

Seiberg-Witten axion

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We present a calculable toy model of an axion coupled to magnetic monopoles and electric charges, with manifest electric-magnetic duality. Our axion-photon coupling incurs periodic corrections due to monopoles (equivalently UV instantons), which are large for light monopoles. Axion periodicity is manifestly duality invariant since the axion itself transforms under duality. Our model is the original Seiberg-Witten theory, which naturally hosts an $\mathcal{R}$ -axion and light monopoles. We calculate the axion-photon amplitude in different duality frames and prove its duality invariance.

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I. INTRODUCTION

QCD axions are widely studied theoretically and experimentally, as they provide an elegant solution to the strong CP problem and may also be the main component of dark matter. As a Goldstone boson (GB) of the anomalous Peccei-Quinn (PQ) symmetry, the axion is an angle, and the action is invariant under $a \rightarrow a + 2\pi f$, where f is the vacuum expectation value (VEV) of the PQ scalar. Correspondingly, the PQ anomaly-induced axion-photon coupling is widely thought to be quantized, forming the target for QCD axion searches. A more general formulation, in which the axions are also coupled to magnetically charged particles, was explored in Refs. [1–3]. The authors found that the inclusion of fermionic monopoles charged under the PQ symmetry modifies the form of the axion-photon coupling and, naïvely, seems to destroy its quantization. However, Ref. [4] claimed that such a lack of quantization is internally inconsistent and proposed a fix for this apparent inconsistency, by ascribing a transformation rule for the field strength F under $a \rightarrow a + 2\pi f$. Unfortunately, the latter fix leads to the theory being strongly ruled out phenomenologically.

In this manuscript, we present a fully calculable toy model of a PQ axion with fermionic magnetic monopoles carrying a nonvanishing PQ charge. This allows us to examine the form of the photon-axion coupling and to better understand the dynamics generating these couplings. We find that the form of the axion coupling to $F_{\mu\nu}\tilde{F}^{\mu\nu}$ contains additional dynamically generated periodic terms, while the IR effective field theory (EFT) is manifestly covariant under electromagnetic (EM) duality. One key aspect that helps reconcile the seeming contradiction between duality and coupling quantization is that the axion itself transforms nonlinearly under the electric-magnetic duality. Moreover, the axion is accompanied by a radial mode (also known as the dilaton or saxion), and the two generally mix under duality. In the electric frame, the contribution of the electric charges appear as the usual anomaly induced linear term, while the contribution of all magnetically charged states appears as nonperturbative effects (which can be identified with instantons). These nonperturbative effects give rise to periodic functions hence automatically invariant under the discrete axion shift symmetry. In the magnetic frame, it is the contribution of the magnetic charges that gives the linear coupling of the *dual axion*, while the contribution of the electric charges appears as nonperturbative effects. As an ultimate test of duality, we show that the canonically normalized axion-photon matrix elements are invariant (up to a phase) under duality transformations, as they must be, since duality gives two different descriptions of the exact same physics [5].

Our axion model is none other than the original $\mathcal{N} = 2$ $SU(2)$ gauge theory without fundamental matter studied by Seiberg and Witten (SW) in their seminal work [6]. Surprisingly, this theory is also a prime example for studying the properties of a strongly coupled axion in a

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calculable, UV complete theory. The theory has a global $U(1)_{\mathcal{R}}$ symmetry which is anomalous under the strongly coupled $SU(2)$ group. This theory has a moduli space of vacua, with the $SU(2)$ gauge group broken to $U(1)$ —“the Coulomb branch.” On this moduli space, the anomalous $U(1)_{\mathcal{R}}$ global symmetry is also spontaneously broken, providing a GB, the axion. Most importantly, at a specific point in the moduli space, a magnetic monopole hypermultiplet becomes massless (while at another point, a dyon becomes massless). The monopole (and the dyon) are charged $U(1)_{\mathcal{R}}$, providing a nontrivial example of an axion model with a light PQ-charged magnetic monopole. The IR dynamics of the model is $\mathcal{N} = 2$ Super Quantum Electrodynamics which is manifestly covariant under EM $SL(2, \mathbb{Z})$ duality. In the electric frame, the axion, a , is identified with the phase of the scalar superpartner of the photon A_{μ} , while in the magnetic frame, the dual axion a_D is the phase of the scalar superpartner of the dual photon A_{μ}^D . By supersymmetry, there are also massless saxions ρ (ρ_D) describing radial fluctuations in the A (A_D). Generically, $a_D(\rho_D)$ is a complex nonlinear function of a , ρ , and their mixing is akin to the mixing of photon helicities under duality [5].

Below, we will see that the axion a in the electric frame couples as

$$-\frac{e^2 F_{\mu\nu} \tilde{F}^{\mu\nu}}{16\pi^2} \left\{ \frac{N_a \bar{a}}{f} - \sum_{k=1}^{\infty} \tilde{d}_k \left(\frac{\sqrt{2}\Lambda}{ef} \right)^{4k} \sin \left(4k \frac{\bar{a}}{f} \right) \right\}, \quad (1)$$

where $\bar{a} = a - (\theta_p/8)f$ is the axion shifted by its VEV and N_a is an integer reflecting the perturbative PQ anomaly. Here, $\tilde{d}_k = (4k-1)(4k-2)d_k$, where $d_k \in \mathbb{R}$ are the well-known instanton coefficients d_k of the prepotential, fully calculable in SW theory. As pointed out in the original paper, the instanton contribution is equivalent to the sum of all quantum contributions from heavy BPS monopoles and dyons.¹ The contributions from an individual BPS state to the anomaly were first calculated in Ref. [10] using EM duality, and the explicit SW solution shows that they must sum up to the periodic terms in (1). In the magnetic dual theory, one obtains an analogous expression, where now the contribution of the magnetic monopole will be interpreted in terms of the perturbative anomaly, while the effects of the electrically charged objects will give another periodic contribution due to nonperturbative effects.

For large VEVs, the scale f of spontaneous R -symmetry breaking is much larger than the scale Λ , at which the theory becomes strongly coupled. In this regime, the SW axion resembles the QCD axion, for which $\Lambda_{\text{QCD}}/f_{\text{PQ}} \ll 1$. As one moves closer to the strongly coupled regime, the scale

separation between f and Λ is lost, and the SW axion becomes an analog of the η' state of QCD (with the caveat that supersymmetry still ensures that the axion and saxion remain massless). In particular, the corrections to the axion coupling become large around the monopole point where $f \lesssim \Lambda$. In this regime, we expect that an addition of supersymmetry breaking would result in axion and saxion masses of the same order as the supersymmetry (SUSY) breaking scale. It would be very interesting to try and model build a scenario in which the masses of the two are split. While our study focuses on a toy model and not a full-fledged phenomenologically viable extension of the Standard Model, it does have two major direct implications for the search for real-world axions and axionlike particles. The first is that the axion coupling need not be quantized in integer multiples of $e^2/16\pi^2$ and can even be *large*, depending on the value of b_k . The second is that axion electrodynamics could be manifestly covariant under EM duality and need not suffer from the phenomenological deal breakers outlined in Ref. [4]. This also implies that the implementation of duality in the seemingly nonduality covariant equations ubiquitous in the literature [11,12] will have to be reexamined, with potential consequences for different search methods [13,14].

