^\mu$ and q^μ do not carry the antisymmetric tensor structure and correspond to nondipole form factors (they are arranged/cancelled by gauge invariance only after summing all diagrams). For matching onto the dimension-six dipole operator, we project onto the $\sigma^{\mu\nu}q_\nu$ structure. Keeping only this piece in Eq. (B32) yields

$$\begin{aligned} I_{B,\sigma}^{(E)\mu}(q) = iM_E\sigma^{\mu\nu}q_\nu P_R \\ \times \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - M_E^2)^2(k^2 - m_{\Phi^+}^2)(k^2 - m_\Sigma^2)}, \end{aligned} \quad (\text{B33})$$

which isolates the dipole contribution quoted in Eq. (B33).

Let us evaluate the integral in Eq. (B33) using Feynman parameters and dimensional regularization. We define the scalar integral

$$J_E := \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - M_E^2)^2(k^2 - m_{\Phi^+}^2)(k^2 - m_\Sigma^2)}. \quad (\text{B34})$$

Using the Feynman-parameter formula

$$\frac{1}{a^2bc} = 6 \int_0^1 dx \int_0^{1-x} dy \frac{xy}{[xa + yb + (1-x-y)c]^4}, \quad (\text{B35})$$

we obtain

$$\begin{aligned} \frac{1}{(k^2 - a)^2(k^2 - b)(k^2 - c)} \\ = 6 \int_0^1 dx \int_0^{1-x} dy \frac{xy}{[\Delta(x, y) - k^2]^4}, \\ \Delta(x, y) = xa + yb + (1-x-y)c, \end{aligned} \quad (\text{B36})$$

and setting $a = M_E^2$, $b = m_{\Phi^+}^2$, and $c = m_\Sigma^2$ yields

$$J_E = 6 \int_0^1 dx x \int_0^{1-x} dy \int \frac{d^n k}{(2\pi)^n} \frac{1}{(\Delta - k^2)^4}. \quad (\text{B37})$$

The dimensional-regularization integral gives

$$\int \frac{d^n k}{(2\pi)^n} \frac{1}{(\Delta - k^2)^4} = \frac{i\Gamma(4 - n/2)}{(4\pi)^{n/2}\Gamma(4)} \frac{1}{\Delta^{4-n/2}} \quad (\text{B38})$$

and taking the $n \rightarrow 4$ limit ($n = 4 - 2\varepsilon$, $\varepsilon \rightarrow 0$) reduces it to

$$\int \frac{d^n k}{(2\pi)^n} \frac{1}{(\Delta - k^2)^4} \rightarrow \frac{1}{6} \times \frac{i}{16\pi^2} \frac{1}{\Delta^2}. \quad (\text{B39})$$

Therefore,

$$\begin{aligned} J_E = \frac{i}{16\pi^2} \int_0^1 dx x \int_0^{1-x} dy \frac{1}{\Delta(x, y)^2}, \\ \Delta(x, y) = xM_E^2 + ym_{\Phi^+}^2 + (1-x-y)m_\Sigma^2 \end{aligned} \quad (\text{B40})$$

follows. To obtain a closed form, we introduce the mass ratios

$$r_\Phi := \frac{m_{\Phi^+}^2}{M_E^2}, \quad r_\Sigma := \frac{m_\Sigma^2}{M_E^2}, \quad (\text{B41})$$

and rewrite

$$\begin{aligned} \Delta(x, y) = xM_E^2 + ym_{\Phi^+}^2 + (1-x-y)m_\Sigma^2 \\ = M_E^2[r_\Sigma + x(1-r_\Sigma) + y(r_\Phi - r_\Sigma)]. \end{aligned} \quad (\text{B42})$$

Then Eq. (B40) becomes

$$\begin{aligned} J_E = \frac{i}{16\pi^2} \frac{1}{M_E^4} \int_0^1 dx x \\ \times \int_0^{1-x} dy \frac{1}{[r_\Sigma + x(1-r_\Sigma) + y(r_\Phi - r_\Sigma)]^2}. \end{aligned} \quad (\text{B43})$$

Performing the y integral gives

$$\int_0^{1-x} dy \frac{1}{[r_\Sigma + x(1-r_\Sigma) + y(r_\Phi - r_\Sigma)]^2} = \frac{1}{r_\Phi - r_\Sigma} \left[\frac{1}{r_\Sigma + x(1-r_\Sigma)} - \frac{1}{r_\Phi + x(1-r_\Phi)} \right]. \quad (\text{B44})$$

Substituting into Eq. (B43) yields

$$J_E = \frac{i}{16\pi^2} \frac{1}{M_E^4} \frac{1}{r_\Phi - r_\Sigma} \times \int_0^1 dx \left[\frac{x}{r_\Sigma + x(1-r_\Sigma)} - \frac{x}{r_\Phi + x(1-r_\Phi)} \right]. \quad (\text{B45})$$

We next evaluate the auxiliary function

$$F(r) := \int_0^1 dx \frac{x}{r + x(1-r)}. \quad (\text{B46})$$

Setting $a := 1 - r$, we write

$$\frac{x}{r + ax} = \frac{1}{a} \left(1 - \frac{r}{r + ax} \right), \quad (\text{B47})$$

and for $r \neq 1$ this gives

$$F(r) = \frac{1}{a} \int_0^1 dx - \frac{r}{a} \int_0^1 dx \frac{1}{r + ax} = \frac{1}{1-r} + \frac{r \ln r}{(1-r)^2}. \quad (\text{B48})$$

Equation (B45) can then be expressed as

$$J_E = \frac{i}{16\pi^2} \frac{1}{M_E^4} \frac{1}{r_\Phi - r_\Sigma} [F(r_\Sigma) - F(r_\Phi)]. \quad (\text{B49})$$

Substituting Eq. (B48) and first organizing the rational terms, we find

$$\frac{1}{r_\Phi - r_\Sigma} \left[\frac{1}{1-r_\Sigma} - \frac{1}{1-r_\Phi} \right] = -\frac{1}{(1-r_\Phi)(1-r_\Sigma)}. \quad (\text{B50})$$

Therefore, the bracket in Eq. (B49) becomes

$$\begin{aligned} & \frac{1}{r_\Phi - r_\Sigma} [F(r_\Sigma) - F(r_\Phi)] \\ &= -\frac{1}{(1-r_\Phi)(1-r_\Sigma)} \\ &+ \frac{1}{r_\Phi - r_\Sigma} \left[\frac{r_\Sigma \ln r_\Sigma}{(1-r_\Sigma)^2} - \frac{r_\Phi \ln r_\Phi}{(1-r_\Phi)^2} \right]. \end{aligned} \quad (\text{B51})$$

Defining

$$\mathcal{I}(r_\Phi, r_\Sigma) := -\frac{1}{(1-r_\Phi)(1-r_\Sigma)} + \frac{1}{r_\Phi - r_\Sigma} \left[\frac{r_\Sigma \ln r_\Sigma}{(1-r_\Sigma)^2} - \frac{r_\Phi \ln r_\Phi}{(1-r_\Phi)^2} \right], \quad (\text{B52})$$

we obtain

$$J_E = \frac{i}{16\pi^2} \frac{1}{M_E^4} \mathcal{I}(r_\Phi, r_\Sigma). \quad (\text{B53})$$

The limit $r_\Phi = r_\Sigma$ can be obtained directly from Eq. (B43), or defined continuously from Eq. (B51) using l'Hôpital's rule. In particular, for $r_\Phi = r_\Sigma = 1$ we have $\Delta = M_E^2$, and Eq. (B40) immediately gives

$$\mathcal{I}(1, 1) = \int_0^1 dx x \int_0^{1-x} dy = \int_0^1 dx x(1-x) = \frac{1}{6}. \quad (\text{B54})$$

Using Eqs. (B33) and (B53), we find

$$\begin{aligned} I_{B,\sigma}^{(E)\mu}(q) &= iM_E \sigma^{\mu\nu} q_\nu P_R \frac{i}{16\pi^2} \frac{1}{M_E^4} \mathcal{I}(r_\Phi, r_\Sigma) \\ &= -\frac{1}{16\pi^2} \frac{1}{M_E^3} \mathcal{I}(r_\Phi, r_\Sigma) \sigma^{\mu\nu} q_\nu P_R. \end{aligned} \quad (\text{B55})$$

Consequently, the dipole part of Eq. (B27) is

$$\begin{aligned} i\mathcal{M}_{B,\sigma}^{(E)} &= (ig'Y_E)(-iy_{\Sigma,\alpha})(-iy_{H,i}^*)(-iy_\sigma^*) \\ &\times \bar{u}_{L_\alpha} \left[-\frac{1}{16\pi^2} \frac{1}{M_E^3} \mathcal{I}(r_\Phi, r_\Sigma) \right] \sigma^{\mu\nu} q_\nu P_R u_{N_i} \varepsilon_\mu. \end{aligned} \quad (\text{B56})$$

Moreover, the factor $q_\nu \varepsilon_\mu$ can be recombined into the field strength. In momentum space, $B_{\mu\nu} = i(q_\mu \varepsilon_\nu - q_\nu \varepsilon_\mu)$, and by the antisymmetry of $\sigma^{\mu\nu}$ one has Eq. (59). Thus the spinor structure $\bar{u}_{L_\alpha} \sigma^{\mu\nu} q_\nu P_R u_{N_i} \varepsilon_\mu$ can be traded for the corresponding field-strength contraction [up to the conventional factor implicit in Eq. (59)]. To make the operator gauge invariant in the symmetric phase, one must insert a Higgs doublet. At the present stage, however, the external scalar is $\tilde{D}$, and the corresponding local EFT operator is identified as

$$\mathcal{O}_{NB,ai}^{(D)} = (\bar{L}_\alpha \sigma^{\mu\nu} P_R N_i) \tilde{D} B_{\mu\nu}. \quad (\text{B57})$$

Accordingly, one defines the Wilson coefficient of this operator by the coefficient of the dipole spinor structure extracted from the UV amplitude.

Finally, we define the Wilson-coefficient contribution corresponding to Fig. 1(a) as the coefficient of $\bar{u}_{L_\alpha} \sigma^{\mu\nu} q_\nu P_R u_{N_i} \varepsilon_\mu$ [cf. Eq. (B13)]:

$$i\mathcal{M}_{B,\sigma}^{(E)} := \bar{u}_{L_\alpha} \sigma^{\mu\nu} q_\nu P_R u_{N_i} \varepsilon_\mu \frac{2C_{NB,ai}^{(D)}|_{(E)}}{\Lambda^2}. \quad (\text{B58})$$

With this definition, Eq. (B56) yields

$$\frac{C_{NB,ai}^{(D)}|_{(E)}}{\Lambda^2} = \frac{g' Y_{E\Sigma,\alpha} y_{H,i}^* \mu_s^*}{16\pi^2 M_E^3} \mathcal{I}\left(\frac{m_{\Phi^+}^2}{M_E^2}, \frac{m_\Sigma^2}{M_E^2}\right). \quad (\text{B59})$$

The remaining diagrams, in which B is attached to the Φ^+ or Σ line [diagrams (b) and (c)] and W is attached to the Σ line [diagram (d)], are treated by the same q expansion, dipole projection, and master-integral reduction, so we do not repeat the intermediate steps here and instead summarize their substitutions and final matching coefficients in the next subsection.

5. One-loop matching results and a unified representation

The detailed derivation for diagram (a) given in Appendix B 4 already establishes the only nontrivial loop integral needed in this class of one-loop matchings. The remaining diagrams differ only in two respects: which internal line carries the external gauge-boson insertion, and the corresponding gauge factor. We therefore do not repeat the q expansion, the projection onto the $\sigma^{\mu\nu} q_\nu$ structure, or the Feynman-parameter evaluation.

For later convenience, we introduce the master scalar integral

$$J(a; b, c) := \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - a)^2 (k^2 - b)(k^2 - c)}. \quad (\text{B60})$$

Using the result already derived in Eq. (B53), one immediately finds

$$J(a; b, c) = \frac{i}{16\pi^2} \frac{1}{a^2} \mathcal{I}\left(\frac{b}{a}, \frac{c}{a}\right), \quad (\text{B61})$$

where the loop function $\mathcal{I}$ is the same as in Eq. (B52). Its degenerate limit is understood by continuity, and in particular Eq. (B54) implies

$$\mathcal{I}(1, 1) = \frac{1}{6}. \quad (\text{B62})$$

With this notation, the remaining one-loop diagrams in Figs. 1(b)–1(d) are obtained from diagram (a) simply by choosing which propagator is squared, i.e. which internal line emits the external gauge boson:

- (a) B attached to E : $(a, b, c) = (M_E^2, m_{\Phi^+}^2, m_\Sigma^2)$,
- (b) B attached to Φ^+ : $(a, b, c) = (m_{\Phi^+}^2, M_E^2, m_\Sigma^2)$,
- (c) B attached to Σ : $(a, b, c) = (m_\Sigma^2, M_E^2, m_{\Phi^+}^2)$,
- (d) W^a attached to Σ : $(a, b, c) = (m_\Sigma^2, M_E^2, m_{\Phi^+}^2)$. (B63)

For diagram (d), the $\Sigma - W^a$ vertex carries the generator

$$\tau^a = \frac{\sigma^a}{2}, \quad (\text{B64})$$

which is the same normalization used in the definition of $\mathcal{O}_{NW}^{(D)}$; hence no additional conversion factor appears.

The Wilson-coefficient contributions to $\mathcal{O}_{NB}^{(D)}$ are therefore

$$\begin{aligned} \frac{C_{NB,ai}^{(D,E)}}{\Lambda^2} &= \frac{g' Y_{E\Sigma,\alpha} y_{H,i}^* \mu_s^*}{16\pi^2} \frac{1}{M_E^3} \mathcal{I}\left(\frac{m_{\Phi^+}^2}{M_E^2}, \frac{m_\Sigma^2}{M_E^2}\right), \\ \frac{C_{NB,ai}^{(D,\Phi^+)}}{\Lambda^2} &= \frac{g' Y_{\Phi^+\Sigma,\alpha} y_{H,i}^* \mu_s^*}{16\pi^2} \frac{M_E}{m_{\Phi^+}^4} \mathcal{I}\left(\frac{M_E^2}{m_{\Phi^+}^2}, \frac{m_\Sigma^2}{m_{\Phi^+}^2}\right), \\ \frac{C_{NB,ai}^{(D,\Sigma)}}{\Lambda^2} &= \frac{g' Y_{\Sigma\Sigma,\alpha} y_{H,i}^* \mu_s^*}{16\pi^2} \frac{M_E}{m_\Sigma^4} \mathcal{I}\left(\frac{M_E^2}{m_\Sigma^2}, \frac{m_{\Phi^+}^2}{m_\Sigma^2}\right). \end{aligned} \quad (\text{B65})$$

Summing the three hypercharge attachments, we obtain

$$\frac{C_{NB,ai}^{(D)}}{\Lambda^2} = \frac{C_{NB,ai}^{(D,E)}}{\Lambda^2} + \frac{C_{NB,ai}^{(D,\Phi^+)}}{\Lambda^2} + \frac{C_{NB,ai}^{(D,\Sigma)}}{\Lambda^2}. \quad (\text{B66})$$

Similarly, since only Σ carries $\text{SU}(2)_L$ charge in the loop, the contribution to $\mathcal{O}_{NW}^{(D)}$ is

$$\frac{C_{NW,ai}^{(D)}}{\Lambda^2} = \frac{g Y_{\Sigma,\alpha} y_{H,i}^* \mu_s^*}{16\pi^2} \frac{M_E}{m_\Sigma^4} \mathcal{I}\left(\frac{M_E^2}{m_\Sigma^2}, \frac{m_{\Phi^+}^2}{m_\Sigma^2}\right). \quad (\text{B67})$$

Thus, Appendix B 4 serves as the explicit worked example, whereas the present subsection only records the diagram-by-diagram substitutions and the final matching coefficients.

APPENDIX C: ONE-LOOP ANOMALOUS DIMENSIONS FOR THE DIPOLE OPERATORS

1. $B - W^3$ vacuum polarization

In Sec. III we used the fact that the mixed $B - W^3$ vacuum polarization has no UV-divergent part in the electroweak-symmetric phase. Here we give the explicit calculation. The relevant one-loop contributions contain light fields charged under both $\text{U}(1)_Y$ and $\text{SU}(2)_L$. Their divergent part is proportional to the group-theory factor $\sum_{\text{light}} Y T^3$, which vanishes after summing over complete $\text{SU}(2)_L$ multiplets. Therefore, the off-diagonal wave-function counterterm $Z_{BW}^{(1)}$ vanishes, and no one-loop mixing between C_{NB} and C_{NW} is induced through gauge-boson wave-function renormalization.

a. One-loop integral with an internal Dirac fermion

We first evaluate the one-loop integral for a Dirac fermion f . Since we are interested only in the amputated two-point function, amputation removes the external fields from the loop integral, and we obtain

$$\begin{aligned}
i\Pi_{BW,f}^{\mu\nu}(q) &= (-1) \int \frac{d^4 k}{(2\pi)^4} \\
&\quad \times \frac{\text{tr}\{i(k+m_f)(ig'Y_f\gamma^\mu)i(k-q+m_f)[ig(\tau^3/2)\gamma^\nu P_L]\}}{(k^2-m_f^2)[(k-q)^2-m_f^2]} \\
&= -g'gY_fT_f^3 \int \frac{d^n k}{(2\pi)^n} \frac{\text{tr}[(k+m_f)\gamma^\mu(k-q+m_f)\gamma^\nu P_L]}{(k^2-m_f^2)[(k-q)^2-m_f^2]}. \quad (C1)
\end{aligned}$$

In the second line we have continued the loop integral to n dimensions.

For the Pauli matrices τ^a and the third component of weak isospin T_f^3 , we use the following relation. Since the $SU(2)_L$ generators are τ^a , acting with τ^3 on a doublet component (the $a = 1, 2$ components do not appear in the present graph) yields

$$T_f^3 = \begin{cases} +1/2, & (f_1 = \nu_L, u_L), \\ -1/2, & (f_2 = e_L, d_L), \\ 0, & (\text{RH singlet}). \end{cases} \quad (C2)$$

Evaluating the trace in the numerator, we find

$$\begin{aligned}
&\text{tr}[(k+m_f)\gamma^\mu(k-q+m_f)\gamma^\nu P_L] \\
&= 2(2k^\mu k^\nu - g^{\mu\nu}k^2 + g^{\mu\nu}k \cdot q - k^\mu q^\nu - k^\nu q^\mu + g^{\mu\nu}m_f^2). \quad (C3)
\end{aligned}$$

Substituting this into Eq. (C1), we obtain

$$\begin{aligned}
i\Pi_{BW,f}^{\mu\nu}(q) &= -2g'gY_fT_f^3 \int \frac{d^n k}{(2\pi)^n} \\
&\quad \times \frac{2k^\mu k^\nu - k^\mu q^\nu - k^\nu q^\mu - g^{\mu\nu}(k^2 - k \cdot q - m_f^2)}{(k^2-m_f^2)[(k-q)^2-m_f^2]}. \quad (C4)
\end{aligned}$$

Applying the Feynman-parameter formula to the denominator,

$$\begin{aligned}
&\frac{1}{(k^2-m_f^2)[(k-q)^2-m_f^2]} \\
&= \int_0^1 dx \frac{1}{(m_f^2 - q^2x + 2k \cdot qx - k^2)^2}, \quad (C5)
\end{aligned}$$

and using the dimensional-regularization integrals, we obtain

$$\begin{aligned}
i\Pi_{BW,f}^{\mu\nu}(q) &= -2g'gY_fT_f^3 \int_0^1 dx \int \frac{d^n k}{(2\pi)^n} \\
&\quad \times \frac{2k^\mu k^\nu - k^\mu q^\nu - k^\nu q^\mu - g^{\mu\nu}(k^2 - k \cdot q - m_f^2)}{(m_f^2 - q^2x + 2k \cdot qx - k^2)^2} \\
&= -\frac{ig'gY_fT_f^3}{16\pi^2} \bar{\epsilon}^{-1} \left[+\frac{2}{3}(g^{\mu\nu}q^2 - q^\mu q^\nu) \right]. \quad (C6)
\end{aligned}$$

Therefore,

$$\Pi_{BW,f}^{\mu\nu}(q) = \frac{g'gY_fT_f^3}{16\pi^2} \bar{\epsilon}^{-1} \left[-\frac{2}{3}(g^{\mu\nu}q^2 - q^\mu q^\nu) \right]. \quad (C7)$$

Here $\bar{\epsilon}^{-1}$, defined in Eq. (83), denotes the UV-divergent part in the $\overline{\text{MS}}$ scheme.

b. One-loop integral with an internal Higgs doublet

Next, we evaluate the one-loop integral for the Higgs doublet H . After amputation, no external fields remain inside the loop integral, and we obtain

$$\begin{aligned}
i\Pi_{BW,\text{bubble}}^{\mu\nu}(q) &= \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2-m_H^2} [ig'Y_H(2k-q)^\mu] \\
&\quad \times \frac{i}{(k-q)^2-m_H^2} [igT_H^3(2k-q)^\nu] \\
&= g'gY_HT_H^3 \int \frac{d^n k}{(2\pi)^n} \frac{4k^\mu k^\nu - 2k^\mu q^\nu - 2q^\mu k^\nu + q^\mu q^\nu}{(k^2-m_H^2)[(k-q)^2-m_H^2]}. \quad (C8)
\end{aligned}$$

Using the Feynman-parameter formula,

$$\begin{aligned}
&\frac{1}{(k^2-m_H^2)[(k-q)^2-m_H^2]} \\
&= \int_0^1 dx \frac{1}{(m_H^2 - q^2x + 2k \cdot qx - k^2)^2}, \quad (C9)
\end{aligned}$$

we find

$$\begin{aligned}
i\Pi_{BW,\text{bubble}}^{\mu\nu}(q) &= g'gY_HT_H^3 \\
&\quad \times \int \frac{d^n k}{(2\pi)^n} \int_0^1 dx \frac{4k^\mu k^\nu - 2k^\mu q^\nu - 2q^\mu k^\nu + q^\mu q^\nu}{(m_H^2 - q^2x + 2k \cdot qx - k^2)^2} \\
&= \frac{ig'gY_HT_H^3}{16\pi^2} \bar{\epsilon}^{-1} \left[2g^{\mu\nu}m_H^2 - \frac{1}{3}(g^{\mu\nu}q^2 - q^\mu q^\nu) \right]. \quad (C10)
\end{aligned}$$

Therefore, we obtain

$$\begin{aligned}
\Pi_{BW,\text{bubble}}^{\mu\nu}(q) &= \frac{g'gY_HT_H^3}{16\pi^2} \bar{\epsilon}^{-1} \left[2g^{\mu\nu}m_H^2 - \frac{1}{3}(g^{\mu\nu}q^2 - q^\mu q^\nu) \right]. \quad (C11)
\end{aligned}$$

c. Seagull diagram with an internal Higgs doublet

The Higgs sector contains yet another type of vacuum polarization. Expanding the Higgs kinetic term,

$$\begin{aligned}\mathcal{L}_{\text{kin}} &= (D_\mu H)^\dagger (D^\mu H), \\ D_\mu &= \partial_\mu - ig'Y_H B_\mu - ig\tau^a W_\mu^a,\end{aligned}\quad (\text{C12})$$

we find

$$\begin{aligned}(D_\mu H)^\dagger (D^\mu H) &\supset 2g'gY_H T_H^3 B_\mu W^{3\mu} H^\dagger H \\ &= 2g'gY_H T_H^3 g^{\mu\nu} B_\mu W_\nu^3 H^\dagger H,\end{aligned}\quad (\text{C13})$$

which implies the four-point interaction (“seagull interaction”)

$$\begin{aligned}V_{BW^3HH}^{\mu\nu} &:= \frac{i\delta^4}{\delta B_\mu \delta W_\nu^3 \delta H \delta H^\dagger} \int d^n x \, 2g'gY_H T_H^3 g^{\rho\sigma} B_\rho W_\sigma^3 H^\dagger H \\ &= +2ig'gY_H T_H^3 g^{\mu\nu}.\end{aligned}\quad (\text{C14})$$

This vertex is shown in Fig. 8(a). Contracting the Higgs legs of Fig. 8(a) yields the tadpole (seagull) contribution to the mixed vacuum polarization shown in Fig. 8(b), which can be written as

$$i\Pi_{BW,\text{seagull}}^{\mu\nu}(q) = V_{BW^3HH}^{\mu\nu} \int \frac{d^n k}{(2\pi)^n} \frac{i}{k^2 - m_H^2}. \quad (\text{C15})$$

Evaluating Fig. 8(b), the UV contribution is

$$\begin{aligned}i\Pi_{BW,\text{seagull}}^{\mu\nu}(q) &= 2ig'gY_H T_H^3 g^{\mu\nu} \int \frac{d^n k}{(2\pi)^n} \frac{-i}{m_H^2 - k^2} \\ &= -\frac{ig'gY_H T_H^3}{16\pi^2} \bar{\epsilon}^{-1} (2g^{\mu\nu} m_H^2),\end{aligned}\quad (\text{C16})$$

and hence

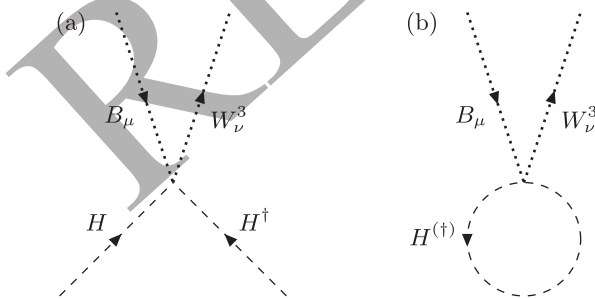


FIG. 8. Seagull contribution induced by the quartic BW^3HH interaction. (a) The BW^3HH seagull vertex. (b) The corresponding tadpole (seagull) contribution to the mixed vacuum polarization $\Pi_{BW}^{\mu\nu}$ obtained by contracting the Higgs legs.

$$\Pi_{BW,\text{seagull}}^{\mu\nu}(q) = \frac{g'gY_H T_H^3}{16\pi^2} \bar{\epsilon}^{-1} (-2g^{\mu\nu} m_H^2). \quad (\text{C17})$$

Combining the bubble (C11) and seagull (C17) contributions, the Higgs-loop vacuum polarization $\Pi_{BW,H}^{\mu\nu}(q)$ is given by

$$\Pi_{BW,H}^{\mu\nu}(q) = \frac{g'gY_H T_H^3}{16\pi^2} \bar{\epsilon}^{-1} \left[-\frac{1}{3} (g^{\mu\nu} q^2 - q^\mu q^\nu) \right]. \quad (\text{C18})$$

d. One-loop integral with an internal FP ghost

Since the gauge-boson two-point function must be transverse, it satisfies the Slavnov-Taylor identity

$$q_\mu \Pi^{\mu\nu}(q) = 0. \quad (\text{C19})$$

This follows from BRST symmetry (gauge invariance of the quantum theory). Therefore, the “mass-pole” terms of the form $g^{\mu\nu} m_H^2 \bar{\epsilon}^{-1}$, which appear in the one-loop computation of the β function, must cancel. If such terms remained, they would imply that the gauge boson acquires a mass, leading to a contradiction.

The Faddeev-Popov (FP) ghosts [93] ($c, \bar{c}$) are scalar fields in the adjoint representation with anticommuting Grassmann statistics. Hence they do not generate mass poles, while the derivative structure of their three-point vertices cancels the ξ -dependent contributions arising from the gauge-boson sector in the kinetic terms (where ξ is the gauge parameter). In other words, they guarantee the ξ independence of the transverse gauge two-point function (gauge invariance of physical quantities). Consequently, FP ghosts must be included in loops in order to maintain gauge invariance.

Since the FP ghosts carry no hypercharge ($Y = 0$), there is no vertex of the form $B_\mu \bar{c} c$ in the present case, and therefore they do not contribute to $\Pi_{BW}^{\mu\nu}(q)$.

e. Derivation of the scalar vacuum polarization (transverse component)

We denote the transverse scalar form factor of the vacuum polarization $\Pi_{BW}^{\mu\nu}(q)$ simply by $\Pi_{BW}(q^2)$. We define the transverse projector $P_T^{\mu\nu}(q)$ by

$$P_T^{\mu\nu}(q) := g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}. \quad (\text{C20})$$

The projector $P_T^{\mu\nu}(q)$ satisfies transversality,

$$q_\mu P_T^{\mu\nu}(q) = 0, \quad P_T^{\mu\nu}(q) q_\nu = 0, \quad (\text{C21})$$

and extracts the transverse part of the Ward-Takahashi identity. It also satisfies the projector property,

$$P_{T\rho}^\mu(q)P_T^{\rho\nu}(q) = P_T^{\mu\nu}(q). \quad (C22)$$

Since the tensor structure of the vacuum polarization can be written as

$$g^{\mu\nu}q^2 - q^\mu q^\nu = q^2 \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) = q^2 P_T^{\mu\nu}(q), \quad (C23)$$

multiplying both sides of

$$\Pi_{BW,f}^{\mu\nu}(q) = (g^{\mu\nu}q^2 - q^\mu q^\nu) \Pi_{BW,f}(q^2) \quad (C24)$$

by the transverse projector $P_{\mu\nu}^T(q)$ yields

$$P_{\mu\nu}^T(q) \Pi_{BW,f}^{\mu\nu}(q) = (n-1)q^2 \Pi_{BW,f}(q^2), \quad (C25)$$

and hence

$$\Pi_{BW,f}(q^2) = \frac{1}{(n-1)q^2} P_{\mu\nu}^T(q) \Pi_{BW,f}^{\mu\nu}(q). \quad (C26)$$

Therefore, to obtain $\Pi_{BW,f}(q^2)$, it suffices to multiply the two-point function $\Pi_{BW,f}^{\mu\nu}(q)$ by the transverse projector $P_{\mu\nu}^T(q)$ and then by the factor $1/[(n-1)q^2]$. In the present case the vacuum polarization has only a transverse component, so the evaluation is straightforward:

$$\Pi_{BW,f}(q^2) = \frac{2}{3} \times \frac{g' g Y_f T_f^3}{16\pi^2} \bar{\epsilon}^{-1}. \quad (C27)$$

Using the identity

$$P_{\mu\nu}^T(q) g^{\mu\nu} = n-1 \quad (C28)$$

gives

$$\Pi_{BW,H}(q^2) = -\frac{1}{3} \times \frac{g' g Y_H T_H^3}{16\pi^2} \bar{\epsilon}^{-1}. \quad (C29)$$

f. UV coefficient of the $B-W^3$ mixing term

From the above results, the UV pole of the $B-W^3$ mixed vacuum polarization is given, for a Dirac fermion and the Higgs field respectively, by

$$\Pi_{BW,f}(q^2) = \frac{2}{3} \times \frac{g' g Y_f T_f^3}{16\pi^2} \bar{\epsilon}^{-1}, \quad (C30)$$

$$\Pi_{BW,H}(q^2) = -\frac{1}{3} \times \frac{g' g Y_H T_H^3}{16\pi^2} \bar{\epsilon}^{-1}. \quad (C31)$$

Noting that both expressions are proportional to YT^3 , the complete $SU(2)_L$ multiplet sums vanish:

$$\sum_{\text{light}} Y_f T_f^3 = 0, \quad \sum_H Y_H T_H^3 = 0. \quad (C32)$$

Indeed, within each left-handed fermion doublet the two components have the same hypercharge and opposite values of T^3 , while right-handed singlets have $T^3 = 0$. The Higgs doublet gives the same cancellation between its two $SU(2)_L$ components. Therefore,

$$\sum_f \Pi_{BW,f}(q^2) = \frac{2}{3} \frac{g' g}{16\pi^2} \bar{\epsilon}^{-1} \sum_f Y_f T_f^3 = 0, \quad (C33)$$

$$\sum_H \Pi_{BW,H}(q^2) = -\frac{1}{3} \frac{g' g}{16\pi^2} \bar{\epsilon}^{-1} \sum_H Y_H T_H^3 = 0. \quad (C34)$$

Consequently,

$$\Pi_{BW}(q^2) = 0, \quad (C35)$$

after summing over the light SM fields in the electroweak-symmetric phase.

2. $B_\mu B_\nu$ self-energy

As shown above, the pole part of the mixed $B-W^3$ vacuum polarization vanishes after summing over the light SM fields, so the corresponding one-loop off-diagonal wave-function counterterm is absent:

$$Z_{BW}^{(1)} = 0. \quad (C36)$$

Consequently, $B-W^3$ wave-function mixing does not contribute to the anomalous dimensions of C_{NB} and C_{NW} at the order considered here.

It remains to compute the diagonal gauge-boson wave-function factors, the lepton-doublet wave-function factor, and the Higgs-doublet wave-function factor. We begin with the self-energy of the hypercharge gauge field B_μ .

a. B self-energy with an internal Dirac fermion

Evaluating the one-loop integral for a Dirac fermion f , we obtain

$$\begin{aligned} i\Pi_{BB,f}^{\mu\nu}(q) &= -g^2 Y_f^2 \int \frac{d^n k}{(2\pi)^n} \frac{\text{tr}[(k+m_f)\gamma^\mu(k-\not{q}+m_f)\gamma^\nu]}{(k^2-m_f^2)[(k-q)^2-m_f^2]}. \end{aligned} \quad (C37)$$

The trace in the numerator is

$$\begin{aligned} \text{tr}[(k+m_f)\gamma^\mu(k-\not{q}+m_f)\gamma^\nu] &= 4(2k^\mu k^\nu - g^{\mu\nu} k^2 + g^{\mu\nu} k \cdot q - k^\mu q^\nu - k^\nu q^\mu + g^{\mu\nu} m_f^2). \end{aligned} \quad (C38)$$

Hence, the integral reduces to the same tensor structure. Continuing the computation of $i\Pi_{BB,f}^{\mu\nu}(q)$, we find

$$i\Pi_{BB,f}^{\mu\nu}(q) = -4g^2 Y_f^2 \int \frac{d^n k}{(2\pi)^n} \times \frac{2k^\mu k^\nu - k^\mu q^\nu - k^\nu q^\mu - g^{\mu\nu}(k^2 - k \cdot q - m_f^2)}{(k^2 - m_f^2)[(k - q)^2 - m_f^2]}. \quad (\text{C39})$$

Applying the Feynman-parameter formula to the denominator, and using the dimensional-regularization integral formulas, we obtain

$$\begin{aligned} i\Pi_{BB,f}^{\mu\nu}(q) &= -4g^2 Y_f^2 \int_0^1 dx \\ &\times \int \frac{d^n k}{(2\pi)^n} \frac{2k^\mu k^\nu - k^\mu q^\nu - k^\nu q^\mu - g^{\mu\nu}(k^2 - k \cdot q - m_f^2)}{(m_f^2 - q^2 x + 2k \cdot qx - k^2)^2} \\ &= -\frac{ig^2 Y_f^2}{16\pi^2} \bar{\epsilon}^{-1} \left[\frac{4}{3} (g^{\mu\nu} q^2 - q^\mu q^\nu) \right], \end{aligned} \quad (\text{C40})$$

and

$$\Pi_{BB,f}^{\mu\nu}(q) = \frac{g^2 Y_f^2}{16\pi^2} \bar{\epsilon}^{-1} \left[-\frac{4}{3} (g^{\mu\nu} q^2 - q^\mu q^\nu) \right]. \quad (\text{C41})$$

b. B self-energy with an internal Higgs doublet

Evaluating the one-loop integral for the Higgs doublet H , we obtain

$$i\Pi_{BB,H}^{\mu\nu}(q) = g^2 Y_H^2 \int \frac{d^n k}{(2\pi)^n} \frac{(2k - q)^\mu (2k - q)^\nu}{(k^2 - m_H^2)[(k - q)^2 - m_H^2]}. \quad (\text{C42})$$

This is the same integral as in Appendix C 1 b. Accounting for the difference in the overall coefficient, the bubble diagram yields

$$\Pi_{BB,\text{bubble}}^{\mu\nu}(q) = \frac{g^2 Y_H^2}{16\pi^2} \bar{\epsilon}^{-1} \left[2g^{\mu\nu} m_H^2 - \frac{1}{3} (g^{\mu\nu} q^2 - q^\mu q^\nu) \right]. \quad (\text{C43})$$

We next compute the seagull contribution. Expanding the Higgs kinetic term generates the quartic interaction $g^2 Y_H^2 B_\mu B^\mu H^\dagger H$. The amputated $BBHH$ vertex is defined by functional differentiation with respect to the external fields; since two identical gauge fields B_μ appear, this yields the combinatorial factor 2,

$$\begin{aligned} V_{BBHH}^{\mu\nu} &:= \frac{i\delta^4}{\delta B_\mu \delta B_\nu \delta H \delta H^\dagger} \int d^n x g^2 Y_H^2 B_\rho B^\rho H^\dagger H \\ &= 2ig^2 Y_H^2 g^{\mu\nu}. \end{aligned} \quad (\text{C44})$$

The one-loop integral then becomes

$$i\Pi_{BB,\text{seagull}}^{\mu\nu}(q) = 2ig^2 Y_H^2 g^{\mu\nu} \int \frac{d^n k}{(2\pi)^n} \frac{i}{k^2 - m_H^2}. \quad (\text{C45})$$

Since this is identical to the integral in Appendix C 1 c, we immediately obtain

$$\Pi_{BB,\text{seagull}}^{\mu\nu}(q) = \frac{g^2 Y_H^2}{16\pi^2} \bar{\epsilon}^{-1} (-2g^{\mu\nu} m_H^2). \quad (\text{C46})$$

Therefore, the sum of the bubble (C43) and seagull (C46) contributions is

$$\Pi_{BB,H}^{\mu\nu}(q) = \frac{g^2 Y_H^2}{16\pi^2} \bar{\epsilon}^{-1} \left[-\frac{1}{3} (g^{\mu\nu} q^2 - q^\mu q^\nu) \right]. \quad (\text{C47})$$

Adding all contributions to the B self-energy, we obtain

$$\begin{aligned} \Pi_{BB}^{\mu\nu}(q) &= \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} \left[-\sum_f \left(\frac{1}{2} \times \frac{4Y_f^2}{3} \right) - \sum_H \frac{Y_H^2}{3} \right] \\ &\times (g^{\mu\nu} q^2 - q^\mu q^\nu). \end{aligned}$$

Since we count chiral (Weyl) fermions, while the one-loop result in Eq. (C48) is written for a Dirac fermion with vector coupling, we include an overall factor of 1/2 (equivalently, $4/3 \rightarrow 2/3$ per Weyl fermion). Evaluating the sums, and noting the contributions from three generations, we find

$$\frac{2}{3} \sum_f Y_f^2 = \frac{2}{3} \times 3 \left(\frac{1}{6} + \frac{4}{3} + \frac{1}{3} + \frac{1}{2} + 1 \right) = \frac{20}{3}, \quad (\text{C48})$$

$$\frac{1}{3} \sum_H Y_H^2 = \frac{1}{3} \times 2 \times \left(\frac{1}{2} \right)^2 = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}. \quad (\text{C49})$$

Substituting these results, we obtain

$$\begin{aligned} \Pi_{BB}^{\mu\nu}(q) &= \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} \left(-\frac{20}{3} - \frac{1}{6} \right) (g^{\mu\nu} q^2 - q^\mu q^\nu) \\ &= \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} \left[-\frac{41}{6} (g^{\mu\nu} q^2 - q^\mu q^\nu) \right]. \end{aligned} \quad (\text{C50})$$

Extracting the transverse scalar component, we find

$$\begin{aligned}\Pi_{BB}(q^2) &= \frac{1}{(n-1)q^2} P_{\mu\nu}^T(q) \Pi_{BB}^{\mu\nu}(q) \\ &= -\frac{g'^2}{16\pi^2} \bar{\epsilon}^{-1} \frac{41}{6}.\end{aligned}\quad (\text{C51})$$

c. One-loop β function of the $U(1)_Y$ gauge coupling g'

We consider a theory consisting of the hypercharge gauge field B_μ , a Dirac fermion ψ , and the Higgs doublet H , and relate bare and renormalized quantities as

$$\begin{aligned}B_{0\mu} &= Z_3^{1/2} B_\mu, & \psi_0 &= Z_\psi^{1/2} \psi, \\ H_0 &= Z_H^{1/2} H, & g'^{(0)} &= Z_{g'} g', \\ m_{0f} &= Z_{m_f} m_f, & m_{0H}^2 &= Z_{m_H^2} m_H^2.\end{aligned}\quad (\text{C52})$$

The bare Lagrangian is

$$\begin{aligned}\mathcal{L}_0 &= -\frac{1}{4} B_{0\mu\nu} B_0^{\mu\nu} + \bar{\psi}_0 (i\gamma^\mu D_\mu^{(0)} - m_{0f}) \psi_0 \\ &\quad + (D_\mu^{(0)} H_0)^\dagger (D^{(0)\mu} H_0) - m_{0H}^2 H_0^\dagger H_0 - \lambda_0 (H_0^\dagger H_0)^2, \\ D_\mu^{(0)} &= \partial_\mu + ig'^{(0)} Y_{f,H} B_{0\mu},\end{aligned}$$

where $Y_{f,H}$ denotes the hypercharge of ψ or H .

