

Coalescence production of sexaquark with three diquarks in high-energy nuclear collisions

Zhi-Lei She¹, An-Ke Lei^{2,3}, Dai-Mei Zhou^{2,*}, Larissa V. Bravina^{3,†}, Evgeny E. Zabrodin^{3,4},
Sonia Kabana^{5,‡} and Vipul Bairathi⁵

¹*School of Mathematical and Physical Sciences, Wuhan Textile University, Wuhan 430200, China*

²*Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*

³*University of Oslo, POB 1048 Blindern, N-0316 Oslo, Norway*

⁴*Skobeltsyn Institute of Nuclear Physics, Moscow State University, Vorob'evy Gory, Moscow RU-119991, Russia*

⁵*Instituto de Alta Investigación, Universidad de Tarapacá, Arica 1000000, Chile*

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The coalescence production of the sexaquark, a hypothetical stable state with quark content ($uuddss$), is investigated using the parton and hadron cascade model PACIAE in pp collisions at $\sqrt{s} = 7$ TeV. In this work, the compact sexaquark bound state of three diquarks is formed in the final partonic state by a two-step approach, which involves “diquark” formation via partonic coalescence and sexaquark construction with the dynamically constrained phase-space coalescence model successively. The yields, yield ratios, and dependences of the spatial parameters (the size of the diquark D_0 and the radius of the sexaquark R_0) of a (anti)sexaquark are predicted. The yields of the hadronic H-dibaryon molecule $H(\Lambda\Lambda)$ generated in the final hadronic state are also compared. These estimates provide references for future sexaquark searches and other exotic state studies, such as dibaryons.

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I. INTRODUCTION

Exotic hadrons with more than the minimal quark content ($q\bar{q}$ or qqq) are allowed by the quark model proposed by Gell-Mann and Zweig [1,2]. In recent years, many unconventional hadron candidates containing multi-quarks have been discovered in experiments. Some examples include the first observation of a tetraquark candidate $X(3872)$ with valence quark content $c\bar{c}u\bar{u}$ by the Belle Collaboration in 2003 [3] and the first observation of pentaquark state candidates $P_c(4380)$ and $P_c(4450)$ with $c\bar{c}uud$ by the LHCb Collaboration in 2015 [4]. However, their exact nature is still largely unclear. For a review, see Refs. [5–7] and references therein.

Among the multi-quark states, the sexaquark (S) with the quark content $uuddss$, a possible stable multi-quark state, was proposed by Farrar [8]. Such a state is assumed to have

a hard-core radius of ~ 0.1 – 0.4 fm and mass lower than 2 GeV, and is a good candidate for dark matter and the composition of neutron stars [9,10]. A recent search for sexaquarks was performed and an upper limit was published on its production in $\Upsilon \rightarrow S\bar{\Lambda}\bar{\Lambda}$ decays by the BABAR Collaboration [11]. Note that the sexaquark is different than the H-dibaryon $H(\Lambda\Lambda)$ with the same quark content, which was introduced by Jaffe [12] and estimated to have a mass of 2150 MeV. In Ref. [13], it was suggested that the H-dibaryon could be a hadronic molecule of two Λ hyperons, while a sexaquark is a more tightly compact bound state of three diquarks bound by color forces. The hadron resonance gas model with multicomponent hard-core repulsion [14,15] was used to study the thermal production rates of sexaquarks in heavy-ion collisions at LHC energies, where sexaquarks may be produced at relatively high rates [16].

The coalescence mechanism is a useful tool allowing the study of exotic multi-quark states [17], such as the coalescence production of the tetraquark $X(3872)$ in heavy-ion collisions [18,19]. A dynamically constrained phase-space coalescence (DCPC) model [20] was successfully used to describe the production of exotic hadrons with the coalescence of final-state hadrons, such as $X(3872)$ [21,22] and P_c states [23] in relativistic nuclear collisions, where the hadronic final states were generated by the parton and hadron

* Contact author: zhoudm@mail.ccnu.edu.cn

† Contact author: larissa.bravina@fys.uio.no

‡ Contact author: Sonja.Kabana@cern.ch

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cascade model PACIAE 2.0 [24]. Recently, PACIAE 3.0 [25] + DCPC model was upgraded to simulate the exotic hadrons under different internal configurations, i.e., the compact multiquarks in the final partonic state and hadronic molecules in the final hadronic state, such as X(3872) [26] and glueball-like particle X(2370) [27].