II. SEIBERG-WITTEN $\mathcal{R}$ AXION

SW theory² has an anomalous $U(1)_{\mathcal{R}}$ symmetry, which is spontaneously broken by the VEV of the adjoint scalar and plays the role of a Peccei-Quinn [15] symmetry. In the UV, the phase of the adjoint then plays the role of the ($\mathcal{R}$) axion. The $\mathcal{R}$ axion has been studied in different contexts [16–25] including collider phenomenology [26–28]. In the low-energy effective theory, there is only³ an $\mathcal{N} = 2$ gauge multiplet, A , whose scalar component has charge 2 under the IR $\mathcal{R}$ symmetry [29]. The IR axion $a(x)$ is then a spatially dependent *phase* of that scalar component of A , i.e., $A(x) = (e(u)/\sqrt{2})[f(u) + \rho(x)] \exp\{ia(x)/f(u) + i\theta_p(u)\}$, where $f(u) = \sqrt{2}|A^v(u)|/e(u)$ is the axion decay constant, and $A^v(u)$ is the (generally complex) VEV of A as a function of the Coulomb branch coordinate u , $\theta_p = \arg A^v$, all chosen so that the axion kinetic term is canonically normalized. Note that we have also included the massless radial mode excitation $\rho(x)$ (i.e., the saxion) which mixes with the axion under $SL(2, \mathbb{Z})$. To reduce notational clutter, for the rest of the discussion, we will suppress the explicit u dependence of A^v , f , e , and θ_p and $\rho(x)$ dependence of A .

²We provide a brief review of SW theory in Appendix A.

³At two isolated singularities on the moduli space, there are additional massless hypermultiplets—these are BPS monopoles/dyons that become massless at the singularities. Moving slightly away from these singularities, these BPS states get a mass, and we can integrate them out.

¹The nonperturbative correction to the axion-photon coupling is analogous to the axion potential calculated in Ref. [7]. It emerges from the UV breaking of a magnetic 1-form symmetry participating in a higher group with PQ symmetry [8,9].

First, note that the SW solution fixes

$$\tau = \frac{4\pi i}{e_\rho^2(\rho)} - \frac{8\bar{a}}{2\pi f} - \frac{1}{2\pi} G(\bar{a}, \rho)$$

$$G(\bar{a}, \rho) \equiv i \sum_{k=1}^{\infty} \tilde{d}_k \left(\frac{\sqrt{2}\Lambda}{e(f+\rho)} \right)^{4k} \exp\left(-i \frac{4k\bar{a}}{f}\right), \quad (2)$$

where $e_\rho^2(\rho) = \pi^2 / \log[\sqrt{2}e(f+\rho)/\Lambda]$ is the perturbative contribution to the coupling and $\theta_p = -8 \arg(A^v)$ is the perturbative theta angle. From the expression (2) for τ , we learn that it has a) a real term which is linear in the axion (this will give the perturbative part of the axion coupling) as well as b) real and imaginary terms that are trigonometric in the axion and nonlinear in the saxion ρ . These constitute an explicit breaking of the axion shift symmetry by the instanton corrections and are automatically consistent with the $a \rightarrow a + 2\pi f$ shift of the axion. However, the a and ρ masses remain protected by SUSY. The attentive reader may note that the real part of τ couples to $F\tilde{F}$ in the Lagrangian, while the imaginary part couples to F^2 . In terms of (2), the relevant terms for the axion-photon coupling in the IR Lagrangian are given by

$$\mathcal{L}_{\text{IR}} \supset \left(-\frac{1}{2e_\rho^2(\rho)} + \frac{\text{Im} G(\bar{a}, \rho)}{16\pi^2} \right) F_{\mu\nu} F^{\mu\nu} + \left(\frac{8\bar{a}}{16\pi^2} + \frac{\text{Re} G(\bar{a}, \rho)}{16\pi^2} \right) F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (3)$$

where the instanton corrections are encapsulated in the $G(\bar{a}, \rho)$ function (see Appendix B for more details about the generation of these terms). We emphasize that θ_p here coincides with the IR θ angle of the theory only at *weak* coupling, while at strong coupling $|A^v| \sim \Lambda$, instanton corrections set $\theta = \text{Re}\{\tau(a=\rho=0)\} \neq \theta_p$. We note that θ_p is physical since these instanton corrections explicitly break the axion shift symmetry.

Expanding τ to linear order in a and ρ , we find the canonically normalized⁴ scalar-photon interactions

$$\mathcal{L}_{\text{IR}} \supset \frac{1}{4} (a \ \rho) \begin{pmatrix} \text{Re}(g_{a\gamma\gamma}) & \text{Im}(g_{a\gamma\gamma}) \\ \text{Im}(g_{a\gamma\gamma}) & -\text{Re}(g_{a\gamma\gamma}) \end{pmatrix} \begin{pmatrix} F_{\mu\nu} \tilde{F}^{\mu\nu} \\ F_{\mu\nu} F^{\mu\nu} \end{pmatrix}, \quad (4)$$

where the complex axion coupling is defined as

$$\frac{g_{a\gamma\gamma}}{4} = \frac{e^2}{16\pi^2 f} c_{a\gamma\gamma} = \frac{\sqrt{2}ie^3}{16\pi} e^{i\theta_p} \frac{\partial \tau}{\partial A} \Big|_{a,\rho=0}. \quad (5)$$

⁴We choose to work in units where electric and magnetic charges are integers. In these units, the photon kinetic term is normalized by $1/2$.