Traditionally, the renormalized coupling g is defined through $g_0 = Z_1 Z_2^{-1} Z_3^{-1/2} g$, where Z_1 relates the bare and renormalized vertices, $Z_2 = Z_\psi$ is the fermion-field renormalization, and Z_3 is the gauge-field renormalization. In an Abelian gauge theory, the Ward-Takahashi identity implies $Z_1 = Z_\psi$ and hence

$$g_0 = Z_3^{-1/2} g, \quad (\text{C53})$$

i.e., $Z_{g'} = Z_3^{-1/2}$ in Eq. (C52). By contrast, in a non-Abelian gauge theory the coupling is often defined from the triple-gauge vertex, which leads to the relation (with $Z_2 = Z_3$)

$$g_0 = Z_1 Z_3^{-3/2} g. \quad (\text{C54})$$

Replacing the bare quantities using Eq. (C52) [and using $Z_{g'} = Z_3^{-1/2}$ for $U(1)_Y$], the renormalized Lagrangian can be written as

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\psi} (i\gamma^\mu \partial_\mu - m_f) \psi + \bar{\psi} g' Y_f \gamma^\mu B_\mu \psi \\ &\quad + (D_\mu H)^\dagger (D^\mu H) - m_H^2 H^\dagger H - \lambda (H^\dagger H)^2, \\ D_\mu &= \partial_\mu + ig' Y_H B_\mu,\end{aligned}\quad (\text{C55})$$

while the counterterm Lagrangian includes, in particular,

$$\begin{aligned}\mathcal{L}_{\text{count.}} &= -\frac{1}{4} (Z_3 - 1) B_{\mu\nu} B^{\mu\nu} \\ &\quad + \bar{\psi} [(Z_\psi - 1) i\gamma^\mu \partial_\mu - (Z_\psi Z_{m_f} - 1) m_f] \psi \\ &\quad + \bar{\psi} g' Y_f \gamma^\mu B_\mu \psi (Z_\psi - 1) \\ &\quad + (Z_H - 1) (D_\mu H)^\dagger (D^\mu H) \\ &\quad - (Z_H Z_{m_H^2} - 1) m_H^2 H^\dagger H,\end{aligned}\quad (\text{C56})$$

where we omit counterterms in the scalar potential, which are irrelevant for the determination of Z_3 and the one-loop running of g' .

Now consider the ultraviolet divergence of the B_μ self-energy,

$$\Pi_{BB}^{\mu\nu}(q) = \frac{g'^2}{16\pi^2} \bar{\epsilon}^{-1} \left[-\frac{41}{6} (g^{\mu\nu} q^2 - q^\mu q^\nu) \right]. \quad (\text{C57})$$

Using the Fourier decomposition

$$\begin{aligned}B_\mu(x) &= \int \frac{d^4 q}{(2\pi)^4} e^{-iqx} B_\mu(q), \\ B_{\mu\nu}(x) &= \int \frac{d^4 q}{(2\pi)^4} (-iq_\mu B_\nu + iq_\nu B_\mu) e^{-iqx},\end{aligned}\quad (\text{C58})$$

the counterterm contribution from Eq. (C56) reads

$$\begin{aligned}& -\frac{1}{4} (Z_3 - 1) B_{\mu\nu} B^{\mu\nu} \\ & \rightarrow -\frac{1}{4} (Z_3 - 1) \int d^4 x B_{\mu\nu} B^{\mu\nu} \\ & = -\frac{1}{2} (Z_3 - 1) \int \frac{d^4 q}{(2\pi)^4} B_\mu(-q) (g^{\mu\nu} q^2 - q^\mu q^\nu) B_\nu(q) \\ & \propto -Z_3^{(1)} (g^{\mu\nu} q^2 - q^\mu q^\nu),\end{aligned}\quad (\text{C59})$$

which must cancel the UV pole in Eq. (C57). Therefore,

$$\Pi_{BB}^{\mu\nu}(q) - Z_3^{(1)} = 0 \Rightarrow Z_3^{(1)} = -\frac{g'^2}{16\pi^2} \bar{\epsilon}^{-1} \frac{41}{6}. \quad (\text{C60})$$

We now use the definition of the β function [94],

$$\beta(\lambda) := \left(Q \frac{\partial}{\partial Q} \right)_0 \lambda, \quad \lambda_0 = Z_\lambda \lambda, \quad (\text{C61})$$

where subscript “0” on the partial derivative indicates that bare quantities are held fixed, and λ denotes a coupling constant. With Eq. (C53) inserted,

$$\begin{aligned}\beta(\lambda) &:= \left(Q \frac{\partial}{\partial Q} \right)_0 \lambda = -\lambda Q \frac{\partial \ln Z_\lambda}{\partial Q} \Big|_{\lambda_0, \epsilon} \\ &= -g' Q \frac{\partial \ln Z_3^{-1/2}}{\partial Q} \Big|_{g'_0, \epsilon},\end{aligned}\quad (\text{C62})$$

where $\bar{\epsilon}^{-1}$ denotes the ultraviolet-divergent part in the $\overline{\text{MS}}$ scheme, defined by (83),

$$\bar{\epsilon}^{-1} := \frac{2}{4-n} - \gamma + \ln 4\pi, \quad \epsilon^{-1} = \frac{2}{4-n}, \quad (\text{C63})$$

and we will use this quantity throughout. Using the formula [94],

$$\left(Q \frac{\partial}{\partial Q} \right)_{\lambda_0, \epsilon} \ln Z_i = -2\hbar \left(\frac{\partial Z_i^{(1)}}{\partial \bar{\epsilon}^{-1}} \right)_{\lambda, Q} + \mathcal{O}(\hbar^2). \quad (\text{C64})$$

From Eq. (C62) we then obtain

$$\begin{aligned} \beta(g') &= +\frac{1}{2} g' Q \frac{\partial \ln Z_3}{\partial Q} \Big|_{g'_0, \epsilon} = -g' \left(\frac{\partial Z_3^{(1)}}{\partial \bar{\epsilon}^{-1}} \right)_{g', Q} \\ &= +\frac{41}{6} \times \frac{g'^3}{16\pi^2}. \end{aligned} \quad (\text{C65})$$

Summarizing the above results, the $U(1)_Y$ gauge-field renormalization $Z_3 =: Z_B$ is given by

$$Z_B^{(1)} = -\frac{41}{6} \times \frac{g'^2}{16\pi^2} \bar{\epsilon}^{-1}, \quad (\text{C66})$$

and the beta function becomes

$$\beta(g') = +\frac{41}{6} \times \frac{g'^3}{16\pi^2}. \quad (\text{C67})$$

Hence, the $U(1)_Y$ gauge theory is not asymptotically free.

d. One-loop running of the $U(1)_Y$ coupling g'

Here, using the resulting beta function, we determine the one-loop running (RG evolution) of the coupling g' . From (C67),

$$\beta(g') = Q \frac{dg'}{dQ} = \frac{b_1}{16\pi^2} g'^3, \quad b_1 = \frac{41}{6}, \quad (\text{C68})$$

and separating variables gives

$$\frac{dg'}{g'^3} = \frac{b_1}{16\pi^2} \frac{dQ}{Q}. \quad (\text{C69})$$

Integrating both sides with the initial condition $Q_i := Q_{\text{match}}$, $g'_i := g'(Q_i)$ yields

$$g'(Q) = \frac{g'_i}{1 - \frac{b_1}{16\pi^2} g'^2_i \ln \frac{Q}{Q_i}}. \quad (\text{C70})$$

3. $W_\mu^3 W_\nu^3$ self-energy

From $Z_{BW}^{(1)} = 0$ shown in Sec. III B, operator mixing $B_\mu \leftrightarrow W_\mu^I$ does not contribute at one loop. This means that,

at the one-loop level, the Wilson-coefficient vector $(C_{NB} C_{NW})^T$ associated with the dimension-six operators $\{\mathcal{O}_{NB}, \mathcal{O}_{NW}\}$ satisfies

$$\begin{aligned} Q \frac{d}{dQ} \begin{pmatrix} C_{NB} \\ C_{NW} \end{pmatrix} &= \frac{1}{16\pi^2} \begin{pmatrix} \gamma_{BB}^{(1)} & \gamma_{BW}^{(1)} \\ \gamma_{WB}^{(1)} & \gamma_{WW}^{(1)} \end{pmatrix} \begin{pmatrix} C_{NB} \\ C_{NW} \end{pmatrix} \\ &= \frac{1}{16\pi^2} \begin{pmatrix} \gamma_{BB}^{(1)} C_{NB} \\ \gamma_{WW}^{(1)} C_{NW} \end{pmatrix}, \end{aligned} \quad (\text{C71})$$

i.e.,

$$Q \frac{d}{dQ} C_{NB} = \frac{\gamma_{BB}^{(1)}}{16\pi^2} C_{NB}, \quad Q \frac{d}{dQ} C_{NW} = \frac{\gamma_{WW}^{(1)}}{16\pi^2} C_{NW}, \quad (\text{C72})$$

which implies that the $U(1)_Y$ and $SU(2)_L$ sectors are completely decoupled at one loop.

a. W^3 self-energy with an internal Dirac fermion

The one-loop integral for a Dirac fermion f is given by

$$\begin{aligned} i\Pi_{WW,f}^{\mu\nu}(q) &= -g^2 (T_f^3)^2 \int \frac{d^n k}{(2\pi)^n} \\ &\times \frac{\text{tr}[(k + m_f) \gamma^\mu P_L (k - q + m_f) \gamma^\nu P_L]}{(k^2 - m_f^2)[(k - q)^2 - m_f^2]}. \end{aligned} \quad (\text{C73})$$

Rewriting the trace in the numerator as

$$\begin{aligned} \text{tr}[(k + m_f) \gamma^\mu P_L (k - q + m_f) \gamma^\nu P_L] \\ = 2[2k^\mu k^\nu - k^\mu q^\nu - k^\nu q^\mu - g^{\mu\nu}(k^2 - k \cdot q)], \end{aligned} \quad (\text{C74})$$

the dimensional-regularization integral becomes identical to that in Appendix C 2 a, up to an overall coefficient and the absence of the $g^{\mu\nu} m_f^2$ term. Since the final result in Appendix C 2 a is independent of m_f , we eventually obtain

$$\Pi_{WW,f}^{\mu\nu}(q) = \frac{g^2 (T_f^3)^2}{16\pi^2} \bar{\epsilon}^{-1} \left[-\frac{2}{3} (g^{\mu\nu} q^2 - q^\mu q^\nu) \right]. \quad (\text{C75})$$

Note that the coefficient is $-2/3$, rather than the value $-4/3$ often quoted in, e.g., Ref. [94], because the gauge group is $SU(2)_L$ (only left-handed fermions couple) rather than a vectorlike $SU(2)$.

b. W^3 self-energy with an internal Higgs doublet

The one-loop integral for the Higgs doublet H reads

$$\begin{aligned} i\Pi_{WW,\text{bubble}}^{\mu\nu}(q) \\ = g^2 (T_H^3)^2 \int \frac{d^n k}{(2\pi)^n} \frac{(2k - q)^\mu (2k - q)^\nu}{(k^2 - m_H^2)[(k - q)^2 - m_H^2]}. \end{aligned} \quad (\text{C76})$$

This is the same integral as in Appendix C 2 b up to an overall coefficient, and therefore

$$\begin{aligned} \Pi_{WW,\text{bubble}}^{\mu\nu}(q) &= \frac{g^2(T_H^3)^2}{16\pi^2} \bar{\epsilon}^{-1} \left[2g^{\mu\nu}m_H^2 - \frac{1}{3}(g^{\mu\nu}q^2 - q^\mu q^\nu) \right]. \quad (\text{C77}) \end{aligned}$$

On the other hand, the seagull contribution arises from the quartic interaction $g^2(T_H^3)^2 W_\mu^3 W^\mu H^\dagger H$. Since two identical gauge fields W_μ^3 appear, the variation is, as in Appendix C 2 b,

$$\begin{aligned} V_{W^3 W^3 H H}^{\mu\nu} &:= \frac{i\delta^4}{\delta W_\mu^3 \delta W_\nu^3 \delta H \delta H^\dagger} \int d^n x g^2 (T_H^3)^2 W_\rho^3 W^{3\rho} H^\dagger H \\ &= 2ig^2 (T_H^3)^2 g^{\mu\nu}, \quad (\text{C78}) \end{aligned}$$

so that the vertex factor is $2ig^2(T_H^3)^2 g^{\mu\nu}$. The one-loop integral then becomes

$$i\Pi_{WW,\text{seagull}}^{\mu\nu}(q) = 2ig^2(T_H^3)^2 g^{\mu\nu} \int \frac{d^n k}{(2\pi)^n} \frac{i}{k^2 - m_H^2}, \quad (\text{C79})$$

which is the same tadpole integral as in Appendix C 2 b. Hence, we immediately obtain

$$\Pi_{WW,\text{seagull}}^{\mu\nu}(q) = \frac{g^2(T_H^3)^2}{16\pi^2} \bar{\epsilon}^{-1} (-2g^{\mu\nu}m_H^2). \quad (\text{C80})$$

Therefore, the sum of the bubble and seagull contributions is

$$\Pi_{WW,H}^{\mu\nu}(q) = \frac{g^2(T_H^3)^2}{16\pi^2} \bar{\epsilon}^{-1} \left[-\frac{1}{3}(g^{\mu\nu}q^2 - q^\mu q^\nu) \right]. \quad (\text{C81})$$

c. $W_\mu^3 W_\nu^3$ self-energy with a $W^\pm$ loop

Fixing to the 't Hooft-Feynman gauge $\xi = 1$, the W -boson propagator is given by $D_{\mu\nu}^{ab}(k) = -i\delta^{ab}g_{\mu\nu}/k^2$. The three-gauge-boson vertex factor, with all momenta taken to flow into the vertex, can be written as [95]

$$\begin{aligned} V_{\mu\nu\rho}^{3ab}(p_1, p_2, p_3) &:= gf^{3ab}[g_{\mu\nu}(p_1 - p_2)_\rho + g_{\nu\rho}(p_2 - p_3)_\mu + g_{\rho\mu}(p_3 - p_1)_\nu]. \quad (\text{C82}) \end{aligned}$$

In this notation, the left vertex satisfies $p_1 = q$, $p_2 = k - q$, and $p_3 = -k$, so that

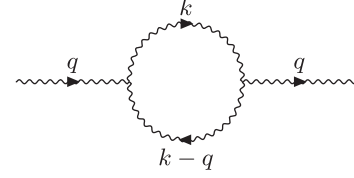


FIG. 9. One-loop W -boson self-energy from two triple-gauge vertices.

$$\begin{aligned} V_{\mu\rho\sigma}^{3ab}(q, k - q, -k) &= gf^{3ab}[g_{\mu\nu}(2q - k)^\rho + g_{\nu\rho}(2k - q)_\mu + g_{\rho\mu}(-k - q)_\nu]. \quad (\text{C83}) \end{aligned}$$

The right vertex satisfies $p_1 = k$, $p_2 = -k + q$, and $p_3 = -q$, so that

$$\begin{aligned} V_{\tau\lambda\nu}^{3cd}(k, -k + q, -q) &= gf^{dc3}[g_{\tau\lambda}(2k - q)_\nu + g_{\lambda\nu}(-k + 2q)_\tau + g_{\nu\tau}(-k - q)_\lambda]. \quad (\text{C84}) \end{aligned}$$

The vacuum polarization which corresponds to the one-loop diagram shown in Fig. 9, including the symmetry factor $1/2$, is then given by

$$\begin{aligned} i\Pi_{WW,W^\pm}^{\mu\nu}(q) &= \frac{g^2}{2} \int \frac{d^n k}{(2\pi)^n} \frac{-i\delta^{ac}g_{\alpha\gamma}}{k^2} \cdot \frac{-i\delta^{bd}g_{\beta\delta}}{(k - q)^2} \\ &\quad \times V_{\mu\alpha\beta}^{3ab}(-q, k, q - k) V_{\nu\gamma\delta}^{3cd}(k - q, -k, q). \quad (\text{C85}) \end{aligned}$$

Extracting the group and vertex factors, and using $f^{dc3} = -f^{3cd}$ together with the value of the quadratic Casimir in the adjoint representation of $SU(2)_L$,

$$\begin{aligned} C_2(G) &:= C_A = \sum_{c,d} \delta^{ac} \delta^{bd} f^{3ab} f^{3cd} \\ &= \sum_{a,b} f^{3ab} f^{3ab} = \sum_{a,b} \epsilon^{3ab} \epsilon^{3ab} = 2 \quad (\text{C86}) \end{aligned}$$

(where ϵ^{3ab} is the totally antisymmetric tensor) yields

$$\begin{aligned} \frac{g^2}{2} \delta^{ac} \delta^{bd} g_{\sigma\tau} g_{\lambda\rho} V_{\mu\rho\sigma}^{3ab}(q, k - q, -k) V_{\tau\lambda\nu}^{3cd}(k, -k + q, -q) \\ = -g^2 [g_{\mu\nu}(2k^2 - 2q \cdot k + 5q^2) \\ + 10k_\mu k_\nu - 5k_\mu q_\nu - 5q_\mu k_\nu - 2q_\mu q_\nu]. \quad (\text{C87}) \end{aligned}$$

Substituting this into the expression for the vacuum polarization, we obtain

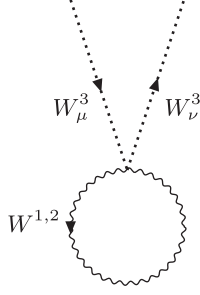


FIG. 10. Seagull contribution to the W^3 self-energy from the quartic gauge interaction.

$$i\Pi_{WW,W^\pm}^{\mu\nu}(q) = g^2 \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2(k-q)^2} \times [g^{\mu\nu}(2k^2 - 2q \cdot k + 5q^2) + 10k^\mu k^\nu - 5k^\mu q^\nu - 5q^\mu k^\nu - 2q^\mu q^\nu]. \quad (\text{C88})$$

Applying the Feynman-parameter formula to the denominator, the dimensional-regularization integral becomes

$$\Pi_{WW,W^\pm}^{\mu\nu}(q) = \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} \left(\frac{19}{6} g^{\mu\nu} q^2 - \frac{11}{3} q^\mu q^\nu \right). \quad (\text{C89})$$

d. Seagull diagram with an internal $W^\pm$ loop

Next, we derive the $4W$ interaction Lagrangian in order to compute the seagull diagram shown in Fig. 10. From the gauge Lagrangian

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu}, \quad W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g f^{abc} W_\mu^b W_\nu^c, \quad (\text{C90})$$

expanding the two factors of $g f^{abc} W_\mu^b W_\nu^c$ yields

$$\mathcal{L}_{4W} = -\frac{g^2}{4} f^{abc} f^{cde} W_\mu^a W_\nu^b W^{c\mu} W^{d\nu}. \quad (\text{C91})$$

The seagull contribution to the W^3 self-energy, taking into account the symmetry factor $1/2$ and the Feynman rule for the quartic gauge vertex [95], is given by

$$i\Pi_{\text{seagull}}^{\mu\nu}(q) = i^2 g^2 \int \frac{d^n k}{(2\pi)^n} \frac{2(n-1)g^{\mu\nu}}{m_W^2 - k^2}. \quad (\text{C92})$$

Here we introduced a dummy mass parameter m_W in the W -boson propagator in order to perform the dimensional-regularization integral. We then obtain

$$\Pi_{\text{seagull}}^{\mu\nu}(q) = \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} \cdot 6g^{\mu\nu} m_W^2 \xrightarrow{m_W \rightarrow 0} 0. \quad (\text{C93})$$

e. W_μ^3 self-energy with an internal Faddeev-Popov ghost

The FP-ghost propagator is $i\delta^{ab}/k^2$, and the vertex factor is $-gf^{3ab}k^\mu$. A minus sign is associated with the loop due to the Grassmann nature of the ghosts. Moreover, since a ghost and an antighost propagate in the loop, there is no symmetry factor. Therefore, the vacuum polarization is

$$i\Pi_{\text{ghost}}^{\mu\nu}(q) = -2g^2 \int \frac{d^n k}{(2\pi)^n} \frac{k^\mu k^\nu - k^\mu q^\nu}{k^2(k-q)^2}, \quad (\text{C94})$$

with $C_A = 2$. Using the Feynman-parameter formula for the denominator, the dimensional-regularization integral yields

$$\Pi_{\text{ghost}}^{\mu\nu}(q) = \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} \left(\frac{1}{6} g^{\mu\nu} q^2 + \frac{1}{3} q^\mu q^\nu \right). \quad (\text{C95})$$

f. Sum of the W_μ^3 self-energy

The Dirac-fermion contribution and the Higgs contribution must each be summed over the third component of weak isospin. Taking into account three generations, we find

$$\sum_f (T_f^3)^2 = 3 \times \left(\frac{3}{2} + \frac{1}{2} \right) = 6, \quad (\text{C96})$$

$$\sum_H (T_H^3)^2 = \left(+\frac{1}{2} \right)^2 + \left(-\frac{1}{2} \right)^2 = \frac{1}{2}. \quad (\text{C97})$$

Therefore,

$$\Pi_{WW,f}^{\mu\nu}(q) = \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} [-4(g^{\mu\nu} q^2 - q^\mu q^\nu)], \quad (\text{C98})$$

$$\Pi_{WW,H}^{\mu\nu}(q) = \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} \left[-\frac{1}{6} (g^{\mu\nu} q^2 - q^\mu q^\nu) \right]. \quad (\text{C99})$$

Since the $SU(2)_L$ gauge interaction acts only on left-handed Weyl-fermion doublets, the fermionic multiplicities entering the sum are counted in units of a single two-component Weyl fermion.

Combining the above results, we obtain

$$\begin{aligned} \Pi_{WW}^{\mu\nu}(q) &= \Pi_{WW,f}^{\mu\nu}(q) + \Pi_{WW,H}^{\mu\nu}(q) \\ &\quad + \Pi_{WW,W^\pm}^{\mu\nu}(q) + \Pi_{WW,\text{ghost}}^{\mu\nu}(q) \\ &= \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} \left[-\frac{5}{6} (g^{\mu\nu} q^2 - q^\mu q^\nu) \right]. \end{aligned} \quad (\text{C100})$$

This is nothing but the W^3 -boson self-energy.

Since analogous computations will appear repeatedly below, we make the $SU(2)_L$ gauge-theory Lagrangian and its counterterms explicit. The bare Lagrangian is

$$\mathcal{L}_0 = -\frac{1}{4} W_{0\mu\nu}^a W_0^{a\mu\nu} + \bar{\psi}_0 (i\gamma^\mu P_L D_{0\mu} - m_0) \psi_0 + \bar{c}_0 \partial_\mu D^{0\mu} c_0, \quad (\text{C101})$$

which, in terms of renormalized quantities, becomes

$$\mathcal{L} = -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} + \bar{\psi} (i\gamma^\mu P_L D_\mu - m) \psi + \bar{c} \partial_\mu D^\mu c. \quad (\text{C102})$$

The counterterm Lagrangian is

$$\begin{aligned} \mathcal{L}_{\text{count.}} = & -\frac{1}{4} (Z_3 - 1) (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)^2 \\ & + (Z_2 - 1) \bar{\psi} i\gamma^\mu P_L \partial_\mu \psi \\ & - (Z_2 Z_m - 1) m \bar{\psi} \psi + (\tilde{Z}_3 - 1) \bar{c}^a \partial_\mu \partial^\mu c^a \\ & - (Z_{W^3} - 1) g f^{abc} \partial_\mu W_\nu^a W^{b\mu} W^{c\nu} \\ & - \frac{1}{4} (Z_{W^4} - 1) g^2 (f^{eab} W_\mu^a W_\nu^b)^2 \\ & + (Z_1 Z_2 Z_3^{-1} - 1) g W_\mu^a \bar{\psi} \gamma^\mu T^a P_L \psi \\ & + (\tilde{Z}_1 - 1) g \partial_\mu \bar{c}^a f^{abc} W_\mu^b c^c. \end{aligned} \quad (\text{C103})$$

Here

$$\begin{aligned} W_{0\mu} &= Z_3^{1/2} W_\mu, & \psi_0 &= Z_2^{1/2} \psi, \\ (c_0, \bar{c}_0) &= \tilde{Z}_3^{1/2} (c, \bar{c}), & g_0 &= Z_1 Z_3^{-3/2} g. \end{aligned} \quad (\text{C104})$$

The relation $g_0 = Z_1 Z_3^{-3/2} g$ for a non-Abelian gauge theory was also mentioned in Appendix C 2 c. Here, however, we directly derive it from the gauge-fermion interaction term in Eq. (C103) as

$$\begin{aligned} Z_1 Z_2 Z_3^{-1} g \bar{\psi} W \psi &= g_0 (\bar{\psi}_0 W_0 \psi_0) \\ &= g_0 (Z_2^{1/2} \bar{\psi} Z_3^{1/2} W Z_2^{1/2} \psi) \\ &= g_0 (Z_2 Z_3^{1/2}) \bar{\psi} W \psi, \end{aligned} \quad (\text{C105})$$

where we suppressed T^a and P_L for notational simplicity.

In the same spirit, we can relate the renormalization of the ghost-gauge interaction to the renormalization constants introduced in Eqs. (C103) and (C104). The bare ghost interaction reads

$$\mathcal{L}_{0,\bar{c}cW} = g_0 \partial_\mu \bar{c}_0^a f^{abc} W_0^{b\mu} c_0^c. \quad (\text{C106})$$

Substituting the field and coupling renormalizations in Eq. (C104) into Eq. (C106) yields

$$\begin{aligned} \mathcal{L}_{0,\bar{c}cW} &= g_0 Z_3^{1/2} \tilde{Z}_3 \partial_\mu \bar{c}^a f^{abc} W^{b\mu} c^c \\ &= g (Z_1 Z_3^{-1} \tilde{Z}_3) \partial_\mu \bar{c}^a f^{abc} W^{b\mu} c^c \\ &= [1 + (Z_1^{(1)} - Z_3^{(1)} + \tilde{Z}_3^{(1)}) + \mathcal{O}(\hbar^2)] \\ &\quad \times g \partial_\mu \bar{c}^a f^{abc} W^{b\mu} c^c. \end{aligned} \quad (\text{C107})$$

On the other hand, on the renormalized-Lagrangian side we parametrize the ghost vertex by $\tilde{Z}_1$ as in Eq. (C103),

$$\begin{aligned} \mathcal{L}_{\bar{c}cW} &= \tilde{Z}_1 g \partial_\mu \bar{c}^a f^{abc} W^{b\mu} c^c \\ &= [1 + \tilde{Z}_1^{(1)} + \mathcal{O}(\hbar^2)] g \partial_\mu \bar{c}^a f^{abc} W^{b\mu} c^c. \end{aligned} \quad (\text{C108})$$

Identifying the coefficients of the common operator $\partial_\mu \bar{c}^a f^{abc} W^{b\mu} c^c$ in Eqs. (C107) and (C108), we obtain the Slavnov-Taylor relation at one-loop order,

$$\tilde{Z}_1^{(1)} = Z_1^{(1)} - Z_3^{(1)} + \tilde{Z}_3^{(1)}. \quad (\text{C109})$$

Equivalently, at the all-orders level one may write $\tilde{Z}_1 = Z_1 Z_3^{-1} \tilde{Z}_3$.

We now aim to extract anomalous dimensions and β functions by determining the vertex renormalization Z_1 , the fermion-field renormalization Z_2 , and the gauge-field renormalization Z_3 . To this end, the ultraviolet divergence in Eq. (C100) must be canceled by adding the counterterm contribution

$$\begin{aligned} & -\frac{1}{4} (Z_3 - 1) W_{\mu\nu}^a W^{a\mu\nu} \\ & \supset -\frac{1}{4} (Z_3 - 1) (\partial^\mu W^{3\nu} - \partial^\nu W^{3\mu})^2 \\ & = -\frac{1}{2} (Z_3 - 1) \int \frac{d^4 q}{(2\pi)^4} W^{3\nu}(-q) (g^{\mu\nu} q^2 - q^\mu q^\nu) W^{3\mu}(q) \\ & \supset -Z_3^{(1)} (g^{\mu\nu} q^2 - q^\mu q^\nu). \end{aligned} \quad (\text{C110})$$

Hence,

$$\Pi_{WW}^{\mu\nu}(q) - Z_3^{(1)} (g^{\mu\nu} q^2 - q^\mu q^\nu) = 0, \quad (\text{C111})$$

i.e.,

$$Z_3^{(1)} = -\frac{5}{6} \times \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1}. \quad (\text{C112})$$

4. Two-point function of a Dirac fermion

Next, in order to determine the fermion field renormalization constant Z_2 for a Dirac fermion f , we compute the two-point function.

The Dirac-fermion self-energy shown in Fig. 11 is obtained by noting that the fermion-gauge-boson vertex is $ig\gamma^\mu T^a$ ($a = 1, 2, 3$). We find

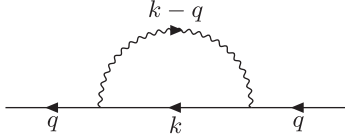


FIG. 11. One-loop self-energy diagram for a Dirac fermion.

$$\begin{aligned}
 i\Sigma_f &= \sum_a \int \frac{d^n k}{(2\pi)^n} \\
 &\times (ig\gamma^\mu T^a P_L) \frac{i(k+m_f)}{k^2 - m_f^2} (ig\gamma^\nu T^a P_L) \frac{-ig_{\mu\nu}}{(k-q)^2} \\
 &= -g^2 C_F \int \frac{d^n k}{(2\pi)^n} \frac{\gamma^\mu P_L (k+m_f) \gamma_\mu P_L}{(k^2 - m_f^2)(k-q)^2} \\
 &= \frac{3}{2} g^2 \int \frac{d^n k}{(2\pi)^n} \frac{k P_L}{(k^2 - m_f^2)(k-q)^2}, \quad (C113)
 \end{aligned}$$

with $C_F := C_2(F)$. In the last step we used the contraction identities $\gamma^\mu \gamma_\mu = 4$ and $\gamma^\mu \gamma^\nu \gamma_\mu = -2\gamma^\nu$ [96]. Here $T^a T^a = C_2(R)\mathbf{1}$ denotes the quadratic Casimir $C_2(R)$ in the representation R of the Dirac field ψ , and it differs from the adjoint value $C_2(G) = C_A = 2$ [94]. For example, the left-handed lepton doublet $L_\alpha = (\nu_{\alpha L} e_{\alpha L})^T$ transforms in the fundamental representation of $SU(2)_L$. Its quadratic Casimir is

$$C_F \mathbf{1} = \sum_a T^a T^a = \sum_a \frac{1}{4} \sigma^a \sigma^a = \frac{1}{4} \mathbf{1} \times 3 = \frac{3}{4} \mathbf{1}, \quad (C114)$$

where σ^a are the Pauli matrices. Hence, $C_F = 3/4$, as used above.

Applying the Feynman-parameter formula to the denominator, we have

$$\begin{aligned}
 &\frac{1}{(k^2 - m_f^2)(k-q)^2} \\
 &= \int_0^1 dx \frac{1}{[m_f^2(1-x) - q^2 x + 2k \cdot xq - k^2]^2}. \quad (C115)
 \end{aligned}$$

Substituting this into the dimensional-regularization integral, we obtain

$$\Sigma_f = \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} \left(\frac{3}{4} q P_L \right). \quad (C116)$$

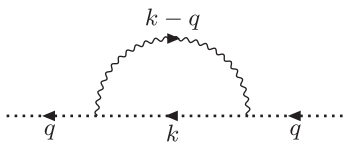


FIG. 12. One-loop self-energy diagram for an FP ghost.

The UV divergence of Σ_f must be canceled by adding the counterterm contribution $Z_2^{(1)} q P_L$, so that

$$\Sigma_f + Z_2^{(1)} q P_L = 0 \Rightarrow Z_2^{(1)} = -\frac{3}{4} \times \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1}. \quad (C117)$$

5. Two-point function of an FP ghost

Next, in order to determine the ghost-field renormalization constant $\tilde{Z}_3$, we compute the two-point function of the Faddeev-Popov ghost.

The gauge-boson propagator is $-i\delta^{ab}g^{\mu\nu}/k^2$, the FP-ghost propagator is $i\delta^{ab}/k^2$, and the gauge-ghost-antighost three-point vertex factor is $-gf^{abc}q^\mu$. Therefore, the ghost two-point function given by the self-energy diagram shown in Fig. 12 is

$$\begin{aligned}
 i\tilde{\Sigma}_3 &= \int \frac{d^n k}{(2\pi)^n} gf^{adc} q^\mu \frac{-i\delta^{ab}g_{\mu\nu}}{k^2} gf^{bdc} k^\nu \frac{i}{k^2} \\
 &= g^2 f^{adc} f^{adc} \int \frac{d^n k}{(2\pi)^n} \frac{k \cdot q}{k^2 (k-q)^2}. \quad (C118)
 \end{aligned}$$

Applying the Feynman-parameter formula to the denominator and using $f^{adc} f^{adc} = 2$, the dimensional-regularization integral yields

$$\begin{aligned}
 i\tilde{\Sigma}_3 &= 2g^2 \int_0^1 dx \int \frac{d^n k}{(2\pi)^n} \frac{k \cdot q}{(-q^2 x + 2k \cdot xq - k^2)^2} \\
 &= \frac{ig^2}{16\pi^2} \bar{\epsilon}^{-1} q^2, \quad (C119)
 \end{aligned}$$

$$\tilde{\Sigma}_3 = \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} q^2. \quad (C120)$$

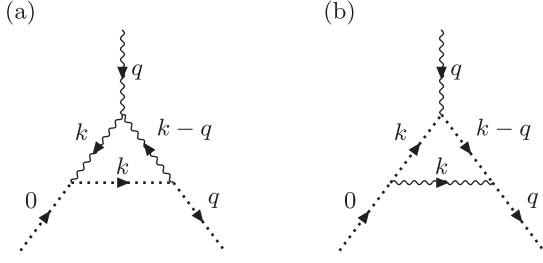
The UV divergence of $\tilde{\Sigma}_3$ must be canceled by adding the counterterm contribution $-\tilde{Z}_3^{(1)} q^2$, so that

$$\tilde{\Sigma}_3 - \tilde{Z}_3^{(1)} q^2 = 0 \Rightarrow \tilde{Z}_3^{(1)} = \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1}. \quad (C121)$$

6. Three-point function of an FP ghost

In order to determine the renormalization constant $\tilde{Z}_1$ for the gauge-ghost vertex, we compute the gauge-ghost-antighost three-point function $\Gamma_{\bar{c}cW}$.