In the present paper, we employ the newest version of PACIAE 4.0 [28] and two coalescence models (a partonic coalescence of diquarks and the DCPC model) to estimate the coalescence production of the sexaquark (S) with a three-diquark configuration ($ud - us - ds$) in pp collisions at LHC energies assuming such states do exist in nature. In this study, we expect to investigate the coalescence production and properties of the exotic (anti) sexaquark.

The paper is organized as follows. In Sec. II we briefly describe the PACIAE 4.0 and two coalescence models. Section III contains the simulated results of the sexaquark state. In Sec. IV we give a summary.

II. MODELS

As presented above, PACIAE 4.0 [28], based on the PYTHIA 8.3 code [29] and two coalescence models (the partonic coalescence of diquarks and the DCPC model), are applied to simulate the production of the S in pp collisions at LHC energies assuming a coalescence production of the sexaquark.

A. PACIAE model

The PACIAE model [24,25,28] is a phenomenological model to simulate relativistic elementary particle collisions and nuclear collisions. It divides the whole collision process into four main stages: parton initiation, parton cascade, hadronization, and hadron cascade.

In the PACIAE 4.0 model C_PY8-simulation framework [28], the initial free partons are produced by turning off hadronization temporarily in PYTHIA 8.3 [29] for each nucleon-nucleon or hadron-hadron collision. Here a special pure quarks-antiquarks initial state is chosen, i.e., a deexcited quarks-antiquarks state.

The parton cascade is further considered by employing the $2 \rightarrow 2$ parton-parton scattering with the massive lowest-order perturbative quantum chromodynamics cross sections [30]. The resulting final partonic state (FPS) comprises numerous (anti)quarks with their four-coordinates and four-momenta.

After the partonic freeze-out, the hadronization is implemented by the Lund string fragmentation regime [31] or the coalescence model [24] to generate an intermediate hadronic state. The coalescence model is employed in this work.

The hadron rescattering is then performed based on the two-body scattering until the hadronic freeze-out, resulting in a final hadronic state (FHS) for a nucleon-nucleon (hadron-hadron) collision.

B. Coalescence models

A two-step approach is used to produce the exotic sexaquark with a three-diquark structure ($ud - us - ds$), as illustrated in Fig. 1.

First, the partonic coalescence model is introduced to produce the “diquarks.”

The “diquarks” (uu, ud, dd, us, ds, ss) are formed via the partonic coalescence model at partonic freeze-out in the FPS, where the varieties, four-coordinates, and four-momenta of partons remain unchanged. In the simulation, three constraints should be satisfied. The first involves the corresponding u, d, s quark components. The second is the “diquark” mass. Here we assume that the masses of “diquarks” are given as the sum of corresponding quark masses, without the spin-splitting term [29]. The third is the relative distance constraint, $D_{\text{diq}} < D_0$. Here, D_{diq} is the spatial distance between components in one diquark and D_0 is a free parameter of the size of the diquark, which can be obtained from theoretical assumptions. In particular, the value of D_0 is 0.4 fm when the diquark-to-quark qq/q rate in PYTHIA 8 Monash 2013 tune [32] is taken into account.

Then, these diquarks (we only consider the diquarks in the lighter mass states, i.e., ud, us, ds) are further used to coalesce into the sexaquark in the DCPC model [20].