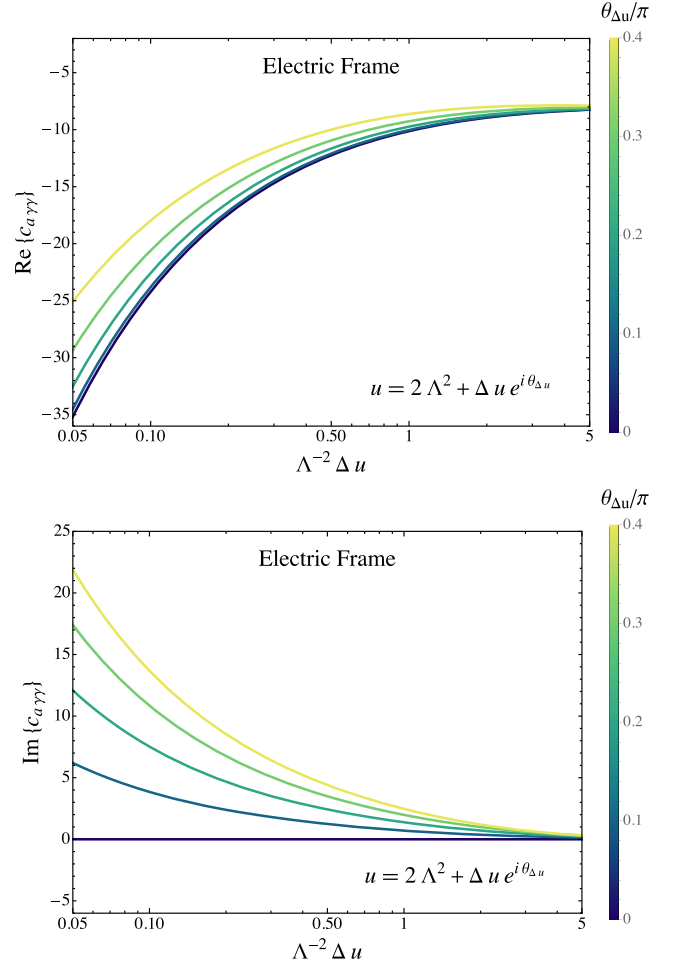


FIG. 1. The real and imaginary parts of the axion coupling coefficient $c_{a\gamma\gamma}$ in the electric duality frame. The coupling is plotted as a function of the distance Δu from the monopole singularity, at different angles in the complex u plane. Evidently, $\text{Re}\{c_{a\gamma\gamma}\} \rightarrow -8$ and $\text{Im}\{c_{a\gamma\gamma}\} \rightarrow 0$ at weak coupling $\Delta u \rightarrow \infty$. $\text{Re}\{c_{a\gamma\gamma}\}$ diverges at the monopole point $\Delta u = 0$. Similarly, $\text{Im}\{c_{a\gamma\gamma}\}$ diverges at the monopole point for $\theta_{\Delta u} \neq 0$.

Here, $\text{Re}\{c_{a\gamma\gamma}\}$ is identified with the coefficient of $(e^2/16\pi^2 f)aF\tilde{F}$, and $\text{Re}\{g_{a\gamma\gamma}\}$ is identified with the coefficient of $(1/4)aF\tilde{F}$, both ubiquitous in the literature. Since we have not set $\theta_p = 0$, we have the additional parity violating aF^2 coupling, which generically appears in the one-loop anomaly of dyonic states [10]. In terms of the prepotential instanton expansion, we can explicitly identify the perturbative and nonperturbative contributions to the axion coupling

$$c_{a\gamma\gamma} = -8 - \sum_{k=1}^{\infty} 4k\tilde{d}_k e^{-4ik\theta_p} \left(\frac{\sqrt{2}\Lambda}{ef} \right)^{4k}. \quad (6)$$

The -8 in (6) is the perturbative anomaly coming from the four gauginos in $W_{\pm}$ which have electric charge 2, whereas the instantons account for the contributions from the BPS

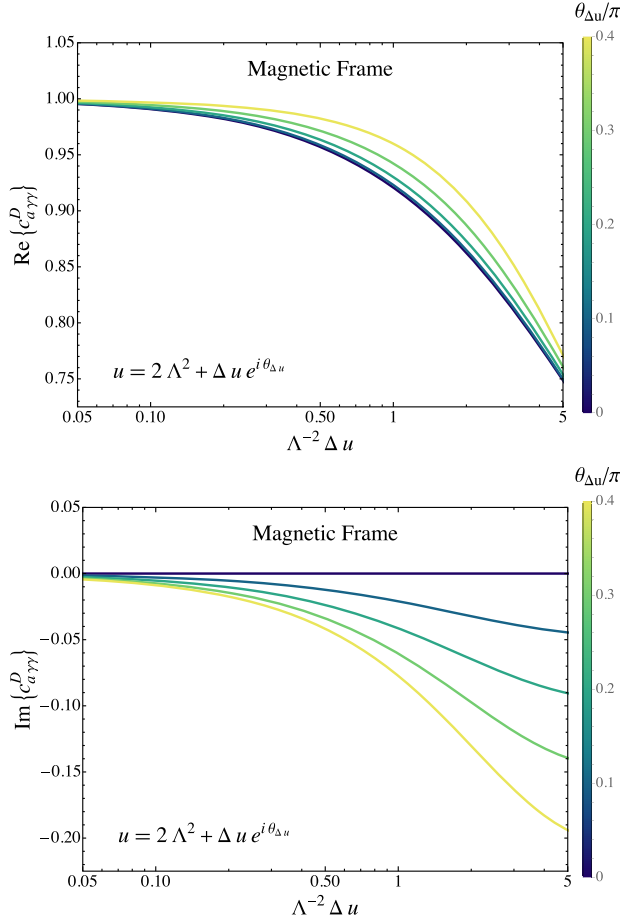


FIG. 2. The real and imaginary parts of the axion coupling coefficient $c_{a\gamma\gamma}^D$ in the magnetic duality frame. The factors are plotted as a function of the distance Δu from the monopole singularity, at different angles in the complex u plane. Evidently, $\text{Re}\{c_{a\gamma\gamma}^D\} \rightarrow 1$ and $\text{Im}\{c_{a\gamma\gamma}^D\} \rightarrow 0$ at the monopole point $\Delta u = 0$ and significantly deviate from these values at weak coupling.

states. In Fig. 1, we present the values of $c_{a\gamma\gamma}$. We find that the coupling diverges near the monopole point where $u \rightarrow 2\Lambda^2$.

III. S-DUAL AXION

To understand this divergence better, we can go to the S -dual description where the monopole is perturbatively coupled to the photon, referred to as the *magnetic duality frame*. The monopole has $\mathcal{R}$ -charge 1 leading to a contribution to the anomaly. Similarly as before, we define the dual axion a_D to be the GB of the anomalous $U(1)_{\mathcal{R}}$ symmetry.⁵ The axion-photon coupling can be discerned from the dual prepotential, which reveals a nonlinear

⁵It is nontrivial to relate the $U(1)_{\mathcal{R}}$ symmetries of the weak ($u \rightarrow \infty$) and strongly coupled ($u \rightarrow 2\Lambda^2$) theories, though we can say for certain they both originate from the same UV $U(1)_{\mathcal{R}}$ symmetry.

relation between a , ρ of electric frame and a_D , ρ_D of the magnetic frame. In other words, the axion and saxion transform (and generally mix) under duality.