As can be seen from the counterterm structure $(\tilde{Z}_1 - 1) \times g\partial_\mu \bar{c}^a f^{abc} W_\mu^b c^c$, the divergent part of this vertex function must be proportional to $\bar{\epsilon}^{-1} f^{abc} q_\mu$, where q_μ is the antighost momentum. We may therefore evaluate the diagrams shown in Fig. 13 with the external ghost momentum set to zero from the outset. Then, for the diagram (a), we obtain

FIG. 13. One-loop diagrams contributing to $\Gamma_{\bar{c}cW}$.

$$\begin{aligned}
 i\Gamma_{\bar{c}cW}^{(a)} &= \int \frac{d^n k}{(2\pi)^n} g f^{cef} q^\rho \frac{i}{k^2} g f^{fdb} k^\nu \frac{-ig_{\rho\sigma}}{(k-q)^2} \\
 &\quad \times V_{\mu\sigma\tau}^{aed}(q, k-q, -k) \frac{-ig_{\tau\nu}}{k^2} \\
 &= -ig^3 f^{cef} f^{fdb} f^{aed} \int \frac{d^n k}{(2\pi)^n} \frac{q_\sigma}{k^2} \frac{1}{(k-q)^2} \\
 &\quad \times [g^{\mu\sigma}(2q-k)^\tau + g^{\sigma\tau}(2k-q)^\mu + g^{\tau\mu}(-k-q)^\sigma] \frac{k_\tau}{k^2}.
 \end{aligned} \quad (C122)$$

The group factor can be evaluated using the adjoint-representation matrices $(T^a)^{bc} := if^{bac}$ and the quadratic Casimir defined by $f^{acd}f^{bcd} = C_A \delta^{ab}$:

$$\begin{aligned}
 f^{cef} f^{fdb} f^{aed} &= -f^{aed}(T^e)^{cf}(T^d)^{fb} = -f^{aed}(T^e T^d)^{cb} \\
 &= \frac{1}{2} f^{aed} [T^d, T^e]^{cb} = \frac{1}{2} f^{aed} if^{def}(T^f)^{cb} \\
 &= -\frac{i}{2} C_A \delta^{af} (T^f)^{cb} = -\frac{1}{2} C_A f^{abc}.
 \end{aligned} \quad (C123)$$

Therefore,

$$\begin{aligned}
 i\Gamma_{\bar{c}cW}^{(a)} &= +\frac{i}{2} g^3 C_A f^{abc} \int \frac{d^n k}{(2\pi)^n} \frac{q_\sigma k_\tau}{k^4 (k-q)^2} \\
 &\quad \times [g^{\mu\sigma}(2q-k)^\tau + g^{\sigma\tau}(2k-q)^\mu + g^{\tau\mu}(-k-q)^\sigma] \\
 &= ig^3 f^{abc} \int \frac{d^n k}{(2\pi)^n} \\
 &\quad \times \frac{[q^\mu(2q \cdot k - k^2) + (k \cdot q)(2k-q)^\mu - k^\mu(q \cdot k + q^2)]}{k^4 (k-q)^2}.
 \end{aligned} \quad (C124)$$

Applying the Feynman-parameter formula to the denominator,

$$\begin{aligned}
 \frac{1}{(k-q)^2 k^4} &= \frac{\Gamma(1+2)}{\Gamma(1)\Gamma(2)} \int_0^1 dx \frac{1-x}{[(k-q)^2 x + k^2(1-x)]^3} \\
 &= \Gamma(3) \int_0^1 dx \frac{1-x}{(-q^2 x + 2k \cdot qx - k^2)^3}
 \end{aligned} \quad (C125)$$

and substituting into the dimensional-regularization integral gives

$$\begin{aligned}
 i\Gamma_{\bar{c}cW}^{(a)} &= i\Gamma(3) g^3 f^{abc} \int_0^1 dx (1-x) \int \frac{d^n k}{(2\pi)^n} \\
 &\quad \times \frac{q^\mu(2q \cdot k - k^2) + (k \cdot q)(2k-q)^\mu - k^\mu(q \cdot k + q^2)}{(-q^2 x + 2k \cdot qx - k^2)^3} \\
 &= \frac{ig^3}{16\pi^2} \bar{\epsilon}^{-1} f^{abc} \left(\frac{3}{4} q^\mu \right).
 \end{aligned} \quad (C126)$$

Thus, we obtain

$$\Gamma_{\bar{c}cW}^{(a)} = \frac{g^3}{16\pi^2} \bar{\epsilon}^{-1} f^{abc} \left(\frac{3}{4} q^\mu \right). \quad (C127)$$

For diagram (b), the vertex function is

$$i\Gamma_{\bar{c}cW}^{(b)} = -ig^3 f^{cfe} f^{bfd} f^{dae} \int \frac{d^n k}{(2\pi)^n} \frac{g_{\rho\sigma} q^\rho k^\sigma (k-q)^\mu}{k^4 (k-q)^2}. \quad (C128)$$

The group factor can be evaluated as in diagram (a):

$$f^{cfe} f^{bfd} f^{dae} = (-f^{cef}) f^{fdb} f^{aed} = +\frac{1}{2} C_A f^{abc}. \quad (C129)$$

Hence,

$$i\Gamma_{\bar{c}cW}^{(b)} = -ig^3 f^{abc} \int \frac{d^n k}{(2\pi)^n} \frac{(k \cdot q)(k-q)^\mu}{k^4 (k-q)^2}. \quad (C130)$$

Applying the Feynman-parameter formula (C125) and substituting into the dimensional-regularization integral, we obtain

$$\Gamma_{\bar{c}cW}^{(b)} = \frac{g^3}{16\pi^2} \bar{\epsilon}^{-1} f^{abc} \left(\frac{1}{4} q^\mu \right). \quad (C131)$$

Summing diagrams (a) and (b), we find

$$\begin{aligned}
 \Gamma_{\bar{c}cW} &= \Gamma_{\bar{c}cW}^{(a)} + \Gamma_{\bar{c}cW}^{(b)} \\
 &= \frac{g^3}{16\pi^2} \bar{\epsilon}^{-1} f^{abc} \left(\frac{3}{4} q^\mu \right) + \frac{g^3}{16\pi^2} \bar{\epsilon}^{-1} f^{abc} \left(\frac{1}{4} q^\mu \right) \\
 &= \frac{g^3}{16\pi^2} \bar{\epsilon}^{-1} f^{abc} q^\mu.
 \end{aligned} \quad (C132)$$

The UV divergence of $\Gamma_{\bar{c}cW}$ must be canceled by adding the counterterm contribution $\tilde{Z}_1^{(1)} g f^{abc} q^\mu$:

$$\Gamma_{\bar{c}cW} + \tilde{Z}_1^{(1)} g f^{abc} q^\mu = 0 \Rightarrow \tilde{Z}_1^{(1)} = -\frac{g^2}{16\pi^2} \bar{\epsilon}^{-1}. \quad (\text{C133})$$

7. β function of the $\text{SU}(2)_L$ gauge theory

The definition of the β function is

$$\beta(\lambda) := \left(Q \frac{\partial}{\partial Q} \right)_0 \lambda. \quad (\text{C134})$$

The subscript “0” on the partial derivative indicates that bare quantities are held fixed, and λ denotes a coupling constant. Taking the logarithm of the coupling renormalization $g_0 = Z_1 Z_3^{-3/2} g$, we obtain

$$\ln g_0 = \ln Z_1 - \frac{3}{2} \ln Z_3 + \ln g. \quad (\text{C135})$$

Acting with $(Q \frac{\partial}{\partial Q})_0$ on both sides yields

$$\beta(g) = -g \left(Q \frac{\partial}{\partial Q} \right)_{g_0, g'_0, \epsilon} \ln(Z_1 Z_3^{-3/2}). \quad (\text{C136})$$

Using the formula

$$\left(Q \frac{\partial}{\partial Q} \right)_{g_0, g'_0, \epsilon} \ln Z_i = -2\hbar \left(\frac{\partial Z_i^{(1)}}{\partial \bar{\epsilon}^{-1}} \right)_{g, g', Q} + \mathcal{O}(\hbar^2), \quad (\text{C137})$$

we find

$$\begin{aligned} \beta(g) &= -g \left(Q \frac{\partial}{\partial Q} \right)_{g_0, g'_0, \epsilon} \left(\ln Z_1 - \frac{3}{2} \ln Z_3 \right) \\ &= +2g \left(\frac{\partial Z_1^{(1)}}{\partial \bar{\epsilon}^{-1}} \right)_{g, g', Q} - 3g \left(\frac{\partial Z_3^{(1)}}{\partial \bar{\epsilon}^{-1}} \right)_{g, g', Q} \end{aligned} \quad (\text{C138})$$

Thus, it suffices to determine $Z_1^{(1)}$ and $Z_3^{(1)}$. While $Z_3^{(1)}$ is given in Eq. (C112), $Z_1^{(1)}$ is not. Rather than computing the W^3 vertex function, we compute the $W\bar{c}c$ vertex and use the Slavnov-Taylor identity [86–88] implied by BRST invariance [89–92],

$$\tilde{Z}_1 = Z_1 Z_3^{-1} \tilde{Z}_3, \quad (\text{C139})$$

or its one-loop approximation derived in Eq. (C109),

$$\tilde{Z}_1^{(1)} = Z_1^{(1)} - Z_3^{(1)} + \tilde{Z}_3^{(1)}, \quad (\text{C140})$$

we obtain, from Eqs. (C112), (C121), and (C133),

$$Z_1^{(1)} = \tilde{Z}_1^{(1)} + Z_3^{(1)} - \tilde{Z}_3^{(1)} = -\frac{17}{6} \times \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1}. \quad (\text{C141})$$

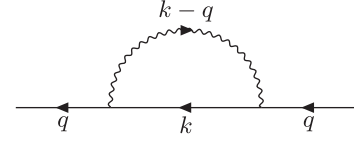


FIG. 14. L_α self-energy from a hypercharge-gauge-boson loop.

Substituting Eqs. (C112) and (C141) into Eq. (C138) then gives

$$\beta(g) = -\frac{19}{6} \times \frac{g^3}{16\pi^2}. \quad (\text{C142})$$

Therefore, the $\text{SU}(2)_L$ gauge theory is asymptotically free.

8. $\bar{L}_\alpha L_\alpha$ self-energy

a. $\bar{L}_\beta L_\alpha$ self-energy from a $B+L$ loop

The $B-L-L$ vertex factor is $ig'\gamma^\mu Y_L$, and the hypercharge of the left-handed lepton doublet $(\nu, e)_L$ is $Y_L = -1/2$. The present two-point function with external legs $\bar{L}_\beta$ and L_α has fixed flavor indices α, β and therefore involves no sum over flavors. The one-loop integral corresponding to the diagram shown in Fig. 14 is then

$$i\Sigma_{LL,B}(q) = -\frac{1}{4} g'^2 \int \frac{d^n k}{(2\pi)^n} \frac{-2k + 4m_\ell}{(k-q)^2(k^2 - m_\ell^2)}. \quad (\text{C143})$$

Applying the Feynman-parameter formula to the denominator and substituting into the dimensional-regularization integral gives

$$\Sigma_{LL,B}(q) = \frac{g'^2}{16\pi^2} \bar{\epsilon}^{-1} \left(\frac{1}{4} \not{q} - m_\ell \right). \quad (\text{C144})$$

The counterterm Lagrangian for the $\text{U}(1)_Y$ gauge theory is given in Eq. (C56). Accordingly, the UV divergence of $\Sigma_{LL,B}(q)$ must be canceled by the lepton wave-function and mass counterterms. We denote these one-loop counterterms by $Z_{L,B}^{(1)}$ and $Z_{m_\ell,B}^{(1)}$, respectively, and impose

$$\Sigma_{LL,B}(q) + Z_{L,B}^{(1)} \not{q} - (Z_{L,B}^{(1)} + Z_{m_\ell,B}^{(1)}) m_\ell = 0. \quad (\text{C145})$$

This gives

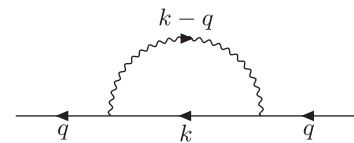
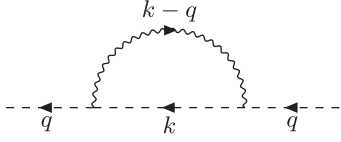


FIG. 15. L_α self-energy from an $\text{SU}(2)_L$ gauge-boson loop.

FIG. 16. $\tilde{H}$ self-energy from a hypercharge-gauge-boson loop.

$$Z_{L,B}^{(1)} = -\frac{1}{4} \times \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1}, \quad Z_{m_{\ell},B}^{(1)} = -\frac{3}{4} \times \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1}. \quad (\text{C146})$$

9. $\tilde{L}_\beta L_\alpha$ self-energy from a $W^I + L$ loop

We consider the diagram shown in Fig. 15. Since the vertex factor is $ig\gamma^\mu T^a P_L$, one finds

$$i\Sigma_{LL,W}(q) = \frac{3}{2}g^2 \int \frac{d^n k}{(2\pi)^n} \frac{k P_L}{(k^2 - m_\ell^2)(k - q)^2}. \quad (\text{C147})$$

Since this is identical to Eq. (C113), the result is

$$\Sigma_{LL,W}(q) = \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} \left(\frac{3}{4} q P_L \right). \quad (\text{C148})$$

The $SU(2)_L$ counterterm Lagrangian is given in Eq. (C103). Thus the UV divergence of $\Sigma_{LL,W}(q)$ must be canceled by the lepton field counterterm. We denote the corresponding one-loop counterterm by $Z_{L,W}^{(1)}$ and require

$$\Sigma_{LL,W}(q) + Z_{L,W}^{(1)} q P_L = 0 \Rightarrow Z_{L,W}^{(1)} = -\frac{3}{4} \times \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1}. \quad (\text{C149})$$

10. $\tilde{H}^\dagger \tilde{H}$ self-energy

a. $\tilde{H}^\dagger \tilde{H}$ self-energy from a $B + \tilde{H}$ loop

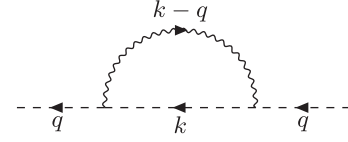
The $B - \tilde{H} - \tilde{H}$ vertex factor is $ig'(k^\mu + q^\mu)Y_{\tilde{H}}$, and the hypercharge of the Higgs doublet H is $Y_H = +1/2$. Therefore, for $\tilde{H} = i\tau^2 H^*$ we have $Y_{\tilde{H}} = -1/2$. Here $i\tau^2$ is an $SU(2)_L$ matrix and hence commutes with $U(1)_Y$. The one-loop integral corresponding to the diagram shown in Fig. 16 is then

$$i\Sigma_{HH,B}(q) = -\frac{1}{4}g'^2 \int \frac{d^n k}{(2\pi)^n} \frac{k^2 + 2k \cdot q + q^2}{(k^2 - m_H^2)(k - q)^2}. \quad (\text{C150})$$

Using the same Feynman-parameter formula as in Appendix C 8 a and substituting into the dimensional-regularization integral gives

$$\Sigma_{HH,B}(q) = -\frac{g'^2}{16\pi^2} \bar{\epsilon}^{-1} \left(\frac{1}{2} q^2 + \frac{1}{4} m_H^2 \right). \quad (\text{C151})$$

The counterterm contribution from Eq. (C56) must cancel the UV pole in Eq. (C151), hence

FIG. 17. $\tilde{H}$ self-energy from an $SU(2)_L$ gauge-boson loop.

$$\Sigma_{HH,B}(q) - Z_{H,B}^{(1)} q^2 - (Z_{H,B}^{(1)} + Z_{m_{H,B}}^{(1)}) m_H^2 = 0. \quad (\text{C152})$$

This gives

$$Z_{H,B}^{(1)} = -\frac{1}{2} \times \frac{g'^2}{16\pi^2} \bar{\epsilon}^{-1}, \quad Z_{m_{H,B}}^{(1)} = \frac{1}{4} \times \frac{g'^2}{16\pi^2} \bar{\epsilon}^{-1}. \quad (\text{C153})$$

11. $\tilde{H}^\dagger \tilde{H}$ self-energy from a $W^I + \tilde{H}$ loop

We consider the diagram shown in Fig. 17. Since the scalar vertex factor is $ig(k^\mu + q^\mu)T^a$ (no P_L appears for a scalar), one finds

$$i\Sigma_{HH,W}(q) = -\frac{3}{4}g^2 \int \frac{d^n k}{(2\pi)^n} \frac{k^2 + 2k \cdot q + q^2}{(k - q)^2 (k^2 - m_H^2)}. \quad (\text{C154})$$

Applying the same Feynman-parameter formula as in Appendix C 8 a, we obtain

$$\Sigma_{HH,W}(q) = -\frac{g^2}{16\pi^2} \bar{\epsilon}^{-1} \left(\frac{3}{2} q^2 + \frac{3}{4} m_H^2 \right). \quad (\text{C155})$$

The counterterm contribution from Eq. (C56) must cancel the UV pole in Eq. (C151), hence

$$\Sigma_{HH,W}(q) - Z_{H,W}^{(1)} q^2 - (Z_{H,W}^{(1)} + Z_{m_{H,W}}^{(1)}) m_H^2 = 0. \quad (\text{C156})$$

This gives

$$Z_{H,W}^{(1)} = -\frac{3}{2} \times \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1}, \quad Z_{m_{H,W}}^{(1)} = \frac{3}{4} \times \frac{g^2}{16\pi^2} \bar{\epsilon}^{-1}. \quad (\text{C157})$$

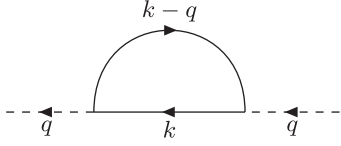
12. $\tilde{H}^\dagger \tilde{H}$ self-energy from a $t_R + Q_3$ loop

In the electroweak-symmetric phase, the Yukawa interaction of the right-handed top quark t_R , the third-generation quark doublet $Q_3 = (t_L b_L)^T$, and the Higgs doublet $H = (\phi^+ \phi^0)^T$ is

$$-\mathcal{L}_t = y_t \bar{Q}_3 \tilde{H} P_R t + y_t \bar{t} P_L \tilde{H}^\dagger Q_3, \quad \tilde{H} = i\sigma^2 H^*, \quad (\text{C158})$$

i.e.,

$$-\mathcal{L}_t = y_t (\bar{t} P_R \phi^{0*} - \bar{b} P_R \phi^-) P_R t + y_t \bar{t} P_L (\phi^0 P_L t - \phi^+ P_L b). \quad (\text{C159})$$

FIG. 18. One-loop $\tilde{H}$ self-energy from a (t_R, Q_3) fermion loop.

Here y_t is the SM top Yukawa coupling. Its complex phase can be removed by a $U(1)$ phase rotation of t_R and Q_3 , so we take y_t to be real. It should not be confused with the (possibly vanishing) neutrino Yukawa coupling y_ν in $\bar{L}_\alpha \tilde{H} N_i$, which is set to zero in the exact Z_2 limit.

From $\mathcal{L}_t$, the vertex factors are $-iy_t P_R$ (left vertex) and $-iy_t P_L$ (right vertex). Consider the diagram shown in Fig. 18. Since the t_R and Q_3 propagators form a fermion loop, the overall sign is multiplied by (-1) and a trace appears. Moreover, quark fields carry a color index $r = 1, 2, 3$. Writing the top Yukawa interaction with explicit color indices,

$$-\mathcal{L}_t = y_t \bar{Q}_{3r} \tilde{H} t_{Rr} + y_t^* \bar{t}_{Rr} \tilde{H}^\dagger Q_{3r}, \quad (\text{C160})$$

we note that each Yukawa vertex is an $SU(3)_c$ singlet and hence carries a factor δ_{rs} , and quark propagators preserve color and also carry δ_{rs} . The one-loop integral is then

$$i\Sigma_{HH,t_R}(q) = -2y_t^2 N_c \int \frac{d^n k}{(2\pi)^n} \frac{k^2 - k \cdot q}{(k^2 - m_t^2)[(k - q)^2 - m_3^2]}. \quad (\text{C161})$$

Here $N_c = \delta_{rr} = 3$ is the number of colors. Applying the Feynman-parameter formula and substituting into the dimensional-regularization integral gives

$$\Sigma_{HH,t_R}(q) = \frac{3y_t^2}{16\pi^2} \bar{\epsilon}^{-1} (q^2 + 2m_t^2 + 2m_3^2). \quad (\text{C162})$$

In the electroweak-symmetric phase, $m_t = 0$ and $m_3 = 0$, and therefore

$$\Sigma_{HH,t_R}(q) = \frac{3y_t^2}{16\pi^2} \bar{\epsilon}^{-1} q^2. \quad (\text{C163})$$

The UV divergence of $\Sigma_{HH,t_R}(q)$ must be canceled by the Higgs wave-function counterterm (see Appendix C 2 c). We denote the corresponding one-loop counterterm by $Z_{H,t}^{(1)}$ and impose

$$\Sigma_{HH,t_R}(q) - Z_{H,t}^{(1)} q^2 = 0. \quad (\text{C164})$$

This gives

$$Z_{H,t}^{(1)} = \frac{3y_t^2}{16\pi^2} \bar{\epsilon}^{-1}. \quad (\text{C165})$$

Applying the following relation

$$\gamma_i = \frac{1}{2} \left(Q \frac{\partial \ln Z_i}{\partial Q} \right)_{g_0, g'_0, \epsilon}, \quad (\text{C166})$$

to the wave-function renormalization constants Z_i obtained above, together with the identity [94]

$$\left(Q \frac{\partial}{\partial Q} \right)_{g_0, g'_0, \epsilon} \ln Z_i = -2\hbar \left(\frac{\partial Z_i^{(1)}}{\partial \bar{\epsilon}^{-1}} \right)_{g, g', Q} + \mathcal{O}(\hbar^2), \quad (\text{C167})$$

one can extract the anomalous dimensions γ_i in the same manner as in the main text. At one loop, we find

$$\gamma_{BW}^{(1)} = 0, \quad (\text{C168})$$

$$\gamma_B^{(1)} = \frac{41}{6} \times \frac{g^2}{16\pi^2}, \quad (\text{C169})$$

$$\gamma_W^{(1)} = \frac{5}{6} \times \frac{g^2}{16\pi^2}, \quad (\text{C170})$$

$$\gamma_L^{(1)} = \frac{1}{16\pi^2} \left(\frac{1}{4} g^2 + \frac{3}{4} g^2 \right), \quad (\text{C171})$$

$$\gamma_H^{(1)} = \frac{1}{16\pi^2} \left(\frac{1}{2} g^2 + \frac{3}{2} g^2 - 3y_t^2 \right). \quad (\text{C172})$$

It then follows, again as in the main text, that the Wilson coefficients obey

$$Q \frac{d}{dQ} C_{NB}(Q) = -\frac{1}{16\pi^2} \left(\frac{91}{12} g^2 + \frac{9}{4} g^2 - 3y_t^2 \right) C_{NB}(Q), \quad (\text{C173})$$

$$Q \frac{d}{dQ} C_{NW}(Q) = -\frac{1}{16\pi^2} \left(\frac{3}{4} g^2 + \frac{37}{12} g^2 - 3y_t^2 \right) C_{NW}(Q). \quad (\text{C174})$$

Formally, their solutions can be written as

$$C_{NX}(Q_{\text{ref}}) = C_{NX}(Q_{\text{match}}) \times \exp \left[\frac{1}{16\pi^2} \int_{\ln Q_{\text{ref}}}^{\ln Q_{\text{match}}} F_X(Q) d \ln Q \right], \quad (\text{C175})$$

where

$$F_B(Q) := \frac{91}{12}g^2(Q) + \frac{9}{4}g^2(Q) - 3y_t^2(Q),$$

$$F_W(Q) := \frac{3}{4}g^2(Q) + \frac{37}{12}g^2(Q) - 3y_t^2(Q). \quad (\text{C176})$$

13. Couplings to be evolved together with the Wilson coefficients C_{NB} and C_{NW}

In this work, we integrate out the heavy new particles E , Σ , D , and Φ^+ at the matching scale Q_{match} and work below that scale in the ν SMEFT, whose dynamical degrees of freedom are the SM fields together with the right-handed neutrinos N_i . Among the dimension-six operators in the ν SMEFT basis, we retain the dipole operators $\mathcal{O}_{NB,ai}$ and $\mathcal{O}_{NW,ai}$ as the operators relevant for the present electromagnetic-leptogenesis analysis. Consequently, the RG evolution over $Q \in [Q_{\text{ref}}, Q_{\text{match}}]$ closes within the couplings of this low-energy EFT. The heavy-sector parameters, such as y_Σ , y_H , μ_s , and μ_{soft} , do not appear explicitly in the EFT evolution; they enter only through the matching conditions for $C_{NB,ai}(Q_{\text{match}})$ and $C_{NW,ai}(Q_{\text{match}})$.

Since the one-loop RGEs (C173) and (C174) involve the coefficient functions in Eq. (C176), evolving C_{NB} and C_{NW} in Q requires at least

$$g'(Q), \quad g(Q), \quad y_t(Q) \quad (\text{C177})$$

as functions of Q . At the accuracy of the present study, the one-loop beta functions for g_s , g , and g' are closed within each coupling. Hence, one can use their analytic one-loop solutions as external inputs, rather than numerically integrating them together with C_{NB} and C_{NW} .

On the other hand, the one-loop RGE for y_t is [97]

$$\beta(y_t) = \mu \frac{dy_t}{d\mu} = \frac{y_t}{16\pi^2} \left[\frac{9}{2}y_t^2 - \left(\frac{17}{20}g^2 + \frac{9}{4}g^2 + 8g_s^2 \right) \right], \quad (\text{C178})$$

where the $8g_s^2$ term is relatively large. Therefore, a consistent evolution of $y_t(\mu)$ requires the running of $g_s(\mu)$ as well. In summary, the minimal set that should be evolved consistently at one loop is

$$\{g_s(Q), g(Q), g'(Q), y_t(Q), C_{NB}(Q), C_{NW}(Q)\}. \quad (\text{C179})$$

Even if g_s does not enter the anomalous-dimension coefficients $\gamma(g', g, y_t)$ directly, it is indispensable for determining $y_t(\mu)$. In fact, raising the scale from $Q_{\text{match}} = 10$ TeV to $Q_{\text{ref}} = 150$ GeV significantly reduces the QCD coupling. If one were to keep g_s fixed at its value at Q_{match} when solving Eq. (C178), the $8g_s^2$ term would be underestimated, leading to an overestimated $y_t(\mu)$. For example, taking as representative values

$$g_s(Q_{\text{ref}} = 150 \text{ GeV}) \simeq 1.17,$$

$$g_s(Q_{\text{match}} = 10 \text{ TeV}) \simeq 0.95, \quad (\text{C180})$$

we obtain

$$\frac{g_s^2(150 \text{ GeV}) - g_s^2(10 \text{ TeV})}{g_s^2(150 \text{ GeV})} \simeq \frac{1.17^2 - 0.95^2}{1.17^2} \simeq 0.34, \quad (\text{C181})$$

i.e. g_s^2 changes by about 34% when running from Q_{match} to Q_{ref} . Hence, approximating $g_s(\mu)$ by a constant can misestimate the dominant $8g_s^2$ contribution in $\beta(y_t)$ by a comparable fraction, and the running of $y_t(\mu)$ over $[Q_{\text{ref}}, Q_{\text{match}}]$ can be significantly affected. An estimate follows from inserting Δg_s^2 into the $8g_s^2$ term in $\beta(y_t)$,

$$\delta \ln y_t \sim \frac{1}{16\pi^2} 8\Delta g_s^2 \ln \frac{Q_{\text{match}}}{Q_{\text{ref}}}. \quad (\text{C182})$$

Moreover, the corresponding change in the coefficient functions is $\Delta F_X(\mu) = -3\Delta(y_t^2(\mu))$. Therefore, once y_t is evolved, g_s must also be evolved.

14. One-loop QCD β function

Finally, we derive the one-loop QCD β function $\beta(g_s)$. This can be obtained with minimal additional effort by reusing the $\text{SU}(2)_L$ computations in Appendix C 3.

In Appendix C 3 a, one replaces $(T_f^3)^2$ by $\text{tr}(T_R^a T_R^b) = T(R)\delta^{ab}$ (with R the fundamental representation of quarks), doubles the coefficient because no chiral projector is present, and multiplies by the number of active flavors $n_f = 6$ for $\mu > m_t \simeq 172$ GeV, to obtain

$$\Pi_{GG,f}^{\mu\nu}(q) = \frac{g_s^2}{16\pi^2} \bar{e}^{-1} T_F n_f \left[-\frac{4}{3} (g^{\mu\nu} q^2 - q^\mu q^\nu) \right]. \quad (\text{C183})$$

Here we used $T(R) = T_F = 1/2$. In Appendix C 3 c, one simply uses the quadratic Casimir $C_A = 3$. The result of Appendix C 3 d can be used as is:

$$\Pi_{GG,G}^{\mu\nu}(q) = \frac{g_s^2}{16\pi^2} \bar{e}^{-1} C_A \left(\frac{19}{12} g^{\mu\nu} q^2 - \frac{11}{6} q^\mu q^\nu \right). \quad (\text{C184})$$

Likewise, using $C_A = 3$ in Appendix C 3 e, we obtain

$$\Pi_{GG,\text{ghost}}^{\mu\nu}(q) = \frac{g_s^2}{16\pi^2} \bar{e}^{-1} C_A \left(\frac{1}{12} g^{\mu\nu} q^2 + \frac{1}{6} q^\mu q^\nu \right). \quad (\text{C185})$$

Summing Eqs. (C183)–(C185) gives

$$\Pi_{GG}^{\mu\nu}(q) = \frac{g_s^2}{16\pi^2} \bar{e}^{-1} \left(\frac{5}{3} C_A - \frac{4}{3} T_F n_f \right) (g^{\mu\nu} q^2 - q^\mu q^\nu). \quad (\text{C186})$$

The counterterm has the same structure as in Eq. (C110), $-Z_s^{(1)}(g^{\mu\nu}q^2 - q^\mu q^\nu)$, hence

$$Z_s^{(1)} = \frac{g_s^2}{16\pi^2} \bar{e}^{-1} \left(\frac{5}{3} C_A - \frac{4}{3} T_F n_f \right). \quad (\text{C187})$$

We next use the analogs of Appendixes C 5 and C 6,

$$\tilde{Z}_s^{(1)} = \frac{1}{2} \times \frac{g_s^2}{16\pi^2} \bar{e}^{-1} C_A, \quad \tilde{Z}_{1,s}^{(1)} = -\frac{1}{2} \times \frac{g_s^2}{16\pi^2} \bar{e}^{-1} C_A, \quad (\text{C188})$$

and the same Slavnov-Taylor relation to obtain

$$\begin{aligned} Z_{1,s}^{(1)} &= \tilde{Z}_{1,s}^{(1)} + Z_s^{(1)} - \tilde{Z}_s^{(1)} \\ &= \frac{g_s^2}{16\pi^2} \bar{e}^{-1} \left(\frac{2}{3} C_A - \frac{4}{3} T_F n_f \right). \end{aligned} \quad (\text{C189})$$

Substituting Eqs. (C187) and (C189) into Eq. (C138), we obtain the one-loop QCD β function,

$$\begin{aligned} \beta(g_s) &= -\frac{g_s^3}{16\pi^2} \left(\frac{11}{3} C_A - \frac{4}{3} T_F n_f \right) \\ &= -\frac{g_s^3}{16\pi^2} \left(\frac{11}{3} \cdot 3 - \frac{4}{3} \cdot \frac{1}{2} \cdot 6 \right) = -\frac{7g_s^3}{16\pi^2}. \end{aligned} \quad (\text{C190})$$

Although Eq. (C190) is computed for $n_f = 6$ (valid for $\mu > m_t \simeq 172$ GeV), we adopt it approximately also for $\mu \geq Q_{\text{ref}} = 150$ GeV. Strictly speaking, $n_f = 5$ for $\mu < m_t$, but since $\ln(m_t/150 \text{ GeV}) \ll 1$, the impact on g_s is a relative effect of order 10^{-3} and can be neglected at one-loop accuracy.

APPENDIX D: CP ASYMMETRIES I: PHOTON MODE WITHOUT $\mathcal{O}_{NW}$ AWAY FROM QUASIDEGENERACY

In this appendix we collect the loop-level details for the photon-mode CP asymmetry in the absence of $\mathcal{O}_{NW}$ and away from the quasidegenerate regime. This is the calculation underlying the nonresonant photon-mode expression quoted in Sec. IV. We include the fermion-flow conventions, the vertex contributions, and the self-energy contributions that generate the loop functions $f_{V_a}(x)$, $f_{S_a}(x)$, and $f_{S_b}(x)$.

The purpose of this appendix is only to make the normalization and signs reproducible. The main text uses the final expressions for the CP asymmetry and the loop functions.

1. Vertex contribution (a) to the photon mode

With these preparations, we now evaluate the vertex contribution in the photon mode. The relevant diagrams are shown in Fig. 19. We begin with diagram (a). The propagators for ν , N_m , and γ are

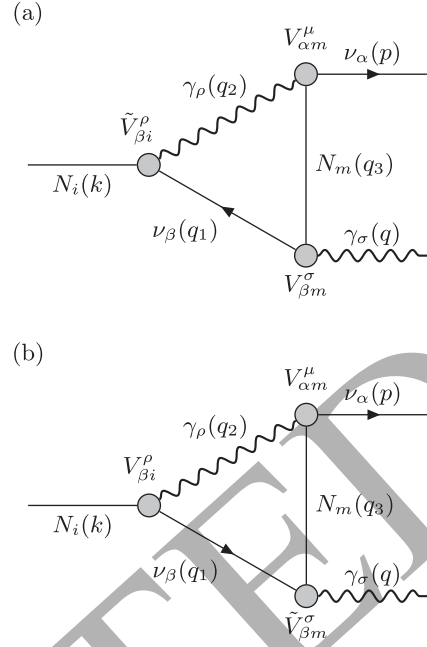


FIG. 19. Two vertex-type one-loop diagrams contributing to the photon-mode CP asymmetry. The diagrams (a) and (b) differ by the fermion-number orientation of the internal light-neutrino line $\nu_\beta(q_1)$, which determines whether the dipole vertex on that line is V or its Hermitian-conjugate form $\tilde{V}$.

$$\begin{aligned} S_\nu(q_1) &\simeq \frac{i q_1}{q_1^2 + i\epsilon}, & S_{N_m}(q_3) &= \frac{i(q_3 + M_m)}{q_3^2 - M_m^2 + i\epsilon}, \\ D_{\mu\nu}(q_2) &= \frac{-i g_{\mu\nu}}{q_2^2 + i\epsilon}, \end{aligned} \quad (\text{D1})$$

where the light neutrino is treated as massless and the chiral structure is enforced by the projectors at the dipole vertices. The $N_i \leftrightarrow \nu_\alpha + \gamma_\rho$ vertex is, for an outgoing photon momentum q ,

$$V_{\alpha i}^\mu(q) = 2\mu_{\alpha i} \sigma^{\mu\nu} q_\nu P_R. \quad (\text{D2})$$

For the three vertices appearing in Fig. 19(a), we thus have

$$\tilde{V}_{\beta i}^\rho(q_2) = 2\mu_{\beta i}^* \sigma^{\rho\lambda} q_{2\lambda} P_L, \quad (\text{D3})$$

$$V_{\alpha m}^\mu(-q_2) = 2\mu_{\alpha m} \sigma^{\mu\eta} (-q_{2\eta}) P_R, \quad (\text{D4})$$

$$V_{\beta m}^\sigma(q) = 2\mu_{\beta m} \sigma^{\sigma\kappa} q_\kappa P_R. \quad (\text{D5})$$

Here we define the Hermitian-conjugate vertex as

$$\tilde{V}_{\alpha i}^\mu(q) := 2\mu_{\alpha i}^* \sigma^{\mu\nu} q_\nu P_L. \quad (\text{D6})$$

With a fixed momentum routing, Denner's rules are then implemented by selecting V versus $\tilde{V}$ at each insertion and, when the routing is opposite to the fermion-number flow on

an internal Dirac line, by evaluating the corresponding propagator as $S(-p)$ rather than $S(p)$. For diagram (a) this prescription is already implemented in Eqs. (D3)–(D5) and in the replacement $S_{\nu_\beta}(q_1) \rightarrow S_{\nu_\beta}(-q_1)$ for the internal ν_β line.

The contribution from the tree and one-loop amplitudes is

$$\begin{aligned} \langle |\mathcal{M}_\gamma|^2 \rangle &:= \frac{1}{2} \sum_{s,s'} |\mathcal{M}_\gamma(s,s')|^2 \\ &\simeq \frac{1}{2} \sum_{s,s'} \sum_{\text{pol.}} |\mathcal{M}_\gamma^{(0)}(s,s')|^2 \\ &\quad + \frac{1}{2} \sum_{s,s'} \sum_{\text{pol.}} \left\{ [\mathcal{M}_\gamma^{(0)}(s,s')]^\dagger [\mathcal{M}_\gamma^{(1)}(s,s')] + \text{c.c.} \right\}. \end{aligned} \quad (\text{D7})$$

Using Eq. (119), the interference term corresponding to diagram (a) becomes

$$\begin{aligned} I_{\text{vert}}^{(a)} &:= \frac{1}{2} \sum_{s,s'} \sum_{\text{pol.}} [\mathcal{M}_\gamma^{(0)}(s,s')]^\dagger [\mathcal{M}_\gamma^{(1)}(s,s')] \\ &= \frac{1}{2} \sum_{s,s'} \sum_{\text{pol.}} [2\mu_{ai}\bar{u}_\alpha(p)\sigma^{\sigma\chi}q_\chi P_R u_i(k)\varepsilon_\sigma^*(q)]^\dagger \\ &\quad \times \int \frac{d^4 q_1}{(2\pi)^4} \bar{u}_\alpha(p) V_{am}^\mu(-q_2) S_{N_m}(q_3) V_{\beta m}^\sigma(q) \\ &\quad \times S_{\nu_\beta}(-q_1) \tilde{V}_{\beta i}^\rho(q_2) u_i(k) D_{\mu\rho}(q_2) \varepsilon_\sigma^*(q). \end{aligned} \quad (\text{D8})$$

We now evaluate $I_{\text{vert}}^{(a)}$. Taking the complex conjugate of the tree amplitude and using $(\sigma^{\mu\nu})^\dagger = \gamma^0 \sigma^{\mu\nu} \gamma^0$, we obtain

$$[\mathcal{M}_\gamma^{(0)}(s,s')]^\dagger = 2\mu_{ai}^* \varepsilon_\sigma(q) \bar{u}_i(k) \sigma^{\nu\chi} q_\chi P_L u_\alpha(p). \quad (\text{D9})$$

Substituting this into Eq. (D8) and carrying out the polarization sum and the spin sums, we find

$$\begin{aligned} I_{\text{vert}}^{(a)} &= -\mu_{ai}^* g_{\nu\sigma} \\ &\quad \times \left\{ \left[\sum_s u_i(k) \bar{u}_i(k) \right] \sigma^\nu_\chi q^\chi P_L \left[\sum_{s'} u_\alpha(p) \bar{u}_\alpha(p) \right] \right. \\ &\quad \times \int \frac{d^4 q_1}{(2\pi)^4} [2\mu_{am} \sigma^{\mu\eta}(-q_{2\eta}) P_R] \frac{i(q_3^2 + M_m^2)}{q_3^2 - M_m^2 + i\epsilon} \\ &\quad \left. \times (2\mu_{\beta m} \sigma^{\sigma\kappa} q_\kappa P_R) \frac{i(-q_1^2)}{q_1^2 + i\epsilon} (2\mu_{\beta i}^* \sigma^{\rho\lambda} q_{2\lambda} P_L) \frac{-ig_{\mu\rho}}{q_2^2 + i\epsilon} \right\}. \end{aligned} \quad (\text{D10})$$

Substituting $A_{\mu,aim}^{(5)}$ from Eq. (114) and using the identity (121), we obtain

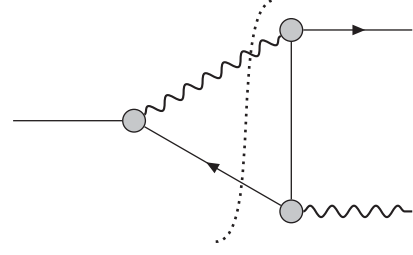


FIG. 20. Cutkosky cut for the vertex contribution. The vertical dashed line indicates the cut, which places the internal photon line and the internal light-neutrino line on shell.

$$\begin{aligned} I_{\text{vert}}^{(a)} &= iA_{\mu,aim}^{(5)} \int \frac{d^4 q_1}{(2\pi)^4} \frac{M_i M_m}{(q_3^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \\ &\quad \times \left\{ -32M_i^2(q \cdot q_1)q_2^2 + 64(q \cdot q_2) \right. \\ &\quad \left. \times [-2(p \cdot q_1)(q \cdot q_2) + 2(p \cdot q_2)(q \cdot q_1) + M_i^2(q_1 \cdot q_2)] \right\}. \end{aligned} \quad (\text{D11})$$

Here we used $k^2 = M_i^2$.

We next evaluate the discontinuity of Eq. (D11) by applying the Cutkosky rules [98]. In the present vertex topology the absorptive part arises from a two-particle cut. The internal masses are (i) q_3 : heavy neutrino N_m , with pole at $q_3^2 = M_m^2 > 0$, (ii) q_1 : light neutrino ν_β , with $q_1^2 \simeq 0$, and (iii) q_2 : photon, with $q_2^2 = 0$.

The only kinematically allowed two-particle cut that can be implemented with positive-energy on-shell states is obtained by cutting the (nearly) massless neutrino line q_1 and the photon line q_2 , as shown in Fig. 20. (In contrast, putting the heavy-neutrino propagator on shell together with another internal line is not possible in this vertex configuration; in particular, on the physical q_1 - q_2 cut the momentum flowing through the N_m line remains off shell.) Therefore, using the Cutkosky replacements, the relevant denominators become

$$\text{Disc}\left(\frac{1}{q_1^2 + i\epsilon}\right) = -2\pi i \delta(q_1^2) \Theta(E_1), \quad (\text{D12})$$

$$\begin{aligned} \text{Disc}\left(\frac{1}{q_2^2 + i\epsilon}\right) &= -2\pi i \delta(q_2^2) \Theta(E_2) \\ &= -2\pi i \delta((k - q_1)^2) \Theta(M_i - E_1). \end{aligned} \quad (\text{D13})$$

Here $q_1 = (E_1, \mathbf{q}_1)$ and $q_2 = (E_2, \mathbf{q}_2) = (M_i - E_1, -\mathbf{q}_1)$.

In the following we evaluate the kinematic part $A_{\mu,aim}^{(5)} J_{\text{vert}}^{(a)} := I_{\text{vert}}^{(a)}$ (i.e., the result with the overall coupling factors stripped off). After applying the Cutkosky rules we can set $\epsilon \rightarrow 0$. In the rest frame of N_i , substitute $q_3 = q_1 - q$ and $q_2 = k - q_1$, denote $x := M_m^2/M_i^2$, and introduce $z := \cos\theta$. Equation (D11) then yields

$$\begin{aligned}
\text{Disc}(J_{\text{vert}}^{(a)}) &= -\frac{i}{8\pi^2} M_i M_m \int d\Omega \\
&\times \frac{M_i^6 (1 + \cos\theta) [-(1 + \cos\theta)^2 + (1 - \cos\theta)^2 + 4]}{2} \\
&\quad - \frac{M_i^2}{2} (1 - \cos\theta) - M_m^2 \\
&= \frac{i}{\pi} M_i^5 M_m \int_{-1}^1 dz \frac{1 - z^2}{1 - z + 2x} \\
&= \frac{i}{\pi} M_i^6 \sqrt{x} \left[2 + 4x - 4x(1+x) \ln \frac{x+1}{x} \right]. \quad (\text{D14})
\end{aligned}$$

Therefore, the CP asymmetry (117) associated with Fig. 19(a) is

$$\begin{aligned}
\epsilon_{ai,\text{vert}}^{\gamma,(a)} &= -\frac{4\Phi_2}{\Gamma_{\gamma,i}^{(0)}} \sum_{m \neq i} \text{Im}(A_{\mu,aim}^{(5)}) \times \frac{1}{2i} \text{Disc}(J_{\text{vert}}^{(a)}) \\
&= -\frac{M_i^2}{2\pi \sum_{\beta} |\mu_{\beta i}|^2} \sum_{m \neq i} \text{Im}[\mu_{ai}^* \mu_{am} (\mu^\dagger \mu)_{im}] \\
&\quad \times \sqrt{x} \left[1 + 2x - 2x(x+1) \ln \frac{x+1}{x} \right], \quad (\text{D15})
\end{aligned}$$

where $x = M_m^2/M_i^2$. Here $\Phi_2 = \|p\|/(8\pi M_i^2)$ with $\|p\| = M_i/2$ is the two-body phase-space factor associated with the on-shell intermediate neutrino and photon generated by the Cutkosky cut. Once the internal lines are put on shell, the corresponding loop integration effectively reduces to an external phase-space integral, and Φ_2 controls the overall size of the interference term.