In the quantum statistical mechanics [33,34], the yield of N -particle (such as the partons in the FPS) clusters can be calculated using

$$Y_N = \int \cdots \int_{E_\alpha \leq H \leq E_\beta} \frac{d\vec{q}_1 d\vec{p}_1 \cdots d\vec{q}_N d\vec{p}_N}{h^{3N}}, \quad (1)$$

where E_α and E_β are the lower and upper energy thresholds of the particle. $\vec{q}_i$ and $\vec{p}_i$ are the i th particle’s three-coordinates and three-momenta, respectively. For instance, the yield of $S(uuddss)$, assumed to consist of three diquarks (ud, us , and ds), is calculated according to the DCPC model using the following integral:

$$Y_{S(uuddss)} = \int \cdots \int \delta_{123} \frac{d\vec{q}_1 d\vec{p}_1 d\vec{q}_2 d\vec{p}_2 d\vec{q}_3 d\vec{p}_3}{h^9}, \quad (2)$$

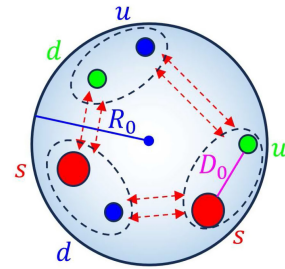


FIG. 1. Possible structure of the sexaquark as a compact bound state of three diquarks ($ud - us - ds$). Here, D_0 is the relative quark-quark distance in the diquark and R_0 is the radius of the sexaquark.

$$\delta_{123} = \begin{cases} 1 & \text{if } 1 \equiv ud, 2 \equiv us, 3 \equiv ds, \\ & m_0 - \Delta m \leq m_{\text{inv}} \leq m_0 + \Delta m, \\ & \text{Max}\{|\vec{q}_{12}|, |\vec{q}_{23}|, |\vec{q}_{13}|\} \leq R_0, \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

$$m_{\text{inv}} = \sqrt{(E_1 + E_2 + E_3)^2 - (\vec{p}_1 + \vec{p}_2 + \vec{p}_3)^2}. \quad (4)$$

In Eq. (3), m_0 represents the rest mass of the S particle and Δm indicates the uncertainty of the mass, which can be set to $m_0 = 1969.5 \text{ MeV}/c^2$ and $\Delta m = 84.5 \text{ MeV}/c^2$ [13]. $|\vec{q}_{12}|$, $|\vec{q}_{23}|$, and $|\vec{q}_{13}|$ denote the distances between each of the three component particles ud , us , and ds under the center-of-mass system, respectively, while $\text{Max}\{\dots\}$ is the maximum distance taken between them. R_0 stands for the radius of S, which can be chosen in the range $0.1 \text{ fm} < R_0 < 0.4 \text{ fm}$ [8].

III. RESULTS AND DISCUSSION

The main PACIAE 4.0 model parameters of the threshold energy of quark deexcitation and gluon splitting in the deexcitation process, i.e., $e_{\text{deex}} = 2.15 \text{ GeV}$ and $e_{\text{split}} = 0.99 \text{ GeV}$, are fixed by fitting the ALICE data [35,36] of $\pi^+ + \pi^-$, $K^+ + K^-$, $p + \bar{p}$, and $\Lambda + \bar{\Lambda}$ yields in pp collisions at $\sqrt{s} = 7 \text{ TeV}$, as shown in Table I. Then, the suitable FHS and the corresponding FPS are obtained with 100 million pp collision events.

After that, the exotic S is constructed from the combination of ud , ds , and us using the coalescence models (the partonic coalescence of the diquark and the DCPC model) in the FPS. We use a set of parameters ($0.3 < D_0 < 0.4 \text{ fm}$, $0.2 < R_0 < 0.4 \text{ fm}$, $m_0 = 1969.5 \text{ MeV}/c^2$, and $\Delta m = 84.5 \text{ MeV}/c^2$) as an illustration in this study.