Explicitly, we find that $A_D(x)$ as $A_D(x) = (e_D/\sqrt{2})[f_D + \rho_D(x)]\exp\{ia_D(x)/f_D(u) + i\theta_{p,D}\}$, where $f_D = \sqrt{2}|A_D^v|/e_D$ and $\theta_{p,D} = \arg A_D^v$. Repeating the same procedure as before, we find the same form for linearized IR Lagrangian (4) with appropriate D subscripts denoting the dual frame fields. Similarly, the linearized scalar photon couplings take the same form as Eq. (5) with

$$c_{a\gamma\gamma}^D = 1 + \sum_{k=1}^{\infty} \frac{k}{2} \tilde{d}_k^D e^{ik\theta_{p,D}} \left(\frac{e_D f_D}{\sqrt{2}\Lambda} \right)^k, \quad (7)$$

where $\tilde{d}_k^D = (k+1)(k+2)d_k^D$ and d_k^D are the instanton coefficients of the dual prepotential (given in Appendix C). Since in this frame the monopole is *weakly* coupled, the constant term in $c_{a\gamma\gamma}^D$ is interpreted as the perturbative chiral anomaly from the magnetic monopole, whereas the instanton contributions describe contributions from the gauginos and other BPS states. In Fig. 2, we present the axion coupling $c_{a\gamma\gamma}^D$ in the magnetic frame. We find that the dominant contribution is the perturbative coupling from the monopole. As we will subsequently show, the seeming tension between $c_{a\gamma\gamma}$ and $c_{a\gamma\gamma}^D$ is resolved by the different prefactors e^2/f and e_D^2/f_D in the two frames.

IV. DUALITY INVARIANCE AND THE $a\gamma\gamma$ AMPLITUDE

The two frames provide equivalent (in this case S -dual) descriptions of the same IR physics. Therefore, we expect that all physical observables computed from these equivalent Lagrangians agree. We show this analytically for the rate of the process $a\gamma\gamma$. Moreover, we will show that this rate becomes *large* close to the monopole singularity, in stark contrast with the standard PQ scenario. Analyzing the situation in both the electric and magnetic duality frame will provide a complete understanding of why this rate becomes large.

Note that in the presence of a nonzero vacuum theta angle θ the axion couples also to FF in (3) (and similarly to $F_D F_D$ in the magnetic frame). Thus, when we compute the rate, we have to sum over the squares of the amplitudes $\mathcal{M}_{\pm\pm}, \mathcal{M}_{\pm\mp}$ generated by $aF\tilde{F}$ and by aFF . Altogether, the rate is

$$\Gamma_{a\gamma\gamma}^{\text{el}} \propto \sum_{h,h'} |\mathcal{M}_{a\gamma(h)\gamma(h')}|^2 \propto |g_{a\gamma\gamma}|^2. \quad (8)$$

In the magnetic frame, we get analogously $\Gamma_{a\gamma\gamma}^{\text{mag}} \propto |g_{a\gamma\gamma}^D|^2$. The point is that $|g_{a\gamma\gamma}|^2$ is in fact *duality invariant*. Using the canonical relation $\tau = -\tau_D^{-1}$, the Seiberg-Witten

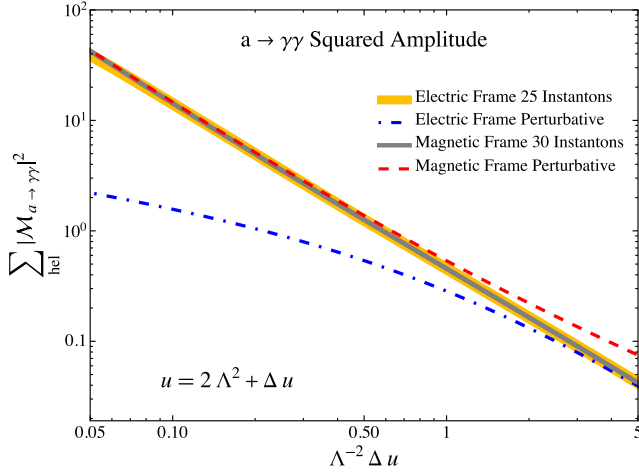


FIG. 3. The duality-invariant sum $|\mathcal{M}_{++}|^2 + |\mathcal{M}_{--}|^2$ as calculated in different frames. Orange: calculated from $\tau(A)$ with 25 instanton terms. Gray: calculated from $\tau_D(A_D)$ with 30 instanton terms. Blue: keeping only the perturbative contribution to $\tau(A)$. Red: keeping only the perturbative contribution to $\tau_D(A_D)$. Near the monopole point, we see that the nonperturbative instanton contributions in the electric frame sum up exactly to the perturbative contribution in the magnetic frame.

relation $\partial A/\partial A_D$, and the holomorphic properties of the prepotential, we find

$$e^3 \frac{\partial \tau}{\partial A} = e^{i\pi + i\theta_p - i\theta_{p,D} - 3i \arg(\tau_D)} e_D^3 \frac{\partial \tau_D}{\partial A_D}, \quad (9)$$

meaning the amplitudes in the two frames are the same up to a phase (for more detail, see Appendix D). Since the different photon helicities are physical observables, this phase indicates the leading-order (linear) mixing of the a, ρ into a_D, ρ_D according to Eq. (4). The mixing of these degrees of freedom is analogous to the mixing of the photon helicities under EM duality [5]. Note that at $\theta_p = 0$ this phase culminates in a relative minus sign between the couplings, which implies no mixing. Consequently, we have $\Gamma_{a\gamma\gamma}^{\text{el}} = \Gamma_{a_D\gamma_D\gamma_D}^{\text{mag}}$, as had to be the case, since the two frames describe the same physics. In Fig. 3, we plot this duality-invariant sum as a function of the Coulomb branch coordinate u at $\theta_p = 0$. Remarkably, we see that the squared amplitude diverges as we approach the monopole point. This is very different from the usual PQ mechanism, due to the large instanton corrections. Furthermore, we see that near the monopole point the decay rate is dominated by the monopole anomaly contribution and far from the monopole point the decay rate is dominated by the gaugino anomaly.