2. Vertex contribution (b) to the photon mode

We next evaluate the interference term corresponding to Fig. 19(b). With the momentum routing fixed as in Fig. 19, the three dipole vertices are

$$V_{\beta i}^\rho(q_2) = 2\mu_{\beta i} \sigma^{\rho\lambda} q_{2\lambda} P_R, \quad (\text{D16})$$

$$V_{am}^\mu(-q_2) = 2\mu_{am} \sigma^{\mu\eta} (-q_{2\eta}) P_R, \quad (\text{D17})$$

$$\tilde{V}_{\beta m}^\sigma(q) = 2\mu_{\beta m}^* \sigma^{\sigma\kappa} q_\kappa P_L. \quad (\text{D18})$$

Here $\tilde{V}$ denotes the vertex originating from the Hermitian-conjugate dipole interaction. Compared to diagram (a), the fermion-number arrow on the internal light-neutrino line is reversed. Following Denner's fermion-flow prescription [99], this change is implemented by

- (i) using V versus $\tilde{V}$ as indicated in Fig. 19, and
 - (ii) evaluating the internal neutrino propagator with the momentum argument consistent with the fixed routing.
- For diagram (b) the fermion-number orientation of ν_β is aligned with the momentum q_1 , hence we use $S_{\nu_\beta}(q_1)$ [in contrast to diagram (a), where one has $S_{\nu_\beta}(-q_1)$ for the same routing].

Proceeding as in Eq. (D8) and using Eq. (D9) for the complex conjugate of the tree amplitude, the interference term reads

$$\begin{aligned}
I_{\text{vert}}^{(b)} &= \frac{1}{2} \sum_{s,s'} \sum_{\text{pol.}} [\mathbf{i} \mathcal{M}_\gamma^{(0)}(s,s')]^\dagger [\mathbf{i} \mathcal{M}_\gamma^{(1)}(s,s')] \\
&= -\mu_{ai}^* g_{\nu\sigma} \left\{ \left[\sum_s u_i(k) \bar{u}_i(k) \right] \sigma^{\nu\chi} q_\chi P_L \left[\sum_{s'} u_a(p) \bar{u}_a(p) \right] \right. \\
&\quad \times \int \frac{d^4 q_1}{(2\pi)^4} V_{am}^\mu(-q_2) S_{N_m}(q_3) \tilde{V}_{\beta m}^\sigma(q) \\
&\quad \left. \times S_{\nu_\beta}(q_1) V_{\beta i}^\rho(q_2) D_{\mu\rho}(q_2) \right\}. \quad (\text{D19})
\end{aligned}$$

It is convenient to define the coupling combination

$$B_{aim}^{(5)} := \mu_{ai}^* \mu_{am} \sum_{\beta} \mu_{\beta m}^* \mu_{\beta i} = \mu_{ai}^* \mu_{am} (\mu^\dagger \mu)_{mi}. \quad (\text{D20})$$

Using the identity (121) to rewrite $\sigma^{\mu\nu} q_\nu$ in terms of γ matrices, and inserting the propagators as in Eq. (D1), we obtain

$$\begin{aligned}
I_{\text{vert}}^{(b)} &= 8i B_{aim}^{(5)} \int \frac{d^4 q_1}{(2\pi)^4} \frac{1}{(q_3^2 - M_m^2 + i\epsilon)(q_2^2 + i\epsilon)(q_1^2 + i\epsilon)} \\
&\quad \times \left(\frac{i}{2} \right)^4 \text{tr}[(k + M_i)(\gamma^\nu \not{q} - \not{q} \gamma^\nu) P_L \not{p} (\gamma^\mu \not{q}_2 - \not{q}_2 \gamma^\mu) P_R \\
&\quad \times (q_3 + M_m)(\gamma_\nu \not{q} - \not{q} \gamma_\nu) P_L \not{q}_1 (\gamma_\mu \not{q}_2 - \not{q}_2 \gamma_\mu) P_R]. \quad (\text{D21})
\end{aligned}$$

Imposing $k^2 = M_i^2$ and $p^2 \simeq 0$ together with the momentum relations $k = p + q$, $q_2 = k - q_1$, and $q_3 = q_1 - q$, the contracted Dirac trace appearing in Eq. (D21) can be reduced to [53]

$$\begin{aligned}
\mathcal{T}_{(b)} &:= \frac{i}{2} \text{tr}[P_R k (\gamma^\nu \not{q} - \not{q} \gamma^\nu) \not{p} (\gamma^\mu \not{q}_2 - \not{q}_2 \gamma^\mu) \\
&\quad \times \not{q}_3 (\gamma_\nu \not{q} - \not{q} \gamma_\nu) \not{q}_1 (\gamma_\mu \not{q}_2 - \not{q}_2 \gamma_\mu)] \\
&= i q^2 q_2^2 [80(k \cdot q_3)(p \cdot q_1) \\
&\quad - 16(k \cdot q_1)(p \cdot q_3) - 16(k \cdot p)(q_1 \cdot q_3)] \\
&\quad + 64\epsilon^{\mu\nu\rho\sigma} p_\mu q_\nu q_{2\rho} q_{3\sigma} [(k \cdot q_2)(q \cdot q_1) - (k \cdot q)(q_1 \cdot q_2)] \\
&\quad - 64\epsilon^{\mu\nu\rho\sigma} k_\mu p_\nu q_{1\rho} q_{3\sigma} (q \cdot q_2)^2 \\
&\quad + 64\epsilon^{\mu\nu\rho\sigma} p_\mu q_\nu q_{1\rho} q_{3\sigma} [(k \cdot q) q_2^2 - (k \cdot q_2)(q \cdot q_2)] \\
&\quad + 64\epsilon^{\mu\nu\rho\sigma} p_\mu q_{1\nu} q_{2\rho} q_{3\sigma} (k \cdot q)(q \cdot q_2) \\
&\quad + 64\epsilon^{\mu\nu\rho\sigma} k_\mu p_\nu q_{1\rho} q_{2\sigma} (q \cdot q_2)(q \cdot q_3) \\
&\quad + 64\epsilon^{\mu\nu\rho\sigma} p_\mu q_\nu q_{1\rho} q_{2\sigma} [(k \cdot q_2)(q \cdot q_3) - (k \cdot q)(q_2 \cdot q_3)] \\
&\quad + 64\epsilon^{\mu\nu\rho\sigma} k_\mu p_\nu q_\rho q_{2\sigma} [(q \cdot q_1)(q_2 \cdot q_3) - (q \cdot q_3)(q_1 \cdot q_2)] \\
&\quad + 64\epsilon^{\mu\nu\rho\sigma} k_\mu p_\nu q_\rho q_{3\sigma} [(q \cdot q_2)(q_1 \cdot q_2) - (q \cdot q_1) q_2^2] \\
&\quad + 64\epsilon^{\mu\nu\rho\sigma} k_\mu p_\nu q_{2\rho} q_{3\sigma} (q \cdot q_1)(q \cdot q_2) \\
&\quad + 64\epsilon^{\mu\nu\rho\sigma} k_\mu p_\nu q_\rho q_{1\sigma} [(q \cdot q_3) q_2^2 - (q \cdot q_2)(q_2 \cdot q_3)], \quad (\text{D22})
\end{aligned}$$

where q^2 and q_2^2 are kept symbolic.

For the photon mode one has $q^2 = 0$, which removes the parity-even part proportional to $q^2 q_2^2$. The remaining totally antisymmetric terms vanish identically, because all momenta $\{k, p, q, q_1, q_2, q_3\}$ lie in the linear span of $\{p, q, q_1\}$ by momentum conservation. Therefore, the contracted Dirac trace vanishes for arbitrary loop momentum q_1 , and the interference term from Fig. 19(b) vanishes,

$$I_{\text{vert}}^{(b)} = 0. \quad (\text{D23})$$

Combining Eqs. (D15) and (D23), the total vertex contribution to the photon-mode CP asymmetry is

$$\varepsilon_{ai,\text{vert}}^{\gamma} = -\frac{M_i^2}{2\pi \sum_{\beta} |\mu_{\beta i}|^2} \sum_{m \neq i} \text{Im}[\mu_{ai}^* \mu_{am} (\mu^\dagger \mu)_{im}] f_{V_a}(x), \quad (\text{D24})$$

where

$$f_{V_a}(x) := \sqrt{x} \left\{ 1 + 2x \left[1 - (x+1) \ln \frac{x+1}{x} \right] \right\}, \quad (\text{D25})$$

with $x := M_m^2/M_i^2$ ($m \neq i$).

3. Self-energy contribution (a) to the photon mode

We now turn to the self-energy contributions to the photon mode. The relevant diagrams are shown in Fig. 21.

We begin with diagram (a). The propagators for the light neutrino, the heavy neutrino N_m , and the photon are

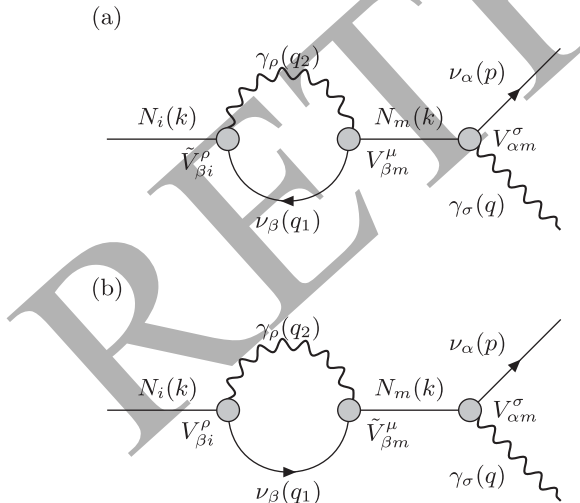


FIG. 21. Self-energy-type one-loop diagrams contributing to the photon mode. Panel (a) shows the fermion-flow contraction that yields the contribution proportional to $(\mu^\dagger \mu)_{im}$, whereas panel (b) shows the alternative contraction proportional to $(\mu^\dagger \mu)_{mi}$.

$$S_\nu(q_1) \simeq \frac{i q_1}{q_1^2 + i\epsilon}, \quad S_{N_m}(k) = \frac{i(k + M_m)}{k^2 - M_m^2 + i\epsilon}, \quad (\text{D26})$$

$$D_{\mu\nu}(q_2) = \frac{-ig_{\mu\nu}}{q_2^2 + i\epsilon}.$$

Unlike the vertex topology, momentum conservation fixes the momentum flowing through the intermediate heavy-neutrino propagator to be the external momentum k ; the loop momentum is carried only by the (ν_β, γ) bubble, with q_1 and $q_2 := k - q_1$.

The three dipole vertices appearing in Fig. 21(a) are

$$\tilde{V}_{\beta i}^\rho(q_2) = 2\mu_{\beta i}^* \sigma^{\rho\lambda} q_{2\lambda} P_L, \quad (\text{D27})$$

$$V_{\beta m}^\mu(-q_2) = 2\mu_{\beta m} \sigma^{\mu\eta} (-q_{2\eta}) P_R, \quad (\text{D28})$$

$$V_{am}^\sigma(q) = 2\mu_{am} \sigma^{\sigma\kappa} q_\kappa P_R. \quad (\text{D29})$$

As in Appendixes D 1 and D 2, the assignments of V versus $\tilde{V}$ and the momentum arguments in Dirac propagators are fixed unambiguously by Denner's fermion-flow prescription [99]. In diagram (a), for the fixed routing shown in Fig. 21 the momentum q_1 runs opposite to the fermion-number orientation of the internal ν_β line, so the corresponding propagator is evaluated as $S_{\nu_\beta}(-q_1)$.

Proceeding analogously to Eq. (D8) and using Eq. (113) together with the identity (121), the interference term corresponding to diagram (a) is given by

$$I_{\text{self}}^{(a)} = \frac{1}{2} \sum_{s,s'} \sum_{\text{pol.}} [i\mathcal{M}_\gamma^{(0)}(s,s')]^\dagger [i\mathcal{M}_\gamma^{(1)}(s,s')] \\ = iA_{\mu,aim}^{(5)} M_i M_m \\ \times \int \frac{d^4 q_1}{(2\pi)^4} \frac{(p \cdot q) [-256(q \cdot q_2)(q_1 \cdot q_2) + 64(q \cdot q_1)q_2^2]}{(k^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)}. \quad (\text{D30})$$

For completeness, we comment on possible fermion-flow signs. In our implementation of Denner's prescription we treat V and $\tilde{V}$ as distinct building blocks originating from the dipole operator and its Hermitian conjugate. With this choice the fermion chain in Eq. (D30) is assembled using the standard vertices, and no additional reversal operation $\Gamma \rightarrow \Gamma^{\text{rev}}$ is required; hence no extra η_Γ sign arises beyond the explicit choice of V versus $\tilde{V}$ and the momentum argument $S_{\nu_\beta}(-q_1)$.

To extract the absorptive part of Eq. (D30), we apply the Cutkosky rules [98]. Since the loop consists only of the ν_β line and the photon line, the only admissible cut places these two propagators on shell, as shown in Fig. 22. We replace

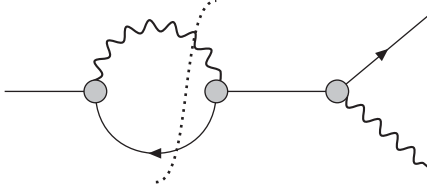


FIG. 22. Cutkosky cut for the self-energy contributions. The vertical dashed line indicates the cut that puts both the internal photon and the internal light-neutrino propagators on shell.

$$\text{Disc}\left(\frac{1}{q_1^2 + i\epsilon}\right) = -2\pi i \delta(q_1^2) \Theta(E_1), \quad (\text{D31})$$

$$\begin{aligned} \text{Disc}\left(\frac{1}{q_2^2 + i\epsilon}\right) &= -2\pi i \delta(q_2^2) \Theta(E_2) \\ &= -2\pi i \delta((k - q_1)^2) \Theta(M_i - E_1), \end{aligned} \quad (\text{D32})$$

where $q_1 = (E_1, \mathbf{q}_1)$ and $q_2 = (E_2, \mathbf{q}_2) = k - q_1 = (M_i - E_1, -\mathbf{q}_1)$. Taking $\epsilon \rightarrow 0$, the discontinuity of $J_{\text{self}}^{(a)}$ defined by $A_{\mu,aim}^{(5)} J_{\text{self}}^{(a)} := I_{\text{self}}^{(a)}$ is

$$\text{Disc}(J_{\text{self}}^{(a)}) = \frac{2iM_i^7 M_m}{\pi(M_i^2 - M_m^2)}. \quad (\text{D33})$$

Therefore,

$$\text{Im}(J_{\text{self}}^{(a)}) = \frac{1}{2i} \text{Disc}(J_{\text{self}}^{(a)}) = \frac{M_i^7 M_m}{\pi(M_i^2 - M_m^2)}. \quad (\text{D34})$$

The part of the self-energy contribution to the CP asymmetry carried by diagram (a) can be written as

$$\begin{aligned} \epsilon_{ai,\text{self}}^{\gamma,(a)} &= -\frac{4\Phi_2}{\Gamma_{\text{tot}}} \sum_{m \neq i} \text{Im}(A_{\mu,aim}^{(5)}) \text{Im}(J_{\text{self}}^{(a)}) \\ &= -\frac{M_i^2}{2\pi \sum_{\beta} |\mu_{\beta i}|^2} \sum_{m \neq i} \text{Im}[\mu_{ai}^* \mu_{am} (\mu^\dagger \mu)_{im}] \frac{\sqrt{x}}{1-x}. \end{aligned} \quad (\text{D35})$$

4. Self-energy contribution (b) to the photon mode

We next evaluate diagram (b) in Fig. 21. In this case the three dipole vertices are

$$V_{\beta i}^\rho(q_2) = 2\mu_{\beta i} \sigma^{\rho\lambda} q_{2\lambda} P_R, \quad (\text{D36})$$

$$\tilde{V}_{\beta m}^\mu(-q_2) = 2\mu_{\beta m}^* \sigma^{\mu\eta} (-q_{2\eta}) P_L, \quad (\text{D37})$$

$$V_{am}^\sigma(q) = 2\mu_{am} \sigma^{\sigma\kappa} q_\kappa P_R. \quad (\text{D38})$$

It is convenient to introduce the coupling combination

$$B_{\mu,aim}^{(5)} := \mu_{ai}^* \mu_{am} \sum_{\beta} \mu_{\beta m}^* \mu_{\beta i} = \mu_{ai}^* \mu_{am} (\mu^\dagger \mu)_{mi}. \quad (\text{D39})$$

Proceeding as in Eq. (D30) and using the identity (121), we obtain the interference term

$$\begin{aligned} I_{\text{self}}^{(b)} &= \frac{1}{2} \sum_{s,s'} \sum_{\text{pol.}} [i\mathcal{M}_\gamma^{(0)}(s,s')]^\dagger [i\mathcal{M}_\gamma^{(1)}(s,s')] \\ &= \frac{1}{2} \sum_{s,s'} \left[\sum_{\text{pol.}} \epsilon_\nu(q) \epsilon_\sigma^*(q) \right] [2\mu_{ai}^* \bar{u}_i(k) \sigma^{\nu\chi} q_\chi P_L u_a(p)] \\ &\quad \times \int \frac{d^4 q_1}{(2\pi)^4} \bar{u}_\alpha(p) V_{am}^\sigma(q) S_{N_m}(k) \tilde{V}_{\beta m}^\mu(-q_2) \\ &\quad \times S_{\nu_\rho}(q_1) V_{\beta i}^\rho(q_2) D_{\mu\rho}(q_2) u_i(k). \end{aligned} \quad (\text{D40})$$

At this point it is useful to spell out how Denner's fermion-flow prescription [99] is implemented. Compared to diagram (a), the fermion-number orientation on the internal light-neutrino line is reversed, while we keep the same momentum routing. Denner's rules then amount to

- (i) using the Hermitian-conjugate dipole vertex on the $N_m - \nu_\beta - \gamma(q_2)$ insertion, i.e. $\tilde{V}_{\beta m}^\mu(-q_2)$, and
- (ii) evaluating the internal Dirac propagator with the momentum argument consistent with this routing, here $S_{\nu_\rho}(q_1)$.

Equivalently, one may insist on using the same vertex orientation but apply Denner's reversal map $\Gamma \rightarrow \Gamma^{\text{rev}} = C\Gamma C^{-1}$ to that insertion; since $(\sigma^{\mu\nu})^{\text{rev}} = -\sigma^{\mu\nu}$, this alternative implementation differs only by an overall minus sign. In what follows we work directly with the explicit vertices V and $\tilde{V}$ in Eqs. (D36)–(D38), so no additional reversal operation is inserted beyond this bookkeeping.

With these conventions, performing the polarization sum and the spin sums yields

$$\begin{aligned} I_{\text{self}}^{(b)} &= \int \frac{d^4 q_1}{(2\pi)^4} \frac{32iM_i^4 B_{\mu,aim}^{(5)}}{(k^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \\ &\quad \times [4(q \cdot q_2)(q_1 \cdot q_2) - (q \cdot q_1)q_2^2 \\ &\quad - 4(k \cdot q_2)(q_1 \cdot q_2) + (k \cdot q_1)q_2^2]. \end{aligned} \quad (\text{D41})$$

The discontinuity of the integral in Eq. (D41) is obtained from the Cutkosky cut in Fig. 22. Setting $q_1 = (E_1, \mathbf{q}_1)$ and using $q_2 = k - q_1$, Eqs. (D31) and (D32) give

$$\text{Disc}(J_{\text{self}}^{(b)}) = \frac{2iM_i^8}{\pi(M_i^2 - M_m^2)}. \quad (\text{D42})$$

Therefore,

$$\text{Im}(J_{\text{self}}^{(b)}) = \frac{1}{2i} \text{Disc}(J_{\text{self}}^{(b)}) = \frac{M_i^8}{\pi(M_i^2 - M_m^2)}. \quad (\text{D43})$$

The part of the self-energy contribution to the CP asymmetry carried by diagram (b) is then

$$\begin{aligned}\varepsilon_{ai,\text{self}}^{\gamma,(b)} &= -\frac{4\Phi_2}{\Gamma_{\text{tot}}} \sum_{m \neq i} \text{Im}(B_{\mu,aim}^{(5)}) \text{Im}(J_{\text{self}}^{(b)}) \\ &= -\frac{M_i^2}{2\pi \sum_{\beta} |\mu_{\beta i}|^2} \text{Im}[\mu_{ai}^* \mu_{am} (\mu^\dagger \mu)_{mi}] \frac{1}{1-x},\end{aligned}\quad (\text{D44})$$

with $x := M_m^2/M_i^2$ ($m \neq i$).

APPENDIX E: CP ASYMMETRIES II: PHOTON MODE IN THE QUASIDEGENERATE REGIME

In this appendix we derive the resonantly regulated self-energy loop functions used in the photon-mode CP asymmetry. The vertex contribution is regular in the quasidegenerate limit and is therefore unchanged from Appendix D. The singular behavior arises only from the self-energy loop functions $f_{S_a}(x)$ and $f_{S_b}(x)$, which are regulated by inserting the Dyson-resummed propagator of the intermediate heavy neutrino.

We keep the derivation here because the intermediate expressions are lengthy and would obscure the main logic of Sec. IV. The main text quotes only the Dyson-resummed propagator, the Breit-Wigner-regulated loop functions, and the resonance condition.

We now derive Eq. (140). Following Eq. (139), we Dyson resum the internal N_m propagator and recompute the self-energy diagrams. We start with Fig. 21(a). The propagators are

$$\begin{aligned}S_\nu(q_1) &\simeq \frac{i\not{q}_1}{q_1^2 + i\epsilon}, \quad S_{N_m}(k) = \frac{i(k + M_m)}{k^2 - M_m^2 + iM_m\Gamma_m + i\epsilon}, \\ D_{\mu\nu}(q_2) &= \frac{-ig_{\mu\nu}}{q_2^2 + i\epsilon},\end{aligned}\quad (\text{E1})$$

and the three dipole vertices are

$$\tilde{V}_{\beta i}^\rho(q_2) = 2\mu_{\beta i}^* \sigma^{\rho\lambda} q_{2\lambda} P_L, \quad (\text{E2})$$

$$V_{\beta m}^\mu(-q_2) = 2\mu_{\beta m} \sigma^{\mu\eta} (-q_{2\eta}) P_R, \quad (\text{E3})$$

$$V_{am}^\sigma(q) = 2\mu_{am} \sigma^{\sigma\kappa} q_\kappa P_R. \quad (\text{E4})$$

We use $A_{\mu,aim}^{(5)}$ defined in Eq. (114), and the interference term for Fig. 21(a) reads

$$\begin{aligned}I_{\text{self}}^{(a)} &= \frac{1}{2} \sum_{s,s'} \sum_{\text{pol.}} [i\mathcal{M}_\gamma^{(0)}(s,s')]^\dagger [i\mathcal{M}_\gamma^{(1)}(s,s')] \\ &= iA_{\mu,aim}^{(5)} M_i M_m \\ &\quad \times \int \frac{d^4 q_1}{(2\pi)^4} \frac{(p \cdot q)[-256(q \cdot q_2)(q_1 \cdot q_2) + 64(q \cdot q_1)q_2^2]}{(k^2 - M_m^2 + iM_m\Gamma_m + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)}.\end{aligned}\quad (\text{E5})$$

According to Denner's fermion-flow prescription [99], the spinor chain can be assembled by inserting the building blocks corresponding to the chosen fermion-flow orientation without introducing any additional reversal operation; hence no extra sign arises beyond the explicit use of V versus $\tilde{V}$ and the momentum assignment $S_{\nu_\mu}(-q_1)$.

To evaluate the discontinuity of Eq. (E5), we apply the Cutkosky rules [98]. As shown in Fig. 22, the only cut that puts both loop propagators on shell is the cut through the fermion propagator with momentum q_1 and the photon propagator with momentum q_2 . Consequently, the denominators in Eq. (E5) become

$$\text{Disc}\left(\frac{1}{q_1^2 + i\epsilon}\right) = -2\pi i \delta(q_1^2) \Theta(E_1), \quad (\text{E6})$$

$$\begin{aligned}\text{Disc}\left(\frac{1}{q_2^2 + i\epsilon}\right) &= -2\pi i \delta(q_2^2) \Theta(E_2) \\ &= -2\pi i \delta((k - q_1)^2) \Theta(M_i - E_1),\end{aligned}\quad (\text{E7})$$

where $q_1 = (E_1, \mathbf{q}_1)$ and $q_2 = (E_2, \mathbf{q}_2) = k - q_1 = (M_i - E_1, -\mathbf{q}_1)$. Working in the N_i rest frame, and taking $\epsilon \rightarrow 0$, the discontinuity of $J_{\text{self}}^{(a)} = I_{\text{self}}^{(a)}/A_{\mu,aim}^{(5)}$ is

$$\text{Disc}(J_{\text{self}}^{(a)}) = \frac{2iM_i^7 M_m (M_i^2 - M_m^2 - iM_m\Gamma_m)}{\pi[(M_i^2 - M_m^2)^2 + M_m^2\Gamma_m^2]}. \quad (\text{E8})$$

Therefore,

$$\begin{aligned}\text{Im}(J_{\text{self}}^{(a)}) &= \frac{1}{2i} \text{Disc}(J_{\text{self}}^{(a)}) \\ &= \frac{M_i^7 M_m}{\pi} \frac{M_i^2 - M_m^2}{(M_i^2 - M_m^2)^2 + M_m^2\Gamma_m^2}.\end{aligned}\quad (\text{E9})$$

The corresponding self-energy contribution to the CP asymmetry is

$$\begin{aligned}\varepsilon_{\text{self},ai}^{\gamma,(a)} &= -\frac{M_i^2}{2\pi \sum_{\beta} |\mu_{\beta i}|^2} \text{Im}[\mu_{ai}^* \mu_{am} (\mu^\dagger \mu)_{im}] \\ &\quad \times \frac{(M_i^2 - M_m^2) M_i M_m}{(M_i^2 - M_m^2)^2 + M_m^2\Gamma_m^2}.\end{aligned}\quad (\text{E10})$$

In terms of $x = M_m^2/M_i^2$ ($m \neq i$), Eq. (E10) can be rewritten as

$$\tilde{f}_{S_a}(x) \simeq \frac{\sqrt{x}(1-x)}{(1-x)^2 + (\Gamma_m/M_i)^2}, \quad (\text{E11})$$

where the last approximation holds in the quasidegenerate limit $x \simeq 1$. This is the Breit-Wigner-type (up to the slowly varying factor $\sqrt{x}$) for the self-energy contribution regulated by the total width Γ_m .

We next evaluate Fig. 21(b). The three vertices are

$$V_{\beta i}^\rho(q_2) = 2\mu_{\beta i}\sigma^{\rho\lambda}q_{2\lambda}P_R, \quad (\text{E12})$$

$$\tilde{V}_{\beta m}^\mu(-q_2) = 2\mu_{\beta m}^*\sigma^{\mu\eta}(-q_{2\eta})P_L, \quad (\text{E13})$$

$$V_{am}^\sigma(q) = 2\mu_{am}\sigma^{\sigma\kappa}q_\kappa P_R. \quad (\text{E14})$$

The corresponding interference term is

$$\begin{aligned} I_{\text{self}}^{(b)} &= \frac{1}{2} \sum_{s,s'} \sum_{\text{pol.}} [\mathbf{i}\mathcal{M}_\gamma^{(0)}(s, s')]^\dagger [\mathbf{i}\mathcal{M}_\gamma^{(1)}(s, s')] \\ &= \frac{1}{2} \sum_{s,s'} \left[\sum_{\text{pol.}} \varepsilon_\nu(q) \varepsilon_\sigma^*(q) \right] [2\mu_{ai}^* \bar{u}_i(k) \sigma^{\nu\chi} q_\chi P_L u_\alpha(p)] \\ &\quad \times \int \frac{d^4 q_1}{(2\pi)^4} \bar{u}_\alpha(p) V_{am}^\sigma(q) S_{N_m}(k) \tilde{V}_{\beta m}^\mu(-q_2) \\ &\quad \times S_{\nu\beta}(q_1) V_{\beta i}^\rho(q_2) D_{\mu\rho}(q_2) u_i(k). \end{aligned} \quad (\text{E15})$$

The fermion-flow bookkeeping is implemented as follows. For the fixed momentum routing, the internal Dirac line is evaluated as $S_{\nu\beta}(q_1)$, and the vertex on that line is the conjugate one $\tilde{V}_{\beta m}^\mu(-q_2)$. No additional reversal operation is required once these building blocks are chosen consistently.

Using $B_{\mu,aim}^{(5)}$ defined in Eq. (D39) and carrying out the Clifford algebra gives

$$\begin{aligned} I_{\text{self}}^{(b)} &= 32iM_i^4 B_{\mu,aim}^{(5)} \\ &\quad \times \int \frac{d^4 q_1}{(2\pi)^4} \frac{1}{(k^2 - M_m^2 + iM_m\Gamma_m + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)} \\ &\quad \times [4(q \cdot q_2)(q_1 \cdot q_2) - (q \cdot q_1)q_2^2 \\ &\quad - 4(k \cdot q_2)(q_1 \cdot q_2) + (k \cdot q_1)q_2^2]. \end{aligned} \quad (\text{E16})$$

The discontinuity of Eq. (E16) follows from the Cutkosky cut in Fig. 22. Setting $q_1 = (E_1, \mathbf{q}_1)$ and $q_2 = k - q_1$, we obtain

$$\text{Disc}(J_{\text{self}}^{(b)}) = \frac{2iM_i^8(M_i^2 - M_m^2 - iM_m\Gamma_m)}{\pi[(M_i^2 - M_m^2)^2 + M_m^2\Gamma_m^2]}. \quad (\text{E17})$$

Therefore,

$$\begin{aligned} \text{Im}(J_{\text{self}}^{(b)}) &= \frac{1}{2i} \text{Disc}(J_{\text{self}}^{(b)}) \\ &= \frac{M_i^8}{\pi} \frac{M_i^2 - M_m^2}{(M_i^2 - M_m^2)^2 + M_m^2\Gamma_m^2}, \end{aligned} \quad (\text{E18})$$

and the corresponding self-energy contribution to the CP asymmetry is

$$\begin{aligned} \varepsilon_{\text{self},ai}^{\gamma,(b)} &= -\frac{M_i^2}{2\pi \sum_\beta |\mu_{\beta i}|^2} \text{Im}[\mu_{ai}^* \mu_{am}(\mu^\dagger \mu)_{mi}] \\ &\quad \times \frac{(M_i^2 - M_m^2)M_i^2}{(M_i^2 - M_m^2)^2 + M_m^2\Gamma_m^2}. \end{aligned} \quad (\text{E19})$$

In terms of $x = M_m^2/M_i^2$, Eq. (E19) can be rewritten as

$$\tilde{f}_{S_b}(x) \simeq \frac{1-x}{(1-x)^2 + (\Gamma_m/M_i)^2}, \quad (\text{E20})$$

where the last approximation holds in the quasidegenerate limit $x \simeq 1$. This is again the universal BW-type regularization of the self-energy contribution by the total width Γ_m .

Combining Eqs. (E11) and (E20) reproduces the desired resonant loop functions in Eq. (140).

APPENDIX F: CP ASYMMETRIES III: $\mathcal{O}_{NW}$ -INCLUSIVE RESONANT REGIME

In this appendix we give the loop-level derivation of the $\mathcal{O}_{NW}$ -inclusive CP asymmetries used in Sec. V. Once $\mathcal{O}_{NW}$ is present, the broken-phase dipole matrices μ^γ , μ^Z , and μ^W are independent in general. Consequently, the external decay channel $V = \gamma, Z, W$ and the internal on-shell state $U = \gamma, Z, W$ must be treated separately.

We collect the vertex contributions, the self-energy contributions, and the quasidegenerate limit in a unified notation. This appendix contains the detailed Cutkosky cuts, angular integrations, and finite-gauge-boson-mass loop functions. The main text quotes only the final channel-resolved CP asymmetries.

1. Vertex contribution (a)

a. Cutkosky rules

We collect the coupling factors as

$$A_{\mu,aim}^{VU} := \mu_{ai}^{V*} \mu_{am}^U (\mu^{U\dagger} \mu^V)_{im} = \mu_{ai}^{V*} \mu_{am}^U \sum_\beta \mu_{\beta i}^{U*} \mu_{\beta m}^V. \quad (\text{F1})$$

Then the interference integral can be written as

$$I_{\text{vert}}^{(a)}(V; U) = A_{\mu,aim}^{VU} J_{\text{vert}}^{(a)}(V; U), \quad (\text{F2})$$

where $A_{\mu,aim}^{VU}$ is independent of the loop momentum, and $J_{\text{vert}}^{(a)}(V; U)$ is given by

$$J_{\text{vert}}^{(a)}(V; U) := i \int \frac{d^4 q_1}{(2\pi)^4} \frac{\mathcal{N}_{\text{vert}}^{(a)}(q_1; k, q)}{(q_3^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 - m_U^2 + i\epsilon)}. \quad (\text{F3})$$

With this convention, the Cutkosky discontinuity of $J_{\text{vert}}^{(a)}$ directly yields the absorptive part needed for the CP asymmetry. We cut the internal lepton line q_1 and the internal gauge-boson line q_2 (see Fig. 20) and use

$$\begin{aligned} \text{Disc}\left(\frac{1}{q_1^2 + i\epsilon}\right) &= -2\pi i \delta(q_1^2) \Theta(E_1), \\ \text{Disc}\left(\frac{1}{q_2^2 - m_U^2 + i\epsilon}\right) &= -2\pi i \delta(q_2^2 - m_U^2) \Theta(E_2). \end{aligned} \quad (\text{F4})$$

These equations give

$$\begin{aligned} \text{Disc} J_{\text{vert}}^{(a)}(V; U) &= -i(2\pi)^2 \int \frac{d^4 q_1}{(2\pi)^4} \\ &\times \frac{\delta(q_1^2) \delta(q_2^2 - m_U^2) \Theta(E_1) \Theta(E_2)}{q_3^2 - M_m^2} \mathcal{N}_{\text{vert}}^{(a)}|_{\text{cut}}. \end{aligned} \quad (\text{F5})$$

b. From the δ -function integrals to an angular integral

We now evaluate the δ -function integrals in Eq. (F5). In the rest frame of N_i , $q_2 = k - q_1$, and hence

$$q_2^2 = (k - q_1)^2 = M_i^2 - 2M_i E_1. \quad (\text{F6})$$

Therefore, $\delta(q_2^2 - m_U^2)$ fixes

$$E_1 = \frac{M_i^2 - m_U^2}{2M_i} = \frac{M_i}{2}(1 - r_U), \quad r_U := \frac{m_U^2}{M_i^2}. \quad (\text{F7})$$

Since $q_1^2 = 0$ implies $\|\mathbf{q}_1\| = E_1$, the remaining integral reduces to an angular integral over $t_1 := \cos \theta_1 := \hat{\mathbf{q}} \cdot \hat{\mathbf{q}}_1$.

To perform the angular integral, we express the remaining denominator $q_3^2 - M_m^2$ as a function of t_1 . From $q_3 = q_1 - q$,

$$\begin{aligned} q_3^2 &= (q_1 - q)^2 = q^2 - 2q_1 \cdot q \\ &= m_V^2 - 2(E_1 E_V - \|\mathbf{q}_1\| \|\mathbf{q}\| t_1). \end{aligned} \quad (\text{F8})$$

Using

$$\begin{aligned} E_V &= \frac{M_i^2 + m_V^2}{2M_i} = \frac{M_i}{2}(1 + r_V), \\ \|\mathbf{q}\| &= \frac{\sqrt{\lambda(M_i^2, 0, m_V^2)}}{2M_i} = \frac{M_i}{2}(1 - r_V), \end{aligned} \quad (\text{F9})$$

and Eq. (F7), i.e., $E_1 = \|\mathbf{q}_1\| = M_i(1 - r_U)/2$, we find

$$q_1 \cdot q = \frac{M_i^2}{4}(1 - r_U)[(1 + r_V) - (1 - r_V)t_1]. \quad (\text{F10})$$

It follows that

$$q_3^2 - M_m^2 = -\frac{M_i^2}{2}(D_0 - D_1 t_1), \quad (\text{F11})$$

with

$$\begin{aligned} D_0 &:= 2x_{mi} + 1 - r_V - r_U - r_V r_U, \\ D_1 &:= (1 - r_V)(1 - r_U). \end{aligned} \quad (\text{F12})$$

Here we denote $x_{mi} := M_m^2/M_i^2$ ($m \neq i$).

We now carry out the δ -function integrals explicitly. Writing q_1^0 for the time component, $q_1^2 = (q_1^0)^2 - E_1^2$,

$$\delta(q_1^2) = \delta((q_1^0)^2 - E_1^2) = \frac{1}{2E_1} [\delta(q_1^0 - E_1) + \delta(q_1^0 + E_1)]. \quad (\text{F13})$$

Therefore,

$$\delta(q_1^2) \Theta(E_1) = \frac{1}{2E_1} \delta(q_1^0 - E_1). \quad (\text{F14})$$

Using this to integrate over q_1^0 , we obtain

$$\begin{aligned} I_{\text{cut}} &:= \int \frac{d^4 q_1}{(2\pi)^4} (2\pi)^2 \delta(q_1^2) \delta(q_2^2 - m_U^2) \Theta(E_1) \Theta(E_2) \mathcal{F}(t_1) \\ &= \int \frac{d^3 q_1}{(2\pi)^2} \frac{1}{2E_1} \delta(q_2^2 - m_U^2) \Theta(E_2) \mathcal{F}(t_1), \end{aligned} \quad (\text{F15})$$

where $t_1 := \cos \theta_1 := \hat{\mathbf{q}} \cdot \hat{\mathbf{q}}_1$, and $\mathcal{F}(t_1)$ denotes the remaining cut integrand, which depends on q_1 only through t_1 once the δ functions are imposed. In the present application we will set $\mathcal{F}(t) = \mathcal{N}_{\text{vert}}^{(a)}|_{\text{cut}}/(q_3^2 - M_m^2)$. Next, with $k = (M_i, \mathbf{0})$ and $q_2 = k - q_1$,

$$q_2^0 = M_i - q_1^0 = M_i - E_1, \quad \|\mathbf{q}_2\| = \|\mathbf{q}_1\| = E_1, \quad (\text{F16})$$

and hence

$$q_2^2 = (q_2^0)^2 - \|\mathbf{q}_2\|^2 = (M_i - E_1)^2 - E_1^2 = M_i^2 - 2M_i E_1. \quad (\text{F17})$$

Thus,

$$\begin{aligned} \delta(q_2^2 - m_U^2) &= \delta(M_i^2 - 2M_i E_1 - m_U^2) \\ &= \frac{1}{2M_i} \delta\left(E_1 - \frac{M_i^2 - m_U^2}{2M_i}\right) =: \frac{\delta(E_1 - E_1^*)}{2M_i}. \end{aligned} \quad (\text{F18})$$

Here

$$E_1^* := \frac{M_i^2 - m_U^2}{2M_i} = \frac{M_i}{2}(1 - r_U), \quad r_U = \frac{m_U^2}{M_i^2}. \quad (\text{F19})$$

In spherical coordinates,

$$\begin{aligned} d^3q_1 &= q_1^2 dq_1 d\Omega = E_1^2 dE_1 d\Omega, \\ d\Omega_1 &= dt_1 d\phi_1, \quad t_1 = \cos \theta_1 \in [-1, 1]. \end{aligned} \quad (\text{F20})$$

Therefore,

$$\begin{aligned} I_{\text{cut}} &= \frac{1}{(2\pi)^2} \int d\Omega_1 \int_0^\infty dE_1 \\ &\times \frac{E_1^2}{2E_1} \frac{\delta(E_1 - E_1^*)}{2M_i} \Theta(M_i - E_1) \mathcal{F}(t_1). \end{aligned} \quad (\text{F21})$$

The E_1 integral is immediate, and for $M_i > m_U$ we obtain

$$I_{\text{cut}} = \frac{1 - r_U}{16\pi} \int_{-1}^1 dt_1 \mathcal{F}(t_1). \quad (\text{F22})$$

As a result,

$$\text{Disc} J_{\text{vert}}^{(a)}(V; U) = -\frac{i}{16\pi} (1 - r_U) \int_{-1}^1 dt_1 \frac{\mathcal{N}_{\text{vert}}^{(a)}|_{\text{cut}}}{q_3^2 - M_m^2}. \quad (\text{F23})$$

The remaining task is the t_1 integral.

c. Denominator: Evaluation of $q_3^2 - M_m^2$

We use

$$q_3 := q_1 - q, \quad x_{mi} := \frac{M_m^2}{M_i^2}, \quad r_V := \frac{m_V^2}{M_i^2} = \frac{q^2}{M_i^2}, \quad (\text{F24})$$

and impose the cut condition $q_1^2 = 0$, which gives

$$q_3^2 - M_m^2 = (q_1 - q)^2 - M_m^2 = q^2 - 2q \cdot q_1 - M_m^2. \quad (\text{F25})$$

Note that q_3^2 itself does not depend on x_{mi} ; the dependence enters only through the subtraction of M_m^2 .