Figure 2 shows the yields of the (anti)sexaquark and the antisexaquark-to-sexaquark ratio values as functions of the spatial parameter R_0 in pp collisions at $\sqrt{s} = 7 \text{ TeV}$. Here, another spatial parameter, D_0 , the size of diquark, is set to 0.4 fm . In Fig. 2(a), one can see that the yields of the (anti)sexaquark linearly increase with R_0 on a semilogarithmic plot, presenting a significant spatial parameter R_0 dependence, i.e.,

TABLE I. PACIAE 4.0 model simulated yields of $\pi^+ + \pi^-$, $K^+ + K^-$, $p + \bar{p}$, and $\Lambda + \bar{\Lambda}$ compared with the ALICE data [35,36] in pp collisions at $\sqrt{s} = 7 \text{ TeV}$.

Particles	ALICE	PACIAE
$\pi^+ + \pi^-$	4.49 ± 0.20	4.49
$K^+ + K^-$	0.572 ± 0.032	0.602
$p + \bar{p}$	0.247 ± 0.018	0.265
$\Lambda + \bar{\Lambda}$	0.152 ± 0.011	0.141

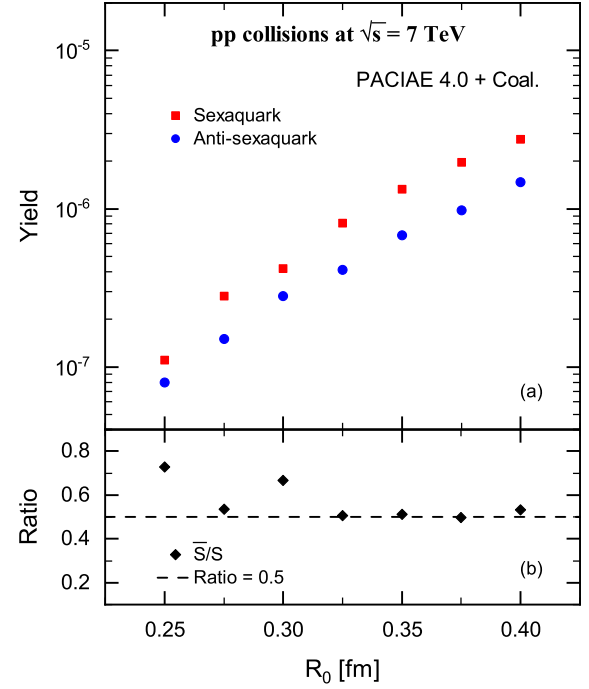


FIG. 2. (a) The yields of the (anti)sexaquark with different spatial parameters R_0 in pp collisions at $\sqrt{s} = 7 \text{ TeV}$. (b) The anti-sexaquark-to-sexaquark ratio values $\bar{S}/S$ with different spatial parameters R_0 . Here the size of the diquark $D_0 = 0.4 \text{ fm}$.

$\ln Y \sim R_0$. The yield values are on the order of 10^{-8} to 10^{-6} from $R_0 = 0.25 \text{ fm}$ to $R_0 = 0.4 \text{ fm}$.

As shown in Fig. 2(b), the antisexaquark-to-sexaquark ratio values ($\bar{S}/S$) are close to 0.5, except for the fluctuation at smaller R_0 . The results reflect that the antisexaquark yield is less than that of its corresponding sexaquark at the same parameter R_0 . This can be interpreted as antiparticle production being harder than that of corresponding particles in high-energy nuclear collisions [37].

The predicted sexaquark-to-hyperon and sexaquark-to-deuteron yield ratios with different spatial parameters D_0 in pp collisions at $\sqrt{s} = 7 \text{ TeV}$ are shown in Fig. 3 and Table II. (Only the cases of $D_0 = 0.3 \text{ fm}$ and $D_0 = 0.4 \text{ fm}$ are shown as examples.) Here the (anti)sexaquark yields are the cumulative values in the range of $0.2 < R_0 < 0.4 \text{ fm}$. In Fig. 3, the yield-ratio values increase with increasing D_0 , reflecting that the ratios of coalescence sexaquarks to strange hadrons and light nuclei depend on the spatial parameters D_0 of the diquark size. More specifically, the ratios of $S/\Lambda(\bar{S}/\bar{\Lambda})$ and $S/d(\bar{S}/\bar{d})$ are, respectively, on the order of 10^{-6} and 10^{-3} for $D_0 = 0.3 \text{ fm}$, while they are 10^{-5} and 10^{-2} for $D_0 = 0.4 \text{ fm}$, as Table II shows.