Lastly, we comment on our identification of the axion and saxion directions in the electric frame. At weak coupling, the $U(1)_{\mathcal{R}}$ is a good symmetry, and we can easily identify the axion and saxion as the GB and radial fluctuation of the VEV A^v . In this paper, we choose to

extrapolate this definition to strong coupling as well—but of course the physics is invariant to our choice of axion vs saxion directions in internal space. Having said that, our identification of the axion and saxion directions at strong coupling is natural in the sense that the gaugino mass $M = e|f + \rho(x)|$ is always independent of the axion. Nevertheless, we could have alternatively defined “non-standard axion electrodynamics” in the spirit of Ref. [4], by *remaining in the electric frame*, yet choosing the axion direction to interpolate to $\arg(A_D)$, rather than $\arg(A)$ at strong coupling. For the latter choice, we would find that gaugino masses at strong coupling do in fact depend on the axion, a phenomenologically disfavored option dubbed the “dual Witten effect.” In our conventions, this dual Witten effect is simply the dependence of gaugino mass on the saxion.

V. CONCLUSIONS

In this paper, we gave a concrete proof of principle showing that axion couplings to photons need not be quantized, as it is often assumed, or even small. The additional contributions originate from nonperturbative effects due to monopoles (and other BPS states), which can also be interpreted as a sum of instanton corrections. These terms are periodic functions of the axion; hence, coupling quantization is trivially satisfied. Theories with magnetic monopoles, like Seiberg-Witten, give a concrete realization of the issues involved and are consistent with the one-loop-exact holomorphic calculation of the anomaly given in Ref. [10]. Our results have many implications for axion searches that we hope to follow up in future work. What are the correct equations of motion for axion electrodynamics when the Bianchi identity is not imposed? What is a safe and self-consistent way to account for these new contributions to the axion coupling when we do not have a nonperturbative calculation of the low-energy effective theory?

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: OVERVIEW OF SEIBERG-WITTEN THEORY

The $\mathcal{N} = 2$ supersymmetric $SU(2)$ gauge theory without fundamental matter fields, analyzed by Seiberg and Witten [6], provides a nice laboratory to explore axion couplings to mutually nonlocal charges.⁶ The IR dynamics of Seiberg-Witten theory involves a continuum of degenerate $\mathcal{N} = 2$ vacua parametrized by a complex coordinate u . In all of these vacua, the $SU(2)$ gauge symmetry is Higgsed to $U(1)$ by the adjoint scalar ϕ in the $SU(2)$ gauge multiplet. For this reason, the continuum of vacua is also called the *Coulomb branch*. The Higgsing gives masses to $W^\pm$ gauge bosons and their four chargino superpartners.

The moduli space of the Coulomb branch is parametrized by a complex coordinate $u \equiv \frac{1}{2}\text{tr}(\phi^2)$, where $u \neq 0$. The IR dynamics is given in terms of the photon⁷ (A, V) or, equivalently, the “dual photon” (A_D, V_D). The two are not separate degrees of freedom but rather nonlinear functions of each other, with the chiral superfield components related by

⁶See Refs. [30,31] for excellent reviews of Seiberg-Witten theory. Here, we follow the conventions of Ref. [31].

⁷The $\mathcal{N} = 2$ gauge multiplet of the photon consists of an $\mathcal{N} = 1$ chiral superfield A and an $\mathcal{N} = 1$ vector superfield V , whose supersymmetric field strength is W_α .

$$A_D = \frac{\partial \mathcal{F}(A)}{\partial A}, \quad A = -\frac{\partial \mathcal{F}_D(A_D)}{\partial A_D}. \quad (\text{A1})$$

Here, $\mathcal{F}(A)$ is the exactly calculable [32] *prepotential*, and $\mathcal{F}_D$ is its exactly calculable Legendre transform, the *dual prepotential*. The VEVs $A^v(u)$ and $A_D^v(u)$ are also known explicitly for Seiberg-Witten theory and are given by

$$A^v(u) = \sqrt{u + 2\Lambda^2} {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}, 1; \frac{4\Lambda^2}{u + 2\Lambda^2}\right) \\ A_D^v(u) = i \frac{u - 2\Lambda^2}{2\Lambda} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}, 2; \frac{2\Lambda^2 - u}{4\Lambda^2}\right), \quad (\text{A2})$$

where ${}_2F_1$ is the *Gauss hypergeometric function*. Note that $A(u)$ has branch points at $u = \pm 2\Lambda^2$, while $A_D(u)$ has a branch point at $u = -2\Lambda^2$. At these branch points, BPS states become exactly massless. From the monodromies of $(A_D(u), A(u))$ from (A2) around these points, we can read off the charges of the BPS states that become massless at these points; at $u = 2\Lambda^2$, this is a monopole hypermultiplet with EM charges $(g, q) = (1, 0)$ (in the language of the weak coupling), while at $u = -2\Lambda^2$, they are dyons of charge $(g, q) = (1, 2)$. Finally, the holomorphic gauge coupling on the moduli space τ is exactly computable on the Coulomb branch and is given by

$$\tau = \frac{\partial^2 \mathcal{F}}{\partial A^2} \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}. \quad (\text{A3})$$

Note that τ is a nonlinear function of A as an $\mathcal{N} = 1$ *chiral superfield*. To get the numerical values of e and θ , we have to substitute the VEV $A(u)$ from (A2). The low-energy physics is completely determined by $\mathcal{F}(A)$, namely,

$$\mathcal{L}_{\text{IR}} = \frac{1}{8\pi i} \int d^4\theta \frac{\partial \mathcal{F}}{\partial A} \bar{A} + \frac{1}{8\pi i} \int d^2\theta \tau(A) W^\alpha W_\alpha + \text{H.c.} \quad (\text{A4})$$

The action is canonically normalized by rescaling the fields as $A \rightarrow eA$. However, this should be done with caution since the prepotential is only holomorphic in the original fields.

We now briefly describe two important regions of the Coulomb branch, namely, weak coupling and the strong coupling singularities.

1. Weak coupling

At large $|u|$, the theory is Higgsed at weak coupling, and the Coulomb branch coordinate u coincides with $\frac{1}{2}\text{tr}\phi^2 = A^2$. In this weak coupling regime, the spectrum of the theory consists of the massless photon multiplet A , two $W^\pm$ multiplets getting their mass from the adjoint VEV $u = \frac{1}{2}\text{tr}\phi^2 = A^2$, and a tower of heavy semiclassical

't Hooft-Polyakov monopoles and their dyonic excitations, all BPS states whose mass satisfies

$$m_{(g,q)}^{BPS} = |A^v(u)q + A_D^v(u)g|, \quad (\text{A5})$$

where (g, q) are the magnetic and electric charges of every state in the tower.