In the rest frame of N_i , with $t_1 = \cos \theta_1 = \hat{\mathbf{q}} \cdot \hat{\mathbf{q}}_1$, we can write

$$\begin{aligned} q_3^2 - M_m^2 &= -\frac{M_i^2}{2} D(t_1), \\ D(t_1) &:= (1 - r_V)(1 - r_U)(1 - t_1) \\ &\quad + 2(x_{mi} - r_V r_U). \end{aligned} \quad (\text{F26})$$

Therefore,

$$\frac{1}{q_3^2 - M_m^2} = -\frac{2}{M_i^2} \frac{1}{D(t_1)}. \quad (\text{F27})$$

Substituting Eq. (F27) into Eq. (F23) yields

$$\text{Disc} J_{\text{vert}}^{(a)}(V; U) = i \frac{1 - r_U}{8\pi M_i^2} \int_{-1}^1 dt_1 \frac{\mathcal{N}_{\text{vert}}^{(a)}|_{\text{cut}}}{D(t_1)}. \quad (\text{F28})$$

The sign flip originates from the overall minus sign in $q_3^2 - M_m^2$.

d. Numerator: Reduction of the Dirac trace to a polynomial in t_1

After reducing the Dirac trace with the $\sigma^{\mu\nu}$ insertions and imposing the on-shell conditions after the cut,

$$q_1^2 = 0, \quad q_2^2 = m_U^2, \quad (\text{F29})$$

the numerator reduces to a quadratic polynomial in t_1 ,

$$\begin{aligned} \mathcal{N}_{\text{vert}}^{(a)}|_{\text{cut}} &= 8M_i^7 M_m (1 - r_U) \\ &\times [(1 + r_V) - (1 - r_V)(r_V + r_U + r_V r_U)t_1 \\ &\quad - (1 - r_U)(1 - r_V)^2 t_1^2] \\ &=: 8M_i^7 M_m (1 - r_U) (A_2 t_1^2 + A_1 t_1 + A_0). \end{aligned} \quad (\text{F30})$$

The coefficients can be written, equivalently, as

$$\begin{aligned} A_0 &= 1 + r_V, \\ A_1 &= r_V^2 (1 + r_U) - (r_V + r_U), \\ A_2 &= -(1 - r_U)(1 - r_V)^2. \end{aligned} \quad (\text{F31})$$

In the limit $r_V = r_U = 0$,

$$\begin{aligned} \text{Disc} J_{\text{vert}}^{(a)}(V; U) &\rightarrow \frac{i}{8\pi M_i^2} \int_{-1}^1 dt_1 \frac{8M_i^7 M_m (1 - t_1^2)}{1 - t_1 + 2x_{mi}} \\ &= \frac{i}{\pi} M_i^5 M_m \int_{-1}^1 dt_1 \frac{1 - t_1^2}{1 - t_1 + 2x_{mi}}, \end{aligned} \quad (\text{F32})$$

in agreement with the photon-mode result for the vertex discontinuity, Eq. (D14).

e. Angular integral

From Eq. (F26), we may write

$$\begin{aligned} D(t_1) &= D_0 - D_1 t_1, \\ D_1 &= (1 - r_V)(1 - r_U), \\ D_0 &= D_1 + 2(x_{mi} - r_V r_U) = 2x_{mi} + 1 - r_V - r_U - r_V r_U. \end{aligned} \quad (\text{F33})$$

Polynomial division yields the decomposition

$$\begin{aligned} & \frac{A_2 t_1^2 + A_1 t_1 + A_0}{D_0 - D_1 t_1} \\ &= (1 - r_V) t_1 + \frac{2x_{mi} + 1}{1 - r_U} + \frac{2B}{1 - r_U} \frac{1}{D_0 - D_1 t_1}, \quad (\text{F34}) \end{aligned}$$

where

$$B := r_V[x_{mi}(1 + r_U) + 1] + r_U x_{mi} - 2x_{mi}(x_{mi} + 1) \quad (\text{F35})$$

is a polynomial in x_{mi} , r_V , and r_U . The first term in Eq. (F34) is odd in t_1 and integrates to zero over $[-1, 1]$. Therefore,

$$\begin{aligned} & \int_{-1}^1 dt_1 \frac{A_2 t_1^2 + A_1 t_1 + A_0}{D(t_1)} \\ &= \frac{2(2x_{mi} + 1)}{1 - r_U} + \frac{2B}{1 - r_U} \int_{-1}^1 \frac{dt_1}{D_0 - D_1 t_1}. \quad (\text{F36}) \end{aligned}$$

The remaining integral is

$$\int \frac{dt_1}{D_0 - D_1 t_1} = -\frac{1}{D_1} \ln(D_0 - D_1 t_1), \quad (\text{F37})$$

and thus

$$\begin{aligned} \int_{-1}^1 \frac{dt_1}{D_0 - D_1 t_1} &= \frac{1}{D_1} \ln \frac{D_0 + D_1}{D_0 - D_1} \\ &= \frac{1}{(1 - r_V)(1 - r_U)} \ln \frac{D_0 + D_1}{D_0 - D_1}. \quad (\text{F38}) \end{aligned}$$

Using Eq. (F33),

$$\begin{aligned} D_0 + D_1 &= 2(x_{mi} + 1 - r_V - r_U), \\ D_0 - D_1 &= 2(x_{mi} - r_V r_U), \quad (\text{F39}) \end{aligned}$$

so

$$\ln \frac{D_0 + D_1}{D_0 - D_1} = \ln \frac{x_{mi} + 1 - r_V - r_U}{x_{mi} - r_V r_U}. \quad (\text{F40})$$

This yields

$$\begin{aligned} & \int_{-1}^1 dt \frac{A_2 t_1^2 + A_1 t_1 + A_0}{D(t_1)} \\ &= \frac{2(2x_{mi} + 1)}{1 - r_U} + \frac{2B}{(1 - r_V)(1 - r_U)^2} \\ & \quad \times \ln \frac{x_{mi} + 1 - r_V - r_U}{x_{mi} - r_V r_U}, \quad (\text{F41}) \end{aligned}$$

and hence

$$\begin{aligned} \text{Disc} J_{\text{vert}}^{(a)}(V; U) &= \frac{2i}{\pi} M_i^6 \sqrt{x_{mi}} \left[(1 - r_U)(2x_{mi} + 1) + \frac{B}{1 - r_V} \right. \\ & \quad \left. \times \ln \frac{x_{mi} + 1 - r_V - r_U}{x_{mi} - r_V r_U} \right]. \quad (\text{F42}) \end{aligned}$$

Thus, we obtain the discontinuity relevant to the vertex contribution from diagram (a).

2. Vertex contribution (b)

The discontinuity $J_{\text{vert}}^{(b)}$ is given by putting q_1 and q_2 on shell and carrying out the phase-space integral:

$$\begin{aligned} & \text{Disc} J_{\text{vert}}^{(b)}(V; U) \\ &= \text{Disc} \int \frac{d^4 q_1}{(2\pi)^4} \frac{\mathcal{T}_{(b)}|_{\text{cut}}}{(q_3^2 - M_m^2 + i\epsilon)(q_1^2 + i\epsilon)(q_2^2 - m_U^2 + i\epsilon)} \\ &= 4i M_i^8 r_V r_U (1 - r_V)(1 - r_U) \\ & \quad \times \int \frac{d^4 q_1}{(2\pi)^4} \frac{(1 - t_1) - 2(r_V + r_U)(1 + t_1)}{-\frac{M_i^2}{2}(D_0 - D_1 t_1)} \\ & \quad \times (-2\pi i)^2 \delta(q_1^2) \Theta(E_1) \delta(q_2^2 - m_U^2) \Theta(E_2). \quad (\text{F43}) \end{aligned}$$

Here the angular integral involves the denominator

$$\begin{aligned} q_3^2 - M_m^2 &= -\frac{M_i^2}{2}(D_0 - D_1 t_1), \\ D_0 &= 2x + 1 - r_V - r_U - r_V r_U, \\ D_1 &= (1 - r_V)(1 - r_U). \quad (\text{F44}) \end{aligned}$$

The angular integral appearing in the absorptive part is

$$\begin{aligned} \text{Disc} J_{\text{vert}}^{(b)}(V; U) &= -\frac{2i}{\pi} M_i^8 r_V r_U (1 - r_V)(1 - r_U) \\ & \quad \times \int dE_1 E_1^2 \frac{1}{2E_1} \frac{\delta(E_1 - E_1^*)}{2M_i} \Theta(M_i - E_1) \\ & \quad \times \int_{-1}^1 dt_1 \frac{(1 - t_1) - 2(r_V + r_U)(1 + t_1)}{-\frac{M_i^2}{2}(D_0 - D_1 t_1)}, \\ &= \frac{i}{2\pi} M_i^6 r_V r_U (1 - r_V)(1 - r_U)^2 \\ & \quad \times \int_{-1}^1 dt_1 \frac{(1 - t_1) - 2(r_V + r_U)(1 + t_1)}{D_0 - D_1 t_1} \\ &= \frac{i}{2\pi} M_i^6 r_V r_U (1 - r_U) \mathcal{I}_{(b)}(x; r_V, r_U), \quad (\text{F45}) \end{aligned}$$

where $E_1^* := \frac{M_i}{2}(1 - r_U)$, and therefore

$$\begin{aligned} \mathcal{I}_{(b)}(x; r_V, r_U) &:= (1 - r_V)(1 - r_U) \\ &\times \int_{-1}^1 dt_1 \frac{1 - t_1 - 2(r_V + r_U)(1 + t_1)}{D_0 - D_1 t_1}. \end{aligned} \quad (\text{F46})$$

This integral can be expressed in closed form:

$$\begin{aligned} \mathcal{I}_{(b)}(x; r_V, r_U) &= 2(2r_U + 2r_V + 1) \\ &+ \frac{2A(x; r_V, r_U)}{(1 - r_U)(1 - r_V)} \ln \left(\frac{x - r_V r_U}{x + 1 - r_V - r_U} \right), \end{aligned} \quad (\text{F47})$$

with

$$\begin{aligned} A(x; r_V, r_U) &:= x(2r_U + 2r_V + 1) + 2(r_U + r_V) \\ &- 2(r_U^2 + r_V^2) - 5r_U r_V. \end{aligned} \quad (\text{F48})$$

Hence, we obtain

$$\begin{aligned} \text{Disc} J_{\text{vert}}^{(b)}(V; U) &= \frac{i}{\pi} M_i^6 r_V r_U \left[(1 - r_U)(2r_V + 2r_U + 1) \right. \\ &+ \frac{2(r_V + r_U)(x + 1) + x - 2(r_V^2 + r_U^2) - 5r_V r_U}{1 - r_V} \\ &\times \ln \left(\frac{x - r_V r_U}{x + 1 - r_V - r_U} \right) \left. \right]. \end{aligned} \quad (\text{F49})$$

3. Self-energy contribution (a)

For diagram type (a) in Fig. 21, we keep the self-energy bubble explicit. Let q_1 be the loop momentum of the internal lepton line and define $q_2 = k - q_1$ as the momentum of the internal gauge boson U . Then the amplitude can be written as

$$\begin{aligned} i\mathcal{M}_{\text{self}}^{(a)}(V; U) &= \sum_{m \neq i} \sum_{\beta} \int \frac{d^4 q_1}{(2\pi)^4} \\ &\times \bar{u}_{\alpha}(p)(V^V)_{\alpha m}^{\rho}(q) S_{N_m}(k)(V^U)_{\beta m}^{\mu}(-q_2) \\ &\times S_{L_{\beta}}(-q_1)(\tilde{V}^U)_{\beta i}^{\nu}(q_2) u_i(k) D_{\mu\nu}^U(q_2) \epsilon_{\rho}^*(q). \end{aligned} \quad (\text{F50})$$

Here $U = \gamma, Z, W$ labels the gauge boson running in the lepton bubble. For $U = \gamma, Z$ the internal lepton is $L_{\beta} = \nu_{\beta}$, while for $U = W$ it is $L_{\beta} = \ell_{\beta}$.

The absorptive part of the self-energy bubble is obtained by cutting the internal lepton and gauge-boson propagators. The Cutkosky rules give

$$\begin{aligned} \text{Disc} \left(\frac{1}{q_1^2 + i\epsilon} \right) &= -2\pi i \delta(q_1^2) \Theta(q_1^0), \\ \text{Disc} \left(\frac{1}{q_2^2 - m_U^2 + i\epsilon} \right) &= -2\pi i \delta(q_2^2 - m_U^2) \Theta(q_2^0), \end{aligned} \quad (\text{F51})$$

with $q_2 = k - q_1$ and $k^2 = M_i^2$. The resulting δ -function integration reduces to the two-body phase space of the on-shell intermediate state $L_{\beta}(q_1) + U(q_2)$ and yields

$$\begin{aligned} &\int \frac{d^4 q_1}{(2\pi)^4} (2\pi)^2 \delta(q_1^2) \delta(q_2^2 - m_U^2) \Theta(q_1^0) \Theta(q_2^0) \mathcal{F}(t_1) \\ &= \frac{1 - r_U}{16\pi} \int_{-1}^1 dt_1 \mathcal{F}(t_1), \end{aligned} \quad (\text{F52})$$

with $t_1 := \cos \theta_1 := \hat{\mathbf{q}} \cdot \hat{\mathbf{q}}_1$. For the present self-energy topologies, the t_1 dependence drops out after angular averaging, as shown below.

After the Dirac trace and the sum over the internal flavor index β , the absorptive part is proportional to the coupling contractions

$$(\mu^{U\dagger} \mu^U)_{im} := \sum_{\beta} \mu_{\beta i}^{U*} \mu_{\beta m}^U, \quad (\mu^{U\dagger} \mu^U)_{mi} = \sum_{\beta} \mu_{\beta m}^{U*} \mu_{\beta i}^U. \quad (\text{F53})$$

The genuinely new ingredient induced by $\mathcal{O}_{NW}$ is the charged-current contribution with $U = W$, that is, $(\mu^{W\dagger} \mu^W)_{im}, (\mu^{W\dagger} \mu^W)_{mi}$.

Next, we define the interference term for diagram type (a) as

$$I_{\text{self}}^{(a)}(V; U) := \frac{1}{2} \sum_{\text{spins, pol}} (i\mathcal{M}_V^{(0)})^{\dagger} i\mathcal{M}_{\text{self}}^{(a)}(V; U). \quad (\text{F54})$$

Since the self-energy bubble depends only on the external momentum k , the propagator denominator of the intermediate heavy neutrino N_m is

$$k^2 - M_m^2 = M_i^2 - M_m^2 = M_i^2(1 - x_{mi}), \quad (\text{F55})$$

where $x_{mi} := M_m^2/M_i^2$, and is independent of the angular variable t_1 . Consequently, after taking the Cutkosky cut the loop integration reduces to the two-body phase-space factor and the angular average of the cut numerator.

Carrying out the same trace algebra as in Sec. IVA for $r_U \neq 0$, we obtain

$$\begin{aligned} \mathcal{N}_{\text{self}}^{(a)}|_{\text{cut}} &= M_i^7 M_m (1 - r_U)^2 \\ &\times \left[\frac{(r_U + 1)(r_U + 2)}{2} + (\text{terms odd in } t_1) \right], \end{aligned} \quad (\text{F56})$$

and the odd terms vanish upon integration over $t_1 \in [-1, 1]$, giving

$$\int_{-1}^1 dt_1 \mathcal{N}_{\text{self}}^{(a)} \Big|_{\text{cut}} = M_i^7 M_m (1 - r_U)^2 (r_U + 1)(r_U + 2). \quad (\text{F57})$$

Combining this with the phase-space factor yields the discontinuity

$$\text{Disc} I_{\text{self}}^{(a)}(V; U) = \frac{i M_i^7 M_m}{\pi (M_i^2 - M_m^2)} (1 - r_U)^2 (r_U + 1)(r_U + 2). \quad (\text{F58})$$

It is convenient to express the corresponding contribution to the channel-normalized CP asymmetry in terms of a loop function. Defining

$$F_{S_a}(x; r_U) := \frac{\sqrt{x}}{1-x} \cdot \frac{(1 - r_U)^3 (r_U + 1)(r_U + 2)}{2}, \quad (\text{F59})$$

which reduces to $f_{S_a}(x; 0) = \sqrt{x}/(1-x)$ for $r_U \rightarrow 0$, so we write

$$\begin{aligned} \epsilon_{\text{self}, ai}^{V, (a)}(U) = & -\frac{M_i^2}{2\pi R_V \sum_{\beta} |\mu_{\beta i}^V|^2} \\ & \times \sum_{m \neq i} \text{Im}[\mu_{ai}^{V*} \mu_{am}^V (\mu^{U\dagger} \mu^U)_{im}] F_{S_a}(x_{mi}; r_U), \end{aligned} \quad (\text{F60})$$

with $R_V = (1 - r_V)^2 (1 + r_V)$ and $x_{mi} = M_m^2/M_i^2$.

4. Self-energy contribution (b)

Diagram type (b) in Fig. 21 corresponds to the alternative contraction of the dipole vertices in the self-energy loop, which interchanges the coupling contraction and yields $(\mu^{U\dagger} \mu^U)_{mi}$. Repeating the same cut integration for $r_U \neq 0$, one obtains

$$F_{S_b}(x; r_U) := \frac{1}{1-x} \cdot \frac{(1 - r_U)^3 (r_U + 2)}{2}, \quad (\text{F61})$$

which reduces to $f_{S_b}(x; 0) = 1/(1-x)$ for $r_U \rightarrow 0$. Accordingly,

$$\begin{aligned} \epsilon_{\text{self}, ai}^{V, (b)}(U) = & -\frac{M_i^2}{2\pi R_V \sum_{\beta} |\mu_{\beta i}^V|^2} \\ & \times \sum_{m \neq i} \text{Im}[\mu_{ai}^{V*} \mu_{am}^V (\mu^{U\dagger} \mu^U)_{mi}] F_{S_b}(x_{mi}; r_U). \end{aligned} \quad (\text{F62})$$

5. Quasidegenerate limit

In the quasidegenerate regime, the x -dependent factors in Eqs. (F59) and (F61) are replaced as in Eq. (140). The loop functions then become

$$\begin{aligned} \tilde{F}_{S_a}(x; r_U) & := \frac{\sqrt{x}(1-x)}{(1-x)^2 + (\Gamma_m/M_i)^2} \cdot \frac{(1 - r_U)^3 (r_U + 1)(r_U + 2)}{2}, \end{aligned} \quad (\text{F63})$$

$$\begin{aligned} \tilde{F}_{S_b}(x; r_U) & := \frac{1-x}{(1-x)^2 + (\Gamma_m/M_i)^2} \cdot \frac{(1 - r_U)^3 (r_U + 2)}{2}, \end{aligned} \quad (\text{F64})$$

where we use the vacuum width Γ_m in the regulator functions $\tilde{F}_{S_{a,b}}$. Throughout we use these width-regulated loop functions in order to capture the resonant enhancement of the self-energy contribution.

APPENDIX G: DETAILS OF THE SCHWINGER-KELDYSH AND 2PI FORMALISM

In this appendix we collect the formal details underlying the compact summary in Sec. VI. We review the one- and two-particle-irreducible effective actions, the CTP generating functional, the Kadanoff-Baym equations, the Wigner transform [72], the leading-gradient expansion, and the Markov/quasiparticle limits [100–102]. These results are standard [66], but we include them to fix conventions and to make the derivation of the density-matrix QKEs reproducible. The main text relies only on the final Wightman-function form of the collision terms and on the projected QKEs.

1. n -particle-irreducible generating functionals

We consider a quantum field theory for a real N -component scalar field φ_a ($a = 1, \dots, N$) with classical action $S[\varphi]$:

$$\begin{aligned} S[\varphi] = & \int_x \left[\frac{1}{2} \partial_\mu \varphi_a(x) \partial^\mu \varphi_a(x) - \frac{m^2}{2} \varphi_a(x) \varphi_a(x) \right. \\ & \left. - \frac{\lambda}{4!N} (\varphi_a(x) \varphi_a(x))^2 \right]. \end{aligned} \quad (\text{G1})$$

Here and in the following, repeated indices are summed. We use $x := (x_0, \mathbf{x})$ and the shorthand notation $\int_x := \int_{\mathcal{C}} dx^0 \int d^d x$, with a time contour $\mathcal{C}$ to be specified below. Accordingly, δ functions such as $\delta^4(x - y)$ should be understood as the contour $\delta_{\mathcal{C}}(x^0 - y^0) \delta^3(\mathbf{x} - \mathbf{y})$.

In the presence of two source terms $\sim J_a(x)$ and $\sim R_{ab}(x, y)$, the partition function $Z[J, R]$ and the generating functional $W[J, R]$ of connected Green's functions are defined by

$$\begin{aligned}
Z[J, R] &= e^{iW[J, R]} \\
&= \int \mathcal{D}\varphi \exp \left\{ i \left[S[\varphi] + \int_x J_a(x) \varphi_a(x) \right. \right. \\
&\quad \left. \left. + \frac{1}{2} \int_{xy} R_{ab}(x, y) \varphi_a(x) \varphi_b(y) \right] \right\}. \quad (G2)
\end{aligned}$$

The macroscopic field ϕ_a and the connected two-point function G_{ab} are defined as functional derivatives of W :

$$\begin{aligned}
\frac{\delta W[J, R]}{\delta J_a(x)} &= \phi_a(x), \\
\frac{\delta W[J, R]}{\delta R_{ab}(x, y)} &= \frac{1}{2} [\phi_a(x) \phi_b(y) + G_{ab}(x, y)]. \quad (G3)
\end{aligned}$$

Before constructing the 2PI effective action, we consider the 1PI effective action. It is obtained by a Legendre transformation with respect to the source term linear in the field (at fixed R):

$$\begin{aligned}
\Gamma^R[\phi] &= W[J, R] - \int_x \frac{\delta W[J, R]}{\delta J_a(x)} J_a(x) \\
&= W[J, R] - \int_x \phi_a(x) J_a(x). \quad (G4)
\end{aligned}$$

We note that:

- (i) $\Gamma^{R=0}[\phi]$ coincides with the standard 1PI effective action.
- (ii) For $R \neq 0$, $\Gamma^R[\phi]$ can be viewed as the 1PI effective action of a theory governed by the modified classical action

$$S^R[\varphi] = S[\varphi] + \frac{1}{2} \int_{xy} R_{ab}(x, y) \varphi_a(x) \varphi_b(y), \quad (G5)$$

i.e., in the presence of a nonconstant “mass term” quadratic in the fields.

As a consequence, relations for the 1PI effective action carry over to $\Gamma^R[\phi]$ with the replacements $S \rightarrow S^R$ and $G_0^{-1} \rightarrow G_0^{-1} - iR$. For instance, the one-loop expression reads

$$\Gamma^{R(1\text{-loop})}[\phi] = S^R[\phi] + \frac{i}{2} \text{Tr} \ln[G_0^{-1}(\phi) - iR], \quad (G6)$$

where the functional trace Tr includes integration over time and spatial coordinates and summation over internal indices. Similarly, the exact inverse propagator is obtained by a second functional derivative,

$$\begin{aligned}
\frac{\delta^2 \Gamma^R[\phi]}{\delta \phi_a(x) \delta \phi_b(y)} &= iG_{ab}^{-1}(x, y) \\
&= i \left[G_{0,ab}^{-1}(x, y; \phi) - iR_{ab}(x, y) - \Sigma_{ab}^R(x, y; \phi) \right]. \quad (G7)
\end{aligned}$$

Here the classical inverse propagator $iG_{0,ab}^{-1}(x, y; \phi) = \delta^2 S[\phi] / \delta \phi_a(x) \delta \phi_b(y)$ is

$$\begin{aligned}
iG_{0,ab}^{-1}(x, y; \phi) &= - \left[\square_x + m^2 + \frac{\lambda}{6N} \phi_c(x) \phi_c(x) \right] \delta_{ab} \delta^4(x - y) \\
&\quad - \frac{\lambda}{3N} \phi_a(x) \phi_b(x) \delta^4(x - y), \quad (G8)
\end{aligned}$$

and $\Sigma_{ab}^R(x, y; \phi)$ denotes the proper self-energy, to which only 1PI Feynman diagrams contribute (i.e. diagrams that cannot be disconnected by cutting a single internal line). These relations will be used below.

To obtain the 2PI effective action, we perform a further Legendre transformation of $\Gamma^R[\phi]$ with respect to the bilocal source R :

$$\begin{aligned}
\Gamma[\phi, G] &= \Gamma^R[\phi] - \int_{xy} \frac{\delta \Gamma^R[\phi]}{\delta R_{ab}(x, y)} R_{ba}(y, x) \\
&= \Gamma^R[\phi] - \frac{1}{2} \int_{xy} R_{ab}(x, y) \phi_a(x) \phi_b(y) - \frac{1}{2} \text{Tr} RG. \quad (G9)
\end{aligned}$$

Here we used Eqs. (G3) and (G4). Since the relation between ϕ and J is R -dependent, i.e., inverting $\phi = \delta W[J, R] / \delta J$ yields $J = J^R(\phi)$, one finds from the definition (G4) that

$$\begin{aligned}
\frac{\delta \Gamma^R[\phi]}{\delta R_{ab}(x, y)} &= \frac{\delta W[J, R]}{\delta R_{ab}(x, y)} + \int_z \frac{\delta W[J, R]}{\delta J_c(z)} \frac{\delta J_c(z)}{\delta R_{ab}(x, y)} \\
&\quad - \int_z \phi_c(z) \frac{\delta J_c(z)}{\delta R_{ab}(x, y)} = \frac{\delta W[J, R]}{\delta R_{ab}(x, y)}. \quad (G10)
\end{aligned}$$

Of course, the two successive Legendre transforms employed above agree with a simultaneous Legendre transform of $W[J, R]$ with respect to both source terms:

$$\begin{aligned}
\Gamma[\phi, G] &= W[J, R] - \int_x \frac{\delta W[J, R]}{\delta J_a(x)} J_a(x) \\
&\quad - \int_{xy} \frac{\delta W[J, R]}{\delta R_{ab}(x, y)} R_{ab}(x, y) \\
&= W[J, R] - \int_x \phi_a(x) J_a(x) \\
&\quad - \frac{1}{2} \int_{xy} R_{ab}(x, y) \phi_a(x) \phi_b(y) - \frac{1}{2} \text{Tr} GR \\
&= \Gamma^R[\phi] - \frac{1}{2} \int_{xy} R_{ab}(x, y) \phi_a(x) \phi_b(y) - \frac{1}{2} \text{Tr} GR.
\end{aligned} \tag{G11}$$

From this, the stationarity conditions follow:

$$\frac{\delta \Gamma[\phi, G]}{\delta \phi_a(x)} = -J_a(x) - \int_y R_{ab}(x, y) \phi_b(y), \tag{G12}$$

$$\frac{\delta \Gamma[\phi, G]}{\delta G_{ab}(x, y)} = -\frac{1}{2} R_{ab}(x, y). \tag{G13}$$

In the absence of sources, $J = 0$ and $R = 0$, these give the equations of motion for ϕ and G .

To get familiar with Eqs. (G9)–(G11), we compute $\Gamma[\phi, G]$ to one-loop order using the above result for $\Gamma^R[\phi]$. Plugging Eq. (G6) into Eq. (G9) yields

$$\Gamma[\phi, G] \simeq S[\phi] + \frac{i}{2} \text{Tr} \ln[G_0^{-1}(\phi) - iR] - \frac{1}{2} \text{Tr} RG. \tag{G14}$$

Defining $G^{-1} := G_0^{-1}(\phi) - iR$, we can rewrite this as

$$\Gamma[\phi, G] \simeq S[\phi] + \frac{i}{2} \text{Tr} \ln G^{-1} + \frac{i}{2} \text{Tr} G_0^{-1}(\phi)G + \text{const}. \tag{G15}$$

Indeed, $\text{Tr} G^{-1}G = \text{Tr} \mathbf{1}$ is a field-independent constant. Using Eq. (G15) together with the stationarity condition (G13), one verifies at this order that

$$\begin{aligned}
\frac{\delta \Gamma[\phi, G]}{\delta G} &\simeq -\frac{i}{2} G^{-1} + \frac{i}{2} G_0^{-1}(\phi) = -\frac{1}{2} R \\
&\Rightarrow G^{-1} = G_0^{-1}(\phi) - iR.
\end{aligned} \tag{G16}$$

Beyond one-loop order, it is convenient to write the exact $\Gamma[\phi, G]$ as a one-loop-type expression plus a remainder:

$$\begin{aligned}
\Gamma[\phi, G] &= S[\phi] + \frac{i}{2} \text{Tr} \ln G^{-1} + \frac{i}{2} \text{Tr} G_0^{-1}(\phi)G \\
&\quad + \Gamma_2[\phi, G] + \text{const}.
\end{aligned} \tag{G17}$$

Here “const.” denotes an irrelevant normalization constant. Varying $\Gamma[\phi, G]$ with respect to G yields

$$G_{ab}^{-1}(x, y) = G_{0,ab}^{-1}(x, y; \phi) - iR_{ab}(x, y) - \Sigma_{ab}(x, y; \phi, G), \tag{G18}$$

with the 1PI self-energy defined by

$$\Sigma_{ab}(x, y; \phi, G) := 2i \frac{\delta \Gamma_2[\phi, G]}{\delta G_{ab}(x, y)}. \tag{G19}$$

Comparing Eqs. (G7) and (G18) shows that Σ_{ab} coincides with the 1PI self-energy in the presence of R , with the understanding that R is eliminated in favor of G via Eq. (G18) at the stationary point.

Defining the “classical” propagator

$$G_{\text{cl}} := (G_0^{-1} - iR)^{-1}, \tag{G20}$$

one has an infinite Dyson series:

$$\begin{aligned}
G &= (G_0^{-1} - iR)^{-1} + (G_0^{-1} - iR)^{-1} \Sigma (G_0^{-1} - iR)^{-1} \\
&\quad + (G_0^{-1} - iR)^{-1} \Sigma (G_0^{-1} - iR)^{-1} \Sigma (G_0^{-1} - iR)^{-1} \\
&\quad + \dots
\end{aligned} \tag{G21}$$

Rewriting this in terms of G_{cl} yields

$$G = G_{\text{cl}} + G_{\text{cl}} \Sigma G_{\text{cl}} + G_{\text{cl}} \Sigma G_{\text{cl}} \Sigma G_{\text{cl}} + \dots \tag{G22}$$

Thus, a single full propagator G already represents the resummation of arbitrarily many self-energy insertions on top of G_{cl} . Consequently, any 1PI diagram drawn with full lines G can be expanded uniquely by replacing each full line with its Dyson series (G22), which generates an infinite family of ordinary 1PI diagrams built from G_{cl} and insertions of Σ along propagator lines. Conversely, resumming those insertions reconstructs the compact representation in terms of the full propagator G .

The definition

$$\Sigma_{ab}(x, y; \phi, G) := 2i \frac{\delta \Gamma_2[\phi, G]}{\delta G_{ab}(x, y)} \tag{G23}$$

has a direct diagrammatic meaning: diagrams contributing to Σ are obtained by opening one propagator line in diagrams contributing to Γ_2 . Since Σ is a 1PI self-energy, Γ_2 cannot contain any one-particle-reducible (1PR) structure. This immediately excludes two-particle-reducible (2PR) contributions from Γ_2 . Indeed, suppose Γ_2 contained a 2PR term of the schematic form $\tilde{\Gamma} G G \tilde{\Gamma}'$, i.e., two propagator lines connecting two subdiagrams $\tilde{\Gamma}$ and $\tilde{\Gamma}'$. In coordinate space (suppressing internal indices for clarity) this may be written as

$$\Gamma_2^{(2\text{PR})}[G] \sim \int dx_1 dx_2 dy_1 dy_2 \times \tilde{\Gamma}(x_1, x_2) G(x_1, y_1) G(x_2, y_2) \tilde{\Gamma}(y_1, y_2). \quad (\text{G24})$$

Taking a functional derivative with respect to the bilocal kernel $G(u, v)$ and using

$$\frac{\delta G(x, y)}{\delta G(u, v)} = \delta^4(x - u) \delta^4(y - v), \quad (\text{G25})$$

one finds that the derivative hits either of the two propagator lines:

$$\begin{aligned} \frac{\delta \Gamma_2^{(2\text{PR})}[G]}{\delta G(u, v)} &\sim \int dx_1 dx_2 dy_1 dy_2 \\ &\times \tilde{\Gamma}(x_1, x_2) \frac{G(x_1, y_1)}{G(u, v)} G(x_2, y_2) \tilde{\Gamma}'(y_1, y_2) \\ &+ \int dx_1 dx_2 dy_1 dy_2 \\ &\times \tilde{\Gamma}(x_1, x_2) G(x_1, y_1) \frac{\delta G(x_2, y_2)}{\delta G(u, v)} \tilde{\Gamma}'(y_1, y_2) \\ &\sim \int dx_2 dy_2 \tilde{\Gamma}(u, x_2) G(x_2, y_2) \tilde{\Gamma}'(v, y_2) \\ &+ \int dx_1 dy_1 \tilde{\Gamma}(x_1, u) G(x_1, y_1) \tilde{\Gamma}'(y_1, v). \end{aligned} \quad (\text{G26})$$

Each of the two terms in Eq. (G26) has the schematic form $\tilde{\Gamma}G\tilde{\Gamma}'$: the two subdiagrams are now connected by a single propagator line, so the resulting contribution is 1PR and hence cannot appear in a 1PI self-energy. Therefore, Γ_2 cannot contain any 2PR term, and $\Gamma_2[\phi, G]$ consists solely of 2PI diagrams.

2. 2PI effective action for fermions

The construction of the 2PI effective action for fermionic fields proceeds along very similar lines as for bosons, with the crucial difference that one has to account for the anti-commuting (Grassmann) nature of fermion fields.

The main difference is already visible in the one-loop part (corresponding to $\Gamma_2 = 0$). For vanishing field expectation values, one encounters the Gaussian functional integrals [103]

$$\begin{aligned} -i \ln \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS_0^{(f)}} &= -i \ln(\det \Delta_0^{-1}) \\ &= -i \text{Tr} \ln \Delta_0^{-1}, \\ -i \ln \int \mathcal{D}\varphi e^{iS_0^{(b)}} &= -i \ln(\det G_0^{-1})^{-1/2} \\ &= \frac{i}{2} \text{Tr} \ln G_0^{-1}. \end{aligned} \quad (\text{G27})$$

Here

$$S_0^{(f)}[\psi] = \int d^4x d^4y \bar{\psi}(x) i\Delta_0^{-1}(x, y) \psi(y) \quad (\text{G28})$$

denotes a fermion action bilinear in the Grassmann fields. For a Dirac fermion with mass $m^{(f)}$, the free inverse propagator is

$$i\Delta_0^{-1}(x, y) = (i\partial_x - m^{(f)})\delta^4(x - y). \quad (\text{G29})$$

For bosons, $S_0^{(b)}$ is given by the quadratic part of Eq. (G1), i.e.,

$$\begin{aligned} S_0^{(b)}[\varphi] &= \frac{1}{2} \int d^4x d^4y \varphi(x) iG_0^{-1}(x, y) \varphi(y), \\ iG_0^{-1}(x, y) &= -(\square_x + m^2)\delta^4(x - y). \end{aligned} \quad (\text{G30})$$

Comparing the two integrals in Eq. (G27), one observes that the factor $1/2$ for bosons is replaced by -1 for fermions due to the anti-commuting (Grassmann) nature. Taking this difference into account, one can follow the same steps as in the bosonic case to obtain the 2PI effective action for fermions in complete analogy to Eq. (G17).

Accordingly, for vanishing fermion field expectation values, $\langle \Psi \rangle = \langle \bar{\Psi} \rangle = 0$, one finds

$$\Gamma[\Delta] = -i \text{Tr} \ln \Delta^{-1} - i \text{Tr} \Delta_0^{-1} \Delta + \Gamma_2[\Delta] + \text{const.} \quad (\text{G31})$$

Here $\Gamma_2[\Delta]$ contains all 2PI diagrams with lines associated to the time-ordered propagator $\Delta(x, y) = \langle T\Psi(x)\bar{\Psi}(y) \rangle$. As for the 1PI effective action, each closed fermion loop contributes an additional minus sign.

As in the bosonic case, the equation of motion for Δ in the absence of external sources is obtained by imposing the stationarity condition of the effective action:

$$\frac{\delta \Gamma[\Delta]}{\delta \Delta(x, y)} = 0. \quad (\text{G32})$$

Using Eq. (G31), this condition can be rewritten as

$$\Delta^{-1}(x, y) = \Delta_0^{-1}(x, y) - \Sigma^{(f)}(x, y; \Delta), \quad (\text{G33})$$

with the 1PI fermion self-energy defined by

$$\Sigma^{(f)}(x, y; \Delta) := -i \frac{\delta \Gamma_2[\Delta]}{\delta \Delta(y, x)}. \quad (\text{G34})$$

APPENDIX H: SCATTERING CROSS SECTIONS AND REACTION DENSITIES

In this appendix we collect the details of the $2 \leftrightarrow 2$ scattering calculation used in Sec. VII. We derive the neutral-current processes $N_i f \leftrightarrow \nu_\alpha f$, including γ

exchange, Z exchange, and γZ interference, as well as the charged-current processes $N_i f \leftrightarrow \ell_a f'$ mediated by $W^\pm$. The main text uses only the resulting reaction densities and the scattering equilibration function $S_i(z)$.

1. Explicit calculation I: Neutral-current (NC) scattering

We consider the $2 \rightarrow 2$ processes

$$N_i(p) + f(k) \rightarrow \nu_a(p') + f(k') \quad (\text{H1})$$

including t -channel γ exchange, Z exchange, and γZ interference, from first principles.