Furthermore, we find that the simulated yield ratios are much lower than that of thermal production in Pb-Pb collisions, e.g., the thermal sexaquark with a mass of 1950 MeV is predicted to have a rate of 25% of the deuteron in Pb-Pb collisions at a center-of-mass energy of 2.76 TeV at the LHC [16]. The difference in the predictions

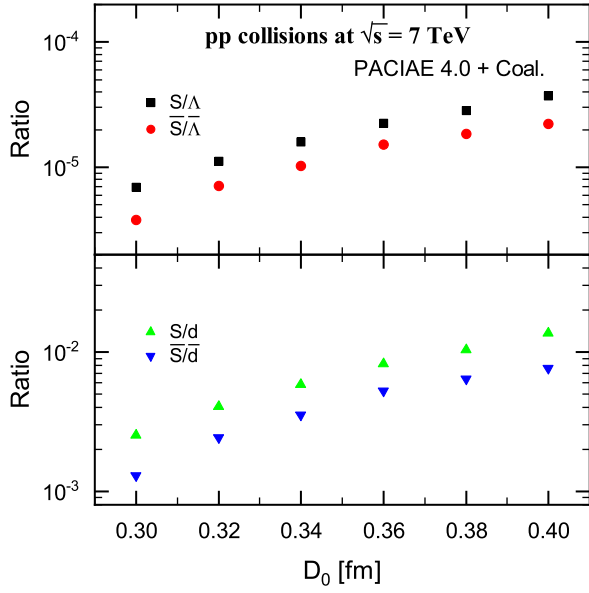


FIG. 3. Yield ratios of sexaquark (S) to hyperon (Λ) and sexaquark to deuteron (d) with different spatial parameters D_0 in pp collisions at $\sqrt{s} = 7$ TeV. Here the radius of the sexaquark R_0 is in the range $0.2 < R_0 < 0.4$ fm.

between the coalescence model and thermal production may be caused by spatial structure constraints, thermal condition selections, and different collision systems.

To compare with two possible exotic hadrons with the same quark content ($uuddss$), i.e., the sexaquark $S(uuddss)$ and H-dibaryon $H(\Lambda\Lambda)$, their coalescence yields and mixed ratios $\bar{S}/S$, $\bar{H}/H$, and $S/H(\bar{S}/\bar{H})$ in pp collisions at $\sqrt{s} = 7$ TeV are listed in Table III. Here the sexaquark yield is evaluated with the spatial parameters $D_0 = 0.4$ fm and $0.2 < R_0 < 0.4$ fm. The H-dibaryon is recombined using the components of two Λ hyperons using the DCPC model in the FHS. The mass and radius of the H-dibaryon as a hadronic molecule, if it could exist, are assumed to be $2055 \leq m_{\text{inv}}(H) \leq 2230$ MeV/ c^2 [12] and $1.0 < R_0 < 4.9$ fm [39], respectively.

TABLE II. Coalescence yield ratios of sexaquark (S) to hyperon (Λ) and sexaquark to deuteron (d) in pp collisions at $\sqrt{s} = 7$ TeV. Here D_0 is the spatial parameter of the diquark size, and the radius of the sexaquark R_0 is in the range $0.2 < R_0 < 0.4$ fm.

Types	Ratios	
	$D_0 = 0.30$ fm	$D_0 = 0.40$ fm
S/Λ	6.91×10^{-6}	3.74×10^{-5}
$\bar{S}/\bar{\Lambda}$	3.77×10^{-6}	2.22×10^{-5}
S/d^a	2.52×10^{-3}	1.35×10^{-2}
$\bar{S}/\bar{d}^b$	1.30×10^{-3}	0.74×10^{-2}

^ad is taken from Ref. [38].