2. Strong coupling

At $u \sim \Lambda^2$, the IR EFT, expressed in terms of the photon A , is strongly coupled. In the vicinity of the $u = 2\Lambda^2$ singularity, we can work with the dual complexified gauge coupling $\tau_D \equiv \frac{\theta_D}{2\pi} + \frac{4\pi i}{e_D^2}$, given by $\tau_D = \partial^2 \mathcal{F}_D(A_D)/\partial A_D^2$ and related to τ via S -duality $\tau_D = -1/\tau$. One can verify that e_D runs logarithmically to 0 at $u = 2\Lambda^2$. In the neighborhood of $u = 2\Lambda^2$, the charge $(1, 0)$ *monopole hypermultiplet* $(M, \tilde{M})$ becomes light, and there is an additional Yukawa term in the superpotential,

$$W = A_D M \tilde{M}. \quad (\text{A6})$$

Exactly at $u = 2\Lambda^2$, $A_D = 0$, and the monopoles become massless. In this regime, we can use an equivalent, S -dual description of the same IR physics in terms of the dual photon A_D and the dual prepotential $\mathcal{F}_D(A_D)$, namely,

$$\begin{aligned} \mathcal{L}_{\text{IR}}^D = & \frac{1}{8\pi i} \int d^4\theta \frac{\partial \mathcal{F}_D}{\partial A_D} \bar{A}_D \\ & + \frac{1}{8\pi i} \int d^2\theta \tau_D(A_D) W_D^\alpha W_{D\alpha} + \text{H.c.}, \end{aligned} \quad (\text{A7})$$

in addition to the monopole term (A6).

The *exact* expressions for the prepotential $\mathcal{F}(A)$ and the dual prepotential $\mathcal{F}_D(A_D)$ [which is the Legendre transform of $\mathcal{F}(A)$] are given by [6,31–34]

$$\begin{aligned} \mathcal{F}(A) = & \frac{1}{2\pi i} \left\{ -4A^2 \left(\ln \frac{2A}{\Lambda} - \frac{3}{2} \right) + A^2 \sum_{k=1}^{\infty} d_k \left(\frac{\Lambda}{A} \right)^{4k} \right\} \\ \mathcal{F}_D(A_D) = & \frac{1}{4\pi i} \left\{ A_D^2 \left(\ln \frac{-iA_D}{32\Lambda} - \frac{3}{2} \right) - 16iA_D \right. \\ & \left. + A_D^2 \sum_{k=1}^{\infty} d_k^D \left(\frac{A_D}{\Lambda} \right)^k \right\}. \end{aligned} \quad (\text{A8})$$

The first term in $\mathcal{F}(A)$ corresponds to a one-loop contribution from the massive gauge multiplets, while the first term in $\mathcal{F}_D(A_D)$ corresponds to the one-loop contribution from the light monopoles $M, \tilde{M}$. We refer to these contributions as perturbative, denoting them with a p subscript. The d_k and d_k^D are known instanton coefficients calculated indirectly in Refs. [29,33] and directly in Ref. [32]. We computed their explicit values up to 25 and 30 instantons, respectively, using the method of Refs. [31,34]. We

demonstrate the convergence of these instanton expansions in Appendix C.

We have seen that the IR dynamics of Seiberg-Witten theory is encapsulated in the analytic structure of the function $A(u), A_D(u)$ given in (A2) (in fact, only from their monodromies around their branch points). Furthermore, this information is preserved under *EM duality transformations* in the duality group $SL(2, \mathbb{Z})$. Two transformations, S and T , generate $SL(2, \mathbb{Z})$. S corresponds to taking $a = d = 0$, $b = 1$, and $c = -1$ and thus implements the textbook symmetry of electromagnetism that interchanges the electric and magnetic fields: $\mathbf{E} \rightarrow \mathbf{B}$, $\mathbf{B} \rightarrow -\mathbf{E}$. T corresponds to $a = b = d = 1$ and $c = 0$, which simply shifts θ by 2π and is well known to be an exact symmetry. A general duality transformation is described by a matrix,

$$\mathcal{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad a, b, c, d \in \mathbb{Z}, \quad (\text{A9})$$

where $ad - bc = 1$, under which (A_D, A) transform as $(A_D, A) \rightarrow \mathcal{M}(A_D, A)$. By (A3), τ transforms under duality as

$$\tau' = \frac{a\tau + b}{c\tau + d}, \quad (\text{A10})$$

while the charges of BPS states transform as

$$\begin{pmatrix} g' \\ q' \end{pmatrix} = \mathcal{M}^{-1T} \begin{pmatrix} g \\ q \end{pmatrix}. \quad (\text{A11})$$

Note that duality preserves the masses of the BPS states (A5) regardless of our identification of the axion in the low energy EFT of SW theory.

APPENDIX B: AXION COUPLING FROM INSTANTONS

The axion-photon coupling receives perturbative and non-perturbative contributions. While the perturbative contribution is readily understood in terms of the triangle anomaly of the $U(1)_{\mathcal{R}}$ current, the non-perturbative, $2\pi f$ -shift-symmetric contributions originate from instantons in the UV theory which generate terms in the holomorphic pre-potential $\mathcal{F}(A)$. In the following we will sketch how the one-instanton contribution is obtained. We work in the weak-coupling regime far out in moduli space, where a semi-classical instanton expansion is valid and where the low-energy degrees of freedom can be trivially identified with the UV degrees of freedom along the unbroken direction. For more details see [35–38].

In $\mathcal{N} = 1$ language the UV theory contains a vector multiplet (v^a, λ^a, D^a) and a chiral multiplet (A^a, ψ^a, F^a) , both in the adjoint representation of $SU(2)$. In the instanton background the gauginos and the fermions in the chiral

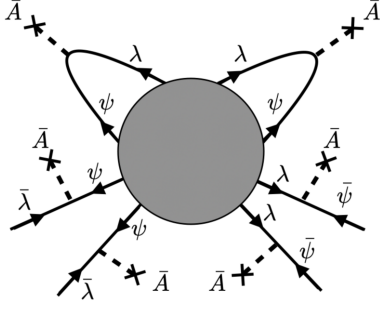


FIG. 4. One-instanton contribution to the correlator $\langle \bar{\psi} \bar{\psi} \bar{\lambda} \bar{\lambda} \rangle$ which can be matched to the operator $\text{Im}\{(\partial^4 \mathcal{F}/\partial A^4) \lambda^2 \psi^2\}$ which allows us to identify one-instanton generated nonperturbative contributions to the prepotential. Arrows denote chirality flow, and crosses denote the insertion of the scalar VEV $\bar{A}$.