2. Feynman rules and kinematics

For an outgoing gauge-boson momentum q , the dipole vertex and its conjugate vertex for $V = \gamma, Z$ are

$$\begin{aligned} (V^V)_{ai}^\rho(q) &= 2\mu_{ai}^V \sigma^{\rho\lambda} q_\lambda P_R, \\ (\tilde{V}^V)_{ai}^\rho(q) &= 2\mu_{ai}^{V*} \sigma^{\rho\lambda} q_\lambda P_L, \end{aligned} \quad (V = \gamma, Z). \quad (\text{H2})$$

The gauge interactions of f are

$$\begin{aligned} -\mathcal{L} \supset & eQ_f A_\mu \bar{f} \gamma^\mu f \\ & + \frac{e}{\sin \theta_W \cos \theta_W} Z_\mu \bar{f} \gamma^\mu (g_L^f P_L + g_R^f P_R) f, \end{aligned} \quad (\text{H3})$$

with Q_f the electric charge in proton units, and

$$g_L^f = T_3^f - Q_f \sin^2 \theta_W, \quad g_R^f = -Q_f \sin^2 \theta_W. \quad (\text{H4})$$

3. Mandelstam variables and inner products

Define

$$q := k' - k = p - p', \quad t := q^2 < 0, \quad s := (p + k)^2. \quad (\text{H5})$$

Using the massless approximation $k^2 = k'^2 = p'^2 = 0$ and $p^2 = M_i^2$,

$$\begin{aligned} s &= (p + k)^2 = p^2 + 2p \cdot k + k^2 = M_i^2 + 2p \cdot k, \\ t &= (p - p')^2 = p^2 - 2p \cdot p' + p'^2 = M_i^2 - 2p \cdot p', \\ u &= (p - k')^2 = p^2 - 2p \cdot k' + k'^2 = M_i^2 - 2p \cdot k'. \end{aligned} \quad (\text{H6})$$

Hence,

$$p \cdot k = \frac{s - M_i^2}{2}. \quad (\text{H7})$$

Squaring momentum conservation $p + k = p' + k'$ gives

$$p' \cdot k' = \frac{s}{2}. \quad (\text{H8})$$

From $u = (p - k')^2$ we obtain

$$p \cdot k' = \frac{M_i^2 - u}{2}. \quad (\text{H9})$$

Eliminating u using $s + t + u = M_i^2$ yields

$$p \cdot k' = \frac{s + t}{2}. \quad (\text{H10})$$

Finally, using $u = (p' - k)^2 = -2p' \cdot k$ and $s + t + u = M_i^2$ gives

$$p' \cdot k = \frac{s + t - M_i^2}{2}. \quad (\text{H11})$$

Defining

$$\Delta := s - M_i^2 > 0, \quad (\text{H12})$$

the t integration range is

$$t \in [t_{\min}, t_{\max}], \quad t_{\min} = M_i^2 - s = -\Delta, \quad t_{\max} = 0. \quad (\text{H13})$$

4. Tree-level amplitudes

The t -channel photon exchange amplitude is

$$\mathcal{M}_\gamma = \frac{2eQ_f \mu_{ai}^\gamma}{t} [\bar{u}_\alpha(p') \sigma^{\mu\nu} q_\nu P_R u_i(p)] [\bar{u}_f(k') \gamma_\mu u_f(k)]. \quad (\text{H14})$$

Similarly, the Z exchange amplitude is

$$\begin{aligned} \mathcal{M}_Z &= \frac{2e\mu_{ai}^Z}{\sin \theta_W \cos \theta_W} \frac{1}{t - m_Z^2} \\ &\times [\bar{u}_\alpha(p') \sigma^{\mu\nu} q_\nu P_R u_i(p)] \\ &\times [\bar{u}_f(k') \gamma_\mu (g_L^f P_L + g_R^f P_R) u_f(k)]. \end{aligned} \quad (\text{H15})$$

5. Simplifying $\sigma^{\mu\nu} q_\nu$ using current conservation

For a Dirac fermion of mass m_f , the Gordon identity gives

$$\begin{aligned} \bar{u}_f(k') \gamma_\mu u_f(k) &= \bar{u}_f(k') \left[\frac{(k' + k)_\mu}{2m_f} + \frac{i\sigma_{\mu\nu} q^\nu}{2m_f} \right] u_f(k) \\ &=: J_\mu^{(\gamma)}, \end{aligned} \quad (\text{H16})$$

with $q = k' - k$. Then

$$q^\mu J_\mu^{(\gamma)} = \bar{u}_f(k') \left[\frac{q \cdot (k' + k)}{2m_f} + \frac{i q^\mu \sigma_{\mu\nu} q^\nu}{2m_f} \right] u_f(k). \quad (\text{H17})$$

The first term vanishes because $q \cdot (k' + k) = k'^2 - k^2 = m_f^2 - m_f^2 = 0$. The second term vanishes because $q^\mu q^\nu$ is symmetric while $\sigma_{\mu\nu}$ is antisymmetric. Hence,

$$q^\mu J_\mu^{(\gamma)} = 0. \quad (\text{H18})$$

Equivalently, using the Dirac equation,

$$q^\mu \bar{u}_f(k') \gamma_\mu u_f(k) = \bar{u}_f(k') (k' - k) u_f(k) = 0. \quad (\text{H19})$$

Using the identity

$$\sigma^{\mu\nu} q_\nu = \frac{i}{2} (\gamma^\mu \not{q} - \not{q} \gamma^\mu) = i(\gamma^\mu \not{q} - q^\mu), \quad (\text{H20})$$

the photon-exchange amplitude contains the contraction

$$[\bar{u}_\alpha(p') \sigma^{\mu\nu} q_\nu P_R u_i(p)] J_\mu^{(\gamma)}. \quad (\text{H21})$$

Substituting (H20) and (H18) gives

$$\begin{aligned} (\bar{u}_\alpha \sigma^{\mu\nu} q_\nu P_R u_i) J_\mu^{(\gamma)} &= i(\bar{u}_\alpha \gamma^\mu \not{q} P_R u_i) J_\mu^{(\gamma)} \\ &\quad - i(\bar{u}_\alpha P_R u_i) (q^\mu J_\mu^{(\gamma)}) \\ &= i(\bar{u}_\alpha \gamma^\mu \not{q} P_R u_i) J_\mu^{(\gamma)}. \end{aligned} \quad (\text{H22})$$

The second term is purely longitudinal and proportional to $q^\mu J_\mu^{(\gamma)}$. Hence, the longitudinal piece does not contribute, and we may project onto the transverse part by the replacement

$$\sigma^{\mu\nu} q_\nu \rightarrow i\gamma^\mu \not{q}, \quad (\text{H23})$$

inside the squared amplitude.

This also makes the infrared behavior manifest: the photon propagator provides $1/t^2$, while the replacement introduces a factor $\not{q}\not{q} = q^2 = t$ after the Dirac algebra, reducing $1/t^2$ to $1/(-t)$. For the γZ interference, the propagator factor $(1/t) \cdot 1/(t - m_Z^2)$ similarly loses the potentially singular $1/t$ because the same Dirac algebra supplies a factor t , so the interference term is infrared finite.

For the Z current,

$$\begin{aligned} J_\mu^{(Z)} &= \bar{u}_f(k') \gamma_\mu (g_V^f P_L + g_A^f P_R) u_f(k) \\ &= \bar{u} \gamma_\mu (g_V^f - g_A^f \gamma_5) u, \end{aligned} \quad (\text{H24})$$

with $g_V^f = (g_L^f + g_R^f)/2$ and $g_A^f = (g_L^f - g_R^f)/2$. Here the subscripts V, A denote vector and axial couplings. The vector part is conserved as for the photon. The axial part satisfies

$$q^\mu \bar{u}(k') \gamma_\mu \gamma_5 u(k) = 2m_f \bar{u}(k') \gamma_5 u(k), \quad (\text{H25})$$

so in the massless limit $m_f \simeq 0$ we again have $q^\mu J_\mu^{(Z)} \simeq 0$.

6. Spin-averaged squared amplitude: γ , Z , and γZ

We define the kinematic polynomial

$$\begin{aligned} K(s, t) &:= 4[(p \cdot k)(p' \cdot k') + (p \cdot k')(p' \cdot k)] \\ &= 2s^2 + 2st + t^2 - 2M_i^2 s - M_i^2 t, \end{aligned} \quad (\text{H26})$$

and

$$\begin{aligned} \tilde{K}(s, t) &:= K(s, t) + M_i^4 - t^2 \\ &= 2s^2 + 2st - 2M_i^2 s - M_i^2 t + M_i^4. \end{aligned} \quad (\text{H27})$$

a. Photon exchange

We start from the identity (H20) and recall that the photon current of the light fermion f ,

$$J_\mu^{(\gamma)} = \bar{u}_f(k') \gamma_\mu u_f(k), \quad (\text{H28})$$

is conserved for on-shell external fermions of equal mass (in particular in the massless limit), (H18).

The tensor $H^{(\gamma)\mu\nu}$ appearing after spin summation is the spin-averaged current product. Equivalently, using spin sums it can be written as a Dirac trace:

$$\begin{aligned} H^{(\gamma)\mu\nu} &:= \frac{1}{2} \sum_{s, s'} J^{(\gamma)\mu}(s, s') J^{(\gamma)\nu}(s, s')^* \\ &= \frac{1}{2} \text{tr}(k' \gamma^\mu k \gamma^\nu). \end{aligned} \quad (\text{H29})$$

Because $q^\mu J_\mu^{(\gamma)} = 0$ holds for each spin configuration, $H^{(\gamma)\mu\nu}$ inherits transversality:

$$q_\mu H^{(\gamma)\mu\nu} = \frac{1}{2} \sum_{s, s'} [q_\mu J^{(\gamma)\mu}(s, s')] J^{(\gamma)\nu}(s, s')^* = 0, \quad (\text{H30})$$

i.e.,

$$q_\nu H^{(\gamma)\mu\nu} = 0. \quad (\text{H31})$$

Consequently, whenever $\sigma^{\mu\nu} q_\nu$ is contracted with $H_{\mu\nu}^{(\gamma)}$, the longitudinal piece proportional to q^μ in (H20) drops out, and we may implement the transverse projection by (H23).

From (H14), the spin-averaged squared amplitude can be written as

$$\langle |\mathcal{M}_\gamma|^2 \rangle = \frac{4e^2 Q_f^2 |\mu_{ai}^\gamma|^2}{t^2} L_{\mu\nu} H^{(\gamma)\mu\nu}, \quad (\text{H32})$$

where

$$\begin{aligned} L_{\mu\nu} &:= \frac{1}{2} \text{tr}[\not{\epsilon}' \gamma_\mu \not{q} P_R (\not{p} + M_i) \not{q} \gamma_\nu P_L], \\ H^{(\gamma)\mu\nu} &:= \frac{1}{2} \text{tr}(k' \gamma^\mu k \gamma^\nu). \end{aligned} \quad (\text{H33})$$

Using the standard trace identity

$$\text{tr}(a \gamma^\mu b \gamma^\nu) = 4(a^\mu b^\nu + a^\nu b^\mu - g^{\mu\nu} a \cdot b), \quad (\text{H34})$$

we obtain

$$H^{(\gamma)\mu\nu} = 2(k'^\mu k^\nu + k'^\nu k^\mu - g^{\mu\nu} k \cdot k'), \quad (\text{H35})$$

which is manifestly symmetric in $\mu \leftrightarrow \nu$ and consistent with the transversality (H31).

Next, the projector kills the mass term:

$$P_R(\not{p} + M_i)P_L = P_R \not{p} P_L + M_i P_R P_L = \not{p} P_L. \quad (\text{H36})$$

Thus,

$$L_{\mu\nu} = \frac{1}{2} \text{tr}(\not{\epsilon}' \gamma_\mu \not{q} \not{p} P_L \not{q} \gamma_\nu). \quad (\text{H37})$$

Since $H^{(\gamma)\mu\nu}$ is symmetric, terms proportional to $\epsilon^{\mu\nu\rho\sigma}$ vanish upon contraction with $H^{(\gamma)\mu\nu}$, so under the contraction we may set $P_L \rightarrow 1/2$:

$$L_{\mu\nu} \rightarrow \frac{1}{4} \text{tr}(\not{\epsilon}' \gamma_\mu \not{q} \not{p} \not{q} \gamma_\nu). \quad (\text{H38})$$

Using

$$\not{q} \not{p} \not{q} = 2(q \cdot p) \not{q} - q^2 \not{p} = 2(q \cdot p) \not{q} - t \not{p}, \quad (\text{H39})$$

and (H34), we find

$$\begin{aligned} L_{\mu\nu} &= \frac{1}{4} [2(q \cdot p) \text{tr}(\not{\epsilon}' \gamma_\mu \not{q} \gamma_\nu) - t \text{tr}(\not{\epsilon}' \gamma_\mu \not{p} \gamma_\nu)] \\ &= 2(q \cdot p)(p'_\mu q_\nu + p'_\nu q_\mu - g_{\mu\nu} p' \cdot q) \\ &\quad - t(p'_\mu p_\nu + p'_\nu p_\mu - g_{\mu\nu} p' \cdot p). \end{aligned} \quad (\text{H40})$$

Now contract with $H^{(\gamma)\mu\nu}$. Using (H40), we first write

$$\begin{aligned} L_{\mu\nu} H^{(\gamma)\mu\nu} &= 2(q \cdot p) [p'_\mu q_\nu H^{(\gamma)\mu\nu} + p'_\nu q_\mu H^{(\gamma)\mu\nu} - (p' \cdot q) H^{(\gamma)\mu}_\mu] \\ &\quad - t [p'_\mu p_\nu H^{(\gamma)\mu\nu} + p'_\nu p_\mu H^{(\gamma)\mu\nu} - (p' \cdot p) H^{(\gamma)\mu}_\mu]. \end{aligned} \quad (\text{H41})$$

The first two terms in the first bracket vanish by the transversality $q_\mu H^{(\gamma)\mu\nu} = q_\nu H^{(\gamma)\mu\nu} = 0$ in (H31). Moreover, since $H^{(\gamma)\mu\nu}$ is symmetric, the second bracket

simplifies as $p'_\mu p_\nu H^{(\gamma)\mu\nu} + p'_\nu p_\mu H^{(\gamma)\mu\nu} = 2p'^\mu p^\nu H^{(\gamma)}_{\mu\nu}$. Hence,

$$\begin{aligned} L_{\mu\nu} H^{(\gamma)\mu\nu} &= -2(q \cdot p)(p' \cdot q) H^{(\gamma)\mu}_\mu \\ &\quad - t[2p'^\mu p^\nu H^{(\gamma)}_{\mu\nu} - (p' \cdot p) H^{(\gamma)\mu}_\mu]. \end{aligned} \quad (\text{H42})$$

First,

$$H^{(\gamma)\mu}_\mu = 2(2k \cdot k' - 4k \cdot k') = -4k \cdot k'. \quad (\text{H43})$$

Since $t = (k' - k)^2 = -2k \cdot k'$ for $k^2 = k'^2 = 0$,

$$H^{(\gamma)\mu}_\mu = 2t. \quad (\text{H44})$$

Next,

$$\begin{aligned} p'^\mu p^\nu H^{(\gamma)}_{\mu\nu} &= 2[(p \cdot k)(p' \cdot k') + (p \cdot k')(p' \cdot k) \\ &\quad - (p \cdot p')(k \cdot k')]. \end{aligned} \quad (\text{H45})$$

Substituting these, the $(p \cdot p')(k \cdot k')$ terms cancel, and we obtain

$$\begin{aligned} L_{\mu\nu} H^{(\gamma)\mu\nu} &= -4t(q \cdot p)(p' \cdot q) \\ &\quad - 4t[(p \cdot k)(p' \cdot k') + (p \cdot k')(p' \cdot k)]. \end{aligned} \quad (\text{H46})$$

Using

$$\begin{aligned} q \cdot p &= p \cdot (p - p') = p^2 - p \cdot p' = M_i^2 - p \cdot p' = \frac{M_i^2 + t}{2}, \\ p' \cdot q &= p' \cdot (p - p') = p' \cdot p = \frac{M_i^2 - t}{2}, \end{aligned} \quad (\text{H47})$$

the first term in Eq. (H46) becomes

$$-4t(q \cdot p)(p' \cdot q) = -4t \frac{M_i^2 + t}{2} \frac{M_i^2 - t}{2} = -t(M_i^4 - t^2). \quad (\text{H48})$$

Therefore, combining this with Eq. (H26),

$$L_{\mu\nu} H^{(\gamma)\mu\nu} = -t[K(s, t) + M_i^4 - t^2] = -t\tilde{K}(s, t). \quad (\text{H49})$$

Substituting into (H32) gives

$$\langle |\mathcal{M}_\gamma|^2 \rangle = 4e^2 Q_f^2 |\mu'_{ai}|^2 \frac{\tilde{K}(s, t)}{-t}. \quad (\text{H50})$$

b. Z exchange

For Z exchange the f vertex has chiral couplings:

$$-\mathcal{L}_{Zff} \supset \frac{e}{\sin \theta_W \cos \theta_W} Z_\mu \bar{f} \gamma^\mu (g_L^f P_L + g_R^f P_R) f. \quad (\text{H51})$$

Define

$$H^{(Z)\mu\nu} := \frac{1}{2} \text{tr} \left[k' \gamma^\mu (g_L^f P_L + g_R^f P_R) k \gamma^\nu (g_L^f P_L + g_R^f P_R) \right]. \quad (\text{H52})$$

Expanding this yields

$$\begin{aligned} H^{(Z)\mu\nu} = & \frac{1}{2} \left[(g_L^f)^2 \text{tr}(k' \gamma^\mu P_L k \gamma^\nu P_L) \right. \\ & + (g_R^f)^2 \text{tr}(k' \gamma^\mu P_R k \gamma^\nu P_R) \\ & + g_L^f g_R^f \text{tr}(k' \gamma^\mu P_L k \gamma^\nu P_R) \\ & \left. + g_L^f g_R^f \text{tr}(k' \gamma^\mu P_R k \gamma^\nu P_L) \right]. \quad (\text{H53}) \end{aligned}$$

In the massless approximation, the mixed terms vanish because $P_L k = k P_R$ and $P_R k = k P_L$, hence

$$\text{tr}(k' \gamma^\mu P_L k \gamma^\nu P_R) = \text{tr}(k' \gamma^\mu k \gamma^\nu P_L P_R) = 0, \quad (\text{H54})$$

and similarly for the other mixed term. Therefore,

$$H^{(Z)\mu\nu} = \frac{(g_L^f)^2 + (g_R^f)^2}{2} H^{(\gamma)\mu\nu}. \quad (\text{H55})$$

Using (H49) we obtain

$$L_{\mu\nu} H^{(Z)\mu\nu} = -t \frac{(g_L^f)^2 + (g_R^f)^2}{2} \tilde{K}(s, t), \quad (\text{H56})$$

and thus,

$$\langle |\mathcal{M}_Z|^2 \rangle = \frac{2e^2 [(g_L^f)^2 + (g_R^f)^2]}{\sin^2 \theta_W \cos^2 \theta_W} |\mu_{ai}^Z|^2 \frac{-t \tilde{K}(s, t)}{(t - m_Z^2)^2}. \quad (\text{H57})$$

c. γZ interference

Define the mixed tensor

$$H^{(\gamma Z)\mu\nu} := \frac{1}{2} \text{tr} [k' \gamma^\mu k \gamma^\nu (g_L^f P_L + g_R^f P_R)]. \quad (\text{H58})$$

In the massless approximation,

$$H^{(\gamma Z)\mu\nu} = \frac{g_L^f + g_R^f}{2} H^{(\gamma)\mu\nu}, \quad (\text{H59})$$

and hence

$$L_{\mu\nu} H^{(\gamma Z)\mu\nu} = -t \frac{g_L^f + g_R^f}{2} \tilde{K}(s, t). \quad (\text{H60})$$

Therefore, the interference term is

$$\begin{aligned} & \langle 2 \text{Re}(\mathcal{M}_\gamma \mathcal{M}_Z^*) \rangle \\ &= -\frac{4e^2 Q_f (g_L^f + g_R^f)}{\sin \theta_W \cos \theta_W} \text{Re}(\mu_{ai}^\gamma \mu_{ai}^{Z*}) \frac{\tilde{K}(s, t)}{t - m_Z^2}. \quad (\text{H61}) \end{aligned}$$

7. Differential cross section $d\sigma/dt$

For $2 \rightarrow 2$ scattering

$$1(p_1) + 2(p_2) \rightarrow 3(p_3) + 4(p_4), \quad (\text{H62})$$

the Lorentz-invariant differential cross section is

$$d\sigma = \frac{1}{4F} |\mathcal{M}|^2 d\Phi_2, \quad (\text{H63})$$

with Møller invariant flux [104]

$$F := \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}, \quad (\text{H64})$$

and two-body phase space

$$d\Phi_2 = \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4). \quad (\text{H65})$$

Using $p_1 \cdot p_2 = (s - m_1^2 - m_2^2)/2$, we find

$$F = \frac{1}{2} \sqrt{\lambda(s, m_1^2, m_2^2)}, \quad (\text{H66})$$

where λ is Källén function

$$\lambda(s, m_1^2, m_2^2) = s^2 + m_1^4 + m_2^4 - 2m_1^2 s - 2m_2^2 s - 2m_1^2 m_2^2. \quad (\text{H67})$$

In the center-of-mass frame,

$$d\Phi_2 = \frac{\|\mathbf{p}_f\|}{8\pi\sqrt{s}} d\Omega, \quad (\text{H68})$$

with

$$\|\mathbf{p}_i\| = \frac{\sqrt{\lambda(s, m_1^2, m_2^2)}}{2\sqrt{s}}, \quad \|\mathbf{p}_f\| = \frac{\sqrt{\lambda(s, m_3^2, m_4^2)}}{2\sqrt{s}}. \quad (\text{H69})$$

Hence,

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2 \|\mathbf{p}_f\|}{64\pi^2 s \|\mathbf{p}_i\|}. \quad (\text{H70})$$

Using $d\Omega = 2\pi d \cos \theta$, and

$$t(\cos \theta) = m_1^2 + m_2^2 - 2E_1 E_3 + 2\|\mathbf{p}_1\| \|\mathbf{p}_3\| \cos \theta$$

$$\Rightarrow \frac{dt}{d \cos \theta} = 2\|\mathbf{p}_i\| \|\mathbf{p}_f\|, \quad (\text{H71})$$

we obtain

$$\frac{d\sigma}{dt} = \frac{|\mathcal{M}(s, t)|^2}{16\pi\lambda(s, m_1^2, m_2^2)}. \quad (\text{H72})$$

For $(m_1, m_2) = (M_i, 0)$, $\lambda(s, M_i^2, 0) = (s - M_i^2)^2 =: \Delta^2$, one has

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\Delta^2} \langle |\mathcal{M}(s, t)|^2 \rangle. \quad (\text{H73})$$

8. NC differential cross sections

Using the results above yields

$$\frac{d\sigma_\gamma}{dt} = \frac{e^2 Q_f^2 |\mu_{ai}^\gamma|^2}{4\pi\Delta^2} \frac{\tilde{K}(s, t)}{-t}, \quad (\text{H74})$$

$$\frac{d\sigma_Z}{dt} = \frac{e^2}{8\pi\Delta^2} \frac{(g_L^f)^2 + (g_R^f)^2}{\sin^2 \theta_W \cos^2 \theta_W} |\mu_{ai}^Z|^2 \frac{-t\tilde{K}(s, t)}{(t - m_Z^2)^2} \quad (\text{H75})$$

$$\frac{d\sigma_{\gamma Z}}{dt} = -\frac{e^2 Q_f (g_L^f + g_R^f)}{4\pi\Delta^2} \frac{\text{Re}(\mu_{ai}^\gamma \mu_{ai}^{Z*})}{\sin \theta_W \cos \theta_W} \frac{\tilde{K}(s, t)}{t - m_Z^2}, \quad (\text{H76})$$

where

$$\Delta := s - M_i^2, \quad (\text{H77})$$

$$\tilde{K}(s, t) := 2s^2 + 2st - 2M_i^2 s - M_i^2 t + M_i^4. \quad (\text{H78})$$

The total differential cross section is

$$\frac{d\sigma}{dt} = \frac{d\sigma_\gamma}{dt} + \frac{d\sigma_Z}{dt} + \frac{d\sigma_{\gamma Z}}{dt}. \quad (\text{H79})$$

9. Total cross section $\sigma(s)$

a. Photon exchange with Debye screening

To obtain the total cross section, we integrate the differential cross sections. The photon-exchange piece is infrared divergent as $t \rightarrow 0$, so we regulate the divergence by Debye screening [105–107], using an infrared regulator $m_D(T)$ and setting

$$t_{\max} \simeq -m_D^2(T), \quad (\text{H80})$$

where $m_D(T)$ denotes the electric (Debye) mass; our implementation follows the standard treatment of gauge-boson scatterings in thermal leptogenesis [108]. For definiteness, we take [107,109]

$$m_D^2(T) := \kappa_D e^2 (Q_{\text{ref}}) T^2, \quad \kappa_D = \frac{1}{3}, \quad (\text{H81})$$

corresponding to the QED expression for unit-charged relativistic fermions. In the full electroweak plasma the coefficient κ_D is modified by the particle content and mixing, but remains $\mathcal{O}(1)$ and does not affect our conclusions. Defining

$$a := 2s - M_i^2, \quad c := 2s^2 - 2M_i^2 s + M_i^4, \quad (\text{H82})$$

gives $\tilde{K}(s, t) = at + c$, so we have

$$\sigma_\gamma(s; T) = \frac{e^2 Q_f^2 |\mu_{ai}^\gamma|^2}{4\pi\Delta^2} \left[-a(\Delta - m_D^2) + c \ln \frac{\Delta}{m_D^2} \right]. \quad (\text{H83})$$

b. Z exchange

For Z exchange, since m_Z^2 in the denominator regulates the infrared, we integrate to $t_{\max} = 0$:

$$\sigma_Z(s) = \int_{-\Delta}^0 dt \frac{d\sigma_Z}{dt}$$

$$= \frac{e^2}{8\pi\Delta^2} \frac{(g_L^f)^2 + (g_R^f)^2}{\sin^2 \theta_W \cos^2 \theta_W} |\mu_{ai}^Z|^2 I_Z(\Delta; m_Z^2), \quad (\text{H84})$$

where

$$I_Z(\Delta; b) := \int_{-\Delta}^0 dt \frac{-t}{(t-b)^2} \tilde{K}(s, t), \quad s = M_i^2 + \Delta. \quad (\text{H85})$$

For $b > 0$ this yields

$$I_Z(\Delta; b) = (c + 2ab) \ln \left(1 + \frac{\Delta}{b} \right)$$

$$- c - a(\Delta + b) + \frac{b(c + ab)}{\Delta + b}. \quad (\text{H86})$$

Therefore,

$$\sigma_Z(s) = \frac{e^2 |\mu_{ai}^Z|^2}{8\pi\Delta^2} \frac{(g_L^f)^2 + (g_R^f)^2}{\sin^2 \theta_W \cos^2 \theta_W}$$

$$\times \left[(c + 2m_Z^2 a) \ln \left(1 + \frac{\Delta}{m_Z^2} \right) \right.$$

$$\left. - c - a(\Delta + m_Z^2) + \frac{m_Z^2 (c + m_Z^2 a)}{\Delta + m_Z^2} \right]. \quad (\text{H87})$$

c. γZ interference

Similarly,

$$\begin{aligned}\sigma_{\gamma Z}(s) &= \int_{-\Delta}^0 dt \frac{d\sigma_{\gamma Z}}{dt} \\ &= -\frac{e^2 Q_f (g_L^f + g_R^f)}{4\pi\Delta^2} \frac{\text{Re}(\mu_{ai}^\gamma \mu_{ai}^{Z*})}{\sin\theta_W \cos\theta_W} I_{\gamma Z}(\Delta; m_Z^2),\end{aligned}\quad (\text{H88})$$

where

$$I_{\gamma Z}(\Delta; b) := \int_{-\Delta}^0 dt \frac{\tilde{K}(s, t)}{t-b}, \quad s = M_i^2 + \Delta. \quad (\text{H89})$$

Using $\tilde{K}(s, t) = at + c$,

$$\frac{\tilde{K}(s, t)}{t-b} = a + \frac{c+ab}{t-b}. \quad (\text{H90})$$

Therefore,

$$I_{\gamma Z}(\Delta; b) = a\Delta - (c+ab) \ln\left(1 + \frac{\Delta}{b}\right), \quad (\text{H91})$$

and

$$\begin{aligned}\sigma_{\gamma Z}(s) &= -\frac{e^2 Q_f (g_L^f + g_R^f)}{4\pi\Delta^2} \frac{\text{Re}(\mu_{ai}^\gamma \mu_{ai}^{Z*})}{\sin\theta_W \cos\theta_W} \\ &\quad \times \left[a\Delta - (c + m_Z^2 a) \ln\left(1 + \frac{\Delta}{m_Z^2}\right) \right].\end{aligned}\quad (\text{H92})$$

In the total cross sections derived above, the t integration runs over $t \in [-\Delta, t_{\max}]$ with $t_{\max} \leq 0$ [and $t_{\max} \simeq -m_D^2(T)$ for the photon-exchange piece], so the momentum transfer carried by the exchanged gauge boson, $q^2 =: t$, is spacelike throughout the integration region. In the CTP/Schwinger-Keldysh description at finite temperature, the gauge-boson spectral density naturally separates into pole contributions (timelike quasiparticle modes) and a Landau-damping (LD) cut supported on the spacelike region. Therefore, the present t -channel result—which probes only $t < 0$ —matches precisely the Landau-damping part of the propagator that enters $\Sigma_{\text{scatt}}^{<}$, and hence accounts for genuine $2 \leftrightarrow 2$ scattering contributions rather than decay or inverse-decay processes associated with pole kinematics.

10. Reaction density $\gamma(T)$

We reduce the reaction density to a one-dimensional integral. After deriving the KB collision term and reducing it to the QKE, we approximate the statistical distributions by Maxwell-Boltzmann (MB). This approximation affects only the distribution functions and the $(1 \pm f)$ factors; it does not affect the pole/LD split itself.

a. Definition and reduction to a single s integral

For a $2 \rightarrow 2$ process we define the reaction density

$$\gamma_{12 \rightarrow 34}(T) := \int d\Pi_1 d\Pi_2 f_1^{\text{eq}}(p_1) f_2^{\text{eq}}(p_2) \cdot 4F \sigma_{12 \rightarrow 34}(s), \quad (\text{H93})$$

where

$$d\Pi_a := g_a \frac{d^3 p_a}{(2\pi)^3 2E_a} \quad (\text{H94})$$

denotes the Lorentz-invariant phase-space measure, and

$$f_a^{\text{eq}}(p_a) = e^{-E_a/T} \quad (\text{H95})$$

is Maxwell-Boltzmann equilibrium distribution, and

$$F := \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} \quad (\text{H96})$$

is Møller invariant flux [104]. Using the Källén function

$$\lambda(s, m_1^2, m_2^2) := s^2 + m_1^4 + m_2^4 - 2m_1^2 s - 2m_2^2 s - 2m_1^2 m_2^2, \quad (\text{H97})$$

we have

$$4F = 2\sqrt{\lambda(s, m_1^2, m_2^2)}. \quad (\text{H98})$$

Insert the identity

$$1 = \int \frac{d^4 P}{(2\pi)^4} (2\pi)^4 \delta^4(P - p_1 - p_2), \quad (\text{H99})$$

and fix $s = P^2$ by

$$1 = \int_{s_{\min}}^{\infty} ds \delta(s - P^2). \quad (\text{H100})$$

Then,

$$\begin{aligned}\gamma_{12 \rightarrow 34}(T) &= \int_{s_{\min}}^{\infty} ds \sigma(s) \\ &\quad \times \int \frac{d^4 P}{(2\pi)^4} e^{-P^0/T} \delta(s - P^2) \mathcal{I}(P; s),\end{aligned}\quad (\text{H101})$$

where

$$\mathcal{I}(P; s) := \int d\Pi_1 d\Pi_2 4F (2\pi)^4 \delta^4(P - p_1 - p_2). \quad (\text{H102})$$

Since $\mathcal{I}$ is Lorentz invariant, we evaluate it in the CM frame $P^\mu = (\sqrt{s}, \mathbf{0})$. Using the standard two-body phase-space integral,

$$\int d\Pi_1 d\Pi_2 (2\pi)^4 \delta^4(P - p_1 - p_2) = \frac{g_1 g_2}{8\pi} \frac{\sqrt{\lambda(s, m_1^2, m_2^2)}}{s}, \quad (\text{H103})$$

we obtain

$$\mathcal{I}(P; s) = 4F \times \frac{g_1 g_2}{8\pi} \frac{\sqrt{\lambda}}{s} = g_1 g_2 \frac{\lambda(s, m_1^2, m_2^2)}{4\pi s}. \quad (\text{H104})$$

The remaining P integral is

$$J(s) := \int \frac{d^4 P}{(2\pi)^4} e^{-P^0/T} \delta(s - P^2) \Theta(P^0), \quad (\text{H105})$$

where $\theta(P^0)$ selects positive energy. Write $\delta(s - P^2) = \delta(s - (P^0)^2 + \|\mathbf{P}\|^2)$ and integrate over P^0 :

$$J(s) = \int \frac{d^3 P}{(2\pi)^4} \frac{e^{-\sqrt{s+\|\mathbf{P}\|^2}/T}}{2\sqrt{s+\|\mathbf{P}\|^2}}. \quad (\text{H106})$$

Perform the angular integral,

$$J(s) = \frac{4\pi}{(2\pi)^4} \int_0^\infty dP \frac{P^2}{2\sqrt{s+P^2}} e^{-\sqrt{s+P^2}/T}. \quad (\text{H107})$$

Change variables to $E := \sqrt{s+P^2}$ (so $PdP = EdE$). Then

$$\frac{P^2}{2E} dP = \frac{1}{2} \sqrt{E^2 - s} dE, \quad (\text{H108})$$

hence

$$J(s) = \frac{4\pi}{(2\pi)^4} \cdot \frac{1}{2} \int_{\sqrt{s}}^\infty dE \sqrt{E^2 - s} e^{-E/T}. \quad (\text{H109})$$

Using the integral representation of the modified Bessel function K_1 ,

$$\int_{\sqrt{s}}^\infty dE \sqrt{E^2 - s} e^{-E/T} = \sqrt{s} T K_1\left(\frac{\sqrt{s}}{T}\right), \quad (\text{H110})$$

we obtain

$$J(s) = \frac{T\sqrt{s}}{8\pi^3} K_1\left(\frac{\sqrt{s}}{T}\right). \quad (\text{H111})$$

Substituting (H104) and (H111) into (H101), we obtain the general formula

$$\gamma_{12 \rightarrow 34}(T) = g_1 g_2 \frac{T}{32\pi^4} \times \int_{s_{\min}}^\infty ds \frac{\lambda(s, m_1^2, m_2^2)}{\sqrt{s}} K_1\left(\frac{\sqrt{s}}{T}\right) \sigma(s). \quad (\text{H112})$$

b. NC reaction density summed over plasma species

For $(m_1, m_2) = (M_i, 0)$,

$$s_{\min} = M_i^2, \quad \lambda(s, M_i^2, 0) = \Delta^2. \quad (\text{H113})$$

Summing over targets f and lepton flavors α , we define

$$\gamma_{S,i}^{\text{NC}}(T) = \sum_\alpha \sum_f g_N g_f \frac{T}{32\pi^4} \int_{M_i^2}^\infty ds \frac{\Delta^2}{\sqrt{s}} K_1\left(\frac{\sqrt{s}}{T}\right) \times [\sigma_\gamma(s; T) + \sigma_Z(s) + \sigma_{\gamma Z}(s)], \quad (\text{H114})$$

where g_N is the internal degree of freedom (d.o.f.) of N (Majorana: $g_N = 2$), and $g_f = 2N_c$ includes spin and color. If both particles and antiparticles are included in the sum, one adds $\bar{f}$ as well (giving an overall factor 2 when chemical potentials are neglected). Only σ_γ is cut off at $t_{\max} = -m_D^2(T)$; σ_Z and $\sigma_{\gamma Z}$ are IR finite and integrated to $t = 0$.

11. Explicit calculation II: Charged-current (CC) scattering

a. Scattering amplitude

We consider the processes

$$N_i(p) + f(k) \rightarrow \ell_\alpha(p') + f'(k'). \quad (\text{H115})$$

The t -channel exchange boson is W , and the N - ℓ_α - W dipole vertex is controlled by μ_{ai}^W . The SM Wff' vertex is purely left-chiral with coupling $g/\sqrt{2}$. Thus, the amplitude can be written as

$$\mathcal{M}_W = \frac{2g_{Wff'} \mu_{ai}^W}{t - m_W^2} [\bar{u}_\ell(p') \sigma^{\mu\nu} q_\nu P_R u_N(p)] \times [\bar{u}_{f'}(k') \gamma_\mu P_L u_f(k)], \quad (\text{H116})$$

where

$$g_{Wff'} = \frac{g}{\sqrt{2}} \times \begin{cases} 1, & (f, f') = (\ell_\beta, \nu_\beta) \text{ or } (\nu_\beta, \ell_\beta), \\ V_{ab}, & (f, f') = (d_b, u_a) \text{ or } (u_a, d_b), \end{cases} \quad (\text{H117})$$

and V_{ab} is the CKM matrix [74,75].

b. Spin-averaged squared amplitude and cross section

In the massless approximation $m_f = m_{f'} = 0$, we again use $\sigma^{\mu\nu} q_\nu \rightarrow i\gamma^\mu \not{q}$. Since the f current is purely left-handed, the f -side trace is half of the pure vector case, as in the Z -exchange discussion.

Carrying out the trace algebra in the same normalization convention as in the NC case, we obtain

$$\langle |\mathcal{M}_W|^2 \rangle = 2|g_{Wff'}|^2 |\mu_{ai}^W|^2 \frac{-t}{(t - m_W^2)^2} \tilde{K}(s, t). \quad (\text{H118})$$

Therefore,

$$\begin{aligned} \frac{d\sigma_W}{dt} &= \frac{1}{16\pi\Delta^2} \langle |\mathcal{M}_W|^2 \rangle \\ &= \frac{|g_{Wff'}|^2 |\mu_{ai}^W|^2}{8\pi\Delta^2} \frac{-t}{(t - m_W^2)^2} \tilde{K}(s, t), \end{aligned} \quad (\text{H119})$$

and since $t \in [-\Delta, 0]$ there is no infrared divergence. Integrating over t gives

$$\sigma_W(s) = \frac{|g_{Wff'}|^2 |\mu_{ai}^W|^2}{8\pi\Delta^2} I_Z(\Delta; m_W^2), \quad (\text{H120})$$

where the same integral $I_Z(\Delta; b)$ defined in Eq. (H86) is used with $b = m_W^2$.

c. Reaction density

The CC reaction density is

$$\begin{aligned} \gamma_{S,i}^{\text{CC}}(T) &= \sum_\alpha \sum_f g_N g_f \frac{T}{32\pi^4} \\ &\times \int_{M_i^2}^\infty ds \frac{\Delta^2}{\sqrt{s}} K_1\left(\frac{\sqrt{s}}{T}\right) \sigma_W(s), \end{aligned} \quad (\text{H121})$$

where the sum over f runs over all initial states that couple to W :

- (i) Leptons: for $\beta = e, \mu, \tau$, include both $f = \ell_\beta$ and $f = \nu_\beta$ (and antiparticles if desired).
- (ii) Quarks: for $a, b = 1, 2, 3$, include both $f = u_a$ and $f = d_b$ with $|V_{ab}|^2$ and the color factor $N_c = 3$.

12. Matrix-valued scattering rate

For completeness, we record the matrix-valued scattering rate in the heavy-neutrino flavor space. Defining

$$(\mu^{V\dagger} \mu^U)_{ij} := \sum_\alpha \mu_{ai}^{V*} \mu_{aj}^U, \quad V, U = \gamma, Z, W, \quad (\text{H122})$$

the Landau-damping contribution to the heavy-neutrino damping matrix can be written as

$$\begin{aligned} (\Gamma_N^{\text{scatt}})_{ij} &= \Gamma_i^\gamma(T) (\mu^{\gamma\dagger} \mu^\gamma)_{ij} + \Gamma_i^Z(T) (\mu^{Z\dagger} \mu^Z)_{ij} + \Gamma_i^W(T) (\mu^{W\dagger} \mu^W)_{ij} \\ &+ \frac{\Gamma_i^{\gamma Z}(T)}{2} [(\mu^{\gamma\dagger} \mu^Z)_{ij} + (\mu^{Z\dagger} \mu^\gamma)_{ij}]. \end{aligned} \quad (\text{H123})$$

The diagonal part of this expression gives Eq. (196) after summing the channel contributions into $\gamma_{S,i}$.