^bd is taken from Ref. [38].

TABLE III. Simulated sexaquark yields with three diquarks and the H-dibaryon with two $\Lambda(uds)$ hyperons, and the mixed ratios $\bar{S}/S$, $\bar{H}/H$, and S/H in pp collisions at $\sqrt{s} = 7$ TeV. Here $D_0 = 0.4$ fm and $0.2 < R_0 < 0.4$ fm for the sexaquark, and $1.0 < R_0 < 4.9$ fm for the H-dibaryon.

Yields	$S(uuddss)$	2.76×10^{-6}
	$\bar{S}(u\bar{u}\bar{d}\bar{d}\bar{s})$	1.47×10^{-6}
	$H(\Lambda\Lambda)$	1.65×10^{-4}
	$\bar{H}(\bar{\Lambda}\bar{\Lambda})$	2.99×10^{-5}
Mixed ratios	$\bar{S}/S$	0.533
	$\bar{H}/H$	0.181
	S/H	1.67×10^{-2}
	$\bar{S}/\bar{H}$	4.92×10^{-2}

Table III shows that the coalescence yields of $S(\bar{S})$ are much less than those of $H(\bar{H})$ for the selected parameters. The mixed ratios $S/H(\bar{S}/\bar{H})$ indicate that the yield of $S(\bar{S})$ is lower than $H(\bar{H})$ by 2 orders of magnitude. It may be related not only to the spatial structures, formation time, etc., but also to the fact that the three-body combinations are more difficult to generate than two-body combinations in the coalescence framework. However, the value of $\bar{S}/S$ (~ 0.533) is larger than that of $\bar{H}/H$ (~ 0.181) simulated by the PACIAE + DCPC model in pp collisions. This phenomenon suggests that, compared with their corresponding particle counterparts, the compact anti-H-dibaryon is harder to produce than that of the antisexaquark. Further studies are needed to fully understand these yield features.

IV. SUMMARY AND OUTLOOK

We employed PACIAE 4.0 and two coalescence models to simulate sexaquark production in pp collisions at $\sqrt{s} = 7$ TeV. In this simulation, the sexaquark is assumed to be a compact bound state of a three-diquark configuration ($ud - ds - us$). The “diquark” formation via partonic coalescence and then sexaquark construction with the DCPC model were executed successively in the FPS.

The (anti)sexaquark yields were computed with different spatial parameter R_0 (the radius of sexaquark selected). The results show a significant spatial parameter R_0 dependence. The values are on the order of 10^{-8} to 10^{-6} from $R_0 = 0.25$ fm to $R_0 = 0.4$ fm. Moreover, the antisexaquark yield is lower than that of its corresponding sexaquark, i.e., the antisexaquark-to-sexaquark ratio values are close to 0.5.

For the yield ratios, the coalescence sexaquarks to hyperons and sexaquarks to light nuclei present a spatial parameter D_0 (the size of diquark) dependence. The values are on the order of 10^{-6} and 10^{-3} in the $D_0 = 0.3$ fm case and 10^{-5} and 10^{-2} in the $D_0 = 0.4$ fm case, which are much lower than that of thermal production in heavy-ion collisions.

Besides, although the sexaquark $S(uuddss)$ and H-dibaryon $H(\Lambda\Lambda)$ have the same quark content ($uuddss$),

the coalescence yields of $S(\bar{S})$ as a compact bound state are much less than those of $H(\bar{H})$ as a hadronic molecule recombined in the FHS, i.e., their yields differ by 2 orders of magnitude. However, the yield ratio of $\bar{S}/S$ (~ 0.533) is larger than that of $\bar{H}/H$ (~ 0.181).

The study should be extended to Pb-Pb collisions at LHC energies. The use of this compact three-diquark configuration for the dibaryons (such as $p\Lambda$, $\Lambda\Lambda$, and $\Omega\Omega$) also deserves to be studied further.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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