multiplet have four zero modes each, two classical and two supersymmetric/superconformal. When the scalar A^a gets a VEV, which we take to be $A^a \equiv A \delta^{a3}$, it breaks $SU(2)$ to $U(1)$, what makes the instanton calculation IR finite and lifts the four superconformal zero modes via the Yukawa interaction $ig\epsilon^{abc}\psi^a\lambda^b\bar{A}^c$ and mixes classically massless $\bar{\lambda} \equiv \bar{\lambda}^3$ and $\bar{\psi} \equiv \bar{\psi}^3$ with supersymmetric zero modes ψ_{SS} and λ_{SS} , respectively. The instanton thus contributes to the $U(1)_{\mathcal{R}}$ violating correlator $\langle \bar{\psi} \bar{\psi} \bar{\lambda} \bar{\lambda} \rangle$ via the diagram shown in Figure 4. Using the diagram, the parametric dependence can be easily estimated using power counting rules (see e.g., [39])

$$\begin{aligned} & \langle \bar{\psi}(x_1) \bar{\psi}(x_2) \bar{\lambda}(x_3) \bar{\lambda}(x_4) \rangle \\ & \propto \int d^4 x_0 \int \frac{d\rho}{\rho^5} (\Lambda\rho)^4 \bar{A}^6 \rho^{12} e^{-4\pi^2 A \bar{A} \rho^2} \prod_{i=1}^4 S_F(x_i - x_0) \\ & \propto \frac{\Lambda^4}{A^6} \int d^4 x_0 \prod_{i=1}^4 S_F(x_i - x_0), \end{aligned} \quad (\text{B1})$$

where $S_F(x_i - x_0)$ is a fermion propagator and we left the spinor indices and gauge couplings implicit. Note that the instanton size integral makes the result holomorphic, providing a nontrivial consistency check for its contribution to the prepotential.

Equation (A4) indeed contains a term

$$\mathcal{L}_{\text{IR}} \supset \frac{1}{4\pi} \text{Im} \left[-\frac{1}{4} \frac{\partial^4 \mathcal{F}}{\partial A^4} \psi^2 \lambda^2 \right], \quad (\text{B2})$$

which induces a fermionic correlator equivalent to Eq. (B1). One can match the two correlators to determine d_1 , the coefficient of the $A^2(\Lambda/A)^4$ term in $\mathcal{F}(A)$. Carefully accounting all coefficients, one finds [36] $d_k = 1/2$, which agrees with the determination from the analytic solution of Seiberg-Witten theory in Eq. (C1), which establishes that the first nonperturbative coefficient in $\mathcal{F}(A)$ is generated by

a one-instanton configuration in the UV theory. Note that the fermion correlator in Eq. (B1) is related to a bosonic correlator, also generated by instantons, containing the axion and two photons through a supersymmetry transformation. However, practically, it is simpler to obtain the fermion correlator, as done above.

APPENDIX C: INSTANTON EXPANSION CONVERGENCE

To demonstrate the convergence of the instanton expansions for the prepotential $\mathcal{F}(A)$ and its Legendre transform $\mathcal{F}_D(A_D)$ given in Eq. (A8), we explicitly show that both expansions yield the same curves (A, A_D) when varying u . In this paper, we used the first 30 instantons coefficients recursively calculated using the method of Refs. [31,34]. Here, we present the first six coefficients,

$$\begin{aligned} d_1 &= \frac{1}{2}, & d_2 &= \frac{5}{64}, & d_3 &= \frac{3}{64}, \\ d_4 &= \frac{1469}{32768}, & d_5 &= \frac{4471}{81920}, & d_6 &= \frac{40397}{524288}, \dots, \end{aligned} \quad (\text{C1})$$

as well as

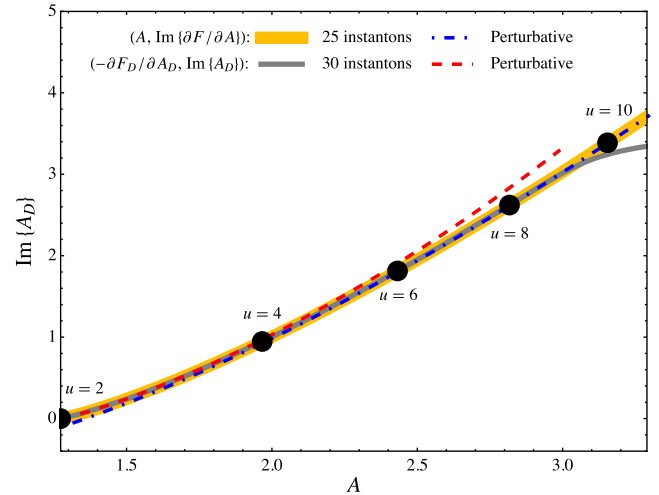


FIG. 5. A_D vs A . Orange: calculated from $\mathcal{F}(A)$ with 25 instanton coefficients. Gray: calculated from $\mathcal{F}_D(A_D)$ with 30 instanton coefficients. Blue: keeping only the perturbative contribution to $\mathcal{F}(A)$. Red: keeping only the perturbative contribution to $\mathcal{F}_D(A_D)$. The black circles are $(A(u), A_D(u))$ sample values of u , as calculated from (A2). We see agreement between the instanton expansions of $\mathcal{F}$ and $\mathcal{F}_D$. The deviation around $u \sim 10$ is attributed to the loss of validity of this finite term asymptotic series and would be alleviated by considering additional instanton terms. Remarkably, we find that the perturbative one-loop pieces of both expansions are the main contributions in the segment $2 \leq u \leq 10$ along the real axis of the moduli space.

$$\begin{aligned}
d_1^D &= \frac{i}{16}, & d_2^D &= -\frac{5}{1024}, & d_3^D &= -\frac{11i}{16384}, \\
d_4^D &= \frac{63}{524288}, & d_5^D &= \frac{527i}{20971520}, & d_6^D &= -\frac{3129}{536870912}, \dots
\end{aligned}
\tag{C2}$$

In Fig. 5. we plot A_D vs A for Seiberg-Witten theory, calculated from (A1) using the explicit expressions (A8). As one can readily see, the instanton expansion converges rapidly for $\partial\mathcal{F}/\partial A$ in the entire region and for $\partial\mathcal{F}_D/\partial A_D$ at $u \lesssim 9\Lambda^2$. Furthermore, we see that the perturbative one-loop pieces of $\mathcal{F}$ and $\mathcal{F}_D$ are able to approximate this curve quite well along the real axis of the moduli space.