APPENDIX I: PROJECTION OF KADANOFF-BAYM EQUATIONS TO DENSITY-MATRIX QKES

In this appendix we collect the technical derivation underlying Sec. VIII. Starting from the heavy-neutrino and lepton Wightman functions, we derive the $\langle, \rangle$ components of the Kadanoff-Baym equations [110,111], perform the Wigner transform [72] and leading-gradient expansion, impose the quasiparticle ansatz, and Hermitize the collision term. We then explain the momentum-averaging step leading to the yield-level heavy-neutrino equation, record the index convention relating $S_N^<$ to the heavy-neutrino density matrix, and derive the CP -odd source and washout terms for the flavor charges. The main text quotes only the compact QKE system used in the numerical analysis.

1. Kadanoff-Baym collision term

We define the Wightman functions for the right-handed Majorana neutrinos N_i as

$$\begin{aligned} iS_{N,ij}^<(x, y) &:= \langle \bar{N}_j(y) N_i(x) \rangle, \\ iS_{N,ij}^>(x, y) &:= -\langle N_i(x) \bar{N}_j(y) \rangle, \end{aligned} \quad (\text{I1})$$

where i, j are matrix indices in the heavy-neutrino generation space.

Decomposing the Schwinger-Dyson equation into the $\langle, \rangle$ components yields the KB equation

$$\begin{aligned} (i\partial_x - M)S_N^<(x, y) &= \int d^4z [\Sigma_N^R(x, z) S_N^<(z, y) + \Sigma_N^<(x, z) S_N^A(z, y)], \end{aligned} \quad (\text{I2})$$

and the analogous equation given by exchanging $\langle \leftrightarrow \rangle$. Here $\Sigma_N^{R,A,<,>}$ denotes the heavy-neutrino self-energy.

2. Wigner transform, leading-gradient expansion, and collision term

Using the center coordinate $X = (x + y)/2$ and the relative coordinate $r = x - y$, we introduce the Wigner transform

$$S_N^<(K, X) := \int d^4r e^{iK \cdot r} S_N^<\left(X + \frac{r}{2}, X - \frac{r}{2}\right). \quad (\text{I3})$$

A convolution becomes a Moyal product $A \star B$. Expanding to first order in the gradient ∂_X yields [112]

$$\begin{aligned} A \star B &\simeq AB + \frac{i}{2} \{A, B\}_{\text{PB}}, \\ \{A, B\}_{\text{PB}} &:= (\partial_{K_\mu} A)(\partial_{X^\mu} B) - (\partial_{X^\mu} A)(\partial_{K_\mu} B). \end{aligned} \quad (\text{I4})$$

With these definitions, the KB equation becomes

$$(\not{K} - M - \Sigma_N^H)S_N^< + \frac{i}{2}\{\not{K} - M - \Sigma_N^H, S_N^<\}_{\text{PB}} \\ = \Sigma_N^<S_N^> - \Sigma_N^>S_N^<, \quad (\text{I5})$$

where $\Sigma_N^H = (\Sigma_N^R + \Sigma_N^A)/2$. The right-hand side is the collision term

$$C_N := \Sigma_N^>S_N^< - \Sigma_N^<S_N^>. \quad (\text{I6})$$

In the following, the computations in terms of cross sections and reaction densities are alternative representations of evaluating C_N in the one-loop approximation and under the quasiparticle approximation.

3. Quasiparticle approximation and Kadanoff-Baym ansatz

We define the spectral function as

$$\rho_N := i(S_N^> - S_N^<), \quad (\text{I7})$$

and adopt the quasiparticle approximation

$$\rho_N(K, X) \simeq 2\pi \text{sgn}(k^0)(\not{K} + M)\delta(K^2 - M^2). \quad (\text{I8})$$

As the Kadanoff-Baym ansatz, we express the Wightman functions in terms of the matrix-valued distribution function f_N [113]:

$$iS_N^<(K, X) \simeq -f_N(K, X)\rho_N(K, X), \\ iS_N^>(K, X) \simeq [1 - f_N(K, X)]\rho_N(K, X). \quad (\text{I9})$$

Here f_N is a matrix-valued distribution function in generation space. Similarly, for the Wightman components of the self-energy, in the approximation where internal lines are kept in equilibrium (one-loop approximation with equilibrium internal propagators), we may write

$$i\Sigma_N^<(K, X) \simeq -f_\Sigma(K)\Pi_N(K, X), \\ i\Sigma_N^>(K, X) \simeq [1 - f_\Sigma(K)]\Pi_N(K, X), \quad (\text{I10})$$

where

$$\Pi_N := i(\Sigma_N^> - \Sigma_N^<), \quad (\text{I11})$$

and f_Σ denotes the equilibrium distribution referenced by the internal lines. If the internal lines are fixed to equilibrium, then $f_\Sigma = f_N^{\text{eq}}$.

4. Collision term: From KB to QKE

a. Gain-loss structure and relaxation form

Substituting (I9) and (I10) into the collision term (I6), we obtain

$$\Sigma_N^>S_N^< = [-i(1 - f_\Sigma)\Pi_N](-if_N\rho_N) \quad (\text{I12})$$

$$= -(1 - f_\Sigma)\Pi_N f_N \rho_N, \quad (\text{I13})$$

$$\Sigma_N^<S_N^> = (-if_\Sigma\Pi_N)[-i(1 - f_N)\rho_N] \quad (\text{I14})$$

$$= -f_\Sigma\Pi_N(1 - f_N)\rho_N. \quad (\text{I15})$$

Therefore,

$$C_N = -(1 - f_\Sigma)\Pi_N f_N \rho_N + f_\Sigma\Pi_N(1 - f_N)\rho_N \\ = f_\Sigma\Pi_N \rho_N - \Pi_N f_N \rho_N. \quad (\text{I16})$$

In the approximation where internal lines are fixed to equilibrium, $f_\Sigma = f_N^{\text{eq}}\mathbf{1}$ (proportional to the identity in generation space), so f_Σ commutes with matrices in generation space. Moreover, after the on-shell projection of ρ_N the commutativity in generation space is preserved (the noncommuting parts can be absorbed into first-order gradient terms). Hence,

$$C_N \simeq \Pi_N \rho_N (f_N^{\text{eq}} - f_N). \quad (\text{I17})$$

Thus, the collision term has a relaxation form that drives $f_N - f_N^{\text{eq}}$ back to equilibrium.

Writing (I17) as

$$C_N := K\Delta f_N, \quad K := \Pi_N \rho_N, \quad \Delta f_N := f_N^{\text{eq}} - f_N, \quad (\text{I18})$$

we note that f_N must be Hermitian in generation space, hence $\dot{f}_N$ must be Hermitian, and the collision term must satisfy $C_N^\dagger = C_N$. However, since K and Δf_N do not commute in general,

$$(K\Delta f_N)^\dagger = (\Delta f_N)K^\dagger \neq K\Delta f_N, \quad (\text{I19})$$

so (I17) does not guarantee Hermiticity by itself.

b. Hermitization and decomposition into (anti)commutators

We therefore redefine the collision term by its Hermitian part,

$$C_N \rightarrow C_N^H := \frac{1}{2}[K\Delta f_N + (K\Delta f_N)^\dagger] \\ = \frac{1}{2}[K\Delta f_N + (\Delta f_N)K^\dagger], \quad (\text{I20})$$

then $C_N^{H\dagger} = C_N^H$. Next, decompose K into its Hermitian and anti-Hermitian parts:

$$K = K^H + K^A, \quad K^H := \frac{K + K^\dagger}{2}, \quad K^A := \frac{K - K^\dagger}{2}, \quad (\text{I21})$$

where $K^{H\dagger} = K^H$ and $K^{A\dagger} = -K^A$. Substituting into (I20) yields

$$\begin{aligned} C_N^H &= \frac{1}{2} [(K^H + K^A) \Delta f_N + \Delta f_N (K^H - K^A)] \\ &= \frac{1}{2} \{K^H, \Delta f_N\} + \frac{1}{2} [K^A, \Delta f_N], \end{aligned} \quad (\text{I22})$$

i.e., a sum of an anticommutator and a commutator. The anticommutator corresponds to dissipation (damping), while the commutator corresponds to a unitary rotation (dispersion).

We define

$$\begin{aligned} K^H &= \Gamma_N, & K^A &= 2i\Omega_N^{\text{coll}}, \\ \Gamma_N^\dagger &= \Gamma_N, & \Omega_N^{\text{coll}\dagger} &= \Omega_N^{\text{coll}}, \end{aligned} \quad (\text{I23})$$

namely

$$\Gamma_N = \frac{K + K^\dagger}{2}, \quad \Omega_N^{\text{coll}} = \frac{K - K^\dagger}{4i}. \quad (\text{I24})$$

Then, (I22) becomes

$$C_N^H = \frac{1}{2} \{\Gamma_N, \Delta f_N\} + i[\Omega_N^{\text{coll}}, \Delta f_N]. \quad (\text{I25})$$

Using $\Delta f_N = f_N^{\text{eq}} - f_N$ gives

$$[\Omega_N^{\text{coll}}, \Delta f_N] = [\Omega_N^{\text{coll}}, f_N^{\text{eq}}] - [\Omega_N^{\text{coll}}, f_N]. \quad (\text{I26})$$

Since the equilibrium distribution matrix is proportional to **1** in generation space,

$$[\Omega_N^{\text{coll}}, f_N^{\text{eq}}] = 0 \Rightarrow [\Omega_N^{\text{coll}}, \Delta f_N] = -[\Omega_N^{\text{coll}}, f_N]. \quad (\text{I27})$$

Therefore,

$$C_N^H[f_N] = -\frac{1}{2} \{\Gamma_N, f_N - f_N^{\text{eq}}\} - i[\Omega_N^{\text{coll}}, f_N]. \quad (\text{I28})$$

5. Drift term: From KB to QKE

After the Wigner transform and the leading-gradient expansion, the KB left-hand side contains the Poisson-bracket term

$$\frac{i}{2} \{\mathcal{K} - M - \Sigma_N^H, S_N^<\}_{\text{PB}}. \quad (\text{I29})$$

The part that generates transport (drift) is $\{\mathcal{K}, S_N^<\}_{\text{PB}}$, because $\mathcal{K}$ represents free propagation and necessarily produces $\partial_X S_N^<$ at first order in gradients.

a. Drift term from the Poisson bracket

Since $\partial_{X^\mu} \mathcal{K} = 0$ at leading order,

$$\frac{i}{2} \{\mathcal{K}, S_N^<\}_{\text{PB}} = \frac{i}{2} (\partial_{K_\mu} \mathcal{K}) (\partial_{X^\mu} S_N^<) = \frac{i}{2} \gamma^\mu \partial_{X^\mu} S_N^<. \quad (\text{I30})$$

Thus, the KB left-hand side contains the time evolution with respect to the center coordinate X .

Using the KB ansatz

$$iS_N^<(K, X) \simeq -f_N(K, X) \rho_N(K, X), \quad (\text{I31})$$

we obtain

$$\partial_{X^\mu} S_N^< = -i(\partial_{X^\mu} f_N) \rho_N - i f_N (\partial_{X^\mu} \rho_N). \quad (\text{I32})$$

The first term gives the transport (drift) of the distribution function directly. The second term represents a slow variation of the spectral function and is later collected into the dispersive term Ω_N . Thus, for the drift term we keep only the first term:

$$\frac{i}{2} \{\mathcal{K}, S_N^<\}_{\text{PB}} \rightarrow \frac{i}{2} \gamma^\mu (\partial_{X^\mu} f_N) \rho_N. \quad (\text{I33})$$

b. On-shell projection

In the quasiparticle approximation,

$$\rho_N(K, X) \simeq 2\pi \text{sgn}(k^0) (\mathcal{K} + M) \delta(K^2 - M^2). \quad (\text{I34})$$

To obtain a kinetic equation for $f_N(\mathbf{k}, X)$ without Dirac structure, we project onto the positive-energy shell $k^0 = E_k$ by a Dirac trace. We define the on-shell projection operator by

$$\mathcal{P}_k[\mathcal{O}] := \frac{1}{4E_k} \int \frac{dk^0}{2\pi} \text{tr}_D[(\mathcal{K} + M) \mathcal{O}(K, X)]|_{k^0=E_k}, \quad (\text{I35})$$

with $E_k := \sqrt{\mathbf{k}^2 + M^2}$. Writing (I33) as

$$\mathcal{O}_{\text{tr}}(K, X) := \gamma^\mu [\partial_{X^\mu} f_N(K, X)] \rho_N(K, X), \quad (\text{I36})$$

the projection gives

$$\begin{aligned} \mathcal{P}_k[\mathcal{O}_{\text{tr}}] &= \frac{1}{4E_k} \int \frac{dk^0}{2\pi} (\partial_{X^\mu} f_N) \text{tr}_D[(\mathcal{K} + M) \gamma^\mu \\ &\quad \times (2\pi) \text{sgn}(k^0) (\mathcal{K} + M) \delta(K^2 - M^2)]|_{k^0=E_k} \\ &= \frac{1}{4E_k} [\partial_{X^\mu} f_N(\mathbf{k}, X)] (8MK^\mu)_{k^0=E_k} \times \frac{1}{2E_k} \\ &\propto K^\mu \partial_{X^\mu} f_N(\mathbf{k}, X). \end{aligned} \quad (\text{I37})$$

Here we used

$$\text{tr}_D[(\mathcal{K} + M)\gamma^\mu(\mathcal{K} + M)] = 8MK^\mu,$$

$$\delta(K^2 - M^2) = \frac{1}{2E_k} [\delta(k^0 - E_k) + \delta(k^0 + E_k)]. \quad (I38)$$

Absorbing the overall normalization into the definition of $\mathcal{P}_k$ yields

$$\gamma^\mu(\partial_{X^\mu} f_N) \rho_N \xrightarrow{\mathcal{P}_k} K^\mu \partial_{X^\mu} f_N(\mathbf{k}, X). \quad (I39)$$

c. Liouville operator in an FLRW background

In a Friedmann-Lemaître-Robertson-Walker (FLRW) background, free propagation implies redshift of the physical momentum,

$$\dot{\mathbf{k}} = -H\mathbf{k}. \quad (I40)$$

Therefore, the transport (drift) term becomes the Liouville operator

$$\frac{df_N}{dt} = \partial_t f_N + \dot{\mathbf{k}} \cdot \partial_{\mathbf{k}} f_N = (\partial_t - H\mathbf{k} \cdot \nabla_{\mathbf{k}}) f_N(\mathbf{k}, X). \quad (I41)$$

d. Final distribution-matrix QKE and definitions

Combining (I28) with (I41), we obtain the QKE [32]

$$(\partial_t - H\mathbf{k} \cdot \nabla_{\mathbf{k}}) f_N + i[\Omega_N, f_N] = -\frac{1}{2} \{\Gamma_N, f_N - f_N^{\text{eq}}\}. \quad (I42)$$

Here the commutator term is moved to the left-hand side and absorbed into an effective Hamiltonian (dispersive term). We decompose

$$\Omega_N(\mathbf{k}) := \Omega_N^{\text{vac}}(\mathbf{k}) + \delta\Omega_N^{\text{med}}(\mathbf{k}) + \Omega_N^{\text{coll}}(\mathbf{k}), \quad (I43)$$

and define the damping term by

$$\begin{aligned} \Gamma_N(\mathbf{k}) &:= \frac{1}{2E_k} \Pi_N(k^0 = E_k, \mathbf{k}) \\ &= \frac{i}{2E_k} (\Sigma_N^> - \Sigma_N^<) \Big|_{k^0=E_k}. \end{aligned} \quad (I44)$$

The terms in Eq. (I43) correspond to the following:

- (i) Ω_N^{vac} : vacuum dispersion due to free propagation (mass splitting);
- (ii) $\delta\Omega_N^{\text{med}}$: medium-induced refractive correction, e.g., $\delta\Omega_N^{\text{med}} \propto (2E_k)^{-1} \text{Re}\Sigma_N^R|_{k^0=E_k}$; and
- (iii) Ω_N^{coll} : collision-induced dispersive term from the Hermitization of the collision term, defined by (I24).

6. Origin of the effective Hamiltonian: $\Omega_N^{\text{vac}}, \delta\Omega_N^{\text{med}}$

The Poisson bracket derived above generates the transport (drift) term. By contrast, the coherent dispersion term $i[\Omega_N, f_N]$ in the QKE arises not from the Poisson bracket at first order in the gradient expansion, but from the zeroth-order (local) term of the Moyal product built from the Hermitian part of the inverse propagator in Wigner space. Here we make explicit how

$$\Omega_N = \Omega_N^{\text{vac}} + \delta\Omega_N^{\text{med}} + \Omega_N^{\text{coll}} \quad (I45)$$

emerges as a natural structural decomposition of the KB equation.

Consider the KB equation for the lesser component $S_N^<(x, y)$ in Wigner space (K, X) with $X = (x + y)/2$ and K the conjugate momentum. Using the Moyal product,

$$\begin{aligned} [\mathcal{K} - M - \Sigma_N^H(K, X)] \star iS_N^<(K, X) \\ - iS_N^<(K, X) \star S_N^H(K, X) = \mathcal{C}_N(K, X), \end{aligned} \quad (I46)$$

where $\Sigma_N^H = (\Sigma_N^R + \Sigma_N^A)/2$ is the Hermitian part of the self-energy, $S_N^H = (S_N^R + S_N^A)/2$, and $\mathcal{C}_N$ is the gain-loss collision term.

Expanding the Moyal product to first order,

$$\begin{aligned} A \star B &= AB + \frac{i}{2} \{A, B\}_{\text{PB}} + \mathcal{O}(\partial_X^2), \\ \{A, B\}_{\text{PB}} &:= (\partial_{K_\mu} A)(\partial_{X^\mu} B) - (\partial_{X^\mu} A)(\partial_{K_\mu} B), \end{aligned} \quad (I47)$$

we see that (I46) splits into: (i) a zeroth-order local part determining the on-shell dispersive structure, and (ii) a first-order Poisson-bracket part generating transport.

Using the KB ansatz

$$iS_N^<(K, X) \simeq -f_N(K, X)\rho_N(K, X), \quad (I48)$$

and the quasiparticle form

$$\rho_N(K, X) \simeq 2\pi \text{sgn}(k^0)(\mathcal{K} + M)\delta(K^2 - M^2), \quad (I49)$$

with $M = \text{diag}(M_1, M_2)$, we define the on-shell projection

$$\mathcal{P}_k[\mathcal{O}] := \frac{1}{4E_k} \int \frac{dk^0}{2\pi} \text{tr}_D[(\mathcal{K} + M)\mathcal{O}(K, X)]|_{k^0=E_k}, \quad (I50)$$

where $E_k = \sqrt{k^2 + M^2}$ may be viewed as a diagonal matrix $E_{k,i}$ in generation space.

Extract the local (zeroth-order) part of (I46):

$$(\mathcal{K} - M - \Sigma_N^H) iS_N^< - iS_N^< S_N^H = \mathcal{C}_N + \mathcal{O}(\partial_X). \quad (I51)$$

Substitute (I48) and focus on the first term:

$$\begin{aligned}
(\mathcal{K} - M - \Sigma_N^H) iS_N^< &\simeq -(\mathcal{K} - M - \Sigma_N^H) f_N \rho_N \\
&= -f_N (\mathcal{K} - M - \Sigma_N^H) \rho_N \\
&\quad - [\mathcal{K} - M - \Sigma_N^H, f_N] \rho_N. \quad (I52)
\end{aligned}$$

Two points are essential:

- (i) Since f_N is a generation-space matrix without Dirac structure, it commutes with $\mathcal{K}$: $[\mathcal{K}, f_N] = 0$.
- (ii) The spectral function is determined by the constraint equation such that $(\mathcal{K} - M - \Sigma_N^H) \rho_N \simeq 0$ on shell. This fixes the dispersion relation and is distinct from the kinetic equation for f_N .

Therefore, the first term in (I52) vanishes on shell and only the noncommutativity in generation space remains:

$$(\mathcal{K} - M - \Sigma_N^H) iS_N^< = [M + \Sigma_N^H, f_N] \rho_N. \quad (I53)$$

Projecting (I53) onto the mass shell using (I50), we obtain

$$\mathcal{P}_k[(\mathcal{K} - M - \Sigma_N^H) iS_N^<] \Rightarrow [\mathcal{P}_k[M] + \mathcal{P}_k[\Sigma_N^H], f_N(\mathbf{k}, X)]. \quad (I54)$$

Indeed, with the definition

$$\Omega_N^{\text{vac}}(\mathbf{k}) := \mathcal{P}_k[M] = E_k, \quad \delta\Omega_N^{\text{med}}(\mathbf{k}) := \mathcal{P}_k[\Sigma_N^H], \quad (I55)$$

we recover the commutator term $i[\Omega_N^{\text{vac}} + \delta\Omega_N^{\text{med}}, f_N]$.

The commutator is insensitive to identity shifts,

$$[\Omega_N + c\mathbf{1}, f_N] = [\Omega_N, f_N], \quad (I56)$$

so only relative phases between generations are physical. For two generations one may subtract the trace and keep only the traceless part,

$$\Omega_N \rightarrow \Omega_N - \frac{\text{tr}\Omega_N}{2} \mathbf{1}. \quad (I57)$$

In the nonrelativistic and quasidegenerate regime ($k \ll M_i$, $M_1 \simeq M_2 \simeq \bar{M}$),

$$\begin{aligned}
E_{k,i} &\simeq M_i + \frac{k^2}{2M_i}, \\
\Omega_N^{\text{vac}} - \frac{\text{tr}\Omega_N^{\text{vac}}}{2} \mathbf{1} &\simeq \frac{E_{k,2} - E_{k,1}}{2} \sigma_3 \simeq \frac{\Delta M^2}{4\bar{M}} \sigma_3, \quad (I58)
\end{aligned}$$

where $\Delta M^2 = M_2^2 - M_1^2$.

Next, $\delta\Omega_N^{\text{med}}$ comes from the Hermitian self-energy:

$$\delta\Omega_N^{\text{med}}(\mathbf{k}) = \mathcal{P}_k[\Sigma_N^H]. \quad (I59)$$

Using retarded/advanced components on the real frequency axis,

$$\Sigma_N^H := \frac{\Sigma_N^R + \Sigma_N^A}{2}, \quad \Sigma_N^A(K) = \Sigma_N^R(K)^\dagger, \quad (I60)$$

and therefore $\Sigma_N^H = \text{Re}\Sigma_N^R$.

Define the spectral component

$$\Sigma_N^\rho(K) := i[\Sigma_N^R(K) - \Sigma_N^A(K)] = i[\Sigma_N^>(K) - \Sigma_N^<(K)]. \quad (I61)$$

On the real axis,

$$\Sigma_N^\rho(K) = -2 \text{Im} \Sigma_N^R(K). \quad (I62)$$

Since Σ_N^R is analytic in the upper half plane, it admits the spectral representation

$$\Sigma_N^R(k^0, \mathbf{k}) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\Sigma_N^\rho(\omega, \mathbf{k})}{k^0 - \omega + i0}. \quad (I63)$$

Using the Sokhotski-Plemelj formula

$$\frac{1}{x + i0} = \mathcal{P}\left(\frac{1}{x}\right) - i\pi\delta(x), \quad (I64)$$

taking the real part gives the Kramers-Kronig relation (Hilbert transform) [114]

$$\text{Re}\Sigma_N^R(k^0, \mathbf{k}) = \mathcal{P} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\Sigma_N^\rho(\omega, \mathbf{k})}{k^0 - \omega}. \quad (I65)$$

Therefore,

$$\begin{aligned}
\delta\Omega_N^{\text{med}}(\mathbf{k}) &= \mathcal{P}_k[\text{Re}\Sigma_N^R] \\
&= \mathcal{P}_k \left[\mathcal{P} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\Sigma_N^\rho(\omega, \mathbf{k})}{k^0 - \omega} \right]_{k^0=E_k}. \quad (I66)
\end{aligned}$$

Finally, the collision kernel can be represented as a complex matrix K_N in generation space, with the Hermitian/anti-Hermitian decomposition

$$\begin{aligned}
\Gamma_N &:= \frac{K_N + K_N^\dagger}{2}, \quad \Omega_N^{\text{coll}} := \frac{K_N - K_N^\dagger}{4i} \\
&\Leftrightarrow K_N = \Gamma_N + 2i\Omega_N^{\text{coll}}. \quad (I67)
\end{aligned}$$

This separates damping Γ_N from collision-induced dispersion Ω_N^{coll} .

For the effective Hamiltonian Ω_N defined in Eq. (I45), if the equilibrium distribution matrix commutes with Ω_N ,

$$[\Omega_N, f_N^{\text{eq}}] = 0, \quad (I68)$$

it is convenient to introduce the deviation from equilibrium

$$\delta f_N := f_N - f_N^{\text{eq}}. \quad (I69)$$

With the gain-loss definition of the collision term, one finds $C_N \propto (f_N^{\text{eq}} - f_N) = -\delta f_N$, and the right-handed neutrinos satisfy

$$(\partial_t - H\mathbf{k} \cdot \nabla_{\mathbf{k}})f_N = -\frac{1}{2}\{\Gamma_N, \delta f_N\} - i[\Omega_N, f_N]. \quad (\text{I70})$$

Equivalently,

$$(\partial_t - H\mathbf{k} \cdot \nabla_{\mathbf{k}})f_N + i[\Omega_N, f_N] = -\frac{1}{2}\{\Gamma_N, f_N - f_N^{\text{eq}}\}. \quad (\text{I71})$$

Using Eq. (I68), the commutator may be written as $[\Omega_N, f_N] = [\Omega_N, \delta f_N]$, while the anticommutator must retain the deviation from equilibrium, $f_N - f_N^{\text{eq}}$. The commutativity condition (I68) is guaranteed in exact thermal equilibrium by the KMS condition [115,116], but it should be kept in mind when f_N^{eq} is prescribed in a particular basis.

7. Density-matrix QKE for Y_N

We start from the momentum-dependent density-matrix QKE for heavy neutrinos (on shell, in an FLRW background),

$$\begin{aligned} &(\partial_t - H\mathbf{k} \cdot \nabla_{\mathbf{k}})f_N(\mathbf{k}, t) + i[\Omega_N(\mathbf{k}, t), f_N(\mathbf{k}, t)] \\ &= -\frac{1}{2}\{\Gamma_N(\mathbf{k}, t), f_N(\mathbf{k}, t) - f_N^{\text{eq}}(\mathbf{k}, t)\}. \end{aligned} \quad (\text{I72})$$

We define the matrix-valued number density and yield by

$$\begin{aligned} n_N(t) &:= g_N \int \frac{d^3k}{(2\pi)^3} f_N(\mathbf{k}, t), \\ Y_N(t) &:= \frac{n_N(t)}{s(t)}, \quad Y_N^{\text{eq}}(t) := \frac{n_N^{\text{eq}}(t)}{s(t)}. \end{aligned} \quad (\text{I73})$$

Assuming f_N falls off sufficiently fast as $\|\mathbf{k}\| \rightarrow \infty$, an integration by parts yields

$$g_N \int \frac{d^3k}{(2\pi)^3} (\partial_t - H\mathbf{k} \cdot \nabla_{\mathbf{k}})f_N = \dot{n}_N + 3Hn_N. \quad (\text{I74})$$

Using entropy conservation $\dot{s} = -3Hs$, we obtain

$$\dot{Y}_N = \frac{1}{s}(\dot{n}_N + 3Hn_N). \quad (\text{I75})$$

We now switch to the dimensionless temperature variable $z := M_i/T$. For approximately constant $g_{\star s}$ one has $dz/dt = zH$, so that

$$\begin{aligned} \frac{dY_N}{dz} &= \frac{1}{zH} \dot{Y}_N = \frac{1}{zHs}(\dot{n}_N + 3Hn_N) \\ &= \frac{g_N}{zHs} \int \frac{d^3k}{(2\pi)^3} (\partial_t - H\mathbf{k} \cdot \nabla_{\mathbf{k}})f_N. \end{aligned} \quad (\text{I76})$$

Multiplying Eq. (I72) by $g_N/(zHs)$ and integrating over $\mathbf{k}$ gives

$$\begin{aligned} \frac{dY_N}{dz} + \frac{ig_N}{zHs} \int \frac{d^3k}{(2\pi)^3} [\Omega_N(\mathbf{k}), f_N(\mathbf{k})] \\ = -\frac{g_N}{2zHs} \int \frac{d^3k}{(2\pi)^3} \{\Gamma_N(\mathbf{k}), f_N(\mathbf{k}) - f_N^{\text{eq}}(\mathbf{k})\}. \end{aligned} \quad (\text{I77})$$

To close the yield equation, we adopt the kinetic-equilibrium (momentum-averaging) ansatz

$$f_N(\mathbf{k}, z) - f_N^{\text{eq}}(\mathbf{k}, z) \simeq f_N^{\text{eq}}(\mathbf{k}, z) \frac{Y_N(z) - Y_N^{\text{eq}}(z)}{Y_N^{\text{eq}}(z)}, \quad (\text{I78})$$

where $f_N^{\text{eq}}(\mathbf{k}, T) = e^{-E_{\mathbf{k}}/T} \mathbf{1}_N$ ($E_{\mathbf{k}} := \sqrt{\mathbf{k}^2 + M^2}$) is proportional to the identity matrix in heavy-neutrino flavor space. Then the anticommutator term factorizes as

$$\begin{aligned} &\frac{g_N}{s} \int \frac{d^3k}{(2\pi)^3} \{\Gamma_N(\mathbf{k}, T), f_N(\mathbf{k}, z) - f_N^{\text{eq}}(\mathbf{k}, z)\} \\ &\simeq \{\bar{\Gamma}_N(T), Y_N(z) - Y_N^{\text{eq}}(z)\}, \\ \bar{\Gamma}_N(T) &:= \frac{g_N}{n_N^{\text{eq}}(T)} \int \frac{d^3k}{(2\pi)^3} f_N^{\text{eq}}(\mathbf{k}, T) \Gamma_N(\mathbf{k}, T), \end{aligned} \quad (\text{I79})$$

where the overbar denotes a thermal (momentum) average over the equilibrium distribution. We further decompose $\Gamma_N = \Gamma_{N,\text{pole}} + \Gamma_{N,\text{LD}}$ and define

$$\begin{aligned} \bar{\Gamma}_N^{\text{dec}}(T) &:= \frac{g_N}{n_N^{\text{eq}}(T)} \int \frac{d^3k}{(2\pi)^3} f_N^{\text{eq}}(\mathbf{k}, T) \Gamma_{N,\text{pole}}(\mathbf{k}, T), \\ \bar{\Gamma}_N^{\text{scatt}}(T) &:= \frac{g_N}{n_N^{\text{eq}}(T)} \int \frac{d^3k}{(2\pi)^3} f_N^{\text{eq}}(\mathbf{k}, T) \Gamma_{N,\text{LD}}(\mathbf{k}, T), \end{aligned} \quad (\text{I80})$$

so that $\bar{\Gamma}_N(T) = \bar{\Gamma}_N^{\text{dec}}(T) + \bar{\Gamma}_N^{\text{scatt}}(T)$. We also introduce the dimensionless rate matrices

$$D(z) := \frac{\bar{\Gamma}_N^{\text{dec}}(T)}{zH(z)}, \quad S(z) := \frac{\bar{\Gamma}_N^{\text{scatt}}(T)}{zH(z)}. \quad (\text{I81})$$

Here the overbar denotes a thermal (momentum) average over the equilibrium distribution. In particular, in the Maxwell-Boltzmann approximation one finds

$$\bar{\Gamma}_N^{\text{dec}}(T) = \Gamma_N^{\text{dec}}(T) \left\langle \frac{M}{E} \right\rangle(z),$$

$$\left\langle \frac{M}{E} \right\rangle(z) := \frac{\int \frac{d^3k}{(2\pi)^3} f_N^{\text{eq}}(\mathbf{k}, T) \frac{M}{E_k}}{\int \frac{d^3k}{(2\pi)^3} f_N^{\text{eq}}(\mathbf{k}, T)} = \frac{K_1(z)}{K_2(z)}, \quad (I82)$$

where the factor M/E_k accounts for time dilation [17] and $z := M/T$. For scatterings we introduce the reaction density $\gamma_{\text{scatt}}(T)$ in the standard way and set $\bar{\Gamma}_N^{\text{scatt}}(T) := \gamma_{\text{scatt}}(T)/n_N^{\text{eq}}(T)$. Since z and $H(z)$ depend only on the background temperature, not on the particle momentum, they are not part of the thermal averaging.

We define the momentum-averaged effective Hamiltonian by

$$\Omega_N(z) := \frac{g_N}{n_N^{\text{eq}}(T)} \int \frac{d^3k}{(2\pi)^3} f_N^{\text{eq}}(\mathbf{k}, T) \Omega_N(\mathbf{k}, T), \quad (I83)$$

so that, within the same kinetic-equilibrium (momentum-averaging) approximation used above, we neglect the residual term $\int d^3k [\Omega_N(\mathbf{k}, T) - \Omega_N(z), f_N(\mathbf{k}, z)]$ and approximate

$$\frac{g_N}{zH(z)s} \int \frac{d^3k}{(2\pi)^3} [\Omega_N(\mathbf{k}, T), f_N(\mathbf{k}, z)] \simeq [\Omega_N(z), Y_N(z)]. \quad (I84)$$

Substituting these averaged forms into Eq. (I77), we finally obtain

$$\begin{aligned} & \frac{dY_N}{dz} + i[\Omega_N(z), Y_N(z)] \\ &= -\frac{1}{2} \{D(z) + S(z), Y_N(z) - Y_N^{\text{eq}}(z)\}. \end{aligned} \quad (I85)$$

8. Derivation of density-matrix QKE for flavor charges I: Source term

a. Matching of indices between the density matrix and Wightman functions

We define the density matrix using creation and annihilation operators of heavy neutrinos. For the positive-energy modes,

$$(f_N(\mathbf{k}))_{ij} := \langle a_i^\dagger(\mathbf{k}) a_j(\mathbf{k}) \rangle. \quad (I86)$$

On the other hand, by definition

$$iS_{N,ij}^<(x, y) := \langle \bar{N}_j(y) N_i(x) \rangle. \quad (I87)$$

Using the mode expansion

$$\begin{aligned} N_i(x) &= \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \\ &\times [a_i(\mathbf{k}) u(\mathbf{k}) e^{-ik \cdot x} + a_i^\dagger(\mathbf{k}) v(\mathbf{k}) e^{ik \cdot x}], \end{aligned} \quad (I88)$$

the positive-energy contribution to $iS_{N,ij}^<$ necessarily contains $\langle a_j^\dagger a_i \rangle$. Thus, after projecting to $k^0 = E_k$ under the quasiparticle approximation,

$$iS_{N,ij}^<(K) \simeq -2\pi\delta(k^0 - E_k) (\mathbf{K} + M) (f_N(\mathbf{k}))_{ji} + \dots \quad (I89)$$

The crucial point is that the density-matrix element appearing in $S_{N,ij}^<$ is $(f_N)_{ji}$, i.e., the transpose, which follows directly from the index order $iS_{N,ij}^< \sim \langle \bar{N}_j N_i \rangle \sim \langle a_j^\dagger a_i \rangle$.

b. KB collision term for the lepton-doublet asymmetry

The lepton-doublet asymmetry is defined by

$$n_{\Delta\ell_a} := n_{\ell_a} - n_{\bar{\ell}_a}. \quad (I90)$$

From the KB equation,

$$\begin{aligned} & \dot{n}_{\Delta\ell_a} + 3Hn_{\Delta\ell_a} \\ &= - \int \frac{d^4P}{(2\pi)^4} \text{tr} \{ P_L [\Sigma_{\ell_a}^>(P) S_{\ell_a}^<(P) - \Sigma_{\ell_a}^<(P) S_{\ell_a}^>(P)] \} \\ & \quad - (\ell \rightarrow \bar{\ell}), \end{aligned} \quad (I91)$$

where the bracket on the right-hand side is the lepton collision term. The goal of this subsection is to extract only the CP -odd source term from the right-hand side.

c. Inserting the one-loop self-energy into the collision term

With the electromagnetic dipole interaction, the one-loop lepton self-energy is

$$\begin{aligned} \Sigma_{\ell,ab}^{<,>}(P) &= - \sum_{i,j} \sum_V \int \frac{d^4Q}{(2\pi)^4} (\bar{\Gamma}^V)_{ai}^{\mu}(Q) S_{N,ij}^{<,>}(P+Q) \\ &\quad \times (\Gamma^V)_{bj}^{\nu}(Q) D_{V,\mu\nu}^{>,<}(Q). \end{aligned} \quad (I92)$$

The essential point is that $\Sigma_{\ell,ab}^{<,>}$ contains $S_{N,ij}^{<,>}$ carrying the heavy-neutrino indices (i, j) . Therefore, substituting (I92) into (I91) necessarily produces a sum over i, j , and $S_{N,ij}^{<,>}$ contains $(f_N)_{ji}$ as in (I89).

d. Collision term in terms of distribution matrices (quasiparticle + KB ansatz)

Under the quasiparticle approximation and the KB ansatz,

$$\begin{aligned} iS_N^<(K) &\simeq -f_N \rho_N, \\ iS_N^>(K) &\simeq (1 - f_N) \rho_N, \end{aligned} \quad (I93)$$

$$\begin{aligned} iS_{\ell_\alpha}^<(P) &\simeq -f_{\ell}^{\text{eq}}(P) \rho_{\ell}(P), \\ iS_{\ell_\alpha}^>(P) &\simeq [1 - f_{\ell}^{\text{eq}}(P)] \rho_{\ell}(P). \end{aligned} \quad (I94)$$

Since our purpose here is to extract the source term, we set the lepton chemical potential to zero, $\mu_{\ell_\alpha} = 0$, and keep the lepton lines in equilibrium. We similarly keep the gauge lines in equilibrium, so the statistical factors in $D_V^{<,>}$ are given by equilibrium distributions. Then, the right-hand side of (I91) becomes linear in f_N with equilibrium weights.

e. Vanishing in equilibrium (KMS) and linear expansion

If all distributions are in equilibrium, the Kubo-Martin-Schwinger (KMS) condition ensures that (I91) vanishes:

$$(\dot{n}_{\Delta\ell_\alpha} + 3Hn_{\Delta\ell_\alpha})_{f_N=f_N^{\text{eq}}, \mu_{\ell}=0} = 0. \quad (I95)$$

Therefore, we expand

$$f_N = f_N^{\text{eq}} + \delta f_N, \quad (I96)$$

so that the leading contribution is linear in δf_N . This is the source term, which has the structure

$$\dot{n}_{\Delta\ell_\alpha}|_{\text{source}} = \sum_{i,j} \int \frac{d^3k}{(2\pi)^3} \Upsilon_{\alpha,ij}(\mathbf{k}) (\delta f_N(\mathbf{k}))_{ji}. \quad (I97)$$

Here $\Upsilon_{\alpha,ij}(\mathbf{k})$ is a phase-space kernel weighted by equilibrium distributions (leptons, gauge bosons, scattering targets) and contains the CP -odd amplitude difference between ℓ_α and $\bar{\ell}_\alpha$ production. The index pairing $(ij)(ji)$ is already fixed by (I89).

f. Definition of $\Delta\Gamma_{\alpha,ij}^{(\ell)}$ and trace form

Define the yield density matrix

$$(Y_N)_{ij} := \frac{1}{s} \int \frac{d^3k}{(2\pi)^3} g_N(f_N(\mathbf{k}))_{ij}, \quad (Y_N^{\text{eq}})_{ij} := \delta_{ij} Y_{N_i}^{\text{eq}}. \quad (I98)$$

Then,

$$(Y_N - Y_N^{\text{eq}})_{ji} = \frac{1}{s} \int \frac{d^3k}{(2\pi)^3} g_N(\delta f_N(\mathbf{k}))_{ji}. \quad (I99)$$

As in the derivation of $D_i(z)$ and $S_i(z)$, we introduce a momentum average (kinetic equilibrium) and reduce the kernel to a thermally averaged coefficient,

$$\Upsilon_{\alpha,ij}(\mathbf{k}) \longrightarrow \Upsilon_{\alpha,ij}^{\text{avg}}(T). \quad (I100)$$

Then, (I97) becomes

$$\dot{n}_{\Delta\ell_\alpha}|_{\text{source}} = s \sum_{i,j} \Upsilon_{\alpha,ij}^{\text{avg}}(T) (Y_N - Y_N^{\text{eq}})_{ji}. \quad (I101)$$

Using $dz/dt = zH$ and $Y_{\Delta\ell_\alpha} = n_{\Delta\ell_\alpha}/s$, we write

$$\begin{aligned} \frac{dY_{\Delta\ell_\alpha}}{dz} \Big|_{\text{source}} &= \frac{1}{zHs} \dot{n}_{\Delta\ell_\alpha} \Big|_{\text{source}} \\ &= \sum_{i,j} \Delta\Gamma_{\alpha,ij}^{(\ell)}(z) (Y_N - Y_N^{\text{eq}})_{ji}, \end{aligned} \quad (I102)$$

where

$$\Delta\Gamma_{\alpha,ij}^{(\ell)}(z) := \frac{1}{zH} \Upsilon_{\alpha,ij}^{\text{avg}}(T). \quad (I103)$$

By definition, $\Delta\Gamma_{\alpha,ij}^{(\ell)}$ is the CP -odd source kernel (the difference between ℓ_α and $\bar{\ell}_\alpha$ production rates), and in practice it contains the CP asymmetry computed in Sec. V.