APPENDIX D: DUALITY INVARIANCE OF $a \rightarrow \gamma\gamma$

We show explicitly that $e^3\partial\tau/\partial A$ is a duality invariant quantity. For a holomorphic function $f(z) = u(z) + iv(z)$ where $u(z)$, $v(z)$ are real functions, the Cauchy Reimann equations imply

$$\frac{1}{v^{3/2}} \frac{\partial u}{\partial z} = -2i \frac{\partial}{\partial z} \frac{1}{\sqrt{v}}, \tag{D1}$$

$$\frac{\partial |f|}{\partial z \sqrt{v}} = \frac{\bar{f}^2}{|f|} \frac{\partial}{\partial z} \frac{1}{\sqrt{v}}. \tag{D2}$$

From (D1) and the canonical relation $\tau = -\tau_D^{-1}$, we find

$$g_{a\gamma\gamma} \propto \frac{e^3}{4\pi} \frac{\partial\tau}{\partial A} = \frac{\partial e}{\partial A} = \frac{\partial A_D}{\partial A} \frac{\partial}{\partial A_D} \frac{\sqrt{4\pi}|\tau_D|}{\sqrt{\text{Im}\tau_D}}. \tag{D3}$$

However, note that in Seiberg-Witten theory $\partial A_D/\partial A = -\tau_D^{-1}$ and with the help of (D2) we find

$$\frac{\partial e}{\partial A} = \left(-\frac{\bar{\tau}_D}{|\tau_D|} \right)^3 \frac{\partial e_D}{\partial A_D}, \tag{D4}$$

and thus

$$g_{a\gamma\gamma} = \left(-\frac{\bar{\tau}_D}{|\tau_D|} \right)^3 g_{a\gamma\gamma}^D. \tag{D5}$$

The upshot is that under duality the $a \rightarrow \gamma\gamma$ amplitude changes only by a nonphysical global phase.

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- [1] A. V. Sokolov and A. Ringwald, *Proc. Sci., EPS-HEP2021* (2022) 178 [arXiv:2109.08503].
 - [2] A. V. Sokolov and A. Ringwald, arXiv:2205.02605.
 - [3] A. V. Sokolov and A. Ringwald, *Ann. Phys. (Berlin)* **536**, 2300112 (2023).
 - [4] B. Heidenreich, J. McNamara, and M. Reece, *J. High Energy Phys.* **01** (2024) 120.
 - [5] K. Colwell and J. Terning, *J. High Energy Phys.* **03** (2016) 068.
 - [6] N. Seiberg and E. Witten, *Nucl. Phys.* **B426**, 19 (1994); **B430**, 485(E) (1994).
 - [7] J. Fan, K. Fraser, M. Reece, and J. Stout, *Phys. Rev. Lett.* **127**, 131602 (2021).
 - [8] C. Cordova and K. Ohmori, *Phys. Rev. X* **13**, 011034 (2023).
 - [9] C. Cordova, S. Hong, and L.-T. Wang, *J. High Energy Phys.* **05** (2024) 325.
 - [10] C. Csaki, Y. Shirman, and J. Terning, *Phys. Rev. D* **81**, 125028 (2010).
 - [11] P. Sikivie, *Phys. Rev. Lett.* **51**, 1415 (1983); **52**, 695(E) (1984).
 - [12] G. Raffelt and L. Stodolsky, *Phys. Rev. D* **37**, 1237 (1988).
 - [13] L. Di Luzio, M. Giannotti, E. Nardi, and L. Visinelli, *Phys. Rep.* **870**, 1 (2020).
 - [14] M. E. Tobar, A. V. Sokolov, A. Ringwald, and M. Goryachev, *Phys. Rev. D* **108**, 035024 (2023).
 - [15] R. D. Peccei and H. R. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977).
 - [16] G. R. Farrar and S. Weinberg, *Phys. Rev. D* **27**, 2732 (1983).
 - [17] A. E. Nelson, *Phys. Lett. B* **369**, 277 (1996).
 - [18] J. Bagger, E. Poppitz, and L. Randall, *Nucl. Phys.* **B426**, 3 (1994).
 - [19] Z. Komargodski and N. Seiberg, *J. High Energy Phys.* **09** (2009) 066.
 - [20] J. Unwin and T. Yildirim, *Phys. Rev. D* **111**, 015048 (2025).
 - [21] G. Dvali, A. Kobakhidze, and O. Sakhelashvili, *Phys. Rev. D* **110**, 086008 (2024).
 - [22] B. A. Dobrescu and K. T. Matchev, *J. High Energy Phys.* **09** (2000) 031.
 - [23] L. M. Carpenter, M. Dine, G. Festuccia, and L. Ubaldi, *Phys. Rev. D* **80**, 125023 (2009).
 - [24] T. Banks, D. B. Kaplan, and A. E. Nelson, *Phys. Rev. D* **49**, 779 (1994).
 - [25] D. J. Miller and R. Nevzorov, arXiv:hep-ph/0309143.
 - [26] H.-S. Goh and M. Ibe, *J. High Energy Phys.* **03** (2009) 049.
 - [27] B. Bellazzini, A. Mariotti, D. Redigolo, F. Sala, and J. Serra, *Phys. Rev. Lett.* **119**, 141804 (2017).
 - [28] E. Arganda, A. D. Medina, N. I. Mileo, R. A. Morales, and A. Szykman, *Phys. Lett. B* **789**, 575 (2019).
 - [29] N. Seiberg, *Phys. Rev. D* **49**, 6857 (1994).
 - [30] A. Bilal and F. Ferrari, *Nucl. Phys.* **B480**, 589 (1996).
 - [31] Y. Tachikawa, *N = 2 Supersymmetric Dynamics for Pedestrians*, Lecture Notes in Physics, Vol. 890 (Springer, Cham, 2014).
 - [32] N. A. Nekrasov, *Adv. Theor. Math. Phys.* **7**, 831 (2003).

- [33] K. Ito and S.-K. Yang, *Phys. Lett. B* **366**, 165 (1996).
- [34] G. Chan and E. D'Hoker, *Nucl. Phys.* **B564**, 503 (2000).
- [35] N. Seiberg, *Phys. Lett. B* **206**, 75 (1988).
- [36] D. Finnell and P. Pouliot, *Nucl. Phys.* **B453**, 225 (1995).
- [37] N. Dorey, V. V. Khoze, and M. P. Mattis, *Phys. Rev. D* **54**, 2921 (1996).
- [38] K. Ito and N. Sasakura, *Phys. Lett. B* **382**, 95 (1996).
- [39] C. Csáki, R. T. D'Agnolo, E. Kuflik, and M. Ruhdorfer, *J. High Energy Phys.* 04 (2024) 074.