To make the interpretation explicit, define

$$\dot{n}_{\Delta\ell_\alpha} := \dot{n}_{\ell_\alpha} - \dot{n}_{\bar{\ell}_\alpha}. \quad (I104)$$

Evaluating the collision term at one loop gives

$$\begin{aligned} \dot{n}_{\Delta\ell_\alpha}|_{\text{source}} &= \sum_{i,j} \int \frac{d^3k}{(2\pi)^3} \Upsilon_{\alpha,ij}^{(\ell)}(\mathbf{k}) (f_N - f_N^{\text{eq}})_{ji} \\ &\quad - \sum_{i,j} \int \frac{d^3k}{(2\pi)^3} \Upsilon_{\alpha,ij}^{(\bar{\ell})}(\mathbf{k}) (f_N - f_N^{\text{eq}})_{ji}, \end{aligned} \quad (I105)$$

so defining

$$\Upsilon_{\alpha,ij}(\mathbf{k}) := \Upsilon_{\alpha,ij}^{(\ell)}(\mathbf{k}) - \Upsilon_{\alpha,ij}^{(\bar{\ell})}(\mathbf{k}) \quad (I106)$$

gives

$$\dot{n}_{\Delta\ell_\alpha}|_{\text{source}} = \sum_{i,j} \int \frac{d^3k}{(2\pi)^3} \Upsilon_{\alpha,ij}(\mathbf{k}) (f_N - f_N^{\text{eq}})_{ji}. \quad (I107)$$

The index ji arises because $S_{N,ij}^< \propto (f_N)_{ji}$ on shell, cf. Eq. (I89).

Define the thermal average

$$\Upsilon_{\alpha,ij}^{\text{avg}}(T) := \frac{\int \frac{d^3k}{(2\pi)^3} \Upsilon_{\alpha,ij}(\mathbf{k}) (f_N - f_N^{\text{eq}})_{ji}}{\int \frac{d^3k}{(2\pi)^3} (f_N - f_N^{\text{eq}})_{ji}}, \quad (I108)$$

so that

$$\Delta\Gamma_{\alpha,ij}^{(\ell)}(z) := \frac{1}{zH} \Upsilon_{\alpha,ij}^{\text{avg}}(T) \quad (\text{I109})$$

is manifestly the difference between ℓ_α and $\bar{\ell}_\alpha$ production. If CP is conserved, $\Upsilon_{\alpha,ij}^{(\ell)} = \tilde{\Upsilon}_{\alpha,ij}^{(\bar{\ell})}$ holds, and hence $\Delta\Gamma_{\alpha,ij}^{(\ell)} = 0$.

Using the trace identity

$$\text{tr}(AB) = \sum_i (AB)_{ii} = \sum_{i,j} A_{ij} B_{ji}, \quad (\text{I110})$$

the source term (I102) can be written as

$$\left. \frac{dY_{\Delta\ell_\alpha}}{dz} \right|_{\text{source}} = \text{tr}[\Delta\Gamma_\alpha^{(\ell)}(z)(Y_N - Y_N^{\text{eq}})]. \quad (\text{I111})$$

g. Mapping the source term from $Y_{\Delta\ell_\alpha}$ to $Y_{\Delta\alpha}$

The quantity directly derived above is the source term for the lepton-doublet asymmetry $Y_{\Delta\ell_\alpha} = (n_{\ell_\alpha} - n_{\bar{\ell}_\alpha})/s$. In the density-matrix QKE we instead use

$$Y_{\Delta\alpha} := \frac{1}{3} Y_B - Y_{L_\alpha}, \quad (\text{I112})$$

where Y_{L_α} is the total lepton asymmetry in flavor α . For the source term the mapping is simple.

Since the dipole interactions do not change baryon number,

$$\left. \frac{dY_B}{dz} \right|_{\text{source}} = 0, \quad (\text{I113})$$

and thus,

$$\left. \frac{dY_{\Delta\alpha}}{dz} \right|_{\text{source}} = - \left. \frac{dY_{L_\alpha}}{dz} \right|_{\text{source}}. \quad (\text{I114})$$

Moreover, the source processes inject one left-handed lepton doublet, so for the source term

$$\left. \frac{dY_{L_\alpha}}{dz} \right|_{\text{source}} = \left. \frac{dY_{\Delta\ell_\alpha}}{dz} \right|_{\text{source}}. \quad (\text{I115})$$

Combining,

$$\left. \frac{dY_{\Delta\alpha}}{dz} \right|_{\text{source}} = - \left. \frac{dY_{\Delta\ell_\alpha}}{dz} \right|_{\text{source}}. \quad (\text{I116})$$

If (I111) holds, then

$$\left. \frac{dY_{\Delta\alpha}}{dz} \right|_{\text{source}} = -\text{tr}[\Delta\Gamma_\alpha^{(\ell)}(z)(Y_N - Y_N^{\text{eq}})]. \quad (\text{I117})$$

Defining

$$\Delta\Gamma_\alpha(z) := -\Delta\Gamma_\alpha^{(\ell)}(z), \quad (\text{I118})$$

we obtain

$$\left. \frac{dY_{\Delta\alpha}}{dz} \right|_{\text{source}} = \text{tr}[\Delta\Gamma_\alpha(z)(Y_N - Y_N^{\text{eq}})]. \quad (\text{I119})$$

This is merely a convention choice: the minus sign is absorbed into the definition of the kernel.

h. Rewriting the source kernel in terms of the CP asymmetry

In the present setup the flavored CP asymmetries ε_{ai} are taken to be temperature independent. The dimensionless decay kernel that enters the QKE is defined from the thermally averaged total decay rate of N_i . We first write

$$\Gamma_i^{\text{dec}} := \sum_{\alpha=e,\mu,\tau} \sum_{V=\gamma,Z,W} \Gamma_{V,ai}^{(0)}, \quad (\text{I120})$$

where the channel widths $\Gamma_{V,ai}^{(0)}$ are given in Secs. IV and V. In the Maxwell-Boltzmann approximation, the thermally averaged decay rate is

$$\bar{\Gamma}_{D,i}(T) = \frac{K_1(z_i)}{K_2(z_i)} \Gamma_i^{\text{dec}}, \quad z_i := \frac{M_i}{T}. \quad (\text{I121})$$

We then define

$$D_i(z_i) := \frac{\bar{\Gamma}_{D,i}(T)}{z_i H(T)} = \frac{\Gamma_i^{\text{dec}}}{z_i H(T)} \frac{K_1(z_i)}{K_2(z_i)}. \quad (\text{I122})$$

We collect these kernels into the diagonal matrix

$$D(z) := \text{diag}(D_i(z_i)). \quad (\text{I123})$$

It is then convenient to factor out this purely kinematic kernel from the source matrix by defining a CP -asymmetry matrix ε_α through

$$(\Delta\Gamma_\alpha)_{ij}(z) := (\varepsilon_\alpha)_{ij} D_j(z), \quad (\varepsilon_\alpha)_{ii} = \varepsilon_{ai}. \quad (\text{I124})$$

With this definition, Eq. (I119) becomes

$$\left. \frac{dY_{\Delta\alpha}}{dz} \right|_{\text{source}} = \text{tr}[\varepsilon_\alpha D(z)(Y_N - Y_N^{\text{eq}})], \quad (\text{I125})$$

which reduces, for diagonal Y_N , to the familiar Boltzmann form

$$\left. \frac{dY_{\Delta\alpha}}{dz} \right|_{\text{source}} = \sum_i \varepsilon_{ai} D_i(z) (Y_{N_i} - Y_{N_i}^{\text{eq}}). \quad (\text{I126})$$

9. Derivation of density-matrix QKE for flavor charges II: Washout term

In this subsection, we show that the washout matrices $W_{\alpha\beta}^{\text{ID}}(z)$ and $W_{\alpha\beta}^{\text{scatt}}(z)$ in the $Y_{\Delta\alpha}$ equation follow from the lepton KB collision term

$$C_{\ell_\alpha} = \Sigma_{\ell_\alpha}^> S_{\ell_\alpha}^< - \Sigma_{\ell_\alpha}^< S_{\ell_\alpha}^>. \quad (\text{I127})$$

The conclusion is that the pole part generates inverse-decay washout and the Landau damping part generates scattering washout. Since they are defined on disjoint spectral domains, no RIS subtraction is needed [67–69,73].

We define $n_{\Delta\ell_\alpha} := n_{\ell_\alpha} - n_{\bar{\ell}_\alpha}$. Taking the equal-time limit yields

$$\begin{aligned} \dot{n}_{\Delta\ell_\alpha} + 3Hn_{\Delta\ell_\alpha} &= - \int \frac{d^4P}{(2\pi)^4} \text{tr} \{ P_L [\Sigma_{\ell_\alpha}^>(P) S_{\ell_\alpha}^<(P) - \Sigma_{\ell_\alpha}^<(P) S_{\ell_\alpha}^>(P)] \} \\ &\quad - (\ell \rightarrow \bar{\ell}). \end{aligned} \quad (\text{I128})$$

Using (177), the one-loop lepton self-energy is

$$\begin{aligned} \Sigma_{\ell,\alpha\beta}^{<,>}(P) &= - \sum_{i,j} \sum_V \int \frac{d^4Q}{(2\pi)^4} (\bar{\Gamma}^V)_{ai}^{\mu}(Q) S_{N,ij}^{<,>}(P+Q) \\ &\quad \times (\Gamma^V)_{\beta j}^{\nu}(Q) D_{V,\mu\nu}^{<,>}(Q), \end{aligned} \quad (\text{I129})$$

with $V = \gamma, Z, W$.

In a thermal medium the gauge Wightman function admits the split

$$D_V^{<,>}(Q) = D_{V,\text{pole}}^{<,>}(Q) + D_{V,\text{LD}}^{<,>}(Q). \quad (\text{I130})$$

Substituting into (1129) defines

$$\begin{aligned} \Sigma_{\ell}^{<,>}(P) &= \Sigma_{\ell,\text{pole}}^{<,>}(P) + \Sigma_{\ell,\text{scatt}}^{<,>}(P), \\ \Sigma_{\ell,\text{scatt}}^{<,>} &:= \Sigma_{\ell}^{<,>}|_{D_V^{<,>} \rightarrow D_{V,\text{LD}}^{<,>}}. \end{aligned} \quad (\text{I131})$$

Accordingly, the collision term separates into a pole contribution and a Landau-damping contribution. The pole part is supported on timelike momenta and reproduces the $1 \leftrightarrow 2$ decay and inverse-decay processes, whereas the Landau-damping cut is supported on spacelike momenta and yields the genuine $2 \leftrightarrow 2$ scattering contribution [105,109]. Because these kinematic domains do not overlap, the separation is free of double counting at this order; in particular, no real-intermediate-state subtraction is required in the effective kinetic equations [67–69,73].

a. Linearization near equilibrium

With Maxwell-Boltzmann distributions, the lepton distributions including chemical potentials are

$$\begin{aligned} f_{\ell_\alpha}(E; \mu_{\ell_\alpha}) &= e^{-(E-\mu_{\ell_\alpha})/T} = f_{\ell}^{\text{eq}}(E) e^{\mu_{\ell_\alpha}/T}, \\ f_{\bar{\ell}_\alpha}(E; \mu_{\ell_\alpha}) &= e^{-(E+\mu_{\ell_\alpha})/T} = f_{\ell}^{\text{eq}}(E) e^{-\mu_{\ell_\alpha}/T}. \end{aligned} \quad (\text{I132})$$

Thus,

$$n_{\ell_\alpha} = n_{\ell}^{\text{eq}} e^{\mu_{\ell_\alpha}/T}, \quad n_{\bar{\ell}_\alpha} = n_{\ell}^{\text{eq}} e^{-\mu_{\ell_\alpha}/T}, \quad (\text{I133})$$

and therefore

$$n_{\Delta\ell_\alpha} = n_{\ell}^{\text{eq}} (e^{\mu_{\ell_\alpha}/T} - e^{-\mu_{\ell_\alpha}/T}) \simeq 2n_{\ell}^{\text{eq}} \frac{\mu_{\ell_\alpha}}{T}. \quad (\text{I134})$$

Equivalently,

$$\frac{\mu_{\ell_\alpha}}{T} \simeq \frac{n_{\Delta\ell_\alpha}}{2n_{\ell}^{\text{eq}}} = \frac{Y_{\Delta\ell_\alpha}}{2Y_{\ell}^{\text{eq}}}, \quad Y_{\Delta\ell_\alpha} := \frac{n_{\Delta\ell_\alpha}}{s}, \quad Y_{\ell}^{\text{eq}} := \frac{n_{\ell}^{\text{eq}}}{s}. \quad (\text{I135})$$

Since the collision term vanishes in equilibrium (KMS), the linearized form is

$$\begin{aligned} \left. \frac{dn_{\Delta\ell_\alpha}}{dt} \right|_{\text{ID}} &= - \sum_{\beta} \mathcal{W}_{\alpha\beta}^{\text{ID}}(T) \frac{\mu_{\ell_\beta}}{T} + \dots, \\ \left. \frac{dn_{\Delta\ell_\alpha}}{dt} \right|_{\text{scatt}} &= - \sum_{\beta} \mathcal{W}_{\alpha\beta}^{\text{scatt}}(T) \frac{\mu_{\ell_\beta}}{T} + \dots. \end{aligned} \quad (\text{I136})$$

b. Definition of the dimensionless washout matrix $W_{\alpha\beta}^{\text{ID,scatt}}(z)$

Rewriting Eq. (I136) in terms of $Y_{\Delta\ell_\alpha} = n_{\Delta\ell_\alpha}/s$ and using $dz/dt = zH$, we obtain

$$\begin{aligned} \left. \frac{dY_{\Delta\ell_\alpha}}{dz} \right|_{\text{ID}} &= - \sum_{\beta} \frac{1}{zHs} \mathcal{W}_{\alpha\beta}^{\text{ID}}(T) \frac{\mu_{\ell_\beta}}{T} + \dots, \\ \left. \frac{dY_{\Delta\ell_\alpha}}{dz} \right|_{\text{scatt}} &= - \sum_{\beta} \frac{1}{zHs} \mathcal{W}_{\alpha\beta}^{\text{scatt}}(T) \frac{\mu_{\ell_\beta}}{T} + \dots. \end{aligned} \quad (\text{I137})$$

Using Eq. (I135) to replace μ_{ℓ_β}/T by $Y_{\Delta\ell_\beta}$, we find

$$\begin{aligned} \left. \frac{dY_{\Delta\ell_\alpha}}{dz} \right|_{\text{ID}} &= - \sum_{\beta} \frac{1}{2Y_{\ell}^{\text{eq}}} \frac{\mathcal{W}_{\alpha\beta}^{\text{ID}}(T)}{zHs} Y_{\Delta\ell_\beta}, \\ \left. \frac{dY_{\Delta\ell_\alpha}}{dz} \right|_{\text{scatt}} &= - \sum_{\beta} \frac{1}{2Y_{\ell}^{\text{eq}}} \frac{\mathcal{W}_{\alpha\beta}^{\text{scatt}}(T)}{zHs} Y_{\Delta\ell_\beta}. \end{aligned} \quad (\text{I138})$$

The lepton-doublet asymmetry $Y_{\Delta\ell_\alpha}$ is mapped to $Y_{\Delta\alpha}$ by the susceptibility matrix $A_{\ell}(T)$ [cf. Table II] [117]:

TABLE II. Representative numerical values of the sphaleron conversion factor $c_{\text{sph}}(T)$ and the electroweak-crossover susceptibility matrix $A_\ell(T)$ across the electroweak crossover. In the numerical implementation we approximate $v(T)$ by its zero-temperature value $v = 246$ GeV when evaluating $x(T) := v(T)/T$ in the electroweak window.

T [GeV]	$x(T)$	$c_{\text{sph}}(T)$	$[A_\ell]_{\alpha\alpha}$	$[A_\ell]_{\alpha\neq\beta}$
$T \rightarrow \infty$	0	28/79	257/711	20/711
160	1.5375	0.34012	0.34238	0.00905
155	1.5871	0.33964	0.34175	0.00841
150	1.6400	0.33915	0.34109	0.00776
145	1.6966	0.33864	0.34041	0.00708
140	1.7571	0.33811	0.33971	0.00637
135	1.8222	0.33757	0.33899	0.00565
130	1.8923	0.33702	0.33824	0.00491
$T \rightarrow 0$	∞	12/37	107/333	-4/333

$$Y_{\Delta\ell_\alpha} = -\sum_\beta [A_\ell(T)]_{\alpha\beta} Y_{\Delta\beta},$$

$$[A_\ell(T)]_{\alpha\beta} = \begin{cases} \frac{963x(T)^2 + 2827}{9[333x(T)^2 + 869]}, & \alpha = \beta, \\ \frac{4[55 - 9x(T)^2]}{9[333x(T)^2 + 869]}, & \alpha \neq \beta, \end{cases} \quad (\text{I139})$$

with $x(T) := v(T)/T$, where $v(T)$ denotes the Higgs expectation value across the electroweak crossover. For simplicity in our numerical implementation we approximate $v(T)$ by its zero-temperature value $v = 246$ GeV over the narrow window $T \simeq 130\text{--}160$ GeV, so that $x(T) \simeq v/T$ in Table II.

Substituting the above relation into Eq. (I138), we define the washout matrices for $Y_{\Delta\beta}$ by

$$\left. \frac{dY_{\Delta\alpha}}{dz} \right|_{\text{washout}} =: -\sum_\beta [W_{\alpha\beta}^{\text{ID}}(z) + W_{\alpha\beta}^{\text{scatt}}(z)] Y_{\Delta\beta}. \quad (\text{I140})$$

We make this general expression explicit by using the reaction densities already obtained (inverse decays plus scatterings) together with the branching ratios B_{ai} , and derive the density-matrix QKE for $Y_{\Delta\alpha}$.

c. Definition of the inverse-decay washout matrix $W_{\alpha\beta}^{\text{ID}}(z)$

We now make the inverse-decay contribution in Eq. (I140) explicit. For each heavy neutrino N_i , we first isolate a scalar factor that depends only on the heavy-neutrino index i , and then separate the flavor projector. We therefore define the scalar inverse-decay kernel, normalized to the equilibrium lepton density, by

$$W_i^{\text{ID}}(z) := \frac{1}{2} \frac{n_{N_i}^{\text{eq}}(z)}{n_\ell^{\text{eq}}(z)} D_i(z) = \frac{1}{4} \frac{\Gamma_i^{\text{dec}}}{H(z)} z K_1(z). \quad (\text{I141})$$

Here $D_i(z)$ is the decay kernel defined above. The second equality follows from detailed balance [17], together with the Maxwell-Boltzmann expressions

$$n_{N_i}^{\text{eq}}(z) = \frac{g_N M_i^3}{2\pi^2} \frac{K_2(z)}{z}, \quad n_\ell^{\text{eq}}(z) = \frac{g_\ell M_i^3}{\pi^2} \frac{1}{z^3}, \quad (\text{I142})$$

with $g_N = 2$ and $g_\ell = 2$. We define

$$n_\ell^{\text{eq}}(T) := \sum_{\text{d.o.f.}} \int \frac{d^3p}{(2\pi)^3} e^{-E/T}, \quad (\text{I143})$$

which is the Maxwell-Boltzmann equilibrium number density of leptons, and $Y_\ell^{\text{eq}} = n_\ell^{\text{eq}}/s$.

To keep the flavor structure explicit, we next introduce the inverse-decay branching ratio, or equivalently the flavor projector,

$$B_{ai}^{\text{ID}}(T) := \frac{\Gamma_{ai}^{\text{dec}}}{\sum_\rho \Gamma_{\rho i}^{\text{dec}}}. \quad (\text{I144})$$

The inverse-decay contribution to the washout matrix is then defined by

$$W_{\alpha\beta}^{\text{ID}}(z) := \sum_i B_{ai}^{\text{ID}}(T) W_i^{\text{ID}}(z) [A_\ell(T)]_{\alpha\beta}. \quad (\text{I145})$$

Thus, $W_i^{\text{ID}}(z)$ depends only on the heavy-neutrino index i , whereas the lepton-flavor dependence is incorporated through B_{ai}^{ID} and the conversion matrix $A_\ell(T)$.

d. Definition of the scattering washout matrix $W_{\alpha\beta}^{\text{scatt}}(z)$

We define the flavor branching ratio for scatterings from the scattering reaction densities. Removing the sum over α from Eqs. (H114) and (H121), we define

$$\gamma_{S,ai}^{\text{NC}}(T) := \sum_f g_N g_f \frac{T}{32\pi^4} \int_{M_i^2}^{\infty} ds \frac{\Delta^2}{\sqrt{s}} K_1\left(\frac{\sqrt{s}}{T}\right) \times [\sigma_\gamma(s; T) + \sigma_Z(s) + \sigma_{\gamma Z}(s)], \quad (\text{I146})$$

$$\gamma_{S,ai}^{\text{CC}}(T) := \sum_f g_N g_f \frac{T}{32\pi^4} \int_{M_i^2}^{\infty} ds \frac{\Delta^2}{\sqrt{s}} K_1\left(\frac{\sqrt{s}}{T}\right) \sigma_W(s). \quad (\text{I147})$$

We further define

$$\gamma_{ai}^{\text{scatt}}(T) := \gamma_{S,ai}^{\text{NC}}(T) + \gamma_{S,ai}^{\text{CC}}(T),$$

$$\gamma_{S,i}(T) := \sum_\rho \gamma_{\rho i}^{\text{scatt}}(T), \quad (\text{I148})$$

and the scattering branching ratio as

$$B_{ai}^{\text{scatt}}(T) := \frac{\gamma_{ai}^{\text{scatt}}(T)}{\sum_{\rho} \gamma_{\rho i}^{\text{scatt}}(T)} = \frac{\gamma_{ai}^{\text{scatt}}(T)}{\gamma_{S,i}(T)}. \quad (\text{I149})$$

Next, following the linearization of the lepton collision term in Eq. (I138), we define the scalar scattering washout kernel by

$$W_i^{\text{scatt}}(z) := \frac{1}{2} \frac{\gamma_{S,i}(T)}{zH(z)n_{\ell}^{\text{eq}}(T)} = \frac{1}{2} \frac{\gamma_{S,i}(T)}{zH(z)sY_{\ell}^{\text{eq}}(T)}. \quad (\text{I150})$$

The normalization differs from the heavy-neutrino equilibration function $S_i(z) = \gamma_{S,i}/(zHn_{N_i}^{\text{eq}})$; the two are related by

$$W_i^{\text{scatt}}(z) = \frac{1}{2} \frac{n_{N_i}^{\text{eq}}(z)}{n_{\ell}^{\text{eq}}(z)} S_i(z). \quad (\text{I151})$$

This difference arises because the Y_N equation is normalized by the N number density, while the $Y_{\Delta\ell}$ equation is normalized by the ℓ number density.

Finally, we define the scattering washout matrix as

$$W_{\alpha\beta}^{\text{scatt}}(z) := \sum_i B_{ai}^{\text{scatt}}(T) W_i^{\text{scatt}}(z) [A_{\ell}(T)]_{\alpha\beta}. \quad (\text{I152})$$

Combining Eqs. (I125) and (I140), we find that $Y_{\Delta\alpha}$ satisfies

$$\begin{aligned} \frac{dY_{\Delta\alpha}}{dz} = & \text{tr}[\varepsilon_{\alpha}(z)D(z)(Y_N - Y_N^{\text{eq}})] \\ & - \sum_{\beta} \left[W_{\alpha\beta}^{\text{ID}}(z) + W_{\alpha\beta}^{\text{scatt}}(z) \right] Y_{\Delta\beta}. \end{aligned} \quad (\text{I153})$$

Together with Eq. (I85), the density-matrix QKE system reads

$$\begin{aligned} \frac{dY_N}{dz} = & -i[\Omega_N(z), Y_N] \\ & - \frac{1}{2} \{D(z) + S(z), Y_N - Y_N^{\text{eq}}\}, \end{aligned} \quad (\text{I154})$$

$$\begin{aligned} \frac{dY_{\Delta\alpha}}{dz} = & \text{tr}[\varepsilon_{\alpha}(z)D(z)(Y_N - Y_N^{\text{eq}})] \\ & - \sum_{\beta} \left[W_{\alpha\beta}^{\text{ID}}(z) + W_{\alpha\beta}^{\text{scatt}}(z) \right] Y_{\Delta\beta}. \end{aligned} \quad (\text{I155})$$

e. Time evolution of the density-matrix QKE

In the broken electroweak phase, sphaleron processes convert $B - L$ into B as long as they remain in thermal equilibrium; after sphaleron freeze-out, B is conserved. Therefore, the frozen-out baryon asymmetry is evaluated at the sphaleron freeze-out temperature T_{sph} as

$$Y_B^{\text{FO}} = c_{\text{sph}}(T_{\text{sph}}) Y_{B-L}(T_{\text{sph}}), \quad Y_{B-L} = \sum_{\alpha} Y_{\Delta\alpha}, \quad (\text{I156})$$

where the sphaleron conversion factor is [117]

$$c_{\text{sph}}(T) = \frac{4[27x(T)^2 + 77]}{333x(T)^2 + 869}, \quad x(T) \simeq \frac{v}{T}. \quad (\text{I157})$$

Here we use the numerical value of $c_{\text{sph}}(T_{\text{sph}})$ listed in Table II.

- [1] A. D. Sakharov, Pis'ma Zh. Eksp. Teor. Fiz. **5**, 32 (1967) [JETP Lett. **5**, 24 (1967)].
- [2] N. S. Manton, Phys. Rev. D **28**, 2019 (1983).
- [3] F. R. Klinkhamer and N. S. Manton, Phys. Rev. D **30**, 2212 (1984).
- [4] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).
- [5] P. Minkowski, Phys. Lett. **67B**, 421 (1977).
- [6] T. Yanagida, in *Proceedings of the Workshop on Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979), p. 95.
- [7] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315.
- [8] R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).
- [9] R. N. Mohapatra and G. Senjanović, Phys. Rev. D **23**, 165 (1981).

- [10] M. Magg and C. Wetterich, Phys. Lett. **94B**, 61 (1980).
- [11] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).
- [12] C. Wetterich, Nucl. Phys. **B187**, 343 (1981).
- [13] G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. **B181**, 287 (1981).
- [14] R. Foot, H. Lew, X. G. He, and G. C. Joshi, Z. Phys. C **44**, 441 (1989).
- [15] E. Ma, Phys. Rev. Lett. **81**, 1171 (1998).
- [16] E. Ma and D. P. Roy, Nucl. Phys. **B644**, 290 (2002).
- [17] W. Buchmüller, P. Di Bari, and M. Plumacher, Ann. Phys. (Amsterdam) **315**, 305 (2005).
- [18] S. Davidson, E. Nardi, and Y. Nir, Phys. Rep. **466**, 105 (2008).
- [19] N. F. Bell, B. J. Kayser, and S. S. C. Law, Phys. Rev. D **78**, 085024 (2008).
- [20] G. Magill, R. Plestid, M. Pospelov, and Y.-D. Tsai, Phys. Rev. D **98**, 115015 (2018).

- [21] D. Barducci, E. Bertuzzo, M. Taoso, and C. Toni, *J. High Energy Phys.* **03** (2023) 239.
- [22] D. Barducci, E. Bertuzzo, M. Taoso, C. A. Ternes, and C. Toni, *J. High Energy Phys.* **10** (2024) 016.
- [23] A. Crivellin, S. Najjari, and J. Rosiek, *J. High Energy Phys.* **04** (2014) 167.
- [24] J. Aebischer, W. Dekens, E. E. Jenkins, A. V. Manohar, D. Sengupta, and P. Stoffer, *J. High Energy Phys.* **07** (2021) 107.
- [25] R. Takada, *J. High Energy Phys.* **12** (2025) 010.
- [26] A. Pilaftsis, *Phys. Rev. D* **56**, 5431 (1997).
- [27] A. Pilaftsis and T. E. Underwood, *Nucl. Phys.* **B692**, 303 (2004).
- [28] P. S. B. Dev, M. Garny, J. Klaric, P. Millington, and D. Teresi, *Int. J. Mod. Phys. A* **33**, 1842003 (2018).
- [29] R. Takada, *J. High Energy Phys.* **04** (2026) 073.
- [30] D. Choudhury, N. Mahajan, S. Patra, and U. Sarkar, *J. Cosmol. Astropart. Phys.* **04** (2012) 017.
- [31] A. Pilaftsis and T. E. J. Underwood, *Phys. Rev. D* **72**, 113001 (2005).
- [32] G. Sigl and G. Raffelt, *Nucl. Phys.* **B406**, 423 (1993).
- [33] M. Garny, A. Kartavtsev, and A. Hohenegger, *Ann. Phys. (Amsterdam)* **328**, 26 (2013).
- [34] Y. Liao and X.-D. Ma, *Phys. Rev. D* **96**, 015012 (2017).
- [35] H.-L. Li, Z. Ren, M.-L. Xiao, J.-H. Yu, and Y.-H. Zheng, *J. High Energy Phys.* **11** (2021) 003.
- [36] A. Datta, J. Kumar, H. Liu, and D. Marfatia, *J. High Energy Phys.* **02** (2021) 015.
- [37] A. Datta, J. Kumar, H. Liu, and D. Marfatia, *J. High Energy Phys.* **05** (2021) 037.
- [38] M. D’Onofrio, K. Rummukainen, and A. Tranberg, *Phys. Rev. Lett.* **113**, 141602 (2014).
- [39] M. D’Onofrio and K. Rummukainen, *Phys. Rev. D* **93**, 025003 (2016).
- [40] E. Nardi, Y. Nir, E. Roulet, and J. Racker, *J. High Energy Phys.* **01** (2006) 164.
- [41] A. Abada, S. Davidson, A. Ibarra, F.-X. Josse-Michaux, M. Losada, and A. Riotto, *J. High Energy Phys.* **09** (2006) 010.
- [42] F. F. Deppisch, J. Harz, W.-C. Huang, M. Hirsch, and H. Päs, *Phys. Rev. D* **92**, 036005 (2015).
- [43] L. Autonne, *Ann. Univ. Lyon, Nouvelle Sér. I* **38**, 1 (1915).
- [44] T. Takagi, *Jpn. J. Math.* **1**, 83 (1924).
- [45] G. ’t Hooft, *NATO Sci. Ser. B* **59**, 135 (1980).
- [46] W. Buchmüller, P. Di Bari, and M. Plumacher, *Nucl. Phys.* **B643**, 367 (2002); **B793**, 362(E) (2008).
- [47] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, J. P. Pinheiro, and T. Schwetz, *J. High Energy Phys.* **12** (2024) 216.
- [48] Y. B. Zeldovich, I. Y. Kobzarev, and L. B. Okun, *Zh. Eksp. Teor. Fiz.* **67**, 3 (1974).
- [49] B. Holdom, *Phys. Rev. D* **27**, 332 (1983).
- [50] H. Fleming, *Phys. Lett.* **146B**, 191 (1984).
- [51] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, *J. High Energy Phys.* **10** (2010) 085.
- [52] I. Brivio and M. Trott, *Phys. Rep.* **793**, 1 (2019).
- [53] S. S. C. Law, Neutrino models and leptogenesis, Ph.D. thesis, Melbourne University, 2008, [arXiv:0901.1232](https://arxiv.org/abs/0901.1232).
- [54] G. Källén, *Elementary Particle Physics* (Addison-Wesley, Reading, MA, 1964).
- [55] M. Flanz, E. A. Paschos, and U. Sarkar, *Phys. Lett. B* **345**, 248 (1995).
- [56] M. Flanz, E. A. Paschos, U. Sarkar, and J. Weiss, *Phys. Lett. B* **389**, 693 (1996).
- [57] L. Covi, E. Roulet, and F. Vissani, *Phys. Lett. B* **384**, 169 (1996).
- [58] L. Covi and E. Roulet, *Phys. Lett. B* **399**, 113 (1997).
- [59] T. Altherr and D. Seibert, *Phys. Lett. B* **333**, 149 (1994).
- [60] A. Pilaftsis, *Int. J. Mod. Phys. A* **14**, 1811 (1999).
- [61] C. H. Albright and S. M. Barr, *Phys. Rev. D* **70**, 033013 (2004).
- [62] P. C. Martin and J. Schwinger, *Phys. Rev.* **115**, 1342 (1959).
- [63] L. V. Keldysh, *Sov. Phys. JETP* **20**, 1018 (1965).
- [64] G. Baym and L. P. Kadanoff, *Phys. Rev.* **124**, 287 (1961).
- [65] G. Baym, *Phys. Rev.* **127**, 1391 (1962).
- [66] J. Berges, *AIP Conf. Proc.* **739**, 3 (2004).
- [67] M. Garny, A. Hohenegger, A. Kartavtsev, and M. Lindner, *Phys. Rev. D* **80**, 125027 (2009).
- [68] M. Beneke, B. Garbrecht, C. Fidler, M. Herranen, and P. Schwaller, *Nucl. Phys.* **B843**, 177 (2011).
- [69] M. Beneke, B. Garbrecht, M. Herranen, and P. Schwaller, *Nucl. Phys.* **B838**, 1 (2010).
- [70] J. Råmmer and H. Smith, *Rev. Mod. Phys.* **58**, 323 (1986).
- [71] A. D. Simone and A. Riotto, *J. Cosmol. Astropart. Phys.* **08** (2007) 002.
- [72] E. Wigner, *Phys. Rev.* **40**, 749 (1932).
- [73] K. Ala-Mattinen, M. Heikinheimo, K. Tuominen, and K. Kainulainen, *Phys. Rev. D* **108**, 096034 (2023).
- [74] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963).
- [75] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
- [76] A. M. Abdullahi *et al.*, *J. Phys. G Nucl. Part. Phys.* **50**, 020501 (2023).
- [77] K. Afanaciev *et al.* (MEG II Collaboration), *Eur. Phys. J. C* **85**, 1177 (2025).
- [78] T. S. Roussy, L. Caldwell, T. Wright, W. B. Cairncross, Y. Shagam, K. B. Ng, N. Schlossberger, S. Y. Park, A. Wang, J. Ye, and E. A. Cornell, *Science* **381**, 46 (2023).
- [79] D. P. Aguillard *et al.* (Muon $g - 2$ Collaboration), *Phys. Rev. Lett.* **131**, 161802 (2023).
- [80] N. Aghanim *et al.* (Planck Collaboration), *Astron. Astrophys.* **641**, A6 (2020).
- [81] M. Aker *et al.* (KATRIN Collaboration), *Science* **388**, adq9592 (2025).
- [82] E. E. Jenkins, A. V. Manohar, and M. Trott, *J. High Energy Phys.* **10** (2013) 087.
- [83] E. E. Jenkins, A. V. Manohar, and M. Trott, *J. High Energy Phys.* **01** (2014) 035.
- [84] J. C. Ward, *Phys. Rev.* **78**, 182 (1950).
- [85] Y. Takahashi, *Nuovo Cimento* (1955–1965) **6**, 371 (1957).
- [86] A. A. Slavnov, *Theor. Math. Phys.* **10**, 99 (1972).
- [87] J. Taylor, *Nucl. Phys.* **B33**, 436 (1971).
- [88] G. ’t Hooft, *Nucl. Phys.* **B33**, 173 (1971).
- [89] C. Becchi, A. Rouet, and R. Stora, *Phys. Lett.* **52B**, 344 (1974).
- [90] C. Becchi, A. Rouet, and R. Stora, *Commun. Math. Phys.* **42**, 127 (1975).
- [91] C. Becchi, A. Rouet, and R. Stora, *Ann. Phys. (N.Y.)* **98**, 287 (1976).

- [92] I. V. Tyutin, [arXiv:0812.0580](#).
- [93] L. D. Faddeev and V. N. Popov, *Phys. Lett.* **25B**, 29 (1967).
- [94] T. Kugo, *Quantum Theory of Gauge Fields*, New Physics Series No. 24 Vol. II (Baifukan, Tokyo, 1989).
- [95] M. D. Schwartz, *Quantum Field Theory and the Standard Model* (Cambridge University Press, Cambridge, England, 2013).
- [96] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, MA, 1995).
- [97] M. E. Machacek and M. T. Vaughn, *Nucl. Phys.* **B236**, 221 (1984).
- [98] R. E. Cutkosky, *J. Math. Phys. (N.Y.)* **1**, 429 (1960).
- [99] A. Denner, H. Eck, O. Hahn, and J. Kublbeck, *Nucl. Phys.* **B387**, 467 (1992).
- [100] J. Knoll, Y. Ivanov, and D. Voskresensky, *Ann. Phys. (N.Y.)* **293**, 126 (2001).
- [101] P. Danielewicz, *Ann. Phys. (N.Y.)* **152**, 239 (1984).
- [102] W. Botermans and R. Malfliet, *Phys. Rep.* **198**, 115 (1990).
- [103] T. Kugo, *Quantum Theory of Gauge Fields*, New Physics Series No. 23 Vol. I (Baifukan, Tokyo, 1989).
- [104] C. Møller, K. Dan. Vidensk. Selsk., Mat.-Fys. Medd. **23**, 1 (1945), Original source for the invariant flux factor (“Møller flux”).
- [105] E. Braaten and R. D. Pisarski, *Nucl. Phys.* **B337**, 569 (1990).
- [106] R. D. Pisarski, *Phys. Rev. Lett.* **63**, 1129 (1989).
- [107] M. Le Bellac, *Thermal Field Theory*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 1996).
- [108] G. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia, *Nucl. Phys.* **B685**, 89 (2004).
- [109] H. A. Weldon, *Phys. Rev. D* **26**, 1394 (1982).
- [110] D. C. Langreth and J. W. Wilkins, *Phys. Rev. B* **6**, 3189 (1972).
- [111] D. C. Langreth, in *Linear and Nonlinear Electron Transport in Solids*, edited by J. T. Devreese and V. E. van Doren (Springer US, Boston, MA, 1976), pp. 3–32.
- [112] J. E. Moyal, *Math. Proc. Cambridge Philos. Soc.* **45**, 99 (1949).
- [113] P. Lipavský, V. Špička, and B. Velický, *Phys. Rev. B* **34**, 6933 (1986).
- [114] J. S. Toll, *Phys. Rev.* **104**, 1760 (1956).
- [115] R. Kubo, *J. Phys. Soc. Jpn.* **12**, 570 (1957).
- [116] H. Araki, *Prog. Theor. Phys. Suppl.* **64**, 12 (1978).
- [117] S. Eijima, M. Shaposhnikov, and I. Timiryasov, *J. Cosmol. Astropart. Phys.* **11** (2017) 030.